Robust Obstacle Avoidance via Vision-Based Hybrid Learning Control

Anonymous Author(s)

Affiliation
Address
email

Abstract: We study the problem of target stabilization with robust obstacle avoidance in robots and vehicles that have access to vision-based sensors generating streams of complex nonlinear data. This problem is particularly challenging due to the topological obstructions induced by the obstacle, which preclude the existence of smooth feedback controllers able to achieve simultaneous stabilization and robust obstacle avoidance. To overcome this issue, we develop a hybrid controller that switches between two different feedback laws depending on the current position of the vehicle. The main innovation of the paper is the incorporation of perception maps, learned from data obtained from the camera, into the hybrid controller. Moreover, under suitable assumptions on the perception map, we establish theoretical guarantees for the trajectories of the vehicle in terms of convergence and obstacle avoidance. The proposed hybrid controller is numerically tested under different scenarios, including under noisy data, sensor failures, and camera occlusions. Mathematical proofs and illustrative simulation videos are included in the supplemental material.

Keywords: Perception, Control Theory, Learning.

1 Introduction

Many autonomous systems rely on processing high-dimensional data in order to extract information from the environment and/or the system. Typical examples include autonomous vehicles and robots equipped with cameras for the purpose of navigation. In these systems, the control and navigation algorithms use information extracted from perceptual sensing modalities in order to make decisions in real time. Therefore, the management of inexact information and processing approximations is a fundamental task for the control system in order to avoid instabilities and potentially unsafe maneuvers. The literature contains a number of examples of systems integrating high-dimensional inputs, like images, in feedback control loops. For example, several successful end-to-end approaches have employed reinforcement learning, including [1], where the state-space construction is automated by learning a state representation directly from camera images. Also, in [2] the authors introduce deep Q-networks, a class of models that allow mappings from images of the state of Atari games directly to a control command, achieving approximate human-level performance. Other works have used deep generative models to synthesize controllers with inputs coming from an embedding space of high-dimensional data, which does not necessarily correspond to an interpretable space (e.g., joint coordinates of the robot). Some examples in this direction include [3, 4, 5, 6, 7]. Finally, in works such as [5, 8, 9], the authors integrated state predictions using robust control tools to handle approximation errors. Nevertheless, even though significant progress has been made during the last years, several challenges remain unsolved when it comes to integrating feedback control with perceptual sensing, particularly in non-trivial control problems where topological obstructions might preclude the implementation of standard linear approaches based on optimal control [10] or robust adaptive control [11].

Motivated by the previous background, in this work we develop a vision-based hybrid controller for robust and resilient obstacle avoidance in mobile robots. We show that, unlike standard smooth feedback controllers, the proposed hybrid algorithm can overcome arbitrarily small and potentially adversarial disturbances, noisy states, sensor failures, as well as camera occlusions. Our approach is motivated by works such as [6, 8], which have incorporated into the controller a class of perception...
maps learned from data to predict the system’s state and the system’s dynamics. However, in contrast
to previous approaches that have used standard gradient flows with suitable navigation functions
[12, 13, 14, 15, 16], we consider hybrid controllers that incorporate continuous-time dynamics as
well as discrete-time dynamics [17], and which have been successfully used for the solution of
obstacle avoidance problems in settings where position measurements are directly available to the
vehicle, e.g., [18, 19]. However, in contrast to these earlier works, in this paper we incorporate for
the first time perception maps (learned from data) into the hybrid controller for the robust solution
of the obstacle avoidance problems using vision sensors, such as cameras. Our main results provide
theoretical guarantees, as well as extensive numerical validations in different scenarios including
settings with noisy sensors, camera occlusions, and leader-follower missions with multiple vehicles.

2 Preliminaries and Problem Formulation

We start by introducing the notation, as well as some preliminaries on hybrid dynamical systems.
After this, we formalize the robust obstacle avoidance problem.

2.1 Definitions & Notation

Given a compact set $A \subset \mathbb{R}^n$ and a vector $z \in \mathbb{R}^n$, we use $[z]_A := \min_{s \in A} ||z - s||_2$ to denote
the minimum distance from $z$ to $A$. A set-valued mapping $M : \mathbb{R}^p \rightrightarrows \mathbb{R}^n$ is said to be: a) outer
semicontinuous (OSC) at $z$ if for each sequence $\{s_i, p_i\} \to (z, s) \in \mathbb{R}^p \times \mathbb{R}^n$ satisfying $s_i \in M(z_i)$
for all $i \in \mathbb{Z}_{\geq 0}$, we have $s \in M(z)$; b) locally bounded at $z$ if there exists an open neighborhood
$N_z \subset \mathbb{R}^p$ of $z$ such that $M(N_z)$ is bounded. We use $r\mathbb{B}$ to denote a closed ball in the Euclidean
space, of radius $r > 0$, and centered at the origin, and we use $p + r\mathbb{B}$ to denote the union of all the
points $p_i$ that satisfy $|p - p_i| \leq r$. Given a set $B$, we use $\text{cl}(B)$ and $\text{bd}(B)$ to denote the closure,
and the boundary, of the set $B$, respectively.

In this paper, we will use the formalism of hybrid dynamical systems [17] for the synthesis and
analysis of robust vision-based control systems. Specifically, a hybrid dynamical system (HDS)
combines continuous-time dynamics, usually modeled by differential equations or inclusions, and
discrete-time dynamics, usually modeled by recursions or difference inclusions. A HDS with state
$z \in \mathbb{R}^n$ is represented by its data $\mathcal{H} := \{C, F, D, G\}$, and the dynamics

$$z \in C, \quad \dot{z} \in F(z), \quad \text{and} \quad z \in D, \quad z^+ \in G(z),$$

(1)

where the set-valued mappings $F : \mathbb{R}^n \rightrightarrows \mathbb{R}^n$ and $G : \mathbb{R}^n \rightrightarrows \mathbb{R}^n$, called the flow map and
the jump map, respectively, describe the evolution of the state $z$ when it belongs to the flow set
$C$, and the jump set $D$, respectively. Solutions to (1) are defined on hybrid time domains, which,
under mild assumptions on the data $\mathcal{H}$, permits the use of graphical convergence notions to establish
sequential compactness results for the solutions of (1), e.g., the limit of a sequence of solutions
is also a solution. Such sequential compactness results play an important role in the robustness
analysis of nonlinear control systems. A set $E \subset \mathbb{R}_{\geq 0} \times \mathbb{Z}_{\geq 0}$ is called a compact hybrid time
domain if $E = (\cup_{j=0}^{J-1}(I_j, I_{j+1})), j$ for some finite sequence of times $0 = t_0 \leq t_1 \ldots \leq t_J$. The
set $E$ is a hybrid time domain if for all $(T, J) \in E, E \cap ([0, T] \times \{0, \ldots, J\})$ is a compact hybrid
time domain. By using the notion of hybrid time domains, the following definition formalizes the
concept of (complete) solutions to HDS of the form (1).

**Definition 1** A function $z : \text{dom}(z) \mapsto \mathbb{R}^n$ is a hybrid arc if dom($z$) is a hybrid time domain and
t $\mapsto z(t, j)$ is locally absolutely continuous for each $j$ such that the interval $I_j := \{t : (t, j) \in \text{dom}(z)\}$ has nonempty interior. A hybrid arc $z$ is a solution to (1) if $z(0, 0) \in C \cup D$, and the
following two conditions hold:

1. For each $j \in \mathbb{Z}_{\geq 0}$ such that $I_j$ has nonempty interior: $z(t, j) \in C$ for all $t \in \text{int}(I_j)$, and
   $\dot{z}(t, j) \in F(z(t, j))$ for almost all $t \in I_j$.

2. For each $(t, j) \in \text{dom}(z)$ such that $(t, j + 1) \in \text{dom}(z)$: $z(t, j) \in D$, and $z(t, j + 1) \in G(z(t, j))$.

A hybrid solution $z$ is said to be forward pre-complete if its domain is compact or unbounded, i.e.,
if the flows do not generate finite escape times. A hybrid solution is said to be forward complete if
its domain is unbounded. A hybrid solution is maximal if there does not exist another solution \( \psi \) to \( \mathcal{H} \) such that \( \text{dom}(\psi) \) is a proper subset of \( \text{dom}(\psi) \), and \( \psi(t, j) = \psi(t, j) \) for all \( (t, j) \in \text{dom}(z) \). □

To establish and exploit suitable robustness properties for our control systems we will impose the following Basic Conditions on the data \( \mathcal{H} \).

**Definition 2** The HDS (1) is said to satisfy the Basic Conditions if: (a) the sets \( C \subset \text{dom}(F) \) and \( D \subset \text{dom}(G) \) are closed; (b) \( F \) is convex-valued, outer-semicontinuous, and locally bounded relative to \( C \); (c) \( G \) is outer-semicontinuous and locally bounded.

### 2.2 The Obstacle Avoidance Problem: Robustness Limitations in Vision-Based Control

In this paper, we are interested in the synthesis and analysis of robust feedback controllers able to autonomously steer a vehicle from any initial position \( p_0 \in \mathbb{R}^2 \) to a final target \( p_T \in \mathbb{R}^2 \), by using real-time data provided by a *generative model* as feedback. Typical examples of generative models include cameras and high-dimensional data generated by the fusion of multiple noisy sensors. To illustrate our controllers, we will consider simple velocity actuated vehicle dynamic, given by an integrator evolving on the plane, with dynamics

\[
\dot{x} = u_x, \quad \dot{y} = u_y, \quad \theta = h(x, y),
\]

where \((x, y)\) are the coordinates in the Cartesian plane, and \( \theta \) corresponds to real-time data generated by \( h \), which can be seen as a map that produces images as functions of the vehicle’s position.

The main goal is to design a feedback law \((u_x, u_y)\) such that the trajectories of the vehicle avoid an obstacle \( \mathcal{N} \subset \mathbb{R}^2 \) contained in a sphere of constant radius, and converge to an arbitrarily small neighborhood of the target destination \( p_T \in \mathbb{R}^2 \). Such type of navigation problems have been extensively studied in the literature via different approaches, including planning and tracking algorithms [20, 21], triangular partitions [22], and barrier functions [23], to name just a few. In contrast to these settings, in this paper we are interested in real-time feedback-based controllers where planning and navigation are simultaneously executed, and where robustness guarantees can be provided under arbitrarily small disturbances.

One of the most popular approaches for the solution of navigation problems in mobile robots is based on implementing navigation functions \( J : \mathbb{R}^2 \to \mathbb{R} \), and gradient-based feedback laws of the form

\[
u_x = k_x \frac{\partial J(x, y)}{\partial x}, \quad u_y = k_y \frac{\partial J(x, y)}{\partial y},
\]

where \( J \) incorporates attractive terms (to converge to the target point) and repulsive terms (to avoid the obstacles); see [12, 13, 14, 15, 16]. Note that the closed-loop dynamics (2)-(3) can be written as

\[
\dot{p} = k \nabla J(\theta), \quad p = [x, y]^T,
\]

where for simplicity we used \( k_x = k_y = k \in \mathbb{R}_{>0} \). As it is standard in the literature of perception-based control [8], we assume the existence of a perception map \( \ell \) that generates imperfect predictions of the state of the vehicle using the images \( \theta \), namely, \( \ell(\theta) = Cp + e \), where \( C \in \mathbb{R}^{2 \times 2} \) is a constant matrix, and \( e \in \mathbb{R}^2 \) is the approximation error. Using this perception map to close the loop between the camera and the vehicle, the resulting system becomes

\[
\dot{p} = k \nabla J(\ell(\theta)), \quad p \in \mathbb{R}^2 \setminus \mathcal{N}.
\]

In this paper, we will use data-driven techniques to learn the perception map \( \ell \) using a sequence of labeled training data \( \mathcal{D} = \{p_i, \theta_i\}_{i=1}^N \). To do this, we will leverage the following assumption.

**Assumption 1** For each compact set \( K \subset \mathbb{R}^2 \), each \( L > 0 \), and each \( \varepsilon > 0 \), there exists a perception map \( \ell \) learned with training data \( \mathcal{D} = \{p_i, \theta_i\}_{i=1}^N \), such that \( K \subset \text{int}(\mathcal{S}_\varepsilon^L) \), where

\[
\mathcal{S}_\varepsilon^L := \bigcup_{(p_d, \theta_d) \in \mathcal{D}} \{ p \in \{p_d\} + r \mathbb{B} : |\ell(\theta_d) - p_d| + L|p - p_d| \leq \varepsilon \}.
\]

In words, Assumption 1 guarantees the existence of sufficient data needed to learn a suitable perception map able to cover any compact set \( K \) of interest. The next lemma, established in [8, Lemma 3.1], will also be instrumental for the characterization of the approximation error of the perception map learned from the data.
Lemma 1 Let $F(p) := (\ell \circ h - I)(p)$, and suppose that $p \mapsto F(p)$ is $L$-Lipschitz continuous. Then, the perception error satisfies $|\ell(\theta) - p| \leq \varepsilon$, for all $(p, \theta)$ such that $p \in S^*_\varepsilon$.

The right plot of Figure 1 illustrates a trajectory of the robot, as well as the predicted states by a perception map $\ell$ that satisfies Lemma 1, learned from data by training a convolutional neural network; see the supplemental material for details. As observed, the predictions of the perception map remain in an $\varepsilon$-neighborhood of the actual trajectory. Since in this work we focus on obstacle avoidance problems on the plane using cameras located vertically over the robots, we can take $C$ to be equal to the identity matrix without loss of generality.

Note that if the learned perception map $p$ satisfies the conditions of Lemma 1, then the closed-loop system (4) is given by the following perturbed dynamical system

$$\dot{p} = k \nabla J(p + e), \quad |e(p)| \leq \varepsilon, \quad \forall p \in S^* \setminus N.$$  \hspace{1cm} (6)

Stability and convergence properties of perturbed systems of the form (6) have been extensively studied in the control's literature [24]. Indeed, for the obstacle avoidance problem, the disturbance $e$ can have a dramatic effect on the trajectories of the vehicle. To illustrate this fact, consider the situation shown in the left plot of Figure 1, where a vehicle, denoted with a white square, aims to converge to the target, denoted with a red circle, while avoiding the obstacle $N$ denoted with a white circle. Note that, in order to arrive to the target, the vehicle must choose a trajectory that goes above the obstacle or below the obstacle. Let $K_1$ denote the set of initial conditions for which the closed-loop system (6) converges to the region $P$ from above, and let $K_2$ denote the initial conditions for which the closed-loop system (6) converges to the region $P$ from below. It then follows that there must exist a set $K$ where the vehicle must make a logic decision. Mathematically, for the obstacle avoidance problem, this behavior is captured by the following assumption; see also [18, 19]:

Assumption 2 There exists $T > 0$ such that for each $\rho > 0$ and each $\tilde{p}_0 \in K$, where $K := K_1 \cap K_2$, there exist points $\tilde{p}_1(0), \tilde{p}_2(0) \in [\tilde{p}_0 + \rho \mathcal{B}]$, for which there exist solutions $\tilde{p}_1$ and $\tilde{p}_2$ of (6) with $e = 0$, satisfying $\tilde{p}_1(t) \in K_1 \setminus K$ and $\tilde{p}_2(t) \in K_2 \setminus K$ for all $t \in [0, T]$.

Under Assumption 2, the next proposition establishes zero margins of robustness against small adversarial perturbations $t \mapsto e(t)$ in the closed-loop system (6). The proposition follows by [25, Thm. 6.5] or [19, Prop. 1]:

Proposition 1 Suppose that Assumption 2 holds. Then for each $\varepsilon, \rho, \rho'' > 0$, and every $\tilde{p}_0 \in K + \varepsilon \mathcal{B}$ such that $\tilde{p}_0 + \rho \mathcal{B} \subset \mathbb{R}^2 \setminus N$ and $\tilde{p}_0 + \rho'' \mathcal{B} \subset (K_1 \cup K_2)$ there exist a piecewise constant function $e : \text{dom}(e) \to \varepsilon \mathcal{B}$ and a (Carathéodory) solution $\tilde{p} : \text{dom}(\tilde{p}) \to \mathbb{R}^2 \setminus N$ to (6) such that $\tilde{p}(t) \in ((K + \varepsilon \mathcal{B}) \setminus (K_1 \cup K_2)) \cap (\tilde{p}_0 + \rho'' \mathcal{B})$, for all $t \in [0, T')$ for some $T' \in (T^*, \infty)$, where $\text{dom} \tilde{p} = \text{dom} \tilde{e}$, $T^* = \min \{|\rho', \rho''| m^{-1}\}$, and $m = \sup \{1 + (|k \nabla J(\eta)|) : \eta \in p_0 + \max \{|\rho', \rho''\}| \mathcal{B} \}$. If $T'$ is finite, then $\lim_{t \to T'} \tilde{p}(t) \notin (K_1 \cup K_2) \cup (\tilde{p}_0 + \rho'' \mathcal{B})$.

The result of Proposition 1 has important implications for perception-based control in systems under topological obstructions, such as obstacles. Namely, it establishes the existence of a set of points $K \subset \mathbb{R}^2$ where arbitrarily small approximations $e$ on the learned perception map $\ell$ can have a
dramatic effect in the stability properties of the controller. Given that, in general, the error in the
perception map $\ell$ can only be guaranteed to be bounded (see Lemma 1), Proposition 1 establishes
that no robust controller based on simple navigation functions exists for the solution of obstacle
avoidance problems with inexact perception maps. Indeed, for navigation functions that combine
attractive fields and repulsive fields, the set $K$ will contain the spurious critical points of the nav-
gation function $J$, which includes any saddle-point\(^1\). In this case, it is even possible to design
adversarial disturbances $t \mapsto \epsilon(t)$ in (6) able to stabilize a spurious equilibrium. The left plot of
Figure 2 illustrates this case for a perception map learned via convolutional neural networks (see
Supplemental Material), implemented in the closed-loop system (6), and having an arbitrarily small
adversarial disturbance $\epsilon$ designed to stabilize a saddle point located at the point $p_s = (-13.8, 0.8)$.
This saddle point emerges due to the combination of repulsive and attractive fields in the navigation
function $J$. As observed, the trajectory of the robot never leaves a neighborhood of $p_s$.

In contrast to the behavior observed in the left plot of Figure 2, the right plot shows a trajectory with
identical initialization and evolving under the same perception map $\ell$, with the same adversarial
disturbance $\epsilon$. However, in this case, the trajectory converges to the target point. In contrast to (6),
this trajectory is generated by a hybrid controller that not only incorporates the perception map,
but also incorporates logic variables in the feedback mechanism. The synthesis and robustness
properties of this controller are presented in the next section.

3 Robust Perception-Based Hybrid Control

3.1 Synthesis of the Controller

To synthesize the hybrid controller, we first characterize the class of admissible obstacles. Through-
out the rest of the paper we use $p_T \in \mathbb{R}^2$ to denote the target point of the robot.

Assumption 3 There exists $\rho \in \mathbb{R}_{>0}$ and $\varepsilon \in \mathbb{R}_{>0}$ such that the obstacle $\mathcal{N} \subset \mathbb{R}^2$ satisfies $\mathcal{N} \subset p_0 + \rho \mathbb{B}$ and $(p_0 + 2\rho\sqrt{2}\mathbb{B}) \cap (\{p_T\} + \varepsilon \mathbb{B}) = \emptyset$, where $p_0 = [x_0, y_0]^T \in \mathbb{R}^2$. \hfill $\Box$

In words, Assumption 3 considers obstacles that are contained in spheres located sufficiently far
away from the target point. Next, to achieve robust obstacle avoidance, we will design a switched
perception-based controller that implements different potential fields in different sub-regions of the
operational space of the vehicle. By using this switching approach, we will be able to rule out the
emergence of problematic sets $K$ such as the one shown in the left plot of Figure 1. To design the
covering of the operational space, for each $p_0 \in \mathbb{R}^2$ and $\rho > 0$, define the set $\mathcal{B}_{p_0, \rho} := \{ p \in \mathbb{R}^2 : || p - p_0 || \leq 2\rho \sqrt{2} \}$, which satisfies $\{ p_0 \} + \rho \mathbb{B} \subset \mathcal{B}_{p_0, \rho} \subset \{ p_0 \} + 2\rho \sqrt{2} \mathbb{B}$. As in the standard
state-based hybrid control [18, 19], we define the sets:

$L_{1a} := \{ p \in \mathbb{R}^2 : y < -x + y_0 + x_0 - 2\rho \sqrt{2} \}, \quad L_{1b} := \{ p \in \mathbb{R}^2 : y < x + y_0 + x_0 + 2\rho \sqrt{2} \}. \\
L_{2a} := \{ p \in \mathbb{R}^2 : y > x + y_0 + x_0 - 2\rho \sqrt{2} \}, \quad L_{2b} := \{ p \in \mathbb{R}^2 : y > -x + y_0 + x_0 + 2\rho \sqrt{2} \},$

\(^1\) The existence of such saddle points in navigation functions with attractive and repulsive fields was established in [12].
as well as the unions $O_1 := L_{1a} \cup L_{1b}$, $O_2 := L_{2a} \cup L_{2b}$, and $O := O_1 \cup O_2$. In this way, $O = \mathbb{R}^2 \setminus B_{\rho_0, p}$ and $N \cap O = \{\emptyset\}$. Figure 3 shows the geometric structure of both sets $O_1$ (left plot) and $O_2$ (right plot). For each of the sets $O_1$ and $O_2$, we will design suitable potential functions $V_q$, $q \in \{1, 2\}$, that can be used in a gradient-based controller of the form (4). The level sets of these functions are also shown in Figure 3. Note that in each of the sets $O_q$ the potential function $V_q$ has a unique critical point located at the position of the target. The controller will then switch between these two potential functions depending on its current location $p$, which will be generated by a perception map $\ell$. Specifically, the potential functions are defined as

$$V_q(p) := J_q(p) - J(p), \quad \forall \ p \in O_q,$$

and

$$V_q(p) := \infty, \quad \forall \ p \notin O_q, \tag{7}$$

where $J$ and $J_q$ satisfy the following assumption.

Assumption 4 The functions $\{V_q\}_{q \in \{1, 2\}}$ satisfy the following: (a) For each $q \in \{1, 2\}$ there exist functions $\alpha_{1,q}, \alpha_{2,q} \in K_{\infty}$, and proper indicators $\tilde{\omega}_q$ of $\{p_T\}$ on $O_q$, such that $\alpha_{1,q}(\tilde{\omega}_q(p)) \leq V_q(p) \leq \alpha_{2,q}(\tilde{\omega}_q(p))$, $\forall \ p \in O_q$; (b) For each $q \in \{1, 2\}$, we have $\{p^* \in O_q : \nabla V_q(p^*) = 0\} = p_T$; (c) For each $q \in \{1, 2\}$, the function $V_q(\cdot)$ is continuously differentiable in $O_q$.

The conditions of Assumption 4 can be readily satisfied using different classes of functions $J$ and $J_q$. For example, they hold when $J$ is given by $J = -(x - x_T)^2 - (y - y_T)^2$, and $J_q$ is given by $J_q(p) := B(\tilde{d}_q(p))$, where $\tilde{d}_q(p) := |p|^2_{\mathbb{R}^2 \setminus O_q}$, and $B(s) := (s - \rho)^2 \log(\frac{1}{s})$, if $s \in [0, \rho]$, and $B(s) := 0$, if $s > \rho$, with $\rho \in (0, 1]$ being a tunable parameter selected sufficiently small.

Remark 1 While the theoretical results of this paper focus mainly on static targets, our framework can be extended to time-varying targets modeling follower-leader settings. In this case, the leader uses potential functions $V_q$ with a minimizer located at the target position $p_T$, while the follower uses potential functions $V_q$ with a minimizer located at the position of the leader. Figure 4 illustrates this scenario. Additionally, we note that the hybrid controller can also be implemented in vehicles with more complex dynamics, including second-order dynamics and non-holonomic vehicles [19].

2For a compact set $A$ contained in an open set $U$, a continuous function $\tilde{\omega} : U \rightarrow \mathbb{R}_{\geq 0}$ is a proper indicator of $A$ on $U$ if $\tilde{\omega}(z) = 0$ if and only if $z \in A$, and $\tilde{\omega}(z_i) \rightarrow \infty$ when $i \rightarrow \infty$ if either $|z_i| \rightarrow \infty$, or the sequence $\{z_i\}_{i \in \mathbb{Z}}$ approaches the boundary of $U$.
3.2 Main Results

Using the above construction, we can now formulate the complete perception-based hybrid control system. Let $\chi \in (1, \infty)$ and $\lambda \in (0, \chi - 1)$ be tunable parameters. The closed-loop hybrid system has states $(p, q) \in \mathbb{R}^2 \times Q$, where $Q = \{1, 2\}$. The continuous-time dynamics are given by

$$
\dot{p} = -k \nabla J_q(\ell(\theta)), \quad \dot{q} = 0,
$$

(8)

which are allowed to evolve in the set

$$
C_{p,q} := \{(\ell(\theta), q) \in \mathcal{O} \times Q : V_q(\ell(\theta)) \leq \chi V_{3-q}(\ell(\theta))\},
$$

(9)

and discrete-time dynamics

$$
p^+ = p, \quad q^+ = 3 - q,
$$

(10)

which are allowed to evolve in the set

$$
D_{p,q} := \{(\ell(\theta), q) \in \mathcal{O} \times Q : V_q(\ell(\theta)) \geq (\chi - \lambda)V_{3-q}(\ell(\theta))\}.
$$

(11)

Note that in (8), (9), and (11), the position of the vehicle is given by the perception map $\ell(\theta)$ rather than the state $p$. The term $(\chi - \lambda)$ in (11) guarantees that the intersection of the sets $C_{p,q}$ and $D_{p,q}$ is not empty. Thus, for initial conditions in $C_{p,q} \cap D_{p,q}$ solutions are not unique. The set $C_{p,q}$ characterizes the points where the vehicle implements the controller (8) with constant state $q$. On the other hand, the set $D_{p,q}$ describes the points in the space where the vehicle toggles the logic state $q$ whenever it approaches the boundary of the respective set $\mathcal{O}_q$. In particular, note that since $\chi > 1$ and $\chi - \lambda > 1$, the robot toggles the potential field $V_q$ whenever its current value exceeds a threshold compared to the other potential fields $V_{3-q}$. After each jump, the robot flows again using now the new potential function $V_{3-q}$, until a new jump (if at all) is triggered. Note that this switching rule describes a hysteresis property in the feedback controller.

The following theorem is the main theoretical result of this paper. It establishes the existence of perception maps $\ell$ under which robust convergence to the target is guaranteed via the hybrid controller. The proof is presented in the supplemental material.

**Theorem 1** Let $\delta > 0$ and $K_0 \subset C_{p,q} \cup D_{p,q}$, where $K_0$ is compact. Suppose that $p \mapsto F(p)$ is $L$-Lipschitz continuous, where $F$ is defined in Lemma 1. Then, there exists a perception map $\ell$ and training data $D = \{(p_i, \theta_i)\}_{i=1}^N$ such that every trajectory of the vehicle generated by the hybrid system (8)-(11) is complete and converges to a $\delta$-neighborhood of the target point $p_T$ while avoiding the obstacle $\mathcal{N}$.

The proof of Theorem 1 is presented in the supplemental material. It leverages structural robustness properties of well-posed hybrid systems, as well as Lemma 1. To the best knowledge of the authors, this is the first result in the literature that integrates perception-based maps and hybrid control theory.

One of the main advantages of hybrid controllers is their robustness with respect to noisy measurements and small bounded disturbances. The perception map, learned from data, enhances this robustness properties due to the generalization properties. In the next section, we numerically test the controller under different scenarios.

4 Numerical Experiments

We test the perception-based hybrid controller by training a perception map using a convolutional neural network. The model’s architecture consists of three sets of Conv-ReLU-MaxPool blocks, with kernel size of $3 \times 3$ and $2 \times 2$, respectively. Finally, a Dense layer, preceded by a Dropout layer with probability 0.5 [26], takes the flattened output of the last Conv-ReLU-MaxPool block of layers and outputs a vector that imperfectly describes the state information. The model was trained using Keras [27]. Training took place for 5 epochs, with batch size 128 and input images with shape $(60, 100, 3)$. The output is a $1 \times 2$ vector describing the predicted $(x, y)$-position of the agent. The optimizer was Adam with learning rate 0.001. The loss function was the Mean Squared Error (MSE) between the predicted and real state of the agent. Mean Absolute Error (MAE) was also used as a validation metric. Further details of the training are given in the supplemental manuscript.

To test the controller, we consider a leader robot (denoted with a white square) aiming to converge to the static target $G$ while avoiding the obstacle $\mathcal{N}$. We also consider a follower robot (denoted with a green square), which tracks the leader. Both employ the hybrid controller, and the follower...
robot uses the learned perception map in order to approximate the leader’s state. Figure 5 shows the trajectories obtained in this scenario under two different initial conditions: (-12, 2) in the left plot, and (-37, -17) in the right plot. On the other hand, Figure 6 shows that the controller can successfully handle scenarios with noisy state measurements (simulating drift on the vehicle dynamics) and sensor failures (e.g., the camera does not transmit data at each time \( t \) with certain probability). Additionally, in Figure 7 we consider a similar scenario, but in this case, the images contains occlusions, e.g., clouds. The left plot shows the hybrid controller performance when using a perception map trained on images without occlusions. It can be seen that the controller successfully handles the increment in prediction error due to inputs to the vision model being generated from a different process. The right plot of Figure 7 also considers this scenario but employing a controller trained on data with occlusions. Further numerical experiments and illustrative videos are presented in the supplemental material.

Figure 5: Trajectories when both vehicles employ the hybrid controller. The plot below shows the evolution of the logic state of the hybrid controllers. The flow and jump sets for this controllers are defined with \( \chi = 1.1 \) and \( \lambda = 0.09 \).

Figure 6: Performance of the controllers when noise \( (\epsilon \sim \mathcal{N}(0, 0.5)) \) is added to the reference robot’s state, and the follower robot sensor’s failure rate is 50% (no photo is taken).

Figure 7: Snapshots of the trajectories of the robots implementing the hybrid controller, but using different perception maps. In the left figure, the perception map was trained on images without occlusions (shown in blue color). In the right plot, the perception map was trained using images with occlusion. In both cases, the hybrid controller is successful.

5 Conclusions

In this paper, we introduced a perception-based hybrid controller for the robust solution of obstacle avoidance problems that use vision-based sensors for the purpose of feedback. Unlike existing results in the literature, our controller incorporates a perception map learned by using supervised learning methods, and which provides a suitable approximation of the position of the vehicle based on images generated by a camera. By leveraging the structural robustness properties of the hybrid controller we establish obstacle avoidance and convergence to the target point from every initial condition. Different numerical experiments were carried out to validate the theoretical results. The numerical tests showed that the hybrid controller with perception map is robust to additional noisy measurements, sensors with low probability of failure, and also to the emergence of occlusions in the perception based sensor. Moreover, the controller is also able to track a leader vehicle that moves sufficiently slow. Future research directions will focus on experimental validations of the results in robots with complex dynamics, such as non-holonomic vehicles.
References


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