TourRank: Utilizing Large Language Models for Documents Ranking with a Tournament-Inspired Strategy

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Abstract

 Large Language Models (LLMs) are increas- ingly employed in zero-shot documents rank- ing, yielding commendable results. However, several significant challenges still persist in LLMs for ranking: (1) LLMs are constrained by limited input length, precluding them from **processing a large number of documents simul-** taneously; (2) The output document sequence is influenced by the input order of documents, 010 resulting in inconsistent ranking outcomes; (3) Achieving a balance between cost and ranking performance is quite challenging. To tackle these issues, we introduce a novel documents ranking method called TourRank, which is in- spired by the tournament mechanism. This ap-**proach alleviates the impact of LLM's limited** input length through intelligent grouping, while the tournament-like points system ensures ro- bust ranking, mitigating the influence of the document input sequence. We test TourRank with different LLMs on the TREC DL datasets and the BEIR benchmark. Experimental results show that TourRank achieves state-of-the-art performance at a reasonable cost.

025 1 Introduction

 Recently, Large Language Models (LLMs) have demonstrated great potential in numerous Natural Language Processing (NLP) tasks, especially under the zero-shot settings. Researchers and practition- ers have also tried to leverage LLMs document ranking, a core task in information retrieval, under the zero-shot settings. Most of the existing LLM- based document ranking methods can be divided into three categories: the *Pointwise* approach that prompts LLMs to independetly assess the relevance 036 of each candidate document [\(Sachan et al.,](#page-8-0) [2022;](#page-8-0) [Liang et al.,](#page-8-1) [2022;](#page-8-1) [Zhuang et al.,](#page-9-0) [2023a;](#page-9-0) [Guo et al.,](#page-8-2) [2024\)](#page-8-2); the *Pairwise* approach that use LLMs to compare each document against all the other docu-ments [\(Qin et al.,](#page-8-3) [2023\)](#page-8-3); and the *Listwise* approach

that instruct LLMs to generate a ranked list of doc- **041** ument labels according to their relevance to the **042** [q](#page-8-5)uery [\(Sun et al.,](#page-9-1) [2023;](#page-9-1) [Ma et al.,](#page-8-4) [2023;](#page-8-4) [Pradeep](#page-8-5) **043** [et al.,](#page-8-5) [2023a](#page-8-5)[,b;](#page-8-6) [Zhuang et al.,](#page-9-2) [2023c\)](#page-9-2).[1](#page-0-0)

044

While these three approaches lead to different **045** trade-offs between effectiveness and efficiency, the **046** listwise approach, such as RankGPT, is considered **047** as the preferred prompting strategy for the LLM- **048** based zero-shot document ranking task. Unlike the **049** pointwise approach, the listwise approach consid- **050** ers multiple documents simultaneously and thus **051** yields better effectiveness in ranking. Meanwhile, **052** listwise ranking eludes the quadratic growing cost **053** exposed in the comparisons of all document pairs, **054** resulting in improved efficiency. **055**

Although the listwise approaches achieve a good **056** trade-off between effectiveness and efficiency and **057** thus are considered preferred prompting strate- **058** gies for LLM-based document ranking, they also **059** face certain challenges: (1) The maximum context **060** length of LLMs limits the number of documents **061** that can be compared in a single prompt; (2) The **062** listwise generation process can not run in parallel, **063** which makes it hard to return the final ranking list 064 under a tight time constraint. (3) The ranking re- **065** sults are highly dependent on the initial order of **066** the candidate documents in the input prompt. **067**

To address these challenges, we need to de- **068** velop a prompting strategy for LLM-based docu- **069** ment ranking that can: (Requirement 1) establish a **070** global ranking for about 100 candidate documents **071** through multiple local comparisons of 2 to 10 doc- **072** uments in a single prompt; (Requirement 2) par- **073** allelize multiple LLM inferences to minimize the **074** overall ranking process time; and (Requirement **075** 3) effectively utilize the initial order of candidate **076** documents set by the first-stage retrieval model **077** without relying too heavily on it. 078

¹See Appendix [A.2](#page-10-0) for a more detailed literature review of the pointwise, pairwise, and listwise approaches for LLMbased document ranking.

Figure 1: The 1982 FIFA World Cup. In the first group stage, 24 teams were divided into six groups, and the top 2 out of 4 teams in each group qualified. In the

second group stage, 12 teams were divided into 4 groups, and only the top 1 out of 3 teams in each group advanced. In knockout stages, only the winner in each two-team match progressed to the next stage.

 Interestingly, we find that using LLMs and prompts to rank documents for a query can be anal- ogous to ranking teams or athletes in a sports tour- nament, as the design of a sports tournament has similar requirements. A tournament in sports is a structured competition involving multiple teams or individual competitors who compete against each other in a series of matches or games, with the goal of determining a champion or ranking the partic- ipants. Figure [1](#page-1-0) shows the format and results of an example tournament, the 1982 FIFA World Cup. The tournament consists of two group stages and two knockout stages (i.e., the semi-finals and the final). Analogous to Requirement 1, each group in the group stages and each two-team match in the knockout stages served as a local comparison; the results of these local matches determined which teams could advance to the next stage and their final rankings in the tournament. To expedite the ranking process, the World Cup organized multiple parallel matches across different groups. This par- allelization allowed the tournament to progress effi- ciently and fit into a tight 4-week schedule, which meets Requirement 2. Regarding Requirement 3, the initial groupings were based on seeding and previous performance, providing an initial order of teams. However, the tournament did not solely rely on these seedings; each team's performance in the group stage and subsequent rounds determined their advancement and final rankings.

 Therefore, inspired by the tournament mecha- nism, we propose a new zero-shot document rank- ing method called TourRank, which can fulfill the three requirements and mitigate the challenges in

Knockout stage We also design a grouping strategy, similar to the **120** existing methods. In TourRank, we regard each can- **113** didate document as a participant in a multi-stage **114** tournament. In each stage, we group the candidate **115** documents and prompt the LLM to select the most **116** relevant documents in each group to advance to **117** the next stage. The LLM inferences across dif- **118** ferent groups in a single stage can be parallelized. **119** seeding strategy in sports tournaments, to make 121 use of the initial document order provided by the **122** first-stage retrieval model in ranking. In addition, **123** to further improve the effectiveness and robustness, **124** we design a point system to assign different points **125** to each candidate document based on its ranking **126** in each round tournament and perform multiple **127** rounds of tournament. In this way, the ranking list **128** can be obtained based on the final accumulated **129** points in descending order. **130**

> To prove the effectiveness of our approach, We **131** test TourRank and baselines on the TREC DL **132** 19 [\(Craswell et al.,](#page-8-7) [2020\)](#page-8-7), TREC DL 20 datasets **133** [\(Craswell et al.,](#page-8-8) [2021\)](#page-8-8), and 8 datasets from BEIR **134** benchmark [\(Thakur et al.,](#page-9-3) [2021\)](#page-9-3). **135**

> To conclude, our contributions can be summa- **136** rized as follows: **137**

- We introduce TourRank, a novel zero-shot **138** documents ranking method based on LLMs. **139** By conducting multiple rounds of tourna- **140** ment, TourRank outperforms existing prompt- **141** ing strategies for zero-shot documents rank- **142** ing. **143**
- TourRank effectively mitigates the shortcom- **144** ings of current methods, particularly their sen- **145** sitivity to the initial candidate documents. **146**
- TourRank strikes a commendable balance be- **147** tween inference cost and effectiveness, further **148** solidifying its advantages. **149**
- Our experimental results confirm that Tour- **150** Rank also achieves SOTA on the open-source **151** models Mistral-7B [\(Jiang et al.,](#page-8-9) [2023\)](#page-8-9) and **152** Llama-3-8B [\(MetaAI,](#page-8-10) [2024\)](#page-8-10). **153**

2 Related Works **¹⁵⁴**

Neural Network Approaches Recent advance- **155** ments in document ranking have been achieved **156** with the help of pre-trained language models like 157 BERT [\(Devlin et al.,](#page-8-11) [2018\)](#page-8-11) and T5 [\(Raffel et al.,](#page-8-12) **158** [2020\)](#page-8-12). Notably, [Nogueira and Cho](#page-8-13) [\(2019\)](#page-8-13) develop **159** a multi-stage text ranking system using BERT, **160** while [Nogueira et al.](#page-8-14) [\(2020\)](#page-8-14) and [Zhuang et al.](#page-9-4) **161** [\(2023b\)](#page-9-4) employ T5 for document ranking. **162** LLMs Approaches Recent studies have utilized LLMs for ranking tasks, employing pointwise, pair- wise, and listwise approaches. Pointwise meth- ods, such as Query Generation (QG) [\(Sachan et al.,](#page-8-0) [2022\)](#page-8-0) and Binary Relevance Generation (B-RG) [\(Liang et al.,](#page-8-1) [2022\)](#page-8-1), use LLMs to compute the probability or likelihood of query-passage pairs. Pairwise approaches, such as Pairwise Ranking Prompting (PRP) [\(Qin et al.,](#page-8-3) [2023\)](#page-8-3), leverage LLMs to conduct pairwise comparisons and ranking of retrieved documents. RankGPT [\(Sun et al.,](#page-9-1) [2023\)](#page-9-1) is a listwise method that adopts a sliding window strategy for document ranking. There are also other listwise methods, like RankVicuna [\(Pradeep et al.,](#page-8-5) [2023a\)](#page-8-5) and RankZephyr [\(Pradeep et al.,](#page-8-6) [2023b\)](#page-8-6), which employ instruction-tuning for documents ranking. Setwise prompting [\(Zhuang et al.,](#page-9-2) [2023c\)](#page-9-2) enhances efficiency by reducing model inferences and prompt token consumption.

182 More introduction of existing works can be seen **183** in the Appendix [A.](#page-10-1)

¹⁸⁴ 3 Method: TourRank

 In this section, we introduce our novel zero-shot ranking approach called TourRank, which is in- spired by the tournament mechanism and includes multiple parallel tournaments. Similar to how play- ers are ranked based on the accumulated points of multiple tournaments in descending order in a season, TourRank gets the ranking order of candi- date documents based on the accumulated points of multi-round tournaments in descending order.

 Next, we first delineate how a basic tournament works in TourRank. Then, we explain how to get the accumulated points of the candidate documents, which are subsequently utilized for document rank- ing. Lastly, we propose a specific grouping method to circumvent the constraints on the input length of LLMs and make full use of the initial ranking **201** order.

202 3.1 A Basic Tournament

203 In one tournament of TourRank, we select N_K documents from N¹ candidates that are most rele- vant to the query in a stage-by-stage manner and each document gets a corresponding point after a whole tournament. As shown in Figure [2](#page-3-0) (a), we choose the documents by stagewise selection $(N_1 \rightarrow N_2 \rightarrow \cdots \rightarrow N_{K-1} \rightarrow N_K)$. In the k-th **selection stage** $(k \in \{1, 2, \dots K - 1\})$, the most top- N_{k+1} relevant documents to the given query

are selected from N_k documents to next selection 212 stage. After the k-th selection stage, the points of **213** the N_{k+1} selected documents are added by 1 to 214 k. And the points of $N_k - N_{k+1}$ documents that 215 are not selected are still $k - 1$. In this way, after a **216** full round of tournament, all candidate documents **217** can get the corresponding points P_{T_r} which is ex- 218 pressed as Table [1.](#page-2-0) In our experiments, the number **219** of candidate documents is 100, and the specific **220** points of all 100 documents after one tournament **221** are shown in Table [9](#page-15-0) in Appendix [G.](#page-14-0) **222**

Number of Docs Points of Docs	
N_K	$K-1$
$N_{K-1}-N_K$	$K-2$
$N_k - N_{k+1}$	$k-1$
$N_1 - N_2$	

Table 1: The points of all candidate documents after one tournament. For example, there are $N_k - N_{k+1}$ documents with a score of $k-1$. ($k \in \{1, 2, \dots K-1\}$)

The parameter r of P_{T_r} represents the r -th round 223 of tournaments. As shown in Figure [3,](#page-3-1) R rounds **224** of tournaments can be performed in parallel, so we **225** have $r \in \{1, 2, \cdots, R\}$. **226**

3.2 Getting The Accumulated Points **227**

Since the points obtained by one tournament are **228** coarse, multiple tournaments are required to ob- **229** tain more fine-grained document points. Figure **230** [3](#page-3-1) illustrates the process of multiple tournaments, **231** where we can see that points of candidate doc- 232 uments $P_{T_r}(r \in \{1, \dots, R\})$ are obtained after 233 each round of the tournament. **234**

Because there are many factors that affect the **235** output content of LLMs, such as decoding strategy, **236** temperature coefficient, and especially the order **237** of documents input to LLMs, may introduce some **238** bias, so each set of points $(P_{T_1}, \dots, P_{T_R})$ obtained 239 by R rounds of tournaments are a little bit different. **240** If these points are added up, the bias of each round **241** tournament could be reduced to some extent, and **242** the accumulated points P_T , which is expressed 243 as Equation [\(1\)](#page-3-2), are more fine-grained and robust. **244** So the final ranking list is obtained according to **245** the accumulated points P_F in descending order. 246 The analysis in Appendix [D](#page-12-0) shows how exactly **247** TourRank-r improves document ranking. **248**

Figure 2: (a) A basic tournament. (b) The grouping strategy in the selection stage of the tournament.

Figure 3: Get the accumulated points of all candidate documents through R tournaments.

R

$$
P_T = \sum_{r=1} P_{T_r} \tag{1}
$$

250 3.3 The Grouping and Selection Strategy

 Considering the limitation of the input length of LLMs, in some stages of TourRank, such as the 253 stage of selecting N_{k+1} documents from N_k candi- dates in Figure [2](#page-3-0) (a), we may not be able to input 255 all N_k documents into LLMs at once. Therefore, 256 we take the approach of assigning N_k candidate documents to several groups and then inputting documents of each group into LLMs separately and simultaneously.

60 **As shown in Figure 2 (b), the** N_k **documents are** divided into G groups, each of which contains n 262 documents. Here the relative order of N_k initial documents is given by the retrieval model, such as BM25 [\(Robertson et al.,](#page-8-15) [2009\)](#page-8-15), etc. When grouping in a sports tournament, the seeded players and the weaker players are evenly assigned into different groups to ensure the fairness of the competition. Similarly, we used a similar strategy to group the documents by evenly distributing the documents

 $Documents$ **and take** \bigotimes **i** \bigoplus \bigoplus **i** \bigoplus \bigoplus \bigoplus \bigoplus \bigoplus \bigoplus \bigoplus \bigoplus \bigoplus \bigoplus in the initial order into different groups as shown **270** in Figure [2](#page-3-0) (b). In this way, there will be some **271** difference in the relevance of the documents within **272** a group, making it easier for LLMs to select the **273**

> External 275 **Additionally, [Liu et al.](#page-8-16)** [\(2024\)](#page-8-16) find that current 275 $\left\{\n\begin{array}{c}\n\vdots \\
> \text{Additionally, Liu et al. (2024) find that current} \\
> \text{language models do not robustly access and use information in long input contexts because of the}\n\end{array}\n\right\}$ language models do not robustly access and use **276** information in long input contexts because of the **277** position bias. In order to eliminate the bias of **278** LLMs on document input order and achieve a ro- **279** bust ranking, the order of documents in each group **280** will be shuffled before entering LLMs and the multiple tournaments will be performed as shown in **282** Figure [3.](#page-3-1) **283**

> > After grouping the documents, we select the **284** most relevant m documents from the n ($m < n$) 285 documents in each group. In Figure [2](#page-3-0) (b), we mark **286** the selected m documents in red in each group, and **287** these documents advance to the next stage. **288**

> > Eventually, through the k-th selection stage of **289** the tournament, N_{k+1} more relevant documents are 290 selected from the N_k documents to advance to the 291 next stage. Benefiting from this smart grouping **292** stage and multi-round tournaments mechanism, we **293** solve the problem of limited input length of LLMs **294** while achieving a more robust selection. 295

3.4 The Overall of TourRank **296**

As the Pseudo-code of TourRank shown in Algo- **297** rithm 1, we perform R parallel tournaments as the **298** process in Figure [3](#page-3-1) for the given query q and the **299** candidate documents list D. In r-th round tourna- **300** ment, we first initialize the points of all N_1 candi- 301 date documents, that is $P_{T_r} = 0$ for N_1 documents. **302** Then, we select and increase the points of the docu- **303** ments in a stage-by-stage way in which $K-1$ times 304 selection stages are executed serially, and this is 305

 corresponds to Figure [2](#page-3-0) (a). In k-th selection stage, we adopt a suitable grouping approach (Figure [2](#page-3-0) 308 (b)) to get the N_{k+1} documents which can advance to the next selection stage, while adding points to 310 the selected N_{k+1} documents. After R rounds tour-311 nament, the points $P_{T_r}, r \in \{1, \cdots, R\}$ can be obtained. We can calculate the final points P_T ac- cording to Equation [\(1\)](#page-3-2). Finally, we re-rank the can- didate documents list according to the final points P_T in descending order.

Algorithm 1 The Pseudo-code of TourRank

- 1: Input: The query q and candidate documents list D
- 2: *Perform R tournaments in parallel*, $r \in \{1, \dots, R\}$:
- 3: *Initialize the points as* $P_{T_r} = 0$ *for* N_1 *documents.*
- 4: *Perform* k*-th selection stages, for* k *in range(*1, K*):*
- 5: *Assign* N^k *documents to* G *groups and each group has* n *documents.*
- 6: *Select* m *documents that are more relevant to the query* q *from* n *in each group in parallel.*
- 7: *Get the selected* N_{k+1} *documents to advance to next stage.*
- 8: *The points* P_{T_r} *of the selected* N_{k+1} *documents add* 1*.*
- 9: *Get a set of points* P_{T_r} *for all* N_1 *documents.*
- 10: After R times parallel tournaments, the final points P_T *can be obtained according to Equation [\(1\)](#page-3-2).*
- 11: *Rank the candidate documents D according to* P_T *in descending order.*
- 12: Output: A ranked list of candidate documents D

316 The specific hyperparameters of TourRank can **317** be seen in Table [8](#page-15-1) in the Appendix [G.](#page-14-0)

³¹⁸ 4 Experiments

319 Our experiments mainly focus on the following **320** research questions:

- **321** RQ.1: Whether TourRank outperforms existing **322** ranking methods based on LLMs.
- **323** RQ.2: Is TourRank sensitive to the candidate **324** documents retrieved by different models and the **325** initial order of documents, that is, does it have a **326** robust ranking?
- **327 RQ.3**: Is TourRank easy to achieve a trade-off be-**328** tween effectiveness and resource consumption?
- **329** RQ.4: Can TourRank achieve the best perfor-**330** mance based on different LLMs (open-source **331** and close-source)?

332 4.1 Experimental Settings

 Datasets We conduct experiments to answer the above research questions on TREC DL datasets [\(Craswell et al.,](#page-8-7) [2020,](#page-8-7) [2021\)](#page-8-8) and BEIR benchmark [\(Thakur et al.,](#page-9-3) [2021\)](#page-9-3). TREC is a widely used benchmark in IR research. We use the test sets **337** of TREC DL 19 and TREC DL 20, which con- **338** tain 43 and 54 queries. BEIR is a heterogeneous **339** [z](#page-9-1)ero-shot evaluation benchmark. Following [Sun](#page-9-1) **340** [et al.](#page-9-1) [\(2023\)](#page-9-1), we select 8 datasets for evaluation, in- **341** cluding Covid, Touche, DBPedia, SciFact, Signal, **342** News, Robust04, and NFCorpus. **343**

Metrics In the next evaluations, we re-rank the **344** top-100 documents retrieved by the first-stage re- **345** trieval model. If not specified, we use BM25 as the **346** default retrieval model and PySerini for implemen- **347** tation.^{[2](#page-4-0)} We use NDCG@ $\{5, 10, 20\}$ as evaluation 348 metrics. 349

Baselines We compare TourRank with sev- **350** eral state-of-the-art baselines in documents rank- **351** ing, including the supervised methods monoBERT **352** [\(Nogueira and Cho,](#page-8-13) [2019\)](#page-8-13) and monoT5 [\(Nogueira](#page-8-14) **353** [et al.,](#page-8-14) [2020\)](#page-8-14), and zero-shot methods based on **354** LLMs: two *pointwise* methods, DIRECT(0, 10) **355** [\(Guo et al.,](#page-8-2) [2024\)](#page-8-2), Binary Relevance Generation **356** (B-RG) [\(Liang et al.,](#page-8-1) [2022\)](#page-8-1), one *pairwise* method **357** PRP [\(Qin et al.,](#page-8-3) [2023\)](#page-8-3), and two *listwise* methods, **358** [S](#page-9-1)etwise [\(Zhuang et al.,](#page-9-2) [2023c\)](#page-9-2) and RankGPT [\(Sun](#page-9-1) **359** [et al.,](#page-9-1) [2023\)](#page-9-1). The detailed introductions of these **360** baselines are in the Appendix [B.](#page-10-2) **361**

4.2 Experimental Results **362**

Results on TREC DL datasets Table [2](#page-5-0) shows **363** the performance of different methods on TREC DL **364** datasets. We compare NDCG@{5, 10, 20}, and the **365** best top-4 results of zero-shot LLM methods are **366** shaded. We reproduce all zero-shot LLM methods **367** with gpt-3.5-turbo API. From the results, we can make the following findings: 369

(1) Our TourRank-10 outperforms all zero-shot **370** ranking baselines. It is worth noting that after **371** two tournaments (TourRank-2) the performance **372** is much better than one tournament (TourRank- **373** 1), and TourRank-2 can significantly outper- **374** form RankGPT. This indicates that TourRank can **375** achieve good results with fewer tournaments. **376**

(2) Generally, the two pointwise methods tend **377** to underperform in comparison to the pairwise **378** method, PRP-Allpair. However, the PRP-Allpair **379** falls short when compared to the listwise methods, **380** Setwise.bubblesort and our TourRank-10. This in- **381** dicates that listwise, which considers multiple doc- **382** uments simultaneously, is generally more effective **383** among LLM-based zero-shot ranking methods. **384**

(3) PRP-Allpair achieves about the same per- **385**

² https://github.com/castorini/pyserini

		TREC DL 19		TREC DL 20			
Methods	NDCG@5	NDCG@10	NDCG@20	NDCG@5	NDCG@10	NDCG@20	
BM25	52.78	50.58	49.14	50.67	47.96	47.21	
Supervised Methods							
monoBERT (340M)	73.25	70.50		70.74	67.28		
mono $T5(220M)$	73.77	71.48		69.40	66.99		
mono $T5(3B)$	73.74	71.83		72.32	68.89		
Zero-Shot LLM Methods							
DIRECT(0, 10)	54.22	54.59	54.15	55.17	55.35	54.73	
$B-RG$	63.33	62.51	60.00	65.04	63.37	60.47	
PRP-Allpair	70.43	68.18	64.61	69.75	66.40	64.03	
Setwise bubblesort	73.58	71.16	67.89	71.66	69.04	65.52	
RankGPT	72.05	68.19	62.21	67.25	63.60	59.12	
TourRank-1	70.95	66.23	62.49	66.65	63.74	60.59	
TourRank-2	72.24	69.54	65.03	67.65	65.20	62.78	
TourRank-10	73.83	71.63	68.37	72.49	69.56	66.13	

Table 2: Performance comparison of different methods on TREC datasets. We reproduce all the zero-shot LLM methods with gpt-3.5-turbo API. The best-performing algorithms for supervised methods and zero-shot LLM methods are bolded, respectively. The best top-4 results of zero-shot LLM methods are shaded in each metric. TourRank- r represents that we perform r times tournaments.

 formance as RankGPT on TREC DL 19, and outperforms RankGPT on TREC DL 20. Set- wise.bubblesort outperforms RankGPT on both datasets and is second only to TourRank-10. However, PRP-Allpair and Setwise.bubblesort achieve relatively good results at the cost of much higher complexity and resource consumption than RankGPT and TourRank. We discuss the effective-ness and cost of them in Section [4.5.](#page-6-0)

 (4) TourRank-10 achieves comparable results to the best supervised methods on TREC DL 19, and on TREC DL 20 TourRank-10 outperforms the best performing supervised method monoT5 (3B). It can be seen that TourRank is the only zero-shot method based on gpt-3.5-turbo API that can do this. We also perform TourRank with other close and open source LLMs in Section [4.6.](#page-7-0)

 Results on BEIR benchmark Table [3](#page-6-1) shows the NDCG@10 of different methods on 8 tasks of BEIR benchmark. The following are some valuable discussions:

407 (1) TourRank-10 achieves the best performance in **408** 6 out of 8 tasks and the best average NDCG@10 **409** across 8 tasks among zero-shot LLM methods.

 (2) The average of TourRank-2 (49.46) outper- forms RankGPT (49.37) in terms of NDCG@10 in Table [3,](#page-6-1) which, together with the better perfor- mance of TourRank-2 over RankGPT on TREC DL datasets shown in Table [2,](#page-5-0) prove that our TourRank algorithm can achieve good results with only a few times tournaments.

417 (3) Note that on the Touche task and Signal task,

all supervised methods and zero-shot methods in **418** Table [3](#page-6-1) perform even worse than BM25. The **419** NDCG@10 of all methods on these two tasks is **420** low, only about 0.3. According to [Thakur et al.](#page-9-5) **421** [\(2024\)](#page-9-5), the poor performance of neural retrieval **422** models is mainly due to the large number of short **423** texts and unlabeled texts in the Touche dataset. **424**

The results on TREC datasets and BEIR bench- **425** mark jointly answer the **RQ.1**. **426**

4.3 Sensitivity Analysis to Initial Ranking **427**

We compare 3 different initial ranking: 1) BM25: **428** Get top-100 documents by BM25; 2) Ran- **429** domBM25: Shuffle the order of BM25; 3) In- **430** verseBM25: Reverse the order of BM25. Figure [4](#page-6-2) **431** shows the results of RankGPT and our TourRank **432** based on 3 initial rankings and all these experi- **433** ments are based on gpt-3.5-turbo API. **434**

From Figure [4,](#page-6-2) we can see that RankGPT is very 435 sensitive to the initial permutation of documents **436** list. When the initial permutation is shuffled or **437** reversed, the performance of RankGPT becomes **438** much worse. This is caused by the ranking mecha- **439** nism of RankGPT, which adjusts the overall permu- **440** tation of documents list through the sliding window **441** strategy. Sliding the window from bottom to top **442** makes it easier for documents that are originally 443 near the top to be ranked at top positions in the final **444** permutation. Whereas documents that are at the **445** bottom of the initial permutation need to be ranked **446** at the top of every comparison in corresponding **447** sliding window in order to be ranked at the top of 448

Methods	Covid	NFCorpus	Touche	DBPedia	SciFact	Signal	News	Robust ₀₄	Average
BM25	59.47	30.75	44.22	31.80	67.89	33.05	39.52	40.70	43.42
Supervised Methods									
monoBERT $(340M)$	70.01	36.88	31.75	41.87	71.36	31.44	44.62	49.35	47.16
monoT5 $(220M)$	78.34	37.38	30.82	42.42	73.40	31.67	46.83	51.72	49.07
mono $T5(3B)$	80.71	38.97	32.41	44.45	76.57	32.55	48.49	56.71	51.36
Zero-Shot LLM Methods									
RankGPT	76.67	35.62	36.18	44.47	70.43	32.12	48.85	50.62	49.37
TourRank-1	77.17	36.35	29.38	40.62	69.27	29.79	46.41	52.70	47.71
TourRank-2	79.85	36.95	30.58	41.95	71.91	31.02	48.13	55.27	49.46
TourRank-10	82.59	37.99	29.98	44.64	72.17	30.83	51.46	57.87	50.94

Table 3: Performance (NDCG@10) comparison of different methods on BEIR benchmark. The best-performing algorithms for supervised methods and zero-shot LLM methods are bolded. TourRank-r represents that we perform r times tournaments.

Figure 4: The sensitivity analysis to initial ranking of TourRank and RankGPT on TREC DL 19 and TREC DL 20.

 the final permutation, otherwise they are left at the bottom or middle of the whole documents list. So, this is the reason why RankGPT is very sensitive to the initial ranking.

 However, our TourRank is quite robust to differ- ent initial orderings, as shown by the fact that shuf- fling and reversing the initial order has almost no effect on TourRank-r. The robustness of TourRank to the initial ranking benefits from the tournament mechanism presented in Figure [2.](#page-3-0) Each tournament is a selection over all candidate documents, not just a fine-tuning of the initial ranking like RankGPT.

461 4.4 Analysis to Different Retrieval Models

462 In addition to BM25, we also obtain top-100 docu-**463** ments based on two more powerful retrieval mod-**464** els, including a dense retriever model Contriever

[\(Izacard et al.,](#page-8-17) [2021\)](#page-8-17) and a neural sparse retrieval **465** model SPLADE++ ED [\(Formal et al.,](#page-8-18) [2022\)](#page-8-18), as the **466** first-stage retrieval model. Then, we perform Tour- **467** Rank and RankGPT to re-rank the top-100 candi- **468** date documents retrieved by different retrieval mod- **469** els based on gpt-3.5-turbo API. The results in Ta- **470** ble [4](#page-6-3) show that TourRank-10 achieves SOTA rank- **471** ing performance based on 3 kinds of different top- **472** 100 initial candidate documents. And TourRank-2 **473** can also outperform RankGPT in general. **474**

Table 4: NDCG@10 of TourRank and RankGPT based on different retrieval models. Here we use gpt-3.5-turbo API for TourRank and RankGPT.

The results in Table [4](#page-6-3) and the analysis in Sec- **475** tion [4.3](#page-5-1) jointly answer the RQ.2, that is, TourRank **476** has the ability of robust ranking. **477**

4.5 The Trade-Off between Effectiveness and **478 Resource Consumption 479**

Table [5](#page-7-1) shows the approximation of the theoretical **480** lowest time complexity of different methods and **481** the number of documents LLMs need to receive. **482** All the contents of Table [5](#page-7-1) are based on the rec- **483** ommended parameters. More detailed discussions **484**

485 on precise time complexity and number of input **486** documents are in Table [7](#page-13-0) in the Appendix [E.](#page-12-1) From **487** Table [5,](#page-7-1) we can see that:

 (1) PointWise has the lowest time complexity and the lowest number of documents received by LLMs, but the experimental results of DIRECT(0, 10) and B-RG in Table [2](#page-5-0) show that PointWise exhibits poor performance.

 (2) Although the pairwise method PRP-Allpair per- forms well in the experiments on TREC datasets, the number of input documents required by PRP-**AllPair is** $N^2 - N$, which will greatly increase the cost of ranking.

 (3) Setwise.bubblesort performs very well on TREC DL datasets in Table [2](#page-5-0) and is second only to TourRank-10. However, the multiple steps of Setwise have dependencies and cannot be run in parallel, resulting in the time complexity of Setwise being extremely high and unacceptable.

 (4) Two listwise methods RankGPT and our Tour- Rank take into account both the time complexity and the number of documents inputted to LLMs. The experimental results in Table [2](#page-5-0) and [3](#page-6-1) show that TourRank-2 can outperform RankGPT. From Table [5,](#page-7-1) we can see that TourRank-2 $(r = 2)$ achieves this goal with about twice as many documents to LLMs as RankGPT but with lower time complexity. We also compare TourRank with running RankGPT multiple iterations in serial (Appendix [F\)](#page-13-1), and Tour- Rank demonstrates better performance and lower consumption.

Table 5: A approximation of the theoretical lowest time complexity of various methods and the number of documents which are inputted to LLMs for each method. N is the number of candidate documents. Setwise rank the top-k ($k < N$) documents through bubblesort. $K - 1$ is the times of the selection stages in a tournament (Figure [2](#page-3-0) (a)) and r is the times of tournaments in TourRank- r . (Note: The approximate contents in this table are based on the recommended parameters.)

516 The above experimental results and the analysis **517** of the trade-offs between effectiveness and effi-**518** ciency jointly answer the RQ.3.

4.6 TourRank Based on Other LLMs **519**

We also perform TourRank based on the open- **520** source models Mistral-7B [\(Jiang et al.,](#page-8-9) [2023\)](#page-8-9) and **521** Llama-3-8B [\(MetaAI,](#page-8-10) [2024\)](#page-8-10), and gpt-4-turbo API. **522**

Table [6](#page-7-2) shows the performance of TourRank **523** with different LLMs. The top-100 candidate doc- **524** uments are retrieved by BM25. The results show **525** that RankGPT performs far worse based on the **526** open-source LLMs than based on OpenAI's APIs. **527** However, the performances of our TourRank based **528** on open-source LLMs are still good. Especially **529** on TREC DL 19 dataset, the performance of **530** TourRank-10 based on Llama 3 8B achieves 72.76, **531** which is higher than the performance of TourRank- 532 10 based on gpt-3.5-turbo (71.63). In addition, **533** TourRank-5 with gpt-4-turbo outperforms all meth- **534** ods based on gpt-3.5-turbo in Table [2,](#page-5-0) which in- **535** dicates that TourRank can achieve higher perfor- **536** mance with fewer tournaments times r based on a 537 stronger model. **538**

Table 6: NDCG@10 of TourRank and RankGPT based on open-source LLMs, Mistral 7B and Llama 3 8B, and gpt-4-turbo API.

These experiments shows that TourRank can **539** achieve good performance not only based on Ope- **540** nAI's API, but also based on open source LLMs, **541** answering the **RQ.4**. 542

5 Conclusions **⁵⁴³**

We introduce TourRank, a novel zero-shot docu- **544** ments ranking method inspired by the tournament **545** mechanism. TourRank addresses challenges in **546** large language models for ranking, such as input 547 length limitations and sensitivity to input order. **548**

Our experiments show that TourRank outper- **549** forms existing LLM-based zero-shot ranking meth- **550** ods, balances effectiveness and cost. This demon- **551** strates that TourRank is a promising approach for **552** future research in zero-shot documents ranking. **553**

⁵⁵⁴ Limitations

555 The performance of TourRank is inherently depen-**556** dent on the capabilities of the underlying LLMs. If

557 the LLMs can't follow the instructions well, it will

558 be difficult to achieve good results.

559 Although multiple tournaments of TourRank can **560** be performed in parallel in a multi-process manner,

- **561** for example, based on the API of OpenAI, it is
- **562** difficult to run the open-source models in a multi-
- **563** process manner under the environment of limited

564 computing resources.

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⁶⁹⁴ Appendix

⁶⁹⁵ A More Related Works

696 A.1 Neural Network Approaches

 Documents ranking has made significant progress, with the help of pre-trained language models, such [a](#page-8-12)s BERT [\(Devlin et al.,](#page-8-11) [2018\)](#page-8-11) and T5 [\(Raffel](#page-8-12) [et al.,](#page-8-12) [2020\)](#page-8-12). [Nogueira and Cho](#page-8-13) [\(2019\)](#page-8-13) present a multi-stage text ranking system using BERT, introducing monoBERT and duoBERT models that offer a balance between quality and latency, achieving state-of-the-art results on MS MARCO and TREC CAR datasets. [Nogueira et al.](#page-8-14) [\(2020\)](#page-8-14) introduce a new method for document ranking using a pre-trained sequence-to-sequence model, T5, which outperforms classification-based mod- els, especially in data-poor scenarios, and demon- strates the model's ability to leverage latent knowl- edge from pretraining for improved performance. [Zhuang et al.](#page-9-4) [\(2023b\)](#page-9-4) introduce RankT5, a method for fine-tuning the T5 model for text ranking us- ing ranking losses, which shows significant per- formance improvements over models fine-tuned with classification losses and demonstrates better zero-shot ranking performance on out-of-domain **718** data.

719 A.2 LLMs Approaches

 Pointwise Approaches There are several works that employ various zero-shot pointwise rankers. Query Generation (QG) [\(Sachan et al.,](#page-8-0) [2022\)](#page-8-0) in- volves rescoring retrieved passages by leveraging a zero-shot question generation model. The model uses a pre-trained language model to compute the probability of the input question, conditioned on a retrieved passage. Binary Relevance Generation (B- RG) [\(Liang et al.,](#page-8-1) [2022\)](#page-8-1) proposes to utilize LLMs to make predictions on a query-passage pair, uti- lizing the likelihood of "Yes/No" responses for the computation of ranking scores. The Rating Scale 0−k Relevance Generation (RS-RG) [\(Zhuang et al.,](#page-9-0) [2023a\)](#page-9-0) incorporates fine-grained relevance labels into the prompts for LLM rankers to better differen- tiate documents of varying relevance levels to the query, thereby achieving more accurate rankings. [Guo et al.](#page-8-2) [\(2024\)](#page-8-2) propose a multi-perspective evalu- ation criteria-based ranking model to overcome the deficiencies of LLM rankers in standardized com- parison and handling complex passages, thereby significantly enhancing the pointwise ranking per-formance. [Guo et al.](#page-8-2) [\(2024\)](#page-8-2) have also considered

the Rating Scale $0 - k$ Directly Score (DIRECT $(0, 743)$ k)) method. This approach prompts the LLM to **744** directly generate the relevance score for each query- **745** passage pair. **746**

Pairwise Approaches [Pradeep et al.](#page-8-19) [\(2021\)](#page-8-19) de- **747** sign a pairwise component to enhance the early **748** precision performance of the text ranking system **749** by employing a pre-trained sequence-to-sequence **750** model (such as T5 [\(Raffel et al.,](#page-8-12) [2020\)](#page-8-12)) to con- **751** duct pairwise comparisons and reranking of re- **752** trieved document pairs. [Qin et al.](#page-8-3) [\(2023\)](#page-8-3) intro- **753** duce a method called Pairwise Ranking Prompting **754** (PRP), which effectively enables LLMs to perform **755** text ranking tasks by simplifying the prompt de- **756** sign and achieving competitive performance across **757** multiple benchmark datasets. **758**

Listwise Approaches LRL [\(Ma et al.,](#page-8-4) [2023\)](#page-8-4) en- **759** hances text retrieval reranking by employing a large **760** language model as a zero-shot listwise reranker, $\frac{761}{26}$ utilizing a simple instruction template and a slid- **762** ing window strategy to process multi-document **763** information. Similarly, [Sun et al.](#page-9-1) [\(2023\)](#page-9-1) introduce **764** a novel instructional permutation generation ap- **765** proach called RankGPT, utilizing a sliding window **766** strategy to effectively enable LLMs (such as Chat- **767** GPT [\(OpenAI\)](#page-8-20) and GPT-4 [\(Achiam et al.,](#page-8-21) [2023\)](#page-8-21)) **768** to be used for relevance ranking tasks in informa- **769** tion retrieval, achieving competitive and even su- **770** perior results on popular IR benchmarks. In addi- **771** tion, both RankVicuna [\(Pradeep et al.,](#page-8-5) [2023a\)](#page-8-5) and **772** RankZephyr [\(Pradeep et al.,](#page-8-6) [2023b\)](#page-8-6) utilize open- **773** source LLMs and employ instruction fine-tuning **774** to achieve zero-shot listwise document reranking, **775** thereby enhancing the ranking performance of **776** smaller LLMs. [Zhuang et al.](#page-9-2) [\(2023c\)](#page-9-2) propose a $\frac{777}{2}$ novel Setwise prompting approach to enhance the **778** efficiency and effectiveness of LLMs in zero-shot **779** document ranking tasks, by reducing the number **780** of model inferences and prompt token consump- **781** tion, which significantly improves computational **782** efficiency while maintaining high ranking perfor- **783** mance. **784**

B Introduction of Baselines **785**

B.1 Supervised Methods **786**

- monoBERT [\(Nogueira and Cho,](#page-8-13) [2019\)](#page-8-13): A rank- **787** ing method with a cross-encoder architecture **788** based on BERT-large, trained on MS MARCO. **789**
- monoT5 [\(Nogueira et al.,](#page-8-14) [2020\)](#page-8-14): A ranking **790** method that calculates the scores using T5 model. $\frac{791}{291}$

(c) Average of 8 tasks on BEIR

Figure 5: The performance of TourRank with different times of tournaments. The abscissa is the times of tournaments, and the ordinate is NDCG@{5, 10, 20, 30, 50}. All the results are based on gpt-3.5-turbo API.

792 B.2 Zero-Shot Methods

- **793** DIRECT(0, 10) [\(Guo et al.,](#page-8-2) [2024\)](#page-8-2): A pointwise **794** method which gives the relevance scores ranging **795** from 0 to 10 to each query-document pair in text **796** format using LLMs. Then, rank the documents **797** according to these scores in descending order.
- **798** [•](#page-8-1) Binary Relevance Generation (B-RG) [\(Liang](#page-8-1) **799** [et al.,](#page-8-1) [2022\)](#page-8-1): A pointwise method which ranks **800** the candidate documents according to the likeli-**801** hood of "Yes or No" on a query-document pair.
- **802** PRP [\(Qin et al.,](#page-8-3) [2023\)](#page-8-3): A pairwise method that **803** reduces the burden on LLMs by using a technique **804** called Pairwise Ranking Prompting.
- **805** Setwise [\(Zhuang et al.,](#page-9-2) [2023c\)](#page-9-2): A listwise **806** method that improves the efficiency of LLM-**807** based zero-shot ranking. The authors introduce **808** two setwise methods, setwise.bubblesort and set-**809** wise.heapsort. Because the former has better **810** performances on experiments, we reproduce set-**811** wise.bubblesort in our experiments (Table [2\)](#page-5-0).

And we set $c = 3$ which is the best value for 812 setwise.bubblesort. *c* is the number of compared 813 documents in a prompt. 814

• RankGPT [\(Sun et al.,](#page-9-1) [2023\)](#page-9-1): A listwise method **815** that uses a sliding window strategy to achieve list- **816** wise ranking based on LLMs. In the experiments, 817 we observed some instability in the performance **818** of RankGPT. So the values in Table [2](#page-5-0) and Figure **819** [4](#page-6-2) are the average by running RankGPT 3 times. **820**

C The Performance of TourRank-r **⁸²¹**

Figure [5](#page-11-0) shows the trend of NDCG@{5, 10, 20, 30, **822** 50} with the increase of the number of tournaments **823** for TourRank on TREC datasets and BEIR bench- **824** mark. We can see that after the first two tourna- **825** ments, TourRank-2 achieves relatively good results **826** on all datasets, outperforming RankGPT on all cor- **827** responding metrics shown. Even in TourRank-10, **828** the metrics still have the potential to continue to **829** increase. **830**

Figure 6: The relationship between the accumulated points P_T and the corresponding labels for TourRank-1 and TourRank-10. The query of this case is "how long is life cycle of flea" which is one of the queries in the TREC DL 19.

 Since the number of tokens consumed by Tour- Rank scales linearly with the number of tourna- ments, we can control the number of consumed tokens by controlling the number of tournaments. Thus, the balance between effectiveness and token consumption can be achieved.

837 **D** Case Study: How Does TourRank **⁸³⁸** Improve the Performance of **839 Documents Ranking?**

 In Figure [6,](#page-12-2) the horizontal coordinate represents the ranking position of top-50 documents, the red lines 842 represent the accumulated points P_T of TourRank- 1 and TourRank-10 respectively, and the blue star points represent the corresponding real labels (inte-gers from 0 to 3).

846 It can be seen that the P_T of TourRank-1 is coarse, and the labels for the top-50 ranked doc- uments are also relatively scattered. However, 849 the accumulated points P_T of TourRank-10 be- come much more fine-grained after 10 tourna- ments, and the labels corresponding to top-50 docu- ments are relatively concentrated. After testing, the NDCG@{10, 50} of the case query have increased from {0.7078, 0.8186} to {0.8715, 0.911}.

 Therefore, as the times of tournaments increases, the accumulated points P_T become more fine- grained. This is how exactly TourRank improves the document ranking performance.

859 **E** The Discussions on Time Complexity **⁸⁶⁰** and Number of Documents Inputted to **⁸⁶¹** LLMs

 Table [7](#page-13-0) is a more precise version of Table [5](#page-7-1) which shows the theoretical lowest time complexity of var- ious methods and the number of documents which are inputted to LLMs for each method. Then, we analysis the content in Table [7.](#page-13-0)

E.1 Time Complexity 867

PointWise and Pairwise Since PointWise scoring a single document and PRP-Allpair comparing **869** a pair of documents can be performed in paral- **870** lel, the theoretical lowest time complexity is $O(1)$. 871 However, since pairwise methods need to compare **872** about $O(N^2)$ pairs of documents, the theoretical 873 minimum time complexity $O(1)$ is difficult to implement. 875

Setwise.bubblesort According to [\(Zhuang et al.,](#page-9-2) **876** [2023c\)](#page-9-2), the time complexity of Setwise.bubblesort **877** is $O(k * \frac{N}{c-1})$. Setwise rank the top- $k (k < N)$ doc- 878 uments through bubblesort, and c is the documents **879** compared in a prompt of Setwise. Considering **880** that Setwise can achieve the best performance with **881** $c = 3$, the time complexity is: 882

$$
O(k * \frac{N}{c-1}) \approx O(\frac{1}{2}k * N)
$$

RankGPT RankGPT uses sliding window strat- **884** egy, so its time complexity is $O(\frac{N-\omega}{s})$ $\frac{-\omega}{s}$). The best 885 window size is $\omega = 20$ and the best step size is 886 $s = 10$ in RankGPT. Based on the optimal pa- 887 rameters ($\omega = 20$ and $s = 10$) and considering 888 that ω is often much smaller than N , the best time **889** complexity of RankGPT is: **890**

$$
O(\frac{N-\omega}{s}) = O(\frac{N-20}{10}) \approx O(\frac{1}{10} * N)
$$

TourRank-r One tournament includes $K - 1$ 892 times selection stages shown in Figure [2,](#page-3-0) so the **893** time complexity of one tournament is $O(K - 1)$. 894 Because r rounds tournaments can be performed **895** in parallel, the time complexity of TourRank-r is **896 also** $O(K − 1)$. 897

Methods	Time Complexity	No. of Docs to LLMs		
PointWise	O(1)	N		
PRP-Allpair	O(1)	N^2-N		
Setwise.bubblesort	$O(k*\frac{N}{c-1}) \approx O(\frac{1}{2}k*N)$	$k * \frac{N}{c-1} * c \approx \frac{3}{2}k * N$		
RankGPT	$O(\frac{N-\omega}{\epsilon}) \approx O(\frac{1}{10} * N)$	$\omega * \frac{N-\omega}{2} \approx 2*N$		
TourRank- r	$O(K-1)$	$\left(\sum_{k=0}^{K-1} \frac{N}{2^k}\right) * r \approx 2r*N$		

Table 7: This Table is a more precise version of Table [5.](#page-7-1) The theoretical lowest time complexity of various methods and the number of documents which are inputted to LLMs for each method. N is the number of candidate documents. Setwise rank the top-k $(k < N)$ documents through bubblesort, and $c = 3$ is the documents compared in a prompt of Setwise. $\omega = 20$ is window size and $s = 10$ is step size in RankGPT. $K - 1$ is the times of the selection stages in a tournament (Figure [2](#page-3-0) (a)) and r is the times of tournaments in TourRank-r. All the approximate contents in this table are based on the recommended parameters.

898 E.2 No. of Docs to LLMs

899 **PointWise** Since the PointWise method scores **900** each document once, the number of documents **901** inputted to LLMs is N.

 Pairwise However, PRP-Allpair needs to form **at least** $\frac{N*(N-1)}{2}$ pairs for N candidate documents, and since one pair of documents is inputted to LLMs each time, the number of documents it inputs 906 to LLMs is $N^2 - N$.

Setwise.bubblesort The time complexity of Set-908 wise.bubblesort is $O(k*\frac{N}{c-1})$ and $c=3$ documents is compared in a prompt, so the number of docu-ments inputted to LLMs for Setwise.bubblesort is:

911
$$
k * \frac{N}{c-1} * c \approx \frac{3}{2}k * N
$$

RankGPT In RankGPT, we know that ω docu- ments need to be inputted into each window, and **a** total $\frac{N-\omega}{s}$ intra-window ranking need to be per- formed, so the number of documents input to LLMs is $\omega * \frac{N-\omega}{s}$ **is** $\omega * \frac{N-\omega}{s}$. The best window size ω given in RankGPT is 20 and the best step size s is 10. Based **b** on the optimal parameters and considering that ω is often much smaller than N, the number of docu-ments inputted into the LLMs of RankGPT is:

$$
\omega * \frac{N - \omega}{s} = 20 * \frac{N - 20}{10} \approx 2 * N
$$

922 TourRank-r In TourRank, if close to half of **923** the documents are selected to advance to the next selection stage in a tournament (that is, $m \approx \frac{1}{2}$ 924 selection stage in a tournament (that is, $m \approx \frac{1}{2}n$), **925** the total number of documents input to LLMs is **926** about:

$$
N + \frac{N}{2} + \dots + \frac{N}{2^{K-2}} = \sum_{k=0}^{K-1} \frac{N}{2^k}
$$

$$
= N * \frac{1 - \left(\frac{1}{2}\right)^{K-1}}{1 - \frac{1}{2}} \tag{928}
$$

$$
\approx 2*N \qquad \qquad 929
$$

The TourRank-r performs r rounds tournaments, **930** so the number of documents inputted to LLMs of **931** TourRank is about: **932**

$$
\left(\sum_{k=0}^{K-1} \frac{N}{2^k}\right) * r \approx 2r * N
$$

F Comparison Between Serial RankGPT **⁹³⁴** and Parallel TourRank-r **⁹³⁵**

We also run RankGPT multiple times in seriality **936** called RankGPT (serial), that is, the documents **937** order obtained by this iteration is used as the ini- **938** tial order for the next iteration. Figure [7](#page-14-1) shows the **939** comparison of RankGPT (serial) and our TourRank. **940** We can see that on both TREC DL 19 and TREC 941 DL 20 datasets, the NDCG@10 of RankGPT (se- **942** rial) goes up for the first three iterations, but stops **943** going up after that. This indicates that RankGPT **944** will reach the upper limit after a few serial runs. 945 However, after multiple iterations (or tournaments) **946** of TourRank-r, the NDCG@10 still continues to **947** rise and performs much better than RankGPT (se- **948** rial). **949**

RankGPT (serial) and TourRank after the same **950** r iterations: (1) The number of documents inputted **951** to LLMs are both about $2r * N$; (2) The time complexity $O(K - 1)$ of TourRank is also significantly 953

Figure 7: The comparison of NDCG@10 between running RankGPT multiple times in serial and running TourRank r in parallel.

less than $O(\frac{r}{10} * N)$ of RankGPT (serial); (3) The performance of TourRank is significantly better than RankGPT (serial). These indicate that Tour- Rank can achieve a better balance between effec-tiveness and efficiency.

 G The Detail Hyperparameters of TourRank

 The detail of hyperparameters of TourRank are shown in Table [8.](#page-15-1)

 Table [9](#page-15-0) shows the specific points of candidate document after 1 time tournament under the setting of our experiments.

H Prompts

 Table [10](#page-15-2) shows the prompt used in the grouping and selction stage (Figure [2](#page-3-0) (b)) of TourRank.

Table 9: The specific points of all documents after one tournament in our experimental settings.

system: You are an intelligent assistant that can compare multiple documents based on their relevancy to the given query.

user: I will provide you with the given query and n documents. Consider the content of all the documents comprehensively and select the m documents that are most relevant to the given query: query.

assistant: Okay, please provide the documents.

user: Document 1: $Doc₁$ assistant: Received Document 1.

user: Document 2: $Doc₂$ assistant: Received Document 2.

(User input more documents to assistant.)

user: The Query is: *query*. Now, you must output the top m documents that are most relevant to the Query using the following format strictly, and nothing else. Don't output any explanation, just the following format: Document 3, ..., Document 1

Table 10: The prompt of the grouping and selection stage of TourRank.