Do Large Language Models Exhibit Cognitive Dissonance? Studying the Difference Between Revealed Beliefs and Stated Answers

Anonymous ACL submission

Abstract

Prompting and Multiple Choices Questions (MCQ) have become the preferred approach to assess the capabilities of Large Language Models (LLMs), due to their ease of manipulation and evaluation. Such experimental appraisals have pointed toward the LLMs' apparent ability to perform causal reasoning or to grasp uncertainty. In this paper, we investigate whether these abilities are measurable outside of tailored prompting and MCQ by reformulating these issues as direct text completion – the foundation of LLMs. To achieve this goal, we define scenarios with multiple possible outcomes and we compare the prediction made by the LLM through prompting (their `Stated Answer`) to the probability distributions they compute over these outcomes during next token prediction (their `Revealed Belief`). Our findings suggest that the `Revealed Belief` of LLMs significantly differs from their `Stated Answer` and hint at multiple biases and misrepresentations that their beliefs may yield in many scenarios and outcomes. As text completion is at the core of LLMs, these results suggest that common evaluation methods may only provide a partial picture and that more research is needed to assess the extent and nature of their capabilities.

1 Introduction

In recent years, Large Language Models (LLMs) have gained significant traction in the research community and the public at large (Zhao et al., 2023; Chang et al., 2024). At their core, LLMs are statistical models of languages that predict, for any given context, a probability distribution over their vocabulary for the occurrence of the next token in a sequence (Bender et al., 2021). Despite this simplicity, a wide array of earlier research has noted that their sophisticated use of natural language (NL) is impressive (Chang et al., 2024; Hu and Levy, 2023), and it has been claimed that they may provide a candidate model for the acquisition of language in humans (Warstadt and Bowman, 2022).

Other studies have gone further and claimed that LLMs have become more than just statistical models, and gained emergent abilities due to their massive training sets and architecture sizes (Bubeck et al., 2023; Wei et al., 2022). Notably, it has been argued that recent LLMs have acquired the capability to perform more complex tasks such as reasoning and probability manipulation (Kıcıman et al., 2023). However, this fact is debated in the research community. While recent LLMs perform well on advanced benchmarks (bench authors, 2023), they can also be tricked by simple questions and adversarial modifications of their prompt (Xu et al., 2023; Zou et al., 2023). This has raised the question of whether LLMs indeed have an aptitude for reasoning, or whether these observations are an illusion that emerges from their mastery of NL and propensity for data memorization.

Indeed, at the heart of these debates is the problem of evaluating LLMs. The most common way to evaluate them is through prompting (see e.g., Brown et al. 2020) and most benchmarks are collections of questions and answers (Hendrycks et al., 2021; bench authors, 2023; Liang et al., 2023), where the LLMs are prompted with a question and the resulting answer is compared to a known ground truth. By nature, this method of evaluation is vulnerable to data contamination, where part of the evaluation set has been observed by the LLMs during training – a problem exacerbated by the fact that the training datasets of most LLMs are generally not accessible to the research community (Deng et al., 2023). Furthermore, since the evaluation of open-ended answers is quite complicated and resource-consuming (Frieder et al., 2024), the questions in these benchmarks are most often Multiple Choice Questions (MCQs) – see e.g., (Liang et al., 2023) – where LLMs are asked to choose between a finite set of answers. The evaluation via MCQs has been shown to suffer from a variety of biases (Pezeshkpour and Hruschka, 2023; Wang et al., 2024b) that compound the contamination issue, resulting in suboptimal assessments.

Recently, (Hu and Levy, 2023) has compared the merits of evaluating LLMs with direct text completion and with the use of prompts. While their experiments focus on the linguistic capabilities of the LLMs and their knowledge of the English language, their findings highlight that prompting is not a substitute for direct text completion, and that it is a key dimension of LLM evaluation that can be used to shed some light on their capabilities. In this paper, we build on this idea and introduce a different paradigm to evaluate LLMs – and in particular their handling of uncertainty in language – named `Revealed Belief`. It forgoes the questions/answer framework and instead relies solely on next-token prediction, LLMs' elementary unit of computation. In this approach, we present an LLM with a piece of text describing a scenario that has multiple possible outcomes (e.g., the throw of a fair die) and use the LLM's text completion to simulate its resolution (e.g., on which face the die lands). Then, the observed distribution over the possible outcomes is compared to the true probability distribution (e.g., a uniform distribution). In line with the "show, don't tell" adage, this approach emphasizes the text generated by LLMs (their `Revealed Belief`), instead of their answer to a predetermined question (their `Stated Answer`) – see Section 3.

We apply this new paradigm in a wide range of scenarios (see Section 4) and make several key observations. First, while the LLMs can perform well in the MCQ setting, as reflected by their `Stated Answer`, we observe that their `Revealed Belief` tend to differ substantially. Second, their `Revealed Belief` highlights numerous biases toward specific outcomes, the disproportionate impact of innocuous language elements on the distribution of outcomes, and the undue effect of unrelated events in the context (see Section 5). As these observations are not compatible with advanced reasoning capabilities, these results hint at the limitation of current evaluation methods and suggest that more research is needed in the study of LLMs' capabilities.

2 Related work

Since the introduction of LLMs, the research community has investigated their reasoning capabilities (Kıcıman et al., 2023). In particular, formal and mathematical reasoning have received significant attention, including the study of graph reasoning (Wang et al., 2024a) and arithmetic reasoning abilities (Mishra et al., 2022).

LLMs and probability. Our proposed empirical evaluation framework revolves indirectly around probabilities and uncertainty, and many previous studies have examined the aptitude of LLMs to handle problems involving probabilities. Recent contributions include (Nafar et al., 2024), which studied the reasoning capabilities of LLMs around text that contains explicit probability values, and (Saeed et al., 2021), which analyzed the capacity of LLMs to deduce soft logic rules (i.e., rules with a probability of being satisfied). While the evaluations performed in these papers indicate good performance by LLMs, more recently, (Jin et al., 2023) proposed a new dataset that contains questions involving probabilities to evaluate the performance of LLMs on causal inference in natural language. Their results point towards LLMs achieving disappointing results, with an accuracy of around 60%. Similar results have been observed in (Frieder et al., 2024), which studied the abilities of LLMs, and GPT-4 in particular, for advanced mathematical problems – including probability problems. The authors used manual expert evaluation of the model's answers and concluded that GPT-4 shows, generally speaking, poor performance at solving advanced mathematics. Compared to this work, it is important to note that the probability problems involved in our experiments are significantly easier, such as the flip of a coin, or the throw of a die. Moreover, all the aforementioned studies evaluate the `Stated Answer` of the LLMs, whereas we investigate their `Revealed Belief` (as explained in Section 3) and observe that LLMs `Revealed Belief` can underperform even on these simpler problems.

Investigating LLM evaluation. Previous work has also questioned the evaluation of LLMs and scrutinized their flaws. In particular, MCQs – one of the most prevalent types of evaluation (Hendrycks et al., 2021; bench authors, 2023; Liang et al., 2023) – have faced a variety of criticisms. For instance, (Pezeshkpour and Hruschka, 2023) has shown that merely reordering the options of MCQ can lead to double-digit performance gaps across multiple benchmarks, while (Alzahrani et al., 2024) additionally showed that a similar phenomenon can be observed by changing the numbering scheme of the provided answers. In addition, (Wang et al., 2024b) pointed out a significant misalignment between the first token predicted by the model in MCQ, which is often used as a proxy to infer a model's answer, and the model's actual answer. Other works have pointed to the problem of data contamination, where the LLMs are shown to have been exposed during training or fine-tuning to the evaluation data used in common benchmarks. Notably, (Deng et al., 2023; Balloccu et al., 2024) have shown, using two different but complementary approaches, that this problem is present in all LLMs but is particularly pronounced in the GPT-series of models.

LLM `Revealed Belief`. Arguably, the paper closest to our work is (Hu and Levy, 2023), where the authors highlight the discrepancy between the direct completion of some text and prompting LLMs to complete the text, with particular emphasis on their mastery of the English language. While `Revealed Belief` and `Stated Answer` follow a similar dichotomy, the scope of our study – scenarios with uncertainty and multiple outcomes – differs significantly, and our method uses the full information of the next token's distributions to investigate the details of the LLM's implicit biases and errors. Furthermore, while (Hu and Levy, 2023) also hypothesised that new LLM capabilities may emerge by studying direct text completion instead of prompting, our findings points toward a more nuanced conclusion, as LLMs' `Revealed Belief` often perform worse than their `Stated Answer` in our experiments, even in simple settings.

3 `Revealed Belief` and `Stated Answer`

In the rest of this paper, we study LLMs through *the direct measurement of the model-derived next token distributions*, similarly to (Hu and Levy, 2023). For any given context, we collect the logits produced by the LLM before any temperature-based sampling, and we compute the exact probability distribution of the next token (or specific combination of tokens), noted $\mathbb{P}_{\text{LLM}}(\,\cdot\,|\text{context})$. This distribution reflects the exact LLM beliefs, in contrast to simply sampling LLM answers.

At the heart of our analysis is the use of scenarios with multiple possible outcomes. Consider for instance the roll of a single, fair, six-faced die. There are six possible results for this throw, 1 to 6, which are equally probable, with probability $1/6$. The commonly used method to assess whether an LLM is knowledgeable of this fact is to first query the model directly, for instance, with the MCQ question "What is the probability that the die falls on face 2?" and a set of possible answers $1/4$, $1/6$, etc. Then, the model's probability of selecting the correct answer is computed from the next token distribution (generally by selecting the most probable token as the answer) and is used as the metric to evaluate the LLM. We refer to this approach as the `Stated Answer` (`StaA`), as it uses the LLMs' prompted answer to evaluate them.

In this work, we propose an alternative method to evaluate LLMs for these multiple outcome scenarios, called `Revealed Belief` (`RevB`). Instead of explicitly querying the LLM, we assess its implicit beliefs about the given scenario by presenting it with an incomplete sentence describing this scenario and ending with the incomplete sentence "the die falls on face ...". The LLMs' probability of choosing any of the possible outcomes (in this case, 1 to 6) as its next token is then computed and the resulting distribution is compared to the true distribution (in this case, a uniform distribution). In other words, instead of asking the model what it knows, we let the model actually show its beliefs, through the distribution it produces over the set of possible outcomes. Figure 1 illustrates both frameworks for the dice scenario. Examples of detailed prompts and contexts can be found in Section 4.

While `RevB` can only be computed for scenarios with multiple outcomes, we argue that it presents an interesting alternative evaluation method that has multiple advantages. First, our approach is centred around text completion, which is the elementary computational unit of LLMs. As such, it can evaluate any LLM, including base models that have not been fine-tuned on dialogue behaviour and can be used to assess the influence of different fine-tuning methods. Furthermore, as it does not involve MCQs, `RevB` avoids their common pitfalls (choice of the answers, ordering, numeration, etc.), as highlighted in Section 2. Second, this evaluation approach incorporates significantly more information than `StaA`, as it allows an in-depth comparison of the difference between the true distribution and the revealed distribution of the outcomes, which can highlight biases towards or against specific outcomes. For instance, our experiments (Section 5) show that despite the LLMs `Stated Answer` that the probability of a die roll resulting in any face equals $1/6$, the `RevB` of these LLMs show an inordinate bias toward the result 1. Finally, it is impor-

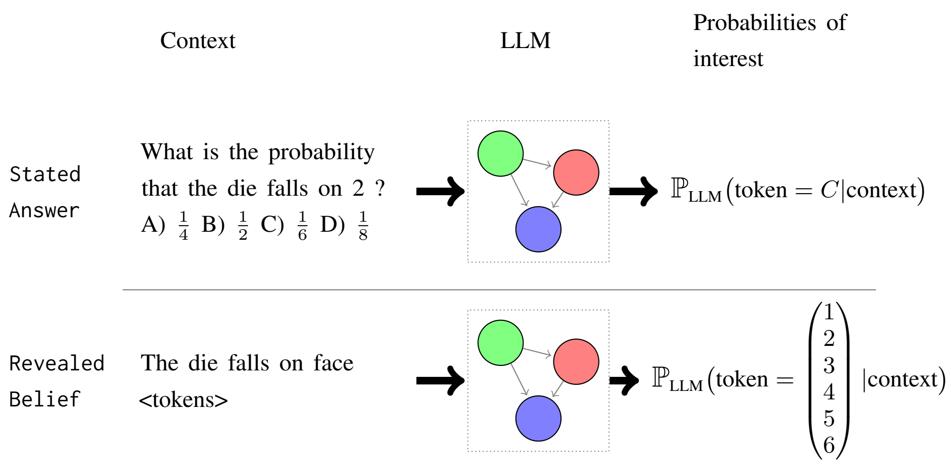


Figure 1: Illustration of the `Revealed Belief` and `Stated Answer` frameworks, for the scenario that involves a regular fair die with six faces. Only a short version of the context is presented here. See Section 3 for more details.

tant to point out that the biases and issues shown in `RevB` can have an important impact on an LLM's behaviour. For instance, one of our experiments highlights the LLMs' preferences towards the first answer in a fair choice between abstract, equiprobable options, which illustrates the bias that LLMs can exhibit when answering MCQs. Importantly, these biases are compounded when the outcomes have multiple meanings, such as the terms "left" and "right". See Section 5 for an in-depth exploration of these issues.

4 Methodology

4.1 Tasks

We examine the `Revealed Belief` of LLMs through four different scenarios: Dice, Coins, Choice, and Preferences. The first two aim at examining the aptitude of LLMs to handle outcome distributions of varying complexity, while the latter two scrutinize some of the possible biases that LLMs may yield in their `RevB`. Each scenario is described below, and Table 1 offers a summary of the different experimental setups.

4.1.1 Scenario 1: Dice

Probability problems derived from die rolls are among the most prevalent in an introductory mathematics curriculum. Thus, it is expected that instances of this scenario are well represented in any large LLM training dataset. As die rolls can yield many probability distributions, we use them as the first scenario to explore the `RevB` of LLMs.

Prompt Example. "A die has 6 faces. The die is equally likely to land on any of its faces. The die is cast. The die lands on face <tokens>".

Variants & Parameters. In the simple variant, several dice (1-3) with four to twelve faces each are rolled once, and the outcome is the total sum of the dice faces. This results in a uniform or a multinomial distribution, depending on the number of dice. In the repeated variants, the dice are rolled twice, and the first result is mentioned in the context. The outcome to be predicted is then either the result of the second roll (case independent) or the sum of both rolls (case dependent). These variants aims at examining the influence of a previous roll on the `RevB`. In the independent case, the expected result should be identical to the simple variant, while in the dependent variant, the distribution should be shifted by the value of the previous roll. Finally, in the observation scenario, some information regarding the die roll result is disclosed to the model – for instance, that the result is an even number. These observations allow manipulating the expected distributions and studying the influence of new information on the `RevB`.

4.1.2 Scenario 2: Coins

Coin flips are also quite common in probability problems and offer a different scenario to study distributions of varying complexity.

Prompt Example. "There are 3 coins. Each coin is biased and is 5 times more likely to land on Heads than on Tails. The coins are flipped and the resulting number of Heads is equal to <tokens>".

Variants & Parameters. Compared to the dice scenario, coins have only two faces (Heads and

Scenario	Variants	Parameters
Dice	Single Roll Repeated (Independent, Dependent) Observations	Number of Dice, Number of Faces Number of Dice, Number of Faces, Result previous roll Number of Dice, Number of Faces, Observation(s)
Coins	Single Flip Repeated (Independent, Dependent)	Number of Coins, Heads or Tails, Bias Number of Coins, Heads or Tails, Bias, Result previous flip
Choice	Single Choice Repeated (Independent)	Number of Options Number of Options, Previous choice
Preference	Single Selection Repeated (Independent)	Option 1, Option 2, Bias Option 1, Option 2, Bias, Previous selection

Table 1: Summary of the scenarios, variants, and parameters.

Tails). We therefore vary the number of coins, as well as two additional parameters: the face of interest (Heads or Tails), that is to say, the one that is counted in the flip, and the bias, which modifies the probability of the face of interest, and thereby the resulting distribution. The Coins scenario includes both the simple (a single flip) and the repeated variants (both independent and dependent).

4.1.3 Scenario 3: Choice

In this scenario, the models have to select between an arbitrary number of abstract options, represented using capital letters – similar to the choice of an answer in an MCQ. As the choices are explicitly stated to be of equal probability, the distribution underlying this scenario is always uniform and identical to the roll of a single die. Here, the interest is to scrutinize the influence of the setting (e.g., dice versus abstract) on simple distributions and distil the raw preferences over abstract choices by discarding the connotations related to the scenarios.

Prompt Example. "A person has to choose randomly between 4 options. The options are A, B, C, and D. All possible options are equally likely. The person chooses at random option <tokens>".

Variants & Parameters. We consider two variants of this scenario: the simple variant, where a single choice is made and the parameter is the number of options; and the repeated independent variant, where the LLM makes a second choice after being presented with the result of a first one.

4.1.4 Scenario 4: Preference

Compared to Choice, the Preference scenario contains only two options, these two options are no longer abstract (for instance, left or right), and their probabilities are no longer necessarily equal. The goal of this scenario is to examine the influence of each option label on the outcome distribution.

Prompt Example. "A person has to choose randomly between two options: Left and Right. The option Left is 2 times more likely to be chosen than the option Right. The person chooses at random option <tokens>".

Variants & Parameters. The Preference scenario contains the same variants as the Choice Scenario: the simple variant and the repeated independent variant. The parameters that are varied are the labels of the options (e.g., Left/Right, Heads/Tails), the explicit bias, the result of the previous selection, and the order of the options in the query.

4.2 Models

We tested the `RevB` framework using 12 models chosen to reflect the state-of-the-art as of May 2024 within three model sizes, according to benchmarking results reported in the Huggingface Open LLM Leaderboard[1] and the LMSYS Chatbot Arena Leaderboard[2]. We only examined open-weight LLMs (in order to analyze their next token distribution) and only selected models where both the base and instruction fine-tuned variants were available. The bracket of small models include Llama-3-8B[3], Yi-1.5-{6B, 9B} (Young et al., 2024), gemma-7B (Mesnard et al., 2024), Qwen2-7B (Bai et al., 2023), and Mistral-7B-v0.3 (Jiang et al., 2023). The family of large models contains Llama-3-70B, Yi-1.5-34B, and Qwen2-72B. The considered mixture of expert models are Qwen2-57B-A14B and Mixtral-8x22B (Jiang et al., 2024). We evaluated the base and instruct versions of each model to compare their respective performance, using the models available on Hugging Face and we query their `Stated Answer` using the instruct version.

[1] huggingface.co/open-llm-leaderboard
[2] huggingface.co/spaces/lmsys/chatbot-arena-leaderboard
[3] github.com/meta-llama/llama3/blob/main/MODEL_CARD.md

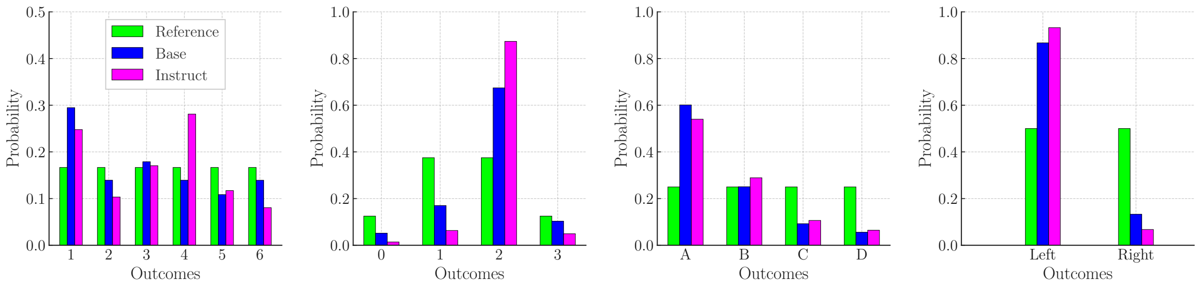
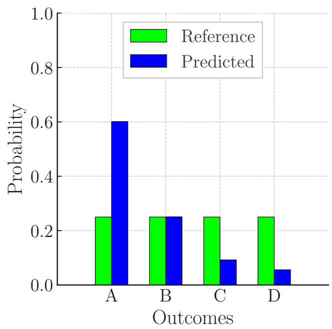


Figure 2: Comparison between the RevB of Llama-3-70B Base (blue), Instruct (magenta), and the true distribution (green) for respectively the simple 6-sided Dice (leftmost), 3 Coins (second left), 4 Choices (second right) and Preferences (rightmost).



	Y6B	M7B	Q7B	L8B	G7B	Y9B	Y34B	Q57xB	L70B*	Q72B	M8x22B
4 faces (RevB)	0.05	0.16	0.09	0.14	0.09	0.06	0.14	**0.04**	0.09	0.09	0.15
4 faces (StaA)	0.62	**0.00**	1.00	0.96	0.94	0.99	**0.00**	0.18	**0.00**	**0.00**	0.01
10 faces (RevB)	**0.06**	0.08	0.19	0.10	0.23	0.14	0.15	0.13	0.13	**0.06**	0.09
10 faces (StaA)	0.03	0.05	0.09	0.70	1.00	1.00	0.85	**0.00**	**0.00**	**0.00**	0.11
2 choices (RevB)	0.42	0.41	0.46	0.41	0.42	0.43	0.39	0.41	**0.32**	0.46	0.38
2 choices (StaA)	**0.00**	**0.00**	**0.00**	0.88	**0.00**	0.05	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**
4 choices (RevB)	0.45	0.48	0.61	0.44	0.54	0.57	**0.32**	0.55	0.35*	0.65	0.34
4 choices (StaA)	**0.00**	1.00	0.04	0.35	0.05	**0.00**	**0.00**	0.01	**0.00**	**0.00**	**0.00**

Figure 3: **Left:** Probability distribution over four abstract choices (Llama-3-70B). **Right:** Rev: Chebyshev distance between predicted and reference distribution for RevB; Stated: Probability of error of the StaA. Represented scenarios: die roll (4, 6, and 10-sided) and abstract choices (2, 4, and 6 options). Best score per scenario in bold.

4.3 Evaluation method

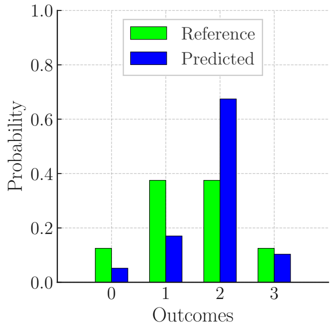
Each LLM is evaluated on all combinations of scenarios, variants, and parameters. First, for each resulting context, the RevB (i.e., the distribution over the possible outcomes) of the base model and the instruct models are computed and normalized. Then, these distributions are compared to the true probability distribution, using three different metrics: the Chebyshev distance, the Manhattan distance and the Kullback-Leibler divergence. While the last two measure the total difference between the different distributions, the Chebyshev distance represents the maximal difference of the weights between the distributions and is thus particularly representative of the bias that RevB can have towards or against a specific outcome. We also report the error of each instruct model Stated Answer, defined as the probability of the model giving a wrong answer, to provide an additional frame of reference for the LLM's performance.

5 Experimental Results

In total, we tested each model in more than 500 different settings. For brevity, we describe in this section the main findings of our experiments and emphasize them using the Chebyshev error metric. We refer the reader to the Appendix for further results of each scenario, as well as additional metrics.

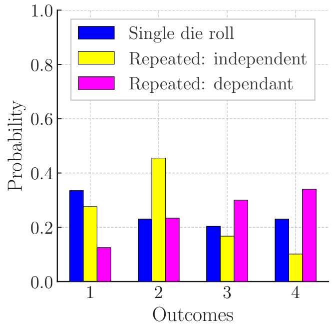
Result #1: Instruction fine-tuning does not improve RevB. Across the different scenarios studied in our experiments, we observe that the RevB of base-LLMs are always at least as good and often better than their instruction fine-tuned counterparts. Figure 2 shows four examples of RevB for both Llama-3-70B Base and Instruct. Consequently, while the fine-tuning of LLMs into instruction models is important for prompting, we observe that it does not improve their RevB and that both variants show similar biases and skews.

Result #2: LLMs RevB favour the first possible outcome in equiprobable scenarios. Figure 3



	Y6B	M7B	Q7B	L8B	G7B	Y9B	Y34B	Q57xB	L70B*	Q72B	M8x22B
2 dice (RevB)	0.07	0.10	0.13	0.20	0.14	0.12	0.09	**0.04**	0.07	0.09	0.08
2 dice (StaA)	**0.03**	**0.03**	0.47	0.87	0.97	0.92	1.00	0.99	1.00	1.00	0.96
2 coins (RevB)	0.20	0.12	0.25	0.18	0.20	0.45	0.14	**0.07**	0.20	0.16	0.16
2 coins (StaA)	0.90	**0.00**	0.38	0.62	1.00	1.00	0.27	1.00	**0.00**	**0.00**	0.02
3 coins (RevB)	0.11	**0.02**	0.14	0.15	0.09	0.19	0.03	0.06	0.30*	0.20	0.14
3 coins (StaA)	**0.00**	1.00	**0.00**	0.79	1.00	**0.00**	1.00	0.06	**0.00**	**0.00**	0.07

Figure 4: **Left:** Probability distribution of Heads for a 3 coins flip (Llama-3-70B). **Right:** Rev: Chebyshev distance between predicted and reference distribution for RevB. Stated: probability of errors of the StaA. Represented scenarios: 4-sided die rolls (2 and 3 dice) and coin flips (2 and 3 coins). Best score per scenario in bold.

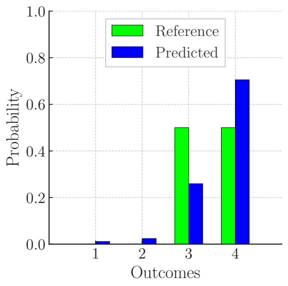


	Y6B	M7B	Q7B	L8B	G7B	Y9B	Y34B	Q57xB	L70B*	Q72B	M8x22B
Indep. prev. face 1 (RevB)	0.17	0.24	0.24	0.21	0.24	0.25	0.20	0.16	0.21*	**0.12**	0.20
Indep. prev. face 1 (StaA)	0.02	0.71	0.93	1.00	**0.00**	0.01	**0.00**	0.53	0.01	**0.00**	0.42
Dep. prev. face 1 (RevB)	0.04	0.06	0.09	0.10	0.29	**0.03**	0.10	0.11	0.12*	0.16	0.11
Dep. prev. face 1 (StaA)	**0.09**	1.00	0.46	0.99	0.12	0.99	1.00	0.98	1.00	1.00	0.87
Indep. prev. 0 heads (RevB)	0.13	0.13	0.15	0.16	**0.06**	0.17	0.17	0.39	0.24	0.17	0.13
Indep. prev. 0 heads (StaA)	0.99	1.00	0.03	0.93	1.00	0.80	0.19	0.41	1.00	**0.00**	0.35
Dep. prev. 0 heads (RevB)	0.13	**0.07**	0.30	0.09	0.13	0.20	0.27	0.28	0.22	0.21	0.18
Dep. prev. 0 heads (StaA)	0.97	**0.05**	0.21	0.63	0.68	0.97	**0.05**	0.93	1.00	1.00	0.35

Figure 5: **Left:** Probability distribution of a four-sided die roll (Llama-3-70B). Blue: no prior roll. Yellow: independent roll after a result on face 1. Magenta: shifted distribution for dependent roll after a result on face 1. **Right:** Rev: Chebyshev distance between predicted and reference distribution of the RevB and probability of errors of the StaA. Represented scenarios: 4-sided die rolls (previous roll landed on faces 1 or 2), and the number of heads when tossing two coins (previous toss resulted in 0 or 1 heads), with both dependent and independent variants.

illustrates the behaviour of the RevB when addressing a scenario yielding a uniform probability distribution (e.g., the roll of a single die or a choice between abstract options). Importantly, while the StaA are almost always correct, with many models exhibiting a 0% error (in particular for the family of large models), the RevB are significantly different from the expected uniform distributions. We observe that in most settings, the first possible option (side 1 of a die, or the abstract option “A”) is favoured by the LLMs. Interestingly, this bias is significantly stronger when predicting the outcome of an abstract choice, resulting in worse scores. This problem is also apparent for multinomial distributions (e.g., multiple dice rolls or coin flips) – see Figure 4. Indeed, many LLMs RevB exhibit a bias towards individual outcomes, such as a value near the midpoint of the distribution, or multiples of 10. Moreover, while most LLMs perform reasonably well when the number of outcomes is low, their RevB show very skewed distributions when this number exceeds a certain threshold (often around 12). While their StaA errors are higher than in the uniform case, large models still exhibit errors close to 0, despite their skewed RevB.

Result #3: Previous results described in a prompt have an undue impact on RevB and StaA. The repeated variant of the scenarios aimed at scrutinizing the influence of a previous realisation of the event on a future realisation. In the independent variant, the previous outcome is explicitly stated as having no bearing on the new outcome, while in the dependent variant, it should only shift the resulting distribution. However, as illustrated by Figure 5, we observe that the RevB of the next die roll is strongly influenced by the previous result. Indeed, depending on the LLM and the presented previous value, the resulting distribution is significantly con-



	Y6B	M7B	Q7B	L8B	G7B	Y9B	Y34B	Q57xB	L70B*	Q72B	M8x22B
Even face (RevB)	0.27	**0.06**	0.21	0.21	0.33	0.28	0.37	0.11	0.22	0.14	0.24
Even face (StaA)	0.98	**0.00**	1.00	0.37	1.00	1.00	**0.00**	0.96	**0.00**	0.73	0.29
Greater than 2 (RevB)	0.06	0.05	0.05	**0.01**	0.27	0.19	0.37	0.05	0.24*	0.12	0.10
Greater than 2 (StaA)	1.00	1.00	0.98	**0.00**	0.88	1.00	0.11	1.00	**0.00**	**0.00**	0.23
Even & > 2 (RevB)	0.12	0.04	0.10	0.03	0.36	0.17	0.40	0.11	0.10	0.04	**0.02**
Even & > 2 (StaA)	1.00	0.99	1.00	1.00	1.00	1.00	**0.00**	1.00	1.00	1.00	0.95
Even & not 1 (RevB)	0.27	0.33	0.29	0.26	0.41	0.30	0.46	**0.10**	0.29	0.12	0.13
Even & not 1 (StaA)	1.00	0.99	0.44	0.11	1.00	0.87	0.05	1.00	**0.00**	**0.00**	0.29

Figure 6: **Left:** Probability distribution of a four-sided die roll, with the observation that the result is greater than two (Llama-3-70B). **Right:** Chebyshev distance between predicted and reference distribution for RevB and errors for StaA. Represented observation: the die landed on an even face, not on face two, or is greater than two.

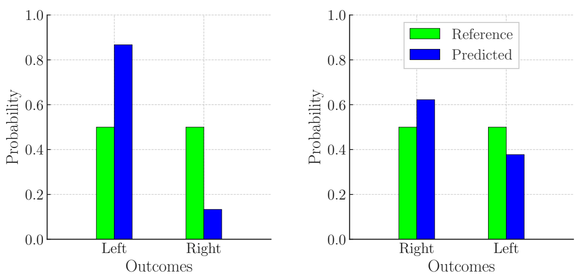


Figure 7: Probability distribution over "Left" and "Right" (Left) vs. "Right" and "Left" (Right) of Llama-3-70B RevB for the preference Scenario.

centrated either around or away from said value. With some rare exceptions, the scores shown in Figure 5 are significantly worse than those for the regular die roll (Figure 3), despite both being uniform distributions. This issue is also reflected in the error of the StaA of the LLMs, which are significantly worse. This shows that repeating an event within the same context window impacts an LLM's next prediction and decreases its reliability in the downstream prediction. We further observe that larger LLMs do not outperform their smaller variants in this scenario, indicating that scaling may not be the solution to this issue.

Result #4: RevB are better than StaA at handling partial information. Interestingly, in the variant with observations, where partial information about the result of a die roll is included in the context (e.g., "the result is even"), LLMs RevB are more accurate than their StaA. For instance, excluded outcomes (i.e., odd numbers in the aforementioned example) are assigned a probability close to zero, showing that the observation stated in the prompt is well integrated into the prediction. Conversely, the StaA of LLMs are generally significantly worse, as shown in Figure 6. This is particularly visible when the prompt contains combinations of observations that exclude multiple outcomes.

Result #5: The labels of the outcomes can strongly bias the RevB. The results of the Preference scenario show that even when the options are explicitly stated to be equiprobable, LLMs' RevB show their inherent pairwise preferences, as illustrated by Figure 7. In this case, "Left" is strongly preferred over "Right" (left figure) and this bias is not equally reciprocated even when the option "Right" is presented first (right figure), indicating that this bias is not due to ordering (Result #2).

6 Conclusion

This paper introduced a novel paradigm to evaluate LLMs in scenarios with multiple outcomes by directly using the probability distributions of next-token predictions and comparing them to the true distribution. Our findings suggest that the revealed beliefs of LLMs significantly differ from their stated answers and hint at a variety of biases and misrepresentations that they may express toward many outcomes and scenarios during text generation. Consequently, further research may be needed in the study of LLMs' capabilities and evaluations. Additionally, the exploration of loaded terms and inherent model biases (Result #5) yielded interesting results and the use of Revealed Belief to further investigate them is a promising direction for future work.

Limitations

We only evaluated the RevB and the StaA of LLMs on around 500 scenarios. While these scenarios were designed to cover many different types of distributions and already hint at many characteristics of the RevB of LLMs, it would be beneficial to study additional cases (e.g., Poisson distributions), as well as other variants (e.g., multiple repeated results instead of a single repeat, or more complex dependencies between results).

Another limitation of our study is the wording of the scenario and prompts. While significant time and effort were spent designing them in order to maximize the LLMs RevB performance, it is always possible that a different wording of the context can have yield better results. However, if such wordings were found, it would also highlight the significant lack of robustness of the RevB of LLMs.

References

Norah Alzahrani, Hisham Abdullah Alyahya, Yazeed Alnumay, Sultan Alrashed, Shaykhah Alsubaie, Yusef Almushaykeh, Faisal Mirza, Nouf Alotaibi, Nora Altwairesh, Areeb Alowisheq, et al. 2024. When benchmarks are targets: Revealing the sensitivity of large language model leaderboards. *arXiv preprint arXiv:2402.01781*.

Jinze Bai, Shuai Bai, Yunfei Chu, Zeyu Cui, Kai Dang, Xiaodong Deng, Yang Fan, Wenbin Ge, Yu Han, Fei Huang, Binyuan Hui, Luo Ji, Mei Li, Junyang Lin, Runji Lin, Dayiheng Liu, Gao Liu, Chengqiang Lu, Keming Lu, Jianxin Ma, Rui Men, Xingzhang Ren, Xuancheng Ren, Chuanqi Tan, Sinan Tan, Jianhong Tu, Peng Wang, Shijie Wang, Wei Wang, Shengguang Wu, Benfeng Xu, Jin Xu, An Yang, Hao Yang, Jian Yang, Shusheng Yang, Yang Yao, Bowen Yu, Hongyi Yuan, Zheng Yuan, Jianwei Zhang, Xingxuan Zhang, Yichang Zhang, Zhenru Zhang, Chang Zhou, Jingren Zhou, Xiaohuan Zhou, and Tianhang Zhu. 2023. Qwen technical report. *arXiv preprint arXiv:2309.16609*.

Simone Balloccu, Patrícia Schmidtová, Mateusz Lango, and Ondřej Dušek. 2024. Leak, Cheat, Repeat: Data Contamination and Evaluation Malpractices in Closed-Source LLMs. *arXiv preprint*. ArXiv:2402.03927 [cs].

BIG bench authors. 2023. Beyond the imitation game: Quantifying and extrapolating the capabilities of language models. *Transactions on Machine Learning Research*.

Emily M Bender, Timnit Gebru, Angelina McMillan-Major, and Shmargaret Shmitchell. 2021. On the dangers of stochastic parrots: Can language models be too big? In *Proceedings of the 2021 ACM conference on fairness, accountability, and transparency*, pages 610–623.

Tom Brown, Benjamin Mann, Nick Ryder, Melanie Subbiah, Jared D Kaplan, Prafulla Dhariwal, Arvind Neelakantan, Pranav Shyam, Girish Sastry, Amanda Askell, et al. 2020. Language models are few-shot learners. *Advances in neural information processing systems*, 33:1877–1901.

Sébastien Bubeck, Varun Chandrasekaran, Ronen Eldan, Johannes Gehrke, Eric Horvitz, Ece Kamar, Peter Lee, Yin Tat Lee, Yuanzhi Li, Scott Lundberg, et al. 2023. Sparks of artificial general intelligence: Early experiments with gpt-4. *arXiv preprint arXiv:2303.12712*.

Yupeng Chang, Xu Wang, Jindong Wang, Yuan Wu, Linyi Yang, Kaijie Zhu, Hao Chen, Xiaoyuan Yi, Cunxiang Wang, Yidong Wang, et al. 2024. A survey on evaluation of large language models. *ACM Transactions on Intelligent Systems and Technology*, 15(3):1–45.

Chunyuan Deng, Yilun Zhao, Xiangru Tang, Mark Gerstein, and Arman Cohan. 2023. Investigating data contamination in modern benchmarks for large language models. *arXiv preprint arXiv:2311.09783*.

Simon Frieder, Luca Pinchetti, Ryan-Rhys Griffiths, Tommaso Salvatori, Thomas Lukasiewicz, Philipp Petersen, and Julius Berner. 2024. Mathematical capabilities of chatgpt. *Advances in Neural Information Processing Systems*, 36.

Dan Hendrycks, Collin Burns, Steven Basart, Andy Zou, Mantas Mazeika, Dawn Song, and Jacob Steinhardt. 2021. Measuring massive multitask language understanding. In *International Conference on Learning Representations*.

Jennifer Hu and Roger P. Levy. 2023. Prompting is not a substitute for probability measurements in large language models.

Albert Q Jiang, Alexandre Sablayrolles, Arthur Mensch, Chris Bamford, Devendra Singh Chaplot, Diego de las Casas, Florian Bressand, Gianna Lengyel, Guillaume Lample, Lucile Saulnier, et al. 2023. Mistral 7b. *arXiv preprint arXiv:2310.06825*.

Albert Q Jiang, Alexandre Sablayrolles, Antoine Roux, Arthur Mensch, Blanche Savary, Chris Bamford, Devendra Singh Chaplot, Diego de las Casas, Emma Bou Hanna, Florian Bressand, et al. 2024. Mixtral of experts. *arXiv preprint arXiv:2401.04088*.

Zhijing Jin, Yuen Chen, Felix Leeb, Luigi Gresele, Ojasv Kamal, LYU Zhiheng, Kevin Blin, Fernando Gonzalez Adauto, Max Kleiman-Weiner, Mrinmaya Sachan, et al. 2023. Cladder: Assessing causal reasoning in language models. In *Thirty-seventh Conference on Neural Information Processing Systems*.

Emre Kıcıman, Robert Ness, Amit Sharma, and Chenhao Tan. 2023. Causal reasoning and large language models: Opening a new frontier for causality. *arXiv preprint arXiv:2305.00050*.

Percy Liang, Rishi Bommasani, Tony Lee, Dimitris Tsipras, Dilara Soylu, Michihiro Yasunaga, Yian Zhang, Deepak Narayanan, Yuhuai Wu, Ananya Kumar, Benjamin Newman, Binhang Yuan, Bobby Yan, Ce Zhang, Christian Alexander Cosgrove, Christopher D Manning, Christopher Re, Diana Acosta-Navas, Drew Arad Hudson, Eric Zelikman, Esin Durmus, Faisal Ladhak, Frieda Rong, Hongyu Ren, Huaxiu Yao, Jue WANG, Keshav Santhanam, Laurel Orr, Lucia Zheng, Mert Yuksekgonul, Mirac Suzgun, Nathan Kim, Neel Guha, Niladri S. Chatterji, Omar Khattab, Peter Henderson, Qian Huang, Ryan Andrew Chi, Sang Michael Xie, Shibani Santurkar, Surya Ganguli, Tatsunori Hashimoto, Thomas Icard, Tianyi Zhang, Vishrav Chaudhary, William Wang, Xuechen Li, Yifan Mai, Yuhui Zhang, and Yuta Koreeda. 2023. Holistic evaluation of language models. *Transactions on Machine Learning Research*. Featured Certification, Expert Certification.

Thomas Mesnard, Cassidy Hardin, Robert Dadashi, Surya Bhupatiraju, Shreya Pathak, Laurent Sifre, Morgane Rivière, Mihir Sanjay Kale, Juliette Love, et al. 2024. Gemma: Open models based on gemini research and technology. *arXiv preprint arXiv:2403.08295*.

Swaroop Mishra, Arindam Mitra, Neeraj Varshney, Bhavdeep Sachdeva, Peter Clark, Chitta Baral, and Ashwin Kalyan. 2022. Numglue: A suite of fundamental yet challenging mathematical reasoning tasks. *arXiv preprint arXiv:2204.05660*.

Aliakbar Nafar, Kristen Brent Venable, and Parisa Kordjamshidi. 2024. Probabilistic Reasoning in Generative Large Language Models. *arXiv preprint*. ArXiv:2402.09614 [cs].

Pouya Pezeshkpour and Estevam Hruschka. 2023. Large language models sensitivity to the order of options in multiple-choice questions. *arXiv preprint arXiv:2308.11483*.

Mohammed Saeed, Naser Ahmadi, Preslav Nakov, and Paolo Papotti. 2021. Rulebert: Teaching soft rules to pre-trained language models. *arXiv preprint arXiv:2109.13006*.

Heng Wang, Shangbin Feng, Tianxing He, Zhaoxuan Tan, Xiaochuang Han, and Yulia Tsvetkov. 2024a. Can language models solve graph problems in natural language? *Advances in Neural Information Processing Systems*, 36.

Xinpeng Wang, Bolei Ma, Chengzhi Hu, Leon Weber-Genzel, Paul Röttger, Frauke Kreuter, Dirk Hovy, and Barbara Plank. 2024b. " my answer is c": First-token probabilities do not match text answers in instruction-tuned language models. *arXiv preprint arXiv:2402.14499*.

Alex Warstadt and Samuel R Bowman. 2022. What artificial neural networks can tell us about human language acquisition. In *Algebraic structures in natural language*, pages 17–60. CRC Press.

Jason Wei, Yi Tay, Rishi Bommasani, Colin Raffel, Barret Zoph, Sebastian Borgeaud, Dani Yogatama, Maarten Bosma, Denny Zhou, Donald Metzler, et al. 2022. Emergent abilities of large language models. *arXiv preprint arXiv:2206.07682*.

Xilie Xu, Keyi Kong, Ning Liu, Lizhen Cui, Di Wang, Jingfeng Zhang, and Mohan Kankanhalli. 2023. An llm can fool itself: A prompt-based adversarial attack. *arXiv preprint arXiv:2310.13345*.

Alex Young, Bei Chen, Chao Li, Chengen Huang, Ge Zhang, Guanwei Zhang, Heng Li, Jiangcheng Zhu, Jianqun Chen, Jing Chang, et al. 2024. Yi: Open foundation models by 01. ai. *arXiv preprint arXiv:2403.04652*.

Wayne Xin Zhao, Kun Zhou, Junyi Li, Tianyi Tang, Xiaolei Wang, Yupeng Hou, Yingqian Min, Beichen Zhang, Junjie Zhang, Zican Dong, et al. 2023. A survey of large language models. *arXiv preprint arXiv:2303.18223*.

Andy Zou, Zifan Wang, J Zico Kolter, and Matt Fredrikson. 2023. Universal and transferable adversarial attacks on aligned language models. *arXiv preprint arXiv:2307.15043*.

A Scenario 1: Dice

We report in these Appendices the detailed results of our evaluation scenarios. Where similar variants could be aggregated (e.g., when asking for the number of Heads and for the number of Tails in a coin toss), we report their average. Within each scenario, we first show the `Revealed Belief` of the base LLMs, then their instruction fine-tuned counterparts, and finally, the scores of the model's `Stated Answer`.

A.1 Base models

A.1.1 Regular, independent, dependent

Chebyshev	Y6B	M7B	Q7B	L8B	G7B	Y9B	Y34B	Q57xB	L70B	Q72B	M8x22B
Regular – 1 die	**0.06**	0.11	0.16	0.11	0.17	0.10	0.10	0.08	0.11	**0.06**	0.10
Regular – 2 dice	0.08	**0.06**	**0.06**	0.08	0.08	0.09	**0.06**	0.09	0.10	0.07	0.10
Regular – 3 dice	0.12	0.08	0.10	**0.06**	0.07	0.09	0.10	**0.06**	0.08	0.10	0.10
Independent 1 die	0.14	0.14	0.41	**0.13**	0.23	0.23	0.25	0.30	0.25	0.14	**0.13**
Independent – 2 dice	0.15	0.12	0.22	0.12	0.16	**0.09**	0.19	0.13	0.18	0.20	0.20
Independent – 3 dice	0.20	**0.12**	0.27	**0.12**	0.17	0.15	0.26	0.17	0.22	0.19	0.20
Dependant 1 die	0.10	**0.07**	0.08	0.09	0.16	0.08	0.09	0.08	0.09	0.08	0.08
Dependant – 2 dice	0.20	0.17	0.18	**0.13**	0.19	0.28	0.23	0.24	0.15	0.22	0.29
Dependant – 3 dice	0.25	0.19	**0.14**	0.15	0.21	0.30	0.17	0.24	0.18	0.33	0.22
L1											
Regular – 1 die	0.23	0.37	0.40	0.33	0.39	0.25	0.27	0.24	0.27	**0.16**	0.35
Regular – 2 dice	0.42	0.45	**0.30**	0.37	0.36	0.36	0.41	0.38	0.57	0.42	0.37
Regular – 3 dice	0.67	0.71	0.66	0.52	0.44	0.56	0.63	**0.39**	0.56	0.66	0.52
Independent 1 die	0.47	0.40	0.92	**0.34**	0.62	0.60	0.66	0.72	0.59	0.38	0.42
Independent – 2 dice	0.65	0.55	0.71	0.55	0.77	**0.52**	0.79	0.57	0.68	0.69	0.72
Independent – 3 dice	0.84	0.76	0.93	**0.74**	0.98	0.85	0.99	0.79	0.87	0.85	0.90
Dependant 1 die	0.30	**0.29**	0.34	0.30	0.56	0.31	0.39	0.32	0.34	**0.29**	0.31
Dependant – 2 dice	0.92	0.81	0.80	**0.64**	0.91	0.91	0.98	0.97	0.80	0.92	1.12
Dependant – 3 dice	1.17	1.05	0.90	**0.88**	1.22	1.07	1.04	1.23	1.06	1.08	1.12
Symmetric KL											
Regular – 1 die	0.09	0.31	0.27	0.17	0.37	0.14	0.12	0.11	0.13	**0.05**	0.26
Regular – 2 dice	0.27	0.38	**0.18**	0.24	0.22	0.22	0.27	0.24	0.58	0.29	0.28
Regular – 3 dice	0.89	0.92	0.82	0.47	0.41	0.67	0.69	**0.26**	0.55	0.94	0.55
Independent 1 die	0.45	0.37	1.23	**0.23**	0.97	0.61	0.72	0.95	0.61	0.28	0.32
Independent – 2 dice	0.88	0.72	1.05	**0.57**	1.15	0.59	1.16	0.66	0.93	0.92	0.96
Independent – 3 dice	1.81	1.48	2.32	**1.21**	1.92	1.73	2.14	1.39	1.78	1.74	1.74
Dependant 1 die	0.18	**0.16**	0.23	0.19	0.60	0.24	0.29	0.17	0.23	0.17	0.17
Dependant – 2 dice	1.49	1.18	1.09	**0.77**	1.54	1.59	1.73	1.80	1.10	1.50	2.30
Dependant – 3 dice	2.86	2.18	**1.56**	1.59	3.08	2.57	2.19	3.05	2.24	2.85	2.55

A.1.2 Observations: One observation

Chebyshev	Y6B	M7B	Q7B	L8B	G7B	Y9B	Y34B	Q57xB	L70B	Q72B	M8x22B
Smaller	0.10	0.07	0.14	0.06	0.11	0.16	0.11	0.07	**0.04**	0.09	0.13
Larger	**0.10**	0.11	0.16	0.12	0.19	0.24	0.29	0.14	0.11	0.14	0.13
Even	0.24	0.14	0.23	0.20	0.23	0.25	0.28	0.17	0.26	**0.10**	0.17
Odd	0.16	0.15	0.10	0.09	0.22	0.22	0.15	0.10	0.13	**0.07**	0.09
Not first	0.49	0.35	0.32	**0.18**	0.30	0.44	0.36	0.27	0.23	0.21	0.38
Not middle	0.09	0.20	0.15	**0.07**	0.15	0.20	0.13	0.17	0.10	0.10	0.10
Smaller – 2 dice	0.13	0.12	**0.08**	0.11	0.12	0.22	0.10	0.12	0.11	0.10	0.11
Larger – 2 dice	0.20	**0.09**	0.18	0.13	0.42	0.19	0.27	0.16	0.16	0.15	0.17
Even – 2 dice	0.25	0.10	0.15	0.10	0.28	0.13	0.19	0.13	0.10	0.14	**0.09**
Odd – 2 dice	0.18	0.11	0.14	0.09	0.18	0.15	0.15	0.09	0.13	**0.08**	0.11
Not first – 2 dice	0.28	0.34	**0.06**	0.10	0.36	0.21	0.67	0.10	0.20	0.10	0.23
Not middle – 2 dice	0.09	0.10	**0.06**	0.07	0.09	0.13	0.20	0.18	0.21	0.09	0.12
Smaller – 3 dice	0.15	0.13	0.14	0.13	0.11	0.15	0.12	0.19	**0.08**	0.24	0.12
Larger – 3 dice	0.14	0.13	0.11	**0.10**	0.34	0.15	0.35	0.16	0.20	0.30	0.33
Even – 3 dice	0.15	0.12	0.18	0.11	0.20	0.12	0.12	**0.10**	0.12	0.13	0.11
Odd – 3 dice	0.14	0.13	0.16	0.12	0.18	0.13	0.12	0.12	**0.11**	**0.11**	0.14
Not first – 3 dice	0.26	0.41	0.11	0.13	0.36	0.42	0.45	**0.09**	0.19	0.12	0.31
Not middle – 3 dice	0.13	0.08	**0.06**	**0.06**	0.08	0.20	0.15	0.07	0.25	0.09	0.11
L1											
Smaller	0.24	0.17	0.34	0.13	0.26	0.34	0.26	0.20	**0.10**	0.20	0.29
Larger	0.39	0.30	0.39	0.32	0.54	0.60	0.72	0.45	**0.25**	0.33	0.32
Even	0.67	0.41	0.54	0.46	0.60	0.65	0.58	0.45	0.61	**0.23**	0.37
Odd	0.52	0.40	0.28	0.24	0.56	0.61	0.35	0.27	0.39	**0.22**	**0.22**
Not first	1.09	1.02	0.81	0.56	0.88	1.05	1.14	0.63	0.62	**0.55**	0.88
Not middle	0.39	0.62	0.42	0.29	0.45	0.59	0.45	0.47	0.42	**0.28**	0.29
Smaller – 2 dice	0.38	0.31	0.28	**0.27**	0.35	0.57	0.41	0.38	0.28	0.29	0.30
Larger – 2 dice	0.61	**0.35**	0.48	0.39	0.95	0.56	0.77	0.54	0.47	0.38	0.45
Even – 2 dice	0.78	**0.35**	0.70	0.52	0.90	0.60	0.68	0.43	0.56	0.49	0.44
Odd – 2 dice	0.67	**0.35**	0.81	0.42	0.81	0.71	0.63	0.36	0.56	0.37	0.43
Not first – 2 dice	0.97	1.11	**0.36**	0.62	1.08	0.82	1.45	0.54	0.72	0.55	0.79
Not middle – 2 dice	0.52	0.50	**0.27**	0.36	0.54	0.53	0.64	0.63	0.72	0.36	0.44
Smaller – 3 dice	0.47	0.31	0.39	0.46	0.43	0.46	0.38	0.59	**0.28**	0.53	0.42
Larger – 3 dice	0.57	0.51	0.41	**0.38**	0.87	0.44	0.92	0.60	0.62	0.72	0.80
Even – 3 dice	0.77	0.67	0.93	0.66	1.10	0.74	0.56	**0.55**	0.74	0.77	0.63
Odd – 3 dice	0.75	0.66	1.06	0.76	1.19	0.95	0.63	**0.62**	0.69	0.70	0.70
Not first – 3 dice	1.19	1.49	0.79	0.92	1.32	1.33	1.37	**0.65**	0.91	0.88	1.21
Not middle – 3 dice	0.62	0.58	**0.40**	0.43	0.72	0.73	0.61	0.47	0.80	0.53	0.60
Symmetric KL											
Smaller	0.42	**0.08**	0.65	0.09	0.68	0.41	0.54	0.50	0.15	0.22	0.33
Larger	1.08	0.30	0.48	**0.27**	1.29	1.19	1.77	0.90	0.50	0.43	**0.27**
Even	1.49	0.65	0.61	0.37	1.11	1.22	0.65	0.94	0.75	0.35	**0.31**
Odd	1.08	0.40	0.31	0.23	1.35	1.19	0.35	0.40	0.72	0.49	**0.16**
Not first	2.09	2.68	1.47	**0.63**	1.63	2.05	3.45	1.07	1.13	0.97	1.42
Not middle	0.59	1.21	0.72	0.23	0.56	0.99	1.27	0.79	0.54	0.93	**0.21**
Smaller – 2 dice	0.41	**0.16**	0.62	0.36	0.37	0.60	0.66	0.59	**0.16**	0.31	0.27
Larger – 2 dice	1.51	0.53	**0.49**	0.60	1.62	0.93	2.09	1.25	0.86	**0.49**	0.70
Even – 2 dice	1.63	**0.36**	2.06	0.86	1.63	1.53	1.00	0.48	0.83	0.86	0.51
Odd – 2 dice	1.67	0.65	2.91	1.23	2.16	3.07	1.11	0.52	0.91	0.96	**0.48**
Not first – 2 dice	2.26	2.55	**0.32**	1.14	2.57	1.77	9.00	0.84	2.51	1.07	1.41
Not middle – 2 dice	0.74	0.90	**0.25**	0.40	0.81	0.73	2.46	1.26	2.72	0.59	0.36
Smaller – 3 dice	0.58	0.23	0.42	0.61	0.52	0.40	0.70	1.10	**0.20**	0.49	0.69
Larger – 3 dice	1.12	0.78	**0.38**	0.55	1.38	0.61	1.95	0.95	0.99	0.97	1.33
Even – 3 dice	1.68	**0.86**	2.88	1.35	2.46	1.91	1.06	1.04	1.17	2.43	1.20
Odd – 3 dice	2.13	**1.06**	4.58	1.77	3.29	3.78	1.30	1.30	1.36	1.79	1.28
Not first – 3 dice	2.88	4.68	1.20	1.81	3.64	3.53	6.62	**1.08**	2.79	1.91	2.97
Not middle – 3 dice	0.89	1.14	**0.38**	0.57	1.26	1.04	1.95	0.62	3.24	0.77	0.82

A.1.3 Observations: Two observations – Single die

Chebyshev	Y6B	M7B	Q7B	L8B	G7B	Y9B	Y34B	Q57xB	L70B	Q72B	M8x22B
Smaller – Even	0.35	0.21	0.23	0.29	0.19	0.31	0.24	0.19	0.31	0.21	**0.17**
Smaller – Odd	0.11	**0.07**	0.13	0.10	0.17	0.16	0.15	0.17	0.11	0.11	**0.07**
Smaller – Not first	0.21	0.25	0.19	0.16	0.22	0.24	0.35	0.24	0.15	0.23	**0.14**
Smaller – Not middle	**0.07**	0.08	0.11	0.09	**0.07**	0.10	0.20	0.16	0.13	0.08	0.13
Larger – Even	0.29	0.24	0.28	0.20	0.37	0.33	0.36	0.22	0.24	0.26	**0.16**
Larger – Odd	0.25	0.17	0.13	0.16	0.26	0.26	0.18	0.19	0.17	**0.09**	**0.09**
Larger – Not first	0.11	0.19	0.14	**0.08**	0.25	0.21	0.35	0.14	0.15	0.13	0.13
Larger – Not middle	**0.12**	0.23	0.19	0.21	0.29	0.22	0.35	**0.12**	0.16	0.13	**0.12**
Even – Not first	0.26	0.42	0.31	0.25	0.37	0.24	0.53	0.14	0.32	**0.07**	0.17
Even – Not middle	0.16	0.18	0.15	0.22	0.19	0.25	0.40	0.14	0.24	**0.09**	0.14
L1											
Smaller – Even	0.55	0.33	0.36	0.41	0.39	0.47	0.46	0.31	0.48	0.33	**0.30**
Smaller – Odd	0.26	0.15	0.28	0.21	0.38	0.32	0.31	0.35	0.22	0.24	**0.14**
Smaller – Not first	0.43	0.59	0.46	**0.33**	0.56	0.55	0.57	0.60	0.35	0.38	**0.33**
Smaller – Not middle	0.19	**0.17**	0.23	0.20	0.23	0.23	0.55	0.46	0.33	0.22	0.30
Larger – Even	0.69	0.50	0.60	0.42	0.85	0.73	0.77	0.52	0.53	0.52	**0.33**
Larger – Odd	0.58	0.37	0.26	0.36	0.60	0.56	0.37	0.39	0.36	0.18	**0.17**
Larger – Not first	0.43	0.68	0.58	**0.25**	0.91	0.75	0.96	0.59	0.68	0.28	0.44
Larger – Not middle	0.46	0.63	0.54	0.51	0.86	0.74	0.97	0.55	0.56	0.37	**0.36**
Even – Not first	0.85	0.94	0.72	0.78	1.05	0.84	1.22	0.42	0.73	**0.20**	0.42
Even – Not middle	0.47	0.49	0.42	0.52	0.58	0.70	0.82	0.48	0.52	**0.22**	0.39
Symmetric KL											
Smaller – Even	2.08	0.34	0.86	0.39	0.99	0.84	0.36	0.42	0.61	**0.17**	**0.17**
Smaller – Odd	0.71	0.21	0.83	0.42	1.81	0.35	0.34	0.45	0.50	0.25	**0.17**
Smaller – Not first	1.04	0.98	1.01	0.22	0.61	1.15	0.46	2.12	0.43	0.19	**0.07**
Smaller – Not middle	0.41	**0.23**	0.51	0.28	0.65	0.93	2.34	2.12	1.34	0.70	0.86
Larger – Even	2.83	0.71	1.21	0.87	3.28	1.79	2.00	1.27	1.18	0.86	**0.41**
Larger – Odd	2.77	0.65	0.69	1.05	2.75	1.27	0.65	1.41	0.90	0.41	**0.22**
Larger – Not first	1.37	3.10	2.76	**0.69**	3.76	3.34	2.51	2.42	3.28	0.82	1.24
Larger – Not middle	1.67	1.04	0.85	**0.57**	2.46	2.33	3.03	2.38	2.64	1.02	0.81
Even – Not first	4.26	1.82	1.06	1.96	2.84	2.70	2.71	1.08	1.18	0.49	**0.48**
Even – Not middle	1.33	0.96	0.75	0.67	1.60	1.83	1.52	1.52	0.75	0.48	**0.43**

A.1.4 Observations: Two observations – Multiple die

Chebyshev	Y6B	M7B	Q7B	L8B	G7B	Y9B	Y34B	Q57xB	L70B	Q72B	M8x22B
Smaller – Even – 2 dice	0.37	0.18	**0.12**	0.32	0.40	0.35	0.29	0.14	0.28	0.15	0.29
Smaller – Odd – 2 dice	0.24	0.08	0.18	0.12	0.35	0.15	0.17	0.12	0.10	**0.07**	0.14
Smaller – Not first – 2 dice	0.23	0.23	0.14	0.26	0.52	0.26	0.19	**0.12**	0.16	0.23	0.23
Smaller – Not middle – 2 dice	0.14	0.09	**0.06**	0.11	0.14	0.10	0.32	0.13	0.14	0.11	0.11
Larger – Even – 2 dice	0.30	0.17	0.33	0.13	0.23	0.23	0.17	0.13	0.20	**0.08**	0.14
Larger – Odd – 2 dice	0.30	**0.10**	0.16	0.17	0.39	0.20	0.27	0.17	0.20	0.14	0.16
Larger – Not first – 2 dice	0.15	**0.07**	0.15	0.12	0.26	0.19	0.15	0.16	0.16	0.11	0.12
Larger – Not middle – 2 dice	0.29	0.20	**0.15**	0.20	0.39	0.29	0.27	0.20	0.21	0.22	0.16
Even – Not first – 2 dice	0.13	0.39	0.12	0.24	0.39	0.15	0.27	**0.11**	0.23	0.25	0.16
Even – Not middle – 2 dice	0.15	0.13	0.12	0.14	**0.11**	0.20	0.12	0.12	0.15	0.16	**0.11**
Smaller – Even – 3 dice	0.26	0.19	0.15	0.28	0.34	0.24	0.19	0.17	0.24	**0.12**	0.23
Smaller – Odd – 3 dice	0.24	0.20	0.19	0.25	0.37	0.21	0.23	0.17	0.21	**0.12**	0.22
Smaller – Not first – 3 dice	0.20	0.19	0.15	0.19	0.38	0.18	**0.14**	0.15	**0.14**	0.20	0.16
Smaller – Not middle – 3 dice	0.22	0.11	0.13	**0.10**	0.15	0.17	0.29	0.18	0.20	0.18	**0.10**
Larger – Even – 3 dice	0.25	0.20	0.21	**0.13**	0.23	0.18	0.17	0.20	0.16	0.20	0.22
Larger – Odd – 3 dice	0.24	0.17	0.16	**0.13**	0.24	0.20	0.19	0.16	0.17	0.20	0.20
Larger – Not first – 3 dice	**0.10**	0.11	0.11	**0.10**	0.14	0.13	0.17	0.14	0.17	0.14	0.14
Larger – Not middle – 3 dice	0.26	0.24	0.21	0.21	0.35	0.34	0.22	**0.16**	0.29	0.36	0.29
Even – Not first – 3 dice	**0.13**	0.35	0.15	0.20	0.39	0.25	0.19	0.14	0.15	0.20	0.28
Even – Not middle – 3 dice	0.14	0.12	**0.09**	0.12	0.13	0.15	0.13	0.12	0.13	0.12	0.13
L1											
Smaller – Even – 2 dice	0.92	0.40	0.54	0.64	0.92	0.80	0.66	0.37	0.58	**0.31**	0.67
Smaller – Odd – 2 dice	0.60	**0.21**	0.76	0.28	0.78	0.37	0.36	0.34	0.22	**0.21**	0.32
Smaller – Not first – 2 dice	0.72	0.59	**0.40**	0.69	1.14	0.77	0.60	0.52	0.50	0.62	0.61
Smaller – Not middle – 2 dice	0.42	0.27	**0.25**	0.36	0.47	0.29	0.73	0.52	0.45	0.37	0.55
Larger – Even – 2 dice	0.82	0.45	0.78	0.38	0.76	0.60	0.45	0.47	0.54	**0.29**	0.45
Larger – Odd – 2 dice	0.94	**0.30**	0.49	0.55	1.07	0.70	0.64	0.55	0.61	0.42	0.49
Larger – Not first – 2 dice	0.66	**0.53**	0.68	0.76	1.02	0.76	0.79	0.77	1.02	0.60	0.66
Larger – Not middle – 2 dice	0.77	0.56	**0.51**	0.53	1.01	0.78	0.96	0.69	0.73	0.55	0.53
Even – Not first – 2 dice	0.61	1.03	0.67	0.83	1.17	0.84	0.91	**0.51**	0.74	0.81	0.68
Even – Not middle – 2 dice	0.72	0.55	0.61	0.61	0.55	0.78	0.51	**0.47**	0.66	0.67	0.50
Smaller – Even – 3 dice	0.81	0.43	0.66	0.59	0.86	0.66	0.50	0.53	0.60	**0.31**	0.58
Smaller – Odd – 3 dice	0.74	0.48	0.87	0.64	1.02	0.59	0.55	0.52	0.53	**0.31**	0.52
Smaller – Not first – 3 dice	0.75	0.71	0.67	0.72	1.13	0.72	**0.45**	0.87	0.63	0.78	0.68
Smaller – Not middle – 3 dice	0.66	**0.30**	0.45	0.34	0.50	0.44	0.81	0.73	0.62	0.60	0.51
Larger – Even – 3 dice	0.82	0.59	0.58	**0.47**	0.82	0.56	0.49	0.64	0.51	0.60	0.71
Larger – Odd – 3 dice	0.91	0.52	**0.49**	0.52	0.91	0.70	0.53	0.55	**0.49**	0.62	0.69
Larger – Not first – 3 dice	**0.53**	0.72	0.81	0.72	0.95	0.67	0.70	0.77	1.01	0.89	0.78
Larger – Not middle – 3 dice	0.79	0.70	0.74	**0.59**	1.00	0.89	0.87	0.64	0.84	0.90	0.78
Even – Not first – 3 dice	0.85	1.39	1.07	1.08	1.53	1.32	1.16	**0.78**	0.91	1.19	1.17
Even – Not middle – 3 dice	0.82	0.65	0.63	0.65	0.81	0.77	0.71	**0.60**	0.73	0.77	0.70
Symmetric KL											
Smaller – Even – 2 dice	4.36	0.63	2.29	1.10	2.21	2.94	1.27	0.74	0.54	**0.46**	0.79
Smaller – Odd – 2 dice	2.81	0.47	3.54	0.73	2.92	1.20	0.52	1.36	**0.32**	0.53	0.44
Smaller – Not first – 2 dice	1.12	0.80	**0.47**	1.04	2.27	1.39	1.10	1.02	0.81	0.74	0.78
Smaller – Not middle – 2 dice	0.76	**0.27**	0.74	0.63	0.97	0.46	3.82	1.54	1.05	0.77	1.86
Larger – Even – 2 dice	3.19	0.83	1.91	1.16	2.72	1.55	0.95	1.61	1.50	**0.77**	1.46
Larger – Odd – 2 dice	4.14	**0.92**	2.04	2.10	5.88	2.97	1.27	2.33	1.86	1.50	1.94
Larger – Not first – 2 dice	**2.13**	2.19	2.84	3.29	4.49	2.36	2.89	2.27	5.03	2.21	2.77
Larger – Not middle – 2 dice	2.20	0.90	1.25	**0.75**	2.17	1.71	3.70	1.62	2.16	0.81	1.26
Even – Not first – 2 dice	1.89	1.93	2.54	2.27	3.31	3.32	2.68	**1.23**	3.01	2.62	1.56
Even – Not middle – 2 dice	2.46	1.34	2.00	0.97	1.59	2.69	1.26	**0.95**	1.30	1.32	1.66
Smaller – Even – 3 dice	3.52	0.65	2.70	1.30	2.42	2.50	1.20	1.70	0.60	**0.42**	0.82
Smaller – Odd – 3 dice	2.57	0.74	3.71	1.17	3.11	1.93	0.62	1.38	0.61	**0.42**	0.78
Smaller – Not first – 3 dice	1.55	1.08	1.06	1.27	2.51	1.34	**0.76**	2.24	1.18	1.36	1.12
Smaller – Not middle – 3 dice	1.26	**0.31**	0.84	0.67	0.93	0.56	3.70	1.87	1.24	1.29	1.28
Larger – Even – 3 dice	3.46	1.17	1.51	1.41	3.04	1.58	**0.92**	1.37	1.04	1.37	2.32
Larger – Odd – 3 dice	4.15	1.17	1.69	1.89	3.81	2.78	**0.92**	1.84	0.95	1.60	2.09
Larger – Not first – 3 dice	**1.29**	2.97	3.49	2.95	4.00	2.02	1.89	2.31	4.67	3.00	2.97
Larger – Not middle – 3 dice	1.68	1.14	1.62	**0.82**	2.27	1.72	2.62	1.31	2.01	1.45	1.50
Even – Not first – 3 dice	2.74	3.80	3.63	2.79	5.62	4.44	4.05	**1.63**	2.21	3.80	3.07
Even – Not middle – 3 dice	3.05	1.65	2.37	1.23	2.48	2.30	2.68	**1.22**	1.53	2.54	2.12

A.2 Instruction fine-tuned models

A.2.1 Regular, independent, dependent

Chebyshev	Y6B	M7B	Q7B	L8B	G7B	Y9B	Y34B	Q57xB	L70B	Q72B	M8x22B
Regular – 1 die	0.13	0.15	0.24	0.18	0.33	0.11	0.09	0.12	0.10	0.08	**0.06**
Regular – 2 dice	0.16	0.12	0.11	0.30	0.31	0.15	0.16	0.25	0.26	0.10	**0.06**
Regular – 3 dice	0.20	0.18	0.16	0.21	0.24	**0.08**	0.27	0.12	0.27	0.13	**0.08**
Independent 1 die	0.28	0.17	0.68	0.22	0.41	0.32	0.46	0.36	0.29	0.16	**0.13**
Independent – 2 dice	0.21	**0.12**	0.73	0.26	0.34	0.17	0.43	0.21	0.24	0.25	0.19
Independent – 3 dice	0.40	**0.13**	0.79	0.38	0.37	0.26	0.42	0.17	0.25	0.21	0.19
Dependant 1 die	0.17	0.11	0.14	0.22	0.35	0.14	0.20	0.17	0.20	0.12	**0.07**
Dependant – 2 dice	0.28	**0.21**	0.30	0.68	0.53	0.39	0.31	0.35	0.22	0.22	0.25
Dependant – 3 dice	0.28	**0.20**	0.29	0.84	0.65	0.45	0.27	0.33	0.29	0.32	0.31
L1											
Regular – 1 die	0.44	0.52	0.53	0.56	1.01	0.45	0.35	0.39	0.42	0.25	**0.22**
Regular – 2 dice	0.64	0.57	0.47	0.82	0.99	0.52	0.70	0.78	0.91	0.60	**0.29**
Regular – 3 dice	0.85	0.77	0.85	0.75	1.08	**0.43**	1.03	0.59	0.86	0.86	0.47
Independent 1 die	0.68	0.49	1.37	0.71	1.06	0.72	1.00	0.83	0.69	0.45	**0.41**
Independent – 2 dice	0.86	**0.61**	1.51	0.94	1.19	0.62	1.17	0.72	0.89	0.80	0.73
Independent – 3 dice	1.17	**0.78**	1.61	1.22	1.24	0.92	1.27	0.83	1.00	0.89	0.92
Dependant 1 die	0.56	0.41	0.50	0.71	1.00	0.50	0.59	0.56	0.62	0.42	**0.24**
Dependant – 2 dice	1.11	0.99	1.02	1.45	1.58	1.12	1.11	1.18	1.00	0.99	**0.94**
Dependant – 3 dice	1.33	1.07	1.22	1.70	1.77	1.29	**1.00**	1.40	1.19	1.13	1.11
Symmetric KL											
Regular – 1 die	0.35	0.69	0.50	0.66	2.78	0.40	0.18	0.28	0.29	0.12	**0.10**
Regular – 2 dice	0.71	0.54	0.40	1.08	2.27	0.49	0.82	0.99	1.97	0.64	**0.14**
Regular – 3 dice	1.63	1.26	1.40	0.94	3.07	**0.43**	1.85	0.62	1.23	1.45	0.45
Independent 1 die	0.95	0.50	2.91	0.90	2.61	0.95	1.50	1.32	0.86	0.38	**0.28**
Independent – 2 dice	1.54	**0.66**	4.13	1.66	3.01	0.72	2.55	1.08	1.68	1.17	0.95
Independent – 3 dice	3.36	**1.26**	6.26	3.15	4.35	2.06	3.67	1.43	2.14	1.78	1.78
Dependant 1 die	0.58	0.27	0.45	0.98	2.26	0.53	0.65	0.51	0.76	0.32	**0.10**
Dependant – 2 dice	2.37	1.72	1.82	4.21	6.47	2.50	2.27	2.98	2.03	1.76	**1.71**
Dependant – 3 dice	3.78	**2.17**	2.92	6.85	9.48	4.12	2.50	4.18	3.16	3.01	2.95

A.2.2 Observations: One observation

Chebyshev	Y6B	M7B	Q7B	L8B	G7B	Y9B	Y34B	Q57xB	L70B	Q72B	M8x22B
Smaller	0.27	**0.08**	0.19	0.15	0.40	0.11	0.16	0.11	0.09	0.15	0.17
Larger	0.29	0.27	0.23	0.32	0.63	**0.17**	0.34	0.18	**0.17**	**0.17**	**0.17**
Even	0.29	0.26	0.24	0.66	0.70	0.29	0.27	0.24	0.26	**0.15**	0.26
Odd	0.26	0.34	0.18	0.41	0.59	0.12	0.23	0.23	0.19	**0.07**	0.14
Not first	0.19	0.40	0.36	**0.16**	0.29	0.28	0.59	0.38	0.27	0.22	0.43
Not middle	0.27	0.22	0.19	0.14	0.45	0.15	0.16	0.23	0.14	0.13	**0.12**
Smaller – 2 dice	0.25	0.20	0.17	0.44	0.31	0.29	0.13	0.24	0.19	0.16	**0.09**
Larger – 2 dice	0.17	0.14	0.25	0.30	0.33	0.12	0.37	0.30	0.31	0.17	**0.07**
Even – 2 dice	0.13	0.17	0.30	0.33	0.34	0.14	0.15	0.26	0.22	0.21	**0.08**
Odd – 2 dice	0.22	0.16	0.21	0.16	0.53	0.21	0.13	0.15	0.26	0.09	**0.07**
Not first – 2 dice	0.34	0.33	**0.08**	0.15	0.31	0.21	0.14	0.26	0.26	0.16	0.17
Not middle – 2 dice	0.16	0.15	**0.07**	0.16	0.21	0.17	0.15	0.27	0.17	0.11	0.13
Smaller – 3 dice	0.22	0.15	0.26	0.21	0.40	0.20	**0.09**	0.30	0.20	0.29	0.12
Larger – 3 dice	**0.14**	0.16	0.22	0.16	0.30	0.18	0.32	0.27	0.22	0.29	0.16
Even – 3 dice	0.13	0.17	0.32	0.18	0.59	0.13	0.17	0.13	0.17	0.15	**0.09**
Odd – 3 dice	0.23	0.10	0.31	0.17	0.69	0.16	0.12	0.18	**0.09**	0.14	0.10
Not first – 3 dice	0.35	0.40	0.16	0.11	0.44	0.37	**0.09**	0.15	0.23	0.16	0.28
Not middle – 3 dice	0.18	0.14	**0.10**	0.12	0.19	0.23	0.14	0.13	0.16	0.12	0.15
L1											
Smaller	0.60	0.23	0.42	0.36	0.82	0.28	0.37	**0.22**	0.24	0.34	0.38
Larger	0.79	0.61	0.52	0.79	1.28	0.51	0.83	0.55	0.45	**0.43**	0.45
Even	0.68	0.57	0.59	1.31	1.40	0.66	0.64	0.63	0.67	**0.34**	0.62
Odd	0.72	0.73	0.42	1.00	1.22	0.35	0.62	0.58	0.49	**0.24**	0.39
Not first	**0.55**	1.22	0.91	0.62	0.85	0.97	1.28	0.80	0.72	0.59	0.93
Not middle	0.76	0.66	0.50	0.55	1.09	0.49	0.56	0.63	0.53	0.40	**0.32**
Smaller – 2 dice	0.78	0.56	0.48	0.98	0.79	0.77	0.38	0.61	0.50	0.42	**0.24**
Larger – 2 dice	0.57	0.39	0.63	0.75	0.88	0.60	0.94	0.77	0.79	0.40	**0.18**
Even – 2 dice	0.49	0.56	0.97	0.98	1.26	0.51	0.62	0.69	0.75	0.55	**0.32**
Odd – 2 dice	0.90	0.45	0.95	0.57	1.17	1.04	0.62	0.44	0.72	0.42	**0.30**
Not first – 2 dice	1.21	1.03	**0.50**	0.73	1.29	0.88	0.71	0.76	0.92	0.65	0.59
Not middle – 2 dice	0.81	0.56	**0.33**	0.53	0.79	0.66	0.72	0.83	0.74	0.45	0.45
Smaller – 3 dice	0.72	0.41	0.63	0.62	1.04	0.50	**0.30**	0.84	0.51	0.59	0.33
Larger – 3 dice	0.50	0.49	0.66	0.50	1.00	0.65	0.87	0.77	0.80	0.79	**0.43**
Even – 3 dice	0.56	0.52	1.20	0.54	1.59	0.64	0.90	0.53	0.58	0.68	**0.50**
Odd – 3 dice	1.03	0.47	1.33	0.69	1.57	1.10	0.69	0.72	**0.43**	0.74	0.53
Not first – 3 dice	1.38	1.32	0.88	0.74	1.62	1.46	**0.73**	0.87	0.94	0.90	1.10
Not middle – 3 dice	0.82	0.59	0.61	**0.51**	0.92	0.76	0.64	0.63	0.78	0.66	0.57
Symmetric KL											
Smaller	0.88	0.15	0.48	0.22	1.74	0.60	1.12	0.19	**0.11**	0.29	0.44
Larger	1.38	0.56	0.66	1.45	4.18	1.53	2.24	0.93	0.43	0.47	**0.40**
Even	1.36	0.76	0.80	4.23	6.07	0.88	0.89	0.98	1.03	**0.31**	0.86
Odd	1.46	0.77	0.47	2.49	3.47	0.59	1.27	0.49	0.45	**0.43**	1.08
Not first	**0.76**	4.01	1.87	0.87	2.62	3.05	2.49	1.14	1.43	0.98	1.39
Not middle	1.43	1.03	0.89	0.50	3.16	1.71	1.25	0.70	1.18	0.96	**0.22**
Smaller – 2 dice	1.16	0.47	0.52	2.23	1.58	1.17	0.31	1.01	0.37	0.33	**0.19**
Larger – 2 dice	0.68	0.31	0.76	0.97	1.48	1.93	2.12	1.35	1.52	0.43	**0.14**
Even – 2 dice	0.87	0.58	2.35	1.63	3.41	0.58	1.54	0.84	1.50	0.70	**0.51**
Odd – 2 dice	1.99	**0.39**	3.89	0.64	3.94	5.03	1.82	0.46	1.30	0.83	0.40
Not first – 2 dice	2.87	1.79	**0.62**	1.12	3.22	2.75	1.62	1.19	2.69	1.11	0.87
Not middle – 2 dice	1.65	0.62	**0.29**	0.57	1.24	0.83	1.22	1.47	1.75	0.81	0.38
Smaller – 3 dice	1.13	0.33	0.58	0.75	2.59	0.53	**0.20**	1.73	0.39	0.58	0.27
Larger – 3 dice	0.64	0.53	0.76	0.49	1.99	0.91	1.65	1.12	1.58	1.04	**0.37**
Even – 3 dice	1.03	**0.49**	3.59	0.63	6.54	1.20	2.62	0.88	0.63	1.46	0.93
Odd – 3 dice	2.35	**0.45**	7.22	0.95	7.75	4.25	2.25	1.20	0.77	1.28	1.00
Not first – 3 dice	3.84	3.41	1.79	**1.09**	5.95	5.35	1.25	1.55	1.96	1.83	2.35
Not middle – 3 dice	1.55	0.82	0.76	0.55	1.93	0.91	1.21	0.77	1.49	1.00	**0.54**

A.2.3 Observations: Two observations – Single die

Chebyshev	Y6B	M7B	Q7B	L8B	G7B	Y9B	Y34B	Q57xB	L70B	Q72B	M8x22B
Smaller – Even	0.29	0.26	0.27	0.33	0.40	0.22	0.29	**0.19**	**0.19**	0.26	**0.19**
Smaller – Odd	0.18	0.15	0.13	0.15	0.29	0.19	0.19	0.17	**0.09**	0.17	0.16
Smaller – Not first	0.27	0.25	**0.17**	0.22	0.50	0.24	0.27	0.25	0.20	0.32	0.23
Smaller – Not middle	0.13	0.20	**0.09**	0.12	0.29	0.10	0.17	**0.09**	**0.09**	**0.09**	0.18
Larger – Even	0.28	0.33	0.34	0.36	0.41	0.28	0.47	0.32	**0.24**	0.29	0.29
Larger – Odd	0.25	0.27	0.16	0.23	0.23	0.21	0.25	0.20	**0.09**	0.12	0.23
Larger – Not first	0.25	0.23	**0.13**	0.40	0.47	0.15	0.35	0.24	0.15	0.20	0.17
Larger – Not middle	0.15	0.28	0.27	0.33	0.50	0.21	0.30	**0.14**	**0.14**	0.15	0.21
Even – Not first	0.21	0.44	0.50	0.30	0.49	0.20	0.23	0.28	0.18	**0.13**	0.25
Even – Not middle	0.26	**0.17**	0.21	0.33	0.49	0.18	0.43	0.19	0.21	**0.17**	0.25
L1											
Smaller – Even	0.39	0.35	0.41	0.46	0.60	0.28	0.47	**0.24**	0.31	0.41	0.33
Smaller – Odd	0.36	0.31	0.28	0.30	0.58	0.39	0.39	0.34	**0.17**	0.35	0.34
Smaller – Not first	0.45	0.46	0.42	0.43	0.81	0.48	0.49	0.45	**0.39**	0.48	0.40
Smaller – Not middle	0.32	0.43	**0.21**	0.31	0.67	0.33	0.43	0.25	**0.21**	0.24	0.42
Larger – Even	0.60	0.65	0.70	0.73	0.82	0.66	0.97	0.69	**0.48**	0.58	0.69
Larger – Odd	0.57	0.55	0.32	0.47	0.46	0.50	0.51	0.40	**0.19**	0.25	0.49
Larger – Not first	0.58	0.60	0.58	1.00	1.12	0.55	0.95	0.69	0.45	**0.43**	0.52
Larger – Not middle	0.44	0.70	0.68	0.74	1.11	0.65	0.86	0.50	0.38	**0.35**	0.57
Even – Not first	0.53	0.95	1.05	0.72	1.15	0.59	0.75	0.62	0.45	**0.29**	0.68
Even – Not middle	0.59	0.48	0.53	0.71	0.98	0.43	0.90	0.47	0.46	**0.36**	0.60
Symmetric KL											
Smaller – Even	0.42	0.12	0.33	0.28	2.44	0.22	0.56	0.09	**0.06**	0.18	0.50
Smaller – Odd	0.33	0.24	0.37	0.24	2.27	0.48	1.57	0.30	**0.09**	0.29	1.06
Smaller – Not first	0.37	0.23	0.66	0.27	1.97	0.67	0.40	0.74	**0.13**	0.30	0.24
Smaller – Not middle	0.31	0.32	0.26	**0.19**	1.32	0.97	1.45	1.02	0.20	0.21	1.00
Larger – Even	1.09	1.19	1.30	1.45	3.52	1.61	2.87	1.15	**0.44**	1.02	1.51
Larger – Odd	2.04	0.87	0.63	0.55	1.47	1.63	1.05	0.72	**0.08**	0.39	1.42
Larger – Not first	0.69	0.71	2.45	2.33	4.37	2.05	3.24	1.76	1.09	**0.53**	1.36
Larger – Not middle	1.00	0.83	0.98	1.10	4.07	1.43	2.57	1.47	0.74	**0.42**	0.94
Even – Not first	0.75	1.88	1.77	1.64	4.63	1.61	2.14	0.86	0.36	**0.24**	1.28
Even – Not middle	0.96	0.66	0.81	1.25	3.53	0.64	1.79	0.83	**0.43**	0.44	0.99

A.2.4 Observations: Two observations – Multiple dice

Chebyshev	Y6B	M7B	Q7B	L8B	G7B	Y9B	Y34B	Q57xB	L70B	Q72B	M8x22B
Smaller – Even – 2 dice	0.46	0.33	**0.19**	0.43	0.66	0.44	0.37	0.29	0.32	0.28	0.23
Smaller – Odd – 2 dice	0.14	0.13	0.13	0.15	0.41	0.19	0.17	0.23	**0.08**	0.11	**0.08**
Smaller – Not first – 2 dice	0.28	0.27	0.18	0.27	0.65	0.53	0.19	0.24	0.18	0.27	**0.17**
Smaller – Not middle – 2 dice	0.22	0.18	0.15	0.27	0.33	0.28	0.13	0.23	0.13	0.16	**0.12**
Larger – Even – 2 dice	0.40	**0.16**	0.34	0.29	0.63	0.38	0.30	0.28	0.41	0.25	0.21
Larger – Odd – 2 dice	0.21	**0.12**	0.18	0.31	0.29	0.26	0.28	0.27	0.34	0.24	**0.12**
Larger – Not first – 2 dice	0.17	0.10	0.17	0.21	0.33	0.17	0.16	0.36	0.30	0.14	**0.07**
Larger – Not middle – 2 dice	0.25	**0.14**	0.22	0.31	0.39	0.29	0.25	0.37	0.33	0.24	**0.14**
Even – Not first – 2 dice	0.17	0.36	0.17	0.57	0.43	0.24	0.45	0.13	0.19	0.37	**0.12**
Even – Not middle – 2 dice	0.22	0.15	0.22	0.23	0.31	0.14	0.21	0.20	0.18	0.24	**0.12**
Smaller – Even – 3 dice	0.39	0.34	0.20	0.34	0.59	0.23	0.29	**0.16**	0.30	0.22	0.21
Smaller – Odd – 3 dice	0.26	0.23	0.19	0.31	0.59	0.23	**0.13**	0.19	0.23	0.19	0.19
Smaller – Not first – 3 dice	0.27	0.20	0.14	0.18	0.47	0.33	0.14	0.18	0.17	0.20	**0.13**
Smaller – Not middle – 3 dice	0.26	**0.13**	0.25	0.19	0.29	0.40	0.24	0.34	0.23	0.27	0.21
Larger – Even – 3 dice	0.32	0.24	0.21	0.22	0.50	0.24	0.23	0.33	0.33	0.31	**0.17**
Larger – Odd – 3 dice	0.28	0.26	**0.14**	0.20	0.49	0.26	0.36	0.32	0.30	0.31	0.15
Larger – Not first – 3 dice	0.15	0.12	0.12	0.14	0.30	0.13	0.21	0.19	0.21	0.17	**0.11**
Larger – Not middle – 3 dice	0.22	**0.17**	0.20	0.18	0.41	0.45	0.26	0.24	0.27	0.40	0.29
Even – Not first – 3 dice	**0.12**	0.43	0.13	0.21	0.39	0.39	0.20	0.16	0.14	0.24	0.23
Even – Not middle – 3 dice	0.16	0.15	0.21	0.21	0.40	0.14	0.15	0.14	0.15	0.17	**0.13**
L1											
Smaller – Even – 2 dice	1.25	0.71	0.60	0.88	1.33	0.91	0.97	0.62	0.68	0.59	**0.46**
Smaller – Odd – 2 dice	0.32	0.26	0.48	0.32	0.88	0.46	0.39	0.50	0.20	0.23	**0.17**
Smaller – Not first – 2 dice	0.80	0.63	0.56	0.67	1.40	1.13	0.71	0.67	0.55	0.75	**0.45**
Smaller – Not middle – 2 dice	0.53	0.46	0.42	0.66	0.88	0.77	0.44	0.70	**0.37**	0.41	0.47
Larger – Even – 2 dice	1.10	**0.43**	0.80	0.65	1.31	0.82	0.74	0.64	0.87	0.58	0.51
Larger – Odd – 2 dice	0.63	**0.30**	0.48	0.70	0.64	0.63	0.66	0.66	0.68	0.61	**0.30**
Larger – Not first – 2 dice	0.52	**0.28**	0.59	0.63	0.75	0.91	0.81	0.95	0.93	0.47	0.32
Larger – Not middle – 2 dice	0.62	0.42	0.54	0.78	0.89	0.90	0.83	0.84	0.82	0.57	**0.33**
Even – Not first – 2 dice	0.58	0.92	0.74	1.31	1.25	0.69	1.09	**0.43**	0.65	1.06	0.45
Even – Not middle – 2 dice	0.80	0.47	0.79	0.66	0.86	0.51	0.77	0.58	0.71	0.74	**0.43**
Smaller – Even – 3 dice	0.99	0.71	0.87	0.72	1.22	0.54	0.68	**0.37**	0.68	0.47	0.44
Smaller – Odd – 3 dice	0.64	0.52	0.85	0.64	1.41	0.54	**0.40**	0.53	0.50	0.45	0.41
Smaller – Not first – 3 dice	0.87	0.66	0.57	**0.51**	1.40	0.99	0.58	0.73	0.58	0.68	0.54
Smaller – Not middle – 3 dice	0.59	**0.36**	0.63	0.56	0.75	1.05	0.64	0.90	0.50	0.65	0.62
Larger – Even – 3 dice	0.86	0.59	0.61	0.62	1.12	0.66	0.58	0.73	0.76	0.72	**0.50**
Larger – Odd – 3 dice	0.90	0.61	0.52	0.62	1.02	0.64	0.79	0.78	0.67	0.73	**0.43**
Larger – Not first – 3 dice	**0.45**	0.46	0.66	0.51	0.83	0.89	0.82	0.73	0.85	0.89	0.54
Larger – Not middle – 3 dice	0.69	0.55	0.69	**0.53**	0.97	1.18	0.78	0.72	0.85	0.97	0.65
Even – Not first – 3 dice	**0.69**	1.25	0.82	0.86	1.68	1.33	0.92	0.76	0.71	1.06	1.04
Even – Not middle – 3 dice	0.80	0.54	0.83	0.58	1.12	0.52	0.75	0.53	0.56	0.69	**0.49**
Symmetric KL											
Smaller – Even – 2 dice	5.42	0.82	1.76	1.45	6.27	1.96	1.72	0.73	0.78	**0.61**	0.64
Smaller – Odd – 2 dice	0.27	0.15	1.91	0.31	2.27	0.61	0.59	0.78	**0.11**	0.32	0.36
Smaller – Not first – 2 dice	1.14	0.82	0.50	0.98	5.18	2.20	2.10	0.96	0.41	0.99	**0.38**
Smaller – Not middle – 2 dice	0.44	0.40	0.82	0.72	1.23	1.68	0.81	1.55	**0.18**	0.47	1.37
Larger – Even – 2 dice	5.04	**0.42**	1.36	0.95	4.15	2.04	1.75	1.02	1.62	0.73	1.02
Larger – Odd – 2 dice	2.26	**0.26**	1.11	1.46	1.56	2.51	1.81	1.42	1.68	1.42	1.17
Larger – Not first – 2 dice	0.68	**0.22**	1.56	0.84	1.53	4.45	3.31	1.60	2.66	1.14	1.09
Larger – Not middle – 2 dice	1.16	**0.32**	0.88	0.95	2.11	2.39	2.17	1.35	1.99	0.70	0.36
Even – Not first – 2 dice	0.91	1.23	1.76	3.25	5.17	1.72	5.94	**0.84**	1.13	2.62	0.97
Even – Not middle – 2 dice	2.18	**0.46**	2.46	0.76	1.64	0.71	1.43	0.74	1.18	1.06	0.86
Smaller – Even – 3 dice	4.64	0.73	4.18	1.27	4.91	0.85	1.27	1.05	0.90	**0.38**	0.55
Smaller – Odd – 3 dice	1.12	0.54	4.06	0.77	5.11	0.64	0.60	0.76	0.40	**0.34**	0.44
Smaller – Not first – 3 dice	1.61	0.72	0.79	0.59	4.81	1.95	1.29	1.56	**0.58**	0.98	0.79
Smaller – Not middle – 3 dice	0.62	**0.24**	0.80	0.56	1.10	2.43	0.95	1.76	0.48	0.87	1.06
Larger – Even – 3 dice	4.05	**0.73**	1.70	0.82	2.83	1.76	1.24	0.88	1.56	1.07	1.33
Larger – Odd – 3 dice	4.42	**1.03**	1.55	1.26	2.37	1.97	1.58	1.45	1.26	1.08	1.18
Larger – Not first – 3 dice	**0.51**	0.83	1.83	0.58	1.42	4.00	2.11	1.54	2.09	2.20	1.51
Larger – Not middle – 3 dice	0.86	**0.53**	1.55	0.66	2.23	2.87	1.25	1.07	1.80	1.54	0.79
Even – Not first – 3 dice	1.77	2.80	2.75	1.52	8.77	3.57	2.66	1.32	**1.18**	2.62	2.66
Even – Not middle – 3 dice	2.53	0.61	1.84	**0.58**	2.72	1.17	1.96	0.70	0.71	1.71	1.01

A.3 `Stated Answer`

A.3.1 Regular, independent, dependent

Chebyshev	Y6B	M7B	Q7B	L8B	G7B	Y9B	Y34B	Q57xB	L70B	Q72B	M8x22B
Regular – 1 die	0.40	0.68	0.56	0.67	0.91	0.90	0.32	0.22	**0.00**	**0.00**	0.14
Regular – 2 dice	0.67	0.76	0.57	0.94	0.88	0.97	0.80	0.86	1.00	**0.51**	0.93
Regular – 3 dice	0.71	0.98	0.76	**0.59**	0.60	0.60	0.90	0.95	1.00	0.70	0.69
Independent 1 die	0.54	0.59	0.66	0.75	0.79	0.32	0.23	0.35	0.20	**0.13**	0.18
Independent – 2 dice	0.64	**0.55**	0.96	0.77	0.95	0.67	0.60	0.80	0.75	0.90	0.74
Independent – 3 dice	0.70	0.85	0.80	**0.62**	0.78	0.89	0.76	0.89	0.80	1.00	0.65
Dependant 1 die	0.80	0.76	0.71	0.78	0.77	0.89	0.77	0.76	**0.65**	0.83	0.70
Dependant – 2 dice	0.89	**0.66**	0.83	0.79	0.84	0.88	0.76	0.91	0.86	0.86	0.79
Dependant – 3 dice	0.73	0.84	0.74	**0.65**	0.66	0.88	0.69	0.76	0.85	0.98	0.78

A.3.2 Observations: One observation

Chebyshev	Y6B	M7B	Q7B	L8B	G7B	Y9B	Y34B	Q57xB	L70B	Q72B	M8x22B
Smaller	0.99	1.00	0.78	0.79	1.00	0.99	0.81	0.85	**0.29**	0.75	0.81
Larger	0.67	0.96	0.88	0.27	0.79	1.00	0.69	0.98	0.32	**0.07**	0.49
Even	0.84	0.64	0.91	0.70	0.71	1.00	0.60	0.97	**0.33**	0.91	0.45
Odd	0.94	0.67	0.84	0.48	0.72	0.96	0.44	1.00	**0.39**	0.67	0.68
Not first	0.02	1.00	0.76	0.99	0.02	**0.00**	**0.00**	0.01	**0.00**	0.04	0.01
Not middle	1.00	1.00	1.00	0.45	0.14	1.00	0.59	0.95	0.08	**0.00**	0.22
Smaller – 2 dice	0.99	0.84	**0.67**	0.91	1.00	1.00	0.97	0.77	0.94	1.00	0.94
Larger – 2 dice	0.91	1.00	0.92	0.85	0.71	0.93	**0.27**	0.99	0.98	1.00	0.97
Even – 2 dice	0.40	0.97	0.40	0.63	1.00	**0.36**	0.76	0.57	0.96	0.97	0.92
Odd – 2 dice	0.91	1.00	0.79	0.58	0.56	0.73	0.83	0.97	**0.54**	1.00	0.96
Not middle – 2 dice	0.37	0.81	0.92	0.83	1.00	0.69	0.43	0.74	0.68	0.70	**0.13**
Smaller – 3 dice	1.00	1.00	0.92	**0.67**	1.00	0.99	0.98	0.80	0.93	1.00	0.94
Larger – 3 dice	0.89	1.00	**0.52**	0.59	0.77	1.00	0.82	0.81	0.78	0.81	0.76
Even – 3 dice	0.90	0.81	**0.67**	0.71	0.91	0.97	0.70	0.73	0.83	0.98	0.69
Odd – 3 dice	0.65	0.93	**0.49**	0.82	0.97	0.99	0.73	0.91	0.91	0.77	0.90

A.3.3 Observations: Two observations – Single die

Chebyshev	Y6B	M7B	Q7B	L8B	G7B	Y9B	Y34B	Q57xB	L70B	Q72B	M8x22B
Smaller – Even	0.76	1.00	0.99	0.78	1.00	1.00	0.83	0.93	**0.25**	0.76	0.98
Smaller – Odd	0.75	1.00	0.98	0.78	1.00	0.98	0.67	0.94	**0.00**	1.00	0.50
Smaller – Not first	0.95	1.00	0.69	0.81	1.00	0.86	0.56	0.97	**0.34**	1.00	0.76
Smaller – Not middle	1.00	1.00	0.96	0.74	1.00	0.78	0.61	0.99	**0.39**	0.51	0.88
Larger – Even	1.00	0.66	1.00	0.39	1.00	0.99	0.66	0.96	**0.33**	1.00	0.69
Larger – Odd	0.69	1.00	1.00	0.81	0.60	1.00	0.72	0.88	**0.25**	1.00	0.53
Larger – Not first	**0.26**	0.93	0.66	0.35	0.67	0.88	0.32	0.54	0.34	0.45	0.59
Larger – Not middle	0.99	**0.13**	0.67	0.54	0.58	0.92	0.83	0.64	0.34	0.44	0.54
Even – Not first	0.89	0.58	0.37	0.62	0.96	0.94	**0.24**	0.88	0.55	0.59	0.60
Even – Not middle	0.96	1.00	0.74	0.75	1.00	0.69	0.43	0.97	**0.28**	0.71	0.74

A.3.4 Observations: Two observations – Multiple dice

Chebyshev	Y6B	M7B	Q7B	L8B	G7B	Y9B	Y34B	Q57xB	L70B	Q72B	M8x22B
Smaller – Even – 2 dice	0.88	1.00	0.92	0.92	1.00	1.00	1.00	0.87	**0.86**	1.00	0.94
Smaller – Odd – 2 dice	1.00	1.00	0.98	0.64	0.51	0.68	0.73	1.00	**0.18**	1.00	0.96
Smaller – Not first – 2 dice	1.00	1.00	0.92	0.87	**0.28**	1.00	0.99	0.96	1.00	1.00	0.83
Smaller – Not middle – 2 dice	0.97	0.50	0.95	0.58	0.99	0.57	0.82	0.54	**0.13**	0.99	0.93
Larger – Even – 2 dice	0.85	**0.44**	0.98	0.81	0.50	1.00	1.00	0.90	1.00	0.58	0.96
Larger – Odd – 2 dice	0.91	1.00	0.99	0.81	**0.70**	0.82	1.00	0.99	0.99	1.00	0.94
Larger – Not first – 2 dice	0.95	1.00	0.95	0.72	**0.57**	1.00	0.71	0.96	0.99	1.00	0.91
Larger – Not middle – 2 dice	0.96	0.93	0.92	0.76	0.77	0.92	1.00	0.87	**0.58**	0.68	0.91
Even – Not middle – 2 dice	0.48	0.99	**0.34**	0.55	0.60	0.98	0.73	0.80	0.75	1.00	0.63
Smaller – Not middle – 3 dice	1.00	1.00	0.98	**0.81**	1.00	0.99	1.00	0.97	0.99	1.00	0.99
Larger – Not first – 3 dice	0.86	0.75	0.72	**0.56**	0.74	0.91	0.67	0.82	1.00	0.81	0.84
Larger – Not middle – 3 dice	0.78	0.95	0.71	0.62	0.65	0.98	**0.50**	0.56	0.81	0.92	0.60
Even – Not first – 3 dice	0.64	0.99	**0.44**	0.82	0.96	0.87	0.60	0.62	0.76	1.00	0.90
Even – Not middle – 3 dice	0.64	0.88	0.46	0.71	0.97	0.89	**0.21**	0.89	0.69	0.89	0.95

B Scenario 2: Coins

B.1 Base models

Chebyshev	Y6B	M7B	Q7B	L8B	G7B	Y9B	Y34B	Q57xB	L70B	Q72B	M8x22B
2 coins Regular	0.22	0.13	0.24	0.17	0.23	0.43	0.12	**0.08**	0.19	0.16	0.14
2 coins Regular 3x Bias	0.15	0.08	**0.04**	0.13	0.05	0.11	0.39	0.09	0.24	0.18	0.13
2 coins Regular 5x Bias	**0.02**	0.24	0.22	0.25	0.11	0.22	0.50	0.24	0.41	0.33	0.25
2 coins Dependant	0.12	**0.07**	0.17	0.09	0.13	0.14	0.16	0.20	0.20	0.16	0.20
2 coins Dependant 3x Bias	0.22	0.24	0.33	0.31	**0.08**	0.36	0.22	0.24	0.30	0.23	0.38
2 coins Dependant 5x Bias	0.36	0.38	0.46	0.45	**0.27**	0.49	0.30	0.36	0.43	0.35	0.49
2 coins Independent	0.17	0.16	0.23	**0.13**	0.16	0.26	0.20	0.33	0.20	0.20	0.14
2 coins Independent 3x Bias	0.22	0.23	**0.11**	0.28	0.21	0.30	0.19	0.18	0.31	0.23	0.28
2 coins Independent 5x Bias	0.32	0.34	**0.19**	0.42	0.28	0.40	0.30	0.28	0.43	0.34	0.34
3 coins Regular	0.12	**0.02**	0.11	0.15	0.10	0.19	0.04	0.06	0.30	0.16	0.13
3 coins Regular 3x Bias	**0.03**	0.17	0.12	0.13	0.09	0.17	0.34	0.17	0.25	0.34	0.31
3 coins Regular 5x Bias	**0.16**	0.31	0.30	0.25	0.23	0.31	0.48	0.34	0.39	0.50	0.42
3 coins Dependant	0.19	0.15	0.25	0.18	**0.05**	0.22	0.20	0.19	0.14	0.12	0.19
3 coins Dependant 3x Bias	0.27	0.29	0.37	0.31	**0.22**	0.41	0.28	0.33	0.40	0.26	0.42
3 coins Dependant 5x Bias	0.39	0.43	0.48	0.45	**0.31**	0.50	0.38	0.43	0.50	0.42	0.52
3 coins Independent	0.19	0.12	0.18	**0.10**	**0.10**	0.14	0.14	0.28	0.19	0.16	0.22
3 coins Independent 3x Bias	0.29	0.28	**0.20**	0.31	0.23	0.34	0.27	0.33	0.33	0.28	0.38
3 coins Independent 5x Bias	0.43	0.41	**0.35**	0.45	**0.35**	0.49	0.43	0.46	0.46	0.41	0.50
L1											
2 coins Regular	0.45	0.27	0.48	0.34	0.45	0.86	0.24	**0.15**	0.37	0.31	0.28
2 coins Regular 3x Bias	0.31	0.15	**0.08**	0.25	0.09	0.21	0.79	0.17	0.49	0.37	0.25
2 coins Regular 5x Bias	**0.04**	0.48	0.45	0.50	0.22	0.45	1.00	0.48	0.83	0.65	0.49
2 coins Dependant	0.24	**0.14**	0.34	0.18	0.26	0.29	0.31	0.40	0.40	0.33	0.39
2 coins Dependant 3x Bias	0.45	0.49	0.67	0.62	**0.16**	0.73	0.43	0.47	0.61	0.45	0.76
2 coins Dependant 5x Bias	0.72	0.76	0.92	0.90	**0.54**	0.97	0.60	0.71	0.87	0.71	0.99
2 coins Independent	0.34	0.31	0.46	**0.25**	0.33	0.52	0.41	0.66	0.40	0.40	0.28
2 coins Independent 3x Bias	0.45	0.47	**0.23**	0.56	0.42	0.59	0.38	0.37	0.62	0.47	0.55
2 coins Independent 5x Bias	0.64	0.68	**0.38**	0.83	0.56	0.79	0.60	0.56	0.86	0.68	0.67
3 coins Regular	0.34	**0.05**	0.34	0.32	0.20	0.54	0.08	0.18	0.60	0.32	0.25
3 coins Regular 3x Bias	**0.07**	0.51	0.27	0.26	0.21	0.35	0.67	0.33	0.51	0.69	0.62
3 coins Regular 5x Bias	**0.32**	0.70	0.59	0.50	0.46	0.62	0.96	0.69	0.78	1.00	0.84
3 coins Dependant	0.39	0.38	0.55	0.44	**0.13**	0.46	0.39	0.38	0.29	0.29	0.39
3 coins Dependant 3x Bias	0.73	0.82	1.02	0.90	0.52	1.13	0.61	0.79	0.86	**0.51**	0.90
3 coins Dependant 5x Bias	0.91	0.99	1.16	1.06	**0.65**	1.25	0.78	0.94	1.04	0.84	1.09
3 coins Independent	0.41	0.25	0.44	0.26	**0.20**	0.28	0.29	0.58	0.39	0.33	0.45
3 coins Independent 3x Bias	0.65	0.66	**0.45**	0.71	0.50	0.74	0.56	0.69	0.73	0.62	0.80
3 coins Independent 5x Bias	0.93	0.86	**0.71**	0.97	0.72	0.97	0.85	0.97	0.97	0.87	1.02
Symmetric KL											
2 coins Regular	0.26	0.11	0.28	0.24	0.23	1.03	0.06	**0.04**	0.26	0.21	0.12
2 coins Regular 3x Bias	0.11	0.03	0.02	0.07	**0.01**	0.05	0.77	0.03	0.26	0.14	0.09
2 coins Regular 5x Bias	**0.00**	0.29	0.29	0.26	0.07	0.21	1.22	0.26	0.75	0.46	0.31
2 coins Dependant	0.10	**0.03**	0.22	0.05	0.11	0.18	0.20	0.26	0.29	0.22	0.26
2 coins Dependant 3x Bias	0.24	0.31	0.72	0.57	**0.03**	0.71	0.23	0.32	0.43	0.26	0.95
2 coins Dependant 5x Bias	0.60	0.75	1.26	1.09	**0.34**	1.21	0.44	0.71	0.85	0.57	1.35
2 coins Independent	0.18	0.14	0.36	**0.10**	0.19	0.35	0.27	0.65	0.19	0.22	0.12
2 coins Independent 3x Bias	0.27	0.33	**0.09**	0.39	0.23	0.65	0.20	0.28	0.56	0.33	0.54
2 coins Independent 5x Bias	0.60	0.71	**0.25**	0.86	0.46	1.02	0.48	0.55	1.07	0.68	0.78
3 coins Regular	0.14	**0.00**	0.16	0.16	0.06	0.46	0.01	0.04	0.41	0.16	0.07
3 coins Regular 3x Bias	**0.01**	0.41	0.13	0.12	0.08	0.20	0.87	0.19	0.35	0.85	0.73
3 coins Regular 5x Bias	**0.15**	0.96	0.63	0.42	0.39	0.63	1.56	0.68	0.77	1.67	1.27
3 coins Dependant	0.20	0.19	0.40	0.27	**0.03**	0.31	0.20	0.22	0.15	0.20	0.20
3 coins Dependant 3x Bias	0.73	0.99	1.46	1.23	**0.40**	1.81	0.58	0.88	1.28	0.47	1.31
3 coins Dependant 5x Bias	1.40	1.81	2.25	2.05	**0.82**	2.72	1.11	1.62	2.13	1.12	2.32
3 coins Independent	0.19	0.09	0.30	0.10	**0.07**	0.12	0.16	0.46	0.25	0.24	0.30
3 coins Independent 3x Bias	0.64	0.72	0.40	0.91	**0.38**	0.99	0.53	0.83	0.91	0.69	1.07
3 coins Independent 5x Bias	1.35	1.31	0.92	1.67	**0.87**	1.81	1.16	1.54	1.62	1.33	1.78

B.2 Instruction fine-tuned models

Chebyshev	Y6B	M7B	Q7B	L8B	G7B	Y9B	Y34B	Q57xB	L70B	Q72B	M8x22B
2 coins Regular	0.19	0.15	0.19	0.39	0.36	0.20	0.20	0.22	0.28	0.25	**0.10**
2 coins Regular 3x Bias	0.10	**0.02**	0.08	0.27	0.39	0.06	0.23	0.25	0.12	0.31	0.09
2 coins Regular 5x Bias	**0.05**	0.14	0.23	0.38	0.25	0.08	0.26	0.33	0.20	0.42	0.13
2 coins Dependant	0.33	0.17	0.20	0.28	0.43	0.33	0.27	0.26	0.27	0.25	**0.14**
2 coins Dependant 3x Bias	0.47	0.24	0.42	0.50	0.43	0.42	0.43	0.34	0.44	**0.18**	0.21
2 coins Dependant 5x Bias	0.58	0.38	0.51	0.61	0.47	0.55	0.54	0.46	0.53	**0.29**	0.37
2 coins Independent	0.26	0.31	0.27	0.34	0.44	0.16	0.19	0.34	0.17	0.34	**0.12**
2 coins Independent 3x Bias	0.32	0.28	0.16	0.46	0.26	0.28	**0.15**	0.28	0.31	0.34	0.23
2 coins Independent 5x Bias	0.39	0.32	**0.23**	0.59	0.28	0.45	**0.23**	0.36	0.42	0.43	0.30
3 coins Regular	0.12	0.22	**0.11**	0.51	0.61	0.35	0.13	0.16	0.43	0.27	0.13
3 coins Regular 3x Bias	0.10	0.16	**0.08**	0.47	0.56	0.18	0.36	0.25	0.52	0.38	0.27
3 coins Regular 5x Bias	**0.19**	0.29	0.23	0.55	0.64	0.32	0.50	0.42	0.56	0.53	0.40
3 coins Dependant	0.40	0.20	0.32	0.37	0.42	0.30	0.32	0.26	0.30	0.16	**0.09**
3 coins Dependant 3x Bias	0.54	0.33	0.43	0.50	0.62	0.42	0.43	0.40	0.50	**0.25**	**0.25**
3 coins Dependant 5x Bias	0.63	0.45	0.54	0.60	0.69	0.53	0.50	0.53	0.59	**0.42**	0.43
3 coins Independent	0.27	0.21	0.22	0.24	0.51	0.16	**0.15**	0.34	0.18	0.21	0.16
3 coins Independent 3x Bias	0.37	0.26	**0.19**	0.43	0.49	0.26	0.29	0.44	0.35	0.32	0.32
3 coins Independent 5x Bias	0.47	0.39	**0.32**	0.56	0.60	0.43	0.43	0.57	0.51	0.45	0.44
L1											
2 coins Regular	0.37	0.31	0.39	0.78	0.72	0.39	0.40	0.45	0.56	0.51	**0.19**
2 coins Regular 3x Bias	0.20	**0.03**	0.16	0.53	0.78	0.11	0.46	0.51	0.24	0.61	0.19
2 coins Regular 5x Bias	**0.10**	0.28	0.45	0.75	0.51	0.15	0.52	0.66	0.40	0.84	0.26
2 coins Dependant	0.67	0.33	0.40	0.57	0.85	0.67	0.53	0.51	0.55	0.49	**0.28**
2 coins Dependant 3x Bias	0.94	0.49	0.84	0.99	0.85	0.83	0.85	0.69	0.88	**0.37**	0.42
2 coins Dependant 5x Bias	1.16	0.75	1.02	1.22	0.94	1.10	1.07	0.92	1.06	**0.59**	0.74
2 coins Independent	0.52	0.62	0.55	0.68	0.88	0.32	0.37	0.69	0.33	0.68	**0.24**
2 coins Independent 3x Bias	0.63	0.56	0.33	0.91	0.52	0.55	**0.31**	0.56	0.61	0.68	0.45
2 coins Independent 5x Bias	0.77	0.64	**0.46**	1.19	0.55	0.90	**0.46**	0.71	0.84	0.86	0.59
3 coins Regular	**0.23**	0.44	0.36	1.03	1.22	0.71	0.32	0.34	0.87	0.54	0.29
3 coins Regular 3x Bias	0.22	0.32	**0.17**	0.95	1.13	0.35	0.73	0.50	1.05	0.76	0.54
3 coins Regular 5x Bias	**0.38**	0.58	0.48	1.11	1.28	0.63	1.00	0.84	1.13	1.05	0.79
3 coins Dependant	0.83	0.48	0.66	0.77	0.88	0.68	0.76	0.52	0.66	0.37	**0.21**
3 coins Dependant 3x Bias	1.19	0.73	1.11	1.12	1.24	1.04	1.04	0.89	1.04	**0.50**	0.51
3 coins Dependant 5x Bias	1.34	0.98	1.25	1.29	1.38	1.23	1.17	1.14	1.21	**0.84**	0.86
3 coins Independent	0.58	0.46	0.51	0.48	1.01	0.36	**0.35**	0.68	0.38	0.45	0.36
3 coins Independent 3x Bias	0.81	0.55	**0.42**	0.91	0.99	0.63	0.58	0.89	0.70	0.68	0.69
3 coins Independent 5x Bias	0.99	0.79	**0.67**	1.15	1.20	0.94	0.86	1.14	1.02	0.91	0.91
Symmetric KL											
2 coins Regular	0.25	0.19	0.27	0.89	1.16	0.39	0.31	0.32	0.47	0.48	**0.06**
2 coins Regular 3x Bias	0.05	**0.01**	0.04	0.32	1.11	0.02	0.25	0.30	0.16	0.41	0.06
2 coins Regular 5x Bias	0.05	0.09	0.27	0.60	0.55	**0.03**	0.36	0.50	0.21	0.77	0.16
2 coins Dependant	0.84	0.21	0.38	0.84	2.04	0.74	0.56	0.48	0.69	0.55	**0.15**
2 coins Dependant 3x Bias	1.30	0.30	1.22	1.62	1.46	1.03	1.12	0.81	1.03	**0.25**	0.27
2 coins Dependant 5x Bias	1.85	0.66	1.78	2.33	1.71	1.74	1.60	1.30	1.44	**0.43**	0.65
2 coins Independent	0.39	0.66	0.49	0.68	1.52	0.14	0.21	0.82	0.22	0.77	**0.09**
2 coins Independent 3x Bias	0.57	0.49	0.19	1.12	0.67	0.41	**0.14**	0.61	0.53	0.86	0.43
2 coins Independent 5x Bias	0.93	0.72	0.37	1.90	0.52	1.01	**0.36**	0.90	0.94	1.30	0.70
3 coins Regular	**0.08**	0.22	0.23	1.83	3.48	0.63	0.22	0.16	0.94	0.45	0.09
3 coins Regular 3x Bias	0.08	0.16	**0.05**	1.83	3.03	0.17	0.92	0.34	2.06	1.10	0.56
3 coins Regular 5x Bias	**0.17**	0.49	0.43	2.24	3.38	0.49	1.41	0.95	2.31	1.87	1.12
3 coins Dependant	1.26	0.41	0.65	1.20	2.12	0.76	0.98	0.45	0.81	0.40	**0.12**
3 coins Dependant 3x Bias	2.10	0.82	1.84	2.37	3.30	1.49	1.83	1.40	1.98	**0.41**	0.48
3 coins Dependant 5x Bias	2.98	1.61	2.68	3.57	4.42	2.57	2.57	2.39	2.96	**1.04**	1.29
3 coins Independent	0.54	0.39	0.40	0.43	1.78	0.23	**0.16**	0.73	0.24	0.58	0.20
3 coins Independent 3x Bias	1.06	0.56	**0.37**	1.78	2.12	0.75	0.59	1.46	1.00	0.96	0.98
3 coins Independent 5x Bias	1.62	1.08	**0.88**	2.91	2.99	1.67	1.12	2.33	1.84	1.58	1.65

B.3 Stated Answer

Chebyshev	*Y6B*	*M7B*	*Q7B*	*L8B*	*G7B*	*Y9B*	*Y34B*	*Q57xB*	*L70B*	*Q72B*	*M8x22B*
2 coins Regular	0.56	0.75	0.53	0.79	0.59	0.65	0.31	0.76	0.25	**0.00**	0.21
2 coins Regular 3x Bias	0.28	0.99	0.04	0.63	0.77	0.24	0.66	0.28	0.02	**0.00**	0.28
2 coins Dependant	0.67	0.85	0.73	0.63	0.63	0.70	**0.49**	0.86	**0.49**	0.52	0.61
2 coins Dependant 3x Bias	**0.26**	0.88	0.41	0.77	0.91	0.42	0.51	0.71	0.50	0.32	0.52
2 coins Independent	0.76	0.86	0.52	0.71	0.46	0.39	0.55	0.59	**0.26**	0.28	0.28
2 coins Independent 3x Bias	**0.40**	0.84	0.45	0.66	0.70	0.47	0.69	0.48	0.47	0.41	0.69
3 coins Regular	0.49	0.96	0.27	0.61	0.74	0.35	0.13	0.04	**0.00**	0.29	0.25
3 coins Dependant	0.64	0.66	0.75	0.60	0.67	0.36	0.67	0.47	**0.29**	0.34	0.54
3 coins Independent	0.67	0.78	0.42	0.68	0.86	0.54	0.55	0.29	0.42	**0.17**	0.34

C Scenario 3: Choice

C.1 Base models

Chebyshev	Y6B	M7B	Q7B	L8B	G7B	Y9B	Y34B	Q57xB	L70B	Q72B	M8x22B
Regular – 2 choices	0.42	0.41	0.46	0.41	0.42	0.43	0.39	0.41	**0.32**	0.46	0.38
Regular – 4 choices	0.45	0.48	0.61	0.44	0.54	0.57	**0.32**	0.55	0.35	0.65	0.34
Regular – 6 choices	0.42	0.40	0.62	0.37	0.50	0.58	0.34	0.54	0.37	0.78	**0.30**
Independent – 2 choices	0.45	0.38	0.42	0.38	0.38	0.41	0.30	0.28	**0.22**	0.33	0.24
Independent – 4 choices	0.31	0.30	0.29	0.30	0.31	0.31	0.24	0.28	**0.21**	0.25	0.22
Independent – 6 choices	0.24	0.20	0.21	0.24	0.21	0.23	0.17	0.20	**0.14**	0.21	0.17
L1											
Regular – 2 choices	0.85	0.82	0.92	0.83	0.85	0.86	0.79	0.83	**0.64**	0.93	0.76
Regular – 4 choices	0.91	0.96	1.22	0.89	1.09	1.14	**0.64**	1.10	0.70	1.30	0.68
Regular – 6 choices	0.85	0.79	1.23	0.74	0.99	1.16	0.68	1.09	0.81	1.57	**0.66**
Independent – 2 choices	0.89	0.76	0.84	0.77	0.77	0.81	0.61	0.55	**0.44**	0.67	0.48
Independent – 4 choices	0.66	0.68	0.62	0.65	0.66	0.61	0.52	0.58	**0.44**	0.52	**0.44**
Independent – 6 choices	0.56	0.53	0.49	0.55	0.49	0.53	0.44	0.45	**0.36**	0.42	0.40
Symmetric KL											
Regular – 2 choices	1.06	0.95	1.43	0.99	1.06	1.14	0.84	0.99	**0.48**	1.50	0.76
Regular – 4 choices	0.97	1.20	1.82	0.94	1.34	1.50	**0.53**	1.46	0.76	2.28	0.59
Regular – 6 choices	1.05	0.85	1.82	0.70	1.15	1.62	0.75	1.44	0.89	3.61	**0.57**
Independent – 2 choices	1.28	0.80	1.10	0.84	0.86	0.95	0.43	0.35	**0.21**	0.64	0.27
Independent – 4 choices	0.84	0.79	0.75	0.70	0.87	0.88	0.50	0.62	**0.33**	0.49	0.37
Independent – 6 choices	0.66	0.52	0.57	0.56	0.55	0.78	0.37	0.42	**0.23**	0.48	0.38

C.2 Instruction fine-tuned models

Chebyshev	Y6B	M7B	Q7B	L8B	G7B	Y9B	Y34B	Q57xB	L70B	Q72B	M8x22B
Regular – 2 choices	**0.21**	0.41	0.49	0.46	0.50	0.47	0.34	0.44	0.32	0.49	0.35
Regular – 4 choices	**0.16**	0.43	0.68	0.51	0.72	0.55	0.42	0.54	0.29	0.72	0.32
Regular – 6 choices	**0.14**	0.36	0.69	0.46	0.72	0.42	0.18	0.53	0.30	0.82	0.29
Independent – 2 choices	0.42	0.32	0.46	0.44	0.50	0.38	0.21	0.38	0.25	0.38	**0.14**
Independent – 4 choices	0.34	0.35	0.31	0.34	0.44	0.31	0.28	0.31	**0.23**	0.27	**0.23**
Independent – 6 choices	0.25	0.25	0.21	0.22	0.50	0.23	0.22	0.21	**0.17**	0.22	0.18
L1											
Regular – 2 choices	**0.41**	0.82	0.97	0.92	1.00	0.95	0.67	0.88	0.64	0.99	0.70
Regular – 4 choices	**0.31**	0.86	1.36	1.01	1.45	1.09	0.84	1.07	0.66	1.45	0.65
Regular – 6 choices	**0.56**	0.72	1.38	0.92	1.44	0.88	**0.56**	1.06	0.76	1.63	0.59
Independent – 2 choices	0.85	0.63	0.92	0.88	1.00	0.77	0.42	0.76	0.51	0.76	**0.27**
Independent – 4 choices	0.78	0.72	0.67	0.81	0.98	0.70	0.58	0.64	**0.46**	0.53	0.47
Independent – 6 choices	0.68	0.59	0.49	0.62	1.09	0.58	0.51	0.46	0.43	0.46	**0.41**
Symmetric KL											
Regular – 2 choices	**0.18**	0.95	2.07	1.43	4.00	1.72	0.55	1.21	0.48	2.60	0.62
Regular – 4 choices	**0.19**	0.90	2.52	1.23	3.89	1.54	1.29	1.38	0.60	3.57	0.48
Regular – 6 choices	**0.49**	0.66	2.45	1.07	3.13	1.18	0.53	1.33	0.92	4.74	0.50
Independent – 2 choices	1.06	0.76	1.47	1.42	4.50	0.84	0.24	0.76	0.29	0.88	**0.08**
Independent – 4 choices	1.22	0.99	1.09	1.35	3.42	1.18	0.65	0.75	**0.37**	0.55	0.39
Independent – 6 choices	1.06	0.72	0.72	0.87	3.22	1.14	0.45	0.43	**0.36**	0.59	**0.36**

C.3 `Stated Answer`

Chebyshev	Y6B	M7B	Q7B	L8B	G7B	Y9B	Y34B	Q57xB	L70B	Q72B	M8x22B
Regular – 2 choices	0.41	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**
Regular – 4 choices	**0.00**	0.61	0.50	0.68	1.00	0.48	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**
Independent – 2 choices	0.34	0.33	0.48	0.24	0.51	**0.00**	**0.00**	0.03	0.03	**0.00**	0.01
Independent – 4 choices	0.40	0.75	0.57	0.66	0.75	0.13	0.30	0.28	0.06	**0.00**	0.04

D Scenario 4: Preference

D.1 Base models

Chebyshev	Y6B	M7B	Q7B	L8B	G7B	Y9B	Y34B	Q57xB	L70B	Q72B	M8x22B
Heads	0.43	**0.19**	0.36	0.32	0.38	0.24	0.29	0.43	0.32	0.36	0.39
Heads 2x	0.25	**0.12**	0.25	0.24	0.21	0.24	0.28	0.15	0.21	0.20	0.24
Heads 3x	0.16	**0.04**	0.16	0.15	0.25	0.16	0.20	0.05	0.13	0.13	0.12
Left	0.32	0.39	0.49	0.35	0.35	0.42	**0.12**	0.47	0.37	0.48	0.39
Left 2x	0.15	0.26	0.32	0.13	0.21	0.28	**0.00**	0.23	0.17	0.26	0.09
Left 3x	0.05	0.16	0.23	0.05	**0.02**	0.19	0.06	0.15	0.07	0.18	**0.02**
Heads Independent	0.21	0.35	0.34	0.38	0.43	**0.18**	0.21	0.30	0.22	0.34	0.30
Heads Independent 2x	0.11	0.13	0.33	0.24	0.36	0.17	**0.09**	0.29	0.23	0.28	0.22
Heads Independent 3x	**0.11**	0.23	0.33	0.24	0.49	0.16	0.15	0.27	0.24	0.36	0.22
Left Independent	0.36	0.28	0.42	0.34	0.42	**0.24**	0.25	0.30	0.26	0.37	0.30
Left Independent 2x	0.19	0.14	0.37	0.24	0.31	**0.12**	0.27	0.28	0.23	0.30	0.25
Left Independent 3x	0.21	0.16	0.37	0.24	0.32	**0.15**	0.36	0.26	0.27	0.29	0.35
L1											
Heads	0.87	**0.39**	0.73	0.63	0.76	0.47	0.58	0.85	0.65	0.72	0.79
Heads 2x	0.50	**0.25**	0.50	0.48	0.43	0.48	0.57	0.30	0.43	0.41	0.48
Heads 3x	0.33	**0.08**	0.31	0.31	0.50	0.31	0.40	0.10	0.26	0.25	0.24
Left	0.64	0.77	0.98	0.70	0.70	0.85	**0.24**	0.95	0.73	0.95	0.79
Left 2x	0.30	0.52	0.63	0.26	0.43	0.56	**0.00**	0.45	0.34	0.51	0.18
Left 3x	0.10	0.32	0.46	0.10	**0.04**	0.38	0.11	0.31	0.14	0.36	**0.04**
Heads Independent	0.43	0.70	0.67	0.76	0.86	**0.36**	0.41	0.60	0.43	0.67	0.60
Heads Independent 2x	0.23	0.26	0.65	0.47	0.71	0.34	**0.18**	0.58	0.46	0.55	0.43
Heads Independent 3x	**0.21**	0.46	0.65	0.47	0.98	0.32	0.30	0.54	0.48	0.73	0.45
Left Independent	0.72	0.56	0.83	0.68	0.83	**0.48**	0.50	0.60	0.53	0.73	0.59
Left Independent 2x	0.38	0.27	0.75	0.49	0.61	**0.24**	0.53	0.57	0.46	0.60	0.50
Left Independent 3x	0.42	0.32	0.73	0.49	0.65	**0.30**	0.73	0.52	0.55	0.58	0.70
Symmetric KL											
Heads	1.14	**0.16**	0.67	0.47	0.76	0.24	0.38	1.08	0.50	0.65	0.83
Heads 2x	0.42	**0.08**	0.42	0.37	0.28	0.37	0.64	0.12	0.28	0.24	0.39
Heads 3x	0.21	**0.01**	0.18	0.18	0.27	0.18	0.37	**0.01**	0.12	0.11	0.10
Left	0.48	0.80	2.30	0.62	0.62	1.06	**0.06**	1.72	0.69	1.75	0.84
Left 2x	0.12	0.49	1.04	0.09	0.28	0.61	**0.00**	0.32	0.16	0.47	0.04
Left 3x	0.01	0.19	0.66	0.01	**0.00**	0.31	0.02	0.18	0.03	0.28	**0.00**
Heads Independent	0.29	0.62	0.59	0.84	1.34	**0.13**	0.18	0.41	0.21	0.66	0.41
Heads Independent 2x	0.10	0.14	0.64	0.27	0.79	0.14	**0.04**	0.43	0.31	0.61	0.27
Heads Independent 3x	**0.06**	0.31	0.86	0.33	2.00	0.15	0.12	0.47	0.45	0.90	0.40
Left Independent	0.69	0.36	1.04	0.63	1.06	**0.27**	0.31	0.46	0.32	0.69	0.41
Left Independent 2x	0.20	0.10	0.87	0.29	0.75	**0.07**	0.30	0.42	0.39	0.53	0.36
Left Independent 3x	0.25	0.21	1.01	0.37	0.98	**0.13**	0.59	0.37	0.57	0.68	0.61

D.2 Instruction fine-tuned models

Chebyshev	Y6B	M7B	Q7B	L8B	G7B	Y9B	Y34B	Q57xB	L70B	Q72B	M8x22B
Heads	0.47	**0.32**	0.38	0.38	0.50	0.45	0.45	0.40	0.34	0.33	0.40
Heads 2x	0.32	**0.04**	0.25	0.25	0.33	0.33	0.30	0.15	0.28	0.32	0.22
Heads 3x	0.24	**0.00**	0.15	0.17	0.25	0.25	0.14	0.09	0.19	0.24	0.14
Left	0.46	0.46	0.50	0.47	0.50	0.45	**0.09**	0.48	0.43	0.50	0.34
Left 2x	0.27	0.31	0.33	0.26	0.33	0.31	0.55	0.26	0.19	0.32	**0.06**
Left 3x	0.17	0.23	0.25	0.18	0.25	0.23	0.70	0.19	0.07	0.24	**0.02**
Heads Independent	0.48	0.42	0.40	0.44	0.50	**0.23**	0.35	0.38	0.36	0.42	0.24
Heads Independent 2x	0.24	0.27	0.40	0.30	0.44	0.21	0.40	0.32	0.23	0.33	**0.13**
Heads Independent 3x	0.29	0.25	0.40	0.30	0.49	0.19	0.48	0.33	0.23	0.35	**0.14**
Left Independent	0.49	0.36	0.43	0.43	0.50	0.29	0.27	0.35	0.41	0.42	**0.19**
Left Independent 2x	0.46	0.30	0.42	0.34	0.44	0.37	0.60	0.30	0.30	0.35	**0.17**
Left Independent 3x	0.46	0.31	0.43	0.35	0.44	0.46	0.68	0.31	0.29	0.36	**0.25**
L1											
Heads	0.93	**0.64**	0.75	0.77	1.00	0.91	0.91	0.81	0.67	0.66	0.80
Heads 2x	0.64	**0.08**	0.50	0.51	0.66	0.66	0.60	0.30	0.56	0.63	0.43
Heads 3x	0.47	**0.01**	0.31	0.33	0.50	0.49	0.29	0.17	0.38	0.47	0.29
Left	0.92	0.93	1.00	0.93	1.00	0.91	**0.19**	0.96	0.86	0.99	0.67
Left 2x	0.55	0.63	0.66	0.51	0.67	0.63	1.11	0.51	0.37	0.63	**0.13**
Left 3x	0.35	0.45	0.50	0.36	0.50	0.45	1.41	0.38	0.14	0.47	**0.04**
Heads Independent	0.96	0.83	0.79	0.87	1.00	**0.46**	0.70	0.76	0.72	0.83	0.48
Heads Independent 2x	0.49	0.53	0.81	0.60	0.88	0.43	0.80	0.65	0.46	0.65	**0.26**
Heads Independent 3x	0.59	0.51	0.80	0.60	0.98	0.38	0.97	0.67	0.46	0.70	**0.28**
Left Independent	0.97	0.73	0.87	0.86	1.00	0.58	0.53	0.70	0.82	0.83	**0.38**
Left Independent 2x	0.92	0.60	0.84	0.69	0.88	0.75	1.20	0.59	0.59	0.69	**0.33**
Left Independent 3x	0.92	0.63	0.86	0.71	0.88	0.92	1.35	0.61	0.58	0.72	**0.50**
Symmetric KL											
Heads	1.58	**0.48**	0.73	0.78	2.99	1.36	1.36	0.91	0.55	0.52	0.87
Heads 2x	1.18	**0.01**	0.42	0.44	2.10	1.50	0.81	0.12	0.61	1.05	0.29
Heads 3x	0.74	**0.00**	0.18	0.21	1.47	1.08	0.15	0.05	0.31	0.75	0.15
Left	1.43	1.50	3.05	1.58	5.50	1.36	**0.03**	1.93	1.14	2.66	0.55
Left 2x	0.56	1.00	1.76	0.47	3.44	1.00	1.53	0.47	0.20	1.09	**0.02**
Left 3x	0.24	0.58	1.21	0.28	2.48	0.60	2.88	0.31	0.03	0.74	**0.00**
Heads Independent	1.95	1.00	0.88	1.31	5.00	0.57	0.62	0.76	0.82	1.02	**0.27**
Heads Independent 2x	0.29	0.41	1.06	0.48	2.29	0.22	0.74	0.55	0.31	0.68	**0.09**
Heads Independent 3x	0.51	0.53	1.21	0.52	2.98	0.22	1.08	0.67	0.29	0.99	**0.14**
Left Independent	2.38	0.73	1.17	1.24	4.24	0.58	0.83	0.66	0.99	1.02	**0.17**
Left Independent 2x	1.57	0.48	1.23	0.64	1.95	0.95	2.26	0.49	0.56	0.64	**0.13**
Left Independent 3x	1.64	0.57	1.49	0.76	2.09	1.21	2.89	0.54	0.68	0.83	**0.29**

D.3 `Stated Answer`

Chebyshev	Y6B	M7B	Q7B	L8B	G7B	Y9B	Y34B	Q57xB	L70B	Q72B	M8x22B
Heads	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**
Heads 3x	0.01	0.57	0.48	0.38	**0.00**	0.42	0.48	0.50	**0.00**	**0.00**	0.02
Left	**0.00**	**0.00**	**0.00**	**0.00**	0.50	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**
Left 3x	0.50	1.00	**0.23**	0.68	0.95	0.39	0.50	0.50	0.45	0.50	0.45
Heads Independent	0.48	0.74	**0.00**	0.04	0.78	**0.00**	**0.00**	0.09	**0.00**	**0.00**	**0.00**
Heads Independent 3x	0.33	0.95	0.54	0.70	0.38	0.65	0.29	0.55	0.28	**0.24**	0.36
Left Independent	0.64	0.51	0.23	0.09	0.03	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	0.01
Left Independent 3x	0.38	1.00	0.77	0.80	0.98	**0.16**	0.60	0.54	0.65	0.28	0.52