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ORIGINAL ARTICLE

A solution of mathematical multi-objective transportation problems using the fermatean fuzzy programming approach

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Abstract In this paper, we present a mathematical model of a traditional transportation problem (TTP) using the fermatean fuzzy parameters (FFPs) and convert it into a crisp form using a new fermatean fuzzy score function (NFFSF). Additionally, we proposed a mathematical model of the multi-objective transportation problem (MOTP) incorporating FFPs, which is similarly transformed into a crisp form using NFFSF under fermatean fuzzy environments (FFE). We extend this approach to develop a mathematical model of a multi-level, multi-objective solid transportation problem (MLMOSTP) using FFPs under FFE. It is also converted into crisp form using NFFSF. The mathematical model of MOTP and MLMOSTP aims to simultaneously minimize three objective functions: total transportation cost, total transportation time, and total carbon emissions. The parameters of mathematical models, including objectives, costs, supply, and demands, are considered as FFPs. The mathematical model of MOTP and MLMOSTP is solved using the fermatean fuzzy programming approach (FFPA) under FFE. The FFPA is particularly suitable for addressing the MOTP and MLMOSTP due to its ability to handle higher uncertainty and vagueness inherent in multi-objective optimization problems. We provided numerical examples to demonstrate the proposed problems' efficiency and practicality. The SciPy optimization library in Python was used to solve these numerical examples and obtain the best compromise solutions. Managerial and practical implications are discussed.

Keywords Fermatean fuzzy parameters · Fermatean fuzzy transportation problems · Multi-objective transportation problems · Fermatean fuzzy programming approach · Score function

1 Introduction

The transportation problem (TP) is a classic optimization problem in industrial engineering and logistics (Ekanayake et al. 2020). It involves determining the most economical method to transport products from suppliers (origins) to destinations (receivers or demand points) while satisfying supply and demand constraints (Qiuping et al. 2023; Sullivan and Novak 2024). However, real-world transportation problems are often more complex, involving multiple, often conflicting objectives that must be optimized simultaneously. The MOTP refers to a variant of the TP where multiple conflicting objectives must be optimized simultaneously (Prathyusha et al. 2024; Almotairi et al. 2024). In contrast to the traditional transportation problem (TTP), which focuses on minimizing transportation costs, MOTP considers additional objectives such as minimizing transportation time, deterioration cost during transportation, minimization of inventory levels, maximization of customer satisfaction, carbon emissions, minimization of the number of vehicles used, maximizing resource utilization, and minimizing congestion.

The solid transportation problem (STP) is an extension of the TTP that considers additional constraints related to the transportation capacity of vehicles or modes of transportation (Saini et al. 2023; Edward and Kaliyaperumal 2024). In STP, the capacity of vehicles is limited not only

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by weight but also by other physical characteristics such as space occupancy (Mondal et al. 2023; Yu et al. 2024). This additional constraint introduces complexities in the optimization process, as it requires considering not only the number of goods to be transported but also their physical properties and compatibility with the vehicles (Kumar and Sharma 2024; Shivani and Rani 2024). Various methods, e.g., can solve TP, the North-West Corner Method, the Least Cost Method, Vogel's Approximation Method, and Heuristic and Metaheuristic Algorithms (Genetic Algorithms, NSGA-2 & 3). In the same manner, the MOTP and STP can be solved by various types of methodologies, e.g., chance constraint programming, fuzzy programming, fuzzy goal programming, multi-objective genetic programming, multi-criteria decision analysis, interactive evolutionary algorithms, goal programming, neutrosophic goal programming approach, and dynamic programming problem.

The FFPA has emerged as a robust approach for handling uncertainties and ambiguities in MOTP (Sindhu et al. 2022; Singh et al. 2024). The concept of fermatean fuzzy sets (FFSs), introduced by Senapati and Yager (2020) enhances the modeling of uncertain environments by incorporating degrees of satisfaction, dissatisfaction, and indeterminacy. The FFPA leverages these properties to provide more flexible and accurate solutions to complex transportation problems. This study aims to improve industrial engineering and logistics decision-making processes by addressing MOTP and MLMOSTP using FFPA under FFEs. By integrating these theoretical advancements, this paper addresses the gap in handling higher-order uncertainties in transportation problems (Senapati and Yager 2020). By incorporating fuzzy logic, decision-makers can better handle the imprecise nature of real-world data, leading to more reliable and effective solutions (Bressane et al. 2024; Khan et al. 2024). This approach is particularly beneficial in today's dynamic and uncertain business environments, where traditional deterministic methods often fall short (Kabashkin 2023).

With the growing need for advanced solution methods to handle the multi-objective, multi-level nature of modern transportation problems under uncertainty, this research is motivated by the increasing globalization and complexity in transportation planning (Arora and Jaggi 2023; Bind et al. 2024). There is a pressing demand for methodologies that can provide optimal and robust solutions, ensuring efficiency and sustainability in logistics operations (Liu et al. 2023). This research aims to develop and validate a solution approach for MOTP and MLMOSTP using FFPA. The goal is to demonstrate that the proposed approach can provide the best solutions compared to traditional methods, thereby contributing to advancing operations research and its applications in logistics and transportation (Kumar and Kumar 2024).

The study significantly contributes to transportation optimization problems by introducing a comprehensive mathematical farmwork that integrates FFPs into MOTP and MLMOSTP. By developing and employing an NFFSF, the research effectively transforms complex fuzzy models into crisp forms, facilitating more straightforward optimization processes. The proposed mathematical models address multiple conflicting objectives—minimizing total transportation cost, time, and carbon emissions—while accommodating the inherent uncertainties in real-world scenarios. By applying the SciPy optimization library in Python, the study validates the proposed approaches' efficiency and practicality and provides valuable numerical examples that highlight their effectiveness. Most studies have focused on single-level frameworks for transportation problems, often neglecting the hierarchical decision- making process inherent in multi-level optimization problems (Lo et al. 2024; van de Berg et al. 2024). Additionally, sustainability needs to be addressed, including factors like minimizing carbon emissions and resource efficiency. Therefore, this study addresses these gaps by developing and validating an FFPA to solve MOTP and MLMOSTP under FFEs. This proposed approach integrates fuzzy logic to handle the imprecise nature of real-world transportation problems and aims to provide more robust and practical solutions (Bouraima et al. 2024; Sarkar and Srivastava 2024). By doing so, this research contributes to the advancement of decision-making in the field of transportation logistics, ensuring both efficiency and sustainability in solving complex transportation problems.

This work's remaining sections are arranged as follows: Sect. 2 follows the literature review of the proposed study. Essential definitions, theorems, and arithmetic operations are presented in Sect. 3. In Sect. 4, the TTP, MOTP, and MLMOSTP mathematical models are shown, and NFFSF is used to transform these models into crisp form. In Sect. 5, we proposed a mathematical modeling approach for FFPA. Section 6 presents the methodology for the proposed problem. Section 7 presents the numerical example of a proposed mathematical model. At last, Sect. 8 discusses the conclusion.

2 Literature review

Fermatean fuzzy programming approach is a nonlinear programming approach designed to handle MOTP, STP, multi-level, multi-objective solid transportation, and other optimization problems. It extends the concept of pythagorean fuzzy programming. Previous research has shown that FFSs provide a more flexible framework for handling uncertainty than traditional fuzzy sets. For instance, Senapati and Yager (2020) introduced FFSs and compared them to pythagorean and intuitionistic fuzzy sets. They proposed a score function to rank FFSs and used Euclidean distance to measure their similarity. They also developed fermatean fuzzy TOPSIS as a decision-making method for complex scenarios. Sharma et al. (2022) developed a ranking function for FFSs and applied it to optimize transportation problems, showcasing its practical applicability and efficiency in uncertain scenarios.

Similarly, Sharma et al. (2024) explored the use of pentagonal and hexagonal fuzzy numbers in transportation problems, highlighting these approaches' flexibility in capturing diverse uncertainties. These studies collectively emphasize the advancements in fuzzy logic methods and their relevance to transportation optimization. By building on these foundations, our study integrates the FFPA to address multi-objective and multi-level transportation problems, filling gaps in existing methodologies and extending their practical applications in logistics and supply chain management.

Srivastava et al. (2024) introduced an innovative method for addressing fuzzy transportation problems using the generalized fuzzy Vogel approximation method. This approach effectively estimates defuzzification, which significantly advances solving fuzzy optimization challenges within transportation planning (Aroniadi and Beligiannis 2024). The method improves solution accuracy and enhances computational efficiency, making it highly suitable for complex decision-making scenarios. Moreover, the FFPA offers a robust alternative for tackling MOTP and other optimization problems in uncertain environments, as highlighted by Senapati and Yager (2019). These advancements collectively provide a solid foundation for developing more practical and scalable solutions in fuzzy optimization.

Akram et al. (2023a, b, c) proposed a fermatean fuzzy data envelopment analysis method to address TP with multiple goals. Their study introduced a novel efficiency measurement technique using fermatean fuzzy score functions, which allowed for better evaluation of transportation routes under uncertain conditions. Their approach simplified the optimization process by converting complex multi-goal problems into single-goal versions and demonstrated improved solution accuracy. Kumar et al. (2023) developed a mathematical model of MOTP under uncertain parameters using a neutrosophic linear programming approach and got efficient solutions. Sharma and Chaudhary (2024) developed a mathematical model of MOTP utilizing time sequential dual hesitant fuzzy sets. This approach allows for consideration of hesitation and uncertainty in decision-making over different periods of the proposed problem. Gütmen et al. (2024) provide a comprehensive analysis of weighted goal programming (WGP) approach, highlighting its superiority over traditional goal programming approach in addressing the MOTP. The study presents a comprehensive mathematical model for the TP, which is essential for understanding the practical applications of the WGP approach. Wang et al. (2021) developed a mathematical model of multi-objective linear programming problems using triangular neutrosophic numbers and solved this model using a neutrosophic approach. This method addresses real-world decision-making scenarios (Adnan et al. 2024; Kacher and Singh 2024). These advancements directly influence our research by providing methodologies for handling uncertainties in transportation parameters and offer a foundation for further extending fermatean fuzzy techniques to multi-level transportation problems.

Apart from this, Akram et al. (2023a, b, c) extended the application of FFSs in decision-making, providing new models for optimizing transportation issues. This body of work collectively supports the growing relevance of advanced fuzzy methodologies in solving multi-objective and hierarchical transportation challenges, forming a strong theoretical and practical basis for the present study. Niksirat (2022) and Akram et al. (2023a, b, c) utilized a Type-1 fermatean fuzzy number (FFN) in an FFE to minimize the factors that needed consideration. It was found that the approaches based on intuitionistic and pythagorean fuzzy environments had limitations when applied to real-world TPs (Bhatia et al. 2023). Sahoo (2021) proposed a novel method for addressing TP that involves fuzzy parameters, particularly fermatean fuzzy ones. The method considers transportation costs, supply, and demand in a fuzzy manner, providing a unique perspective for decision-making. It addresses the uncertainty and imprecision frequently arising in transportation issues, mainly during volatile economic conditions. Kokila and Deepa (2024) proposed an innovative mathematical programming approach to solving fuzzy MOTP by transforming them into trapezoidal MOTP using fuzzy arithmetic operations. The vital objective of their method is to reduce the number of iterations needed to solve the fuzzy MOTP, enhancing computational efficiency while maintaining solution accuracy.

Given the significance of addressing uncertainties in transportation planning, several studies have proposed novel approaches that directly inform the current research. Sahoo (2023) addressed complex TP by incorporating FFP and developed a mathematical framework that transforms these fuzzy parameters into well-defined TP using score and accuracy functions. This approach not only simplifies the problem-solving process but also enhances the applicability of FFPs in real-world decision-making scenarios. The simplicity and practicality of this method are key for the current study, which aims to extend FFP-based models to MOTP. Bouraima et al. (2024) introduced a model based on fermatean fuzzy logic to support sustainable urban transportation decisions. Their approach prioritizes strategies for improving city transport sustainability by considering multiple factors, which aligns with the objectives of the current research in evaluating transportation models with a focus on sustainability and environmental impact. Incorporating fermatean fuzzy logic in their model offers valuable insights for developing a multi-objective framework that includes sustainability as a key objective, as seen in this study's focus on minimizing transportation cost, time, and carbon emissions. Chaudhary et al. (2024) proposed a multi-objective STP model using TS-PFHS parameters to account for randomness and imprecision. Their approach demonstrates the benefits of incorporating fuzzy sets to model uncertainties, which is highly relevant to the current research, as it seeks to incorporate FFPs to handle similar types of uncertainty in transportation problems. Kumar (2024) expanded the TTP frameworks by introducing type-2 intuitionistic fuzzy sets in the context of STPs. By utilizing the intuitionistic fuzzy programming approach, Kumar's work enhances the flexibility of transportation models in handling vagueness and imprecision. It directly applies to the current study, which aims to provide a more flexible and comprehensive solution to MOTP and MLMOSTP using FFSs.

In addition to the previous studies, several key contributions have further advanced the understanding of MOTP under uncertain conditions, which are highly relevant to the current research. Maity et al. (2016) and Agrawal and Singhal (2024) explored MOTPs with uncertain parameters, mainly focusing on transportation costs, supply, and demand. They introduced the concept of reliability to account for the dependability of transportation costs, recognizing that costs could fluctuate due to uncertainties in supply and demand. It aligns with the current study''s goal of incorporating fuzzy parameters to handle uncertainty in MOTP and MLMOSTP, especially in minimizing cost, time, and carbon emissions. Garg and Rizk-Allah (2021) proposed a new mathematical model for MOTP, solving it using the alpha-cuts method. Their approach, which aids decision-making in uncertain environments, contributes valuable insights for solving complex transportation problems. Abd El-Wahed (2001) and Maity and Kumar Roy (2016) developed various mathematical models under uncertain conditions and converted these models into deterministic forms using fuzzy programming approaches. Their findings demonstrated the superiority of fuzzy programming, mainly when dealing with multiple goals and constraints, over traditional interactive procedures. Fathy and Ammar (2023) and Kacher and Singh (2024) explored multi-level, multi-objective models under uncertain conditions and solved the models using interval programming techniques. Their research emphasizes the importance of addressing uncertainty in transportation systems, which resonates with the focus of this study on managing uncertainty through FFPs. Kaur et al. (2024) introduced a novel two-level STP that prioritizes transportation routes based on their importance, aiming to minimize transportation time. This hierarchical approach, which classifies routes into high-priority and low-priority categories, enhances decision-making by prioritizing the most efficient routes. It aligns with the current study's objective of developing an MLMOSTP incorporating various transportation objectives, including minimizing time and cost.

Traditional transportation methods often need help to address the uncertainties and complexities inherent in real-world scenarios, particularly in multi-objective and multi-level decision-making optimization problems. Many studies in the transportation planning sector have focused on single-level frameworks, which fail to incorporate hierarchical decision processes or effectively model uncertainties using advanced fuzzy logic techniques (Jalil et al. 2018; Rodríguez-Segade et al. 2024). While FFSs offer a promising approach to managing uncertainties, their application to complex, multi-objective, and multi-level transportation problems remains underexplored. Most existing research has not thoroughly investigated how FFSs can be integrated into MOTP and MLMOSTP under fuzzy environments. The motivation for this study stems from the need to develop a comprehensive, robust framework that can address these challenges. Specifically, this research aims to investigate the application of FFSs in solving transportation problems under FFEs, including TTP, MOTP, and MOMLSTP. The study is focused on optimizing multiple objectives, such as minimizing cost, transportation time, and carbon emissions, while simultaneously addressing uncertainties in supply, demand, and transportation parameters.

Furthermore, sustainability considerations—often neglected in traditional optimization frameworks—will be integrated into the model to enhance the practical applicability of the solution. The primary objective of this research is to develop an FFPA that can provide effective, practical solutions for optimizing transportation systems in uncertain, multi-objective, and multi-level settings. Through this, the study seeks to answer key research questions: How can FFSs be used to model and solve MOTP and MLMOSTP? How can sustainability be incorporated into the optimization framework to achieve environmentally and economically efficient solutions?

3 Preliminaries and definitions

The basic definitions of the farmatean fuzzy programming, which are used in our proposed work, which is given below:

Fermatean fuzzy sets are an advanced form introduced to handle higher degrees of uncertainty and vagueness in decision-making problems. An FFS is characterized by a membership function, a non-membership function, and a hesitation degree, which provide a more flexible framework than traditional and intuitionistic fuzzy sets.

Definition 3.1 Farmatean fuzzy sets (Senapati and Yager 2020): An FFS is a set that characterizes uncertainty and vagueness in terms of satisfaction and dissatisfaction degrees. It is formally defined as:

$$\widetilde{\mathcal{F}} = \{\langle \omega, \alpha_{\widetilde{\mathcal{F}}}(\omega), \beta_{\widetilde{\mathcal{F}}}(\omega) : \omega \in Y \rangle\}$$

where $\alpha_{\widetilde{\mathcal{F}}}(\omega) : Y \to [0,1]$ is the degree of satisfaction, and $\beta_{\widetilde{\mathcal{F}}}(\omega) : Y \to [0,1]$ is the degree of dissatisfaction, including the conditions. $0 \le \alpha_{\widetilde{\mathcal{F}}}(\omega)^3 + \beta_{\widetilde{\mathcal{F}}}(\omega)^3 \le 1 \forall \omega \in Y$. For any FFSs $\widetilde{\mathcal{F}}$ and $\omega \in Y$, $\pi_{\widetilde{\mathcal{F}}}(\omega) = \sqrt[3]{1 - (\alpha_{\widetilde{\mathcal{F}}}(\omega))^3 - (\beta_{\widetilde{\mathcal{F}}}(\omega))^3}$ is identified as the degree of indeterminacy of $\omega \in Y$ to $\widetilde{\mathcal{F}}$.

Figure 1 depicts the graphical representation of an FFS. The blue curve represents the degree of satisfaction $\alpha_{\widetilde{\mathcal{F}}}(\omega)$, the red curve represents the degree of dissatisfaction $\beta_{\widetilde{\mathcal{F}}}(\omega)$, and the green curve represents the degree of indeterminacy $\pi_{\widetilde{\mathcal{F}}}(\omega)$. The shaded areas indicate the respective degrees for each value of ω within the range [0,1].

Definition 3.2 Fermatean fuzzy number: An FFN is a particular case of an FFS where the universe of discourse is the set of real numbers $\mathbb{R}$. It represents uncertain or imprecise quantities within the real number system. A FFN is formally defined as:

$$\widetilde{A} = \{\langle y, \alpha_{\widetilde{A}}(y), \beta_{\widetilde{A}}(y) : y \in \mathbb{R} \rangle\}$$

where y is an element from the real number set $\mathbb{R}$, $\alpha_{\widetilde{A}}(y) : \mathbb{R} \to [0,1]$ is the degree of satisfaction for y, $\beta_{\widetilde{A}}(y) : \mathbb{R} \to [0,1]$ is the degree of satisfaction for y. The condition $0 \le \alpha_{\widetilde{\mathcal{F}}}(\omega)^3 + \beta_{\widetilde{\mathcal{F}}}(\omega)^3 \le 1$ holds for all $y \in \mathbb{R}$, ensuring that the degrees of satisfaction and dissatisfaction are valid and bounded.

Definition 3.3 "Let $\widetilde{\mathcal{F}} = \langle \alpha_{\widetilde{\mathcal{F}}}, \beta_{\widetilde{\mathcal{F}}} \rangle$, $\widetilde{\mathcal{F}}_1 = \langle \alpha_{\widetilde{\mathcal{F}}_1}, \beta_{\widetilde{\mathcal{F}}_1} \rangle$, and $\widetilde{\mathcal{F}}_2 = \langle \alpha_{\widetilde{\mathcal{F}}_2}, \beta_{\widetilde{\mathcal{F}}_2} \rangle$ be three FFSs on the universal set y, and $\zeta > 0$ be any scalar, then arithmetic operations of FFSs is as follows with numerical examples. Let $\widetilde{\mathcal{F}} = \langle 0.4, 0.7 \rangle$, $\widetilde{\mathcal{F}}_1 = \langle 0.8, 0.6 \rangle$ and $\widetilde{\mathcal{F}}_2 = \langle 0.2, 0.9 \rangle$ be three FFSs and $\zeta = 2$ be any scalar quantity.

3.1 The addition of two FFSs $\widetilde{\mathcal{F}}_1$ and $\widetilde{\mathcal{F}}_2$ is defined as

$$\widetilde{\mathcal{F}}_1 \oplus \widetilde{\mathcal{F}}_2 = \left(\sqrt[3]{\alpha_{\widetilde{\mathcal{F}}_1}{}^3 + \alpha_{\widetilde{\mathcal{F}}_2}{}^3 - \alpha_{\widetilde{\mathcal{F}}_1}{}^3 \alpha_{\widetilde{\mathcal{F}}_2}{}^3}, \beta_{\widetilde{\mathcal{F}}_1} \beta_{\widetilde{\mathcal{F}}_2} \right).$$

$$\begin{aligned} &\langle 0.8, 0.6 \rangle \oplus \langle 0.2, 0.9 \rangle \\ &= \left(\sqrt[3]{0.8^3 + 0.2^3 - 0.8^3 \times 0.2^3}, 0.6 \times 0.9 \right) \\ &= (0.8020, 0.54). \end{aligned}$$

3.2 The multiplication of two FFSs $\widetilde{\mathcal{F}}_1$ and $\widetilde{\mathcal{F}}_2$ is defined as

$$\widetilde{\mathcal{F}}_1 \otimes \widetilde{\mathcal{F}}_2 = \left(\alpha_{\widetilde{\mathcal{F}}_1} \alpha_{\widetilde{\mathcal{F}}_2}, \sqrt[3]{\beta_{\widetilde{\mathcal{F}}_1}{}^3 + \beta_{\widetilde{\mathcal{F}}_2}{}^3 - \beta_{\widetilde{\mathcal{F}}_1}{}^3 \beta_{\widetilde{\mathcal{F}}_2}{}^3} \right).$$

$$\langle 0.8, 0.6 \rangle \otimes \langle 0.2, 0.9 \rangle = \left(0.8 \times 0.2, \sqrt[3]{0.6^3 + 0.9^3 - 0.6^3 \times 0.9^3} \right) = (0.16, 0.923).$$

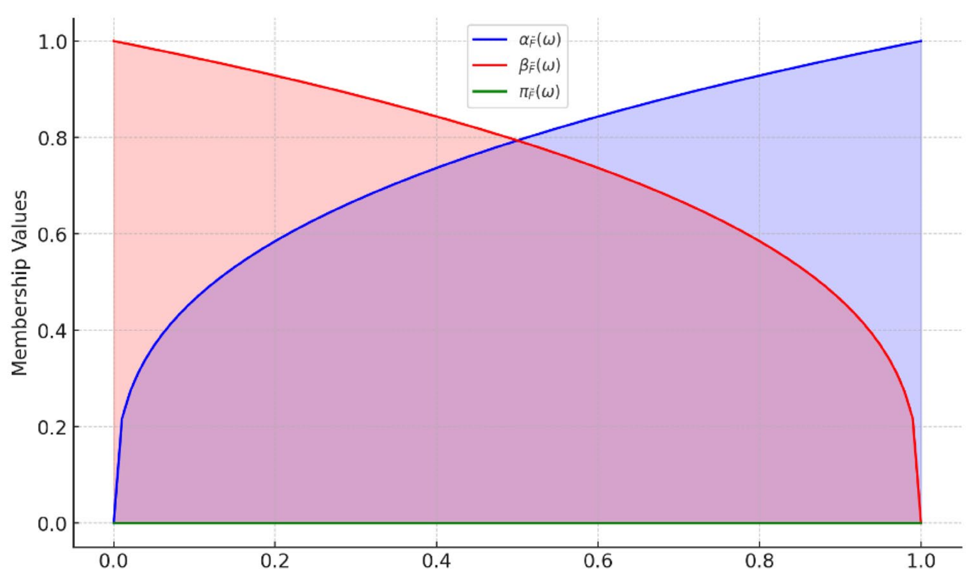

Fig. 1 Shows the graphical representation of FFSs

3.3 The scalar multiplication of an FFS $\widetilde{\mathcal{F}}$ by a scalar ζ is defined as

$$\zeta \odot \widetilde{\mathcal{F}} = \left(\sqrt[3]{1-(1-\alpha_{\widetilde{\mathcal{F}}}{}^{3})^{\zeta}}, \beta_{\widetilde{\mathcal{F}}}{}^{\zeta}\right)$$

$$2 \odot \langle 0.4, 0.7\rangle = \left(\sqrt[3]{1-(1-0.4^{3})^{2}}, 0.7^{2}\right) = (0.498, 0.49)$$

3.4 Exponentiation of an FFS $\widetilde{\mathcal{F}}$ by a scalar ζ is defined as

$$\widetilde{\mathcal{F}}^{\zeta} = \left(\alpha_{\widetilde{\mathcal{F}}}{}^{\zeta}, \sqrt[3]{1-\left(1-\beta_{\widetilde{\mathcal{F}}}{}^{3}\right)^{\zeta}}\right)$$

$$\langle 0.4, 0.7\rangle^{2} = \left(0.4^{2}, \sqrt[3]{1-\left(1-0.7^{3}\right)^{2}}\right) = (0.064, 0.828).$$

Definition 3.4 "Let $\widetilde{\mathcal{F}} = \langle \alpha_{\widetilde{\mathcal{F}}}, \beta_{\widetilde{\mathcal{F}}}\rangle$, $\widetilde{\mathcal{F}}_1 = \langle \alpha_{\widetilde{\mathcal{F}}_1}, \beta_{\widetilde{\mathcal{F}}_1}\rangle$, and $\widetilde{\mathcal{F}}_2 = \langle \alpha_{\widetilde{\mathcal{F}}_2}, \beta_{\widetilde{\mathcal{F}}_2}\rangle$ be three FFSs on the universal set Y, and $\zeta > 0$ be any scalar, then their arithmetic operations of FFSs define as follows.

3.5 The union of two FFSs $\widetilde{\mathcal{F}}_1$ and $\widetilde{\mathcal{F}}_2$ is defined as

$$\begin{aligned}\widetilde{\mathcal{F}}_1 \bigcup \widetilde{\mathcal{F}}_2 &= \left(\max\left\{\alpha_{\widetilde{\mathcal{F}}_1}, \alpha_{\widetilde{\mathcal{F}}_2}\right\}, \min\left\{\beta_{\widetilde{\mathcal{F}}_1}, \beta_{\widetilde{\mathcal{F}}_2}\right\}\right)\\ &= (\max\{\langle 0.8, 0.6\rangle\}, \min\{\langle 0.2, 0.9\rangle\}) = (0.8, 0.2)\end{aligned}$$

3.6 The intersection of two FFSs $\widetilde{\mathcal{F}}_1$ and $\widetilde{\mathcal{F}}_2$ is defined as

$$\begin{aligned}\widetilde{\mathcal{F}}_1 \bigcap \widetilde{\mathcal{F}}_2 &= \left(\min\left\{\alpha_{\widetilde{\mathcal{F}}_1}, \alpha_{\widetilde{\mathcal{F}}_2}\right\}, \max\left\{\beta_{\widetilde{\mathcal{F}}_1}, \beta_{\widetilde{\mathcal{F}}_2}\right\}\right)\\ &= (\min\{\langle 0.8, 0.6\rangle\}, \max\{\langle 0.2, 0.9\rangle\}) = (0.2, 0.6)\end{aligned}$$

3.7 The complement of an FFS $\widetilde{\mathcal{F}}$ is defined as

$$\widetilde{\mathcal{F}}^{c} = \left(\beta_{\widetilde{\mathcal{F}}}, \alpha_{\widetilde{\mathcal{F}}}\right) = \langle 0.4, 0.7\rangle^{c} = (0.7, 0.4).$$

3.8 Accuracy function

The accuracy function measures an FFS''s combined degree of satisfaction and dissatisfaction. It helps assess an FFS's overall reliability or precision in representing uncertain data.

Let $\widetilde{\mathcal{F}} = \langle \alpha_{\widetilde{\mathcal{F}}}, \beta_{\widetilde{\mathcal{F}}}\rangle$ is an FFS, then the accuracy function of FFSs can be represented as follows.

$$A_{\widetilde{\mathcal{F}}}\left(\widetilde{\mathcal{F}}\right) = \left(\alpha_{\widetilde{\mathcal{F}}}{}^{3} + \beta_{\widetilde{\mathcal{F}}}{}^{3}\right).$$

3.9 Score function

The score function for an FFS provides a single-valued representation of the set by balancing the degree of satisfaction and dissatisfaction.

Theorem 1 *Let $\widetilde{\mathcal{F}}$ be an FFS $\widetilde{\mathcal{F}} = \langle \alpha_{\widetilde{\mathcal{F}}}, \beta_{\widetilde{\mathcal{F}}}\rangle$ then the score function $\widetilde{\mathcal{F}}$ is represented as follows*:

$$S_{\widetilde{\mathcal{F}}}{}^{*}\left(\widetilde{\mathcal{F}}\right) = \frac{1}{2}\left(1+\alpha_{\widetilde{\mathcal{F}}}{}^{3}-\beta_{\widetilde{\mathcal{F}}}{}^{3}\right).\left(\min\left(\alpha_{\widetilde{\mathcal{F}}}, \beta_{\widetilde{\mathcal{F}}}\right)\right).$$

Property 1 *Consider an FFS $\widetilde{\mathcal{F}} = \langle \alpha_{\widetilde{\mathcal{F}}}, \beta_{\widetilde{\mathcal{F}}}\rangle$, then* $S_{\widetilde{\mathcal{F}}}{}^{*}\left(\widetilde{\mathcal{F}}\right) \in [0,1]$.

Proof According to the ortho-pair definition, $\alpha_{\widetilde{\mathcal{F}}}, \beta_{\widetilde{\mathcal{F}}} \in [0,1]$. Then, $\min\left(\alpha_{\widetilde{\mathcal{F}}}, \beta_{\widetilde{\mathcal{F}}}\right) \in [0,1]$, and also $\alpha_{\widetilde{\mathcal{F}}}{}^{3} \geq o$, $\beta_{\widetilde{\mathcal{F}}}{}^{3} \geq 0, \alpha_{\widetilde{\mathcal{F}}}{}^{3} \leq 1$, $\beta_{\widetilde{\mathcal{F}}}{}^{3} \leq 1$;

$\Rightarrow 1-\beta_{\widetilde{\mathcal{F}}}{}^{3} \geq 0 \Rightarrow 1+\alpha_{\widetilde{\mathcal{F}}}{}^{3}-\beta_{\widetilde{\mathcal{F}}}{}^{3} \geq 0, \therefore \frac{1}{2}\left(1+\alpha_{\widetilde{\mathcal{F}}}{}^{3}-\beta_{\widetilde{\mathcal{F}}}{}^{3}\right).\left(\min\left(\alpha_{\widetilde{\mathcal{F}}}, \beta_{\widetilde{\mathcal{F}}}\right)\right) \geq 0$, again $\alpha_{\widetilde{\mathcal{F}}}{}^{3}-\beta_{\widetilde{\mathcal{F}}}{}^{3} \leq 1$, add one both sides.

$\Rightarrow 1+\alpha_{\widetilde{\mathcal{F}}}{}^{3}-\beta_{\widetilde{\mathcal{F}}}{}^{3} \leq 2(\because \alpha_{\widetilde{\mathcal{F}}}{}^{3} \geq 0) \Rightarrow \frac{1}{2}\left(1+\alpha_{\widetilde{\mathcal{F}}}{}^{3}-\beta_{\widetilde{\mathcal{F}}}{}^{3}\right).$
$\left(\min\left(\alpha_{\widetilde{\mathcal{F}}}, \beta_{\widetilde{\mathcal{F}}}\right)\right) \leq 1\left(\because \min\left(\alpha_{\widetilde{\mathcal{F}}}, \beta_{\widetilde{\mathcal{F}}}\right) \leq 1\right)$

Hence, $S_{\widetilde{\mathcal{F}}}{}^{*}\left(\widetilde{\mathcal{F}}\right) \in [0,1]$.

Theorem 2 *Let $\widetilde{\mathcal{F}}$ be an FFS $\widetilde{\mathcal{F}} = \langle \alpha_{\widetilde{\mathcal{F}}}, \beta_{\widetilde{\mathcal{F}}}\rangle$ then the NFFSF $\widetilde{\mathcal{F}}_{1d}$ represented simply as follows*;

$$S_{\widetilde{\mathcal{F}}}{}^{*}\left(\widetilde{\mathcal{F}}_{1d}\right) = \frac{1}{2}\left(1+\alpha_{\widetilde{\mathcal{F}}}-\beta_{\widetilde{\mathcal{F}}}\right).\left(\min\left(\alpha_{\widetilde{\mathcal{F}}}, \beta_{\widetilde{\mathcal{F}}}\right)\right)^{2}.$$

Property 2 *Consider an FFS $\widetilde{\mathcal{F}} = \langle \alpha_{\widetilde{\mathcal{F}}}, \beta_{\widetilde{\mathcal{F}}}\rangle$, then* $S_{\widetilde{\mathcal{F}}}{}^{*}\left(\widetilde{\mathcal{F}}_{1d}\right) \in [0,1]$.

Proof According to the ortho-pair definition, $\alpha_{\widetilde{\mathcal{F}}}, \beta_{\widetilde{\mathcal{F}}} \in [0,1]$. Then, $\min\left(\alpha_{\widetilde{\mathcal{F}}}, \beta_{\widetilde{\mathcal{F}}}\right) \in [0,1]$, and also $\alpha_{\widetilde{\mathcal{F}}} \geq o$, $\beta_{\widetilde{\mathcal{F}}} \geq 0$, $\alpha_{\widetilde{\mathcal{F}}}, \beta_{\widetilde{\mathcal{F}}} \leq 1; \Rightarrow 1-\beta_{\widetilde{\mathcal{F}}} \geq 0 \Rightarrow 1+\alpha_{\widetilde{\mathcal{F}}}-\beta_{\widetilde{\mathcal{F}}} \geq 0 \therefore \frac{1}{2}\left(1+\alpha_{\widetilde{\mathcal{F}}}-\beta_{\widetilde{\mathcal{F}}}\right).\left(\min\left(\alpha_{\widetilde{\mathcal{F}}}, \beta_{\widetilde{\mathcal{F}}}\right)\right)^{2} \geq 0$, again, $\alpha_{\widetilde{\mathcal{F}}} \leq 1$, and $\beta_{\widetilde{\mathcal{F}}} \leq 1$, $\alpha_{\widetilde{\mathcal{F}}}-\beta_{\widetilde{\mathcal{F}}} \leq 1$, add one both sides. $\Rightarrow 1+\alpha_{\widetilde{\mathcal{F}}}-\beta_{\widetilde{\mathcal{F}}} \leq 2 \Rightarrow \left(\min\left(\alpha_{\widetilde{\mathcal{F}}}, \beta_{\widetilde{\mathcal{F}}}\right) \leq 1\right)$
$\Rightarrow \left(\min\left(\alpha_{\widetilde{\mathcal{F}}}, \beta_{\widetilde{\mathcal{F}}}\right)\right)^{2} \leq 1 \Rightarrow \frac{1}{2}\left(1+\alpha_{\widetilde{\mathcal{F}}}-\beta_{\widetilde{\mathcal{F}}}\right).\left(\min\left(\alpha_{\widetilde{\mathcal{F}}}, \beta_{\widetilde{\mathcal{F}}}\right)\right)^{2}$
$\leq 1, \left(\because \left(\min\left(\alpha_{\widetilde{\mathcal{F}}}, \beta_{\widetilde{\mathcal{F}}}\right)\right)^{2} \leq 1\right)$

Hence, $S_{\widetilde{\mathcal{F}}}{}^{*}\left(\widetilde{\mathcal{F}}_{1d}\right) \in [0,1]$.

Theorem 3 *Let $\widetilde{\mathcal{F}}$ be a FFSs $\widetilde{\mathcal{F}} = \langle \alpha_{\widetilde{\mathcal{F}}}, \beta_{\widetilde{\mathcal{F}}}\rangle$ then the Type* 1 *score function $\widetilde{\mathcal{F}}_1$ represented as follows*:

Type-1 fermatean fuzzy score function $S_{\widetilde{\mathcal{F}}}^{*}\left(\widetilde{\mathcal{F}}_{11}\right)=\frac{1}{2}\left(1+\alpha_{\widetilde{\mathcal{F}}}^{2}-\beta_{\widetilde{\mathcal{F}}}^{2}\right)$.

According to the ortho-pair definition, $\alpha_{\widetilde{\mathcal{F}}},\beta_{\widetilde{\mathcal{F}}}\in[0,1]$, *and* $\alpha_{\widetilde{\mathcal{F}}}^{2}\geq o$, $\beta_{\widetilde{\mathcal{F}}}^{2}\geq 0$, $\alpha_{\widetilde{\mathcal{F}}}^{2}\leq 1$, *and* $\beta_{\widetilde{\mathcal{F}}}^{2}\leq 1$; $\Rightarrow 1-\beta_{\widetilde{\mathcal{F}}}^{2}\geq 0\Rightarrow 1+\alpha_{\widetilde{\mathcal{F}}}^{2}-\beta_{\widetilde{\mathcal{F}}}^{2}\geq 0 \therefore \frac{1}{2}\left(1+\alpha_{\widetilde{\mathcal{F}}}^{2}-\beta_{\widetilde{\mathcal{F}}}^{2}\right)\geq 0$, *Now, again* $\alpha_{\widetilde{\mathcal{F}}}^{2}-\beta_{\widetilde{\mathcal{F}}}^{2}\leq 1$, *add one both sides.*

$$\Rightarrow 1+\alpha_{\widetilde{\mathcal{F}}}^{2}-\beta_{\widetilde{\mathcal{F}}}^{2}\geq 2(\because \alpha_{\widetilde{\mathcal{F}}}^{2}\geq 0)$$

$$\Rightarrow \frac{1}{2}\left(1+\alpha_{\widetilde{\mathcal{F}}}^{2}-\beta_{\widetilde{\mathcal{F}}}^{2}\right)\geq 1\left(\because\langle\alpha_{\widetilde{\mathcal{F}}},\beta_{\widetilde{\mathcal{F}}}\rangle\leq 1\right)$$

Hence, $S_{\widetilde{\mathcal{F}}}^{*}\left(\widetilde{\mathcal{F}}_{11}\right)\in[0,1]$. *Similarly,*

Type-2 fermatean fuzzy score function $S_{\widetilde{\mathcal{F}}}^{*}\left(\widetilde{\mathcal{F}}_{12}\right)=\frac{1}{3}\left(1+2\alpha_{\widetilde{\mathcal{F}}}^{3}-\beta_{\widetilde{\mathcal{F}}}^{3}\right)$.

Type -3 fermatean fuzzy score function $S_{\widetilde{\mathcal{F}}}^{*}\left(\widetilde{\mathcal{F}}_{13}\right)=\frac{1}{2}\left(1+\alpha_{\widetilde{\mathcal{F}}}^{2}-\beta_{\widetilde{\mathcal{F}}}^{2}\right).\left|\alpha_{\widetilde{\mathcal{F}}}-\beta_{\widetilde{\mathcal{F}}}\right|$

"*Let* $\widetilde{\mathcal{F}}_1=\langle\alpha_{\widetilde{\mathcal{F}}_1},\beta_{\widetilde{\mathcal{F}}_1}\rangle$, *and* $\widetilde{\mathcal{F}}_2=\langle\alpha_{\widetilde{\mathcal{F}}_2},\beta_{\widetilde{\mathcal{F}}_2}\rangle$ *be two FFSs, then the following operations will be satisfied*:

$S_{\widetilde{\mathcal{F}}}^{*}\left(\widetilde{\mathcal{F}}_1\right)\geq S_{\widetilde{\mathcal{F}}}^{*}\left(\widetilde{\mathcal{F}}_2\right)$ *with* $A_{\widetilde{\mathcal{F}}}\left(\widetilde{\mathcal{F}}_1\right)>A_{\widetilde{\mathcal{F}}}\left(\widetilde{\mathcal{F}}_2\right)$ *iff* $\widetilde{\mathcal{F}}_1>\widetilde{\mathcal{F}}_2$.

$S_{\widetilde{\mathcal{F}}}^{*}\left(\widetilde{\mathcal{F}}_1\right)\leq S_{\widetilde{\mathcal{F}}}^{*}\left(\widetilde{\mathcal{F}}_2\right)$ *with* $A_{\widetilde{\mathcal{F}}}\left(\widetilde{\mathcal{F}}_1\right)<A_{\widetilde{\mathcal{F}}}\left(\widetilde{\mathcal{F}}_2\right)$ *iff* $\widetilde{\mathcal{F}}_1<\widetilde{\mathcal{F}}_2$.

$S_{\widetilde{\mathcal{F}}}^{*}\left(\widetilde{\mathcal{F}}_1\right)=S_{\widetilde{\mathcal{F}}}^{*}\left(\widetilde{\mathcal{F}}_2\right)$ *with* $A_{\widetilde{\mathcal{F}}}\left(\widetilde{\mathcal{F}}_1\right)=A_{\widetilde{\mathcal{F}}}\left(\widetilde{\mathcal{F}}_2\right)$ *iff* $\widetilde{\mathcal{F}}_1=\widetilde{\mathcal{F}}_2$.

Example 1 Let $\widetilde{\mathcal{F}}_1=\langle 0.7,0.6\rangle$ and $\widetilde{\mathcal{F}}_2=\langle 0.8,0.5\rangle$ be two FFSs; then, we will see the following mathematical operations:

By using the score function $S_{\widetilde{\mathcal{F}}}^{*}\left(\widetilde{\mathcal{F}}\right)=\frac{1}{2}\left(1+\alpha_{\widetilde{\mathcal{F}}}^{3}-\beta_{\widetilde{\mathcal{F}}}^{3}\right).\left(\min\left(\alpha_{\widetilde{\mathcal{F}}},\beta_{\widetilde{\mathcal{F}}}\right)\right)$.

$$S_{\widetilde{\mathcal{F}}}^{*}\left(\widetilde{\mathcal{F}}_1\right)=\frac{1}{2}\left(1+0.7^3-0.6^3\right).(\min(0.7,0.6))=0.337$$

$$S_{\widetilde{\mathcal{F}}}^{*}\left(\widetilde{\mathcal{F}}_2\right)=\frac{1}{2}\left(1+0.8^3-0.5^3\right).(\min(0.8,0.5))=0.346$$

Hence $S_{\widetilde{\mathcal{F}}}^{*}\left(\widetilde{\mathcal{F}}_1\right)<S_{\widetilde{\mathcal{F}}}^{*}\left(\widetilde{\mathcal{F}}_1\right)\Rightarrow\widetilde{\mathcal{F}}_1<\widetilde{\mathcal{F}}_2$. The score function $S_{\widetilde{\mathcal{F}}}^{*}\left(\widetilde{\mathcal{F}}\right)$ compares FFSs by evaluating their membership and non-membership values. In this example, $\widetilde{\mathcal{F}}_1$ is ranked lower than $\widetilde{\mathcal{F}}_2$ based on the score function values, indicating that $\widetilde{\mathcal{F}}_2$ is preferred.

Example 2 Let $\widetilde{\mathcal{F}}_1=\langle 0.9,0.8\rangle$ and $\widetilde{\mathcal{F}}_2=\langle 0.6,0.5\rangle$ be two FFS; then the following operations are represented.

By using this score function, $S_{\widetilde{\mathcal{F}}}^{*}\left(\widetilde{\mathcal{F}}\right)=\frac{1}{2}\left(1+\alpha_{\widetilde{\mathcal{F}}}^{3}-\beta_{\widetilde{\mathcal{F}}}^{3}\right).\left(\min\left(\alpha_{\widetilde{\mathcal{F}}},\beta_{\widetilde{\mathcal{F}}}\right)\right)$.

$$S_{\widetilde{\mathcal{F}}}^{*}\left(\widetilde{\mathcal{F}}_1\right)=\frac{1}{2}\left(1+0.9^3-0.8^3\right).(\min(0.9,0.8))=0.486$$

$$S_{\widetilde{\mathcal{F}}}^{*}\left(\widetilde{\mathcal{F}}_2\right)=\frac{1}{2}\left(1+0.6^3-0.5^3\right).(\min(0.6,0.5))=0.022$$

Hence $S_{\widetilde{\mathcal{F}}}^{*}\left(\widetilde{\mathcal{F}}_1\right)>S_{\widetilde{\mathcal{F}}}^{*}\left(\widetilde{\mathcal{F}}_1\right)\Rightarrow\widetilde{\mathcal{F}}_1>\widetilde{\mathcal{F}}_2$. In this example, the score function is applied to two FFSs, $\widetilde{\mathcal{F}}_1$ and $\widetilde{\mathcal{F}}_2$. The score values indicate that $\widetilde{\mathcal{F}}_1$ has a higher score than $\widetilde{\mathcal{F}}_2$, implying that $\widetilde{\mathcal{F}}_1$ is preferred over $\widetilde{\mathcal{F}}_2$.

4 Formulation of mathematical models

4.1 Mathematical model of traditional transportation problem

The mathematical model of TTP is represented as follows:

$$\begin{aligned}&Minf=\sum_{i=1}^{M}\sum_{j=1}^{N}C_{ij}y_{ij}\\&S.t\\&\sum_{j=1}^{N}y_{ij}\leq s_i(i=1,2,...,M)\\&\sum_{i=1}^{M}y_{ij}\geq d_j(j=1,2,....,N)\\&y_{ij}\geq 0,\forall i=1,2,...,M,j=1,2,....,N\end{aligned}\tag{1}$$

We apply the FFPs in the above mathematical model under the FFE. The mathematical model of TTP with FFPs is represented as follows:

$$\begin{aligned}&Minf^{*}=\sum_{i=1}^{M}\sum_{j=1}^{N}C_{ij}^{\widetilde{\mathcal{F}}}.y_{ij}\\&S.t\\&\sum_{j=1}^{N}y_{ij}\leq s_i^{\widetilde{\mathcal{F}}},(i=1,2,...,M)\\&\sum_{i=1}^{M}y_{ij}\geq d_j^{\widetilde{\mathcal{F}}},(j=1,2,....,N)\end{aligned}\tag{2}$$

Such that

$$s_i^{\widetilde{\mathcal{F}}} = \left(\alpha_{\widetilde{s}_i}, \beta_{\widetilde{s}_i}\right) \; where \quad 0 \le \alpha_{\widetilde{s}_i}{}^3 + \beta_{\widetilde{s}_i}{}^3 \le 1,$$

$$d_j^{\widetilde{\mathcal{F}}} = \left(\alpha_{\widetilde{d}_j}, \beta_{\widetilde{d}_j}\right) \; where \quad 0 \le \alpha_{\widetilde{d}_j}{}^3 + \beta_{\widetilde{d}_j}{}^3 \le 1,$$

$$C_{ij}^{\widetilde{\mathcal{F}}} = \left(\alpha_{\widetilde{C}_{ij}}, \beta_{\widetilde{C}_{ij}}\right) \; where \quad 0 \le \alpha_{\widetilde{C}_{ij}}{}^3 + \beta_{\widetilde{C}_{ij}}{}^3 \le 1,$$

$$y_{ij} \ge 0, \forall i = 1,2,\dots,M, j = 1,2,\dots.,N.$$

Now, we convert the mathematical model of TTP with FFPs represented in Eq. 2 into the crisp form using the NFFSF under the FFE. The crisp mathematical model of TTP is represented as follows:

$$\begin{aligned} &Minf^* = \sum_{i=1}^{M}\sum_{j=1}^{N} S\left(C_{ij}^{\widetilde{\mathcal{F}}}\right).y_{ij} \\ &S.t \\ &\sum_{j=1}^{N} y_{ij} \le S\left(s_i^{\widetilde{\mathcal{F}}}\right), (i = 1, 2, \dots, M) \\ &\sum_{i=1}^{M} y_{ij} \ge S(d_j^{\widetilde{\mathcal{F}}}), (j = 1, 2\dots., N) \\ &y_{ij} \ge 0, \forall i = 1, 2, \dots, M, j = 1, 2, \dots., N \end{aligned} \tag{3}$$

4.2 Mathematical model of multi-objective transportation problem

The mathematical model of a MOTP with FFPs under the FFEs is represented as follows:

$$\begin{aligned} &Minf_t^* = \sum_{i=1}^{M}\sum_{j=1}^{N} \left(C_{ij}^{\widetilde{\mathcal{F}}}\right)_t .y_{ij}, \forall t = 1, 2, \dots, T \\ &S.t \\ &\sum_{j=1}^{N} y_{ij} \le s_i^{\widetilde{\mathcal{F}}}, (i = 1, 2, \dots, M) \\ &\sum_{i=1}^{M} y_{ij} \ge d_j^{\widetilde{\mathcal{F}}}, (j = 1, 2\dots., N) \end{aligned} \tag{4}$$

Such that

$$s_i^{\widetilde{\mathcal{F}}} = \left(\alpha_{\widetilde{s}_i}, \beta_{\widetilde{s}_i}\right) \; where \quad 0 \le \alpha_{\widetilde{s}_i}{}^3 + \beta_{\widetilde{s}_i}{}^3 \le 1,$$

$$d_j^{\widetilde{\mathcal{F}}} = \left(\alpha_{\widetilde{d}_j}, \beta_{\widetilde{d}_j}\right) \; where \quad 0 \le \alpha_{\widetilde{d}_j}{}^3 + \beta_{\widetilde{d}_j}{}^3 \le 1,$$

$$C_{ij}^{\widetilde{\mathcal{F}}} = \left(\alpha_{\widetilde{C}_{ij}}, \beta_{\widetilde{C}_{ij}}\right) \text{ where } \quad 0 \le \alpha_{\widetilde{C}_{ij}}{}^3 + \beta_{\widetilde{C}_{ij}}{}^3 \le 1,$$

$$y_{ij} \ge 0, \forall i = 1,2,\dots,M, j = 1,2,\dots.,N.$$

Where $s_i^{\widetilde{\mathcal{F}}} = (\alpha_{\widetilde{s}_i}, \beta_{\widetilde{s}_i})$ units are available at the i^{th} supply node, and $d_j^{\widetilde{\mathcal{F}}} = (\alpha_{\widetilde{d}_j}, \beta_{\widetilde{d}_j})$ units are in demand on the j^{th} demand node. Let the transportation cost $C_{ij}^{\widetilde{\mathcal{F}}} = \left(\alpha_{\widetilde{C}_{ij}}, \beta_{\widetilde{C}_{ij}}\right)$ is the unit fermatean fuzzy transportation cost and the i^{th} source node to the j^{th} demand node, and δ_{ij} is the number of items that are carried from the i^{th} source node to the j^{th} demand node. Now, we convert the mathematical model of MOTP with FFPs, represented in Eq. 4, into the crisp form using the proposed NFFSF. The crisp mathematical model of MOTP is represented as follows:

$$\begin{aligned} &Minf_t^* = \sum_{i=1}^{M}\sum_{j=1}^{N} S\left(\left(C_{ij}^{\widetilde{\mathcal{F}}}\right)_t\right).y_{ij}, t = 1, 2, \dots, T \\ &S.t \\ &\sum_{j=1}^{N} y_{ij} \le S\left(s_i^{\widetilde{\mathcal{F}}}\right), (i = 1, 2, \dots, M) \\ &\sum_{i=1}^{M} y_{ij} \ge S\left(d_j^{\widetilde{\mathcal{F}}}\right), (j = 1, 2\dots., N) \\ &y_{ij} \ge 0, \forall i = 1,2,\dots,M, j = 1,2,\dots.,N. \end{aligned} \tag{5}$$

4.2.1 *Mathematical model of multi-level, multi-objective solid transportation problem*

The mathematical model of MLMOSTP is an advanced extension of the TTP, incorporating multiple decision-making levels and objectives and considering solid (or bulk) goods rather than discrete units. MLMOSTP models the transportation network across several hierarchical levels, such as central depots, regional warehouses, and local distribution centers. Each level aims to optimize its specific objectives, including minimizing transportation costs, minimizing transportation time, minimizing carbon emissions, minimizing delivery time, maximizing service levels, and balancing load capacities. The objectives often conflict, requiring trade-offs to achieve an optimal solution. The mathematical formulation of MLMOSTP involves defining decision variables that represent the quantity of goods transported between various nodes at different levels, subject to supply and demand constraints at each node, capacity restrictions, and specific transportation policies. The mathematical model of MLMOSTP aims to optimize multiple objectives across multiple levels of decision-making. The mathematical model of an MLMOSTP is represented as follows:

For level I

$$f_1 = \min_{\bar{y}_1} \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{k=1}^{K} C^1_{ijk}.y_{ijk}$$

For level II

$$f_2 = \min_{\bar{y}_2} \sum_{i=1}^{N} \sum_{j=1}^{M} \sum_{k=1}^{K} C^2_{ijk}.y_{ijk}$$

$$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$$

For level t

$$f_t = \min_{\bar{y}_t} \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} C^t_{ijk}.y_{ijk} \tag{6}$$

S.t

$$\sum_{j=1}^{N} \sum_{k=1}^{K} y_{ijk} \le s_i, i = 1, 2, ..., M$$

$$\sum_{i=1}^{M} \sum_{k=1}^{K} y_{ijk} \le d_j, j = 1, 2, ..., N$$

$$\sum_{i=1}^{M} \sum_{j=1}^{N} y_{ijk} \le e_k, k = 1, 2, ..., K$$

$$y_{ijk} \ge 0, i = 1, 2, ..., Mj = 1, 2, ..., Nk = 1, 2, ..., K.$$

The presented mathematical model of an MLMOSTP addresses the optimization of transportation activities across different levels of a transportation network. At each level of t, the objective is to minimize the total cost of transportation, where y_{ijk} represents the quantity of goods transported from the i^{th} supplier to the j^{th} demand point via the k^{th} transportation mode. The cost of transportation is represented by C^t_{ijk}, which is specific to each level t of the objective function. Each level t is subject to constraints ensuring that the total quantity transported from suppliers to demand points does not exceed supplier capacities, the total demand at each destination is met, and the total transportation capacity for each mode is not exceeded. This hierarchical model allows for optimization at different decision-making levels, reflecting the complexity and hierarchy present in real-world transportation systems.

We use the FFPs in the mathematical model of MLMOSTP, represented in Eq. (6). The mathematical model of MLMOSTP with FFPs under the FFE is represented as follows:

For level I

$$f_1 = \min_{\bar{y}_1} \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{k=1}^{K} \left(C^1_{ijk}\right)^{\tilde{\mathcal{F}}}.y_{ijk}$$

For level II

$$f_2 = \min_{\bar{y}_2} \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{k=1}^{K} \left(C^2_{ijk}\right)^{\tilde{\tilde{\mathcal{F}}}}.y_{ijk}$$

$$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$$

For level t

$$f_t = \min_{\bar{y}_t} \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{k=1}^{K} \left(C^t_{ijk}\right)^{\tilde{\tilde{\mathcal{F}}}}.y_{ijk} \tag{7}$$

S.t

$$\sum_{j=1}^{N} \sum_{k=1}^{K} y_{ijk} \le s_i^{\tilde{\mathcal{F}}}, i = 1, 2, ..., M$$

$$\sum_{i=1}^{M} \sum_{k=1}^{K} y_{ijk} \le d_j^{\tilde{\mathcal{F}}}, j = 1, 2, ..., N$$

$$\sum_{i=1}^{M} \sum_{j=1}^{N} y_{ijk} \le e_k^{\tilde{\mathcal{F}}}, k = 1, 2, ..., K$$

Such that.

$$s_i^{\tilde{\mathcal{F}}} = (\alpha_{\tilde{s}_i}, \beta_{\tilde{s}_i}) \text{ where } 0 \le {\alpha_{\tilde{s}_i}}^3 + {\beta_{\tilde{s}_i}}^3 \le 1,$$
$$d_j^{\tilde{\mathcal{F}}} = (\alpha_{\tilde{d}_j}, \beta_{\tilde{d}_j}) \text{ where } 0 \le {\alpha_{\tilde{d}_j}}^3 + {\beta_{\tilde{d}_j}}^3 \le 1,$$
$$e_k^{\tilde{\mathcal{F}}} = (\alpha_{\tilde{e}_k}, \beta_{\tilde{e}_k}) \text{ where } 0 \le {\alpha_{\tilde{e}_k}}^3 + {\beta_{\tilde{e}_k}}^3 \le 1,$$
$$(C^t_{ijk})^{\tilde{\tilde{\mathcal{F}}}} = \left(\alpha_{(\tilde{C}^t_{ijk})}, \beta_{(\tilde{C}^t_{ijk})}\right) \text{where } 0 \le {\alpha_{(\tilde{C}^t_{ijk})}}^3 + {\beta_{(\tilde{C}^t_{ijk})}}^3 \le 1,$$
$$y_{ijk} \ge 0, i = 1,2, ..., I, j = 1,2, ..., J, k = 1,2, ..., K.$$

This mathematical model cannot be directly solved because the FFPs are involved in this mathematical model. Now convert the mathematical model of MLMOSTP into the crisp form using the new fematean fuzzy score function under the FFE. The crisp mathematical model of the MLMOSTP is represented as follows:

For level I

$$f_1 = \min_{\bar{y}_1} \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{k=1}^{K} S\left(C^1_{ijk}\right)^{\tilde{\tilde{\mathcal{F}}}} . y_{ijk}$$

For level II

$$f_2 = \min_{\bar{y}_2} \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{k=1}^{K} S\left(C^2_{ijk}\right)^{\tilde{\tilde{\mathcal{F}}}} . y_{ijk}$$

$$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$$

For level t

$$f_t = \min_{\bar{y}_t} \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} S\left(C^t_{ijk}\right)^{\tilde{\tilde{F}}} . y_{ijk} \tag{8}$$

S.t

$$\sum_{j=1}^{N} \sum_{k=1}^{K} y_{ijk} \leq S\left(s_i^{\tilde{\mathcal{F}}}\right), i = 1, 2, \ldots, M$$

$$\sum_{i=1}^{M} \sum_{k=1}^{K} y_{ijk} \leq S\left(d_j^{\tilde{\mathcal{F}}}\right), j = 1, 2, \ldots, N$$

$$\sum_{i=1}^{M} \sum_{j=1}^{N} y_{ijk} \leq S\left(e_k^{\tilde{\mathcal{F}}}\right), k = 1, 2, \ldots, K$$

$$y_{ijk} \geq 0, i = 1, 2, \ldots, M, j = 1, 2, \ldots, N, k = 1, 2, \ldots, K.$$

5 Proposed fermatean fuzzy programming approach

The complexity of real-world decision-making problems often involves multiple objectives and hierarchical decision levels, compounded by uncertainties and imprecise information. To address these challenges, the FFPA offers a robust framework by incorporating FFSs, which handle higher degrees of uncertainty more effectively than intuitionistic and pythagorean fuzzy sets. This approach integrates fuzzy logic with multi-objective optimization, enabling a comprehensive representation of uncertainty through membership and non-membership functions. The following sections provide a detailed explanation and mathematical formulation of the FFPA for solving multi-objective and MLMOSTP under uncertainty.

Senapati and Yager (2020) introduced FFSs as an extension of intuitionistic fuzzy sets. They compared them comprehensively with pythagorean and intuitionistic fuzzy sets when the sum of truth and false grades exceeds 1. However, the truth grade and false grade square sum is less than or equal to 1. FFS is considered more realistic and capable of handling more significant uncertainty than intuitionistic and pythagorean fuzzy sets. They discussed the fundamental properties of FFSs, including the complement operator and the entire set of operations. Silambarasan (2020) examined the algebraic properties of these operators, providing valuable insights into the mathematical foundation of FFSs. He expands upon the theoretical framework of FFSs and provides a deeper understanding of their operational characteristics. Akram et al. (2022) evaluated interval-valued FFSs as a robust approach for handling uncertain and incomplete data. They also proposed a novel method for directly solving interval-valued fermatean fuzzy fractional TP, avoiding the need to convert the original problem into a crisp equivalent, streamlining the solution process. It enhances the resilience and efficiency of addressing uncertainties in TP.

Zimmermann (1978) applied fuzzy linear programming (FLP) to the linear vector maximum issue. Zimmermann discusses the effects of using different techniques to combine distinct objective functions to find the best solution. He also provides valuable insights into the effectiveness of FLP in tackling multi-objective optimization problems (MOOP) and offers guidance on selecting appropriate approaches for achieving optimal compromise solutions. This approach utilizes linear, exponential, or hyperbolic truth functions to achieve optimal solutions to problems through compromise. An intuitionistic fuzzy programming approach has been developed for MOOP. This environment allows truth and false grades to be represented as linear, exponential, or hyperbolic functions. Similar challenges in a fuzzy environment can also be solved with the pythagorean fuzzy programming approach. The nonlinear programming method, FFPA, is now presented to find a compromise optimal solution for MOOP in FFE and other contexts. This approach allows simultaneous consideration of all objectives and is defined as follows:

For the objective function $f_t^*(y)$, FFPA incorporates upper bounds U_t and lower bounds L_t. It involves the membership function $\mu\left(f_t^*(y)\right)$ and non-membership function $\theta\left(f_t^*(y)\right)$ for the objective function $f_t^*(y)$. This model aims to optimize decision-making under uncertainty, leveraging FFSs to handle imprecision and uncertainty in objective functions subject to constraints. Including upper and lower bounds and membership and non-membership functions allows for a comprehensive representation of uncertainty, enabling robust decision-making in scenarios lacking precise information. Then, the proposed mathematical model for FFPA is represented as follows:

$$
\begin{aligned}
& Max\,\lambda \delta_1^3 - \delta_2^3 \\
& S.t \\
& \mu\left(f_t^*(y)\right)^3 \geq \delta_1^3, \forall t \\
& \theta\left(f_t^*(y)\right)^3 \leq \delta_2^3, \forall t \\
& \mu\left(f_t^*(y)\right) = \begin{cases} 1, & if\, f_t^*(y) \leq L_t \\ \frac{U_t - f_t^*(y)}{U_t - L_t} & if\, L_t \leq f_t^*(\lambda) \leq U_t \\ 0, & if\, f_t^*(y) \geq U_t \end{cases} \\
& \theta\left(f_t^*(y)\right) = \begin{cases} 0, & if\, f_t^*(y) \leq L_t \\ \frac{f_t^*(y) - L_t}{U_t - L_t} & if\, L_t \leq f_t^*(\lambda) \leq U_t \\ 1, & if\, f_t^*(y) \geq U_t \end{cases} \\
& i.e., (U_t - f_t^*(y))^3 \geq d_t^3 \delta_1^3, \\
& \left(f_t^*(y) - L_t\right)^3 \leq d_t^3 \delta_2^3, \\
& where\; d_t = U_t - L_t \\
& y_{11} + y_{12} + \ldots\ldots\ldots\ldots\ldots\ldots + y_{1N} \leq s_1 \\
& y_{21} + y_{22} + \ldots\ldots\ldots\ldots\ldots\ldots + y_{2N} \leq s_2 \\
& \vdots \\
& y_{M1} + y_{M2} + \ldots\ldots\ldots\ldots\ldots\ldots + y_{MN} \leq s_M \\
& y_{11} + y_{21} + \ldots\ldots\ldots\ldots\ldots\ldots + y_{M1} \leq d_1 \\
& y_{12} + y_{22} + \ldots\ldots\ldots\ldots\ldots\ldots + y_{M2} \leq d_2 \\
& \vdots \\
& y_{1M} + y_{2M} + \ldots\ldots\ldots\ldots\ldots\ldots + y_{NM} \leq d_N \\
& \sum_{i=1}^{M} s_i = \sum_{j=1}^{N} d_j, y_{ij} \geq 0, 0 \leq \delta_1^3, \delta_2^3 \leq 1, \leq \delta_1^3 + \delta_2^3 \leq 1, \delta_1^3 \geq \delta_2^3.
\end{aligned}
\tag{9}
$$

6 Solution methodology

We propose a comprehensive methodology for addressing the mathematical models of MOTP and MLMOST within the FFPA framework. The methodology enhances efficiency and robustness in solving MOTP and MLMOST instances. The following essential steps are part of the suggested methodology:

Step 1 Formulate the MOTP and MLMOST model using FFPs within the FFE.

Step 2 Then, convert the mathematical model of MOTP and MLMOST into the crisp form using the NFFSF.

Step 3 At this point, individually deal with this problem for all objectives. We obtain possible primary responses for every objective function.

Step 4 Develop a pay-off matrix to capture objective-performance relationships in the FFE. Calculate upper U_t and lower L_t bounds for each objective $f_t^*(y)$ using fermatean fuzzy aggregation techniques applied to the pay-off matrix.

Step 5 A problem model will be built using the proposed FFPA and solved using the SciPy optimization library in the Python programming language. In Fig. 2, the architecture of the proposed methodology is displayed.

7 Numerical illustrations

To demonstrate the mathematical model of the multi-objective transportation problem and multi-level, multi-objective solid transportation problem is presented with the following description and data.

7.1 Numerical example 1

In this numerical example, we aim to minimize three key objectives: total transportation cost, time, and carbon emissions. In this framework, we consider a network of suppliers and demand places where adequate transportation of commodities is required. Each supplier has associated transportation costs, times, and carbon emissions for delivering goods to each demand point. These parameters are expressed as FFPs, representing the uncertainty inherent in real-world transportation scenarios. We employ the NFFSF to facilitate analysis and convert the fermatean fuzzy data into a crisp form. This conversion enables us to quantify and optimize our objectives effectively. For instance, the total transportation cost objective is calculated by summing the crisp transportation costs from each supplier to each demand point.

Similarly, the total transportation time and total carbon emissions objectives are determined. By integrating fuzzy parameter handling with multi-objective optimization, our approach offers a robust methodology for tackling transportation logistics problems, promoting efficiency and sustainability in the transportation sector. Tables 1, 2, 3, 4 and 5 represents the fermatean fuzzy data of the proposed problem. The values represented in Tables 1, 2, 3, 4 and 5 are the fermatean fuzzy evaluations of transportation costs, time, carbon emission, supply and demand between sources (α_i) and destinations (β_j). The first value indicates the degree of satisfaction, while the second represents dissatisfaction, offering a balanced view under FFEs.

We apply the NFFSF in Tables 1, 2, 3, 4 and 5 and convert these data into crisp form. Tables 6, 7, 8, 9 and 10 represents the crisp data of the proposed MOTP.

Since$\sum_{i=1}^{M} S(s_i^{\widetilde{\mathcal{F}}}) = \sum_{j=1}^{N} S(d_j^{\widetilde{\mathcal{F}}}) = 0.18$. The best individual compromise solutions for each objective are determined as follows:

For the first objective function (Total transportation cost):

Fig. 2 Shows the flow chart of the proposed study

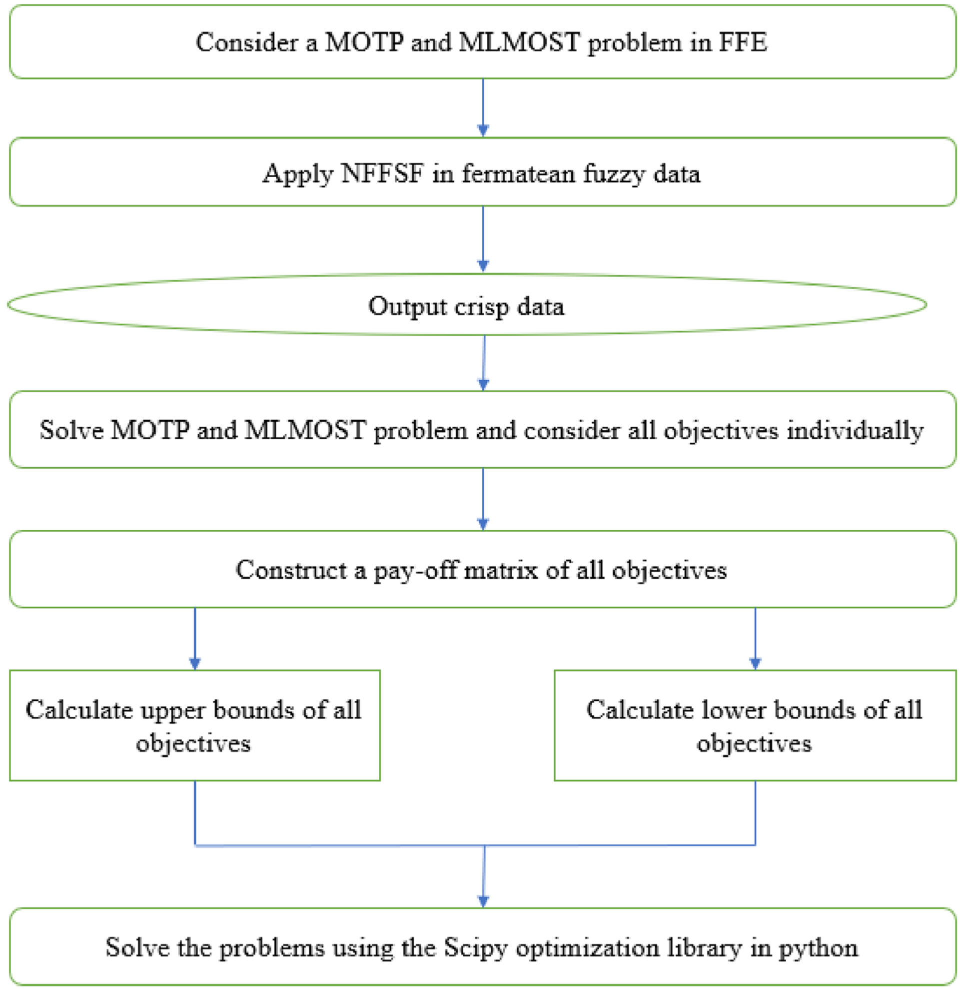

Table 1 Total transportation cost

Source	β_1	β_2	β_3	β_4
α_1	(0.8, 0.7)	(0.7, 0.2)	(0.1, 0.6)	(0.2, 0.9)
α_2	(0.5, 0.8)	(0.1, 0.9)	(0.2, 0.6)	(0.2, 0.1)
α_3	(0.3, 0.4)	(0.7, 0.99)	(0.1, 0.8)	(0.7, 0.9)

Table 2 Total transportation time

Source	β_1	β_2	β_3	β_4
α_1	(0.4, 0.8)	(0.7, 0.50)	(0.2, 0.9)	(0.6, 0.9)
α_2	(0.7, 0.5)	(0.1, 0.99)	(0.6, 0.8)	(0.4, 0.7)
α_3	(0.6, 0.8)	(0.8, 0.6)	(0.5, 0.1)	(0.3, 0.9)

$$\begin{aligned} f_1{}^*(y) = & 0.2695y_{11} + 0.03y_{12} + 0.0225y_{13} + 0.006y_{14} \\ & + 0.0875y_{21} + 0.001y_{22} + 0.012y_{23} + 0.0055y_{24} \\ & + 0.0405y_{31} + 0.1739y_{32} + 0.0015y_{33} + 0.196y_{34} \end{aligned}$$

Table 3 Total carbon emissions cost

Source	β_1	β_2	β_3	β_4
α_1	(0.5, 0.7)	(0.6, 0.8)	(0.2, 0.7)	(0.8, 0.7)
α_2	(0.4, 0.5)	(0.1, 0.2)	(0.8, 0.1)	(0.4, 0.7)
α_3	(0.8, 0.4)	(0.6, 0.4)	(0.4, 0.9)	(0.5, 0.9)

Table 4 Supply cost of the transportation

i	α_1	α_2	α_3
$\left(\alpha_{\mathfrak{F}i}, \beta_{\mathfrak{F}i}\right)$	(0.3, 0.5)	(0.4, 0.8)	(0.6, 0.4)

Table 5 Demand cost of the transportation

j	β_1	β_2	β_3	β_4
$\left(\alpha_{\mathfrak{F}j}, \beta_{\mathfrak{F}j}\right)$	(0.4, 0.7)	(0.2, 0.5)	(0.6, 0.4)	(0.2, 0.5)

Table 6 Total transportation cost

Source	β_1	β_2	β_3	β_4
α_1	(0.2695)	(0.0300)	(0.0025)	(0.006)
α_2	(0.0875)	(0.0010)	(0.0120)	(0.0055)
α_3	(0.0405)	(0.1739)	(0.0015)	(0.1960)

Table 7 Total transportation time

Source	β_1	β_2	β_3	β_4
α_1	(0.048)	(0.1500)	(0.006)	(0.126)
α_2	(0.150)	(0.00055)	(0.144)	(0.056)
α_3	(0.144)	(0.2160)	(0.007)	(0.018)

Table 8 Total carbon emissions cost

Source	β_1	β_2	β_3	β_4
α_1	(0.100)	(0.144)	(0.0100)	(0.2695)
α_2	(0.072)	(0.0045)	(0.0085)	(0.056)
α_3	(0.112)	(0.096)	(0.0400)	(0.075)

Table 9 Supply cost of the transportation

i	α_1	α_2	α_3
$(\alpha_{\tilde{\mathcal{F}}i}, \beta_{\tilde{\mathcal{F}}i})$	(0.036)	(0.048)	(0.096)

Table 10 Demand cost of the transportation

j	β_1	β_2	β_3	β_4
$\left(\alpha_{\tilde{\mathcal{F}}j}, \beta_{\tilde{\mathcal{F}}j}\right)$	(0.056)	(0.014)	(0.096)	(0.014)

S . t $y_{11} + y_{12} + y_{13} + y_{14} \leq 0.036$, $y_{21} + y_{22} + y_{23} + y_{24} \leq 0.048, y_{31} + y_{32} + y_{33} + y_{34} \leq 0.096$, $y_{11} + y_{21} + y_{31} \leq 0.056$, $y_{11} + y_{22} + y_{32} \leq 0.014$, $y_{13} + y_{23} + y_{33} \leq 0.096$, $y_{14} + y_{24} + y_{34} \leq 0.014$, $\sum_{i=1}^{M} s_i = \sum_{j=1}^{N} d_j, y_{ij} \geq 0$.

We solve the first objective function individually using the Python SciPy library and obtain the optimal solutions. The resulting compromise optimal solutions are as follows:

Optimal value $f_1{}^*(y)$: 1.6563000993102234e-13.

Optimal solution: y_{11} = 6.701462050627256e-14, y_{12} = 5.447537900889634e-13, y_{13} = 6.540701226004457e-13, y_{14} = 2.3433877894449986e-12, y_{21} = 1.6269086281778882e-13, y_{22} = 2.8037183688672324e-12, y_{23} = 1.1917212705073402e-12, y_{24} = 2.4672960645959706e-12, y_{31} = 3.4491566715606586e-13, y_{32} = 8.160054271726014e-14, y_{33} = 8.95759410927497e-12, y_{34} = 8.134841315142161e-14.

For the second objective function (Total transportation time):

$$
\begin{aligned}
f_2{}^*(y) =& 0.048y_{11} + 0.15y_{12} + 0.006y_{13} + 0.126y_{14} \\
&+ 0.15y_{21} + 0.00055y_{22} + 0.144y_{23} + 0.056y_{24} \\
&+ 0.144y_{31} + 0.216y_{32} + 0.007y_{33} + 0.018y_{34}
\end{aligned}
$$

S . t $y_{11} + y_{12} + y_{13} + y_{14} \leq 0.036$, $y_{21} + y_{22} + y_{23} + y_{24} \leq 0.048, y_{31} + y_{32} + y_{33} + y_{34} \leq 0.096$, $y_{11} + y_{21} + y_{31} \leq 0.056$, $y_{11} + y_{22} + y_{32} \leq 0.014$, $y_{13} + y_{23} + y_{33} \leq 0.096$, $y_{14} + y_{24} + y_{34} \leq 0.014$, $\sum_{i=1}^{M} s_i = \sum_{j=1}^{N} d_j, y_{ij} \geq 0$.

We solve the second objective function individually using the Python SciPy library and obtain the optimal solutions. The resulting compromise optimal solutions are as follows:

Optimal value $f_2{}^*(y)$: 4.211826137490883e-12.

Optimal solution: y_{11} = 8.263438891810004e-12, y_{12} = 2.5838370001640527e-12, y_{13} = 6.335818003711425e-11, y_{14} = 3.0146972331010592e-12, y_{21} = 2.5299779571885494e-12, y_{22} = 8.790645598327858e-12, y_{23} = 2.669450392021306e-12, y_{24} = 6.670724613805137e-12, y_{31} = 2.6400866542766385e-12, y_{32} = 1.800596343315708e-12, y_{33} = 5.36625998899776e-11, y_{34} = 2.1142895443476957e-11.

For the third objective function (Total carbon emissions):

$$
\begin{aligned}
f_3{}^*(y) =& 0.1y_{11} + 0.144y_{12} + 0.01y_{13} + 0.2695y_{14} \\
&+ 0.072y_{21} + 0.0045y_{22} + 0.0085y_{23} + 0.056y_{24} \\
&+ 0.112y_{31} + 0.096y_{32} + 0.04y_{33} + 0.075y_{34}
\end{aligned}
$$

S . t $y_{11} + y_{12} + y_{13} + y_{14} \leq 0.036$, $y_{21} + y_{22} + y_{23} + y_{24} \leq 0.048, y_{31} + y_{32} + y_{33} + y_{34} \leq 0.096$, $y_{11} + y_{21} + y_{31} \leq 0.056$, $y_{11} + y_{22} + y_{32} \leq 0.014$, $y_{13} + y_{23} + y_{33} \leq 0.096$, $y_{14} + y_{24} + y_{34} \leq 0.014$, $\sum_{i=1}^{M} s_i = \sum_{j=1}^{N} d_j, y_{ij} \geq 0$

We solve the third objective function individually using the Python SciPy library and obtain the optimal solutions. The resulting compromise optimal solutions are as follows:

Optimal value $f_3{}^*$(y): 7.110957493321067e-12.

Optimal solution: $y_{11} = 6.1746254985444975e-12$, y_{12} = 4.317295196854511e-12, y_{13} = 6.194365813830885e-11, y_{14} = 2.30864320123096e-12, y_{21} = 8.587199756168197e-12, y_{22} = 7.545198531183348e-11, y_{23} = 6.803357681972516e-11, y_{24} = 1.0962059083018053e-11, y_{31} = 5.5603385632255506e-12, y_{32} = 6.461011159919284e-12, y_{33} = 1.5425548584548254e-11, y_{34} = 8.269047323003344e-12. After determining the solution for each objective individually, we can formulate the pay-off matrix in the following manner.

Table 11 Total transportation cost by k_1

Source	β_1	β_2	β_3	β_4
α_1	(20.0, 2.0)	(18.0, 1.5)	(18.0, 2.0)	(13.0, 1.0)
α_2	(19.0, 1.0)	(13.0, 1.0)	(16.0, 1.5)	(18.0, 2.0)
α_3	(15.0, 2.0)	(11.0, 2.0)	(17.0, 1.0)	(12.0, 1.0)
α_4	(14.0, 1.5)	(14.0, 1.5)	(16.0, 2.0)	(13.0, 1.5)

$$f_1^*\ f_2^*\ f_3^*$$

$$Pay-offmatrix = \begin{array}{c} f_1^* \\ f_2^* \\ f_3^* \end{array}\begin{pmatrix} 0.000003 & 0.000888 & 0.001341 \\ 0.001852 & 0.000025 & 0.009956 \\ 0.011559 & 0.014590 & 0.000043 \end{pmatrix}$$

Finding the upper and lower bounds for each objective function and $d_t = U_t - L_t$, which are as follows: $L_1 = 0.000003$, $U_1 = 0.001341$, $d_1 = 0.001337$; $L_2 = 0.000025$, $U_2 = 0.009956$, $d_2 = 0.009930$; $L_3 = 0.000043$, $U_3 = 0.014590, d_3 = 0.014546$.

Now, we apply the FFPA to the proposed mathematical problem and obtain the best compromise solutions.

$$Max\lambda\delta_1{}^3 - \delta_2{}^3$$

S.t

$$\mu\left(f_t^*(y)\right)^3 \geq \delta_1{}^3, \forall t$$

$$\theta\left(f_t^*(y)\right)^3 \leq \delta_2{}^3, \forall t$$

i.e., $(U_t - f_t^*(y))^3 \geq d_t{}^3\delta_1{}^3$

$(f_t^*(y) - L_t)^3 \leq d_t{}^3\delta_2{}^3, where d_t = U_t - L_t$

For upper bound $\Rightarrow \left(0.001341 - f_1^*\right)^3 \geq 0.0000000023897975\delta_1^3$, $(0.009956 - f_2{}^*)^3 \geq 0.0000009791466 57\delta_1{}^3$, $(0.014590 - f_3{}^*)^3 \geq 0.0000030777316 43\delta_1{}^3$.

For lower bound $\Rightarrow \left(f_1^* - 0.000003\right)^3 \geq 0.0000000023897975\delta_2^3$, $(f_2{}^* - 0.000025)^3 \geq 0.000000979146657\delta_2{}^3$, $(f_3{}^* - 0.000043)^3 \geq 0.000003077731643\delta_2{}^3$.

$$y_{11} + y_{12} + \dots\dots\dots + y_{1N} \leq s_1$$
$$y_{21} + y_{22} + \dots\dots\dots + y_{2N} \leq s_2$$
$$\vdots$$
$$y_{M1} + y_{M2} + \dots\dots\dots + y_{MN} \leq s_M$$
$$y_{11} + y_{21} + \dots\dots\dots + y_{M1} \leq d_1$$
$$y_{12} + y_{22} + \dots\dots\dots + y_{M2} \leq d_2$$
$$\vdots$$
$$y_{1M} + y_{2M} + \dots\dots\dots + y_{NM} \leq d_N$$
$$\sum_{i=1}^{M} s_i = \sum_{j=1}^{N} d_j, y_{ij} \geq 0, 0 \leq \delta_1^3, \delta_2^3 \leq 1, \leq \delta_1^3 + \delta_2^3 \leq 1, \delta_1^3 \geq \delta_2^3.$$

To solve this problem using the Python SciPy optimization library and obtain the best compromise solutions of all objective functions, such that:

$f_1{}^* = 0.004191$, $f_2{}^* = 0.003977$, $f_3{}^* = 0.002018$, $\delta_1 = 0.001289$, $\delta_2 = 0.000660$, $y_{11} = 0.305732$, $y_{12} = 0.028454$, $y_{13} = 0.0011005$, $y_{14} = 0.001008$, $y_{21} = 0.017421$, $y_{22} = 0.00101$, $y_{23} = 0.001146$, $y_{24} = 0.001063$, $y_{31} = 0.017471$, $y_{32} = 0.001054$, $y_{33} = 0.020575$, $y_{34} = 0.000999$.

7.2 Numerical example 2

This section considers a numerical example of the MLMOSTP under FFEs. To illustrate the numerical example of MLMOSTP, consider a logistics firm aiming to optimize its transportation planning by minimizing total costs, time, and carbon emissions during transportation. This problem involves a hierarchical decision-making process with three levels: the first focuses on minimizing total transportation costs, the second on minimizing total time, and the third on minimizing the total carbon emission during transportation. The logistic firm has two conveyance options, k_1, and k_2, with capacities c_1 and c_2, respectively. There are four source points with supply capacities and four destination points with demand requirements. The parameters in the problem are FFNs following an FFPA under the FFE. Tables 11, 12, 13, 14, 15, 16, 17 and 18 represents the fermatean fuzzy data of the proposed MLMOSTP. The values represented in Tables 11, 12, 13, 14, 15, 16, 17 and 18 are the fermatean fuzzy evaluations of transportation costs, time, carbon

Table 12 Total transportation cost by k_2

Source	β_1	β_2	β_3	β_4
α_1	(30.0, 1.0)	(35.0, 1.5)	(20.0, 2.0)	(20.0, 2.0)
α_2	(22.0, 1.5)	(27.0, 2.0)	(27.0, 2.0)	(23.0, 1.0)
α_3	(25.0, 1.5)	(23.0, 1.0)	(16.0, 2.0)	(16.0, 2.0)
α_4	(30.0, 2.0)	(21.0, 1.0)	(26.0, 1.5)	(25.0, 1.5)

Table 13 Total transportation time by k_1

Source	β_1	β_2	β_3	β_4
α_1	(30.0, 2.0)	(34.0, 1.0)	(34.0, 1.5)	(34.0, 1.0)
α_2	(26.0, 1.5)	(24.0, 2.0)	(31.0, 1.0)	(29.0, 1.0)
α_3	(21.0, 1.5)	(20.0, 2.0)	(26.0, 1.5)	(29.0, 1.5)
α_4	(21.0, 2.0)	(22.0, 1.0)	(25.0, 2.0)	(31.0, 1.5)

Table 14 Total transportation time by k_2

Source	β_1	β_2	β_3	β_4
α_1	(38.0, 1.0)	(38.0, 1.0)	(32.0, 1.5)	(30.0, 2.0)
α_2	(25.0, 1.5)	(28.0, 2.0)	(28.0, 2.0)	(28.0, 1.5)
α_3	(29.0, 1.0)	(33.0, 1.5)	(34.0, 2.0)	(22.0, 1.0)
α_4	(30.0, 2.0)	(28.0, 2.0)	(24.0, 1.5)	(24.0, 1.5)

Table 15 Total carbon emission by k_1

Source	β_1	β_2	β_3	β_4
α_1	(9.0, 1.5)	(10.0, 1.0)	(8.0, 1.0)	(6.0, 1.0)
α_2	(8.0, 1.0)	(11.0, 1.5)	(7.0, 1.5)	(7.0, 1.0)
α_3	(11.0, 2.0)	(10.0, 1.5)	(7.0, 2.0)	(11.0, 1.5)
α_4	(11.0, 1.5)	(10.0, 1.5)	(10.0, 1.0)	(12.0, 1.5)

Table 16 Total carbon emission by k_2

Source	β_1	β_2	β_3	β_4
α_1	(7.0, 2.0)	(10.0, 1.0)	(9.0, 1.0)	(8.0, 1.5)
α_2	(9.0, 1.5)	(8.0, 1.5)	(7.0, 1.0)	(11.0, 1.5)
α_3	(9.0, 1.0)	(8.0, 1.5)	(11.0, 1.5)	(11.0, 1.0)
α_4	(8.0, 1.0)	(7.0, 2.0)	(10.0, 2.0)	(10.0, 1.0)

Table 17 Supply cost of the transportation

i	α_1	α_2	α_3	α_4
$\left(\alpha_{\widetilde{\mathcal{F}}i}, \beta_{\widetilde{\mathcal{F}}i}\right)$	(25.0, 1.5)	(30.0, 1.5)	(32.0, 2.0)	(28.0, 2.0)

Table 18 Demand cost of the transportation

j	β_1	β_2	β_3	β_4
$\left(\alpha_{\widetilde{\mathcal{F}}j}, \beta_{\widetilde{\mathcal{F}}j}\right)$	(10.0, 1.5)	(14.0, 1.0)	(22.0, 1.0)	(18.0, 1.0)

emission with conveyance k_1 and k_2, and supply and demand between sources (α_i) and destinations (β_j). The first value indicates the degree of satisfaction, while the second represents dissatisfaction, offering a balanced view under FFEs.

Now convert the above data, presented in Tables 11, 12, 13, 14, 15, 16, 17 and 18, into the crisp form using the NFFSF under FFE. Tables 19, 20, 21, 22, 23, 24, 25 and 26 represents the crisp data of the proposed MLMOSTP.

Since$\sum_{i=1}^{M} S(s_i^{\widetilde{\mathcal{F}}}) \neq \sum_{j=1}^{N} S(d_j^{\widetilde{\mathcal{F}}})$. Firstly, we calculate the individual best solution for each objective function at each level using the Python SciPy optimization library. The individual best solutions for each objective are presented as follows;

Table 19 Total transportation cost by k_1

Source	β_1	β_2	β_3	β_4
α_1	(38.0)	(21.20)	(34.0)	(6.50)
α_2	(9.50)	(6.50)	(18.95)	(34.0)
α_3	(28.0)	(20.0)	(8.50)	(6.00)
α_4	(15.18)	(15.18)	(30.0)	(14.06)

Table 20 Total transportation cost by k_2

Source	β_1	β_2	β_3	β_4
α_1	(15.0)	(38.81)	(38.0)	(38.0)
α_2	(24.18)	(52.0)	(52.0)	(11.50)
α_3	(27.56)	(11.50)	(30.0)	(30.0)
α_4	(58.0)	(10.50)	(28.68)	(27.56)

Table 21 Total transportation time by k_1

Source	β_1	β_2	β_3	β_4
α_1	(58.0)	(17.0)	(37.68)	(17.0)
α_2	(28.68)	(46.0)	(15.50)	(14.50)
α_3	(23.06)	(38.0)	(28.68)	(32.06)
α_4	(40.0)	(11.0)	(48.0)	(34.31)

Table 22 Total transportation time by k_2

Source	β_1	β_2	β_3	β_4
α_1	(19.0)	(19.0)	(35.43)	(58.0)
α_2	(27.56)	(54.0)	(54.0)	(30.93)
α_3	(14.50)	(36.56)	(66.0)	(11.0)
α_4	(58.00)	(54.00)	(26.43)	(26.43)

Table 23 Total carbon emission by k_1

Source	β_1	β_2	β_3	β_4
α_1	(9.56)	(5.0)	(4.00)	(3.00)
α_2	(4.00)	(11.81)	(7.31)	(3.50)
α_3	(20.0)	(10.68)	(12.0)	(11.81)
α_4	(13.33)	(10.68)	(5.00)	(14.45)

Table 24 Total carbon emission by k_2

Source	β_1	β_2	β_3	β_4
α_1	(3.00)	(5.00)	(4.50)	(8.43)
α_2	(9.56)	(8.43)	(3.50)	(11.81)
α_3	(4.50)	(8.43)	(11.81)	(5.50)
α_4	(4.00)	(12.0)	(18.00)	(5.00)

Table 25 Supply cost of the transportation

i	α_1	α_2	α_3	α_4
$(\alpha_{\widetilde{\mathcal{F}}i}, \beta_{\widetilde{\mathcal{F}}i})$	(27.56)	(33.18)	(62.0)	(54.0)

Table 26 Demand cost of the transportation

j	β_1	β_2	β_3	β_4
$(\alpha_{\widetilde{\mathcal{F}}j}, \beta_{\widetilde{\mathcal{F}}j})$	(10.68)	(7.0)	(11.0)	(9.0)

$f_1{}^* \; f_2{}^* \; f_3{}^*$

$$Pay - offmatrix = \begin{array}{c} f_1{}^* \\ f_2{}^* \\ f_3{}^* \end{array} \begin{pmatrix} 1013.04 & 1011.35 & 1025.76 \\ 1479.93 & 1587.32 & 1597.11 \\ 432.89 & 489.67 & 467.450 \end{pmatrix}$$

Finding the upper and lower bounds for every objective function and $d_t = U_t - L_t$, which are as follows: $L_1 = 1013.04$, $U_1 = 1025.76$, $d_1 = 12.72$; $L_2 = 1587.32$, $U_2 = 1597.11$, $d_2 = 9.79$; $L_3 = 467.450$, $U_3 = 489.67$, $d_3 = 22.22$. We now solved the proposed numerical example using the FFPA.

$Max \lambda \delta_1{}^3 - \delta_2{}^3$

S.t

$$\mu\left(f_t^*(y)\right)^3 \geq \delta_1{}^3, \forall t$$

$$\theta\left(f_t^*(y)\right)^3 \leq \delta_2{}^3, \forall t$$

i.e., $(U_t - f_t^*(y))^3 \geq d_t{}^3 \delta_1{}^3$, $(f_t{}^*(y) - L_t)^3 \leq d_t{}^3 \delta_2{}^3$, where $d_t = U_t - L_t$.

For upper bound $\Rightarrow \left(1025.76 - f_1^*\right)^3 \geq 2058.07\delta_1^3$, $(1597.11 - f_2{}^*)^3 \geq 938.31\delta_1{}^3$, $(489.67 - f_3{}^*)^3 \geq 10970.64\delta_1{}^3$. For lower bound $\Rightarrow \left(f_1^* - 1013.04\right)^3 \geq 2058.07\delta_2^3$, $(f_2{}^* - 1587.32^3 \geq 938.31\delta_2{}^3$, $(f_3{}^* - 467.450)^3 \geq 10970.64\delta_2{}^3$.

$$\sum_{j=1}^{N} \sum_{k=1}^{K} y_{ijk} \leq S(s_i^{\widetilde{\mathcal{F}}}), i = 1,2,\ldots,M$$

$$\sum_{i=1}^{M} \sum_{k=1}^{K} y_{ijk} \leq S\left(d_j^{\widetilde{\mathcal{F}}}\right), j = 1,2,\ldots,N$$

$$\sum_{i=1}^{M} \sum_{j=1}^{N} y_{ijk} \leq S\left(e_k^{\widetilde{\mathcal{F}}}\right), k = 1,2,\ldots,K$$

$y_{ijk} \geq 0, i = 1,2,\ldots,M, j = 1,2,\ldots,N, k = 1,2,\ldots,K.$

Then, solve this numerical example using the Python SciPy optimization library and obtain the best compromise solutions for first, second, and third levels objectives such that:

$f_1{}^* = 1129.97$, $f_2{}^* = 1937.573$, $f_3{}^* = 669.052$, $\delta_1 = 1.0$, $\delta_2 = 0.00539$, $y_{11} = 0$, $y_{12} = 6.273$, $y_{13} = 0$, $y_{14} = 9.576$, $y_{21} = 12.3701$, $y_{22} = 0.1$, $y_{23} = 0$, $y_{24} = 14.780$, $y_{31} = 0.490$, $y_{32} = 7.971$, $y_{33} = 16.82$, $y_{34} = 0$, $y_{41} = 2.453$, $y_{42} = 0$, $y_{43} = 8.178$, $y_{44} = 10.347$.

8 Conclusion

This study introduced the FFPA to solve various mathematical models in industrial engineering, operations research, logistics, and transportation planning. First, we developed the mathematical models of TTP, MOTP, and MOMLST with FFPs and converted them into a crisp form using the NFFSF within the FFE. The advantage of this mathematical programming approach lies in its flexibility to handle different types of uncertainties, such as intuitionistic, pythagorean, and FFSs. Two numerical examples were presented to demonstrate the proposed study's efficiency and practicality. The first example is based on a MOTP, and the second numerical example is based on MLMOSTP. In both numerical examples, the objective was to minimize the total cost, time, and carbon emission of the proposed problem. The numerical examples demonstrate that the proposed approach is versatile and practical in transportation planning. The results from these numerical examples highlight the influence of FFPA on decision-makers when addressing complex MOTP and MLMOSTP.

The numerical example of MOTP minimizes transportation cost, time, and carbon emissions for a network of suppliers and demand points. Using the FFPA, the optimal transportation cost is 0.004191, with the transportation time optimized to 0.003977 and carbon emissions reduced to 0.002018. These results validate the FFPA's capability to handle uncertainty and provide efficient solutions for MOTP. The second example of MLMOSTP deals with a hierarchical multi-level transportation problem, focusing on transportation cost, time, and emissions across levels. The optimal compromise solutions included minimizing transportation costs to 1129.97, total time to 1937.573, and emissions to 669.052. These results illustrate the robustness of FFPA in optimizing multi-layered transportation systems, balancing economic and environmental objectives effectively.

The decision-makers are advised to utilize the FFPA within an FFE to derive solutions for these objective functions. The proposed feramtean fuzzy approach effectively identifies a compromise optimal solution for MOTP and MOMLST. Thus, the proposed FFPA offers an alternative solution approach for MOTP and MOMLST in different environments. Moreover, the FFPA easily applies the different types of mathematical models and obtains the best compromise solutions. In addition to the practical implications for decision-makers, the proposed FFPA offers a significant contribution to the theoretical advancement in mathematical modeling under uncertainty. By integrating FFSs into the solution framework, this study opens new avenues for handling more complex and ambiguous situations in real-world applications. The flexibility of the approach in addressing various uncertainties, such as intuitionistic and pythagorean fuzzy environments, makes it a robust tool for industries seeking more accurate and adaptable solutions to multi-objective optimization problems.

8.1 Managerial and practical implications

The proposed FFPA provides a robust framework for addressing complex transportation problems, offering significant managerial and practical benefits (Bouraima et al. 2024). Incorporating FFPs and an NFFSF enhances logistics and supply chain management decision-making by effectively handling uncertainties and imprecisions in real-world data (Singh et al. 2024; Rodríguez-Segade et al. 2024). Managers can use this methodology to achieve more reliable and optimal solutions for transportation planning, minimizing costs, time, and carbon emissions simultaneously. This approach benefits multi-level decision-making processes, enabling strategic, tactical, and operational decisions to be aligned and optimized coherently. Moreover, implementing FFPA using Python's SciPy optimization library ensures that the solutions are computationally efficient and accessible, providing decision-makers with practical tools to tackle complex transportation challenges in dynamic and uncertain environments.

8.2 Limitations and future research

Despite its advantages, the FFPA has limitations, including the potential loss of information when transforming fuzzy data into crisp values and the significant computational resources required for large-scale problems (Srivastava et al. 2024; Lo et al. 2024). The approach's effectiveness is also contingent on the accuracy and relevance of the FFPs and the score function, which may vary with specific applications (van de Berg et al. 2024). Future research should address these limitations by developing more sophisticated data transformation methods, integrating FFPA with advanced optimization techniques like metaheuristics or machine learning, and expanding its application to other domains, such as production planning and inventory management. Empirical studies with real-world case studies and industry collaboration are essential for validating practical applicability, and further incorporating sustainability considerations can enhance the approach's impact on environmentally and socially responsible decision-making in logistics and supply chain management.

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Declarations

Conflict of interest There are no competing interests to declare.

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