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# 000 001 002 003 004 005 006 007 008 009 010 011 012 013 014 015 016 017 018 019 020 021 022 023 024 025 026 027 028 029 030 031 032 033 034 035 036 037 038 039 040 041 042 043 044 045 046 047 048 049 050 051 052 053 Managing Conflicts Among Black-Box RAN Apps via Multi-Fidelity Game-Theoretic Optimization

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## Abstract

Modern open and softwarized networks—such as O-RAN telecom systems—host independently developed applications, known as xApps, with distinct and potentially conflicting objectives. Coordinating their behavior to ensure stable operation is challenging, especially when each application’s utility is only accessible via costly black-box evaluations. In this work, we consider a centralized controller that suggests joint configurations to multiple apps, modeled as strategic players, with the goal of aligning their incentives toward a stable outcome. This setting is modeled as a Stackelberg game in which the central controller (leader) lacks analytical forms of the players’ utility functions, and must learn them through sequential, multi-fidelity queries. We propose MF-UCB-PNE, a novel multi-fidelity Bayesian optimization strategy that efficiently approximates a pure Nash equilibrium (PNE) under a limited query budget. MF-UCB-PNE balances exploration of cheap, lower-fidelity utility observations with exploitation of high-fidelity evaluations, enabling convergence to incentive-compatible configurations. We provide theoretical guarantees in terms of no-regret learning of equilibrium, as well as empirical results on wireless networking problems, demonstrating that MF-UCB-PNE identifies high-quality equilibrium solutions under strict cost budgets.

## 1 Introduction

### 1.1 Context and Contribution

Open and software-defined network architectures allow multiple independent applications to control shared resources. These applications, or apps for short, often have *conflicting* objectives, which can lead to unstable behavior. For instance, in an O-RAN telecom network, specialized applications, known as xApps, run on a centralized controller, each generally optimizing a different goal [1]. For example, as seen in Fig. 1, one xApp might increase transmit power to maximize throughput, while another may try to shut down radios to save energy, causing a conflict over the use of shared resources.

As illustrated in Fig. 1, the central controller can mitigate such conflicts by recommending joint configurations for all apps, aiming for a working point where no app has incentive to deviate from it on its own. We model this interaction as a hierarchical game in which the central controller plays the role of a *leader* that proposes a configuration profile, and the  $N$  apps are *followers* that respond according to their individual utility functions. A stable operating point corresponds to a *pure Nash equilibrium* (PNE) of this game, i.e., a joint configuration of actions from which no single player can unilaterally improve its utility. The controller’s goal is to find a PNE, so that its recommendation will be self-enforcing.

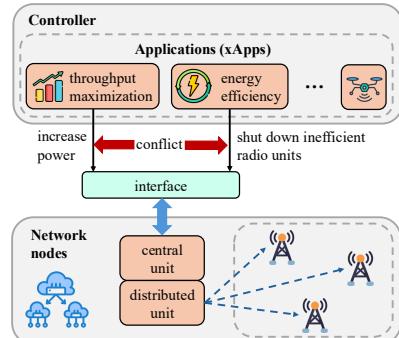


Figure 1: **O-RAN scenario with two conflicting xApps.** Using the methods introduced in this paper, a joint configuration can be chosen by the controller so that neither app has incentive to deviate unilaterally.

However, a key challenge is that each player's utility function is a black-box from the perspective of the controller: it is not given in closed form and can only be observed by running experiments or simulations. Accordingly, each query can be *expensive* in time or resources. To address these challenges, in this work we consider the *multi-fidelity* learning setting illustrated in Fig. 2. In it, the controller can observe utility values at various fidelity levels. Low-fidelity evaluations, e.g., coarse simulations or partial measurements, are cheaper but less accurate, while high-fidelity evaluations, e.g., detailed simulations or field experiments, provide accurate feedback at higher cost. By judiciously using low- and high-fidelity evaluations, the controller can homing in on a stable configuration.

In this context, we introduce **MF-UCB-PNE**, a multi-fidelity Bayesian optimization algorithm to efficiently learn players' utilities and approximate a PNE under a total query cost constraint. We show that this approach leads to provably no-regret convergence to an approximate equilibrium as the budget grows. We also demonstrate the effectiveness of MF-UCB-PNE on two wireless network case studies (power control and random access), where it outperforms single-fidelity baselines in finding high-quality equilibria within limited budgets.

## 1.2 Related Work

**Single-objective black-box optimization.** Bayesian optimization (BO) [2] provides a sample-efficient sequential decision-making framework that is widely adopted in expensive-to-evaluate black-box optimization problems. Uncertainty-aware acquisition functions include upper confidence bound (UCB) [3], expected improvement (EI) [4], and entropy search (ES) [5].

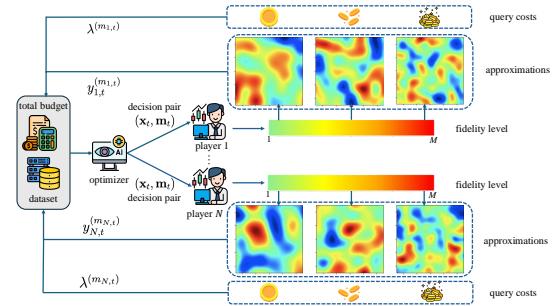
**Multi-fidelity single-objective black-box optimization.** In order to reduce the query cost incurred in the acquisition process, multi-fidelity BO (MFBO) constructs multi-fidelity surrogate models and define the cost-aware acquisition functions to adaptively prioritize queries across fidelity levels [6, 7].

**Nash equilibrium (NE) evaluation with unknown utility functions.** Reference [8] characterized the query complexity associated with evaluating a NE for unknown utilities across various game classes. Subsequent works in [9, 10] proposed a BO-based optimizer that can approximate NE solutions with formal regret guarantees. However, these methods assume access to a single fidelity of utility feedback and do not explore the potential benefits of multi-fidelity observations, which can offer more cost-effective and informative sampling strategies. A longer version of this work can be found in the unpublished manuscript [11].

## 2 Problem Formulation

**Strategic game model.** We consider  $N$  players, representing apps in a softwarized wireless system, with app  $n$  characterized by a configuration setting  $x_n$ , also referred to as action, from its strategy set  $\mathcal{X}_n$ . For example, for a random access app, the configuration  $x_n$  may represent the transmission probability or a back-off parameter. Let  $\mathbf{x} = (x_1, \dots, x_N) \in \mathcal{X} = \mathcal{X}_1 \times \dots \times \mathcal{X}_N$  denote the joint *action profile*.

The utility for player  $n$  is an unknown function  $u_n(\mathbf{x})$ , which depends on all players' actions. We assume the utilities  $u_n(\mathbf{x})$  to be *black-box* functions: the controller cannot directly observe  $u_n(\mathbf{x})$  except by querying the outcome of action profile  $\mathbf{x}$ . This is a typical situation in wireless systems in



**Figure 2: Multi-fidelity Stackelberg game setup.** A central optimizer aims at identifying a joint configuration for the apps that constitute an approximate Nash equilibrium for the utilities of the apps, while having access only to multi-fidelity estimators of expensive-to-evaluate black-box utility functions.

108 which *key performance indicators* (KPIs) can be only measured based on simulations or real-world  
 109 developments.

110 A recommended action profile  $\mathbf{x}$  is considered stable if no player can significantly increase its utility  
 111 by deviating. Accordingly, the *dissatisfaction* of player  $n$  at  $\mathbf{x}$  is defined as

$$113 \quad f_n(\mathbf{x}) = \max_{\mathbf{x}'_n \in \mathcal{X}_n} u_n(\mathbf{x}'_n, \mathbf{x}_{-n}) - u_n(\mathbf{x}) \geq 0, \quad (1)$$

115 where  $\mathbf{x}_{-n}$  denotes the actions of all players except  $n$ . The function  $f_n(\mathbf{x})$  can be interpreted as  
 116 quantifying the incentive for player  $n$  to unilaterally deviate from  $\mathbf{x}$ . A joint action  $\mathbf{x}^*$  is a *pure*  
 117 *Nash equilibrium (PNE)* if  $f_n(\mathbf{x}^*) = 0$  for all players  $n$ —i.e., no player can improve its utility by  
 118 deviating.

119 In many games a PNE may not exist or may be hard to find, so we consider approximate equilibria.  
 120 For  $\epsilon \geq 0$ , an  $\epsilon$ -*PNE* is an action profile  $\mathbf{x}^{(\epsilon)}$  such that the dissatisfaction levels satisfy the inequalities  
 121  $f_n(\mathbf{x}^{(\epsilon)}) \leq \epsilon$  for all  $n$ . In an  $\epsilon$ -PNE, no player can gain more than  $\epsilon$  by unilaterally deviating, so  
 122  $\epsilon$  quantifies the stability of the configuration. Let  $\epsilon^*$  be the smallest achievable dissatisfaction level

$$123 \quad \epsilon^* = \inf\{\epsilon \in \mathbb{R} \mid \mathcal{X}^{(\epsilon)} \neq \emptyset\}, \quad (2)$$

124 where

$$125 \quad \mathcal{X}^{(\epsilon)} := \{\mathbf{x}^{(\epsilon)} \in \mathcal{X} \mid f_n(\mathbf{x}^{(\epsilon)}) \leq \epsilon \text{ for all } n\} \quad (3)$$

126 is the corresponding  $\epsilon$ -PNE set. Our goal is to identify an action profile  $\mathbf{x}^*$  that lies in the  $\epsilon^*$ -PNE  
 127 set, i.e., an approximately stable joint configuration with minimum possible  $\epsilon^*$ .

128 **Multi-fidelity utility observations.** We assume the controller can sequentially query the utilities  
 129 by proposing action profiles and observing the resulting payoffs with chosen fidelity. Specifically,  
 130 at each time step  $t = 1, 2, \dots$ , the controller selects a joint action  $\mathbf{x}_t$  and a fidelity level  $m_{n,t} \in$   
 131  $\mathcal{M} = \{1, \dots, M\}$  for each player  $n$ . We write  $\mathbf{m}_t = (m_{1,t}, \dots, m_{N,t})$ . As a result of this query,  
 132 the controller observes noisy utility feedback  $y_{n,t}^{(m_{n,t})}$  for each player  $n$  as

$$134 \quad y_{n,t}^{(m_{n,t})} = u_n^{(m_{n,t})}(\mathbf{x}_t) + \zeta_{n,t}, \quad (4)$$

135 where  $u_n^{(m)}(\mathbf{x})$  denotes a *fidelity- $m$  approximation* of the true utility  $u_n(\mathbf{x})$ , and  $\zeta_{n,t} \sim \mathcal{N}(0, \sigma^2)$  is  
 136 observational noise. We assume the highest fidelity  $m = M$  corresponds to the actual utility, i.e.,  
 137  $u_n^{(M)}(\mathbf{x}) = u_n(\mathbf{x})$ , whereas lower fidelities  $m < M$  provide cheaper but biased estimates.

138 Each fidelity level  $m$  has an associated cost  $\lambda^{(m)} > 0$  representing the resource expenditure (e.g.,  
 139 computation time or experimental cost) for one evaluation at fidelity  $m$ . We assume  $0 < \lambda^{(1)} \leq$   
 140  $\lambda^{(2)} \leq \dots \leq \lambda^{(M)}$ , so that higher fidelity is costlier. The overall querying process is subject to the  
 141 total cost budget  $\Lambda$  as

$$144 \quad \sum_{t=1}^T \sum_{n=1}^N \lambda^{(m_{n,t})} \leq \Lambda, \quad (5)$$

145 where  $T$  is the (random) total number of queries performed. This budget constraint enforces that the  
 146 controller balance the benefit of information against the cost of its queries.

147 In summary, the controller faces a sequential decision problem: it must adaptively select which joint  
 148 configuration  $\mathbf{x}_t$  and fidelity levels  $\mathbf{m}_t$  to query at each step, based on past observations, in order to  
 149 efficiently learn the utility functions  $u_n(\mathbf{x})$  and approach an  $\epsilon^*$ -PNE solution within the budget  $\Lambda$ .

### 152 3 The MF-UCB-PNE Algorithm

153 MF-UCB-PNE is a BO scheme designed for the hierarchical game setting described in the previous  
 154 section. The algorithm maintains a probabilistic surrogate model for the unknown utilities and uses  
 155 it to decide on informative queries. We leverage a multi-output Gaussian process (MOGP) model to  
 156 jointly capture all players' utility functions. The GP prior provides an uncertainty estimate for each  
 157 utility at any candidate action, which we exploit to guide exploration.

158 MF-UCB-PNE proceeds in a sequence of episodes, indexed by  $j = 1, 2, \dots$ . In each *episode  $j$* ,  
 159 the algorithm first executes an *exploration phase* by collecting a batch of low-cost observations, and  
 160 then it conducts an *evaluation phase* by querying the highest-fidelity utilities to evaluate a candidate

162 action. At the start of episode  $j$ , let  $\mathcal{D}_{j,0}$  be the set of observations collected in all previous episodes.  
 163 The exploration phase of episode  $j$  consists of a series of queries  $(\mathbf{x}_{j,\tau}, \mathbf{m}_{j,\tau})$  for  $\tau = 1, 2, \dots, t_j$ ,  
 164 where at least one player  $n$  is sampled at lower than max fidelity, i.e.,  $m_{n,j,\tau} < M$ . These points  
 165 are chosen by an acquisition function that trades off information gain and cost, i.e.,

$$167 \quad (\tilde{\mathbf{x}}_{j,\tau}, \tilde{\mathbf{m}}_{j,\tau}) = \arg \max_{\mathbf{x} \in \mathcal{X}, \mathbf{m} \in \mathcal{M}} \frac{\sum_{n=1}^N I(y_n^{(m_n)}; u_n(\mathbf{x}) | \mathbf{x}, m_n, \mathcal{D}_{n,j,\tau-1})}{\sum_{n=1}^N \lambda^{(m_n)}} \quad (6a)$$

$$170 \quad \text{s.t.} \quad \sum_{n=1}^N \lambda^{(m_n)} \leq \Lambda - \sum_{j'=1}^{j-1} \Lambda_{j'} - \sum_{n=1}^N \sum_{\tau'=1}^{\tau-1} \lambda^{(m_{n,j,\tau'})} - N\lambda^{(M)}, \quad (6b)$$

173 where details on computing information gain  $I(y_n^{(m_n)}; u_n(\mathbf{x}) | \mathbf{x}, m_n, \mathcal{D}_{n,j,\tau-1})$  via MOGP can be  
 174 found in the longer version [11].

175 By (6), MF-UCB-PNE adopts an information-theoretic criterion adapted to the multi-fidelity setting:  
 176 at each exploration step, it selects the joint action and fidelity that maximize the information-to-cost  
 177 ratio (6a), until a certain stopping condition is met [11]. Intuitively, the exploration phase continues  
 178 as long as lower-fidelity queries can significantly reduce uncertainty about the optimal  $\epsilon^*$ -PNE,  
 179 without exceeding a cost threshold.

180 After exploration, the algorithm performs an *evaluation* step by selecting the most promising action  
 181  $\tilde{\mathbf{x}}_j$  based on the surrogate MOGP model. This is done by selecting the action that appears to min-  
 182 imize the maximum dissatisfaction  $\max_n f_n(\mathbf{x})$  using an upper confidence bound (UCB) criterion.  
 183 This yields an evaluation of the actual joint utility outcome and the corresponding dissatisfaction  
 184 levels  $f_n(\tilde{\mathbf{x}}_j)$ . If the maximum dissatisfaction  $\max_n f_n(\tilde{\mathbf{x}}_j)$  is below a pre-specified threshold  $\eta$ ,  
 185 the algorithm terminates. Otherwise, the knowledge gained from this evaluation is incorporated into  
 186 the dataset, and a new episode begins if budget remains.

187 **Theoretical guarantees.** To analyze the theoretical performance guarantee of MF-UCB-PNE, we  
 188 first define the *loss* accrued by the selection  $(\mathbf{x}, \mathbf{m})$  as

$$190 \quad \ell(\mathbf{x}, \mathbf{m}) = \begin{cases} \max_n f_n(\mathbf{x}) & \text{if } \mathbf{m} = M \cdot \mathbf{1} \quad (\text{evaluation}) \\ 191 \quad C & \text{otherwise} \quad (\text{exploration}) \end{cases} \quad (7)$$

192 with constant  $C$  representing the cost of exploration at lower fidelity levels, satisfying  $C \geq$   
 193  $\max_{n,\mathbf{x}} f_n(\mathbf{x})$ . Defining the *regret* after using budget  $\Lambda$  as  $R(\Lambda) = \sum_{t=1}^T \ell(\mathbf{x}_t, \mathbf{m}_t) - \frac{\Lambda \epsilon^*}{N \lambda^{(M)}}$ ,  
 194 we have the following upper bound on the regret.

196 **Theorem 1 (Regret Bound of MF-UCB-PNE)** *Assume the utility function  $u_n(\mathbf{x})$  for each player  
 197  $n = 1, \dots, N$  lies in the reproducing kernel Hilbert space (RKHS) associated with the same kernel  
 198 function assumed by the MOGP prior. The regret incurred by MF-UCB-PNE is upper bounded as  
 199  $\tilde{O}(1/\sqrt{\Lambda})$  (up to log factors) with probability at least  $1 - N\delta$  for  $\delta \in (0, 1/N)$ .*

200 Theorem 1 implies that as the budget  $\Lambda$  grows, MF-UCB-PNE converges to an  $\epsilon^*$ -PNE with high  
 201 probability, obtaining, asymptotically, a vanishing regret. Due to space constraints, we refer to the  
 202 full paper [11] for a detailed statement and a proof of Theorem 1.

## 204 4 Experiments

207 We demonstrate the performance of MF-UCB-PNE on two wireless network problems: a power  
 208 control game in an interference network, and a random access game for medium access control. In  
 209 each case, we compare our approach against baseline schemes, including a single-fidelity BO algo-  
 210 rithm (UCB-PNE) that always queries at the highest fidelity [9], and the Probability-of-Equilibrium  
 211 (PE) strategy [12] that greedily uses full-fidelity evaluations without exploration.

212 **Power allocation game.** We consider  $N = 20$  interfering wireless links (transmitter-receiver pairs).  
 213 Each link  $n$  chooses a transmit power level  $x_n$  in the range  $[-13, 23]$  dBm. The utility  $u_n(\mathbf{x})$   
 214 is defined as the long-term average spectral efficiency (rate) of link  $n$  minus a penalty term pro-  
 215 portional to its power usage. Specifically, the utility is  $u_n(\mathbf{x}) = \mathbb{E}[\log_2(1 + \text{SINR}_n)] - \xi_n x_n$ ,  
 where  $\text{SINR}_n$  is the signal-to-interference-plus-noise ratio for link  $n$  given the power profile  $\mathbf{x}$ ,

and  $\xi_n = 0.1$  is a weight reflecting the importance of power consumption. The expectation is with respect to random channel gains: each link has a Rayleigh fading channel, and interference between any two links is governed by an interference gain factor  $\psi$ . In our setup, the direct channel gains are  $\mathcal{CN}(0, 1)$  variables and interference gains are  $\mathcal{CN}(0, \psi)$  variables, with  $\psi$  varying from  $-30$  dB (negligible interference) up to  $0$  dB (strong interference). Higher values of parameter  $\psi$  means more interference coupling between links, making the game more challenging.

To emulate multi-fidelity feedback, we let fidelity level  $m \in 1, 2, 3, 4, 5$  correspond to averaging the SINR over  $\lambda^{(m)}$  i.i.d. channel realizations, where  $\lambda^{(1)} = 1$  sample (cheapest, no averaging) and  $\lambda^{(5)} = 100$  samples (highest cost). Thus, the utility  $u_n^{(m)}(\mathbf{x})$  is the sample average of the rate  $\log_2(1 + \text{SINR}_n)$  over  $\lambda^{(m)}$  draws, providing an unbiased estimate of the true expected rate. We assign query costs equal to the number of samples, i.e. cost  $\lambda^{(m)}$  for fidelity  $m$ . A high-fidelity query ( $m = 5$ ) is  $100 \times$  more costly than a low-fidelity one ( $m = 1$ ).

In Figure 3, we plot the *sum spectral efficiency* (sum of all links' rates) achieved by the equilibrium solutions found, as a function of the interference gain  $\psi$ . All methods were given the same total budget  $\Lambda = 36000$ . At low interference, the links' utilities are nearly independent, and all algorithms find similar near-optimal power allocations. In moderate interference regimes, MF-UCB-PNE (green solid line) significantly outperforms UCB-PNE (orange dashed) and the greedy PE baseline (blue dash-dotted), achieving higher sum-rate by finding a better balance between interference and power use. At very high interference, all schemes see degraded performance, but MF-UCB-PNE still maintains a slight advantage. These results show that multi-fidelity exploration allows MF-UCB-PNE to discover improved equilibria especially when interference coupling is non-trivial, by efficiently using the budget on informative low-cost queries.

**Random access game.** Our second experiment involves optimizing a slotted ALOHA random access protocol. We consider  $N = 5$  mobile terminals sharing a wireless channel. Each terminal  $n$  has two action parameters:  $x_{n,1} \in [0, 1]$  is the probability the terminal is active in a slot (duty cycle), and  $x_{n,2} \in [0, 1]$  is the conditional probability of attempting a transmission when active. If terminal  $n$  transmits while another is also transmitting, a collision occurs. The success (packet reception) probability for terminal  $n$  is given by  $T_n(\mathbf{x}) = \frac{x_{n,1}x_{n,2}}{1-x_{n,1}x_{n,2}} \prod_{n' \neq n} (1 - x_{n',1}x_{n',2})$ , which accounts for the event that  $n$  transmits and no other active terminal transmits in the same slot [13]. The expected energy consumed by terminal  $n$  in a slot is  $E_n(\mathbf{x}) = x_{n,1}(c_1 + c_2 x_{n,2})$ , where  $c_1 = 50$  and  $c_2 = 70$  (in normalized units) are the energy costs for being active and for transmitting, respectively. We set per-terminal maximum energy budgets such that  $E_n(\mathbf{x}) \leq 60, 55, 50, 45, 40$  for  $n = 1, \dots, 5$ . Each terminal's utility is  $u_n(\mathbf{x}) = T_n(\mathbf{x}) - \xi_n E_n(\mathbf{x})$ , where  $\xi_n = 6.5 \times 10^{-4}$  is a weight tuning the importance of energy consumption relative to throughput.

We define three fidelity levels for simulation: at fidelities 1, 2, and 3, the throughput function  $T_n(\mathbf{x})$  is replaced by a coarser approximation using factors  $\omega^{(m)}$  instead of the true parameters, while fidelity 4 uses the exact  $T_n(\mathbf{x})$ . The costs are  $\lambda^{(1)} = 1$ ,  $\lambda^{(2)} = 5$ ,  $\lambda^{(3)} = 10$ , and  $\lambda^{(4)} = 20$ .

Figure 4 shows the *simple PNE regret* of each method as a function of the total query budget  $\Lambda$ . Regret here is defined as the gap between the achieved performance and the ideal equilibrium. We

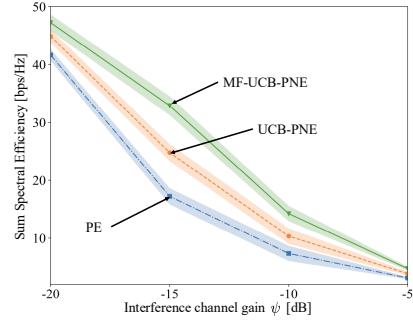


Figure 3: Sum spectral efficiency attained at the approximate equilibrium vs. interference channel gain  $\psi$ , in a power allocation setting for an interference channel.

Figure 4: Simple PNE regret (maximum dissatisfaction) in a random access setting as a function of total query budget  $\Lambda$ .

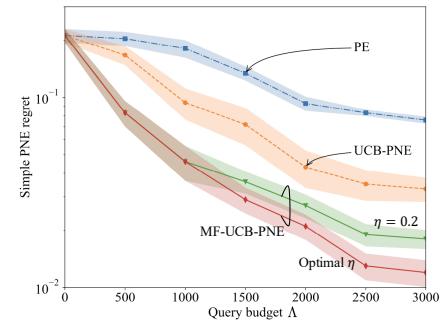


Figure 4: Simple PNE regret (maximum dissatisfaction) in a random access setting as a function of total query budget  $\Lambda$ .

270 compare MF-UCB-PNE with a fixed dissatisfaction threshold  $\eta = 0.2$  (green line) and with an  
 271 optimized threshold  $\eta$  (red line), against UCB-PNE (orange dashed) and PE (blue dash-dotted).  
 272 MF-UCB-PNE attains the lowest regret across all budget values, substantially outperforming both  
 273 baselines. Notably, UCB-PNE and PE, which always use high-fidelity queries, exhaust their budgets  
 274 quickly and then stagnate at higher regret. In contrast, MF-UCB-PNE continues to improve as  
 275 it efficiently allocates budget to informative low-cost queries. For example, at  $\Lambda = 3000$ , MF-  
 276 UCB-PNE achieves a regret of about  $5 \times 10^{-3}$ , whereas UCB-PNE stays above  $1.5 \times 10^{-2}$ . This  
 277 demonstrates that multi-fidelity exploration enables finding better approximate equilibria under tight  
 278 budget constraints, compared to strategies that naively focus on high-fidelity evaluations.  
 279

## 280 5 Conclusions

282 We have proposed MF-UCB-PNE, a novel MFBO strategy that efficiently approximates PNE solu-  
 283 tions by leveraging cheap low-fidelity evaluations for broad exploration, transitioning to high-fidelity  
 284 queries for refined exploitation. Theoretical analysis proves that MF-UCB-PNE obtains a vanishing  
 285 regret under suitable conditions, while empirical results on wireless networking problems validate  
 286 the high-quality equilibrium identified by MF-UCB-PNE under limited cost budgets. Looking for-  
 287 ward, it would be intriguing to extend the strategy to the evaluation of mixed Nash equilibria [9].  
 288

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