SPARSESWAPS: TRACTABLE LLM PRUNING MASK REFINEMENT AT SCALE

Anonymous authors
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ABSTRACT

The resource requirements of Neural Networks can be significantly reduced through pruning—the removal of seemingly less important parameters. However, with the rise of Large Language Models (LLMs), full retraining to recover pruning-induced performance degradation is often prohibitive and classical approaches such as global magnitude pruning are suboptimal on Transformer architectures. State-of-the-art methods hence solve a layer-wise *mask selection problem*, the problem of finding a pruning mask which minimizes the per-layer pruning error on a small set of calibration data. Exactly solving this problem to optimality using Integer Programming (IP) solvers is computationally infeasible not only due to i) the size of the search space, but also because ii) caching all intermediate values of the matrix multiplication needed to specify the optimization objective is already prohibitive. Existing approaches therefore rely on approximations or heuristics. In this work, we demonstrate that the mask selection problem can be made drastically more tractable at LLM scale. To that end, we leverage three key insights: a) enforcing equal sparsity levels per row decouples the rows without harming performance, b) the dimensionality of the problem can be reduced by leveraging the unitary invariance of the Frobenius norm objective and transforming the calibration data accordingly, and c) computing optimal *1-swaps* (exchanging one kept and one pruned weight) can be realized efficiently. These insights enable us to implement a tractable and simple 1-swap algorithm that warm starts from any pruning mask, runs efficiently on GPUs at LLM scale, and is essentially hyperparameter-free. We demonstrate that our approach reduces per-layer pruning error by up to 60% over Wanda (Sun et al., 2023) and consistently improves perplexity and zero-shot accuracy across state-of-the-art GPT architectures.

1 INTRODUCTION

Pruning after training (Han et al., 2015; Gale et al., 2019; Lin et al., 2020; Hoefler et al., 2021; Zimmer et al., 2025) is a state-of-the-art technique to reduce the resource requirements of neural networks. A simple yet effective approach to obtain such *sparse* models starts from a pretrained *dense* model, removes seemingly unimportant parameters based on their magnitude, and requires retraining to compensate for pruning-induced performance degradation. However, while the inexpensive, data-free magnitude criterion has often achieved strong performance on traditional architectures (Gale et al., 2019; Zimmer et al., 2023b), pruning has undergone a paradigm shift with the rise of large pretrained foundation models, particularly LLMs.

First, the size of the models has shifted the focus toward retraining-free pruning criteria, as retraining is often computationally expensive if not infeasible, with parameter-efficient fine-tuning (Lialin et al., 2023; Zimmer et al., 2023a) being an exception. Secondly, systematic activation outliers (Dettmers et al., 2022) and highly important *super-weights* (Yu et al., 2025) in sufficiently large *Transformers* (Vaswani et al., 2017) have rendered magnitude pruning no better than random pruning for LLMs (Sun et al., 2023; Yin et al., 2023). Lastly, state-of-the-art methods (Frantar & Alistarh, 2023; Sun et al., 2023; Zhang et al., 2024) prune *layer-wise*: they split the pruning problem into per-layer subproblems, pruning layers sequentially and independently using a small calibration dataset to estimate parameter importance. Rather than optimizing the *global* loss, such approaches minimize a per-layer *local* pruning loss. Specifically, for a single layer with calibration input matrix

$X \in \mathbb{R}^{d_{in} \times B}$ and weights $W \in \mathbb{R}^{d_{out} \times d_{in}}$, the objective becomes

$$\min_{M} \|WX - (M \odot W)X\|_F^2, \tag{1}$$

where $M \in \{0,1\}^{d_{out} \times d_{in}}$ is a binary pruning mask achieving a desired level of sparsity, e.g., $\|M\|_0 \leq k$ for unstructured sparsity, and $\odot$ denotes the element-wise multiplication or Hadamard product. Here, $B = N \cdot L$ with N being the number of samples in the calibration batch and L being the sequence length.

Solving this combinatorial *mask selection problem* to optimality is NP-hard due to feature correlations: selecting k of $d_{out} \cdot d_{in}$ weights yields a cardinality-constrained binary quadratic program (a best-subset selection variant). Even for a single row i, the problem reduces to

$$\min_{m_i} \left\| w_i^\top X - (m_i \odot w_i)^\top X \right\|_2^2 = \min_{m_i} \sum_{k=1}^{B} \left(\sum_{j=1}^{d_{in}} (1 - m_{ij}) w_{ij} X_{jk} \right)^2,$$

where $w_i \in \mathbb{R}^{d_{in}}$ and $m_i \in \{0,1\}^{d_{in}}$ denote the i-th row of W and M, respectively. Solving the mask selection problem requires caching all $B \cdot d_{in}$ intermediate products $w_{ij}X_{jk}$, the summands of $w_i^\top X$. In a standard forward pass, these terms are computed on the fly and immediately discarded, but optimal pruning must retain them to assess weight importance. To illustrate the scale, consider a single row of the largest matrix in a LLaMA-2-7B Transformer block: the `up_proj` matrix with input dimension $d_{in} = 4096$. With $N = 128$ samples and sequence length $L = 4096$ (so $B = N \cdot L = 524{,}288$), caching all products $w_{ij}X_{jk}$ for that row requires $524{,}288 \times 4096 \approx 2.15$ billion float32 values (about 8.6GB); across all 11,008 rows this totals about 94.6TB. Thus, while IP solvers could theoretically provide optimal solutions, the bottleneck is not only the combinatorial search over mask entries but also the prohibitive memory requirements for caching the intermediates. In practice, existing methods therefore relax Equation 1 or approximate it.

However, with deployed LLMs now serving millions of users, it becomes increasingly worthwhile to invest substantial resources to obtain pruned models that reach high performance, because the pruning cost is paid once during training whereas inference costs scale with the number of requests. In this work, we revisit the per-layer mask selection problem and demonstrate that it can be operationalized at LLM scale, enabling monotone improvements with each optimization step rather than relying on proxy importance scores. To that end, we make multiple key observations: (1) enforcing equal sparsity-level across rows must not be detrimental and ensures *row-wise separability* that yields independent objectives, and (2) the *unitary invariance* of the Frobenius objective can be leveraged to drastically reduce the dimensionality of the problem, making intermediate caching feasible without changing the objective. Taken together, these insights make the problem drastically more tractable. Instead of trying to obtain exact solutions via IP solvers, we opt for and propose a GPU-accelerated local optimization algorithm based on (3) an *exact and efficient local refinement with incremental cost updates* through optimal *1-swaps* (exchanging one kept and one pruned weight) that monotonically decreases the objective from any warm start.

The resulting method, which we term SparseSwaps, can start from any warm-start mask, evaluates the exact per-row quadratic loss, and is scalable, parallelizable across rows, almost hyperparameter-free, and deterministic for a fixed warm start. With only few 1-swap iterations, it can reduce the per-layer pruning error by up to 60% compared to Wanda and improves final perplexity and zero-shot accuracy across architectures. Our approach is thus a post-hoc refinement of existing pruning methods that can significantly improve upon the state of the art for unstructured, per-row, or $N{:}M$ sparsity.

Contributions. Our contributions are as follows:

1. **Novel insights that make the problem tractable.** We identify three simple ideas that make the pruning mask selection objective tractable at LLM scale: row-wise separability of the loss, Singular Value Decomposition (SVD)-based compression of calibration features to enable caching of necessary intermediates, and exact 1-swap evaluation with efficient incremental cost updates.
2. **SparseSwaps: a practical post-hoc pruning algorithm.** Building on these insights, we propose SparseSwaps, a plug-and-play 1-swap refinement that starts from any warm-start

mask and monotonically decreases the exact per-row objective under per-row or $N{:}M$ constraints. It delivers large reductions in local pruning error (up to 60% per-layer error reduction over Wanda) and strong perplexity and zero-shot gains on state-of-the-art Generative Pretrained Transformer (GPT) architectures.

Further related work. *Post-training pruning* has a long history, and while *magnitude pruning* (Janowsky, 1989; Han et al., 2015) is among the most popular criteria, it is not the only one (cf. LeCun et al., 1989; Hassibi & Stork, 1993; Molchanov et al., 2016; Yeom et al., 2019); see Hoefler et al. (2021) for a comprehensive review. Despite their simplicity, magnitude-based methods have been shown to produce sparse models competitive with far more complex algorithms for convolutional architectures (Gale et al., 2019; Zimmer et al., 2023b). For LLMs, however, magnitude pruning is argued to be unsuitable (Yin et al., 2023). Consequently, there is growing interest in criteria beyond magnitude that achieve high performance on LLMs, and do so without requiring an expensive retraining procedure (Kwon et al., 2022; Frantar & Alistarh, 2023; Sun et al., 2023). In this work, we develop a post-hoc refinement of existing methods, rather than proposing a new criterion. A related approach, DSnoT (Zhang et al., 2023), also performs iterative weight swaps but differs significantly in its optimization strategy. Inspired by *dynamic sparse training* (cf. Evci et al., 2020), DSnoT prunes and regrows weights based on expected reconstruction-error improvements, using feature means and variances as surrogates. While effective, it does not guarantee a monotonic decrease in the true pruning error, whereas our method does. We compare the two empirically and find that SparseSwaps consistently outperforms DSnoT.

Subset selection and IP approaches. To solve Equation 1 to global optimality, which can be formulated as a *mixed-integer nonlinear program (MINLP)*, several efficient open-source solvers are available, including SCIP (Bolusani et al., 2024), Bonmin (Bonami et al., 2008), and SHOT (Lundell et al., 2022), among others. In particular, the recently introduced Boscia solver (Hendrych et al., 2025) is particularly well-suited, as it exploits the problem's combinatorial structure. While we demonstrate how the problem can be made drastically more tractable, explicit solution remains very time-consuming for large instances; we therefore opt for a GPU-friendly 1-swap approach that avoids moving large tensors to the CPU for IP solvers. We leave such an extension for future work.

2 Methodology

In the following, we use uppercase letters for matrices (W, X, M) and lowercase letters for scalars and vectors. Matrix entries are denoted W_{ij} for the element in row i, column j. Rows of matrices are denoted with lowercase subscripts: w_i represents the i-th row of matrix W. Row and column slices use colon notation: $X_{j,:}$ for the j-th row and $X_{:,k}$ for the k-th column. We use $\odot$ for element-wise multiplication, $\|\cdot\|_F$ for Frobenius norm, and $\|\cdot\|_2$ for ℓ_2 norm.

2.1 Preliminaries and insights

Before describing our proposed method, we make several crucial assumptions and observations that make the problem tractable.

2.1.1 Insight 1: Equal sparsity-level across rows must not be detrimental

First, note that the objective in Equation 1 decomposes into a sum of d_{out} row-wise quadratics,

$$\|WX - (M \odot W)X\|_F^2 = \sum_{i=1}^{d_{out}} \left\|w_i^\top X - (m_i \odot w_i)^\top X\right\|_2^2 \tag{2}$$

where $w_i \in \mathbb{R}^{d_{in}}$ and $m_i \in \{0,1\}^{d_{in}}$ denote the i-th row of W and M, respectively. This alone does not make the corresponding minimzation problem row-separable under unstructured sparsity, since the matrix cardinality constraint couples rows. In contrast, semi-structured patterns like per-row sparsity (keep k per row) or $N{:}M$ (prune $M{-}N$ per block of M weights) enforce equal per-row sparsity and fully decouples rows which can now be solved independently. We therefore focus on

the decoupled case, allowing to treat each row separately and reducing the problem to

$$\min_{m_i} \left\| w_i^\top X - (m_i \odot w_i)^\top X \right\|_2^2 = \min_{m_i} \sum_{k=1}^{B} \left(\sum_{j=1}^{d_{in}} (1 - m_{ij}) w_{ij} X_{jk} \right)^2 \tag{3}$$

for each row $i \in \{1, \dots, d_{out}\}$. Note that, for LLMs, Sun et al. (2023) observe that row-wise sparsity benefits performance for both Wanda and magnitude pruning. We therefore argue that enforcing per-row sparsity rather than unstructured sparsity is justified and need not harm final performance, at least for LLMs. For semi-structured sparsity, the rows are decoupled anyway.

As a side note, since positive scaling preserves minima and by applying Jensen's inequality, one can now easily derive the Wanda criterion:

$$\min_{m_i} \sum_{k=1}^{B} \left(\sum_{j=1}^{d_{in}} (1 - m_{ij}) w_{ij} X_{jk} \right)^2 \leq \min_{m_i} \sum_{k=1}^{B} \left(\sum_{j=1}^{d_{in}} (1 - m_{ij})^2 w_{ij}^2 X_{jk}^2 \right) \tag{4}$$

$$= \min_{m_i} \sum_{j=1}^{d_{in}} (1 - m_{ij})^2 w_{ij}^2 \|X_{j,:}\|_2^2. \tag{5}$$

Equation 5 is solved by pruning entries with the smallest saliency $|w_{ij}| \cdot \|X_{j,:}\|_2$, i.e., precisely the Wanda criterion. Thus, Wanda optimizes an upper bound to the original problem that ignores within-row interactions, making the combinatorial problem tractable.

2.1.2 Insight 2: Unitary invariance of the mask selection problem

A significant issue is that if the context length L and consequently $B = N \cdot L$ is large, operating with the full data matrix $X \in \mathbb{R}^{d_{in} \times B}$ is computationally infeasible. Typically, B is much larger than d_{in} for LLMs; X has many more columns than rows. To significantly reduce the computational cost and render our approach feasible, we leverage the fact that the Frobenius norm used in our pruning objective is *unitarily invariant*: for any matrix A and unitary matrix U (i.e., $U^{-1} = U^\top$), we have $\|AU\|_F = \|A\|_F$. This property enables significant computational savings through SVD compression.

Precisely, let $X = U\Sigma V^\top$ be the SVD of calibration data $X \in \mathbb{R}^{d_{in} \times B}$. Since $B > d_{in}$, we can write $\Sigma = [\Sigma' | 0]$ with Σ' being the square matrix containing the d_{in} singular values on its diagonal. We construct a compressed representation of the data matrix as follows:

$$X' = U\Sigma' U^\top \in \mathbb{R}^{d_{in} \times d_{in}}. \tag{6}$$

Letting $W_p = W - M \odot W$ for brevity, the key insight is that pruning decisions remain equivalent under this compression:

$$\begin{aligned} \|W_p X\|_F^2 &= \left\|W_p U \Sigma V^\top\right\|_F^2 = \|W_p U \Sigma\|_F^2 = \|W_p U [\Sigma' \mid 0]\|_F^2 \\ &= \|W_p U \Sigma'\|_F^2 = \left\|W_p U \Sigma' U^\top\right\|_F^2 = \|W_p X'\|_F^2, \end{aligned}$$

where we used that U and V are unitary and the Frobenius norm is invariant under unitary transformations. In practice, since we do not need the right singular vectors V, we efficiently compute U and Σ' via eigendecomposition of the symmetric matrix $XX^\top$.

If now $d_{in} \ll B$, the computational and memory savings by working with X' instead of X are significant, effectively solving the issues of caching intermediate summands. Returning to our example from the introduction, with SVD compression the memory requirement per row reduces from caching $B \times d_{in} = 524{,}288 \times 4096 \approx 2.15$ billion products to caching $d_{in} \times d_{in} = 4096 \times 4096 \approx 16.8$ million products, a 128× reduction from about 8.6GB to about 67MB per row.

2.1.3 Insight 3: 1-swap local search is computationally tractable

While the global mask selection problem is NP-hard, we can still make efficient progress via local search. Consider a single row (omitting its index i) and let $d := d_{in}$. Starting from any feasible

mask $m \in \{0,1\}^d$, the idea is to iteratively perform 1-swaps that exchange one kept and one pruned weight to reduce the objective while preserving the sparsity level. The key insight is that we evaluate swaps efficiently by precomputing and caching intermediate summands instead of recomputing the full objective after each iteration.

Recall that our objective from Equation 1 is to minimize the reconstruction error from pruned weights. For a single row, this becomes minimizing

$$\left\|((\mathbb{1} - m) \odot w)^\top X\right\|_2^2 = \left\| \sum_{j:m_j=0} w_j X_{j,:} \right\|_2^2 \tag{7}$$

where $(\mathbb{1} - m)$ identifies the pruned weights and the sum is over all pruned weight indices. We precompute the intermediate value matrix:

$$S = \begin{bmatrix} w_1 X_{1,1} & w_1 X_{1,2} & \cdots & w_1 X_{1,B} \\ w_2 X_{2,1} & w_2 X_{2,2} & \cdots & w_2 X_{2,B} \\ \vdots & \vdots & \ddots & \vdots \\ w_d X_{d,1} & w_d X_{d,2} & \cdots & w_d X_{d,B} \end{bmatrix} \in \mathbb{R}^{d \times B} \tag{8}$$

where row j contains the weighted contribution $w_j X_{j,:}$ from input dimension j. After SVD compression (Insight 2), $B = d$, so S is square; we retain the B notation for clarity.

Let $\mathcal{P} \subseteq \{1, \ldots, d\}$ and $\mathcal{U} \subseteq \{1, \ldots, d\}$ denote the set of pruned and unpruned weight indices, respectively. Our objective then becomes minimizing $\|s\|_2^2$ where

$$s = \sum_{j \in \mathcal{P}} S_{j,:} \tag{9}$$

is the sum over rows of S corresponding to pruned weights, and $\|s\|_2^2$ is the squared reconstruction error from pruned weights. To evaluate a 1-swap that removes index p from the pruned set $\mathcal{P}$ and adds index u from the unpruned set $\mathcal{U}$, the new objective vector is:

$$s^{\text{new}} = s - S_{p,:} + S_{u,:} \tag{10}$$

and the improvement in objective is $\|s\|_2^2 - \|s^{\text{new}}\|_2^2$, computable in $\mathcal{O}(B)$ time using the cached rows from S. By systematically testing all $(d - |\mathcal{P}|) \times |\mathcal{P}|$ possible 1-swap operations (adding one of $|\mathcal{U}| = d - |\mathcal{P}|$ unpruned weights to $\mathcal{P}$, removing one of $|\mathcal{P}|$ pruned weights from $\mathcal{P}$) evaluating the improvement using the above expression, we iteratively pick a best swap and update the mask until we have reached a satisfactory solution or one optimal w.r.t. 1-swap operations.

Cost-effective swap evaluation and the p–u interaction. Crucially, after accepting a swap (p, u) (and before the next swap evaluation) we update the cost vector incrementally via $s \leftarrow s - S_{p,:} + S_{u,:}$ (cost $\mathcal{O}(B)$) rather than recomputing $s = \sum_{j \in P} S_{j,:}$ from scratch (cost $\mathcal{O}(|P|\, B)$). This running-sum update is one of the main efficiency gains of our approach.

Expanding the squared norm makes the computational benefit and the dependency between the removed and added indices explicit. For any candidate pair (p, u) with $p \in \mathcal{P}$ and $u \in \mathcal{U}$ and denoting $s_p := s - S_{p,:}$,

$$\|s - S_{p,:} + S_{u,:}\|_2^2 = \|s_p + S_{u,:}\|_2^2 = \|s_p\|_2^2 + \|S_{u,:}\|_2^2 + 2\, s_p^\top S_{u,:}. \tag{11}$$

For fixed p, the first term is constant and $\|S_{u,:}\|_2^2$ can be precomputed. Evaluating all swaps that share p reduces to dot products $\{s_p^\top S_{u,:}\}_{u \in \mathcal{U}}$, computed efficiently via matrix operations. Over all $p \in \mathcal{P}$, this yields an exhaustive 1-swap search in $\mathcal{O}(|\mathcal{P}|\, |\mathcal{U}|\, B)$ without recomputing full objectives from scratch.

Why picking p and u separately is suboptimal. The cross term $2\, s_p^\top S_{u,:}$ in Equation 11 shows that the best u depends on the chosen p (and vice versa). Consequently, selecting the best p based on $\|s_p\|_2^2$ and then the best u based on $\|s_u\|_2^2$ can result in a suboptimal solution as the following example for the scalar case with $B = 1$ and $d = 4$ shows. Let the current pruned-set contributions

be $\{+10, -1\}$, so $s = 9$, and the unpruned candidates be $\{+9, -9\}$. The best 1-swap is to remove $p = -1$ and add $u = -9$, giving $s^{\text{new}} = 10 - 9 = 1$ and objective $1^2 = 1$. However, if, instead, we greedily remove the best p in isolation, we drop $p = +10$ since $\|9 - 10\|^2 = 1$. We must then add one index; the best addition in isolation is $u = -9$, leading to $s^{\text{new}} = -1 + (-9) = -10$ and objective 100. The error stems precisely from ignoring the interaction term in Equation 11.

2.2 The SparseSwaps algorithm

Building upon the three key insights, we present our complete algorithm. The method takes as input a weight matrix $W \in \mathbb{R}^{d_{out} \times d_{in}}$, calibration data $X \in \mathbb{R}^{d_{in} \times B}$, and a warmstart pruning mask $M^{\text{init}} \in \{0,1\}^{d_{out} \times d_{in}}$ that already satisfies the desired sparsity constraints, e.g., obtained from Wanda or RIA (Zhang et al., 2024).

The algorithm enforces any sparsity pattern that operates per-row, including per-row sparsity (fixed number of zeros per row, cf. Sun et al. (2023)) and structured $N{:}M$ sparsity patterns (e.g., 2:4 or 4:8, Mishra et al. (2021)). All swap operations maintain the sparsity constraints throughout optimization; for $N{:}M$ sparsity, swaps are restricted to occur only within the same $N{:}M$ blocks, while for per-row sparsity, the total number of pruned weights per row remains constant. Even though each swap only changes two mask entries, the cumulative effect of multiple swaps can dramatically reduce reconstruction error compared to the initial solution.

Algorithm 1 SparseSwaps: 1-Swap Pruning Optimization

Require: Weight matrix $W \in \mathbb{R}^{d_{out} \times d_{in}}$, calibration data $X \in \mathbb{R}^{d_{in} \times B}$, warmstart mask M^{init}
Ensure: Improved pruning mask M

1: $X \leftarrow U\Sigma' U^\top$ ▷ SVD compression to reduce memory
2: $M \leftarrow M^{\text{init}}$ ▷ Initialize with warmstart solution
3: **for** $i = 1$ to d_{out} **do** ▷ Process each row independently
4: $w \leftarrow W_{i,:}, m \leftarrow M_{i,:}$ ▷ Extract row weights and mask
5: $S \leftarrow \text{diag}(w)X$ ▷ Cache weighted contributions
6: $\mathcal{P} \leftarrow \{j : m_j = 0\}, \mathcal{U} \leftarrow \{j : m_j = 1\}$ ▷ Pruned and unpruned sets
7: $s \leftarrow \sum_{j \in P} S_{j,:}$ ▷ Current cost vector
8: **for** $t = 1$ to $T_{\max}$ **do**
9: $(p^*, u^*) \leftarrow \arg\min \ \|s - S_{p,:} + S_{u,:}\|_2^2$ ▷ over feasible 1-swaps respecting sparsity
10: **if** swap (p^*, u^*) reduces $\|s\|_2^2$ **then**
11: $m_{p^*} \leftarrow 1, m_{u^*} \leftarrow 0$ ▷ Perform swap
12: $\mathcal{P} \leftarrow \mathcal{P} \setminus \{p^*\} \cup \{u^*\}, \mathcal{U} \leftarrow \mathcal{U} \setminus \{u^*\} \cup \{p^*\}$
13: $s \leftarrow s - S_{p^*,:} + S_{u^*,:}$ ▷ Update cost vector
14: **else**
15: **break** ▷ Local optimum reached
16: **end if**
17: **end for**
18: $M_{i,:} \leftarrow m$ ▷ Store optimized row
19: **end for**

We explain the main phases of the algorithm:

Preparation (Lines 1-2): We compress the calibration data $X \leftarrow U\Sigma' U^\top$ via SVD to achieve the memory reduction from Insight 2, then initialize with the warmstart mask M^{init}.

Row processing (Lines 4-7): For each row i, we extract weights w and current mask m, precompute the weighted contributions $S = \text{diag}(w)X$, define pruned and unpruned index sets $\mathcal{P}$ and $\mathcal{U}$, and compute the current reconstruction cost vector s.

1-Swap optimization (Lines 8-17): We iteratively choose the swap (p^*, u^*) minimizing $\|s - S_{p,:} + S_{u,:}\|_2^2$ among feasible pairs. If this improves the reconstruction error $\|s\|_2^2$, we perform it and update the sets accordingly, otherwise we terminate. At all times, the swaps are appropriately constrained: per-row sparsity allows any swap maintaining $|P|$ constant, while $N{:}M$ sparsity restricts swaps to within the same $N{:}M$ blocks.

Computational complexity: Theoretically, the algorithm has complexity $\mathcal{O}(d_{out} \cdot T_{\max} \cdot |\mathcal{P}| \cdot |\mathcal{U}| \cdot d_{in})$ per layer, where $T_{\max}$ being the maximum number of swap iterations per row, and the d_{in} factor comes from computing norms over the compressed calibration data. However, several complexity factors can be reduced in practice. First, we find that even setting $T_{\max} = 1$ or $T_{\max} = 2$ can drastically reduce the local pruning error; values around $T_{\max} = 25$ often suffice to significantly lower model perplexity, with diminishing returns beyond $T = 100$. Second, row-wise processing can be batched and vectorized across multiple dimensions, enabling parallel swap cost computations and mask updates, and rows can be distributed across GPUs if needed. Third, after precomputing weighted contributions S, cost vector updates $s \leftarrow s - S_{p^*,:} + S_{u^*,:}$ are simple vector additions with $\mathcal{O}(d_{in})$ complexity.

Optional Weight Reconstruction. After mask optimization, we can optionally apply local weight reconstruction to further reduce reconstruction error. Following Frantar & Alistarh (2023), for each row, let $\mathcal{U}$ denote the unpruned column indices and $X_{\mathcal{U}}$ the corresponding rows of X. The least-squares problem $\min_{\hat{w}} \left\| w^\top X - \hat{w}^\top X_{\mathcal{U}} \right\|_2^2$ has the solution:

$$w^*_{\mathcal{U}} = (X_{\mathcal{U}} X_{\mathcal{U}}^\top)^{-1} X_{\mathcal{U}} (w^\top X)^\top.$$

Note that $w^\top X = \sum_{j=1}^{d} S_{j,:}$ is available from our already-cached weighted contributions $S = \text{diag}(w)X$ without additional matrix multiplications.

3 Experimental Results

We outline our general experimental approach, detailing datasets, architectures, and metrics. To enable reproducibility, our code will be publicly released. Our study focuses on language modeling within Natural Language Processing (NLP). We use pretrained models from HuggingFace (Wolf et al., 2020), specifically LLaMA-3.1-8B (Grattafiori et al., 2024), Gemma-2-9B (Riviere et al., 2024), Yi-1.5-9B (Young et al., 2025), DeepSeek-7B-base (Bi et al., 2024), and Qwen2.5-7B (Yang et al., 2025). For calibration, we randomly draw sequences of 2048 tokens from the *C4* dataset (Raffel et al., 2020). For validation, we similarly pick 100 sequences from the validation split. The model performance is assessed via perplexity on the *WikiText* dataset (Merity et al., 2016) and zero-shot accuracy on the EleutherAI evaluation set (Gao et al., 2023). Following Sun et al. (2023), we prune all linear layers, excluding the embedding and final linear head, with uniform sparsity allocation across layers. We provide experiments for unstructured and semi-structured sparsity patterns (Mishra et al., 2021). We use multiple random seeds throughout our experiments.

3.1 Mask refinement at scale

We begin by verifying the effectiveness of SparseSwaps. We make the following observations:

SparseSwaps consistently improves state-of-the-art methods. Table 1 summarizes the main results and reports perplexity (upper half, lower is better) and zero-shot accuracy (lower half, higher is better) for warmstart masks (Wanda, RIA) as well as their refinements using DSnoT and SparseSwaps. For both 60% unstructured and 2:4 semi-structured sparsity, SparseSwaps (with 100 1-swap iterations) consistently reduces perplexity and improves zero-shot accuracy over Wanda and RIA warm start masks. While DSnoT similarly yields improvements, it falls short of SparseSwaps. Note that we left the pruning criterion of DSnoT, which partially uses the Wanda saliency, unchanged, even when using RIA warmstart. For unstructured RIA, we report results when enforcing a per-row sparsity constraint; while RIA yields good (and slightly better) results when enforcing truely unstructured sparsity, we decided to include the results for the per-row setting as this allows direct refinement of the mask with SparseSwaps and DSnoT.

SparseSwaps successfully optimizes the per-layer pruning loss. Figure 1 shows the per-layer reductions in local pruning error relative to a Wanda Warmstart, grouping layers by their corresponding Transformer block of LLaMA-3.1-8B. We observe drastic improvements of close to 70% compared to Wanda, demonstrating that SparseSwaps is able to successfully optimize the local loss. The `attn.o_proj` seems to consistently benefit the most across blocks, with reductions of the objective in Equation 1 ranging between 40%-60%.

Large local error reductions do not always imply reduced perplexity. From Table 1 we observe substantial perplexity gains, especially when sparsity more strongly degrades model quality (cf.

Table 3 in the appendix, which shows more drastic improvements when using magnitude pruning, which more strongly degrades model quality). In contrast, when quality is less affected (e.g., at 50% sparsity where Wanda performs well), SparseSwaps yields limited perplexity gains despite significant local error reductions: Table 2 reports perplexity and average relative error reduction (%) versus the number of 1-swap iterations. Zero iterations correspond to the Wanda warm start; one or more iterations correspond to SparseSwaps from Wanda. At 50% sparsity, a single 1-swap iteration lowers relative error by 6.34%, and 200 iterations by nearly 40%, yet perplexity does not improve, but rather slightly increases. This suggests further reducing local error can overfit the calibration data and may not translate to better perplexity, although we note that the perplexity increase is relatively small. These results emphasize that while the reduction of local error is a useful proxy for perplexity reduction when pruning has a higher negative impact on the model, the local error of Equation 1 remains an approximation to the reconstruction error of the entire model.

Table 1: Perplexity (↓, lower is better) and zero-shot accuracy (↑, higher is better) comparison on WikiText and EleutherAI evaluation set. We report DSnoT and SparseSwaps refinement with Wanda and RIA warmstart for unstructured 60% sparsity and semi-structured 2:4 sparsity. Best values are highlighted in **bold**. We omit standard deviations for legibility.

Perplexity ↓		**LLaMA-3.1**	**Gemma-2**	**Yi-1.5**	**DeepSeek**	**Qwen2.5**
Method	Sparsity	8B	9B	9B	7B	7B
Wanda	60%	21.94	16.74	11.40	11.41	13.75
+ DSnoT	60%	21.94	16.69	11.38	11.40	13.75
+ SparseSwaps	60%	**19.75**	**16.01**	**10.07**	**10.93**	**13.16**
RIA	60%	19.73	16.19	10.73	11.80	12.63
+ DSnoT	60%	19.73	16.22	10.73	11.80	12.63
+ SparseSwaps	60%	**18.47**	**15.44**	**9.98**	**10.79**	**12.47**
Wanda	2:4	24.82	17.45	11.76	11.77	14.53
+ DSnoT	2:4	22.79	16.79	10.84	11.70	14.40
+ SparseSwaps	2:4	**20.17**	**16.30**	**10.73**	**11.70**	**13.95**
RIA	2:4	23.96	16.88	11.29	12.03	13.58
+ DSnoT	2:4	24.26	16.82	10.57	12.03	13.85
+ SparseSwaps	2:4	**20.90**	**16.33**	**10.50**	**11.80**	**13.28**
Accuracy ↑		**LLaMA-3.1**	**Gemma-2**	**Yi-1.5**	**DeepSeek**	**Qwen2.5**
Method	Sparsity	8B	9B	9B	7B	7B
Wanda	60%	48.18%	63.39%	53.59%	50.74%	59.26%
+ DSnoT	60%	48.18%	63.49%	53.79%	50.75%	59.26%
+ SparseSwaps	60%	**50.78%**	**63.84%**	**54.84%**	**51.02%**	**60.15%**
RIA	60%	49.56%	64.37%	52.81%	50.92%	59.84%
+ DSnoT	60%	49.56%	**64.43%**	52.96%	50.83%	59.81%
+ SparseSwaps	60%	**51.02%**	64.32%	**54.45%**	**51.47%**	**61.22%**
Wanda	2:4	46.80%	63.73%	**52.58%**	**51.02%**	59.52%
+ DSnoT	2:4	47.01%	63.66%	52.16%	50.78%	59.09%
+ SparseSwaps	2:4	**48.83%**	**64.70%**	52.43%	50.36%	**59.92%**
RIA	2:4	47.87%	63.87%	**52.68%**	51.22%	58.66%
+ DSnoT	2:4	47.13%	64.17%	51.36%	49.86%	59.72%
+ SparseSwaps	2:4	**49.90%**	**64.60%**	52.30%	**51.46%**	**60.31%**

3.2 Efficiency and hyperparameter ablations

Resource requirements. SparseSwaps is more resource-intensive than DSnoT and, as a drop-in refinement, requires at least the resources of the chosen warm-start method. Beyond that, SparseSwaps needs memory to store the matrix S (cf. Equation 8) and compute to perform the 1-swaps; see the preceding section for the theoretical complexity. While we have argued in the introduction that the additional compute can be justified when amortized over many LLM inference requests,

Figure 1: Per-layer relative reduction in local pruning error compared to Wanda. The plot shows result for LLAMA-3.1-8B, 60% unstructured sparsity and 100 1-swap iterations.

we note that the overhead grows only linearly with the number of 1-swap iterations $T_{\max}$. Table 2 shows that few iterations already yield substantial gains in both perplexity and local error reduction, especially at higher sparsity.

Effect of the number of reconstruction samples. Figure 2 in the appendix shows the perplexity versus the number of reconstruction samples for 50% and 60% unstructured sparsity when using Wanda as well as SparseSwaps with a Wanda warmstart. We observe that the perplexity decreases drastically when using more samples, which leads to SparseSwaps slightly outperforming Wanda for 50% sparsity, despite its advantage typically being larger at higher sparsity. We emphasize that the number of reconstruction samples does not affect SparseSwaps's efficiency: after SVD compression, the size of X (and thus compute/memory cost) is independent of the original sample count.

Table 2: LLAMA-3.1-8B: Perplexity (↓) and mean relative reduction in pruning error (↑) versus number of 1-swap iterations for 50% and 60% unstructured sparsity using Wanda warmstart.

		Number of 1-swap iterations								
Sparsity	**Metric**	0	1	2	5	10	25	50	100	200
50%	Avg. rel. error reduction (%)	0.00	6.34	8.77	12.51	16.38	23.52	30.04	36.48	38.95
	Perplexity	10.13	10.31	10.40	10.41	10.39	10.38	10.27	10.30	10.34
60%	Avg. rel. error reduction (%)	0.00	8.04	11.04	15.34	19.64	26.92	33.58	39.99	43.74
	Perplexity	21.52	21.26	21.51	21.17	21.01	20.38	19.74	18.96	19.17

4 CONCLUSION

We revisited the mask selection problem for post-training pruning and showed that it can be made substantially more tractable, even at LLM scale. We leveraged three central insights—row decoupling via equal per-row sparsity, SVD-based compression exploiting the unitary invariance of the Frobenius objective, and exact 1-swap evaluation with incremental cost updates—to enable tractable optimization of the true row-wise quadratic loss on GPUs. The resulting method, SparseSwaps, is warm-start agnostic, nearly hyperparameter-free, and scalable. It consistently reduces per-layer pruning error and improves perplexity and zero-shot accuracy across modern GPT architectures.

Our work is not without limitations. While per-row sparsity is not necessarily detrimental for LLMs, our approach is restricted to that setting and only partially adapts to truly unstructured sparsity; in its current form, the algorithm can handle unstructured sparsity but cannot reallocate sparsity levels across rows. Furthermore, runtime and memory remain non-trivial for large architectures.

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A Appendix

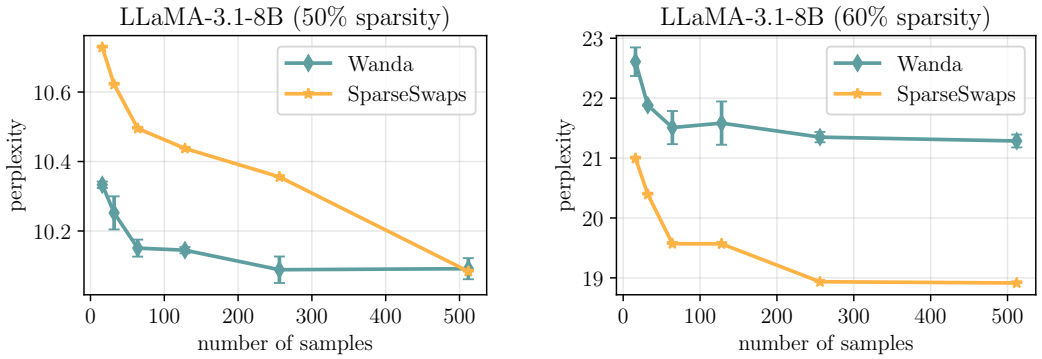
A.1 Use of Large Language Models

Large language models were used to aid in writing (polishing text) as well as to help with the implementation of code components, including both the methods and the generation of plots. They also served as a tool for brainstorming research ideas and refining development approaches to address the challenges explored in this paper.

A.2 Further Results

Table 3: Perplexity (↓, lower is better) comparison on WikiText. We report SparseSwaps refinement with magnitude warmstart for 50% and 60% sparsity. Best values are highlighted in **bold**. We omit standard deviations for legibility.

Perplexity ↓		**LLaMA-3.1**	**Gemma-2**	**DeepSeek**
Method	Sparsity	8B	9B	7B
Magnitude	50%	68.89	31.87	25.05
+ SparseSwaps	50%	52.26	19.11	16.23
Magnitude	60%	3486.26	184.52	330.07
+ SparseSwaps	60%	264.92	60.04	80.24



(a) 50% unstructured sparsity

(b) 60% unstructured sparsity

Figure 2: Perplexity versus the number of reconstruction samples for unstructured sparsity using Wanda warmstart.