On a Geometry of Interbrain Networks

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Abstract

Effective analysis in neuroscience benefits significantly from robust conceptual frameworks. Traditional metrics of interbrain synchrony in social neuroscience typically depend on fixed, correlation-based approaches, restricting their explanatory capacity to descriptive observations. Inspired by the successful integration of geometric insights in network science, we propose leveraging discrete geometry to examine the dynamic reconfigurations in neural interactions during social exchanges. Unlike conventional synchrony approaches, our method interprets inter-brain connectivity changes through the evolving geometric structures of neural networks. This geometric framework is realized through a pipeline that identifies critical transitions in network connectivity using entropy metrics derived from curvature distributions. By doing so, we significantly enhance the capacity of hyperscanning methodologies to uncover underlying neural mechanisms in interactive social behavior.

Keywords: Discrete Geometry, Graph Curvature, Inter-brain Networks, Hyperscanning, Social Neuroscience, Network Dynamics

1. Introduction

Interbrain synchrony (IBS) metrics, such as the Phase Locking Value (PLV), have dominated social neuroscience research, providing practical but fundamentally descriptive measures of neural interactions (Hakim et al., 2023). These methods typically neglect dynamic transitions between brain network states that could provide insight into social interaction mechanisms. Recent advancements in geometric machine learning highlight discrete geometric methods as powerful tools for characterizing complex network structures and dynamics (Weber, 2025). The present opinion piece is motivated by the idea that transient connectivity patterns govern flexible cognitive processes (Sporns, 2010). Such processes have been previously analyzed with geometric tools; however, these works explored primarily intra-brain structural and functional networks (Chatterjee et al., 2021; Weber et al., 2019). In this article, we propose the application of geometric methods to time-varying interbrain networks during social interaction. Specifically, our proposed approach leverages discrete graph curvatures to address the unique challenges of dynamic interbrain networks in hyperscanning research (Hinrichs et al., 2025); it aims to overcome the limitations of correlation-based metrics by providing richer, more mechanistic insights into how brain networks dynamically reorganize during social interactions.

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2. A Graph Geometry Toolkit

Central to our proposal are discrete curvatures, one example of which is the Forman-Ricci curvature (FRC). Developed initially to characterize geometric properties of discrete spaces parametrized as cell complexes (Forman, 2003), a specialization of FRC to graphs quantifies the expansion and contraction of information across the network by examining the network's connectivity patterns. Specifically, the FRC of an edge e connecting nodes i and j in a weighted network is defined as

$$F(e) = w_e \left(\frac{z_i}{w_e} + \frac{z_j}{w_e} - \sum_{e_i \sim i, e_i \neq e} \frac{z_i}{\sqrt{w_e w_{e_i}}} - \sum_{e_j \sim j, e_j \neq e} \frac{z_j}{\sqrt{w_e w_{e_j}}} \right), \tag{1}$$

where z_i, z_j represent node weights and w_e denotes edge weights corresponding to neural connectivity strength. Positive curvature values typically identify edges in densely connected regions, whereas negative curvature highlights edges that bridge highly connected network modules.

Ollivier-Ricci curvature (ORC) represents an alternative notion of discrete Ricci curvature (Ollivier, 2009), which provides a comparable characterization of network geometry (Samal et al., 2018); we defer a formal definition to Appendix A. Its definition via Markov chains lends itself to another interpretation in the context of inter-brain connectivity: The curvature of an edge provides a proxy for its tendency to *attract* information flow, in the sense that negative curvature indicates more attraction (Wang et al., 2022). Regions with a high density of edges with low (negative) curvature promote shortest-path traversal, while regions with higher (positive) curvature promote diffusion.

In the next section, we investigate how a toolkit based on discrete Ricci curvature can be fruitfully applied to social neuroscience.

3. The Case of Hyperscanning

Hyperscanning, defined as the simultaneous recording of neural signals from interacting individuals (Montague et al., 2002), has reshaped social neuropsychology (Schilbach and Redcay, 2025) and clinical neuroscience alike (Adel et al., 2025). Despite these advances, the analytical methods applied in hyperscanning remain heavily reliant on purely correlational approaches (Hamilton, 2021), inherently restricting their explanatory power. *We contend that the curvature-based analysis of interbrain coupling networks can move hyperscanning studies closer toward mechanistic explanations.*

3.1. Interbrain Networks and their Synchrony

Interbrain networks represent the joint neural connectivity of two or more individuals as interconnected nodes within weighted graphs, constructed via hyperscanning, with each node typically corresponding to a neural region and the edge weights derived by computing IBS metrics (e.g., PLV), from the neural activity in these regions (Hakim et al., 2023). These studies have been limited in the mechanistic inferences they afford researchers; at best, correlations between brain regions of interacting subjects can be interpreted in terms of the computational-cognitive roles ascribed to these regions, with detailed mechanisms and their dynamic evolution – as social behavior unfolds over time – remaining speculative. We propose extending studies of time-varying interbrain networks with graph curvatures to detect meaningful phase transitions in interpersonal neural dynamics and provide insight into the information routing strategies interbrain networks use to accomplish joint behavioral tasks. We explore these applications in detail in the following sections.

3.2. Capturing Phase Transitions

Suppose the timing of task-related behavioral transitions or events, such as cooperative engagements, misunderstandings, or conflict resolutions, is synchronized with the timing of phase transitions in interbrain networks as identified by graph curvatures. In that case, investigators can more confidently make inferences about the neural mechanisms of behavior (Steyn-Ross and Steyn-Ross, 2010). To capture significant dynamic shifts in network configurations, we examine divergences over time in the differential entropy of graph curvature distributions of IBS, H_{RC}, defined as

$$H_{RC}(G_t) = -\int_{\mathbb{R}} f^t_{RC}(x) \log \left[f^t_{RC}(x) \right] \, dx, \tag{2}$$

where $f^t_{RC}(x)$ describes the probability density of discrete curvature values across the network configuration G_t at time t (Znaidi et al., 2023). In Figure 3.3, we apply this method to detect phase transitions in a toy model of time-varying brain networks with small-world topology. We show that as the rewiring probability used to generate the networks evolves from zero to unity, the differential entropy of the FRC distribution undergoes a divergence between $p = 10^{-3}$ and $p = 10^{-1}$ as the network transitions between a regular lattice and a random network. Panels E–F show a sharp rise in entropy once $p \gtrsim 10^{-2}$ and a widening curvature distribution (95th-percentile jump), due to increased neighborhood overlap and shortcut formation, marking a transition from a segregated, lattice-like topology to a more integrated small-world/random regime; see Table 1 in Appendix B for modality- and condition-specific expectations that map these geometric signatures to EEG, fNIRS, and fMRI hyperscanning in resting and experimental task conditions.

3.3. Capturing Information Routing Strategies

Theoretical work on information routing in brain networks has used Markov chains to model a spectrum of information routing strategies between shortest-path traversal to a target node at one extreme, and random diffusion at the other (Avena-Koenigsberger et al., 2019). Thus, when applied to IBCs, the ORC distribution of the network can be interpreted as identifying the information routing strategy adopted by its subnetworks.

Recent work in deep learning has shown that FRC can identify information bottlenecks that distort information flow during message-passing in graph neural networks (Topping et al., 2022; Fesser and Weber, 2023). These results suggest that FRC could be a valuable tool for assessing information flow in brain networks, supplementing mechanistic models of information flow at the level of neuronal populations (Clark and Beiran, 2025).

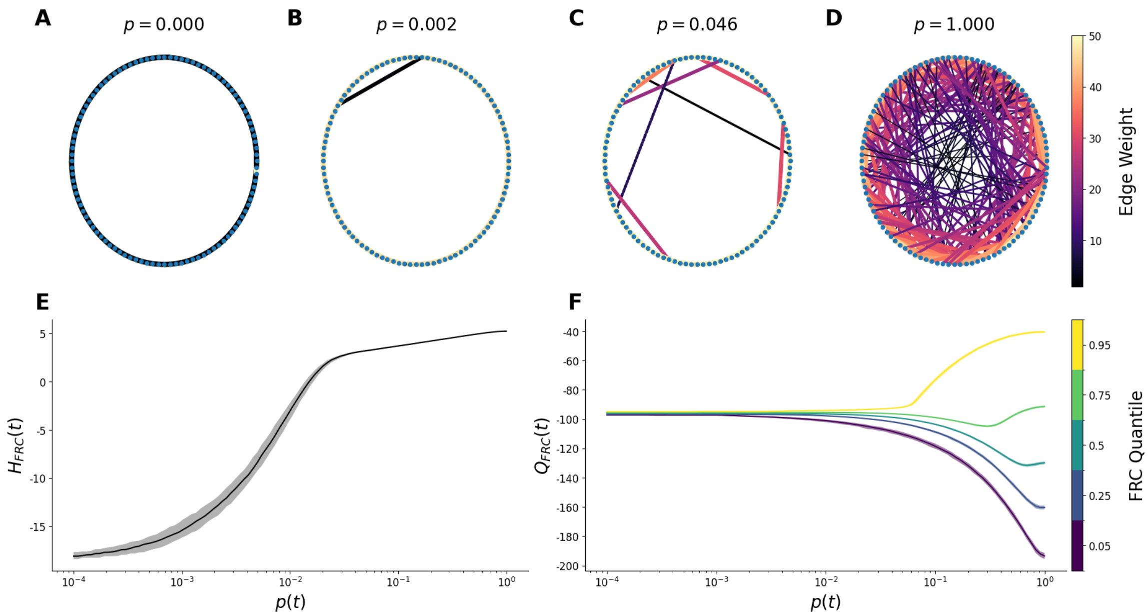

Figure 1: Simulations of time-varying brain networks modeled as weighted small-world networks with varying rewiring probability. **A–D:** Four examples with $N = 100$, mean degree $K = 5$, and different p, generated using Muldoon et al. (2016). **E:** Entropy of the FRC distribution as p evolves from 0 to 1 for $N = 1000$, $K = 50$ (note phase transition around $p = 10^{-2}$). **F:** Corresponding quantiles of the FRC distribution. Solid curves show the median over 200 replications; shaded areas mark 0.05 and 0.95 quantiles.

4. Towards an Interbrain Geometry

Adopting a geometric framework within neuroscience offers methodological and conceptual advancements over traditional IBS-based analyses. *Geometric Hyperscanning* could address the inability of correlation-based metrics alone to capture dynamic network reconfigurations and characterize real-time information routing strategies within and between socially interacting brains.

Discrete curvature distributions could summarize constraints on network dynamics, with divergences in the entropy of the distribution indicating said network reorganization events; while this does not intrinsically resolve the confounding factors that arise in IBS-based approaches, it provides a complementary network-level description of interbrain interactions, enabling further inferences required to construct mechanistic explanations.

This direction accords with Kulkarni and Bassett (2024)'s call for minimal, principled models of brain-network complexity and with Sporns (2010)'s emphasis on meso-scale features (hubs, clusters, bridges), reinforcing discrete curvatures as indicators of structural transitions during social interaction. Curvature-based analyses could allow researchers to explore the information routing implications of IBS and how they reorganize dynamically throughout real-time interactions, as captured in hyperscanning data, paving the way for a deeper mechanistic understanding of the social brain.

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References

Lena Adel, Lisane Moses, Elisabeth Irvine, Kyle T. Greenway, Guillaume Dumas, and Michael Lifshitz. A systematic review of hyperscanning in clinical encounters. *Neuroscience & Biobehavioral Reviews*, 176:106248, 2025. doi: 10.1016/j.neubiorev.2025.106248. URL https://doi.org/10.1016/j.neubiorev.2025.106248.

Andrea Avena-Koenigsberger, Xiaoran Yan, Artemy Kolchinsky, Martijn P. van den Heuvel, Patric Hagmann, and Olaf Sporns. A spectrum of routing strategies for brain networks. *PLOS Computational Biology*, Mar 2019. URL https://journals.plos.org/ploscompbiol/article?id=10.1371%2Fjournal.pcbi.1006833.

Tanima Chatterjee, Réka Albert, Stuti Thapliyal, Nazanin Azarhooshang, and Bhaskar DasGupta. Detecting network anomalies using forman–ricci curvature and a case study for human brain networks. *Scientific reports*, 11(1):8121, 2021.

David G. Clark and Manuel Beiran. Structure of activity in multiregion recurrent neural networks. *Proceedings of the National Academy of Sciences*, 122(10):e2404039122, 2025. doi: 10.1073/pnas.2404039122. URL https://www.pnas.org/doi/abs/10.1073/pnas.2404039122.

Lukas Fesser and Melanie Weber. Mitigating over-smoothing and over-squashing using augmentations of forman-ricci curvature. In *Learning on Graphs Conference*, 2023.

R. Forman. Bochner's method for cell complexes and combinatorial ricci curvature. *Discrete and Computational Geometry*, 29:323–374, 2003. doi: https://doi.org/10.1007/s00454-002-0743-x. URL https://doi.org/10.1007/s00454-002-0743-x.

U. Hakim, S. De Felice, P. Pinti, X. Zhang, J. A. Noah, Y. Ono, P. W. Burgess, A. Hamilton, J. Hirsch, and I. Tachtsidis. Quantification of inter-brain coupling: A review of current methods used in haemodynamic and electrophysiological hyperscanning studies. *NeuroImage*, 280:120354, 2023. doi: 10.1016/j.neuroimage.2023.120354. URL https://doi.org/10.1016/j.neuroimage.2023.120354.

A. F. C. Hamilton. Hyperscanning: Beyond the hype. *Neuron*, 109(3):404–407, 2021. doi: 10.1016/j.neuron.2020.11.008. URL https://doi.org/10.1016/j.neuron.2020.11.008.

Nicolás Hinrichs, Mahault Albarracin, Dimitris Bolis, Yuyue Jiang, Leonardo Christov-Moore, and Leonhard Schilbach. Geometric hyperscanning of affect under active inference. *arXiv preprint arXiv:2506.08599*, 2025. URL https://arxiv.org/abs/2506.08599.

Suman Kulkarni and Dani S Bassett. Towards principles of brain network organization and function. *arXiv preprint arXiv:2408.02640*, 2024. URL https://arxiv.org/abs/2408.02640.

P. Read Montague, Gregory S. Berns, Jonathan D. Cohen, Samuel M. McClure, Giuseppe Pagnoni, Mukesh Dhamala, Michael C. Wiest, Igor Karpov, Richard D. King, Nathan Apple, and Ronald E. Fisher. Hyperscanning: Simultaneous fmri during linked social interactions. *NeuroImage*, 16(4):1159–1164, 2002. doi: 10.1006/nimg.2002.1150. URL https://doi.org/10.1006/nimg.2002.1150.

Sarah Feldt Muldoon, Eric W Bridgeford, and Danielle S Bassett. Small-World propensity and weighted brain networks. *Sci. Rep.*, 6(1):22057, Feb 2016.

Y. Ollivier. Ricci curvature of markov chains on metric spaces. *Journal of Functional Analysis*, 256(3):810–864, 2009.

Areejit Samal, RP Sreejith, Jiao Gu, Shiping Liu, Emil Saucan, and Jürgen Jost. Comparative analysis of two discretizations of ricci curvature for complex networks. *Scientific reports*, 8(1):8650, 2018.

Leonhard Schilbach and Elizabeth Redcay. Synchrony across brains. *Annual Review of Psychology*, 76:883–911, 2025. doi: 10.1146/annurev-psych-080123-101149. URL https://doi.org/10.1146/annurev-psych-080123-101149.

Olaf Sporns. *Networks of the Brain*. MIT Press, Cambridge, MA, 2010.

D. Alistair Steyn-Ross and Moira Steyn-Ross, editors. *Modeling Phase Transitions in the Brain*. Springer, New York, NY, 2010. ISBN 978-1-4419-0795-0 978-1-4419-0796-7. doi: 10.1007/978-1-4419-0796-7.

Jake Topping, Francesco Di Giovanni, Benjamin Paul Chamberlain, Xiaowen Dong, and Michael M. Bronstein. Understanding over-squashing and bottlenecks on graphs via curvature. In *International Conference on Learning Representations*, 2022.

Yaoli Wang, Zhou Huang, Ganmin Yin, Haifeng Li, Liu Yang, Yuelong Su, Yu Liu, and Xv Shan. Applying ollivier-ricci curvature to indicate the mismatch of travel demand and supply in urban transit network. *International Journal of Applied Earth Observation and Geoinformation*, 106:102666, Feb 2022. ISSN 1569-8432. doi: 10.1016/j.jag.2021.102666. URL https://www.sciencedirect.com/science/article/pii/S0303243421003731.

Melanie Weber. Geometric machine learning. *AI Magazine*, 46(1):e12210, 2025. doi: https://doi.org/10.1002/aaai.12210. URL https://onlinelibrary.wiley.com/doi/abs/10.1002/aaai.12210.

Melanie Weber, Johannes Stelzer, Emil Saucan, Alexander Naitsat, Gabriele Lohmann, and Jürgen Jost. Curvature-based methods for brain network analysis. *arXiv*, 2019. URL https://arxiv.org/abs/1707.00180.

Mohamed Ridha Znaidi, Jayson Sia, Scott Ronquist, Indika Rajapakse, Edmond Jonckheere, and Paul Bogdan. A unified approach of detecting phase transition in time-varying complex networks. *Scientific Reports*, 13(1):17948, October 2023. ISSN 2045-2322. doi: 10.1038/s41598-023-44791-3. URL https://www.nature.com/articles/s41598-023-44791-3.

Appendix A. Ollivier's Ricci curvature

We provide a formal definition of Ollivier's Ricci curvature, which was discussed in the main text.

Consider the 1-hop neighborhoods of two adjacent nodes u and v in a network and equip each with uniform measures defined as follows: Let $m_u(i) := \frac{z_i}{\sum_{j \in \mathcal{N}_u} z_j}$, where i is a neighbor of u, z_i its weight and $\mathcal{N}_u$ denotes u's 1-hop neighborhood. An analogous measure can be defined on the neighborhood of v. The cost of transporting mass between these two node neighborhoods along the edge $e = (u, v)$ is quantified by the Wasserstein-1 distance between the measures, namely

$$W_1(m_u, m_v) = \inf_{m \in \Gamma(m_u, m_v)} \int_{(z,z') \in V \times V} d(z, z')\, m(z, z')\; dz\; dz' \,, \tag{3}$$

where $\Gamma(m_u, m_v)$ is the set of all measures over $V \times V$ whose marginals are m_u and m_v. The *Ollivier-Ricci curvature* (Ollivier, 2009) is then defined as

$$\kappa(u, v) := 1 - \frac{W_1(m_u, m_v)}{d_G(u, v)} \,, \tag{4}$$

with $d_G(u, v)$ denoting the shortest-path distance between u and v in G.

Appendix B. Hyperscanning modalities across conditions

We provide a comparative overview pairing illustrative values to empirical hyperscanning modalities across usual conditions, as drawn from our simulations.

Mod./Cond.	**Edge-Weight Range**	**Empiric implication**
EEG – Task	PLV $\approx$ 0.2–0.6	fast, captures rapid behaviour
EEG – Resting	PLV $\approx$ 0.1–0.4	fast, spontaneous activity
fNIRS – Task	Corr. $\approx$ 0.1–0.3	0.1–1 s, suited to slow tasks
fNIRS – Resting	Corr. $<$ 0.2	slow, long-term fluctuations
fMRI – Task	Cohe. $\approx$ 0.2–0.5	1–2 s, block tasks only, too slow for fast actions
fMRI – Resting	Cohe. $<$ 0.2	very slow, long-term networks

Table 1: Modalities across conditions, their canonic edge-weight ranges, and characteristic spatiotemporal scale and empiric implication.

The modality-dependent spatiotemporal sampling rates and signal strengths frame the core challenge addressed by our pipeline: detecting network reconfigurations only when the neural signal is sampled at a distinct ratio which resolves the target behaviour.