# FORGET BUT RECALL: INCREMENTAL LATENT RECTI FICATION IN CONTINUAL LEARNING

Anonymous authors

Paper under double-blind review

#### Abstract

Intrinsic capability to continuously learn a changing data stream is a desideratum of deep neural networks (DNNs). However, current DNNs suffer from catastrophic forgetting, which hinders remembering past knowledge. To mitigate this issue, existing Continual Learning (CL) approaches either retain exemplars for replay, regularize learning, or allocate dedicated capacity for new tasks. This paper investigates an unexplored CL direction for incremental learning called Incremental Latent Rectification or ILR. In a nutshell, ILR learns to propagate with correction (or rectify) the representation from the current trained DNN backward to the representation space of the old task, where performing predictive decisions is easier. This rectification process only employs a chain of small representation mapping networks, called rectifier units. Empirical experiments on several continual learning benchmarks, including CIFAR10, CIFAR100, and Tiny ImageNet, demonstrate the effectiveness and potential of this novel CL direction compared to existing representative CL methods.

024 025 026

027

004

010 011

012

013

014

015

016

017

018

019

021

#### 1 INTRODUCTION

Humans exhibit the innate capability to incrementally learn novel concepts while consolidating
acquired knowledge into long-term memories (Rasch & Born, 2007). More general Artificial
Intelligence systems in real-world applications would require similar imitation to capture the dynamic
of the changing data stream. These systems need to acquire knowledge incrementally without
retraining, which is computationally expensive and exhibits a large memory footprint (Rebuffi et al.,
2016). Nonetheless, existing learning approaches are yet to match human learning in this so-called
Continual Learning (CL) problem due to catastrophic forgetting (McCloskey & Cohen, 1989). These
systems encounter difficulty balancing the capability of incorporating new task knowledge while
maintaining performance on learned tasks, or the plasticity-stability dilemma.

Representative CL approaches in the literature usually involve the use of memory buffer for rehearsal 037 (Ratcliff, 1990; Chaudhry et al., 2019a; Buzzega et al., 2020; Caccia et al., 2022; Bhat et al., 038 2023; Arani et al., 2022), auxiliary loss term for learning regularization (Kirkpatrick et al., 2017; Ebrahimi et al., 2020; Zenke et al., 2017; Schwarz et al., 2018), or structural changes such as 040 pruning or model growing (Rusu et al., 2016; Mallya & Lazebnik, 2018; Fernando et al., 2017; Yan 041 et al., 2021). These methods share the common objective of discouraging the deviation of learned 042 knowledge representation. Rehearsal-based methods allow the model to revisit past exemplars to 043 reinforce previously learned representations. Alternatively, regularization-based methods prevent 044 changes in parameter spaces by formulating additional loss terms. However, both approaches present shortcomings, including keeping a rehearsal buffer of all past tasks during the model lifetime or infusing ad-hoc inductive bias into the regularization process. Meanwhile, structure-based methods 046 utilize the over-parameterization property of the model by pruning, masking, or adding parameters to 047 reduce new task interferences. 048

This paper studies a novel approach for CL named Incremental Latent Rectification (ILR), where we allow the model to "forget" knowledge of old tasks but then "recall" or rectify such "catastrophic forgetting" during inference using a sequence of lightweight knowledge mapping networks. These lightweight knowledge mapping networks, called rectifiers, help significantly reduce information loss on learned tasks by incrementally correcting the changes in the representation space. Specifically, for each new task, we add a small, simple, and computationally inexpensive auxiliary unit that will

rectify the representation from the current task to the previous task. Our method differs from many network expansion methods, where additional parameters are allocated to minimize changes to the old parameters. Instead, we iteratively recover past task representations by backwardly propagating current representations through a series of mapping networks. Through this mechanism, ILR allows
the optimal adaptation of a new task (plasticity) while separately mitigating catastrophic forgetting. In addition, different from various CL approaches that heavily modify the training process, ILR imposes minimal changes to new task learning as modifications are mainly performed after the training process has been completed. Hence, ILR can be easily integrated into the existing CL pipelines.

**Contributions.** We propose a new direction for CL by sequentially correcting the representation of the current task into the past task's representation using a chain of lightweight rectifier units:

- We propose ILR, a novel approach to continual learning that separates catastrophic forgetting mitigation with new task learning via a sequence of lightweight rectifier units.
- To train the rectifier unit, we rely on either data samples from task t 1 or the current task t; when such data is unavailable (e.g., due to memory constraint or privacy concerns), a generative model that synthesizes task t - 1's data can also be utilized. At inference time, for the taskincremental setting, we construct a chain of rectifiers based on the provided task identity and forward the latent representation and inputs to correct the representation. For the class incremental setting, ILR forms the final prediction from an ensemble of predictions based on the reconstructed representations.
- We empirically evaluate our approach on three widely-used continual learning benchmarks (CIFAR10, CIFAR100, and Tiny ImageNet) to demonstrate that our approach achieves comparable performance with the existing representative CL directions.

This paper unfolds as follows. Section 2 discusses the literature on the continual learning problems, and Section 3 describes our Incremental Latent Rectification method. Finally, Section 4 provides the empirical evidence for the effectiveness of our proposed solution.

081

062

063

064 065

066

067

068

069

071

073

074

075

076

077

079

#### 2 RELATED WORK

082 083

Catastrophic forgetting is a critical concern in artificial intelligence and is arguably one of the most 084 prominent questions to address for DNNs. This phenomenon presents significant challenges when 085 deploying models in different applications. Continual learning addresses this issue by enabling agents to learn throughout their lifespan. This aspect has gained significant attention recently (Sun 087 et al., 2022; Hu et al., 2021; Kirichenko et al., 2021; Balaji et al., 2020). Considering a model 880 well-trained on past tasks, we risk overwriting its past knowledge by adapting it for new tasks. The 089 problem of knowledge loss can be addressed using different methods, as explored in the literature (Yin et al., 2020; Farajtabar et al., 2020; Kirkpatrick et al., 2017; Li & Hoiem, 2017; Chaudhry 091 et al., 2019a; Bhat et al., 2023; Rusu et al., 2016; Yan et al., 2021). These methods aim to mitigate 092 knowledge loss and improve task performance through three main approaches: (1) Rehearsal-based methods, which involve reminding the model of past knowledge by using selective exemplars; (2) Regularization-based methods, which penalize changes in past task knowledge through regularization 094 techniques; (3) Parameter-isolation and Dynamic Architecture methods, which allocate sub-networks 095 or expand new sub-networks, respectively, for each task, minimizing task interference and enabling 096 the model to specialize for different tasks.

Rehearsal-based. Experience replay methods build and store a memory of the knowledge learned so
far (Rebuffi et al., 2016; Lopez-Paz & Ranzato, 2017; Shin et al., 2017; Riemer et al., 2018; Rios
& Itti, 2018; Zhang et al., 2019). As an example, Averaged Gradient Episodic Memory (A-GEM)
(Chaudhry et al., 2019a) builds an episodic memory of parameter gradients, while ER-Reservoir
(Chaudhry et al., 2019) uses a reservoir sampling method to maintain the episodic memory. These
methods have shown strong performance in recent studies. However, they require a significant amount
of memory to store the examples.

Regularization-based. A popular early work using regularization is the elastic weight consolidation (EWC) method (Kirkpatrick et al., 2017). Other methods (Zenke et al., 2017; Aljundi et al., 2018; Van et al., 2022; Nguyen et al., 2018; Ahn et al., 2019) propose different criteria to measure the "importance" of parameters. A later study showed that many regularization-based methods are

variations of Hessian optimization (Yin et al., 2020). These methods typically assume there are
 multiple optima in the updated loss landscape in the new data distribution. One can find a good
 optimum for both the new and old data distributions by constraining the deviation from the original
 model weights.

112 Parameter Isolation. Parameter isolation methods allocate different subsets of the parameters to each 113 task (Rusu et al., 2016; Jerfel et al., 2019; Rao et al., 2019; Li et al., 2019). From the stability-plasticity 114 perspective, these methods implement gating mechanisms that improve stability and control plasticity 115 by activating different gates for each task. Masse et al. (2018) proposes a bio-inspired approach for a 116 context-dependent gating that activates a non-overlapping subset of parameters for any specific task. 117 Supermask in Superposition (Wortsman et al., 2020) is another parameter isolation method that starts 118 with a randomly initialized, fixed base network and, for each task, finds a sub-network (supermask) such that the model achieves good performance. 119

120 Dynamic Architecture. Different from Parameter Isolation, which allocates subnets for tasks in a 121 fixed main network, this approach dynamically expands the network structure. Yoon et al. (2018) 122 proposes a method that leverages the network structure trained on previous tasks to effectively learn 123 new tasks, while dynamically expanding its capacity by adding or duplicating neurons as needed. Other methods (Xu & Zhu, 2018; Qin et al., 2021) reformulate CL problems into reinforcement 124 125 learning (RL) problems and leverage RL methods to determine when to expand the architecture when learning new tasks. Yan et al. (2021) introduces a two-stage learning method that first expands the 126 previous frozen task feature representations by a new feature extractor, then re-trains the classifier 127 with current and buffered data. 128

129 130

## **3** PROPOSED FRAMEWORK

131 132

133 We consider the task-incremental and class-incremental learning scenarios, where we sequentially observe a set of tasks  $t \in \{1, \ldots, N\}$ . The neural network comprises a single task-agnostic feature 134 extractor f and a classifier w with task-specific heads  $w^{(t)}|_{t=1}^{N}$ . The architecture of f is fixed; 135 however, its parameters are gradually updated as new tasks arrive. At task t, the system receives the 136 training dataset  $\mathcal{D}_t^{\text{train}}$  sampled from the data distribution  $\mathcal{D}_t$  and learns the updated parameters of 137 the feature extractor f and w. For easier discussion, the feature extractor and classifier obtained after 138 learning at task t are denoted as  $f_t$  and  $w_t$ , respectively. Thus, after learning on task t, we obtain the 139 evolved feature extractor  $f_t$  and classifier  $w_t$  We call the latent space created by the feature extractor 140 trained with  $\mathcal{D}_t^{\text{train}}$  as the t-domain. Catastrophic forgetting occurs as the feature extractor  $f_{t'}$  is 141 updated into  $f_t$ , t' < t, which causes the t'-domain to be overwritten by the t-domain. This domain 142 shift degrades the model's performance over time. 143

To overcome catastrophic forgetting, we propose a new CL paradigm: learning a latent rectification mechanism. This mechanism relies on a lightweight rectifier unit  $r_t$  that learns to align the representations from the *t*-domain to the (t-1)-domain. Intuitively, this module "corrects" the representation change of a sample from the old task t-1 due to the evolution of the feature extractor f when learning the newer task t. These rectifier units will establish a chain of corrections for the representation of any task's input, allowing the model to predict the rectified representation better. Figure 1 provides a visualization of the inference process on a task-t sample, after learning N tasks.

Learning the latent rectification mechanism is central to our proposed framework. In general, each rectifier unit should be small compared to the size of the final model or the feature extractor f, and its learning process should be resource-efficient. The following sections present and describe our solution for learning this mechanism.

154

157

#### 156 3.1 LEARNING THE RECTIFIER UNIT

As the training dataset  $\mathcal{D}_t^{\text{train}}$  of task t arrives, we first update the feature extractor  $f_t$  and the classifier head  $w_t$ . The primary goal herein is to find  $(f_t, w_t)$  that has high classification performance for task t, and the secondary goal is to choose  $f_t$  that can reduce the catastrophic forgetting on previous tasks. To combat catastrophic forgetting, we will first discuss the objective function for learning the lightweight rectifier unit  $r_t$  and the potential alignment training data (or alignment set)  $S_t$ .



Figure 1: At task t, the feature extractor  $f_t$  and classifier head  $w_t$  are optimized on the dataset  $D_t^{\text{train}}$ . 172 During inference for a test sample from task t, we forward the input data  $x \in D_t^{\text{test}}$  through the 173 feature extractor and classifier head to obtain the logits. After learning all N tasks, the DNN loses 174 performance on task t due to catastrophic forgetting. Therefore, the latent representation  $f_N(x)$ 175 is propagated through a series of rectifiers  $r_N, \ldots, r_{t+1}$  to perform incremental latent rectification 176 and obtained approximated representations  $f_{N-1}, \ldots, f_t$ . The logits can be obtained by passing the 177 recovered representation to the respective classifier head. 178

#### 3.1.1 ALIGNMENT LOSS 180

179

186

187 188

181 The goal of  $r_t$  is to reduce the discrepancy between task t's representation  $f_t(x_i)$  and the previous 182 data representation  $f_{t-1}(x_i)$ , for  $x_i \sim \mathcal{D}_{t-1}$ ; i.e.,  $r_t(f_t(x_i), x_i) \approx f_{t-1}(x_i)$ . One simple choice is 183 the  $l_2$  error between  $f_t(x_i)$  and  $r_t(f_t(x_i), x_i)$ . Let s be a function with parameters  $\theta_s$  that encodes inputs  $x_i$  into its respective past representation in domain t-1. We define the alignment loss as: 185

$$\mathcal{L}_{\text{align}}(\theta_s; s, \mathcal{S}_t, f_{t-1}) = \mathbb{E}_{x_i \sim \mathcal{S}_t} \left[ \| s(x_i) - f_{t-1}(x_i) \|_2^2 \right].$$

$$\tag{1}$$

In practice, we could either store the value of  $f_{t-1}(x_i)$  together with  $x_i$  in memory or  $f_{t-1}$  directly.

#### 3.1.2 ALIGNMENT SET 189

190 The alignment set  $S_t$  is used as the training data for the rectifier unit  $r_t$ , enabling the rectifier unit to 191 efficiently learn the mapping from the t-domain back to the t-1-domain. The design of ILR enables 192 several options for selecting the alignment set, including  $\mathcal{D}_{t-1}^{\text{train}}$ ,  $\mathcal{D}_t^{\text{train}}$ , or a generative method. 193 Table 1 demonstrates the difference of alignment sets.

194 **Past task** t - 1 data. (ILR-P) The simplest 195 choice for the alignment set  $S_t$  is the  $\mathcal{D}_{t-1}^{\text{train}}$  (i.e., 196 the training data from the previous task t - 1), 197 which is sampled directly from the task t - 1's distribution. With this option, each element in 199  $S_t$  is a pair  $(x_i, \hat{z}_i)$ , where  $x_i \in \mathcal{D}_{t-1}^{\text{train}}$  is chosen 200 randomly and  $\hat{z}_i = f_{t-1}(x_i)$  is the associated 201 latent representation of  $x_i$  under the feature extractor  $f_{t-1}$ . It is worth noting that this option 202 does not keep data samples from all past tasks 203  $t \in \{1, \ldots, N\}$  like the rehearsal-based meth-204 ods (Verwimp et al., 2021). 205

Table 1: At task t, different alignment sets require temporarily storing different components of the training process, which impose different trade-offs in terms of performance, number of parameters, and privacy.

Variation	t-1 samples	$f_{t-1}$	$G_{t-1}$
ILR-P ( $\mathcal{S}_t \subset \mathcal{D}_{t-1}^{\mathrm{train}}$ )	$\checkmark$	-	-
ILR-C ( $S_t = D_t^{\text{train}}$ )	-	$\checkmark$	-
ILR-G ( $S_t \approx D_{t-1}$ )	-	$\checkmark$	$\checkmark$

206 **Current task** t data. (ILR-C) Another potential option for  $S_t$  is task-t's data. If we expect the tasks' data not to be completely unrelated, using data from  $\mathcal{D}_t^{\text{train}}$  to train  $r_t$  is reasonable. As we show 207 in Section 4, we could achieve comparable performance to strong rehearsal-based methods while 208 remaining *data-free* when setting  $S_t = \mathcal{D}_t^{\text{train}}$ . Additionally, for this option, since we do not have 209 access to t-1-domain data, we need to keep a copy of  $f_{t-1}$  to approximate  $\hat{z}_i = f_{t-1}(x_i)$  with 210  $x_i \in \mathcal{D}_t^{\text{train}}.$ 211

212 **Generated task** t - 1 **data. (ILR-G)** Generative methods provide a potential option for creating training data for the rectifier unit  $r_t$ . Instead of keeping the alignment set  $S_t \subseteq \mathcal{D}_{t-1}^{\text{train}}$ , we could train 213 a generative neural network  $G_{t-1}$  that learns the task t-1 distribution. Unlike generative continual 214 learning methods,  $G_{t-1}$  only needs to remember the task t-1 distribution instead of all past tasks. 215 Thus, LRB can easily integrate with existing generative methods.

In addition, we could fill  $S_t$  with randomly initialized samples. Nonetheless, our experiments indicate that this approach is ineffective. Therefore, we will focus our discussion on the first three options and leave the exploration for other choices of  $S_t$  for future works.

**Distiction from buffer-based methods.** Rehearsal-based methods retain the data from all past tasks  $t \in \{1, ..., N\}$  during the lifetime of the DNN. In contrast, depending on the choice of alignment set  $\mathcal{S}_t$ , ILR can be considered strictly data-free if  $\mathcal{S}_t = \mathcal{D}_t^{\text{train}}$  (ILR-C) or if it uses additional generative model (ILR-G). When  $\mathcal{S}_t \subseteq \mathcal{D}_{t-1}^{\text{train}}$ , ILR-P can still be argued as a data-free method since task t-1data is only retained until the end of task t.

225 226

227

228

229 230

231

3.2 INCREMENTAL LATENT ALIGNMENT

The latent alignment mechanism relies on a chain of task-specific rectifier units  $(r_t)_{t=2}^N$  that aims to correct the distortion of the representation space as the extractor f learns a new task.

#### 3.2.1 LATENT ALIGNMENT

For an input x at task t - 1, its feature representation under the feature extractor  $f_{t-1}$  is  $f_{t-1}(x)$ . One can heuristically define the (t - 1)-domain as the representation of the input under the feature extractor  $f_{t-1}$ . Unfortunately, the (t - 1)-domain is brittle under extractor update: as the subsequent task t arrives, the feature extractor is updated to  $f_t$ , and the corresponding feature representation of the same input x will be shifted to  $f_t(x)$ . Likely, the t-domain and the (t - 1)-domain do not coincide, and  $f_t(x) \neq f_{t-1}(x)$ .

The feature rectifier unit  $r_t$  aims to offset this representation shift. To do this,  $r_t$  takes x, and its t-domain representation  $f_t(x)$  as input, and it outputs the rectified representation that satisfies

240 241

 $r_t(f_t(x), x) \approx f_{t-1}(x),\tag{2}$ 

With this formulation, we can effectively minimize the difference between the rectified representation  $r_t(f_t(x), x)$  and the original representation  $f_{t-1}(x)$ . In practice, we only want to train the rectifier unit  $r_t$  and retain the learned feature extractor  $f_t$ ; therefore, let  $s(x) = r_t(f_t(x), x)$ , we can minimize the difference by using  $L_{\text{align}}(\theta_{r_t}; s, \mathcal{S}_t, f_{t-1})$  as in Equation (1).

246 247

#### 3.2.2 RECTIFIER ARCHITECTURE

248 The proposed rectifier comprises two 249 trainable components: a *weak feature* 250 extractor  $h_t$ , and a gate function  $g_t$ . 251 The size of the rectifier units increases linearly with respect to the number 253 of tasks, similar to the classification 254 heads. However, since the rectifier unit is lightweight, this is trivial com-255 pared to the size of the full model. 256 Figure 2 visualizes the feature recti-257 fier unit. Alternative designs of the 258 rectifier unit that have been explored 259 are provided in the Appendix. 260



Figure 2: The rectifier unit includes a weak feature extractor  $h_t$ , and a sigmoid autoencoder  $g_t$ . The sigmoid autoencoder acts as an element-wise gate function that filters information from (t-1)-domain knowledge in  $f_t$ , while  $h_t$  compensates for the loss of information in  $f_t$  due to catastrophic forgetting.

Weak feature extractor  $h_t$ . The weak feature extractor  $h_t$  processes the input data x to generate a simplified representation  $h_t(x)$ .  $h_t$  is distilled from  $f_{t-1}$  to compress the knowledge of  $f_{t-1}$  into a more compact, low-capacity parameter-efficient network. For our experiment, we choose the *simplest and most naive* design of a weak feature extractor composed of only two 3x3 convolution layers and two max pooling layers. Instead of processing the full-size image, we use max-pooling to down-sample the input to 16x16 images before feeding into  $h_t$ . The weak feature extractor is a small network compared to the main model ( $h_t$ 's architecture is provided in Table 5 in the Appendix).

**Gate function**  $g_t$ . Due to catastrophic forgetting, the original representation of  $f_{t-1}(x)$  will deteriorate as f is updated. The gate function  $g_t$  offsets the information loss by computing an element-wise gating weight  $0 \le g_t(f_t(x)) \le 1$  of the representation  $f_t(x)$  to capture only task t - 1 relevant

information. We use the sigmoid autoencoder similar to TAMiL (Bhat et al., 2023) comprised of a linear encoder with ReLU activation and a linear decoder with sigmoid activation as the gate function.

The weak feature extractor  $h_t$  will compensate for the remaining missing information with weight 1 -  $g_t$ . Computing the element-wise weighted average of both representations, we obtain the rectified representation  $r_t(x_i, f_t(x_i))$ .

- 276
- 277 278

279

281

282

283

284 285

286 287

288

289

290 291 292

299 300

321

322

 $r_t(x, f_t(x)) = g_t(f_t(x)) \odot f_t(x) + (1 - g_t(f_t(x))) \odot h_t(x)$ (3)

**Distiction from network-expansion approach.** It could be argued that one can, instead, separately train a weak feature extractor  $h_t$  for each task, making it a network-expansion CL approach. However, because  $h_t$  is a small and low-capacity network, this approach is ineffective; specifically, our experiments demonstrate that the task-incremental average accuracy across all tasks of this approach on CIFAR100 falls below 53%. Furthermore, for network expansion approaches, the dedicated parameters are allocated for new task learning, which fundamentally differs from ILR's objective to correct representation changes. The new task's knowledge is acquired by  $f_t$  and  $w_t$ .

3.3 TRAINING PROCEDURE

**Network training.** Similar to conventional DNN training, the performance of the feature extractor  $f_t$  and the classifier head  $w_t$  is measured by the standard multi-class cross-entropy loss:

$$\mathcal{L}_{\rm CE}(\theta_{f_t}, \theta_{w_t}; f_t, w_t, \mathcal{D}_t^{\rm train}) = \mathbb{E}_{(x_i, y_i) \sim \mathcal{D}_t^{\rm train}} \left[ -\sum_{c=1}^{M_t} y_i \log(\hat{y}_i) \right], \tag{4}$$

where  $M_t$  is the number of classes of task t,  $\hat{y}_i$  is the probability-valued network output for the input  $x_i$  that depends on the feature extractor  $f_t$  and the classifier  $w_t$  as  $\hat{y}_i = w_t \circ f_t(x_i)$ .

Furthermore, we use the past presentation from the alignment set to enforce task t-1 representation consistency, reduce forgetting, and enable more effective rectification by training and regularizing  $f_t$ on  $\mathcal{D}_t^{\text{train}}$  and  $\mathcal{S}_t$ , respectively. Let  $s(x) = f_t(x)$ , then we can similarly use  $\mathcal{L}_{\text{align}}(\theta_{f_t}; s, \mathcal{S}_t, f_{t-1})$ in Equation (1) with hyperparameter  $\alpha$ :

$$\mathcal{L}_{\text{train}}(\theta_{f_t}, \theta_{w_t}) = \mathcal{L}_{\text{CE}}(\theta_{f_t}, \theta_{w_t}; f_t, w_t, \mathcal{D}_t^{\text{train}}) + \alpha \mathcal{L}_{\text{align}}(\theta_{f_t}; s, \mathcal{S}_t, f_{t-1}).$$
(5)

This is different from the rehearsal method since f only visits  $\mathcal{D}_{t-1}$  samples at task t-1 and task t. After task t, f never see  $\mathcal{D}_{t-1}$  again, while for rehearsal method, f observe samples from  $\mathcal{D}_{t-1}$  throughout its lifetime, risk overfitting on stored exemplars.

**Rectifier training.** Training the rectifier follows two main steps: train the weak feature extractor  $h_t$  at task t-1 and then the gate function  $g_t$  at task t. The weak feature extractor  $h_t$  is distilled from  $f_{t-1}$  as task t-1 training is completed using  $\mathcal{L}_{align}(\theta_{h_t}; s, \mathcal{D}_{t-1}^{train}, f_{t-1})$  as in Equation (1) with  $s(x) = h_t(x)$ . Similarly, after task t training is completed, we also train  $g_t$  using  $\mathcal{L}_{align}(\theta_{g_t}; s, \mathcal{S}_t, f_{t-1})$  as in Equation (1) with  $s(x) = g_t(f_t(x)) \odot f_t(x)$ . Details of ILR's training algorithm are provided in Algorithm 1.

310 **Algorithm 1:** Full training framework at task  $t \in \{1, 2, ..., N\}$ 311 **Input** :Training dataset  $\mathcal{D}_t^{\text{train}}$ , hyperparameter  $\alpha$ , alignment set  $\mathcal{S}_t$ 312 1 for  $\{x_i, y_i\} \in \mathcal{D}_t^{\text{train}}$  do 313 Optimize  $\theta_{f_t}$  and  $\theta_{w_t}$  on  $\mathcal{D}_t^{\text{train}}$  with  $\mathcal{L}_{\text{train}}(\theta_{f_t}, \theta_{w_t})$  [Equation (5)] 2 314 3 for  $\{x_i, y_i\} \in \mathcal{D}_t^{\text{train}}$  do 315 Distill  $\theta_{h_{t+1}}$  with  $\mathcal{L}_{\text{align}}(\theta_{h_{t+1}}; s, \mathcal{D}_t^{\text{train}}, f_t)$  [Equation (1)] and  $s(x) = h_{t+1}(x)$ 4 316  $\mathfrak{s}$  if t > 1 then 317 for  $\{x_i, f_{t-1}(x_i)\} \in \mathcal{S}_t$  do 6 318 Optimize  $\theta g_t$  using  $\mathcal{L}_{align}(\theta_{g_t}; s, \mathcal{S}_t, f_{t-1})$  [Equation (1)] with  $s(x) = g_t(f_t(x)) \odot f_t(x)$ 319 320

3.4 INFERENCE PROCEDURE

We now describe how to stack multiple rectifier units  $r_t$  into a chain for inference. As a new task arrives, our model dynamically extends an additional rectifier unit, forming a sequence of rectifiers.

Method	$ \mathcal{B} $	$ \mathcal{S}_t $	S-C	S-CIFAR10 S-CIFAR100		S-TinyImg		
TIL			NP	AA	NP	AA	NP	ĂA
Joint			11.17м	98.46±0.07	11.22м	86.37±0.17	11.27м	81.86
Finetuning	-	-	11.17м	$64.16{\scriptstyle \pm 2.40}$	11.22м	$24.01 \pm 2.14$	11.27м	13.79
o-EWC			11.17м	69.60±5.22	11.22м	36.61±3.82	11.27м	15.67
LwF.mc	-	-	11.17м	$60.96{\scriptstyle \pm 1.48}$	11.22м	41.00±1.01	11.27м	23.24
AGEM			11.17м	$90.37{\scriptstyle\pm1.05}$	11.22м	63.35±1.47	11.27м	37.14
ER		-	11.17м	$94.24{\scriptstyle\pm0.24}$	11.22м	67.41±0.70	11.27м	46.07
DER++	500		11.17м	$92.49{\scriptstyle \pm 0.55}$	11.22м	$68.52 \pm 0.91$	11.27м	50.84
ER-ACE	500		11.17м	$94.52{\scriptstyle\pm0.13}$	11.22м	$67.26 \pm 0.50$	11.27м	47.72
TAMiL			22.68м	$94.89{\scriptstyle \pm 0.16}$	22.77м	$76.39{\scriptstyle \pm 0.29}$	23.20м	64.24
CLS-ER			33.52м	$95.35{\scriptstyle \pm 0.34}$	33.66м	$77.03{\scriptstyle \pm 0.81}$	33.81м	54.69
ILR-P	-	500	12.00м	86.27±2.89	12.05м	76.23±0.53	13.13м	61.89
AGEM			11.17м	91.68±1.48	11.22м	67.43±1.37	11.27м	46.94
ER			11.17м	$95.25{\scriptstyle\pm0.07}$	11.22м	69.69±1.49	11.27м	54.54
DER++	1000		11.17м	$93.76{\scriptstyle\pm0.23}$	11.22м	$72.27 \pm 1.13$	11.27м	58.67
ER-ACE	1000	-	11.17м	$94.69{\scriptstyle \pm 0.25}$	11.22м	$72.46 \pm 0.58$	11.27м	57.37 <del>.</del>
TAMiL			22.68м	$95.22{\scriptstyle\pm0.42}$	22.77м	$78.72 \pm 0.31$	23.20м	70.89
CLS-ER			33.52м	$96.05{\scriptstyle\pm0.11}$	33.66м	$79.36{\scriptstyle \pm 0.20}$	33.81м	65.00
ILR-P	-	1000	12.00м	90.66±0.97	12.05м	78.14±0.18	13.13м	66.83 <u>-</u>
ILR-P	-	5000	12.00м	$92.77{\scriptstyle\pm0.25}$	12.05м	81.50±0.13	13.13м	72.14
Alternative	e alignme	nt sets						
1 1000000000000000000000000000000000000	0							
ILR-C	-	$S_t = D_t$	-	$89.08{\scriptstyle\pm0.96}$	-	79.25±0.30	-	66.65

Table 2: Task-Incremental Average Accuracy across all tasks after CL training. **Joint**: the upper bound accuracy when jointly training on all tasks (i.e., multi-task learning). **Finetuning**: the lower bound accuracy when learning without CL techniques.  $|\mathcal{B}|$  is the buffer of all past tasks data, while  $|\mathcal{S}_t|$  is the alignment training data set, which only contains data from task t - 1. NP is the number of parameters (lower is better), and AA is the average accuracy of all tasks (higher is better).

**Task-Incremental.** We consider a task-incremental learning setting where a test sample  $x_i$  is coupled with a task identifier  $t_i \in \{1, ..., N\}$ . To classify  $x_i$ , we can recover  $\hat{f}_{t_i}(x)$  by forwarding the current latent variable  $f_N(x)$  through a chain of  $N - t_i$  rectifiers. We then pass this recovered latent variable through classifier head  $w_{t_i}$  to make a prediction. The output  $\hat{y}_i$  is computed as

 $\hat{y}_i = w_{t_i}(\hat{f}_{t_i}(x_i))$  where  $\hat{f}_{t_i}(x_i) = r_{t_i+1}(\hat{f}_{t+i}(x), x)$  with  $t_i < N, \hat{f}_N = f_N$ 

**Class-Incremental.** ILR relies on the task identity to reconstruct the appropriate sequence of rectifier units for propagating the latent representation to the original space. However, no identity is provided for the CL method in the class-incremental learning setting. We provided a simple method for inference without task identity, which demonstrates the method's extension to class-incremental learning; however, more robust task-identity inference methods could also be incorporated.

We obtain the class-incremental probabilities by forming an ensemble that averages the class probabilities over all domains. From the current task t's domain, we iteratively rectified the latent back to task t - 1, task t - 2, ..., task 1's domain. At each domain, we obtain the rectified representation corresponding with the domain, which we forward through the respective classifier. We then average the softmax probabilities of each domain, essentially forming an ensemble of  $w_i(f_i)|_{i=1}^t$ .

371 372 373

374

375

354 355

356

357

358

359 360

361

362

363

364

366

367

368

369

370

4 EXPERIMENTS

Our implementation <sup>1</sup> is based partially on the Mammoth (Boschini et al., 2022; Buzzega et al., 2020) repository, TAMiL (Bhat et al., 2023) repository, and CLS-ER (Arani et al., 2022) repository.

<sup>376</sup> 377

<sup>&</sup>lt;sup>1</sup>Source code will be publicly released after paper acceptance.



Figure 3: The performance of various CL methods at each task training (lighter color is better). The horizontal axis represents the task on which the model has been trained. The vertical axis represents the task accuracy. ILR-P demonstrates a forgetting rate comparable to or better than other rehearsal-based methods without revisiting past task samples. ILR-P with 1000 sample examples exhibits less forgetting than DER++ and ER-ACE. ILR-P with 5000 sample examples exhibits similar forgetting to CLS-ER and TAMIL.

403 404

405

4.1 EVALUATION PROTOCOL

Datasets. We select three standard continual learning benchmarks for our experiments: Sequential
 CIFAR10 (S-CIFAR10), Sequential CIFAR100 (S-CIFAR100), and Sequential Tiny ImageNet (S-TinyImg). Specifically, we divide S-CIFAR10 into five binary classification tasks, S-CIFAR100 into
 five tasks with 20 classes each, and S-TinyImg into 20 tasks with 20 classes each.

410 Baselines. We evaluate ILR against representa-411 tive continual learning methods, including EWC 412 (online) (Schwarz et al., 2018), and LwF (multi-413 class) (Li & Hoiem, 2017), ER (Chaudhry 414 et al., 2019b), AGEM (Chaudhry et al., 2019a), 415 DER++ (Buzzega et al., 2020), ER-ACE (Caccia et al., 2022), CLS-ER (Arani et al., 2022), 416 TAMiL (Bhat et al., 2023). We further provide 417 an upper and lower bound for all methods by 418 joint training on all tasks' data and fine-tuning 419 without catastrophic forgetting mitigation. We 420 employ ResNet18 (He et al., 2016) as the feature 421 extractor for all benchmarks. The classifier com-422 prises a fixed number of separate linear heads 423 for each task. 424

Table 3: Class-Incremental Average Accuracy across all tasks after CL training. The settings are similar to Table 2.

Method	$ \mathcal{B} $	$ \mathcal{S}_t $	S-CIFAR100		
CIL			NP	AA	
Joint			11.22м	$71.07{\scriptstyle\pm0.27}$	
Finetuning	-	-	11.22м	$17.50 \pm 0.09$	
DER++			11.22м	46.96±0.17	
ER-ACE	1000		11.22м	$47.09{\scriptstyle\pm1.16}$	
TAMiL	1000	-	22.77м	$51.83{\scriptstyle\pm0.41}$	
CLS-ER			33.66м	$51.13{\scriptstyle \pm 0.12}$	
ILR-P	-	1000	12.25м	$44.45{\scriptstyle\pm0.48}$	
ILR-P	-	5000	12.25м	$47.96{\scriptstyle \pm 0.50}$	

Further details on datasets, implementation, and hyperparameters are provided in the Appendix.

426 427

4.2 Results

428

**Task-incremental.** Table 2 shows the performance of ILR-P and other CL methods, including rehearsal-based and regularization-based methods, on multiple sequential datasets, including S-CIFAR10, S-CIFAR100, and S-TinyImg. For ILR-P, we create an alignment set from 500, 100, and 5000 samples of  $\mathcal{D}_{t-1}^{\text{train}}$ . As can be observed from the table, ILR-P achieves comparable results on

<sup>425</sup> 

S-CIFAR10, compared to the baselines. On S-CIFAR100 and S-TinyImg, ILR-P is equivalent to or
 outperforms all the baselines, including strong rehearsal-based methods such as TAMiL and CLS-ER,
 given a sufficient alignment set, indicating its ability to rectify representation changes incrementally.

Alignment set choices. Table 2 also demonstrates the results of different alignment set choices. As can be observed, training with data from  $\mathcal{D}_{t-1}^{\text{train}}$  (ILR-P) expectedly achieves the best performance since the data is sampled directly from the data distribution  $\mathcal{D}_t$  of the previous task; increasing the number of samples from  $\mathcal{D}_{t-1}^{\text{train}}$  yields better performance results. The generative network (ILR-G) also yields comparable results due to its ability to synthesize data from  $\mathcal{D}_t$ . Furthermore, training with  $\mathcal{D}_t^{\text{train}}$  (ILR-C) is an attractive choice for its competitive performance and the fact that we do not need to keep a copy of the previous task's data.

Class-incremental. Table 3 demonstrates the extension of ILR to class-incremental settings. As
 the class-incremental probabilities are obtained through averaging, we can still achieve comparable
 performance to other rehearsal-based methods given a sufficient alignment set.

Long rectification chain. Continual learning methods, including rehearsal-based approaches, often
 experience performance degradation over long task sequences. In Figure 3, we demonstrate that ILR
 exhibit less forgetting than several continual learning methods across the ten tasks of S-TinyImg.

448 449 450

462 463

464

4.3 PARAMETER GROWTH COMPARISON

451 This section studies the network-size footprint of 452 our framework. The base ResNet-18 has 11.17 mil-453 lion parameters. We report the network sizes after 454 5, 10, and 20 tasks for ILR and the two baselines, 455 CSL-ER and TAMIL, in Table 4. As we can ob-456 serve, ILR exhibits a linear memory growth and has the smallest memory footprint among the three 457 baselines. Further analysis reveals that the gate 458 function accounts for 0.06 million parameters while 459 the weak feature extractor account contributes 0.14 460 million parameters per task. 461

Table 4: Number of parameters (in millions) of different methods after N tasks measured on the S-TinyImg. The ResNet-18 network with no classifier head is 11.17 million parameters

Methods	5 tasks	10 tasks	20 tasks
ResNet-18	11.27м	11.27м	11.27м
TAMiL	22.87м	23.20м	23.85м
CLS-ER	33.81м	33.81м	33.81м
ILR-P	12.30м	13.13м	15.41м

#### 5 LIMITATIONS

We have shown the potential and high utility of ILR's continual learning mechanism in this paper. 465 Nevertheless, ILR also has some limitations. One limitation is that ILR still maintains additional 466 parameters, i.e., the rectifier, which incurs an additional overhead as the number of tasks increases. 467 Inference cost for a long chain would be costly, which can be further explored with modified chaining 468 methods such as skipping (i.e., building a rectifier every two tasks). Additionally, the best performance 469 is achieved with access to task t - 1's data. Ideally, we would want to remove this requirement; 470 thus, future research should focus on creating the alignment training data. We have attempted to 471 demonstrate that generative methods are a viable option. Furthermore, since ILR relies on the 472 task identity to reconstruct the rectifier sequence, application to class-incremental learning settings 473 requires either inferring task identity or forming an ensemble of predictions. The proposed ensemble 474 solution might suffer from over-confident or under-confident classifiers. Class-incremental learning 475 is still an open research, where more effective adaptations of our framework can be discovered.

476 477

478

## 6 CONCLUSION

This work proposes a new CL paradigm, ILR, for task incremental learning. ILR tackles catastrophic
forgetting through its novel backward-recall mechanism that learns to align the newly learned
presentation of past data to their correct representations. Unlike existing CL methods, it requires
neither a replay buffer nor intricate training modifications. Our experiments validate that the proposed
ILR achieves comparable results to the performance of existing CL baselines for task-incremental
and class-incremental learning.

## 486 REFERENCES

- Hongjoon Ahn, Sungmin Cha, Donggyu Lee, and Taesup Moon. Uncertainty-based continual learning with adaptive regularization. In *Advances in Neural Information Processing Systems*, pp. 4394–4404, 2019.
- Rahaf Aljundi, Francesca Babiloni, Mohamed Elhoseiny, Marcus Rohrbach, and Tinne Tuytelaars.
   Memory aware synapses: Learning what (not) to forget. In *Proceedings of the European conference* on computer vision (ECCV), pp. 139–154, 2018.
- Elahe Arani, Fahad Sarfraz, and Bahram Zonooz. Learning fast, learning slow: A general continual learning method based on complementary learning system. In *International Conference on Learning Representations*, 2022.
- Yogesh Balaji, Mehrdad Farajtabar, Dong Yin, Alex Mott, and Ang Li. The effectiveness of memory
   replay in large scale continual learning. *arXiv preprint arXiv:2010.02418*, 2020.
- Prashant Shivaram Bhat, Bahram Zonooz, and Elahe Arani. Task-aware information routing from common representation space in lifelong learning. In *The Eleventh International Conference on Learning Representations*, 2023.
- Matteo Boschini, Lorenzo Bonicelli, Pietro Buzzega, Angelo Porrello, and Simone Calderara. Class incremental continual learning into the extended der-verse. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 2022.
- <sup>507</sup> Pietro Buzzega, Matteo Boschini, Angelo Porrello, Davide Abati, and Simone Calderara. Dark experience for general continual learning: a strong, simple baseline. In H. Larochelle, M. Ranzato, R. Hadsell, M.F. Balcan, and H. Lin (eds.), *Advances in Neural Information Processing Systems*, volume 33, pp. 15920–15930. Curran Associates, Inc., 2020.
- Lucas Caccia, Rahaf Aljundi, Nader Asadi, Tinne Tuytelaars, Joelle Pineau, and Eugene Belilovsky.
   New insights on reducing abrupt representation change in online continual learning. In *International Conference on Learning Representations*, 2022.
- Arslan Chaudhry, Marc'Aurelio Ranzato, Marcus Rohrbach, and Mohamed Elhoseiny. Efficient
   lifelong learning with A-GEM. In *International Conference on Learning Representations*, 2019a.
- Arslan Chaudhry, Marcus Rohrbach, Mohamed Elhoseiny, Thalaiyasingam Ajanthan, P Dokania, P Torr, and M Ranzato. Continual learning with tiny episodic memories. In *Workshop on Multi-Task* and Lifelong Reinforcement Learning, 2019b.
- Arslan Chaudhry, Marcus Rohrbach, Mohamed Elhoseiny, Thalaiyasingam Ajanthan, Puneet K.
   Dokania, Philip H. S. Torr, and Marc'Aurelio Ranzato. On tiny episodic memories in continual
   learning. *arXiv preprint arXiv:1902.10486*, 2019.
- Sayna Ebrahimi, Mohamed Elhoseiny, Trevor Darrell, and Marcus Rohrbach. Uncertainty-guided continual learning with bayesian neural networks. In *International Conference on Learning Representations*, 2020. URL https://openreview.net/forum?id=HklUCCVKDB.
- Mehrdad Farajtabar, Navid Azizan, Alex Mott, and Ang Li. Orthogonal gradient descent for continual learning. In *International Conference on Artificial Intelligence and Statistics*, pp. 3762–3773.
   PMLR, 2020.
- Chrisantha Fernando, Dylan Banarse, Charles Blundell, Yori Zwols, David Ha, Andrei A Rusu,
   Alexander Pritzel, and Daan Wierstra. Pathnet: Evolution channels gradient descent in super neural
   networks. *arXiv preprint arXiv:1701.08734*, 2017.
- Kaiming He, Xiangyu Zhang, Shaoqing Ren, and Jian Sun. Deep residual learning for image recognition. In 2016 IEEE Conference on Computer Vision and Pattern Recognition (CVPR), pp. 770–778, 2016.
- Huiyi Hu, Ang Li, Daniele Calandriello, and Dilan Gorur. One pass imagenet. In *NeurIPS 2021 Workshop on ImageNet: Past, Present, and Future*, 2021. URL https://openreview.net/forum?id=mEgL92HSW6S.

547

564

565

566

570

577

578

579 580

581

582

588

- Ghassen Jerfel, Erin Grant, Thomas L. Griffiths, and Katherine A. Heller. Reconciling meta-learning and continual learning with online mixtures of tasks. In *NeurIPS*, 2019.
- 543 Minguk Kang and Jaesik Park. ContraGAN: Contrastive Learning for Conditional Image Generation.
   544 2020.
- Minguk Kang, Woohyeon Shim, Minsu Cho, and Jaesik Park. Rebooting ACGAN: Auxiliary Classifier GANs with Stable Training. 2021.
- 548 MinGuk Kang, Joonghyuk Shin, and Jaesik Park. StudioGAN: A Taxonomy and Benchmark of
   549 GANs for Image Synthesis. *IEEE Transactions on Pattern Analysis and Machine Intelligence* 550 (*TPAMI*), 2023.
- Polina Kirichenko, Mehrdad Farajtabar, Dushyant Rao, Balaji Lakshminarayanan, Nir Levine, Ang Li, Huiyi Hu, Andrew Gordon Wilson, and Razvan Pascanu. Task-agnostic continual learning with hybrid probabilistic models. 2021. URL https://openreview.net/forum?id= ZbSeZKdqNkm.
- James Kirkpatrick, Razvan Pascanu, Neil Rabinowitz, Joel Veness, Guillaume Desjardins, Andrei A Rusu, Kieran Milan, John Quan, Tiago Ramalho, Agnieszka Grabska-Barwinska, et al. Overcoming catastrophic forgetting in neural networks. *Proceedings of the National Academy of Sciences*, 114 (13):3521–3526, 2017.
- Xilai Li, Yingbo Zhou, Tianfu Wu, Richard Socher, and Caiming Xiong. Learn to grow: A
   continual structure learning framework for overcoming catastrophic forgetting. *arXiv preprint arXiv:1904.00310*, 2019.
- <sup>563</sup> Zhizhong Li and Derek Hoiem. Learning without forgetting. *arXiv preprint arXiv:1606.09282*, 2017.
  - David Lopez-Paz and Marc'Aurelio Ranzato. Gradient episodic memory for continual learning. In *Advances in Neural Information Processing Systems*, pp. 6467–6476, 2017.
- Arun Mallya and Svetlana Lazebnik. Packnet: Adding multiple tasks to a single network by iterative pruning. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*, pp. 7765–7773, 2018.
- Nicolas Y Masse, Gregory D Grant, and David J Freedman. Alleviating catastrophic forgetting using
   context-dependent gating and synaptic stabilization. *Proceedings of the National Academy of Sciences*, 115(44):10467–10475, 2018.
- 574 Michael McCloskey and Neal J Cohen. Catastrophic interference in connectionist networks: The
  575 sequential learning problem. In *Psychology of learning and motivation*, volume 24, pp. 109–165.
  576 Elsevier, 1989.
  - Cuong V. Nguyen, Yingzhen Li, Thang D. Bui, and Richard E. Turner. Variational continual learning. In *International Conference on Learning Representations*, 2018.
  - Qi Qin, Wenpeng Hu, Han Peng, Dongyan Zhao, and Bing Liu. Bns: Building network structures dynamically for continual learning. *Advances in Neural Information Processing Systems*, 34: 20608–20620, 2021.
- Dushyant Rao, Francesco Visin, Andrei Rusu, Razvan Pascanu, Yee Whye Teh, and Raia Hadsell.
   Continual unsupervised representation learning. In *Advances in Neural Information Processing Systems*, pp. 7645–7655, 2019.
  - Björn Rasch and Jan Born. Maintaining memories by reactivation. *Current Opinion in Neurobiology*, 17(6):698–703, 2007.
- Roger Ratcliff. Connectionist models of recognition memory: Constraints imposed by learning and forgetting functions. *Psychology Review*, 97(2):285–308, April 1990.
- Sylvestre-Alvise Rebuffi, Alexander I Kolesnikov, Georg Sperl, and Christoph H. Lampert. iCaRL:
   Incremental classifier and representation learning. 2017 IEEE Conference on Computer Vision and Pattern Recognition (CVPR), pp. 5533–5542, 2016.

594 595 596	Matthew Riemer, Ignacio Cases, Robert Ajemian, Miao Liu, Irina Rish, Yuhai Tu, and Gerald Tesauro. Learning to learn without forgetting by maximizing transfer and minimizing interference. <i>arXiv</i> preprint arXiv:1810.11910, 2018.
597 598 599	Amanda Rios and Laurent Itti. Closed-loop GAN for continual learning. <i>arXiv preprint arXiv:1811.01146</i> , 2018.
600 601 602	Andrei A Rusu, Neil C Rabinowitz, Guillaume Desjardins, Hubert Soyer, James Kirkpatrick, Koray Kavukcuoglu, Razvan Pascanu, and Raia Hadsell. Progressive neural networks. <i>arXiv preprint arXiv:1606.04671</i> , 2016.
603 604 605 606	Jonathan Schwarz, Wojciech Czarnecki, Jelena Luketina, Agnieszka Grabska-Barwinska, Yee Whye Teh, Razvan Pascanu, and Raia Hadsell. Progress & compress: A scalable framework for continual learning. In <i>International Conference on Machine Learning</i> , pp. 4528–4537. PMLR, 2018.
607 608	Hanul Shin, Jung Kwon Lee, Jaehong Kim, and Jiwon Kim. Continual learning with deep generative replay. In <i>Advances in Neural Information Processing Systems</i> , pp. 2990–2999, 2017.
609 610 611 612	Shengyang Sun, Daniele Calandriello, Huiyi Hu, Ang Li, and Michalis Titsias. Information-theoretic online memory selection for continual learning. In <i>International Conference on Learning Representations (ICLR)</i> , 2022.
613 614	Hung-Yu Tseng, Lu Jiang, Ce Liu, Ming-Hsuan Yang, and Weilong Yang. Regularing generative adversarial networks under limited data. In <i>CVPR</i> , 2021.
615 616 617 618	Linh Ngo Van, Nam Le Hai, Hoang Pham, and Khoat Than. Auxiliary local variables for improving regularization/prior approach in continual learning. In <i>Pacific-Asia Conference on Knowledge Discovery and Data Mining</i> , pp. 16–28. Springer, 2022.
619 620 621	Eli Verwimp, Matthias De Lange, and Tinne Tuytelaars. Rehearsal revealed: The limits and merits of revisiting samples in continual learning. In <i>Proceedings of the IEEE/CVF International Conference on Computer Vision</i> , pp. 9385–9394, 2021.
622 623 624	Mitchell Wortsman, V. Ramanujan, Rosanne Liu, Aniruddha Kembhavi, Mohammad Rastegari, J. Yosinski, and Ali Farhadi. Supermasks in superposition. <i>arXiv preprint arXiv:2006.14769</i> , 2020.
625 626	Ju Xu and Zhanxing Zhu. Reinforced continual learning. <i>Advances in Neural Information Processing Systems</i> , 31, 2018.
627 628 629 630	Shipeng Yan, Jiangwei Xie, and Xuming He. Der: Dynamically expandable representation for class incremental learning. In <i>Proceedings of the IEEE/CVF conference on computer vision and pattern recognition</i> , pp. 3014–3023, 2021.
631 632	Dong Yin, Mehrdad Farajtabar, and Ang Li. SOLA: Continual learning with second-order loss approximation. <i>arXiv preprint arXiv:2006.10974</i> , 2020.
633 634	Jaehong Yoon, Eunho Yang, Jungtae Lee, and Sung Ju Hwang. Lifelong learning with dynamically expandable networks. In <i>Sixth International Conference on Learning Representations</i> . ICLR, 2018.
636 637	Friedemann Zenke, Ben Poole, and Surya Ganguli. Continual learning through synaptic intelligence. In <i>International Conference on Machine Learning</i> , pp. 3987–3995. PMLR, 2017.
638 639 640	Mengmi Zhang, Tao Wang, Joo Hwee Lim, and Jiashi Feng. Prototype reminding for continual learning. <i>arXiv preprint arXiv:1905.09447</i> , 2019.
642 643	
644 645 646	

## 648 A DETAILED EXPERIMENTAL SETUP

#### A.1 BASELINES

650

651

As detailed in Section 4.1, we evaluate ILR against EWC (online version), LwF (multi-class) version, ER, AGEM, DER++, ER-ACE, CLS-ER, and TAMiL.

For extensive comparison, we provide rehearsal-based methods with a buffer with a max capacity of 500 and 1,000 samples, respectively. Since our method does not rely on a buffer of all task data but only an alignment set of task t - 1 data, the forgetting can be more significant, which is not a fair comparison of ILR against other rehearsal-based methods. Therefore, we provide ILR with an alignment set of 500, 1,000, and 5,000 samples.

We replicate training settings: For ER, DER++, ER-ACE, TAMiL, and CLS-ER, we employ the
reservoir sampling strategy to remove the reliance on task boundaries as in the original implementation.
On the other hand, ILR, AGEM, and TAMiL rely on the task boundary to learn the rectifier, modify
the buffer, and add a new task-attention module, respectively. For TAMiL, we use the best-reported
task-attention architecture. For CLS-ER, we perform inference using the stable model per the original
formulation.

#### A.2 DATASETS

668 To demonstrate the effectiveness of our method, we perform empirical evaluations on three stan-669 dard continual learning benchmarks: Sequential CIFAR10 (S-CIFAR10), Sequential CIFAR100 670 (S-CIFAR100), and Sequential Tiny ImageNet (S-TinyImg). The datasets are split into 5, 5, and 10 671 tasks containing 2, 20, and 20 classes, respectively. The dataset of S-CIFAR10 and S-CIFAR100 672 each includes 60000  $32 \times 32$  images splitter into 50000 training images and 10000 test images, with 673 each task occupying 10000 training images and 2000 testing images. The dataset S-TinyImg contains 674 1100000  $64 \times 64$  images with 100000 training images and 10000 test images divided into ten tasks 675 with 10000 training images and 1000 test images each. We augment random horizontal flips and 676 random image cropping for each training and buffered image.

677 678

679

683

666

667

#### A.3 RECTIFIER DESIGNS

Table 5: Architecture of the weak feature extractor  $h_t$ . We use ReLU activation after each convolution layer. For each task, a weak feature extractor  $h_t$  is distilled from the current feature extractor  $f_t$ .

Layer	Channel	Kernel	Stride	Padding	Output size
Input	3				$16 \times 16$
Conv 1	64	$3 \times 3$	2	1	$8 \times 8$
MaxPool			2		$4 \times 4$
Conv 2	128	$3 \times 3$	2	1	$2 \times 2$
MaxPool			2		$1 \times 1$
Linear	512				

Rectifier. We have explored several options for the rectifier unit design and arrived at a gated rectifier
 unit, which is the current design, and a compress-combine rectifier unit as in Figure 4. Both designs
 demonstrate similar performance as shown in Table 6. However, the gated rectifier unit was selected
 because it is more parameter-efficient than the compress-combine rectifier unit.

<sup>697</sup> The compress-combine design includes a linear layer  $a_t$  to reduce f(t)'s representation to a lower <sup>698</sup> dimension, essentially forming a bottleneck to filter task t - 1 information and a linear layer  $b_t$  to <sup>699</sup> combine both  $h_t(x)$  representation and reduced  $a_t(f_t(x))$  representation.

$$r_t(f_t(x), x) = b_t(\text{concatenate}(a_t(f_t(x)), h_t))$$
(6)



Figure 4: Compress-combine rectifier unit design. The Compress layer forms a bottleneck to select the remaining (t - 1)-domain knowledge in  $f_t$ , while  $h_t$  extracts compensation information for the loss information in  $f_t$ . The Combine layer aggregates and transforms the information from both  $h_t$  and  $f_t$  to form the rectified representation.

Table 6: Average accuracy of ILR-P with  $|S_t| = 1000$  using different rectifier design.

Rectifier design	S-CIFAR10	S-CIFAR100	S-TinyImg
Gated	90.66±0.97	$78.14{\scriptstyle \pm 0.18}$	$66.83{\scriptstyle \pm 0.55}$
Compress-Combine	$91.02 \pm 1.76$	$78.53{\scriptstyle \pm 0.25}$	$66.79{\scriptstyle \pm 0.64}$

# 719 A.4 TRAINING 720

702

704 705

706

708 709

710

711

712

713 714

730

736

**Settings.** The training set of each task is divided into 90%-10% for training and validation. All methods are optimized by the Adam optimizer available in PyTorch with a learning rate of  $5 \times 10^{-4}$ . As the validation loss plateau for three epochs, we reduce the learning rate by 0.1. Each task is trained for 40 epochs. For ILR, we train  $h_t$  and  $r_t$  using the same formulation with Adam optimizer at a learning rate of  $5 \times 10^{-4}$  for 40 epochs.

GAN training. We use the StudioGan repository's default implementation Kang et al. (2023; 2021);
Kang & Park (2020) of the BigGAN LeCam Tseng et al. (2021) to train the network on each task of
S-CIFAR100. The FID score for each task is between 17 and 23. The BigGAN network has nearly
95 million parameters. During ILR training, we sampled directly from the BigGAN network.

#### 731 A.5 HYPERPARAMETER SEARCH

For all methods, experiments, and datasets, we perform a grid search over the following hyper-parameters using a validation set. Some hyperparameters are obtained directly from their original implementation to narrow the search range.

- Joint, Finetuning, LwF.mc, ER, AGEM, ER-ACE: No hyperparameters
  o-EWC:
- 738  $-\lambda \in \{10, 20, 50, 100\}$ 739  $-\gamma \in \{0.9,1\}$ 740 • DER++: 741  $-\alpha \in \{0.1, 0.2, 0.5, 1\}$ 742 -  $\beta \in \{0.1, 0.2, 0.5, 1\}$ 743 744 • CLS-ER: 745  $-r_p \in \{0.5, 0.9\}$ 746  $-r_s \in \{0.1, 0.5\}$ 747  $- \alpha_p \in \{0.999\}$ 748  $-\alpha_s \in \{0.999\}$ 749 • TAMiL: 750  $-\alpha \in \{0.2, 0.5, 1\}$ 751 -  $\beta \in \{0.1, 0.2, 1\}$ 752  $- \theta \in \{0.1\}$ 753 754 • ILR:  $-\alpha \in \{1, 2, 3\}$

Method	$ \mathcal{B} $	$ \mathcal{S}_t $	S-CIFAR10	S-CIFAR100	S-TinyImg
o-EWC	_	_	$\lambda=100, \gamma=0.9$	$\lambda=50, \gamma=0.1$	$\lambda = 20, \gamma = 0.9$
DER++ TAMiL CLS-ER			$\begin{array}{l} \alpha = 0.5, \beta = 0.1 \\ \alpha = 1.0, \beta = 1.0 \\ r_p = 0.5, r_s = 0.1 \end{array}$	$\begin{array}{l} \alpha = 0.2, \beta = 0.1 \\ \alpha = 1.0, \beta = 1.0 \\ r_p = 0.9, r_s = 0.1 \end{array}$	$ \begin{aligned} \alpha &= 0.5, \beta = 0.1 \\ \alpha &= 1.0, \beta = 0.5 \\ r_p &= 0.5, r_s = 0. \end{aligned} $
ILR-P	-	500	$\alpha = 2$	$\alpha = 3$	$\alpha = 3$
DER++ TAMiL CLS-ER			$ \begin{aligned} \alpha &= 1.0, \beta = 0.1 \\ \alpha &= 1.0, \beta = 1.0 \\ r_p &= 0.5, r_s = 0.1 \end{aligned} $	$\begin{array}{c} \alpha = 0.2, \beta = 0.1 \\ \alpha = 1.0, \beta = 1.0 \\ r_p = 0.5, r_s = 0.1 \end{array}$	$\alpha = 1.0, \beta = 0.1  \alpha = 1.0, \beta = 0.5  r_p = 0.9, r_s = 0.7 $
ILR-P	-	1000	$\alpha = 3$	$\alpha = 3$	$\alpha = 3$
ILR-P	-	5000	$\alpha = 3$	$\alpha = 3$	$\alpha = 3$

 Table 7: Hyperparameters for method in Table 2

#### **B** VERSATILITY OF ILR FRAMEWORK

In ILR, as the tasks arrive, conventional fine-tuning or training on the new task happens without any CL's intervention. ILR only augments or adds to this process with a separate training of the backward-recall mechanism. The attractiveness of this framework is twofold. First, ILR allows the best adaptation on the new task to possibly achieve maximum plasticity while the backward-recall mechanism mitigates catastrophic forgetting. Second, unlike previous CL approaches that heavily modify the sequential training process, ILR minimally changes the fine-tuning process, allowing the users to more flexibly incorporate this framework into their existing machine learning pipelines.

Relationship to Memory Linking. ILR's process of mapping newly learned knowledge representation resembles the popular humans' mnemonic memory-linking technique, which establishes associations of fragments of information to enhance memory retention or recall.<sup>2</sup> As the model learns a new task, the feature rectifier unit establishes a mnemonic link from the new representation of the sample from the past task to its past task's correct representation.







<sup>&</sup>lt;sup>2</sup>https://en.wikipedia.org/wiki/Mnemonic\_link\_system