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Thought calibration: Efficient and confident test-time scaling

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Abstract

Reasoning large language models achieve impressive test-time scaling by thinking for longer, but this performance gain comes at significant compute cost. Directly limiting test-time budget hurts overall performance, but not all problems are 015 equally difficult. We propose thought calibration to decide dynamically when thinking can be terminated. To calibrate our decision rule, 018 we view a language model's growing body of 019 thoughts as a nested sequence of reasoning trees, 020 where the goal is to identify the point at which novel reasoning plateaus. We realize this framework through lightweight probes that operate on top of the language model's hidden representations, which are informative of both the reason-025 ing structure and overall consistency of response. Based on three reasoning language models and 027 four datasets, thought calibration preserves model 028 performance with up to a 60% reduction in think-029 ing tokens on in-distribution data, and up to 20% 030 in out-of-distribution data.

1. Introduction

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035 Test-time scaling presents a new paradigm for improving language model reasoning by expending large amounts of compute during inference (Kaplan et al., 2020; Wei et al., 038 2022). Though the strategies for eliciting reasoning vary – from large-scale reinforcement learning (Guo et al., 2025a) to explicit tree search (Zhang et al., 2024a;b) - a common 041 effect is that language models improve by sampling substantially more tokens. This may result in wasted compute 043 on easy problems (Chen et al., 2024; Sui et al., 2025), but naively limiting the generation length leads to pronounced 045 drops in accuracy (Muennighoff et al., 2025). This motivates 046 early stopping strategies that reduce the inference budget 047

without significantly degrading performance, and control the extent of impact, if performance must be compromised.

Numerous methods have been proposed for teaching language models to be economical with their token budgets (Han et al., 2024; Arora & Zanette, 2025; Sui et al., 2025), or for identifying opportune stopping points (Yang et al., 2025; Zhang et al., 2025). While these methods demonstrate strong empirical performance, they lack strict statistical guarantees about when they could fail. In an orthogonal direction, conformal prediction has been adapted to equip language models with calibrated confidences about the quality or consistency of their generations (Mohri & Hashimoto, 2024; Quach et al., 2024; Rubin-Toles et al., 2025b;a; Cherian et al., 2024). However, most of these algorithms operate through post-hoc filtering and require external LLM-based validation for scoring intermediate steps - rendering them unsuitable for actively terminating generation.

In this work, we jointly pursue an effective and calibrated decision rule to determine when a language model can stop "thinking." To do so, we introduce the notion of a reasoning tree, where at each step of sampling, a language model either adds a new leaf, walks along the tree, or backtracks to a previous step. Notably, identifying when the thoughts have converged is equivalent to detecting when this reasoning tree stops growing. Inspired by this concept, we approach early stopping as multiple hypothesis testing problem. At each generation step, we test whether the current tree is expected to change, based on the predictions of lightweight probes over the language model's hidden representations. Our algorithm is based on the Learn then Test framework (Angelopoulos et al., 2021), which provides finite-sample, distribution-free guarantees for controlling the risk of our decisions.

We evaluate this strategy, thought calibration, based on its ability to guide efficient reasoning, and whether its decisions are well-calibrated. Our experiments consider two empirical settings: we may or may not have access to training and calibration data from the true test distribution. In the first setting, we train variants of thought calibration using three reasoning lanugage models (DeepSeek-R1 distilled Owen 32B and Llama 70B (Guo et al., 2025a; Yang et al., 2024; Grattafiori et al., 2024), QwQ 32B (Team, 2025)), evaluated

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on a helf-out split of s1K-1.1 (Muennighoff et al., 2025). Here, we are able to halve the number of thinking tokens 057 across accuracy levels, with a maximum reduction of 60%. 058 Then we evaluate Qwen 32B-based thought calibration on 059 three test datasets: AIME 24, GPQA Diamond (Rein et al., 060 2024), MATH-500 (Lightman et al., 2023)). Though these 061 datasets vary in format and difficult, thought calibration 062 is still able to reach up to a 20% reduction in thinking 063 tokens, and in the worst case, is always as efficient as naive budget constraints. In summary, this work has three main 064 065 contributions. 066

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 1. We interpret LLM reasoning through the lens of an abstract reasoning tree, where the problem of early exiting is equivalent to identifying when this tree stops growing.
 - 2. This view allows us to calibrate the decision rule for actively terminating generation.
 - Based on multiple language models and reasoning benchmarks, we provide empirical evidence that thought calibration is effective for efficient test-time scaling.

2. Background

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081082**2.1. Test-time scaling**

083 Efficient inference Since current reasoning models are 084 post-trained through reinforcement learning (Guo et al., 085 2025a), a number of works address the overthinking prob-086 lem (Sui et al., 2025) as part of the reinforcement learning 087 process (Han et al., 2024; Arora & Zanette, 2025; Hou 088 et al., 2025). Other works focus on the inference-time prob-089 lem of predicting when a language model should stop gen-090 erating (Yang et al., 2025; Zhang et al., 2025; Ma et al., 091 2025). This papers falls under the latter category, which on 092 a whole, is compatible with methods that reduce a language 093 model's verbosity during post-training. Finally, another op-094 tion is to achieve efficiency in terms of model architecture. 095 Some works dynamically adapt compute cost (Lei, 2021; 096 Leviathan et al., 2023), while others employ only a subset of 097 all modules during sampling (Kim & Cho, 2021; Liu et al., 098 2022; Schuster et al., 2022). While these strategies operate 099 over the Transformer stack, rather than the generation se-100 quence length, many high-level ideas are broadly applicable to early exiting in our situation.

Self-consistency Self-consistency has been widely used to provide a self-supervised form of confidence during the sampling process (Wang et al., 2022). These methods aim to improve the quality of generated samples, often in situations where multiple samples may be sequentially generated (Mitchell et al., 2022; Madaan et al., 2023; Shi et al., 2023; Weng et al., 2023; Guo et al., 2025b). Consistency can also provide feedback for reasoning-focused reinforcement learning (Wang et al., 2024b). Several recent works have observed that confidence scores can be probed and calibrated from internal representations, to prioritize reasoning trajectories for subsequent runs (Li et al., 2024; Huang et al., 2025; Xie et al., 2024) or for early exiting, similar to this work (Zhang et al., 2025). Our key departure is that we calibrate the *decision rule* to terminate generation, rather than the probabilistic outputs of a probe. This reflects the online setting, where a probe is used to actively guide generation, rather than to filter trajectories post-hoc.

2.2. Conformal prediction and risk control

Conformal prediction quantifies the uncertainty in machine learning models by generating set-valued predictions (Shafer & Vovk, 2008; Angelopoulos & Bates, 2021). These methods are distribution-free and valid under finite samples, which makes them particularly attractive in realworld applications. Specifically, for an input x, a candidate output space \mathcal{Y} , and a predetermined error level ϵ , conformal prediction tests each potential outcome $y \in \mathcal{Y}$ by evaluating the null hypothesis: "output y corresponds to input x." The final prediction set consists of the outputs y for which this null hypothesis fails to be rejected, where the test statistic is known as a nonconformity score. Split conformal prediction leverages a separate training set to learn this nonconformity score (Vovk et al., 2005; Papadopoulos, 2008). The true outcome is included with probability at least $1 - \epsilon$, with guarantees that are typically marginal over draws of the test set and an exchangeable calibration set.

In the context of language modeling, conformal prediction has been adapted to calibrate the factuality (Mohri & Hashimoto, 2024; Cherian et al., 2024), reasoning consistency (Rubin-Toles et al., 2025b), and quality of generations (Quach et al., 2024; Qiu & Miikkulainen, 2024). Here, x may represent an input sequence of text, while y may be a language model output. Of these works, Rubin-Toles et al. (2025b) also introduces the idea of reasoning as coherency over a graph structure, based on logical deducibility. However, this and other methods are primarily designed for post-processing text that has already been generated, and they rely on external language models as scoring functions. As a result, these approaches are not calibrated to be used as decision rules for iterative testing, and the latency required to compute nonconformity scores renders them unsuitable for early exiting.

More recently, the Learn then Test (LTT) framework (Angelopoulos et al., 2021) extends the ideas in conformal prediction to control the risk of arbitrary loss functions, with guarantees over draws of the calibration set. One application of LTT is to convert model outputs into a calibrated

decision rule, by viewing hyperparameter selection (e.g. dis-111 cretization thresholds) as multiple hypothesis testing. Our 112 method and several works in early exiting are built atop the 113 LTT framework. Quach et al. (2024) calibrates a language 114 model's sampling of output sets, similar to this work. Their 115 goal is to generate sufficient outputs y until certain admis-116 sibility criteria have been fulfilled, e.g. correctness and 117 diversity of information. However, the sampling process 118 in Quach et al. (2024) is interactive, in the sense that each 119 step requires an external verifier, and text may be added or 120 removed at any point. As a result, this strategy is unsuitable 121 for providing online decisions about when to stop. Schuster 122 et al. (2022) also leverages LTT to calibrate a stopping rule to exit from a Transformer stack. Their method operates 124 on individual tokens, similar in spirit to applications like 125 speculative decoding (Leviathan et al., 2023). Our focus is 126 on large, coherent thoughts for reasoning, where token-level 127 uncertainties are less informative.

3. Thought Calibration

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131 Given an input $x \in \mathcal{X}$, a reasoning language model gen-132 erates a series of thoughts $y \in \mathcal{Y}$, before synthesizing the 133 final output $z \in \mathcal{Z}$. For example, x may represent a math 134 question; y is a sequence of reasoning steps; and z is the 135 model's attempt at solving the question (Figure 1A). Manip-136 ulating the budget allocated to generating y directly impacts 137 the quality of z (Muennighoff et al., 2025), but as the length 138 of y increases, so too does the cost of inference. Our goal is 139 to identify the point at which growing y no longer improves 140 z.

141 To formalize these ideas, we introduce the notion of an ab-142 stract reasoning graph G, where nodes represent thoughts 143 and directed edges represent entailment relationships (Mac-144 Cartney & Manning, 2014). This graph is rooted at x, the 145 input question. Nodes can be *serialized* into textual descriptions, and different paraphrases of the same idea represent a 147 single node. Where it is clear, we refer to the abstract node 148 and its textual representation interchangeably. 149

150 **Definition 3.1.** A reasoning trajectory z is a root-to-leaf 151 walk in the reasoning graph G.

An arbitrary z need not be "complete" or "correct" with respect to the original question x. We use z^* to denote a walk that starts at x and ends at the right answer, which we assume to be incontrovertible. G uniquely determines the set of all root-to-leaf walks $\{z\}$, and thus, whether a language model has any chance of being correct in its final attempt.

160 **Definition 3.2.** A set of *thoughts* y is a walk, rooted at x, on 161 the augmented graph G' in which every node is connected 162 to each of its ancestors.

¹⁶³ At each stage of sampling, a large language model either

adds a leaf to G (novel thought), or takes one step in G' (backtracking or redundant generation). Let G_t be the reasoning graph at time t. If a language model terminates thinking at this point, it is expected to answer correctly if there exists a path in G_t that yields z^* . Thus, it would be ideal we could calibrate the language model such that with high probability,

$$\mathbb{P}\left(\mathbb{E}\left[\mathbb{1}[z^* \notin G_t] \le \delta\right]\right) \ge 1 - \epsilon \tag{1}$$

for some risk tolerance δ and error level $\epsilon \in (0, 1)$. In principle, a language model could enumerate the space of graphs in a combinatorial search. However, it is far from guaranteed that this graph can be tractably found. Instead, we focus on the consistency between reasoning graphs.

Definition 3.3. Thoughts y and y' are *consistent* if they can be represented by the reasoning graph G.

In particular, if a language model repeatedly revisits a step to arrive at the same conclusion, or traverses the same ideas in a different order, the resultant graph does not change (Figure 1C). Let $y_t := [y^{(i)} \dots y^{(t)}]$ and G_t be the in-progress thoughts and reasoning graph after t steps, and let T be the maximum inference budget (token or model limit). Instead of enforcing that G_t contains z^* , it is more reasonable to guarantee that

$$\mathbb{P}\left(\mathbb{E}\left[\mathbb{1}[G_t \neq G_T] \le \delta\right]\right) \ge 1 - \epsilon.$$
(2)

Due to the sequential nature of generation, G_t is always a (not necessarily strict) subset of G_T .

We now describe how we calibrate the decision rule for terminating language model generation (Section 3.1), and then introduces three strategies for practically estimating the quantities described by Equations 1 and 2 (Section 3.2).

3.1. Calibrating the stopping rule

Suppose we have a calibration dataset \mathcal{D}_{cal} , which contains exchangeable points $\{(x_i, y_i)\}_{i=1}^n$. Given a new example x, let y_t denote the language model's thoughts after t sampling steps, and let y_T denote the maximum set of thoughts. Our goal is to find the smallest t that fulfills Equations 1 or 2, based on the distribution of \mathcal{D}_{cal} . During the sampling process, however, we do not know z^* or G_T , so we must estimate the quantities inside the expectation using a surrogate function f. Here, \mathcal{D}_{cal} serves to calibrate f such that

$$\mathbb{P}\left(\mathbb{E}\left[R(y_t) \le \delta \mid \mathcal{D}_{cal}\right]\right) \ge 1 - \epsilon \tag{3}$$

where R is a bounded risk function associated with f. For example, f may be a linear probe on the hidden representations of thought steps $y^{(i)}$, and its output may be a binary prediction. A potential decision rule could take the form of a threshold λ , where if $f(y^{(t)}) \geq \lambda$, we terminate thinking.



Figure 1: Overview of the problem and our goal. Illustrated example based on s1K-1.1 (Muennighoff et al., 2025).

Similar to Schuster et al. (2022) and Quach et al. (2024), we follow the Learn then Test framework to select a valid set of λ s that provide our desired guarantees (Angelopoulos et al., 2021). On a high level, hyperparameter selection is viewed as a multiple hypothesis testing problem. Let Λ be a finite set of configurations, where each $\lambda_j \in \Lambda$ is associated with the null hypothesis,

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$$H_{i}: \mathbb{E}[R(y_{t}) > \delta]. \tag{4}$$

198 The set of valid $\Lambda_{\text{valid}} \subseteq \Lambda$ is the set of λ_j for which we *fail* 199 *to reject* H_j . In particular, selecting the earliest stopping 200 time is equivalent to identifying the smallest $\lambda \in \Lambda_{\text{valid}}$.

Theorem 3.4 (Adapted from theorem 1 in (Angelopoulos et al., 2021)). Suppose p_j is super-uniform under H_j for all j. Let A be a family-wise error rate (FWER) controlling algorithm at level ϵ . Then $\Lambda_{valid} = \mathcal{A}(p_1, \dots, p_m)$ satisfies Equation 3.

207 Theorem 3.4 specifies that any FWER-controlling algorithm 208 \mathcal{A} can be used with an appropriate p-value to identify Λ_{valid} . 209 While Angelopoulos et al. (2021) proposes several algo-210 rithms to search over Λ , we follow the fixed sequence test-211 ing method, since in principle, our risks are expected to be 212 monotonic ($G_t \subseteq G_T$).

Specifically, let $\Lambda = \{\lambda_1, \dots, \lambda_m\}$ be a descending grid of parameters. Intuitively, larger λ correspond to more permissive thresholds, e.g. allowing a language model to generate for longer.

1. For each j, we compute a valid p-value p_j , e.g. the

binomial tail bound p-value, following (Quach et al., 2024):

$$p_{\lambda}^{BT} := \mathbb{P}(\text{Binom}(n,\epsilon) \le n\hat{R}_n(\lambda)).$$
 (5)

2. If $p_j \leq \epsilon$, we reject H_j and continue. Otherwise, we return λ_{j-1} as the smallest valid threshold for error rate ϵ .

This process yields the binarization threshold for f, where we stop generating when $f(y_t) \ge \lambda_{j-1}$.

3.2. Estimating empirical risk

On a high level, the surrogate function f should reflect the consistency of y_t with expected future generations. Ideally, we would be able to access the graphical structure of G_t , as any repetitions or redundant walks in y_t would be immediately evident. However, since autoregressive language models generate left-to-right, without explicitly conforming to any higher-level structure, we cannot operate directly over G. Instead, we introduce three approaches for designing f in practice.

We first briefly consider the simple case suggested by Equation 1: if we terminate thinking now, is the language model able to answer correctly? That is, we could define

$$f_{\text{correct}}(y_t) := \mathbb{P}(\text{LLM is correct based on } y_t)$$
(6)

$$R_{\text{correct}}(y_t) := \mathbb{I}\{\text{LLM is correct}\} \cdot (1 - f_{\text{correct}}(y_t))$$

$$+ \mathbb{I}\{\text{LLM is wrong}\} \cdot f_{\text{correct}}(y_t).$$
(7)

However, there are several drawbacks of this implementa-221 tion. By construction, the calibration dataset only contain 222 questions that can eventually be answered, which is not true 223 in general. Though the space of graphs is countable, it is 224 unlikely that a language model can efficiently explore the 225 entire space. In other words, the language model may realistically *never* answer correctly. Thus, setting $\lambda = 1$ is 227 not guaranteed to be risk controlling. With this definition 228 of $R_{\text{correct}}(y)$, calibrating based on correctness also requires 229 supervised labels. While this is not an issue on standard 230 benchmarks, it is harder to obtain labels (user feedback) in 231 practice. 232

To address these challenges, we introduce two additional strategies for estimating graph consistency. First, a language model's final attempt z can be viewed as a distillation of its overall reasoning structure. Thus, we compare the language model's attempt z_t after t steps, to the eventual attempt z_T at the maximum reasoning budget. This yields

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$$f_{\text{consistent}}(y_t) := \mathbb{P}(z_t \text{ is the same as } z_T)$$

$$R_{\text{consistent}}(y_t) := \mathbb{1}\{\text{consistent}\} \cdot (1 - f_{\text{consistent}}(y_t))$$

$$+ \mathbb{1}\{\text{inconsistent}\} \cdot f_{\text{consistent}}(y_t)$$
(9)

These values can be determined even for intractable problems, as long as the extended reasoning produces no new insights, and does not require labels of correctness.

Finally, any particular z only represents a single walk through G. Due to stochasticity, two differing attempts could be sampled from the same graph, which is no longer changing. Towards this end, we observed that language models often reiterate redundant information, after having reached the correct answer or the extent of its abilities. Probing for novelty should suffice to capture this phenomena. In practice, however, we found that the following formulation was easier for our verifier to implement, as checking for novelty involves long context reasoning over all previous thoughts, which can be challenging (Wang et al., 2024a).

$$f_{\text{novel leaf}}(y_t) := \mathbb{P}(y^{(t)} \text{ is leaf}) \cdot (1 - \mathbb{P}(y^{(t)} \text{ is novel}))$$
(10)

$$R_{\text{novel leaf}}(y_t) := \mathbb{1}\{\text{LLM inconsistent}\} \cdot f_{\text{novel leaf}}(y_t)$$

+
$$\mathbb{1}\{\text{LLM consistent}\} \cdot (1 - f_{\text{novel leaf}}(y_t)).$$
 (11)

We reuse the labels for consistency due to ease of verification compared to novelty.

3.3. Implementation details

To separate a reasoning trajectory y into individual steps $\{y^{(i)}\}$, we use sections delimited by n n, which also contain either wait or but. We observed that individual tokens representations can vary significantly. Thus, each step uses the mean last-layer representation of its tokens, followed by dimensionality reduction via PCA to d = 256.

To estimate each of quantities in Equations (6) to (11), we train linear probes on these step-level representations. The final probabilities are averaged over a window of 10 steps for smoothness, before calibration. For evaluation, we use a grid of ϵ ranging from 0.05 to 0.5, with precise thresholds selected to roughly match the token range of baselines. During development, we experimented with more complex architectures, e.g. Transformer to predict leaves as a sequence labeling task (Appendix B.1). However, to avoid overfitting on our limited training set, we chose to focus on simple and efficient linear probes. Concurrent work (Zhang et al., 2025) also finds that model confidence can often be extracted linearly. In our experiments, we use three reasoning models: DeepSeek-R1 distilled Qwen 2.5 32B and Llama 3.3 70B (Guo et al., 2025a; Grattafiori et al., 2024; Yang et al., 2024), and QwQ 32B (Team, 2025).

The ground truth labels for these probes are obtained by prompting a separate language model (Qwen 3 32B). Correct: We truncate thinking trajectories to desired lengths, append the <\think> token, and prompt the language model for the final answer, which is compared to the ground truth (Muennighoff et al., 2025). Consistent: The same outputs can be used to check whether G_t is consistent with G_T . by comparing intermediate attempts z_t to maximum budget attempt z_T . Leaf: We annotate whether each step $y^{(i)}$ is a leaf in G by asking a separate language model to identify whether it makes an attempt to answer the original question x, regardless of correctness. Novel: We provide a separate language model with all previous thoughts $y^{(1)} \dots y^{(i-1)}$ and ask whether the new step $y^{(i)}$ provides additional information. All prompts can be found in Appendix A and were run on 4 A6000 GPUs using vLLM (Kwon et al., 2023) and Imdeploy (Contributors, 2023).

We evaluate the correctness of all final attempts using the GPT 4.1 API, between April 15, 2025 and May 15, 2025. For datasets that have no ambiguity (multiple choice, numeric answers), we trimmed the final attempts to 200 characters, to prevent the LLM from "cheating" by using additional thinking budget after the

4. Experiments

4.1. Settings

Datasets. Our experiments focus on efficient language model reasoning across tasks which vary in content, format, and difficulty. In particular, we leverage the following datasets.

s1K-1.1 (Muennighoff et al., 2025) is a curated *training set* for distilling reasoning abilities through data. This dataset contains 1000 difficult math and science questions, along with thought trajectories generated by DeepSeek-R1 (Guo et al., 2025a). As a proof of concept, we split the s1K-



Figure 2: On in-distribution data (held-out test split on s1K), variants of thought calibration achieve up to a 60% reduction in thinking tokens while maintaining full performance. Top right point: Complete DeepSeek-R1 thought trajectory from (Muennighoff et al., 2025). Crop: Fix thinking budget at 512, 1024, 2048, 4096, and 8192 tokens. Supervised: exit based on predicted likelihood of correctness. Consistent, and Leaf Novelty: exit based on predicted consistency of answer or graph. Supervised is over confident, since the test set contains unsolvable problems.



Figure 3: We applied thought calibration probes for DeepSeek-distilled Qwen-2.5 32B on standard math and science benchmarks, which may be out-of-distribution compared to the training and calibration sets, drawn from s1K. We achieve up to a 20% reduction in thinking tokens. While Consistent generally remains below the predetermined error rates, Supervised is overconfident (as expected).

304 1.1 dataset into training, testing, and calibration (500, 50, 305 450, in dataset order). We use the training set to develop 306 our probes, which are calibrated on the calibration set and evaluated on the testing set.

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308 We also consider three common reasoning benchmarks 309 solely for testing. AIME-24 is the 2024 iteration of the 310 American Invitational Mathematics Examination.¹ This 311 dataset contains math questions whose answers take on in-312 tegers between 0 and 999. GPQA Diamond (Rein et al., 313 2024) is a PhD-level math and science reasoning benchmark 314 with multiple choice answers. MATH 500 (Hendrycks 315 et al., 2021; Lightman et al., 2023) is a curated subset of 316 the MATH dataset, which competition math questions of various levels. Note that while s1K-1.1 contains examples 318 of both mathematical and scientific questions, the format 319 and subsequent reasoning patterns may vary. For example, 320 while s1K-1.1 is open-ended, the various choices in GPQA must be compared. Thus, we view these three datasets as 322 "out of distribution" from s1K-1.1, which is itself diverse.

Models. We evaluate the three variants of thought cali-325 bration: the supervised probe for correctness (Equation 6, 326 Supervised); the consistency probe (Equation 8, Consis-327

tent); and the lack of novelty probe (Equation 10, Novel Leaf). To contextualize our experimental results, we also consider a naive budget-forcing baseline (Crop). Specifically, we set a fixed token budget for thinking (ranging from 1024 to the full trajectory). Once the language model reaches this budget, thinking is immediately terminated and the model is prompted for a final answer. This reflects both the practical use case of setting a limit on maximum generation tokens, and the strategy employed by Muennighoff et al. (2025). Finally, concurrent work has also observed that probes for correctness (Zhang et al., 2025) are effective for early exiting. While this design may not be valid for risk control in practice (LLMs are not guaranteed to ever answer correctly), the Supervised baseline is similar to this work.

4.2. In-distribution setting

We start with the case where we have access to samples xthat are drawn from the same distribution as our eventual application. For example, a model provider may possess typical examples of user data. Our goals are to lower the overall test-time budget while maintaining accuracy, and to control any necessary drops in performance based on our predetermined error levels. In Figure 2, we observe that these probes are able to reduce the number of thinking

¹https://maa.org/maa-invitational-competitions/

tokens by over half for all three models, with minimal impact to overall performance. With respect to calibration, the Supervised probe is quite poorly calibrated, especially at lower values of ϵ . All other probes are well calibrated at $\epsilon < 0.1$, though variance is higher outside of this range. This may be due to distribution shift, resulting from the small test split (to maximize training and calibration data for subsequent evaluations).

339 4.3. Generalization setting

340 Next, we consider the case in which the data we have is re-341 lated, but not drawn from the same distribution as our even-342 tual application. To emulate this setting, we apply the super-343 vised and consistent Qwen 32B probes, developed on the s1K-1.1 dataset, to common reasoning benchmarks (Rein 345 et al., 2024; Lightman et al., 2023). Overall, we are able to improve (AIME 24, GPQA) or match (MATH 500) the effi-347 ciency of the budget forcing baseline - even achieving slight 348 gains in performance on AIME 24, perhaps by trimming 349 distracting thoughts (Figure 4). Notably, even though the 350 Supervised probe had access to more information (ground 351 truth answers), the Consistent probe consistently gener-352 alizes better, both in terms of efficiency and calibration. 353 Here, the Consistent probe fulfills the theoretical guarantees, 354 while the Supervised probe remains over-confident. 355

357 4.4. Additional analysis

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358 Figure 4 illustrates that thought calibration probes prioritizes 359 the termination of problems which cannot be solved, even 360 at full budget - perhaps hinting that the language model 361 may have been stuck in a cycle of reasoning, without novel 362 progress. Compared to the naive cropping strategy, thought 363 calibration's input-dependent decision also demonstrate sig-364 nificant variance in the amount of tokens across different 365 problems.

We also examine a specific instance from our s1K-1.1 testing 367 split in Figure 5 (s1K is a distillation dataset, so this diagram does not leak real test examples). The language model 369 reaches the correct answer after 38 steps (out of 48 steps). 370 As the model backtracks, the predicted consistency (with 371 the expected final answer) drops; and as the model returns to the answer, confidence increases, higher than before. This reaffirms that self-consistency is indeed a powerful indica-374 tion of correctness, both distilled into a predictive model, 375 and over the course of sampling. 376

5. Limitations

There are several limitations of our work. Since our method is built atop the Learn then Test framework (Angelopoulos et al., 2021), our theoretical guarantees are only valid over draws of the calibration set. In practice, this means that the calibration data must be sufficiently similar to the actual application. Furthermore, due to our small training and calibration datasets, we implement our framework primarily through linear probes. In Appendix B.1, we found that more complex architectures may lead to slightly better performance in some cases, and the gap is expected to be larger if more training data can be gathered. We leave further investigations regarding the probe architecture to future work. Finally, this paper only addresses the problem of exiting early from reasoning. The broader question of how to calibrate the *steering* of reasoning models remains unanswered, and is an interesting area for further research.



Figure 5: DeepSeek-R1 distilled Llama 70B Consistency probe on s1K-1.1 example from our test split, where color intensity is proportional to $\mathbb{P}(\text{consistent})$. The language model first reaches the correct answer in Step 38, backtracks with lower confidence, and returns to the answer in Step 41.

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	g prompt was used to force the model (DeepSeek-R1 distilled Qwen 32B and Llama 70B, QwQ 32B) to producter a fixed number of thinking steps. Following the recommendation of Guo et al. (2025a) and Team (2025), which a system prompt. We apply the chat template to user prompt before concatenating the "in-progress" though m (Muennighoff et al., 2025).
<bos><user2< th=""><th>></th></user2<></bos>	>
{question}	
Please reason <assistant></assistant>	step by step, and put your final answer within \\boxed{{}}.
<think></think>	
{thoughts}	
Final Answer:	
	ag prompt was used to obtain labels for $\mathbb{P}(\text{correct})$ (Equation 6) using Qwen 3 32B. This prompt was also us nswers using GPT 4.1. Adapted from (Muennighoff et al., 2025).
attempt. You there should h wrong. If the is correct, con	I assistant for grading a science problem. The user will provide you with the question itself, the correct answer, and the student r job is to judge whether the attempt is correct by comparing it with the correct answer. If the correct answer is a number or choic be no ambiguity, and you should directly compare the answer and the final result. If the attempt is incomplete, you should mark it correct answer involves going through the entire reasoning process, you should judge the result based on whether the reasoning proce- mpared to correct answer.
-	to solve the problem yourself. Only grade the attempt based on the correct answer. provide the attempt and the correct answer in the following format:
# Problem	provide the attempt and the correct answer in the following format.
{problem}	
## Correct a	nswer
{solution}	
## Student a	lttempt
{attempt}	reasoning concisely, and end your response on a new line with only "Yes" or "No" (without quotes).
judge whether should directl the same conc	i assistant for grading a science problem. The user will provide you with the question itself and two student attempts. Your job is to r the two students arrive at the same answer. If question asks for a single numerical answer, there should be no ambiguity, and you y compare the two answers. If the question asks for multiple parts, the two attempts are identical if only if all of the parts arrive a clusion. To solve the problem yourself. Only grade whether the two attempts are the same.
The user will # Problem	provide the problem and two attempts in the following format:
# Problem {problem}	
<pre># Problem {problem} ## Attempt</pre>	
<pre># Problem {problem} ## Attempt {attempt1}</pre>	1
<pre># Problem {problem} ## Attempt</pre>	
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<pre># Problem {problem} ## Attempt {attempt1} ## Attempt {attempt2} Explain your</pre>	1 2
<pre># Problem {problem} ## Attempt {attempt1} ## Attempt2} Explain your The followin You are an Ab</pre>	1 2 reasoning concisely, and end your response on a new line with only "Yes" or "No" (without quotes). ng prompt was used to obtain labels for $\mathbb{P}(\text{leaf})$ (Equation 10) using Qwen 3 32B. I assistant for parsing LLM outputs. The user will provide you with the question and an intermediate reasoning step. Your job is
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<pre># Problem {problem} ## Attempt {attempt1} ## Attempt2} Explain your The followin You are an Al judge whether Do NOT attemp </pre>	1 2 reasoning concisely, and end your response on a new line with only "Yes" or "No" (without quotes). Ing prompt was used to obtain labels for $\mathbb{P}(\text{leaf})$ (Equation 10) using Qwen 3 32B. It assistant for parsing LLM outputs. The user will provide you with the question and an intermediate reasoning step. Your job is the given step contains an attempt at a final answer.
<pre># Problem { problem } ## Attempt { attempt1 } ## Attempt { attempt2 Explain your The followin You are an At judge whether Do NOT attem The user will</pre>	1 2 reasoning concisely, and end your response on a new line with only "Yes" or "No" (without quotes). Ing prompt was used to obtain labels for $\mathbb{P}(\text{leaf})$ (Equation 10) using Qwen 3 32B. It assistant for parsing LLM outputs. The user will provide you with the question and an intermediate reasoning step. Your job is the given step contains an attempt at a final answer. mpt to solve the problem yourself. It does not matter if the answer is correct. Only comment on whether an attempt has been mad
<pre># Problem { problem } ## Attempt { attempt1 } ## Attempt { attempt2 Explain your } The followin You are an Ab judge whether Do NOT attem The user will # Problem</pre>	1 2 reasoning concisely, and end your response on a new line with only "Yes" or "No" (without quotes). Ing prompt was used to obtain labels for $\mathbb{P}(\text{leaf})$ (Equation 10) using Qwen 3 32B. It assistant for parsing LLM outputs. The user will provide you with the question and an intermediate reasoning step. Your job is the given step contains an attempt at a final answer. mpt to solve the problem yourself. It does not matter if the answer is correct. Only comment on whether an attempt has been mad provide the problem and reasoning steps in the following format:
<pre># Problem { problem} ## Attempt { attempt1} ## Attempt { attempt2} Explain your : The followin You are an Ai judge whether Do NOT attem The user will # Problem { problem}</pre>	1 2 reasoning concisely, and end your response on a new line with only "Yes" or "No" (without quotes). Ing prompt was used to obtain labels for $\mathbb{P}(\text{leaf})$ (Equation 10) using Qwen 3 32B. If assistant for parsing LLM outputs. The user will provide you with the question and an intermediate reasoning step. Your job is the given step contains an attempt at a final answer. mpt to solve the problem yourself. It does not matter if the answer is correct. Only comment on whether an attempt has been mad provide the problem and reasoning steps in the following format: step
<pre># Problem { problem} ## Attempt { attempt1} ## Attempt { attempt2} Explain your : The followin You are an Ai judge whether Do NOT attem The user will # Problem { problem} # Reasoning st { reasoning st }</pre>	<pre>1 2 reasoning concisely, and end your response on a new line with only "Yes" or "No" (without quotes). ng prompt was used to obtain labels for P(leaf) (Equation 10) using Qwen 3 32B. [assistant for parsing LLM outputs. The user will provide you with the question and an intermediate reasoning step. Your job is to the given step contains an attempt at a final answer. mpt to solve the problem yourself. It does not matter if the answer is correct. Only comment on whether an attempt has been mad provide the problem and reasoning steps in the following format: step p} p? reasoning, and end your response on a new line with only "Yes" or "No" indicating whether or the given step makes an attempt of the problem step makes an attempt of the problem step makes an attempt of the problem step contains and reasoning steps in the following format:</pre>
<pre># Problem { problem} ## Attempt { attempt1} ## Attempt { attempt2} Explain your : The followin You are an Ai judge whether Do NOT atten The user will # Problem { problem} # Reasoning ste Explain your providing the</pre>	<pre>1 2 reasoning concisely, and end your response on a new line with only "Yes" or "No" (without quotes). ng prompt was used to obtain labels for P(leaf) (Equation 10) using Qwen 3 32B. [assistant for parsing LLM outputs. The user will provide you with the question and an intermediate reasoning step. Your job is to the given step contains an attempt at a final answer. mpt to solve the problem yourself. It does not matter if the answer is correct. Only comment on whether an attempt has been made provide the problem and reasoning steps in the following format: step p} p} reasoning, and end your response on a new line with only "Yes" or "No" indicating whether or the given step makes an attempt at a stempt of the problem step contains an attempt on the problem and reasoning steps in the following format:</pre>
<pre># Problem { problem} ## Attempt { attempt1} ## Attempt { attempt2} Explain your : The followin You are an Ai judge whether Do NOT atten The user will # Problem { problem} # Reasoning ste Explain your providing the</pre>	<pre>1 2 reasoning concisely, and end your response on a new line with only "Yes" or "No" (without quotes). 1 1 2 reasoning concisely, and end your response on a new line with only "Yes" or "No" (without quotes). 1 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1 2</pre>

- 660 You are an AI assistant for assessing the quality of logical reasoning. The user will provide you with the question and an incomplete attempt, 661 consisting of a series of reasoning steps. Your job is to judge whether current step appears to provide additional information, compared to the previous steps. If the current step is correct and novel, it is useful. If the current step is wrong or redundant, then it is not useful.
- 662 Do NOT try to solve the problem yourself. It does not matter if the attempt is not complete. Only comment on whether the current step is useful. 663 The user will provide the problem and reasoning steps in the following format:
- # Problem 664 {problem} 665 # Reasoning 666 ## step 1 667 {reasoning step 1} 668 ## step 2 {reasoning step 2} 669 670 ## step k 671 {reasoning step k} 672

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current step

{current reasoning step}

Explain your reasoning, and end your response on a new line with only "Yes" if the current step provides new information or "No" otherwise (without quotes).

B. Implementation details

B.1. Design and implementation of model probes

We tried several architectures, before deciding upon linear probes for simplicity and to avoid overfitting. The differences in performance are not always consistent and the generalization gap is quite large (Table 1). Since our main focus is on calibration, and it requires significant compute to produce and evaluate scaling curves, we consider more exhaustive exploration of alternate architectures as future work.

MLP The input is a single representation $h^{(t)}$ corresponding to single reasoning step $y^{(t)}$, and the output is a binary label $\in \{0, 1\}$. We train until AUC fails to improve for 10 epochs on 10% of the training set (randomly sampled). We report the best calibration set performance of the following hyperparameters. We use the sklearn defaults otherwise (Pedregosa et al., 2011).

• Layers: 1, 2

• FFN dimension: 32, 64, 128

Transformer The input is a sequence of representations, $h^{(1)} ldots h^{(t)}$ corresponding to thoughts $y_t = y^{(1)} ldots y^{(t)}$. The output is either a binary label $\in \{0, 1\}$ for $\mathbb{P}(\text{correct})$ and $\mathbb{P}(\text{consistent})$, or a sequence of labels $\in \{0, 1\}^t$ for $\mathbb{P}(\text{novel})$ and $\mathbb{P}(\text{leaf})$. For the former, we treat the embeddings as a set (i.e. if *any* representation is sufficient to answer correctly, or be consistent). For the latter, we apply a left-to-right causal attention mask during training, and we use sinusoidal positional encodings to encode the index of each reasoning step. We report the best calibration set performance of the following hyperparameters. In contrast to the linear and MLP models, we find that the Transformer performs best if we *do not* apply PCA and instead operate over the original model dimension.

- Layers: 1, 2
- Model dimension: 16, 32, 64
- FFN dimension: 64, 128
- Number of heads: 4, 8
 - Epochs: 5, 10
- 713 714

Table 1: Probe architecture performance on s1K-1.1 train and calibration splits. Metric: Binary AUROC.

	Quantity	Linear		MLP		Transformer	
Model		Train	Cal	Train	Cal	Train	Cal
DeepSeek-R1	$\mathbb{P}(\text{correct})$	0.936 0.919	$0.788 \\ 0.788$	0.990 0.994	0.779 0.747	0.994 0.991	0.760
distilled Qwen 2.5 32B	<pre>P(consistent) P(leaf) P(novel)</pre>	0.919 0.868 0.874	0.788 0.839 0.686	0.994 0.936 0.980	0.747 0.815 0.692	0.991 0.933 0.896	0.773 0.852 0.774
DeepSeek-R1 distilled Llama 3.3 70B	P(correct) P(consistent) P(leaf) P(novel)	0.937 0.921 0.864 0.872	0.765 0.745 0.819 0.686	0.987 0.994 0.970 0.981	0.746 0.743 0.802 0.702	0.991 0.993 0.923 0.915	0.803 0.748 0.848 0.774
QwQ 32B	$ \begin{array}{c} \mathbb{P}(\text{correct}) \\ \mathbb{P}(\text{consistent}) \\ \mathbb{P}(\text{leaf}) \\ \mathbb{P}(\text{novel}) \end{array} $	0.943 0.950 0.869 0.876	0.848 0.699 0.840 0.677	0.986 0.988 0.942 0.952	0.838 0.704 0.822 0.690	0.948 0.939 0.913 0.895	0.848 0.756 0.857 0.792

B.2. LLM experiments

We ran DeepSeek-R1 distilled Qwen 2.5 32B and Llama 70B, and QwQ 32B using Imdeploy (Contributors, 2023) with recommended defaults for each model. Imdeploy natively supports the saving of last layer representations, so it was used for almost all experiments. We ran Qwen 3 32B using vLLM (Kwon et al., 2023) due to early support. Due to computational constraints, we report the mean over a single run.

We downloaded all model weights from transformers between April 1, 2025 and May 1, 2025.

C. Additional analysis

Figure 6 illustrates the early exit probabilities for each of the three probes. The supervised ("correct") probe reaches high exit probabilities the fastest, but it is also the most overconfident (Figure 2D).



Figure 6: Likelihoods of thought calibration probes over s1K-1.1 test set (10 examples). The "No Leaf" variant is the least monotonic. This could potentially indicate that after reaching the answer, the language model explores new knowledge that is irrelevant to the task.