FAIR REPRESENTATION LEARNING THROUGH IMPLICIT PATH ALIGNMENT

Anonymous authors
Paper under double-blind review

ABSTRACT

We considered a fair representation learning perspective, where optimal predictors, on top of the data representation, are ensured to be invariant with respect to different subgroups. Specifically, we formulated the problem as a bi-level optimization, where the representation is learned in the outer-level, and invariant optimal group predictors are updated in the inner-level. To avoid the high computational and memory cost of differentiating in the inner-level optimization, we proposed the implicit path alignment algorithm, which only relies on the solution of inner optimization and the implicit differentiation rather than the exact optimization path. Moreover, the proposed bi-level objective is demonstrated to fulfill the *sufficiency rule*, which is desirable in various practical scenarios but was not commonly studied in fair representation learning. We further analyzed the error gap of the implicit approach and empirically validated the proposed method in both classification and regression settings. Experimental results show the consistently better trade-off in prediction performance and fairness measurement.

1 INTRODUCTION

Machine learning has been widely adopted in the real world decision-making practice such as job candidate screening (Raghavan et al., 2020) and credit application. However, it has been observed that learning algorithms treated some groups of population unfavorably, for example, denying credit on the grounds of gender, age or ethnicity (Hardt et al., 2016). To this end, algorithmic fairness that is to mitigate the *prediction bias* for different subgroups has recently received tremendous attentions.

With the rapid advancement of representation learning (LeCun et al., 2015), learning a fair embedding (Zemel et al., 2013) has been recently highlighted. Specifically, the learned fair representation can easily transfer the unbiased prior knowledge to the downstream tasks, with various successful applications in computer vision (Kim et al., 2019; Kehrenberg et al., 2020), language understanding (Chang et al., 2019; Ethayarajh, 2020) and artificial intelligence for health (Fletcher et al., 2021). Typically, the fair representation learning is achieved by adding various statistical fair metrics during the training process.

Based on this, most existing fair representation approaches in classification or regression principally aim to meet the *independence* or *separation* rule, e.g., (Madras et al., 2018; Song et al., 2019; Chzhen et al., 2020). However, in various real-world scenarios, the *sufficiency rule* is preferable. For example, health systems rely on commercial algorithms to identify and help patients with complex health needs. The algorithm outputs a healthcare need score, where a higher score indicates the patient is sicker and requires more healthcare. Obermeyer et al. (2019) revealed that a widely used algorithm, typical of this industry-wide approach and affecting millions of patients, exhibits significant racial bias. At a given predicted healthcare need score $\hat{Y} = t$, Black patients are considerably sicker than White patients ($\mathbb{E}_{\text{black}}[Y|\hat{Y} = t] > \mathbb{E}_{\text{white}}[Y|\hat{Y} = t]$). Obermeyer et al. (2019) also pointed out that remedying the disparity would increase the percentage of Black patients receiv-

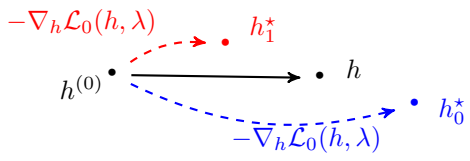
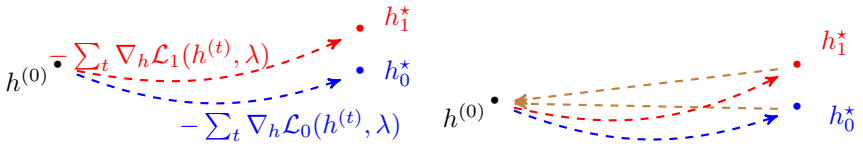


Figure 1: Unfair representation leads to different optimization path and non-invariant optimal predictors on the latent space $\mathcal{Z}$.



(a) Explicit Path (b) Implicit Path

Figure 2: Explicit and Implicit path alignment. (a) The considered fair representation learning criteria lies in ensuring the invariant optimal predictor w.r.t. different subgroups on $\mathcal{Z}$ ($h_0^\star = h_1^\star$). Since the gradient based approach is adopted to optimize h, the explicit path alignment aims to learn a representation λ to enforce the identical *optimization path* w.r.t. h. (b) The proposed implicit path alignment only requires the last iteration point and approximate the gradient w.r.t. λ from the last update of h (the brown arrow).

ing additional healthcare from 17.7 to 46.5%. Moreover, it has been theoretically justified (Barocas et al., 2019) that the Sufficiency rule is generally not compatible with Independence and Separation. Thus learning the fair representation w.r.t. the sufficiency rule is promising in both the algorithmic design and real-world applications.

In this paper, we address the sufficiency rule by considering the following intuition: given a fixed representation function, if the *optimal* predictor that learned on the embedding space are *invariant* from different sub-groups, then the corresponding representation function is fair. Fig. 1 provides an illustrative example. when the representation function $\lambda : \mathcal{X} \to \mathcal{Z}$ is unfair and we adopt gradient descent to learn the predictor $h : \mathcal{Z} \to R$. The optimal predictors of different subgroups (blue, red) are not invariant, resulting in biased predictions. We will later demonstrate such an intuition ensures the learned representation satisfying the sufficiency rule (Liu et al., 2019; Chouldechova, 2017).

The aforementioned intuition can be naturally formulated as a bi-level optimization problem, where we aim to adjust the representation λ (in the outer-level) to satisfy the invariant optimal predictor h (in the inner-level). Thus, when we adopt the gradient-based approach in solving the bi-level objective, a straightforward solution is to learn the representation λ to fulfill the identical *explicit* gradient-descent directions in learning predictor $h^\star$ of different groups, shown in Fig. 2(a). Intuitively, if the inner gradient descent step of each sub-group is identical, their final predictors (as the approximation of $h^\star$) will be invariant. However, the corresponding algorithmic realization is challenging in deep learning: 1) It requires storing the whole gradient steps, which induces a high memory burden. 2) the embedding function λ is optimized via backpropagation from the whole gradient optimization path, which induces a high computational complexity.

To this end, we propose an *implicit* path alignment, shown in Fig. 2(b). Notably, we only consider the final (t-th) update of the predictor $h^{(t)}$, then we update representation function λ by approximating its gradient at point $h^{(t)}$ through the implicit function (Bengio, 2000). By using the gradient approximation, it is no more required to store the whole gradient step and conduct the backpropagation through the entire path. Overall, the highlights in this paper are as follows:

Fair-representation learning to satisfy the sufficiency rule Instead of enforcing the independence or separation rule, the considered fair-representation criteria is proved to satisfy the sufficiency rule in both classification and regression. We also find such a criteria is intrinsically consistent with the recent Invariant Risk Minimization (IRM) (Arjovsky et al., 2019; Bühlmann, 2020), which aims to eliminate suspicious correlations while keeping robust correlations that are invariant across different environments. Intuitively, reducing the correlation w.r.t. the protected attributes enables the fair representation.

Principled and efficient algorithm We proposed a novel implicit path alignment algorithm to learn the fair representation, which addressed the prohibitive memory and computational cost in the original bi-level objective. Besides, we analyzed the approximation error gap of the proposed implicit algorithm, which induces a trade-off between the correct gradient estimation and fairness measures.

Improved fairness in classification and regression We evaluated the implicit algorithm in both classification and regression with tabular, computer vision and NLP datasets. Compared to the baselines, the implicit algorithm effectively improved the fairness with a smaller sufficiency gap.

2 PRELIMINARIES

We suppose the input $X \in \mathcal{X}$, the ground truth label $Y \in \mathcal{Y}$, and the algorithmic output $\hat{Y} \in \mathcal{Y}$. Throughout the paper, we only consider binary sensitive attribute (i.e, two sub-groups) with distributions $\mathcal{D}_0$ and $\mathcal{D}_1$. Then based on (Liu et al., 2019), the **sufficiency rule** is defined as:

$$\mathbb{E}_{\mathcal{D}_0}[Y|\hat{Y}=t] = \mathbb{E}_{\mathcal{D}_1}[Y|\hat{Y}=t], \ \ \forall t \in \mathcal{Y} \tag{1}$$

To measure the fairness w.r.t. the sufficiency rule, we propose the *sufficiency gap* as the metric. Since we aim to evaluate the fairness in both binary classification ($Y \in \{-1, 1\}$) and regression ($Y \in \mathbb{R}$), the metric is separately defined on these two scenarios.

Sufficiency gap in binary classification Based on the sufficiency rule, the sufficiency gap in binary classification is naturally defined as:

$$\Delta\text{Suf}_C = \sum_{y\in\{-1,1\}} |\mathcal{D}_0(Y=y|\hat{Y}=y) - \mathcal{D}_1(Y=y|\hat{Y}=y)| \tag{2}$$

ΔSuf_C encourages the two subgroups with identical Positive predicted value (PPV) and Negative predicted value (NPV). On the practical side, considering the healthcare evaluation system outputs either *High Risk* or *Low Risk*, Obermeyer et al. (2019) essentially revealed $\mathcal{D}_{\text{black}}(Y = \text{High Risk}|\hat{Y} = \text{Low Risk}) > \mathcal{D}_{\text{white}}(Y = \text{High Risk}|\hat{Y} = \text{Low Risk})$: the severity of Black patients is actually underestimated. Thus if ΔSuf_C is small, the racial discrimination can be remedied.

Sufficiency gap in regression Based on the sufficiency rule and (Kuleshov et al., 2018), the sufficiency gap in regression is defined as:

$$\Delta\text{Suf}_R = \int_{t\in\mathcal{Y}} |\mathcal{D}_0(Y \le t|\hat{Y} \le t) - \mathcal{D}_1(Y \le t|\hat{Y} \le t)|dt \tag{3}$$

$\Delta\text{Suf}_R \in [0, 1]$ is an approximation of $|\mathcal{D}_0(Y=y|\hat{Y}=y) - \mathcal{D}_1(Y=y|\hat{Y}=y)|$, $\forall y \in \mathbb{R}$, since the latter is difficult to estimate. From the practical aspect, assuming the health system outputs a *real-value* healthcare score $\hat{Y} = t$ (higher indicates sicker), Obermeyer et al. (2019); Sjoding et al. (2020) observed $\mathcal{D}_{\text{black}}(Y > t|\hat{Y} \le t) > \mathcal{D}_{\text{white}}(Y > t|\hat{Y} \le t)$: for the patients whose predicted healthcare score is less than t, the actual proportion of sicker ($Y > t$) in Black patients is considerably higher than White patients. Therefore a small ΔSuf_R suggests an improved disparity.

3 PROBLEM SETUP

We denote the representation function λ that maps the input X into the latent variable Z, the prediction function h such that $h : \mathcal{Z} \to \mathbb{R}$ for regression and $h : \mathcal{Z} \to \{-1, 1\}$ for binary classification. We then denote the prediction loss as ℓ, the prediction loss on subgroup $\mathcal{D}_0, \mathcal{D}_1$ is expressed as:

$$\mathcal{L}_0(h, \lambda) = \mathbb{E}_{(x,y)\sim\mathcal{D}_0}\ell(h \circ \lambda(x), y), \quad \mathcal{L}_1(h, \lambda) = \mathbb{E}_{(x,y)\sim\mathcal{D}_1}\ell(h \circ \lambda(x), y)$$

According to the intuition, we aim to solve the following bi-level objective:

$$\min_{\lambda} \ \mathcal{L}_0(h_0^\star, \lambda) + \mathcal{L}_1(h_1^\star, \lambda) \qquad \text{(Outer level)}$$

$$\text{s.t.} \ \ h_0^\star = h_1^\star, \ h_0^\star \in \operatorname*{argmin}_h \mathcal{L}_0(h, \lambda), \ h_1^\star \in \operatorname*{argmin}_h \mathcal{L}_1(h, \lambda). \qquad \text{(Inner level)}$$

Specifically, in the outer level, we aim to find a representation λ for minimizing the prediction error, given the optimal predictor $(h_0^\star, h_1^\star)$ on the embedding space $\mathcal{Z}$. As for the inner level, given a fixed representation λ, $h_0^\star$, $h_1^\star$ are the optimal predictor for each sub-group. The constraints $h_0^\star = h_1^\star$ additionally encourage the invariant optimal predictors from $\mathcal{D}_0$, $\mathcal{D}_1$.

Relation to the explicit path alignment In deep learning we adopt the gradient-based approach to minimize the loss, therefore $h^\star$ in the inner level is approximated as $h^{(t+1)}$, the t-th update in the gradient descent: $h_0^\star \approx h^{(0)} - \sum_t \nabla_h \mathcal{L}_0(h^{(t)}, \lambda)$, $h_1^\star \approx h^{(0)} - \sum_t \nabla_h \mathcal{L}_1(h^{(t)}, \lambda)$, where $h^{(0)}$ is the common initialization. Thus the invariant optimal predictor is equivalent to:

$$\sum_t \nabla_h \mathcal{L}_0(h^{(t)}, \lambda) = \sum_t \nabla_h \mathcal{L}_1(h^{(t)}, \lambda).$$

The aforementioned equation suggests learning a representation λ that ensures the identical optimization path w.r.t. h for each sub-group, which recovers the explicit path alignment.

Relation to the Sufficiency rule We further demonstrate the relation between the bi-level objective and Sufficiency rule.

Proposition 1. *If we specify the prediction loss ℓ as logistic regression loss in the classification $\log(1+\exp(-yh(z)))$ with $\mathcal{Y}=\{-1,1\}$ and the square loss in the regression $(h(z)-y)^2$ with $\mathcal{Y}\subset\mathbb{R}$. Then minimizing the inner-level loss is equivalent to:*

$$\mathbb{E}_{\mathcal{D}_0}[Y|Z=z]=\mathbb{E}_{\mathcal{D}_1}[Y|Z=z],\quad \mathbb{E}_{\mathcal{D}_0}[Y|\hat{Y}=h^\star(z)]=\mathbb{E}_{\mathcal{D}_1}[Y|\hat{Y}=h^\star(z)],$$

where $h^\star=h_0^\star=h_1^\star$ and $z=\lambda(x)$.

Proposition 1 reveals that the objective of inner-level loss is to encourage the sufficiency rule.

4 Practical Algorithms

In this section, we propose an implicit alignment in deep learning, where λ and h are implemented by the neural network. We also reformulate as the original objective through Lagrangian relaxation:

$$\min_{\lambda}\ \mathcal{L}_0(h_0^\star,\lambda)+\mathcal{L}_1(h_1^\star,\lambda)+\frac{\kappa}{2}\|h_0^\star-h_1^\star\|_2^2 \qquad \text{(Outer level)}$$

$$\text{s.t. } h_0^\star\in\operatorname*{argmin}_h \mathcal{L}_0(h,\lambda),\ h_1^\star\in\operatorname*{argmin}_h\mathcal{L}_1(h,\lambda), \qquad \text{(Inner level)}$$

where $\kappa>0$ is the coefficient to control the fairness. Then we will drive the approximated gradient w.r.t. λ, which contains the following key elements.

Solving the inner optimization Given a fixed representation λ, we find h_0^ϵ, h_1^ϵ such that:

$$\|h_0^\star-h_0^\epsilon\|\le\epsilon,\quad \|h_1^\star-h_1^\epsilon\|\le\epsilon,$$

where ϵ is the optimization tolerance. Besides, $h_1^\star$ and h_1^ϵ are essentially the function of λ, i.e., h_1^ϵ depends on the predefined representation function λ.

Computing the gradient of λ Given the approximate solution h_0^ϵ, h_1^ϵ, we can compute the gradient w.r.t. λ (referred as $\tilde{\text{grad}}(\lambda)$) [1] in the outer-level:

$$\begin{aligned}\tilde{\text{grad}}(\lambda)=&\nabla_\lambda\mathcal{L}_0(h_0^\epsilon,\lambda)+\left(\nabla_\lambda h_0^\epsilon\right)^T\left(\nabla_{h_0}\mathcal{L}_0(h_0^\epsilon,\lambda)+\kappa(h_0^\epsilon-h_1^\epsilon)\right)\\&+\nabla_\lambda\mathcal{L}_1(h_1^\epsilon,\lambda)+\left(\nabla_\lambda h_1^\epsilon\right)^T\left(\nabla_{h_1}\mathcal{L}_1(h_1^\epsilon,\lambda)-\kappa(h_0^\epsilon-h_1^\epsilon)\right).\end{aligned}$$

Where $\nabla_{h_0}\mathcal{L}_0(h_0^\epsilon,\lambda)$ is the partial derivative in the loss w.r.t. the first term (about h_0), evaluated at h_0^ϵ. Also $\nabla_\lambda\mathcal{L}_0(h_0^\epsilon,\lambda)$ is the partial derivative w.r.t. the second term (about λ).

Implicit function for approximating the gradient In order to compute $\tilde{\text{grad}}(\lambda)$ in `autograd`, we need to estimate $\nabla_\lambda h_0^\epsilon$ and $\nabla_\lambda h_1^\epsilon$. We herein adopt the implicit function (Bengio, 2000) to approximate $\nabla_\lambda h_0^\epsilon$, which has been adopted in the hyperparameter optimization (Pedregosa, 2016) and meta-learning (Rajeswaran et al., 2019).

Concretely, if the prediction loss is smooth and there exist stationary points to achieve optimal, we have: $\nabla_{h_0}\mathcal{L}_0(h_0^\star(\lambda),\lambda)=0,\nabla_{h_1}\mathcal{L}_0(h_1^\star(\lambda),\lambda)=0$. Then differentiating w.r.t. λ will induce: $\mathbf{d}\left(\nabla_{h_0}\mathcal{L}_0(h_0^\star(\lambda),\lambda)\right)/\mathbf{d}\lambda=\nabla^2_{h_0}\mathcal{L}_0(h_0^\star,\lambda)\nabla_\lambda h_0^\star+\nabla_\lambda\nabla_{h_0}\mathcal{L}_0(h_0^\star,\lambda)=0$.[2] Thus we have $\nabla_\lambda h_0^\star=-\left(\nabla^2_{h_0}\mathcal{L}_0(h_0^*,\lambda)\right)^{-1}\left(\nabla_\lambda\nabla_{h_0}\mathcal{L}_0(h_0^*,\lambda)\right)$, where the Hessian matrix $\nabla^2_{h_0}\mathcal{L}_0(h_0^*,\lambda)$ is assumed to be invertible.

Through the implicit function, we can approximate $\nabla_\lambda h_0^\epsilon$ as:

$$\nabla_\lambda h_0^\epsilon\approx-\left(\nabla^2_{h_0}\mathcal{L}_0(h_0^\epsilon,\lambda)\right)^{-1}\left(\nabla_\lambda\nabla_{h_0}\mathcal{L}_0(h_0^\epsilon,\lambda)\right)$$

As for $\nabla_\lambda h_1^\epsilon$, we have the similar result: $\nabla_\lambda h_1^\epsilon\approx-\left(\nabla^2_{h_1}\mathcal{L}_1(h_1^\epsilon,\lambda)\right)^{-1}\left(\nabla_\lambda\nabla_{h_1}\mathcal{L}_1(h_1^\epsilon,\lambda)\right)$.

[1] We denote the ground truth gradient as $\text{grad}(\lambda)$ if we adopt optimal predictor $h_0^\star,h_1^\star$ in the computation.

[2] $\mathbf{d}(\cdot)/\mathbf{d}\lambda$ denotes the total derivative.

Algorithm 1 Implicit Path Alignment Algorithm

Ensure: Representation function λ, predictor h_0, h_1, datasets from two sub-groups $\mathcal{D}_0$. $\mathcal{D}_1$.
1: **for** mini-batch of samples from $(\mathcal{D}_0, \mathcal{D}_1)$ **do**
2: Solving the inner-level optimization with tolerance ϵ. Obtaining $h_0^\epsilon, h_1^\epsilon$.
3: Solving Eq. (4) with tolerance δ. Obtaining $\mathbf{p}_0^\delta$ and $\mathbf{p}_1^\delta$.
4: Computing $\tilde{\text{grad}}^\delta(\lambda)$ (gradient of representation λ)
5: Updating λ through `autograd`: $\lambda \leftarrow \lambda - \tilde{\text{grad}}^\delta(\lambda)$
6: **end for**
7: **return** λ, h_0^ϵ, h_1^ϵ

Efficient and numerical stable gradient estimation Plugging in the approximations, the gradient w.r.t λ is approximated as:

$$\tilde{\text{grad}}(\lambda) \approx \nabla_\lambda \mathcal{L}_0(h_0^\epsilon, \lambda) - (\nabla_\lambda \nabla_{h_0} \mathcal{L}_0(h_0^\epsilon, \lambda))^T \underbrace{\left(\nabla^2_{h_0} \mathcal{L}_0(h_0^\epsilon, \lambda)\right)^{-1} (\nabla_{h_0} \mathcal{L}_0(h_0^\epsilon, \lambda) + \kappa(h_0^\epsilon - h_1^\epsilon))}_{\mathbf{p}_0}$$
$$+ \nabla_\lambda \mathcal{L}_1(h_1^\epsilon, \lambda) - (\nabla_\lambda \nabla_{h_1} \mathcal{L}_1(h_1^\epsilon, \lambda))^T \underbrace{\left(\nabla^2_{h_1} \mathcal{L}_1(h_1^\epsilon, \lambda)\right)^{-1} (\nabla_{h_1} \mathcal{L}_1(h_1^\epsilon, \lambda) - \kappa(h_0^\epsilon - h_1^\epsilon))}_{\mathbf{p}_1}$$

However, the current form is still computationally expensive due to the computation of inverse Hessian matrix. To this end, we denote $\mathbf{p}_0$ and $\mathbf{p}_1$ as the inverse-Hessian vector product. Then computing $\mathbf{p}_0$ and $\mathbf{p}_1$ is equivalent to solve the following quadratic programming (QP):

$$\begin{aligned} &\text{argmin}_{\hat{\mathbf{p}}_0} \frac{1}{2} \hat{\mathbf{p}}_0^T \left(\nabla^2_{h_0} \mathcal{L}_0(h_0^\epsilon, \lambda)\right) \hat{\mathbf{p}}_0 - \hat{\mathbf{p}}_0^T \left(\nabla_{h_0} \mathcal{L}_0(h_0^\epsilon, \lambda) + \kappa(h_0^\epsilon - h_1^\epsilon)\right) \\ &\text{argmin}_{\hat{\mathbf{p}}_1} \frac{1}{2} \hat{\mathbf{p}}_1^T \left(\nabla^2_{h_1} \mathcal{L}_1(h_1^\epsilon, \lambda)\right) \hat{\mathbf{p}}_1 - \hat{\mathbf{p}}_1^T \left(\nabla_{h_1} \mathcal{L}_1(h_1^\epsilon, \lambda) - \kappa(h_0^\epsilon - h_1^\epsilon)\right) \end{aligned} \tag{4}$$

Since it is a typical QP problem and we adopt conjugate gradient method (Concus et al., 1985; Rajeswaran et al., 2019), which can be updated efficiently through `autograd` via computing the Hessian-vector product. We additionally suppose the optimization error in the QP as δ, i.e.: $\|\mathbf{p}_0 - \mathbf{p}_0^\delta\| \leq \delta$, $\|\mathbf{p}_1 - \mathbf{p}_1^\delta\| \leq \delta$, then the gradient w.r.t representation λ can be finally expressed as:

$$\tilde{\text{grad}}^\delta(\lambda) = \nabla_\lambda \mathcal{L}_0(h_0^\epsilon, \lambda) - (\nabla_\lambda \nabla_{h_0} \mathcal{L}_0(h_0^\epsilon, \lambda))^T \mathbf{p}_0^\delta + \nabla_\lambda \mathcal{L}_1(h_1^\epsilon, \lambda) - (\nabla_\lambda \nabla_{h_1} \mathcal{L}_1(h_1^\epsilon, \lambda))^T \mathbf{p}_1^\delta$$

The $\tilde{\text{grad}}^\delta(\lambda)$ can be also efficiently estimated through Hessian vector product via `autograd` without explicitly computing the Hessian matrix.

Proposed algorithm Based on the key elements, the proposed algorithm is shown in Algo. 1.

4.1 The cost of Implicit algorithm: Approximation-Fair Trade-Off

Theorem 1 (Approximation Error Gap)**.** *Suppose that (1)* Smooth Predictive Loss*. The first-order derivatives and second-order derivatives of $\mathcal{L}$ are Lipschitz continuous; (2)* Non-singular Hessian matrix*. We assume $\nabla_{h_0,h_0} \mathcal{L}_0(h_0, \lambda), \nabla_{h_1,h_1} \mathcal{L}_1(h_1, \lambda)$, the Hessian matrix of the inner optimization problem, are invertible. (3)* Bounded representation and predictor function*. We assume the λ and h are bounded, i.e., $\|\lambda\|, \|h\|$ are upper bounded by the predefined positive constants. Then the approximation error between the ground truth and algorithmic estimated gradient w.r.t. the representation is be upper bounded by:*

$$\|grad(\lambda) - \tilde{grad}^\delta(\lambda)\| = \mathcal{O}(\kappa\epsilon + \epsilon + \delta).$$

The proof is delegated in Appendix B. We also discuss the assumptions to guarantee the convergence of Algorithm 1, shown in Appendix C.

Theorem 1 reveals that the gradient approximation error depends on the two-level optimization tolerance ϵ, δ and the coefficient of fair constraints κ. Specifically, the error gap reveals the inherent trade-off in accurate gradient estimation and fair-representation learning. If we fix the optimization tolerance ϵ and δ, a smaller κ indicates a better approximation of the gradient, which yields weak fair constraints. Thus the implicit alignment introduces a trade-off in the prediction performance (i.e., correct approximation of the gradient) and fairness measurement.

5 Related Work

Fair Machine Learning Below we only list the most related work in the *fairness* and refer to the survey paper (Mehrabi et al., 2021) for details in the algorithmic fairness. In the *classification*, various methods in learning fair representations have been proposed. Specifically, a common strategy is to introduce the statistical constraints as the regularization during the training, e.g., demographic parity (DP) (Zhang et al., 2018; Madras et al., 2018; Song et al., 2019; Jiang et al., 2020; Kehrenberg et al., 2020) or equalized odds (EO) (Song et al., 2019; Gupta et al., 2021) as the proxy of the separation and independence rule. Another direction is to disentangle the data for factorizing meaningful representations such as (Locatello et al., 2019). Intuitively, the disentangled embedding is independent of the sensitive attribution, thus reflecting a fair representation w.r.t. the independence rule, which can be potentially problematic when the label distributions of subgroups vary dramatically (Zhao et al., 2019).

Fairness has also been extended to the fields beyond classification. For instance, in the *regression* problem (Komiyama et al., 2018; Agarwal et al., 2019), the bounded group loss has been proposed as the fair measure: if prediction loss in each subgroup is smaller than ϵ, the regression is ϵ-level fair. In fact, the fair criteria in our paper is *not equivalent* to ϵ-fair. Given a fixed λ, the ϵ-level fair does not guarantee the *optimal* and *invariant* predictor for each subgroup and vice versa.

The sufficiency rule has also been discussed in the previous work. Notably, Chouldechova (2017); Liu et al. (2019) proposed the sufficiency gap *in classification* for measuring fairness w.r.t. the sufficiency rule. Liu et al. (2019) also discussed the inequivalence between the sufficiency gap and probabilistic calibration (Guo et al., 2017) (referred as calibration gap). According to Pleiss et al. (2017), the calibration rule is a stronger condition than sufficiency rule while it simultaneously hurts the prediction performance. Throughout this paper, we only consider the sufficiency rule. The triple trade-off between the calibration rule, sufficiency rule, and prediction performance will be left as future work.

Invariant Risk Minimization The analyzed fair-representation criteria shares a quite similar spirit to the IRM (Arjovsky et al., 2019; Bühlmann, 2020; Creager et al., 2021), where an algorithm IRM_v1 is proposed to enable the out-of-distribution (OOD) generalization. The key difference between our work and (Arjovsky et al., 2019) lies in the algorithmic aspect: it has been theoretically justified that the originally proposed IRM_v1 does not necessarily capture the invariance (Rosenfeld et al., 2020). By contrast, we directly solve the bi-level objective in the context of deep-learning and propose an efficient practical algorithm with better empirical performance than IRM_v1. Besides, based on Chen et al. (2021), the proposed algorithm does not provably guarantee the OOD generalization property due to the limited subgroups ($N = 2$) considered within the paper.

6 Experiments

6.1 Experimental setup

In the paper, we adopt the sufficiency gap as fair metrics, where $\hat{Y}$ is denoted as:

$$\hat{Y} = \begin{cases} h_0^\epsilon \circ \lambda(X), & X \in \mathcal{D}_0 \\ h_1^\epsilon \circ \lambda(X), & X \in \mathcal{D}_1 \end{cases}$$

Then in the binary classification, we can estimate $\Delta\text{Suf}_C = \sum_{y\in\{-1,+1\}} |\mathcal{D}_0(Y = y|\hat{Y} = y) - \mathcal{D}_1(Y = y|\hat{Y} = y)|$ from the data.

As for regression, the sufficiency gap $\Delta\text{Suf}_R = \int_t |\mathcal{D}_0(Y \le t|\hat{Y} \le t) - \mathcal{D}_1(Y \le t|\hat{Y} \le t)|$ (shown in Fig. 3, the orange region) is difficult to estimate due to the integration. To address this, we sample multiple values $\{t_1, \ldots, t_m\}$ and compute its average difference as the approximation of the integration. $\Delta\text{Suf}_R \approx \frac{1}{m}\sum_{i=1}^m |\mathcal{D}_0(Y \le t_i|\hat{Y} \le t_i) - \mathcal{D}_1(Y \le t_i|\hat{Y} \le t_i)|$

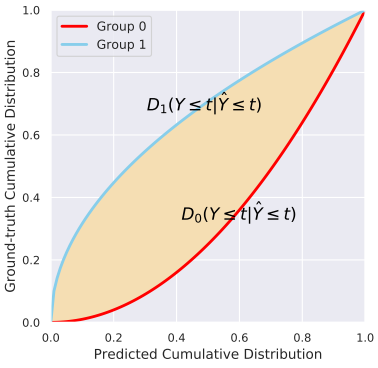


Figure 3: Sufficiency gap (ΔSuf_R) in regression

Method	Accuracy (↑)	ΔSuf_C (↓)
ERM (I)	0.768 ± 0.004	0.173 ± 0.008
Adv_debias (II)	0.760 ± 0.008	0.291 ± 0.006
Mixup (III)	0.758 ± 0.003	0.343 ± 0.022
IRM_v1 (IV)	0.753 ± 0.004	0.057 ± 0.015
One_step (V)	0.755 ± 0.007	0.048 ± 0.008
Implicit	0.760 ± 0.007	0.051 ± 0.012

Table 1: Toxic comments dataset. Accuracy and ΔSuf_C in different approaches.

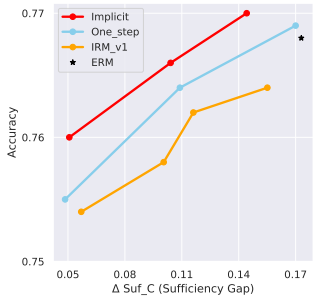


Figure 4: Toxic. Accuracy-Fair Trade-off

Concretely, for a given t_i in each group, we compute the percentile ($\hat{Y}_0$) at point t: $\mathcal{D}_0(\hat{Y}_0 \leq t_i)$, then we compute the corresponding ground truth cumulative distribution (Y) at the same point t_i: $\mathcal{D}(Y \leq t_i|\hat{Y} \leq t_i)$. Through the aforementioned approximation, we can compute $|\mathcal{D}_0(Y \leq t_i|\hat{Y} \leq t_i) - \mathcal{D}_1(Y \leq t_i|\hat{Y} \leq t_i)|$.

Baselines We consider the baselines that add fairness constraints during the training process. Specifically, we compare our method with (I) empirical risk minimization (ERM) that trains the model without considering fairness; (II) adversarial debiasing (Zhang et al., 2018); (III) fair mix-up (Chuang & Mroueh, 2021), a recent data-augmentation and effective approach in the fair representation learning. In fact, the baselines (II) and (III) are DP-based fair approaches, which is designed to demonstrate the general **non-compatibility** in addressing the sufficiency based fairness.

Besides, we include two additional baselines that have the similar objective but different algorithmic realizations. (IV) the original IRM regularization (referred as IRM_v1) (Arjovsky et al., 2019), which adds a gradient penalty to encourage the invariance. (V) One-step explicit alignment. In the inner-level optimization, we suppose to conduct the one-step gradient descent for each sub-group. Then in the outer-level optimization, we add a gradient-incoherence constraint to encourage the identical (one-step) optimization path: $\min_\lambda \|\nabla_{h_0}\mathcal{L}_0(h_0, \lambda) - \nabla_{h_1}\mathcal{L}_1(h_1, \lambda)\|_2^2$. All the results are reported by averaging five repetitions and additional experimental details are delegated in the Appendix.

6.2 Empirical Results

6.2.1 Toxic Comments

The toxic comments dataset (Jigsaw, 2018) is a binary **classification** task in NLP to predict whether comment is toxic or not. The original label is actually not binary since the comments is decided by multiple annotators, where the labelling discrepancy generally occurs. To this end, we conduct a simple strategy to decide comment is toxic if at least one annotator marks it. In this dataset, a portion of comments have been labeled with identity attributes, including gender and race. It has also been revealed that the race identity (e.g., black) is correlated with the toxicity label, which can lead to the predictive discrimination. Thus we adopted the *race* as the protected group by selecting two subgroups of Black and Asian. For the sake of computational simplicity, we first applied the pretrained BERT (Devlin et al., 2018) to extract the word embedding with 748 dimensional vector. Then we adopt representation function λ as two fully-connected layers with hidden dimension 200 with Relu activation and classifier h as a linear predictor. We report the test-set sub-group average accuracy and sufficiency gap (ΔSuf_C) in Tab. 1 and Fig. 4.

The results reveal several interesting facts. (1) The Demographic Parity (DP) based fair constraints are generally non-compatible with the sufficiency rule. Specifically, baseline (II,III) even increase ΔSuf_C with higher value than ERM. (2) For the baselines that track the sufficiency rule (IV,V), the sufficiency gap ΔSuf_C is improved with a similar accuracy, shown in Tab.1. We also change the regularization coefficient in (IV,V) and κ in the implicit approach. We observe that the implicit approach demonstrates a consistent better Accuracy-Fair trade-off, shown in Fig. 4.

Method	Accuracy (↑)	ΔSuf_C (↓)
ERM (I)	0.780 ± 0.015	0.210 ± 0.022
Adv_debias (II)	0.785 ± 0.022	0.165 ± 0.028
Mixup (III)	0.792 ± 0.011	0.160 ± 0.010
IRM_v1 (IV)	0.795 ± 0.012	0.086 ± 0.015
One_step (V)	0.797 ± 0.006	0.086 ± 0.012
Implicit	0.794 ± 0.027	0.074 ± 0.020

Table 2: CelebA dataset. Accuracy and predictive parity in different approaches.

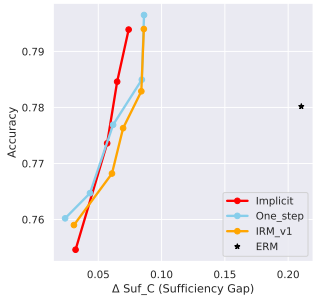


Figure 5: CelebA. Accuracy-Fair Trade-off

6.2.2 CELEBA DATASET

The CelebA dataset (Liu et al., 2015) contains around 200K images of celebrity faces, where each image is associated with 40 human-annotated binary attributes including gender, hair color, young, etc. In this paper, we designate *gender* as the sensitive attribute, and *attractive* as the binary **classification** task. We randomly select around 82K and 18K images as the training and validation set. Then we adopt representation function λ as pre-trained ResNet-18 (He et al., 2016) and classifier h as two-fully connected layers. We report the test-set sub-group average accuracy and sufficiency gap (ΔSuf_C) in Tab. 2 and Fig. 5.

The results in the CelebA show similar behaviors with the Toxic comments. Specifically, the DP based fair approaches (II, III) did not effectively improve ΔSuf_C, shown in Tab. 2. In contrast, the sufficiency can be significantly improved in baselines (IV, V) and implicit approach without largely losing the accuracy. Specifically, Fig. 5 visualizes the accuracy-fair trade-off curve, where the later three approaches show quite similar behaviors.

6.2.3 LAW DATASET

The Law Dataset is a **regression** task to predict a students GPA (real value, ranging from $[0, 4]$), where the data is utilized from the School Admissions Councils National Longitudinal Bar Passage Study (Wightman, 1998) with 20K examples. In the regression task, we adopt the square loss and *race* as the protected attribute (white versus non-white). We adopt λ as the one fully connected layer with hidden dimension 100 and Relu activation and predictor h as a linear predictor. We report the test-set subgroup average MSE (Mean Square Error) and sufficiency gap (ΔSuf_R) in Tab. 1 and Fig. 4.

Compared to the classification task, the results show similar behaviors in the regression. Specifically, the DP based fair approaches (II, III) still increase ΔSuf_R in the regression. In contrast, the gap is significantly improved in our proposed approach and baseline (IV,V). Specifically, Fig. 7 visualizes the sufficiency-gap of different approaches, where the implicit approach significantly mitigate the sufficiency gap. Besides, Fig. 6 shows the MSE-sufficiency gap curve, which still reveals the implicit approach benefits a better trade-off between the performance and fairness.

Method	MSE (↓)	ΔSuf_R (↓)
ERM (I)	0.190 ± 0.005	0.160 ± 0.007
Adv_debias (II)	0.223 ± 0.008	0.188 ± 0.012
Mixup (III)	0.216 ± 0.012	0.172 ± 0.007
IRM_v1 (IV)	0.208 ± 0.006	0.096 ± 0.006
One_step (V)	0.204 ± 0.007	0.125 ± 0.010
Implicit	0.198 ± 0.005	0.091 ± 0.011

Table 3: Law dataset. MSE and sufficiency gap in different approaches.

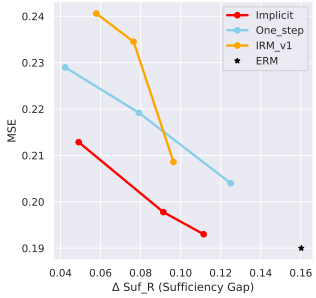
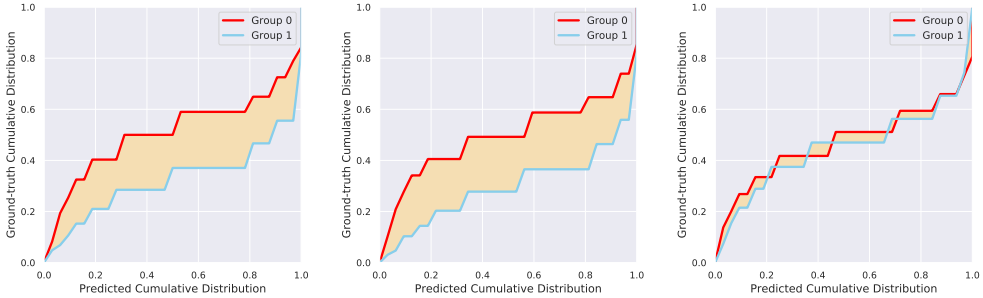


Figure 6: Law. MSE-Fair Trade-off



(a) ERM (b) Fair Mix-up (c) Implicit

Figure 7: Illustration of the sufficiency gap (ΔSuf_R) in Law dataset (regression). The ERM and Fair mix-up suffer a high ΔSuf_R, while the proposed implicit alignment can significantly mitigate the sufficiency gap.

Method	MSE (↓)	ΔSuf_R (↓)
ERM (I)	1.939 ± 0.021	0.246 ± 0.019
Adv_debias (II)	1.982 ± 0.016	0.252 ± 0.020
Mixup (III)	1.979 ± 0.025	0.246 ± 0.023
IRM_v1 (IV)	1.927 ± 0.031	0.077 ± 0.009
One_step (V)	1.904 ± 0.027	0.090 ± 0.019
Implicit	1.906 ± 0.019	0.051 ± 0.005

Table 4: NLSY dataset. MSE and sufficiency gap in different approaches.

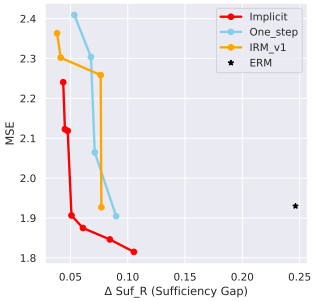


Figure 8: NLSY. MSE-Fair Trade-off

6.2.4 NLSY Dataset

The National Longitudinal Survey of Youth (NLSY, 2021) dataset is a **regression** task with around 7K dataset, which involves the survey results of the U.S. Bureau of Labor Statistics. It is intended to gather information on the labor market activities and other life events of several groups for predicting the income y of each person. We treat the *gender* as the sensitive attribute. We also normalize the output y by diving the $10,000$, then the final output y ranges around $[0, 8]$. The prediction loss is also the square loss. We adopt representation λ as the two fully connected layers with hidden dimension 200 and Relu activation and predictor h as a linear predictor. We report the test-set sub-group average MSE (Mean Square Error) and Sufficiency Gap (ΔSuf_R) in Tab. 4 and Fig. 8.

Tab. 4 provides similar trends with other datasets. Baselines (IV,V) and implicit approach effective control the sufficiency gap, while the DP based approach generally fails to improve the gap. Fig. 8 reveals a slightly better approximation-fair trade off for the implicit approach. Finally, Fig. 11 (in Appendix) visualizes the sufficiency gap of different algorithms. The gap is actually significantly improved while the calibration gap still exists, which is consistent with (Liu et al., 2019). Therefore it can be quite interesting and promising to analyze the triple trade-off between the sufficiency gap, calibration gap and prediction performance in the regression.

7 Conclusion

We considered the fair representation learning from a novel perspective through encouraging the invariant optimal predictors on the top of data representation. Then we formulated this problem as a bi-level optimization and proposed an implicit alignment algorithm. We further demonstrated the bi-level objective is to fulfil the sufficiency rule. Besides, we also analyzed the error gap of the implicit algorithm. The empirical results in both classification and regression settings suggest the improved fairness measurement. Finally, we think the future work can include developing computationally efficient explicit algorithms for avoiding the biased gradient computation.

ETHICS STATEMENT

This paper proposed a novel fair representation algorithm, which aims to address the potential prediction discrimination towards several subgroups. The proposed approach may also introduce the potential negative impact: we merely address the fairness with respect to the sufficiency rule in the paper, which is not always the preferable criteria in several specific scenarios.

REPRODUCIBILITY STATEMENT

We provided a demo source code in the supplementary material for a better understanding the proposed algorithm. Besides, the detailed experimental descriptions and theoretical proofs are also provided in the appendix.

REFERENCES

Alekh Agarwal, Miroslav Dudík, and Zhiwei Steven Wu. Fair regression: Quantitative definitions and reduction-based algorithms. In *International Conference on Machine Learning*, pp. 120–129. PMLR, 2019.

Martin Arjovsky, Léon Bottou, Ishaan Gulrajani, and David Lopez-Paz. Invariant risk minimization. *arXiv preprint arXiv:1907.02893*, 2019.

Solon Barocas, Moritz Hardt, and Arvind Narayanan. *Fairness and Machine Learning*. fairmlbook.org, 2019. `http://www.fairmlbook.org`.

Yoshua Bengio. Gradient-based optimization of hyperparameters. *Neural computation*, 12(8):1889–1900, 2000.

Stephen Boyd, Stephen P Boyd, and Lieven Vandenberghe. *Convex optimization*. Cambridge university press, 2004.

Peter Bühlmann. Invariance, causality and robustness. *Statistical Science*, 35(3):404–426, 2020.

Kai-Wei Chang, Vinod Prabhakaran, and Vicente Ordonez. Bias and fairness in natural language processing. In *Proceedings of the 2019 Conference on Empirical Methods in Natural Language Processing and the 9th International Joint Conference on Natural Language Processing (EMNLP-IJCNLP): Tutorial Abstracts*, Hong Kong, China, November 2019. Association for Computational Linguistics. URL `https://aclanthology.org/D19-2004`.

Yining Chen, Elan Rosenfeld, Mark Sellke, Tengyu Ma, and Andrej Risteski. Iterative feature matching: Toward provable domain generalization with logarithmic environments. *arXiv preprint arXiv:2106.09913*, 2021.

Alexandra Chouldechova. Fair prediction with disparate impact: A study of bias in recidivism prediction instruments. *Big data*, 5(2):153–163, 2017.

Ching-Yao Chuang and Youssef Mroueh. Fair mixup: Fairness via interpolation. *arXiv preprint arXiv:2103.06503*, 2021.

Evgenii Chzhen, Christophe Denis, Mohamed Hebiri, Luca Oneto, and Massimiliano Pontil. Fair regression with wasserstein barycenters. *arXiv preprint arXiv:2006.07286*, 2020.

P. Concus, G. Golub, and Gérard Meurant. Block preconditioning for the conjugate gradient method. *Siam Journal on Scientific and Statistical Computing*, 6, 01 1985. doi: 10.1137/0906018.

Elliot Creager, Jörn-Henrik Jacobsen, and Richard Zemel. Environment inference for invariant learning. In *International Conference on Machine Learning*, pp. 2189–2200. PMLR, 2021.

Jacob Devlin, Ming-Wei Chang, Kenton Lee, and Kristina Toutanova. Bert: Pre-training of deep bidirectional transformers for language understanding. *arXiv preprint arXiv:1810.04805*, 2018.

Kawin Ethayarajh. Is your classifier actually biased? measuring fairness under uncertainty with bernstein bounds. In *Proceedings of the 58th Annual Meeting of the Association for Computational Linguistics*, pp. 2914–2919, Online, July 2020. Association for Computational Linguistics. doi: 10.18653/v1/2020.acl-main.262. URL https://aclanthology.org/2020.acl-main.262.

Richard Ribón Fletcher, Audace Nakeshimana, and Olusubomi Olubeko. Addressing fairness, bias, and appropriate use of artificial intelligence and machine learning in global health. *Frontiers in Artificial Intelligence*, 3:116, 2021. ISSN 2624-8212. doi: 10.3389/frai.2020.561802. URL https://www.frontiersin.org/article/10.3389/frai.2020.561802.

Chuan Guo, Geoff Pleiss, Yu Sun, and Kilian Q Weinberger. On calibration of modern neural networks. In *International Conference on Machine Learning*, pp. 1321–1330. PMLR, 2017.

Umang Gupta, Aaron Ferber, Bistra Dilkina, and Greg Ver Steeg. Controllable guarantees for fair outcomes via contrastive information estimation. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 35, pp. 7610–7619, 2021.

Moritz Hardt, Eric Price, and Nati Srebro. Equality of opportunity in supervised learning. *Advances in neural information processing systems*, 29:3315–3323, 2016.

Kaiming He, Xiangyu Zhang, Shaoqing Ren, and Jian Sun. Deep residual learning for image recognition. In *Proceedings of the IEEE conference on computer vision and pattern recognition*, pp. 770–778, 2016.

Ray Jiang, Aldo Pacchiano, Tom Stepleton, Heinrich Jiang, and Silvia Chiappa. Wasserstein fair classification. In *Uncertainty in Artificial Intelligence*, pp. 862–872. PMLR, 2020.

Jigsaw. Toxic comment classification challenge, 2018. URL https://www.kaggle.com/c/jigsaw-toxic-comment-classification-challenge/overview/description.

Thomas Kehrenberg, Myles Bartlett, Oliver Thomas, and Novi Quadrianto. Null-sampling for interpretable and fair representations. In *European Conference on Computer Vision*, pp. 565–580. Springer, 2020.

Byungju Kim, Hyunwoo Kim, Kyungsu Kim, Sungjin Kim, and Junmo Kim. Learning not to learn: Training deep neural networks with biased data. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, pp. 9012–9020, 2019.

Junpei Komiyama, Akiko Takeda, Junya Honda, and Hajime Shimao. Nonconvex optimization for regression with fairness constraints. In Jennifer Dy and Andreas Krause (eds.), *Proceedings of the 35th International Conference on Machine Learning*, volume 80 of *Proceedings of Machine Learning Research*, pp. 2737–2746. PMLR, 10–15 Jul 2018. URL https://proceedings.mlr.press/v80/komiyama18a.html.

Volodymyr Kuleshov, Nathan Fenner, and Stefano Ermon. Accurate uncertainties for deep learning using calibrated regression. In *International Conference on Machine Learning*, pp. 2796–2804. PMLR, 2018.

Yann LeCun, Yoshua Bengio, and Geoffrey Hinton. Deep learning. *nature*, 521(7553):436–444, 2015.

Lydia T. Liu, Max Simchowitz, and Moritz Hardt. The implicit fairness criterion of unconstrained learning. In Kamalika Chaudhuri and Ruslan Salakhutdinov (eds.), *Proceedings of the 36th International Conference on Machine Learning*, volume 97 of *Proceedings of Machine Learning Research*, pp. 4051–4060. PMLR, 09–15 Jun 2019. URL https://proceedings.mlr.press/v97/liu19f.html.

Ziwei Liu, Ping Luo, Xiaogang Wang, and Xiaoou Tang. Deep learning face attributes in the wild. In *Proceedings of International Conference on Computer Vision (ICCV)*, December 2015.

Francesco Locatello, Gabriele Abbati, Tom Rainforth, Stefan Bauer, Bernhard Schölkopf, and Olivier Bachem. On the fairness of disentangled representations. *arXiv preprint arXiv:1905.13662*, 2019.

David Madras, Elliot Creager, Toniann Pitassi, and Richard Zemel. Learning adversarially fair and transferable representations. In *International Conference on Machine Learning*, pp. 3384–3393. PMLR, 2018.

Ninareh Mehrabi, Fred Morstatter, Nripsuta Saxena, Kristina Lerman, and Aram Galstyan. A survey on bias and fairness in machine learning. *ACM Computing Surveys (CSUR)*, 54(6):1–35, 2021.

NLSY. National longitudinal survey of youth, 2021. URL `https://www.bls.gov/nls/`.

Ziad Obermeyer, Brian Powers, Christine Vogeli, and Sendhil Mullainathan. Dissecting racial bias in an algorithm used to manage the health of populations. *Science*, 366(6464):447–453, 2019. doi: 10.1126/science.aax2342. URL `https://www.science.org/doi/abs/10.1126/science.aax2342`.

Fabian Pedregosa. Hyperparameter optimization with approximate gradient. In *International conference on machine learning*, pp. 737–746. PMLR, 2016.

Geoff Pleiss, Manish Raghavan, Felix Wu, Jon Kleinberg, and Kilian Q Weinberger. On fairness and calibration. *arXiv preprint arXiv:1709.02012*, 2017.

Manish Raghavan, Solon Barocas, Jon Kleinberg, and Karen Levy. Mitigating bias in algorithmic hiring: Evaluating claims and practices. In *Proceedings of the 2020 conference on fairness, accountability, and transparency*, pp. 469–481, 2020.

Aravind Rajeswaran, Chelsea Finn, Sham Kakade, and Sergey Levine. Meta-learning with implicit gradients. In *Advances in neural information processing systems*, 2019.

Elan Rosenfeld, Pradeep Ravikumar, and Andrej Risteski. The risks of invariant risk minimization. *arXiv preprint arXiv:2010.05761*, 2020.

Michael W Sjoding, Robert P Dickson, Theodore J Iwashyna, Steven E Gay, and Thomas S Valley. Racial bias in pulse oximetry measurement. *New England Journal of Medicine*, 383(25):2477–2478, 2020.

Jiaming Song, Pratyusha Kalluri, Aditya Grover, Shengjia Zhao, and Stefano Ermon. Learning controllable fair representations. In *The 22nd International Conference on Artificial Intelligence and Statistics*, pp. 2164–2173. PMLR, 2019.

Linda F. Wightman. Lsac national longitudinal bar passage study, 1998.

Rich Zemel, Yu Wu, Kevin Swersky, Toni Pitassi, and Cynthia Dwork. Learning fair representations. In *International conference on machine learning*, pp. 325–333. PMLR, 2013.

Brian Hu Zhang, Blake Lemoine, and Margaret Mitchell. Mitigating unwanted biases with adversarial learning. In *Proceedings of the 2018 AAAI/ACM Conference on AI, Ethics, and Society*, pp. 335–340, 2018.

Wenbin Zhang and Eirini Ntoutsi. Faht: an adaptive fairness-aware decision tree classifier. *arXiv preprint arXiv:1907.07237*, 2019.

Han Zhao, Amanda Coston, Tameem Adel, and Geoffrey J Gordon. Conditional learning of fair representations. *arXiv preprint arXiv:1910.07162*, 2019.

A PROPOSITION 1

We consider the regression and classification separately.

Regression According to the definition, given a fixed and deterministic representation λ, we have

$$\mathcal{L}_0(h, \lambda) = \mathbb{E}_{\mathcal{D}_0}(h(z) - y)^2$$

It is noted as a typical regression problem with square error. We set the derivative as zero: $\nabla_h \mathcal{L}_0(h, \lambda) = 0$, we have $h_0^\star(z) = \mathbb{E}_{\mathcal{D}_0}[Y|Z = z]$. As for $\mathcal{D}_1$, we apply the same strategy with $h_1^\star(z) = \mathbb{E}_{\mathcal{D}_1}[Y|Z = z]$. Based on the invariant optimal predictor, we have $\mathbb{E}_{\mathcal{D}_0}[Y|Z = z] = \mathbb{E}_{\mathcal{D}_1}[Y|Z = z]$ with $z = \lambda(x)$.

Classification According to the definition, we have:

$$\mathcal{L}_0(h, \lambda) = \mathbb{E}_{\mathcal{D}_0} \log(1 + \exp(-yh(z)))$$

Since the optimal predictor on the logistic loss is the log-conditional density ratio: $h_0^\star(z) = \log\left(\frac{\mathcal{D}_0(Y=1|Z=z)}{\mathcal{D}_0(Y=-1|Z=z)}\right)$. Observe that in the binary classification with $Y = \{-1, 1\}$, we have $\mathcal{D}_0(Y = 1|Z = z) = \frac{1}{2}(1 + \mathbb{E}_{\mathcal{D}_0}[Y|Z = z])$ and $\mathcal{D}_0(Y = -1|Z = z) = \frac{1}{2}(1 - \mathbb{E}_{\mathcal{D}_0}[Y|Z = z])$, then we have:

$$h_0^\star(z) = \log\left(\frac{1 + \mathbb{E}_{\mathcal{D}_0}[Y|Z = z]}{1 - \mathbb{E}_{\mathcal{D}_0}[Y|Z = z]}\right)$$

As for $\mathcal{D}_1$, we adopt the same strategy and we have $\log\left(\frac{1+\mathbb{E}_{\mathcal{D}_0}[Y|Z=z]}{1-\mathbb{E}_{\mathcal{D}_0}[Y|Z=z]}\right) = \log\left(\frac{1+\mathbb{E}_{\mathcal{D}_1}[Y|Z=z]}{1-\mathbb{E}_{\mathcal{D}_1}[Y|Z=z]}\right)$, then we have $\mathbb{E}_{\mathcal{D}_0}[Y|Z = z] = \mathbb{E}_{\mathcal{D}_1}[Y|Z = z]$.

As for the predictive parity, since we have $\mathbb{E}_{\mathcal{D}_0}[Y|Z = z] = \mathbb{E}_{\mathcal{D}_1}[Y|Z = z]$ and $h^\star = h_1^\star = h_2^\star$, then we have $\mathbb{E}_{\mathcal{D}_0}[Y|h^\star(z)] = \mathbb{E}_{\mathcal{D}_1}[Y|h^\star(z)]$.

B APPROXIMATION ERROR

Theorem 2 (Approximation Error Gap)**.** *Suppose that (1)* ***Smooth Predictive Loss****. The first-order derivatives and second-order derivatives of $\mathcal{L}$ are Lipschitz continuous; (2)* ***Non-singular Hessian matrix****. We assume $\nabla_{h_0,h_0}\mathcal{L}_0(h_0, \lambda), \nabla_{h_1,h_1}\mathcal{L}_1(h_1, \lambda)$, the Hessian matrix of the inner optimization problem, are invertible. (3)* ***Bounded representation and predictor function****. We assume the λ and h are bounded, i.e., $\|\lambda\|, \|h\|$ are upper bounded by the predefined positive constants. Then the approximation error between the ground truth and algorithmic estimated gradient w.r.t. the representation is be upper bounded by:*

$$\|grad(\lambda) - \tilde{grad}^{\delta}(\lambda)\| = \mathcal{O}(\kappa\epsilon + \epsilon + \delta).$$

Proof. We denote grad(λ) as the ground truth gradient w.r.t. λ in outer-level loss (given the optimal predictor $h_0^\star$, $h_1^\star$). Then we aim to bound

$$\|\text{grad}(\lambda) - \tilde{\text{grad}}^{\delta}(\lambda)\|$$

We first introduce the following terms for facilitating the proof:

$A_0^\epsilon = \nabla_{h_0}\nabla_\lambda \mathcal{L}_0(h_0^\epsilon, \lambda), A_1^\epsilon = \nabla_\lambda \nabla_{h_1}\mathcal{L}_1(h_1^\epsilon, \lambda), A_0^\star = \nabla_\lambda \nabla_{h_0}\mathcal{L}_0(h_0^\star, \lambda), A_1^\star = \nabla_\lambda \nabla_{h_1}\mathcal{L}_1(h_1^\star, \lambda),$
$B_0^\epsilon = \nabla_\lambda \mathcal{L}_0(h_0^\epsilon, \lambda), B_1^\epsilon = \nabla_\lambda \mathcal{L}_1(h_1^\epsilon, \lambda), B_0^\star = \nabla_\lambda \mathcal{L}_0(h_0^\star, \lambda), B_1^\star = \nabla_\lambda \mathcal{L}_1(h_1^\star, \lambda),$

$$\mathbf{p}_0^\star = \left(\nabla_{h_0}^2 \mathcal{L}_0(h_0^\star, \lambda)\right)^{-1} \left(\nabla_{h_0}\mathcal{L}_0(h_0^\star, \lambda) + \kappa(h_0^\star - h_1^\star)\right),$$

$$\mathbf{p}_1^\star = \left(\nabla_{h_1}^2 \mathcal{L}_1(h_1^\star, \lambda)\right)^{-1} \left(\nabla_{h_1}\mathcal{L}_1(h_1^\star, \lambda) - \kappa(h_0^\star - h_1^\star)\right).$$

Then the approximation error gap can be expressed as:

$$\begin{aligned}\|\text{grad}(\lambda) - \tilde{\text{grad}}^{\delta}(\lambda)\| &= \| \left(B_0^\star - A_0^\star \mathbf{p}_0^\star + B_1^\star - A_1^\star \mathbf{p}_1^\star\right) - \left(B_0^\epsilon - A_0^\epsilon \mathbf{p}_0^\delta + B_1^\epsilon - A_1^\epsilon \mathbf{p}_1^\delta\right) \| \\ &\leq \sum_{i=0}^{1} \|B_i^\star - B_i^\epsilon\| + \sum_{i=0}^{1} \|A_i^\star \mathbf{p}_i^\star - A_i^\delta \mathbf{p}_i^\delta\|\end{aligned}$$

Due to the symmetric of $\mathcal{D}_0$ and $\mathcal{D}_1$, we only focus on the term on $i = 0$, the the upper bound in $i = 1$ can be derived analogously.

As for bounding $\|B_0^\star - B_0^\epsilon\|$, since we assume first order derivative of the loss is Lipschitz functions (with constant L_1), then we have :

$$\|B_0^\star - B_0^\epsilon\| \leq L_1 \|h_0^\star - h_0^\epsilon\| \leq \epsilon L_1$$

Then the second term can be upper bounded by three terms:

$$\|A_0^\star \mathbf{p}_0^\star - A_0^\delta \mathbf{p}_0^\delta\| \leq \underbrace{\|A_0^\star \mathbf{p}_0^\star - A_0^\star \mathbf{p}_0\|}_{(1)} + \underbrace{\|A_0^\star \mathbf{p}_0 - A_0^\epsilon \mathbf{p}_0\|}_{(2)} + \underbrace{\|A_0^\epsilon \mathbf{p}_0 - A_0^\epsilon \mathbf{p}_0^\delta\|}_{(3)}$$

Before estimating the upper bound, we first demonstrate $\|A_0^\epsilon\|$ and $\|A_0^\star\|$ are also bounded.

Since we assume λ and h are bounded (assuming the bounded constant as η and ϕ), the second order derivative are Lipschitz (with constant L_2). Then we consider another fixed point $(\lambda', h_0^\star(\lambda'))$ with bounded second order derivative: $A_0 = \nabla^2_{h_0,\lambda} \mathcal{L}_0(h_0^\star(\lambda'), \lambda')$ and $\|A_0\| \leq A$. We have:

$$\|A_0^\star - A_0\|_2 \leq L_2 \|[h_0^\star(\lambda), \lambda] - [h_0^\star(\lambda'), \lambda']\|_2 \leq L_2 \sqrt{\eta^2 + \phi^2}$$

Thus we have $\|A_0^\star\| \leq A + L_2\sqrt{\eta^2 + \phi^2} = A_{\text{sup}}^\star$. As for the second derivative at point h_0^ϵ, it can be upper bounded analogously with a similar constant A_{sup}^ϵ.

The upper bound of term (1) We have:

$$\|A_0^\star \mathbf{p}_0^\star - A_0^\star \mathbf{p}_0\| \leq \|A_0^\star\| \|\mathbf{p}_0^\star - \mathbf{p}_0\|$$

We have proved $\|A_0^\star\|$ is upper bounded by $A_{\text{sup}}^\star$. We additionally introduce the following auxiliary terms:

$$P_0^\star = \left(\nabla^2_{h_0} \mathcal{L}_0(h_0^\star, \lambda)\right)^{-1}, P_0^\epsilon = \left(\nabla^2_{h_1} \mathcal{L}_1(h_1^\star, \lambda)\right)^{-1}.$$
$$b_0^\star = \nabla_{h_0} \mathcal{L}_0(h_0^\star, \lambda) + \kappa(h_0^\star - h_1^\star), b_0^\epsilon = \nabla_{h_0} \mathcal{L}_0(h_0^\epsilon, \lambda) + \kappa(h_0^\epsilon - h_1^\epsilon)$$

Then we have:

$$\begin{aligned}
\|\mathbf{p}_0^\star - \mathbf{p}_0\| &= \|P_0^\star b_0^\star - P_0^\epsilon b_0^\epsilon\| \\
&\leq \|P_0^\star b_0^\star - P_0^\star b_0^\epsilon\| + \|P_0^\star b_0^\epsilon - P_0^\epsilon b_0^\epsilon\| \\
&\leq \|P_0^\star\| \|b_0^\star - b_0^\epsilon\| + \|b_0^\epsilon\| \|P_0^\star - P_0^\epsilon\|
\end{aligned}$$

As for the $\|P_0^\star\|$, since we assume the Hessian matrix is invertible thus its norm is upper bounded by some constant (denoted as A_{-1}). As for $\|b_0^\star - b_0^\epsilon\|$, we have:

$$\begin{aligned}
\|b_0^\star - b_0^\epsilon\| &\leq \|\nabla_{h_0} \mathcal{L}_0(h_0^\star, \lambda) - \nabla_{h_0} \mathcal{L}_0(h_0^\epsilon, \lambda)\| + 2\kappa\epsilon \\
&\leq \epsilon L_1 + 2\kappa\epsilon
\end{aligned}$$

Thus we have $\|P_0^\star\| \|b_0^\star - b_0^\epsilon\| \leq A_{-1}(\epsilon L_1 + 2\kappa\epsilon)$.

As for $\|b_0^\epsilon\|$, we can easily verify that it is indeed bounded by some constant b. For the first term, we can adopt the same strategy in proving bounded $\|A_0^\star\|$. As for the second term in b_0^ϵ, it is upper bounded by $2\kappa\phi$, due to the bounded predictor.

We now demonstrate $\|P_0^\star - P_0^\epsilon\|$. Denoting $\Delta = (P_0^\star)^{-1} - (P_0^\epsilon)^{-1}$, then according to the second order Lipschitz assumption, we have: $\|\Delta\| \leq \epsilon L_2$. Plugging in the result, we have:

$$\|P_0^\star - P_0^\epsilon\| = \|(P_0^\star)\Delta(P_0^\epsilon)\| \leq \|P_0^\star\| \|\Delta\| \|P_0^\epsilon\| \leq (A_{-1})^2 L_2 \epsilon$$

We still adopt the assumption that the bounded Hessian-inverse matrix by A_{-1}.

Plugging in all the results, we have:

$$(1) \leq A_1(\epsilon L_1 + 2\kappa\epsilon) + b(A_1)^2 L_2 \epsilon := \mathcal{O}(\kappa\epsilon + \epsilon)$$

The upper bound of term (2) We have:

$$\|A_0^\star \mathbf{p}_0 - A_0^\epsilon \mathbf{p}_0\| \leq \|\mathbf{p}_0\|_2 \|A_0^\star - A_0^\epsilon\|$$

Since we assume the loss is second-order Lipschitz, thus we have

$$\|A_0^\star - A_0^\epsilon\| = \|\nabla_\lambda \nabla_{h_0} \mathcal{L}_0(h_0^\star, \lambda) - \nabla_\lambda \nabla_{h_0} \mathcal{L}_0(h_0^\epsilon, \lambda)\| \leq L_2 \|h_0^\star - h_0^\epsilon\| \leq \epsilon L_2$$

We can also demonstrate $\|\mathbf{p}_0\|$ is bounded. According to the definition we have:

$$\begin{aligned}\|\mathbf{p}_0\| &\leq \| \left(\nabla_{h_0}^2 \mathcal{L}_0(h_0^\epsilon, \lambda)\right)^{-1} \| \| \left(\nabla_{h_0} \mathcal{L}_0(h_0^\epsilon, \lambda) + \kappa(h_0^\epsilon - h_1^\epsilon)\right) \| \\ &\overset{(i)}{\leq} A_{-1}(L_1 \|h_0^\star - h_0^\epsilon\|_2 + 2\kappa\phi) \\ &\overset{(ii)}{\leq} A_{-1}(\epsilon L_1 + 2\kappa\phi)\end{aligned}$$

For (i), we assume: 1) the Hessian matrix is invertible thus its norm is surely upper bounded by some constant (denoted as A_{-1}), 2) the first-order derivative is Lipschitz (bounded by L_1), 3) the predictor h is bounded. For (ii), we adopt the definition of h_0^ϵ.

Therefore, the upper bound for Term (2) is formulated as:

$$(2) \leq \epsilon L_2 A_{-1}(\epsilon L_1 + 2\kappa\phi) := \mathcal{O}(\kappa\epsilon)$$

The upper bound of term (3) We have:

$$\|A_0^\epsilon \mathbf{p}_0 - A_0^\epsilon \mathbf{p}_0^\delta\| \leq \|A_0^\epsilon\| \|\mathbf{p}_0 - \mathbf{p}_0^\delta\| \leq \delta A_{\text{sup}}^\epsilon = \mathcal{O}(\delta)$$

Through the upper bound in (1)-(3), we finally have the error between the estimated and ground-truth gradient:

$$\|\text{grad}(\lambda) - \tilde{\text{grad}}^\delta(\lambda)\| = \mathcal{O}(\kappa\epsilon + \epsilon + \delta)$$

□

C THE CONVERGENCE BEHAVIOR

For the sake of completeness, we provide the convergence analysis of the proposed algorithm.

Proposition 2. *We execute the implicit alignment algorithm (Algo. 1), obtaining a sequence of $\lambda_1, \ldots, \lambda_k, \ldots$. Supposing the fair constraint κ is fixed. The optimization tolerances are summable: $\sum_k \epsilon_k^2 \leq +\infty$ and $\sum_k \delta_k^2 \leq +\infty$, then λ_k is proved to be converged with*

$$\lim_{k \to \infty} \lambda_k = \lambda^\star.$$

If the stationary point $\lambda^\star$ is also within the bounded norm, then we have:

$$\textit{grad}(\lambda^\star) = 0.$$

Proof. We denote the entire outer-level loss w.r.t. λ as $\mathcal{L}(\lambda)$, by the assumption the β-smooth loss $\mathcal{L}$. Then at iteration $k+1$ and k, we have:

$$\begin{aligned}\mathcal{L}(\lambda_{k+1}) &\leq \mathcal{L}(\lambda_k) - \text{grad}(\lambda_k)^T(\lambda_k - \lambda_{k+1}) + \frac{\beta}{2}\|\lambda_{k+1} - \lambda_k\|^2 \\ &= \mathcal{L}(\lambda_k) - \left(\text{grad}(\lambda_k) - \tilde{\text{grad}}^\delta(\lambda_k) + \tilde{\text{grad}}^\delta(\lambda_k)\right)^T(\lambda_k - \lambda_{k+1}) + \frac{\beta}{2}\|\lambda_{k+1} - \lambda_k\|^2 \\ &= \mathcal{L}(\lambda_k) - \left(\text{grad}(\lambda_k) - \tilde{\text{grad}}^\delta(\lambda_k)\right)^T(\lambda_k - \lambda_{k+1}) - \tilde{\text{grad}}^\delta(\lambda_k)(\lambda_k - \lambda_{k+1}) + \frac{\beta}{2}\|\lambda_{k+1} - \lambda_k\|^2\end{aligned}$$

Since we assume the representation is within the bounded norm, the projection onto the convex set are non-expansive operators (Boyd et al., 2004). Then for any point p, q, we have $\|\text{proj}(p) - \text{proj}(q)\|^2 \leq (p-q)^T(\text{proj}(p) - \text{proj}(q))$. Then we set λ_k and $\lambda_{k+1} = \lambda_k - \frac{1}{\beta}\tilde{\text{grad}}^\delta(\lambda_k)$, we have:

$$\|\lambda_k - \lambda_{k+1}\|^2 \leq \frac{1}{\beta}(\tilde{\text{grad}}^\delta(\lambda_k))^T(\lambda_k - \lambda_{k+1})$$

Plugging into the results, we have:

$$\begin{aligned}\mathcal{L}(\lambda_{k+1}) &\leq \mathcal{L}(\lambda_k) - \left(\text{grad}(\lambda_k) - \tilde{\text{grad}}^{\delta}(\lambda_k)\right)^T (\lambda_k - \lambda_{k+1}) - \frac{\beta}{2}\|\lambda_{k+1} - \lambda_k\|^2 \\ &\leq \mathcal{L}(\lambda_k) + \|\text{grad}(\lambda_k) - \tilde{\text{grad}}^{\delta}(\lambda_k)\|\|\lambda_k - \lambda_{k+1}\| - \frac{\beta}{2}\|\lambda_{k+1} - \lambda_k\|^2\end{aligned}$$

Rearranging the inequality, we have:

$$\frac{\beta}{2}\|\lambda_{k+1} - \lambda_k\|^2 - \|\text{grad}(\lambda_k) - \tilde{\text{grad}}^{\delta}(\lambda_k)\|\|\lambda_k - \lambda_{k+1}\| + (\mathcal{L}(\lambda_{k+1}) - \mathcal{L}(\lambda_k)) \leq 0$$

Then we have:

$$\|\lambda_{k+1} - \lambda_k\| \leq \frac{1}{\beta}\left(\|\text{grad}(\lambda_k) - \tilde{\text{grad}}^{\delta}(\lambda_k)\| + \sqrt{\|\text{grad}(\lambda_k) - \tilde{\text{grad}}^{\delta}(\lambda_k)\|^2 - 2\beta\left(\mathcal{L}(\lambda_{k+1}) - \mathcal{L}(\lambda_k)\right)}\right)$$

By denoting $B_k = \|\text{grad}(\lambda_k) - \tilde{\text{grad}}^{\delta}(\lambda_k)\|$ and $C_k = \mathcal{L}(\lambda_{k+1}) - \mathcal{L}(\lambda_k)$. Then we have:

$$\begin{aligned}\|\lambda_{k+1} - \lambda_k\|^2 &\leq \frac{1}{\beta^2}\left(B_k^2 + B_k^2 - 2\beta C_k + 2B_k\sqrt{B_k^2 - 2\beta C_k}\right) \\ &\leq \frac{1}{\beta^2}\left(B_k^2 + B_k^2 - 2\beta C_k + B_k^2 + B_k^2 - 2\beta C_k\right) \\ &= \frac{4}{\beta^2}[\|\text{grad}(\lambda_k) - \tilde{\text{grad}}^{\delta}(\lambda_k)\|_2^2 - 2\beta\left(\mathcal{L}(\lambda_{k+1}) - \mathcal{L}(\lambda_k)\right)]\end{aligned}$$

Taking sum over k, we have:

$$\begin{aligned}\sum_{k=1}^{+\infty}\|\lambda_{k+1} - \lambda_k\|^2 &\leq \frac{4}{\beta^2}\sum_{k=1}^{+\infty}\|\text{grad}(\lambda_k) - \tilde{\text{grad}}^{\delta}(\lambda_k)\|_2^2 - \frac{8}{\beta}(\lim_{k\to\infty}\mathcal{L}(\lambda_{k+1}) - \mathcal{L}(\lambda_1)) \\ &\leq \frac{4}{\beta^2}\sum_k[(C+\kappa)^2\epsilon_k^2 + \delta_k^2] - \frac{8}{\beta}\left(\lim_{k\to\infty}\mathcal{L}(\lambda_{k+1}) - \mathcal{L}(\lambda_1)\right) < +\infty\end{aligned}$$

Since 1) the first term on the right side is finite, because the optimization tolerance is summable; 2) the second term is also finite, because the loss is assumed to be bounded. Then the upper bound is finite. In order to satisfy this condition, on the left side we should have:

$$\lim_{k\to\infty}\lambda_{k+1} - \lambda_k = 0$$

By adopting the definition $\lambda_{k+1} = \text{Proj}(\lambda_k - \tilde{\text{grad}}^{\delta}(\lambda_k))$ and $\lim_{k\to\infty}\tilde{\text{grad}}^{\delta}(\lambda_k) = \text{grad}(\lambda_k)$ (Based on theorem 1, the limit of the optimization tolerance is zero), then we have:

$$\lambda^\star = \text{proj}(\lambda^\star - \text{grad}(\lambda^\star))$$

Where $\lambda^\star = \lim_{k\to+\infty}\lambda_{k+1} = \lim_{k\to+\infty}\lambda_k$. Since the projection is on the bounded norm L_{norm} and $\lambda^\star$ is within the bounded norm space, thus if $\lambda^\star - \text{grad}(\lambda^\star)$ is within the bounded norm space, we have:

$$\text{grad}(\lambda^\star) = 0$$

Else if $\lambda^\star - \text{grad}(\lambda^\star)$ is outside the bounded norm space, then according to the definition, the projection of $\lambda^\star - \text{grad}(\lambda^\star)$ is surely on the *boundary* of the L_{norm} space, with $\|\text{proj}(\lambda^\star - \text{grad}(\lambda^\star))\| = L_{\text{norm}}$. However, we have assumed the $\lambda^\star$ is *within* the bounded norm space with $\|\lambda^\star\| < L_{\text{norm}}$, which leads to the contradiction. Based on these discussions, we finally have:

$$\text{grad}(\lambda^\star) = 0$$

□

D Additional Experimental details and Results

D.1 Additional Details

Toxic Comments We split the training, validation and testing set as 70%, 10% and 20%. We adopt Adam optimizer with learning rate 10^{-3} and eps 10^{-3}. The batch-size is set as 500 for each subgroup and we use sampling with replacement to run the explicit algorithm with maximum epoch 100. The fair coefficient is generally set as $\kappa = 0.1 \sim 0.001$. As for the inner-optimization step, the iteration number is 20 and the iteration in running conjugate gradient approach is 10.

CelebA The training/validation/test set are around 82K, 18K and 18K. We also adopt the Adam optimizer with learning rate on $\lambda : 10^{-5} \sim 10^{-4}$ and $h : 10^{-3}$. The batch-size is set as 64 for each subgroup and we iterate the whole dataset as one epoch. The maximum running epoch is set as 20 and the iteration in running conjugate gradient approach is 10.

Law We split the training, validation and testing set as 70%, 10% and 20%. Then we adopt Adam optimizer with learning rate 10^{-3} and eps 10^{-3}. The batch-size is set as 500 for each subgroup and we use sampling with replacement to run the implicit algorithm, with the maximum epoch 100. We adopt the MSE loss in the regression. The fair coefficient is generally set as $\kappa = 0.1 \sim 10^{-4}$. As for the inner-optimization, the iteration number is 20 and the iteration in running conjugate gradient is 10. In computing the sufficiency gap in the regression, we sample 33 points to compute the gap.

NLSY We split the training, validation and testing set as 70%, 10% and 20%. Then we adopt Adam optimizer with learning rate 10^{-3} and eps 10^{-3}. The batch-size is set as 500 for each subgroup and we use sampling with replacement to run the implicit algorithm, with maximum epoch 100. We adopt the MSE loss in the regression. The fair coefficient is generally set as $\kappa = 0.1 \sim 10^{-4}$. As for the inner-optimization, the iteration number is 20 and the iteration in running conjugate gradient is 10. In computing the sufficiency gap, we sample 33 points to compute the sufficiency gap.

D.2 Additional Empirical results

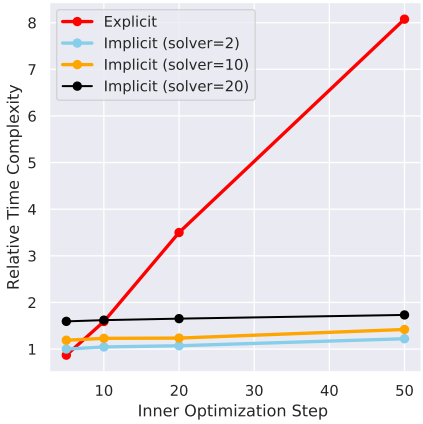


Figure 9: Computational time between T-step explicit and implicit approach in CelebA. Specifically, solver $= 2$ indicates the the conjugate gradient is executed 2 iterations. The results reveals the benefits of implicit approach: avoiding the back-propagation through the inner-optimization path. In contrast, the time complexity in explicit approach linearly increases with the inner-optimization step, which is consistent with our analysis.

Computational complexity To show the efficiency of the implicit approach, we empirically compare the computational complexity of the T-step explicit alignment and implicit approach (for different iterations of conjugate gradient solver.) The experimental results verified the efficiency of the implicit approach, where a significant large inner-optimization step does not considerably increase the computational time.

Gradient evolution We also visualize the gradient norm of the representation λ in the Toxic dataset, shown in Fig. 10. The results verify the convergence behavior and the gradient norm finally tends to zero.

D.3 Discussion with non-deep learning baselines

In order to show the effectiveness of the proposed approach, we additionally compare the FAHT (Zhang & Ntoutsi, 2019), a decision tree based fair classification approach. We evaluated the empirical performance on Toxic comments dataset.

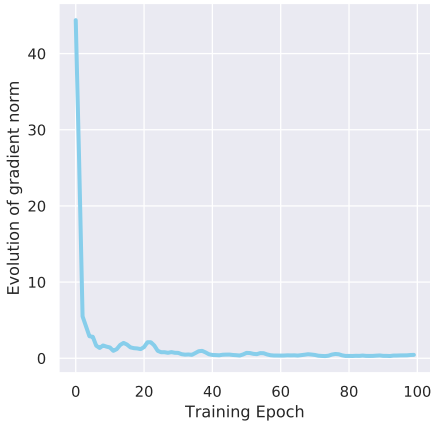


Figure 10: Gradient Norm evolution w.r.t. representation λ in Toxic comments dataset. We visualize the norm of $\tilde{\text{grad}}^{\delta}(\lambda)$ at each training epoch, which suggests a convergence behavior and the gradient finally tends to zero.

Table 5: Comparison with Fairness Aware Decision Tree

Method	Accuracy (↑)	ΔSuf_C (↓)
FAHT	0.596	0.397
Implicit	0.760	0.051

The implicit approach demonstrates the considerable better results, which may come from two aspects: (1) the Toxic task is a high-dimensional classification problem ($x \in \mathbb{R}^{748}$), where the deep learning based approach is more effective in handling the high-dim dataset. (2) The FAHT aims to realize the statistical parity (the independence rule), which is *not compatible* with the sufficiency. According to the analysis of (Barocas et al., 2019), when the sensitive attribute (A) and label (Y) are not independent (This has been justified by computing their Pearson Correlation coefficient), the sufficiency and independence cannot both hold.

D.4 SUFFICIENCY GAP IN REGRESSION

We visualize the sufficiency gap of NLSY dataset.

E COMPLEMENTARY TECHNICAL DETAILS

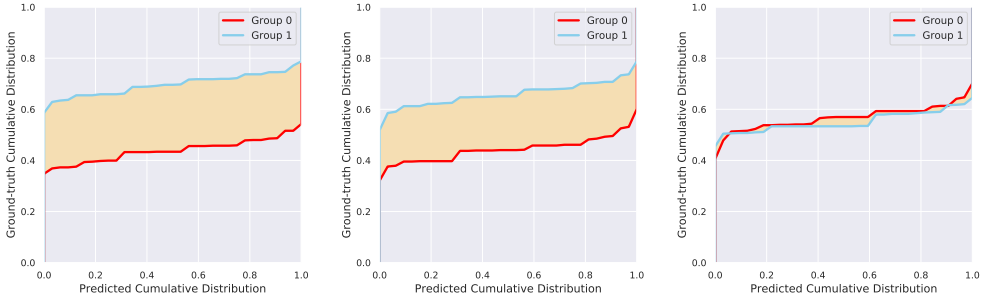
We present complementary details that are related to the paper.

E.1 CONJUGATE GRADIENT METHOD

We present the Conjugate Gradient (CG) algorithm in Algo. 2 through `autograd`. In the conventional CG algorithm with objective $\frac{1}{2}x^T AX - bX$, we need to estimate AX and compute its residual and update X. Since in our problem setting, the $A = \nabla^2_{h_0}\mathcal{L}_0(h_0^\epsilon, \lambda)$, then computing AX can be realized through Hessian-vector product through `autograd`, denoted as function F in the paper. i.e., $\nabla^2_{h_0}\mathcal{L}_0(h_0^\epsilon, \lambda)X = F(x)$.

Below we provided a simple PyTorch code for realizing the Hessian Vector product.

```
import torch
def hessain_vector_product(loss,model,vector):
    # loss: the defined loss
    # model: the model in computing the Hessian
    # vector: the required vector in computing Hessian-vector product
```



(a) ERM (b) Fair Mix-up (c) Implicit

Figure 11: Illustration of the *sufficiency gap* in NLSY dataset. The ERM and mix-up suffer the high predictive sufficiency-gap, while the proposed implicit alignment can significantly mitigate the sufficiency gap. In contrast, the probability calibration is not improved. This results also verifies the inequivalence between the sufficiency gap and calibration gap (Liu et al., 2019).

Algorithm 2 Conjugate Gradient Method

Ensure: Function F that computes Hessian-vector product through `autograd`, initial value X_0, bias vector B.

Computing Residual: $r_0 = B - F(X_0)$
Set $p_0 = r_0$
for inner_iterations k **do**
  Computing $\alpha_k \leftarrow \frac{r_k^T r_k}{p_k^T F(p_k)}$
  $X_{k+1} \leftarrow X_k + \alpha_k p_k$
  $r_{k+1} \leftarrow r_k - \alpha_k F(p_k)$
  If r_{k+1} is sufficiency small, then stop.
  $\beta_k \leftarrow \frac{r_{k+1}^T r_{k+1}}{r_k^T r_k}$
  $p_{k+1} \leftarrow r_{k+1} + \beta_k p_k$
end for
return X_{k+1}

```
    partial_grad = torch.autograd.grad(loss, model_parameters(),
    create_graph=True)
    flat_grad = torch.cat([g.contiguous().view(-1) for g in partial_grad
    ])
    h = torch.sum(flat_grad * vector_to_optimize)
    hvp = torch.autograd.grad(h, model.parameters())
    return hvp
```

Listing 1: Simple demo in computing Hessian vector product

E.2 Calibration Gap in the Regression

Based on Kuleshov et al. (2018), we first compute the predicted cumulative distribution ($\hat{Y}_0$) of at point t: $D_0(\hat{Y}_0 \leq t) = \alpha$, then we compute the corresponding ground truth cumulative distribution (Y_0) at point t. By changing t, we obtain several points on function $\mathcal{D}_0(Y \leq t|\hat{Y}_0 \leq t) = \beta$. Then the regression is probabilistic calibrated when $\alpha \equiv \beta$. From this perspective, the zero calibration gap can guarantee a zero sufficiency gap. But the inverse is not necessarily true, as our experimental results suggest, a small sufficiency gap can lead to either small or large calibration gap. Thus it can be quite promising to explore their inherent relations and trade-off in the fair regression.