G1: Teaching LLMs to Reason on Graphs with Reinforcement Learning

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Abstract

Although Large Language Models (LLMs) have demonstrated remarkable progress, their proficiency in graph-related tasks remains notably limited, hindering the development of truly general-purpose models. Previous attempts, including pretraining graph foundation models or employing supervised fine-tuning, often face challenges such as the scarcity of large-scale, universally represented graph data. We introduce G1, a simple yet effective approach demonstrating that Reinforcement Learning (RL) on synthetic graph-theoretic tasks can significantly scale LLMs' graph reasoning abilities. To enable RL training, we curate Erdős, the largest graph reasoning dataset to date comprising 50 diverse graph-theoretic tasks of varying difficulty levels, 100k training data and 5k test data, all drived from real-world graphs. With RL on Erdős, G1 obtains substantial improvements in graph reasoning, where our finetuned 3B model even outperforms Qwen2.5-72B-Instruct (24x size). RL-trained models also show strong zero-shot generalization to unseen tasks, domains, and graph encoding schemes, including other graph-theoretic benchmarks as well as real-world node classification and link prediction tasks, without compromising general reasoning abilities. Our findings offer an efficient, scalable path for building strong graph reasoners by finetuning LLMs with RL on graph-theoretic tasks, which combines the strengths of pretrained LLM capabilities with abundant, automatically generated synthetic data, suggesting that LLMs possess graph understanding abilities that RL can elicit successfully.

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1. Introduction

Large Language Models (LLMs) have achieved widespread success (Brown et al., 2020; Guo et al., 2025) but exhibit notable limitations in reasoning about graph-structured data, a critical capability for achieving general-purpose intelligence. Proficient graph reasoning is essential for numerous applications, yet even state-of-the-art LLMs like OpenAI's o1 (OpenAI et al., 2024) demonstrate significant deficiencies, with reported accuracies as low as 58.49% on graph connectivity tests (Yuan et al., 2025).

Initial efforts to enhance LLMs' graph understanding explored various natural language encoding schemes (Fatemi et al., 2023; Chu et al., 2025b; Das et al., 2024), but these yielded only modest improvements. Alternative strategies have involved instruction tuning (Luo et al., 2024; Ye et al., 2024) or preference tuning (Chen et al., 2024; Wang et al., 2024a) on curated graph datasets. Others attempted to build specialized graph foundation models through pretraining (Mao et al., 2024; Kong et al., 2025; Liu et al., 2024); however, these are often limited by the lack of large-scale, universal graph representations suitable for diverse graphs. See more discussions on related works in Appendix A. Different from prior work, we believe LLMs pretrained on Internet-scale data already possess graph reasoning ability and can be elicited through trial and error without human data.

In this work, we are the first to explore the use of Reinforcement Learning (RL) to solve graph reasoning tasks. We chose graph-theoretic problems as a testbed as they allow direct verification of generated answers to produce rule-based rewards for RL training, which is shown to be key for the success of DeepSeek R1 in math and coding problems (Guo et al., 2025). We collect the largest-to-date graph-theoretic problem set, Erdős, with either groundtruth answers or automatic verification programs. As illustrated in Table 1, these tasks span a wide spectrum of difficulty levels, from basic graph properties like node counting to NP-hard problems such as finding the maximal independent set. Another advantage of adopting graph-theoretic tasks is its circumvention of scarce human-annotated data; the model learns through exploration and reinforcement on synthetic tasks where ground-truth outcomes provide direct reward signals, similar to the AlphaGo-Zero paradigm (Silver et al., 2017). Besides data construction, we also study various aspects of the training process, such as the influence of data

Table 1: An overview of 50 graph-theoretic tasks in our dataset Erdős (100k train, 5k test), alongside with the difficulty distribution, and the accuracy of the base model Qwen2.5-7B-Instruct and our RL-trained G1-7B model. A complete description of tasks are in Appendix G.2.

Difficulty	Tasks	Ratio	Base Model Acc	G1 Acc
Easy	Node Number, Dominating Set, Common Neighbor, Edge Number, Neighbor, BFS, Has Cycle, DFS, Minimum Spanning Tree, Edge Existence, Is Regular, Degree, Is Tournament, Density	29.16%	57.16%	**95.07%**
Medium	Adamic Adar Index, Clustering Coefficient, Connected Component Number, Bipartite Maximum Matching, Local Connectivity, Jaccard Coefficient, Min Edge Covering, Is Eularian, Degree Centrality, Is Bipartite, Resource Allocation Index	22.91%	42.55%	**88.91%**
Hard	Max Weight Matching, Closeness Centrality, Traveling Salesman Problem, Strongly Connected Number, Shortest Path, Center, Diameter, Barycenter, Radius, Topological Sort, Periphery, Betweenness Centrality, Triangles, Average Neighbor Degree, Harmonic Centrality, Bridges	33.33%	18.87%	**50.44%**
Challenging	Isomophic Mapping, Global Efficiency, Maximal Independent Set, Maximum Flow, Wiener Index, Hamiltonian Path, Min Vertex Cover	14.58%	3.29%	**23.57%**

mixture, supervised initialization, and the use of chain-of-thought (Wei et al., 2022). Our results confirm that RL with synthetic graph-theoretic task is a powerful and scalable approach to improving graph reasoning abilities of LLMs.

Our work makes the following key contributions:

- We are the first to apply reinforcement learning (RL) framework to improving LLMs on graph reasoning tasks. The resulting model G1 significantly enhances the graph reasoning abilities of LLMs across a diverse set of synthetic tasks, demonstrating that appropriately finetuned LLMs can become stronger graph reasoners.
- We introduce Erdős, the largest-scale and most comprehensive graph-theoretic dataset that comprises 50 distinct tasks of varying complexities, uniquely constructed from diverse real-world graphs, providing a reliable platform for training and evaluating graph reasoning.
- We empirically demonstrate that G1 achieves substantial performance improvements on our Erdős benchmark, with gains of up to 46% over baseline models. Notably, our finetuned G1-7B model attains competitive performance with state-of-the-art reasoning models like OpenAI's o3-mini and G1-3B easily rivals Qwen2.5-72B-Instruct by noticeable margins.
- G1 models exhibit strong zero-shot generalization on unseen graph tasks and domains, improving base models' performance on other graph-theoretic benchmarks (GraphWiz and GraphArena) and real-world graphs (Cora and PubMed) without deteriorating general reasoning ability (GSM8K, MATH, and MMLU-pro), indicating a synergetic improvement of LLMs' graph reasoning abilities through RL.

G1 charts a data-efficient and scalable course for developing LLMs with strong graph reasoning. By demonstrating that RL can unlock latent graph understanding within general-purpose LLMs using synthetic data, our work suggests a possible paradigm shift away from reliance on heterogeneous real-world graphs to build graph foundation models. This paves the way for more versatile AI systems capable of sophisticated reasoning across diverse data modalities.

2. Erdős: A Comprehensive Collection of Graph-theoretic Reasoning Tasks on Real-world Graphs

To facilitate rule-based Reinforcement Learning of LLMs (aka. Reinforcement Learning from Verifiable Rewards (RLVR)) on graphs, we construct a diverse, large-scale collection of graph-theoretic reasoning tasks. We name it Erdős to remember Paul Erdős, a seminal figure with diverse contributions to graph theory. Compared to real-world graph tasks, these graph-theoretic tasks allow clear rule-based determination of rewards for the answers sampled from LLMs. We categorize these tasks into **Easy, Medium, Hard, and Challenging**, based on their inherent problem complexity as well as current LLMs' ability to solve them (see a full list in Table 1). For the training split, there are a total of 100,000 question-answer pairs, evenly distributed across tasks with 2,000 examples each. We also reserve 5,000 test pairs with different questions for evaluation. We include a detailed comparison of Erdős with other graph reasoning benchmarks in Appendix G.1. Erdős can serve as a dataset for training LLMs as well as a benchmark for evaluating LLMs on graph-theoretic tasks. We will release all task prompts, problems, chain-of-thought exemplars, and solution verification programs for public use. Below is a more detailed description of the data collection process.

Graph-theoretic Tasks. We curate 50 graph-theoretic reasoning tasks available on NetworkX (Hagberg et al., 2008), one of the most widely used library for graph processing, and construct, as we know, the most comprehensive collection so far. In the difficulty level, the tasks vary from easy determination of graph attributes like node number counting, to well-known NP-hard problems like the traveling salesman problem. This collection includes both tasks for general graphs and tasks specific to directed graphs or weighted graphs, and covers a wide range of answer types including boolean, integer, float, node list, edge list, and node mapping.

Answer Generation. To generate the golden answer for each problem, we utilize the default solvers of NetworkX to automatically solve the problem. If there are multiple solutions to each question, we use NetworkX-based programs to verify the correctness of each generated solution. The procedure ensures rigorous rewarding attribution, avoiding both costly human labeling and potential bias and hacking brought by LLM judges.

Graph Sources. Most previous graph-theoretic datasets or benchmarks (Wang et al., 2023; Luo et al., 2024; Chen et al., 2024) consider random graphs, following Erdős-Rényi model (Erdös, 1959) or Barabási–Albert model (Barabási & Albert, 1999). However, these random graph models are often far from graphs encountered in real-world practice. To mitigate this gap, we utilize the real-world graphs from the Network Repository (Rossi & Ahmed, 2015), the largest network repository with thousands of donations in 30+ domains. As these graphs can be very large and infeasible for LLMs, we downsample the graphs by random walk with a restart strategy, generating subgraphs with sizes from 5 to 35 nodes, following common settings in previous work (Wang et al., 2023; Yuan et al., 2025; Tang et al., 2025).

Language Encoding. There are multiple ways to translate the graph structure into languages that LLMs can understand. Previous works explore serialized formats such as adjacency matrix, edge list, or graph embeddings (Fatemi et al., 2023; Dai et al., 2025; Ye et al., 2024), but fail to find a consistently good method. Here, we choose to describe the graph structure in a unified edge list format, *e.g.*, $(1, 2), (2, 3), \ldots$. In later experiments of Section 4.2, we show that our model trained on a single graph description method can even positively transfer to other formats.

3. Training LLMs to Reason on Graphs

In this section, we introduce the training pipeline that we explored for training `G1`. We design proper rule-based rewards for different graph tasks, while intentionally keeping the RL algorithm general and consistent with previous work. Similar to DeepSeek R1 (Guo et al., 2025), the training of `G1` is very simple: it consists of a Reinforcement Learning phase for rewarding correct rollouts with the GRPO algorithm (Shao et al., 2024), and an *optional* SFT phase for warming up the model in the beginning (without which we call `G1`-Zero). We find that the SFT phase is generally beneficial for learning more challenging tasks, whose initial accuracy with the base model is close to zero.

3.1. Reinforcement Learning of LLMs on Graphs

Rule-based Rewards on Graphs. We design the following rule-based outcome reward model (ORM) for our training on graph-theoretic tasks, with a combination of value match, set matching, and algorithmic verification for different problems:

- *Strict value matching*. For tasks that have a unique ground truth value, e.g., node counting, the policy receives a reward of +1 only when the generated answer is identical to the ground truth in terms of numerical value, e.g., 0.5 and 1/2, otherwise it receives a reward of 0.

- *Jaccard Index for set matching.* For problems whose answer is not a single value $\hat{s}$ but an unordered set, e.g., common neighbors of two nodes, the reward is defined as the Jaccard Index between the generated set $\hat{s}$ and the ground truth s, i.e., $|s \cap \hat{s}|/|s \cup \hat{s}|$. In this way, the model can receive intermediate rewards for imperfect solutions.

- *Algorithmic verification.* Lastly, for problems that have multiple correct solutions (e.g., shortest paths) and it is not feasible to enumerate all of them, we implement algorithmic verifiers to check correctness of the proposed solutions. For instance, we determine the validness of a Hamiltonian path proposed by the policy by checking whether all the edges in the path exist and each node is visited exactly once.

RL Algorithm. Following common practice (Guo et al., 2025), we use the Group Relative Policy Optimization (GRPO) (Shao et al., 2024) algorithm for RL training. Specifically, for each question $q \sim P(Q)$ drawn from the training set, GRPO first samples a set of responses $\{o_i\}_{i=1}^G$ from the policy model. The responses receive rewards $\{r_i\}_{i=1}^G$, which enables calculating the group relative advantages $\{A_i\}_{i=1}^G$:

$$A_i = \frac{r_i - \mathrm{mean}(\{r_1, r_2, \cdots, r_G\})}{\mathrm{std}(\{r_1, r_2, \cdots, r_G\})}. \tag{1}$$

Next, the policy model π_θ is updated by maximizing the following objective:

$$\begin{aligned} \mathcal{J}_{\text{GRPO}}(\theta) = \mathbb{E}_{q,\{o_i\}_{i=1}^G} \frac{1}{G} \sum_{i=1}^{G} \Bigg[\min \Bigg(\frac{\pi_\theta(o_i|q)}{\pi_{\theta_{\text{old}}}(o_i|q)} A_i, \\ \text{clip} \left(\frac{\pi_\theta(o_i|q)}{\pi_{\theta_{\text{old}}}(o_i|q)}, 1-\epsilon, 1+\epsilon \right) A_i \Bigg) \Bigg] \\ - \beta \mathbb{D}_{\text{KL}} \left(\pi_\theta || \pi_{\text{ref}} \right), \end{aligned} \tag{2}$$

where the expectation is taken over $q \sim P(Q)$ and $\{o_i\}_{i=1}^{G} \sim \pi_{\theta_{old}}(O|q)$. The KL divergence to the reference policy π_{ref} (base model) prevents large deviation from the pretrained model and circumvents severe overfitting. Besides, ϵ controls the clipping range of the probability ratios.

3.2. Optional Warm-up with Supervised Fine-tuning

During RL training, we have noticed that for some challenging tasks like isomorphic mapping (see Table 1), the initial accuracy of the base model is often so low that we frequently end up with only incorrect rollouts, producing no useful signal for RL training. This issue can be mitigated by using a stronger base model with higher initial performance; for example, R1 uses DeepSeek V3 (671B parameters) as its base model, although this inevitably increases compute cost. We find that introducing a short warm-up phase with supervised fine-tuning, aimed at teaching the model basic reasoning skills before the RL phase, effectively improves overall learning efficiency. Specifically, in this paper we consider two types of supervised fine-tuning.

Direct-SFT. The first is direct supervised fine-tuning on question-answer pairs (q, a), where q is the textual description of the problem and a is the final answer without any intermediate reasoning steps. As discussed above, for graph-theoretic tasks, these question-answer pairs can often be synthesized by programming. However, this approach does not include the reasoning steps leading to the answers, meaning we cannot use it to explicitly teach the model reasoning processes.

CoT-SFT. Secondly, we can collect reasoning trajectories via sampling (q, c, a) triplets from another model (Yuan et al., 2023), where c represents the Chain-of-Thought (CoT) reasoning steps in natural language that lead to the final answer a, and use them to fine-tune the base model. Specifically, we instruct a base model to generate potential solutions for each question q, and only keep the correct responses that pass verification. This process is also called Rejection Sampling Fine-tuning (RFT) (Yuan et al., 2023). In practice, we use Qwen2.5-32B-Instruct (Team, 2024), a more capable model for generating candidate solutions more reliably, ending up with around 4,500 training examples for the SFT phase.

4. Experiments

4.1. Benchmarking G1 on Graph-theoretic Reasoning Tasks

Setup. As shown in Table 2, in the interest of academic compute budgets, we focus on comparing relatively small models. We include strong proprietary models (of unknown sizes) like GPT-4o-mini (non-reasoning) and OpenAI o3-mini (state-of-the-art reasoning), open-source instruction models like Qwen2.5-Instruct series (3B, 7B, 72B) (Team,

Table 2: Test accuracy (%) comparison of different LLMs of varying sizes on our Erdős benchmark tasks. In all experiments we use Qwen2.5-Instruct models as our base model (marked below). We report the average accuracy across all tasks in the *Average* column, and full results for each task are provided in Appendix E.5.

Model	Easy	Medium	Hard	Challenging	**Average**
		Proprietary (Unknown Parameters)			
GPT-4o-mini	76.20	72.07	28.81	3.34	47.60
OpenAI o3-mini (w/ tool use)	74.83	83.49	59.28	43.22	64.90
		3B Parameters			
Llama-3.2-3B-Instruct	36.50	21.45	6.81	1.14	17.32
Qwen2.5-3B-Instruct (base model)	45.71	30.18	9.44	1.29	22.72
Direct-SFT-3B (Ours)	74.43	75.27	**43.69**	**14.43**	53.78
CoT-SFT-3B (Ours)	65.57	67.64	29.44	4.57	43.56
G1-3B (Ours)	**94.86**	**84.64**	41.25	7.57	**59.76** (+37.04)
		7B Parameters			
Llama-3.1-8B-Instruct	49.21	30.45	13.69	1.43	25.10
Qwen2.5-7B-Instruct (base model)	57.36	42.55	18.87	3.29	32.06
Qwen2.5-Math-7B-Instruct	52.79	39.64	14.82	2.46	28.94
DeepSeek-R1-Distill-Qwen-7B	71.79	73.73	39.12	16.57	51.64
GraphWiz-7B-RFT	14.57	13.73	1.38	0.47	7.70
GraphWiz-7B-DPO	20.36	19.09	1.44	0.78	10.59
Direct-SFT-7B (Ours)	73.57	75.91	39.12	10.71	51.76
CoT-SFT-7B (Ours)	72.57	75.73	38.50	11.00	51.34
G1-7B (Ours)	**95.07**	**88.91**	**50.44**	**23.57**	**66.16** (+34.10)
		70B Parameters			
Llama-3.1-70B-Instruct	68.07	55.45	31.87	4.44	42.28
Qwen2.5-72B-Instruct	71.71	67.81	33.37	8.22	47.16

2024), Qwen2.5-Math-Instruct (Yang et al., 2024), LLaMA-3 series (3B, 8B, 70B) (AI, 2024), and a strong baseline DeepSeek-R1-Distill-Qwen-7B (Guo et al., 2025) that is distilled from DeepSeek R1 with 671B parameters. Additionally, for reference, we incorporate previous training strategies for graph reasoning tasks such as GraphWiz-RFT and GraphWiz-DPO (Chen et al., 2024). We finetune our model from Qwen2.5-Instruct models (3B and 7B) for 300 steps with batch size 512 on a cluster of $8\times$A800 GPUs, using our dataset Erdős. More experimental details can be found in Appendix C.

Performance. As shown in Table 2, our proposed model G1-7B consistently outperforms most proprietary, open-source, and graph training counterparts by significant margins across all difficulty levels. With a notable average accuracy of 66.16%, G1-7B outperforms GPT-4o-mini (47.60%) by 18.56%, reaching competitive performance to a cutting-edge reasoning model like o3-mini (64.90%) that underwent much heavier training. Notably, our small variant G1-3B, delivers a strong average performance of 59.76%, surpassing open-source models including Qwen2.5-72B-Instruct (47.16%) and Llama-3.1-70B-Instruct (42.28%) with $20\times$ parameters.

Remark on SFT baselines. Interestingly, Direct-SFT emerges as a surprisingly strong baseline in Table 2. The 3B and 7B versions of Direct-SFT both outperform larger open-source models with 53.78% and 51.76% accuracy, suggesting that LLMs can discover some effective patterns by directly fitting targets. However, we also observe that with Direct-SFT, the 7B model yields no extra gain over the 3B model, while CoT-SFT and G1 (initialized with CoT-SFT)

Table 3: Test accuracy (%) by computational complexity on the GraphWiz benchmark.

Model	Linear	Poly	NP-Complete	Avg.
Llama-3.2-3B-Instruct	29.80	3.00	2.50	19.80
Qwen2.5-3B-Instruct (base)	40.25	9.58	**69.12**	36.44
G1-3B	**58.06**	**26.75**	**69.12**	**50.08**
Llama-3.1-8B-Instruct	54.00	5.67	32.12	33.03
DeepSeek-R1-Distill-Qwen-7B	57.69	31.42	70.88	51.86
GraphWiz-7B-RFT	67.56	29.83	43.38	49.61
GraphWiz-7B-DPO	63.88	**36.25**	39.50	49.25
Qwen2.5-7B-Instruct (base)	49.06	17.92	**76.12**	44.69
G1-7B	**68.00**	32.25	72.62	**57.11**

Table 4: Test accuracy (%) by computational complexity on the GraphArena benchmark.

Model	Poly-Time		NP-Complete		Avg.
	Easy	Hard	Easy	Hard	
Llama-3.2-3B-Instruct	22.25	6.75	8.00	0.66	8.40
Qwen2.5-3B-Instruct (base)	31.50	14.50	17.33	1.50	14.85
G1-3B	**57.50**	**26.75**	**24.66**	**1.83**	**24.80**
Llama-3.1-8B-Instruct	47.00	21.25	22.00	2.16	20.90
DeepSeek-R1-Distill-Qwen-7B	66.0	22.75	34.83	1.50	28.65
GraphWiz-7B-RFT	2.25	0.75	0.83	0.00	0.85
GraphWiz-7B-DPO	0.25	1.00	0.66	0.16	0.49
Qwen2.5-7B-Instruct (base)	62.00	35.75	28.83	2.16	28.84
G1-7B	**77.50**	**44.25**	**47.33**	**8.50**	**41.10**

Table 5: Test accuracy (%) on Node Classification and Link Prediction benchmarks.

Model	Node		Link		Avg.
	Cora	PubMed	Cora	PubMed	
Llama-3.2-3B-Instruct	68.77	75.20	60.40	57.60	64.79
Qwen2.5-3B-Instruct (base)	70.83	75.08	62.15	58.38	65.66
CoT-SFT-3B	75.97	81.47	75.70	**71.52**	75.12
G1-3B	**77.25**	**83.88**	**78.97**	69.75	**75.16**
Llama-3.1-8B-Instruct	70.90	75.00	50.60	46.10	59.53
DeepSeek-R1-Distill-Qwen-7B	76.50	81.25	68.03	78.72	78.80
Qwen2.5-7B-Instruct (base)	**79.30**	85.35	**88.22**	88.67	85.50
CoT-SFT-7B	73.20	83.25	64.70	68.12	73.17
G1-7B	79.20	**86.20**	87.98	**91.88**	**87.29**

Table 6: Test accuracy (%) on reasoning benchmarks beyond graph-related tasks.

Model	GSM8K	MATH	MMLU-pro
Llama-3.2-3B-Instruct	71.03	42.40	13.50
Qwen2.5-3B-Instruct (base)	**81.95**	**62.20**	**38.53**
CoT-SFT-3B	75.36	56.00	34.85
G1-3B	79.30	61.80	37.11
Llama-3.1-8B-Instruct	74.45	44.80	32.02
DeepSeek-R1-Distill-Qwen-7B	86.03	**87.20**	37.21
Qwen2.5-7B-Instruct (base)	86.27	69.80	45.75
CoT-SFT-7B	83.85	65.80	44.79
G1-7B	**87.49**	71.80	**48.56**

performance scales with larger models. This indicates that even though the CoT-SFT performance may appear low compared to Direct-SFT (possibly because of limited data size with about 100 examples per task), CoT-SFT could have better scaling and generalization properties.

4.2. Transferability of G1 to Unseen Tasks and Domains

In this section, we evaluate *zero-shot* generalization of G1 to unseen domains, tasks, and data formats. Detailed benchmark description and complete evaluation setups are provided in Appendix D.

4.2.1. G1's Transferability to Other Graph Reasoning Benchmarks

We consider two additional graph reasoning benchmarks, *GraphWiz* (Chen et al., 2024) and *GraphArena* (Tang et al., 2025), which bring three major shifts that challenge our model: 1) different distributions of the underlying graphs 2) tasks unseen during training 3) unfamiliar graph encoding formats, e.g., the GraphArena benchmark represents nodes with human names instead of integers.

The performance across models is reported in Table 3 and Table 4. On the GraphWiz benchmark, G1-7B achieves the highest overall accuracy (57.11%) among all models, outperforming DeepSeek-R1-Distill-Qwen-7B (51.86%) and even models specifically trained on GraphWiz data such as GraphWiz-7B-RFT (49.61%). The smaller variant G1-3B also achieves comparable performance with DeepSeek-R1-Distill-Qwen-7B. Similar results can be found on the GraphArena benchmark (Table 4) with a different graph encoding scheme. These results demonstrate that G1 has strong zero-shot generalization ability to unseen graph encoding methods, graph distributions, and graph tasks. Full results for GraphWiz and GraphArena are shown in Appendix E.2 and Appendix E.4.

4.2.2. G1 on Real-world, Non-graph-theoretic Graph-reasoning Tasks

For real-world graph tasks, we consider two standard problems: node classification and link prediction. We adopt the benchmarks introduced by Wang et al. (2025), which are constructed by subsampling from the widely used Cora and PubMed citation graphs. As shown in Table 5, our model G1 significantly outperforms both open-source and distilled baselines across tasks and model sizes. In the 3B model category, G1-3B surpasses the base model (Qwen2.5-3B-Instruct) by a large margin—especially in link predic-

Table 7: Test accuracy (%) on our benchmark. ⋆ denotes the tasks are excluded in model training. G1-Hard-3B is only RL-trained on Hard and Challenging tasks.

Category	Model	Easy	Medium	Hard	Challenging	Average
Base Model	Qwen2.5-3B-Instruct	45.71	30.18	9.44	1.29	22.72
Ours	Direct-SFT-3B	74.43	75.27	<u>43.69</u>	<u>14.43</u>	<u>53.78</u>
	G1-3B	**94.86**	**84.64**	41.25	7.57	**59.76**
	G1-Hard-3B	69.36⋆	70.64⋆	**48.50**	**17.43**	53.30

tion on Cora (+16.82%) and node classification on PubMed (+8.8%). In the 7B model category, G1-7B achieves the highest average score of 87.29%, ranking first on PubMed dataset in both node classification and link prediction tasks. Overall, G1 consistently demonstrates strong generalization across real-world graph tasks where graph-text reasoning is required.

4.2.3. G1's Reasoning Ability beyond Graphs

We next extend our investigations of G1's abilities beyond graph-based tasks. We consider two mathematics benchmarks, GSM8K (Cobbe et al., 2021b) and MATH (Hendrycks et al., 2021), and a massive multi-task benchmark MMLU-Pro (Wang et al., 2024b). In table 6, we first notice that the CoT-SFT training on graph reasoning trajectories leads to a non-negligible degradation in general abilities, which could be attributed to the fact that SFT *memorizes* pattern instead of incentivizing truly generalizable skills (Chu et al., 2025a). Remarkably, the subsequent reinforcement learning stage—despite being trained exclusively on graph tasks—restores the reasoning abilities of both the 3B and the 7B model. G1-7B even surpasses the performance of the initial Qwen-7B checkpoint in all of the three benchmarks (87.49% v.s. 86.27% for GSM8K, 72.8% v.s. 69.8% for MATH, and 48.56% v.s. 45.75% for MMLU-pro). Interestingly, G1-7B also outperforms Qwen-7B-Instruct on several non-STEM tasks like Economy (68.76 v.s. 46.87), which are intuitively less related to graph reasoning (see Appendix E.3 for full MMLU-Pro results).

4.3. Training Analysis

In this section, we further analyze the influence of two training factors on G1's reasoning performance.

Data Mixture. In Table 2, we observe that although G1-3B achieves strong overall performance, it is outperformed by Direct-SFT-3B on the *Hard* and *Challenging* subsets. We hypothesize that this gap arises from imbalanced reward signals across different difficulty levels during RL training. Since correct rollouts are much easier to obtain on simpler tasks, the policy tends to allocate more of its constrained probability ratios as well as KL budget to optimize for *Easy* and *Medium* tasks, thereby maximizing the overall rewrad. To test this hypothesis, we introduce G1-Hard-3B, which is trained exclusively on *Hard* and *Challenging* tasks during

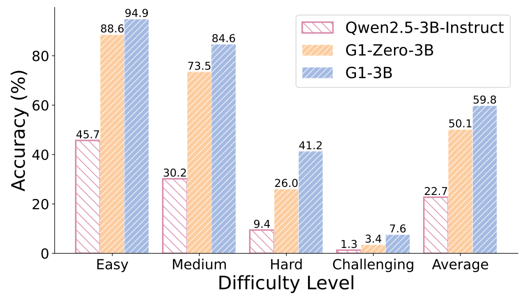


Figure 1: Test accuracy comparison of G1-3B and G1-Zero-3B on our benchmark. Results for -7B are in Appendix E.1.

RL. As shown in Table 7, this model achieves the highest accuracy on *Hard* (48.50%) and *Challenging* (17.43%) tasks, surpassing both G1 and Direct-SFT. These results support our claim, suggesting that the suboptimal performance of G1-3B on challenging tasks is a natural consequence of the uniformly weighted reward function, rather than a shortcoming of G1 training pipeline. Notably, despite being trained only on hard tasks, G1-Hard-3B also generalizes to *Easy* and *Medium* tasks (69.36% and 70.64%), far exceeding the baseline Qwen2.5-3B-Instruct. This indicates that learning to solve difficult tasks confers transferable reasoning skills that benefit performance on simpler problems. To better balance the optimization process across difficulty levels, we further explore reward-weighting strategies in Appendix F.

SFT Warmup. We study the role of SFT as a cold-start mechanism for RL, evaluating its impact on both performance and response behavior. To isolate the effect of SFT, we compare two variants: G1-Zero-3B that is directly trained from the base model Qwen2.5-3B-Instruct with RL, and G1-3B that initializes RL from the CoT-SFT checkpoint. As shown in Figure 1, training RL directly from the base model achieves surprisingly strong performance, aligning with recent findings in Deepseek-R1-Zero (Guo et al., 2025). Meanwhile, initializing RL with CoT-SFT provides clear and consistent improvements across all difficulty levels, with an average accuracy of 59.8% compared to 50.1% of G1-Zero-3B. Besides, we notice that relative improvements become larger as the difficulty increases. In addition to performance gains, we also observe that models initialized by CoT-SFT present more precise reasoning patterns, illustrated by the case study in the following section.

4.4. Understanding the Benefits of RL Training for Graph Reasoning

To understand how RL training helps graph reasoning, we take *shortest path* as a case study. Specifically, we study the behaviors of three models: Qwen2.5-3B-Instruct (base), G1-Zero-3B (RL only), and G1-3B (SFT & RL).

We identify three primary approaches adopted by the models to solve the problem: 1) Breadth-First Search (BFS),

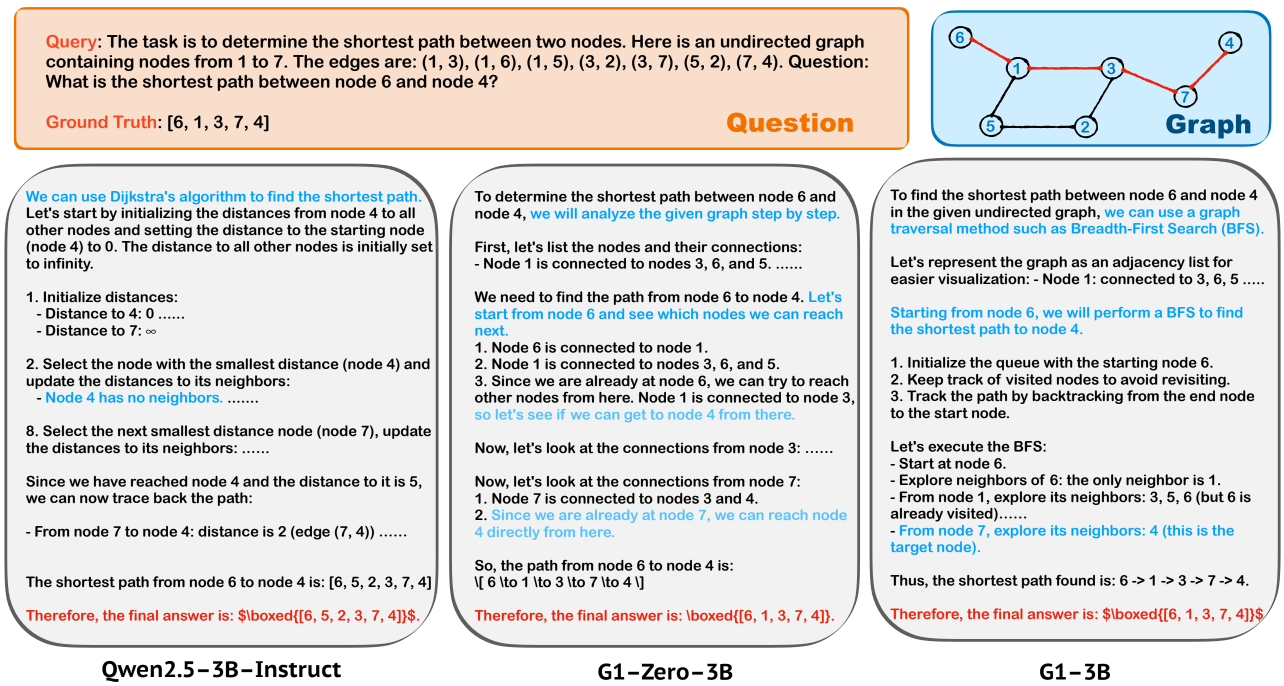
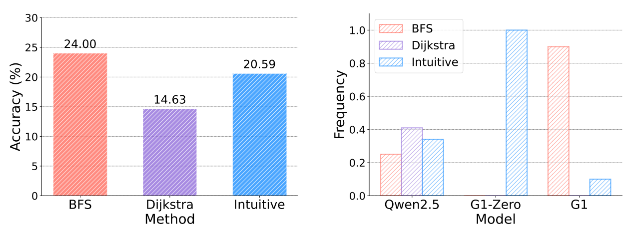


Figure 2: An intuitive illustration of the differences in solution strategies employed by Qwen2.5-3B-Instruct, G1-Zero-3B, and G1-3B for a shortest path problem.



(a) The accuracy of different graph reasoning patterns for shortest path on Qwen2.5-3B-Instruct.

(b) Frequency of different graph reasoning patterns for Qwen2.5-3B-Instruct, G1-Zero-3B and G1-3B.

Figure 3: Reasoning patterns for the shortest path task.

2) Dijkstra's algorithm, and 3) Intuitive deductions. Figure 3a shows the distribution of these approaches alongside their corresponding accuracies for Qwen2.5-3B-Instruct. On *unweighted* graphs, BFS is the most efficient method and yields the highest performance. In contrast, Dijkstra's algorithm is best suited for *weighted* graphs, where it correctly accounts for edge costs. However, its reliance on a min-priority queue and a distance list introduces computational complexity, which appears to challenge Qwen2.5-3B-Instruct and results in its lowest observed accuracy. For example, as shown in Figure 2 (left), the model falsely states that node 4 has no edges (node 4 is connected to node 7) while updating the distance list. Interestingly, intuitive approaches—where the model attempts to visually estimate or heuristically trace paths—can also produce correct answers by a noticeable accuracy, particularly on small graphs.

We proceed by observing that RL training significantly reshapes the models' graph reasoning strategies: RL-trained models largely abandon Dijkstra and prefer a combination of BFS and intuitive search. As shown in Figure 3b and Figure 2 (middle), G1-Zero-3B navigates the graph in a manner akin to human heuristics—sequentially checking neighbors and adjusting paths dynamically. G1-3B primarily adopts a neat BFS-style algorithm as in Figure 3b and Figure 2 (right), executing it with high precision, occasionally resorting to intuitive strategies for simple graphs. To conclude, our case study highlights how RL training enhances graph reasoning by guiding LLMs toward more model-aware strategies that are adaptive to their inherent capabilities (Wu et al., 2025).

5. Discussion

In this paper, we explored the use of RL to improve LLMs' reasoning abilities on gragh reasoning and demonstrate significant improvements across a spectrum of tasks with various difficulty levels, showing that graph reasoning of LLMs can be elicited via RL training (even with only 300 steps). We also comprehensively evaluate the transferability of RL-trained models to unseen graph reasoning tasks, real-world graph tasks, and general reasoning tasks, observing strong zero-shot generalization. These results support that training LLMs on diverse synthetic graph-theoretic tasks via RL offers a scalable, generalizable path toward robust graph reasoning. As a first step, this approach may guide the development of efficient, general-purpose graph reasoners.

Impact Statement

This paper presents work whose goal is to advance the field of Large Language Models on graph reasoning. There are many potential societal consequences of our work, none which we feel must be specifically highlighted here.

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A. Related Work

Graph Reasoning. Graph reasoning problems fall into two categories: domain-specific, which require understanding both graph structures and node/link attributes, *e.g.*, node classification, link prediction, and knowledge-based QA (Hamilton et al., 2017; Zhang & Chen, 2018; Huang et al., 2019); and domain-agnostic, also called *graph-theoretic problems*, which focus solely on structural reasoning but find a lot of practical uses in various domains, *e.g.*, shortest paths, Hamiltonian paths, graph isomorphism (Xu et al., 2019a; Sato et al., 2019). For the latter problems that we study in this paper, people have studied the use of RL (Mirhoseini et al., 2021; Wang et al., 2020) or unsupervised learning (Karalias & Loukas, 2020), often in conjunction with Graph Neural Networks (GNNs) (Kipf & Welling, 2016; Xu et al., 2018) that align with the solution structure (Xu et al., 2019b). Yet these models are often built to solve each problem alone. Recently, Sanford et al. (2024) prove and validate the priority of the transformer models compared to GNNs on complex graph reasoning tasks requiring long-range dependencies. In this work, we focus on building general-purpose graph reasoners that could solve a range of graph-theoretic problems by exploiting the strength of LLM pretraining, and find that the ability also generalizes to the former domain-specific graph tasks.

Benchmarking LLMs on Graph Reasoning. There is a growing interest in evaluating LLMs' graph reasoning abilities. NLGraph (Wang et al., 2023) evaluate LLMs on graph-theoretic tasks and discover preliminary yet brittle reasoning abilities in the face of spurious correlations and large graphs. Later, GraphArena (Tang et al., 2025) and GraCoRe (Yuan et al., 2025) include a broader task coverage and recently released LLMs, finding that even OpenAI o1-mini struggles a lot with complex tasks. Moreover, GraphEval2000 (Wu et al., 2024) and ProGraph (Li et al., 2024) emphasize code-oriented problem solving using library-based prompts, and GraphOmni (Xu et al., 2025) unify varying graph types, encodings, and prompt styles for a comprehensive evaluation. Overall, these benchmarks suggest that LLMs overall demonstrate moderate success on simple tasks but struggle with abstraction, generalization, and larger or more complex graph instances. Nevertheless, these datasets are either too small (e.g., thousands of examples) or not diverse enough (e.g., 8 tasks in NLGraph) for training general-purpose graph reasoners, which motivates the design of `Erdős`.

Improving LLMs on Graph Reasoning. A major concern when using LLMs for graph tasks is the mismatch of data structure: LLMs take text sequences as input, while graphs have no natural order. Fatemi et al. (2023) analyzed different graph encoding schemes for LLMs, such as adjacency lists and real-name networks, revealing that no single strategy proved universally optimal across all tasks and models. Subsequent explorations with different linearization orders (Chu et al., 2025b), graph embeddings (Perozzi et al., 2024), or input modalities (Das et al., 2024) have generally resulted in only modest improvements. Another thread of research proposes post-training LLMs using instruction tuning (Luo et al., 2024; Ye et al., 2024) or preference tuning (Chen et al., 2024; Wang et al., 2024a; Veličković et al., 2020) on curated datasets of graph problems. However, the creation of diverse, high-quality instruction datasets at scale is challenging and expensive and requires extra supervision. Furthermore, models trained via distillation may only learn to memorize patterns and overfit to graph tasks (Chu et al., 2025a); in Section 4.2, we show that previous instruction-tuned models exhibit dramatic failures when generalizing to other data formats and reasoning tasks, while our RL training yields consistently better performance.

Reinforcement Learning for LLMs Reasoning. Recent advances have demonstrated that LLMs can attain strong reasoning abilities in math and coding domains through RL, with representative work like OpenAI o1 (OpenAI et al., 2024) and DeepSeek R1 (Guo et al., 2025). However, as discussed above, even o1 struggles a lot with graph reasoning tasks (Yuan et al., 2025) and it is thus yet unclear whether RL can reliably and scalably improve LLMs' graph reasoning abilities. Our findings on `G1` first confirm the effectiveness of RL on graph reasoning as well and suggest that applying RL to diverse graph-theoretic tasks with verifiable rewards is a scalable path for eliciting generalizable graph reasoning abilities of LLMs.

B. Optional Warm-up with Supervised Fine-tuning

During RL training, we have noticed that for some challenging tasks like isomorphic mapping (see Table 1), the initial accuracy of the base model is often so low that we frequently end up with only incorrect rollouts, producing no useful signal for RL training. This issue can be mitigated by using a stronger base model with higher initial performance; for example, R1 uses DeepSeek V3 (671B parameters) as its base model, although this inevitably increases compute cost. We find that introducing a short warm-up phase with supervised fine-tuning, aimed at teaching the model basic reasoning skills before the RL phase, effectively improves overall learning efficiency. Specifically, in this paper we consider two types of supervised fine-tuning.

Direct-SFT. The first is direct supervised fine-tuning on question-answer pairs (q, a), where q is the textual description of the problem and a is the final answer without any intermediate reasoning steps. As discussed above, for graph-theoretic tasks, these question-answer pairs can often be synthesized by programming. However, this approach does not include the reasoning steps leading to the answers, meaning we cannot use it to explicitly teach the model reasoning processes.

CoT-SFT. Secondly, we can collect reasoning trajectories via sampling (q, c, a) triplets from another model (Yuan et al., 2023), where c represents the Chain-of-Thought (CoT) reasoning steps in natural language that lead to the final answer a, and use them to fine-tune the base model. Specifically, we instruct a base model to generate potential solutions for each question q, and only keep the correct responses that pass verification. This process is also called Rejection Sampling Fine-tuning (RFT) (Yuan et al., 2023). In practice, we use Qwen2.5-32B-Instruct (Team, 2024), a more capable model for generating candidate solutions more reliably, ending up with around 4,500 training examples for the SFT phase.

C. Training Details

C.1. Setups for evaluation on Erdős

As shown in Table 2, in the interest of academic compute budgets, we focus on comparing relatively small models. We include strong proprietary models (of unknown sizes) like GPT-4o-mini (non-reasoning) and OpenAI o3-mini (state-of-the-art reasoning), open-source instruction models like Qwen2.5-Instruct series (3B, 7B, 72B) (Team, 2024), Qwen2.5-Math-Instruct (Yang et al., 2024), LLaMA-3 series (3B, 8B, 70B) (AI, 2024), and a strong baseline DeepSeek-R1-Distill-Qwen-7B (Guo et al., 2025) that is distilled from DeepSeek R1 with 671B parameters. Additionally, for reference, we incorporate previous training strategies for graph reasoning tasks such as GraphWiz-RFT and GraphWiz-DPO (Chen et al., 2024). We finetune our model from Qwen2.5-Instruct models (3B and 7B) for 300 steps with batch size 512 on a cluster of 8$\times$A800 GPUs, using our dataset Erdős. More experimental details can be found in Appendix C.

C.2. Rejection Sampling

We randomly extract a subset with 100 examples per task from the training dataset, and use Qwen2.5-32B-Instruct to sample on the subset for k = 8 times with a temperature of 1.0. We filter the responses by keeping the reasoning steps that lead to the right answer. If the task is difficult and the filtered responses are insufficient, we resample the subset with a different random seed and repeat the process above. In the end, we obtain around 4,500 training examples ($\sim$90 per task) for the SFT phase.

C.3. Supervised Fine-tuning

The detailed training configurations of Naive SFT and RFT are presented in Table 8.

Table 8: Training configurations of Naive-SFT and RFT. In this table, batch size is abbreviated to BSZ, Max-Length refers to the maximum response length during training and Data Num. reports the number of training examples.

Setting	LR	Weight Decay	BSZ	Max-Length	Data Num.	Epoch
Naive-SFT	1e-5 w/ 1% warm-up	1e-2	64	512	98.7k	1
RFT	1e-5 w/ 1% warm-up	1e-2	64	3072	4.4k	2

C.4. Reinforcement Learning

Configurations for training and evaluation. Our experiments primarily adopt Qwen-2.5-3B/7B-Instruct (Qwen et al., 2025) for their moderate sizes and strong reasoning performance. For GRPO training, we set ϵ to be 0.02, β to be 0.001, group size G to be 5, and context length to be 4096 unless otherwise specified. We additionally incorporate an entropy loss of weight 0.001 to encourage the policy to explore. Lastly, we train the models on 8xA800 GPUs with batch size of 512. During evaluation, we use the vLLM (Kwon et al., 2023) engine for efficient inference. For DeepSeek-R1-Distill-Qwen-7B, we set the maximum token generation length to 4096 tokens except for DeepSeek-R1-Distill-Qwen-7B, which is extended to 30768 for its prolonged thinking process. Sampling is configured with a temperature of 0.6, top-p of 0.95, and top-k of 30.

The detailed RL training configurations are presented in Table 9.

Table 9: Training configurations for Naive-SFT and RFT. For abbreviation, we refer the coefficient for entropy loss as Ent. in this table. We report (batch size)/(number of gradient accumulation steps) in the BSZ column, and the temperature for on-policy sampling as T.

Model	LR	ϵ	$\|G\|$	β	γ	T	Ent.	BSZ	Max-Length	Data Num.	Steps
RL-3B	1e-6	0.2	5	1e-3	1.0	1.0	1e-3	512/4	4096	98.7k	300
SFT-RL-3B	1e-6	0.2	5	1e-3	1.0	1.0	1e-3	512/4	4096	98.7k	300
SFT-RL-Hard-3B	1e-6	0.2	16	5e-4	1.0	1.0	5e-4	512/8	8192	49.3k	150
SFT-RL-7B	1e-6	0.2	5	1e-3	1.0	1.0	1e-3	512/8	4096	98.7k	300

D. Evaluation Details

D.1. Benchmark Introduction

GraphWiz (Chen et al., 2024). GraphWiz employs the Erdős-Rényi (ER) model to generate random graphs and describe graphs in the edge-list formation like (u, v). The tasks include four linear complexity tasks, *Connectivity*, *Cycle Detection*, *Bipartite Graph Checking*, and *Topological Sort*; three polynomial complexity tasks, *Shortest Path*, *Maximum Triangle Sum*, and *Maximum Flow*; and two NP-Complete tasks: *Hamilton Path* and *Subgraph Matching*. A prompt example is shown in the following:

Maximum Triangle Sum Example in GraphWiz

Find the maximum sum of the weights of three interconnected nodes. In an undirected graph, [i, k] means that node i has the weight k. (i,j) means that node i and node j are connected with an undirected edge. Given a graph, you need to output the maximum sum of the weights of three interconnected nodes. Q: The nodes are numbered from 0 to 4, weights of nodes are: [0, 8] [1, 5] [2, 3] [3, 6] [4, 3], and the edges are: (0, 4) (0, 3) (0, 1) (1, 3) (1, 2) (3, 4). What is the maximum sum of the weights of three nodes?

GraphArena (Tang et al., 2025). GraphArena samples subgraphs from real-world graphs, including knowledge graphs, social networks, and molecular structures. The tasks include four polynomial-time tasks, *Common Neighbor*, *Shortest Distance*, *Connected Component*, *Graph Diameter*, and six NP-complete tasks, *Maximum Clique Problem (MCP)*, *Maximum Independent Set (MIS)*, *Minimum Vertex Cover (MVC)*, *Maximum Common Subgraph (MCS)*, *Graph Edit Distance (GED)*, and *Traveling Salesman Problem (TSP)*. Each problem is contextualized within the real-world setting of the graph with an example presented as below:

Connected Component Example in GraphArena

You are required to identify all connected components in the given social network and output one representative node from each component. Within a connected component, any node can be reached from any other node through the edges in the graph. Different connected components are isolated from each other.

Problem to Solve
- Names in the network: Veronica Garcia, Katherine Brennan, Angel Chavez, Steven Martin, Brett Johnson, Megan Banks, Julia Dominguez, Rachel Mitchell - Fiendship connections: Veronica Garcia to Brett Johnson, Veronica Garcia to Megan Banks, Katherine Brennan to Brett Johnson, Katherine Brennan to Megan Banks, Angel Chavez to Megan Banks, Angel Chavez to Rachel Mitchell, Steven Martin to Megan Banks, Brett Johnson to Megan Banks, Megan Banks to Julia Dominguez, Megan Banks to Rachel Mitchell.

Identify all connected components in this network. Note that for each connected component, you should only output one of its nodes. Present your answer in the following format: [UserA, UserB, UserC, UserD, ...]

Node Classification and Link Prediction (Wang et al., 2025). We adopt the benchmarks introduced by Wang et al. (2025), which are constructed by subsampling from the widely used Cora and PubMed citation graphs. Each instance includes a description of the target node (or node pair) containing the paper ID and title, along with the textual and structural information of neighboring nodes. For node classification, we consider two cases that the description includes the attributes of the target node and those of its 2-hop neighbors, with or without labels. For link prediction, we consider two cases where target nodes are described using their own node attributes along with those of their 2-hop neighbors (excluding the other targeting node), with or without titles. For each task, we randomly sample 2,000 examples per case from the benchmark and report the average performance. A representative example for node classification is shown below:

Node Classification Example

You are a good graph reasoner. Give you a graph language that describes a graph structure and node information from pubmed dataset. You need to understand the graph and the task definition and answer the question.

Target node: Paper id: 10695 Title: Haplotype structures and large-scale association testing of the 5' AMP-activated protein kinase genes PRKAA2, PRKAB1, and PRKAB2 [corrected] with type 2 diabetes.

Known neighbor papers at hop 1 (partial, may be incomplete):
Paper id: 1155 Title: Computational disease gene identification: a concert of methods prioritizes type 2 diabetes and obesity candidate genes. Label: Type 2 diabetes

Known neighbor papers at hop 2 (partial, may be incomplete):
Paper id: 9816 Title: Mitochondrial dysfunction and type 2 diabetes. Label: Type 2 diabetes
Paper id: 1683 Title: A genome-wide search for type II diabetes susceptibility genes in Chinese Hans. Label: Type 2 diabetes
Paper id: 9916 Title: Genomewide search for type 2 diabetes-susceptibility genes in French whites: evidence for a novel susceptibility locus for early-onset diabetes on chromosome 3q27-qter and independent replication of a type 2-diabetes locus on chromosome 1q21-q24.
Paper id: 3793 Title: Association of amino acid variants in the activating transcription factor 6 gene (ATF6) on 1q21-q23 with type 2 diabetes in Pima Indians. Label: Type 2 diabetes
Paper id: 4788 Title: Altered glycolytic and oxidative capacities of skeletal muscle contribute to insulin resistance in NIDDM. Label: Type 2 diabetes

Please predict the most likely type of the Target node. Your answer should be chosen from: Type 1 diabetes Type 2 diabetes Experimentally induced diabetes

GSM8K (Cobbe et al., 2021a). GSM8K is a dataset of 8.5K high quality linguistically diverse grade school math word problems created by human problem writers. We report the accuracies on the 1K test problems and the dataset is downloaded via `https://huggingface.co/datasets/openai/gsm8k`.

Example in GSM8K

Natalia sold clips to 48 of her friends in April, and then she sold half as many clips in May. How many clips did Natalia sell altogether in April and May?

MATH500. The dataset contains a subset of 500 problems from the MATH benchmark that OpenAI created in their Let's Verify Step by Step paper (Lightman et al., 2023). We download the dataset via `https://huggingface.co/datasets/HuggingFaceH4/MATH-500`.

Example in MATH500

Let $z = 2 + \sqrt{2} - (3 + 3\sqrt{2})i$, and let $c = 2 - 3i$. Let w be the result when z is rotated around c by $\frac{\pi}{4}$ counter-clockwise.
[asy]
unitsize(0.6 cm);
pair C, W, Z;
Z = (2 + sqrt(2), -3 - 3*sqrt(2));
C = (2,-3);
W = rotate(45,C)*(Z);
draw(Z–C–W);
dot("c", C, N);
dot("w", W, SE);
dot("z", Z, S);
label("$\frac{\pi}{4}$", C + (0.6,-1));
[/asy]
Find w.

MMLU-Pro. MMLU-Pro is enhanced version of the Massive Multitask Language Understanding benchmark. It covers a wide range of disciplines, including Math, Law, Engineering, Health, Phycology, etc. We download the dataset via `https://huggingface.co/datasets/TIGER-Lab/MMLU-Pro/viewer/default/test?q=Health&row=5903`.

Health Example in MMLU-pro

Question: Food supplements, including trace minerals and vitamins are frequently advertised with promising health benefits. Which of the following substance could be consumed in excess, i.e. well above the recommended daily requirement?

Options: ["Vitamin C", "Vitamin D", "Zinc", "Vitamin A"]

D.2. Inference Configuration

For inference, we adopt the vLLM framework (Kwon et al., 2023). We set the temperature to be 0.06 and the context window to be 4096 for our evaluations unless otherwise specified.

D.3. Prompt and Answer Extraction

To facilitate answer extraction, we adopt the prompt shown in D.3 to guide the models to reason step by step and place their answers within \boxed{}. We extract the last \boxed{} shown in the model responses and do necessary format normalizations to retrieve the answer, which includes operations like converting LaTeX-style fraction numbers to float numbers.

Problem Instructions

{Question Description}
Approach the problem methodically. Ensure all conclusions are based on precise calculations and logical deductions.
Feel free to explore various solution methods and cross-check results for consistency. Maintain dynamic thinking and always verify each step of your reasoning.
Present the final answer in \boxed{} format, like this: $\boxed{ANSWER}$, where ANSWER is the final result or expression.
Think carefully and break down the problem step by step.

E. Additional Experiment Results

E.1. Results for G1-Zero-7B

In Section 4.3, we study the role of SFT as a cold-start mechanism for RL by comparing two variants: G1-Zero-3B that is directly trained from the base model Qwen2.5-3B-Instruct with RL, and G1-3B that initializes RL from the CoT-SFT checkpoint. We observe that G1-Zero-3B already achieves surprisingly strong performance, while G1-3B presents clear and consistent improvements across all difficulty levels. Here, we provide additional results for comparing G1-Zero-7B and G1-7B. As shown in Figure 4, for *Easy* and *Medium* tasks, the benefit brought by CoT-SFT initialization is marginal, with G1-Zero-7B (96.9%) even surpassing G1-7B (95.1%) on *Easy* tasks. However, on *Hard* and *Challenging* tasks, CoT-SFT as a preliminary step has definite benefits by improving G1-Zero-7B from 13.7% to 23.6% on *Challenging* tasks. This observation agrees with the case in -3B. Moreover, the average gap between G1-Zero-7B and G1-7B is less than -3B case, indicating G1-7B can possibly be further improved with CoT-SFT generated by a stronger teacher model rather than Qwen2.5-32B-Instruct. We leave this exploration for further work.

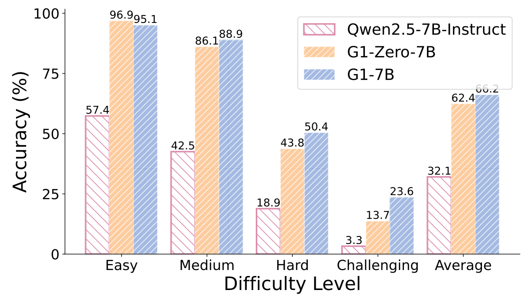

Figure 4: Test accuracy comparison of G1-7B and G1-Zero-7B on our benchmark.

E.2. Detailed Results for GraphWiz

We present the test accuracy for each task in the GraphWiz benchmark in Table 10. G1-7B achieves the highest overall accuracy (57.11%) among all models and reaches the top in 5/7 tasks. It outperforms DeepSeek-R1-Distill-Qwen-7B (51.86%) and even models specifically trained on GraphWiz data such as GraphWiz-7B-RFT (49.61%). Moreover, the smaller variant G1-3B ranks first on all tasks among models of similar parameters, surpassing the base model (Qwen2.5-3B-Instruct) by 13.64% on average and achieves comparable performance with DeepSeek-R1-Distill-Qwen-7B. The results in the GraphWiz benchmark verify the strong zero-shot generalization ability of our G1 models.

Table 10: Test accuracy (%) on the GraphWiz benchmark.

Model	cycle	connect	bipartite	topology	shortest	triangle	flow	hamilton	subgraph	Avg.
Llama-3.2-3B-Instruct	32.00	53.75	25.75	7.50	2.75	3.75	2.50	38.25	12.00	19.80
Qwen2.5-3B-Instruct (base)	58.00	60.50	38.50	4.00	5.75	15.50	7.50	75.00	**63.25**	36.44
G1-3B (Ours)	**91.00**	**64.00**	**64.25**	**13.00**	**14.00**	**23.25**	**43.00**	**96.00**	42.25	**50.08**
GraphWiz-RFT-7B	88.00	**90.25**	72.25	19.75	**28.00**	36.75	24.75	2.50	**84.25**	49.61
GraphWiz-DPO-7B	86.50	82.25	71.75	15.00	26.75	**37.00**	45.00	0.00	79.00	49.25
Llama-3.1-8B-Instruct	64.75	81.00	58.75	11.50	3.50	4.25	9.25	19.25	45.00	33.03
DeepSeek-R1-Distill-Qwen-7B	87.00	90.00	42.75	11.00	18.25	36.00	40.00	84.75	57.00	51.86
Qwen2.5-7B-Instruct (base)	79.00	72.25	40.75	4.25	13.50	28.75	11.50	91.25	61.00	44.69
G1-7B (Ours)	**92.00**	80.00	**75.75**	**24.25**	21.00	29.50	**46.25**	**95.25**	50.00	**57.11**

E.3. Detailed Results for MMLU-Pro

We present the detailed results for our evaluations on MMLU-Pro in Table 11. We first notice that although G1 models share close accuracies with their base model on average, they excel at notably different disciplines: G1-3B does the best in Physics (56.18%) while G1-7B is good at CS (53.32%). Interestingly, RL training on graph problems in some cases improves G1over Qwen on non-STEM subjects such as Health (53.0% v.s. 37.65%) for 3B models and Business (62.76% v.s. 53.91%) for 7B models.

Table 11: Test accuracy (%) on the MMLU-Pro benchmark.

Model	Physics	Chem.	Econ.	Other	Math	Philo.	History	Busi.	Psycho.	Law	Engin.	Health	CS	Bio.	Avg.
Llama-3.2-3B-Instruct	7.18	14.79	15.91	13.39	6.50	13.69	18.54	11.28	23.91	15.40	9.89	14.03	13.25	9.71	13.51
Qwen2.5-3B-Instruct (base)	38.49	31.18	**46.21**	37.34	**58.92**	31.06	31.23	**45.25**	**46.24**	18.07	19.40	37.65	**41.22**	**54.25**	**38.54**
CoT-SFT-3B	35.70	13.99	32.25	38.72	53.29	34.41	25.65	30.04	18.16	**42.71**	28.08	39.22	36.34	46.03	34.23
G1-3B (Ours)	**56.18**	**42.46**	16.26	**43.73**	37.78	**44.55**	**36.10**	31.80	41.46	20.95	**34.42**	**53.00**	28.86	30.18	37.12
Llama-3.1-8B-Instruct	28.79	17.13	33.96	34.03	32.28	41.83	24.91	18.80	43.89	46.45	35.28	36.10	31.75	28.26	32.02
DeepSeek-R1-Distill-Qwen-7B	39.75	11.72	19.20	49.81	40.80	19.95	23.35	25.65	**47.39**	30.30	**72.76**	36.84	34.59	49.51	37.21
Qwen2.5-7B-Instruct (base)	44.17	48.53	46.87	**55.89**	**65.80**	21.44	**54.53**	53.91	27.04	49.50	42.64	**53.66**	33.27	35.96	45.75
CoT-SFT-7B	44.36	**55.51**	44.61	29.82	51.08	**64.84**	45.97	41.45	46.42	37.01	33.87	45.61	21.44	**52.01**	44.54
G1-7B (Ours)	**46.43**	51.19	**68.76**	40.94	47.70	53.90	32.40	**62.76**	25.61	**49.88**	51.50	51.71	**53.32**	36.07	**48.56**

E.4. Detailed Results for GraphArena

We report the detailed results for evaluations on the easy/hard problems from GraphArena in Table 12 and Table 13 respectively. We observe that `G1` models perform equally or better compared to the other models on all tasks but *Distance*, in which `G1` performs slightly worse than the Qwen models.

Table 12: Test accuracy (%) on the **easy** problems from the GraphArena benchmark.

Model	Connected	Diameter	Distance	Neighbor	GED	TSP	MCP	MCS	MIS	MVC
Llama-3.2-3B-Instruct	8.00	16.00	15.00	50.00	9.00	2.00	15.00	10.00	7.00	5.00
Qwen2.5-3B-Instruct (base)	20.00	11.00	**47.00**	48.00	**37.00**	**17.00**	3.00	**41.00**	4.00	2.00
G1-3B (Ours)	**52.00**	**42.00**	**47.00**	**89.00**	30.00	**17.00**	**27.00**	20.00	**32.00**	**22.00**
LLaMA2-7B-RFT	0.00	7.00	1.00	1.00	4.00	0.00	0.00	1.00	0.00	0.00
LLaMA2-7B-DPO	0.00	1.00	0.00	0.00	3.00	0.00	0.00	1.00	0.00	0.00
Llama-3.1-8B-Instruct	33.00	29.00	45.00	81.00	24.00	14.00	32.00	18.00	24.00	20.00
DeepSeek-R1-Distill-Qwen-7B	77.00	41.00	64.00	82.00	22.00	30.00	44.00	40.00	**56.00**	17.00
Qwen2.5-7B-Instruct (Ours)	79.00	15.00	**70.00**	84.00	22.00	22.00	39.00	41.00	28.00	21.00
G1-7B (Ours)	**86.00**	**63.00**	62.00	**99.00**	**30.00**	**38.00**	**52.00**	**51.00**	50.00	**63.00**

Table 13: Test accuracy (%) on the **hard** problems from the GraphArena benchmark.

Model	Connected	Diameter	Distance	Neighbor	GED	TSP	MCP	MCS	MIS	MVC
Llama-3.2-3B-Instruct	0.00	1.00	7.00	19.00	3.00	0.00	0.00	0.00	0.00	1.00
Qwen2.5-3B-Instruct (base)	4.00	4.00	**28.00**	22.00	**7.00**	0.00	**1.00**	0.00	0.00	1.00
G1-3B (Ours)	**19.00**	**12.00**	25.00	**51.00**	3.00	0.00	0.00	0.00	**1.00**	**7.00**
LLaMA2-7B-RFT	0.00	2.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
LLaMA2-7B-DPO	0.00	3.00	0.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00
Llama-3.1-8B-Instruct	8.00	4.00	19.00	54.00	**3.00**	1.00	2.00	0.00	0.00	7.00
DeepSeek-R1-Distill-Qwen-7B	18.00	4.00	33.00	36.00	1.00	0.00	3.00	0.00	1.00	4.00
Qwen2.5-7B-Instruct (base)	27.00	4.00	**44.00**	68.00	2.00	0.00	**5.00**	0.00	1.00	5.00
G1-7B (Ours)	**31.00**	**27.00**	35.00	**84.00**	**3.00**	0.00	3.00	0.00	**6.00**	**39.00**

E.5. Detailed Results for **Erdős**

In Table 14, we show the performance for each task in `Erdős`for our models and baselines in detail.

Table 14: Accuracy comparison of models across tasks

Task	GPT-4o	o3-mini	Llama-3B	Qwen-3B	DSFT-3B	CSFT-3B	G1-3B	Llama-8B	Qwen-7B	Math-7B	R1-7B	GWiz-R	GWiz-D	DSFT-7B	CSFT-7B	G1-7B	Llama-70B	Qwen-72B
node_number	100.00	100.00	83.00	94.00	100.00	97.00	100.00	99.00	99.00	94.00	100.00	0.00	0.00	100.00	100.00	100.00	100.00	100.00
dominating_set	29.41	64.71	57.00	23.00	72.00	31.00	99.00	37.00	27.00	21.00	27.00	34.00	28.00	74.00	68.00	99.00	24.00	44.00
common_neighbor	73.68	73.68	23.00	44.00	71.00	71.00	91.00	56.00	52.00	48.00	79.00	0.00	0.00	76.00	80.00	93.00	91.52	89.99
edge_number	72.22	77.78	9.00	31.00	31.00	59.00	96.00	16.00	58.00	38.00	74.00	0.00	0.00	39.00	34.00	97.00	72.00	66.00
neighbor	84.21	63.16	26.00	36.00	87.00	82.00	91.00	42.00	65.00	64.00	94.00	4.00	2.00	89.00	89.00	93.00	93.05	98.53
bfs	52.17	17.39	0.00	3.00	52.00	30.00	95.00	5.00	12.00	9.00	12.00	0.00	0.00	43.00	44.00	98.00	25.00	53.00
has_cycle	80.00	100.00	51.00	51.00	98.00	63.00	89.00	46.00	55.00	54.00	83.00	65.00	83.00	95.00	93.00	98.00	64.00	54.00
dfs	73.33	33.33	0.00	9.00	61.00	43.00	100.00	12.00	27.00	10.00	23.00	0.00	0.00	52.00	50.00	99.00	29.00	49.00
minimum_spanning_tree	38.10	42.86	5.00	8.00	62.00	17.00	81.00	17.00	15.00	14.00	46.00	0.00	0.00	65.00	65.00	66.00	28.00	39.00
edge_existence	100.00	100.00	60.00	80.00	100.00	97.00	100.00	73.00	96.00	82.00	100.00	52.00	56.00	98.00	97.00	100.00	98.00	100.00
is_regular	100.00	95.00	88.00	95.00	98.00	98.00	100.00	92.00	96.00	99.00	98.00	27.00	58.00	99.00	99.00	100.00	99.00	100.00
degree	95.45	100.00	26.00	58.00	94.00	93.00	95.00	72.00	77.00	79.00	96.00	0.00	0.00	88.00	82.00	99.00	94.00	82.00
is_tournament	100.00	88.89	47.00	75.00	99.00	99.00	99.00	80.00	86.00	87.00	100.00	22.00	58.00	100.00	100.00	99.00	99.00	94.00
density	68.18	90.91	36.00	33.00	17.00	38.00	92.00	42.00	38.00	40.00	81.00	0.00	0.00	12.00	15.00	97.00	51.00	51.00
adamic_adar_index	92.31	88.46	1.00	6.00	74.00	89.00	94.00	12.00	39.00	22.00	82.00	3.00	1.00	76.00	75.00	98.00	52.00	64.00
clustering_coefficient	72.22	94.44	13.00	31.00	71.00	56.00	82.00	25.00	44.00	36.00	65.00	6.00	10.00	67.00	66.00	88.00	49.00	69.00
connected_component_number	60.87	82.61	9.00	27.00	85.00	63.00	79.00	34.00	35.00	30.00	79.00	0.00	0.00	80.00	81.00	92.00	64.00	66.00
bipartite_maximum_matching	40.74	48.15	3.00	19.00	53.00	47.00	82.00	13.00	12.00	3.00	42.00	0.00	0.00	76.00	73.00	87.00	29.00	37.00
local_connectivity	96.15	100.00	57.00	62.00	93.00	86.00	90.00	53.00	74.00	79.00	82.00	53.00	66.00	97.00	98.00	96.00	77.00	69.00
jaccard_coefficient	100.00	100.00	23.00	48.00	81.00	84.00	95.00	44.00	77.00	70.00	95.00	3.00	5.00	78.00	76.00	100.00	87.00	93.00
min_edge_covering	10.53	31.58	1.00	2.00	23.00	17.00	51.00	0.00	1.00	1.00	37.00	0.00	0.00	18.00	17.00	50.00	16.00	15.00
is_eularian	86.36	95.45	78.00	81.00	95.00	89.00	98.00	82.00	81.00	89.00	92.00	33.00	59.00	93.00	93.00	97.00	90.00	80.00
degree_centrality	71.43	85.71	0.00	7.00	81.00	79.00	89.00	4.00	8.00	23.00	87.00	0.00	0.00	80.00	84.00	97.00	49.00	88.00
is_bipartite	68.00	92.00	49.00	39.00	92.00	55.00	79.00	53.00	52.00	43.00	76.00	51.00	67.00	93.00	90.00	80.00	62.00	67.00
resource_allocation_index	94.12	100.00	2.00	10.00	80.00	79.00	92.00	15.00	45.00	40.00	86.00	2.00	2.00	77.00	80.00	92.00	36.00	78.00
max_weight_matching	11.11	27.78	2.00	3.00	25.00	22.00	24.00	7.00	12.00	2.00	40.00	0.00	0.00	25.00	25.00	43.00	24.00	26.00
closeness_centrality	0.00	31.58	0.00	1.00	8.00	6.00	9.00	4.00	3.00	3.00	14.00	0.00	0.00	4.00	5.00	11.00	13.00	11.00
traveling_salesman_problem	36.84	89.47	8.00	24.00	29.00	40.00	43.00	17.00	41.00	41.00	62.00	3.00	1.00	25.00	20.00	51.00	47.00	43.00
strongly_connected_number	13.33	73.33	4.00	5.00	63.00	24.00	58.00	3.00	11.00	7.00	35.00	0.00	0.00	55.00	56.00	59.00	9.00	10.00
shortest_path	69.23	38.46	11.00	19.00	74.00	51.00	62.00	31.00	35.00	11.00	62.00	3.00	0.00	77.00	78.00	70.00	62.00	60.00
center	19.05	66.67	4.00	8.00	25.00	13.00	25.00	6.00	8.00	9.00	26.00	0.00	0.00	24.00	25.00	35.00	29.72	41.48
diameter	17.65	94.12	12.00	8.00	55.00	31.00	46.00	14.00	31.00	27.00	39.00	3.00	4.00	41.00	39.00	49.00	5.00	0.00
barycenter	7.69	69.23	9.00	15.00	56.00	26.00	39.00	20.00	22.00	11.11	29.00	1.01	1.01	49.00	50.00	47.00	53.71	47.61
radius	68.75	87.50	12.00	23.00	66.00	47.00	56.00	26.00	34.00	35.00	52.00	1.00	2.00	63.00	58.00	68.00	5.00	1.00
topological_sort	60.00	48.00	10.00	14.00	76.00	38.00	67.00	25.00	25.00	21.00	64.00	6.00	5.00	74.00	71.00	78.00	73.00	74.00
periphery	29.41	58.82	1.00	3.00	33.00	16.00	22.00	1.00	11.00	6.00	25.00	0.00	0.00	27.00	29.00	31.00	50.06	47.78
betweenness_centrality	18.18	50.00	4.00	4.00	38.00	30.00	39.00	24.00	1.00	5.00	6.00	1.00	2.00	38.00	37.00	39.00	7.00	4.00
triangles	35.29	58.82	13.00	4.00	54.00	48.00	67.00	12.00	30.00	21.00	54.00	0.00	0.00	42.00	40.00	79.00	43.00	55.00
avg_neighbor_degree	66.67	61.11	16.00	17.00	36.00	55.00	68.00	26.00	30.00	29.00	58.00	3.00	6.00	31.00	36.00	82.00	62.00	64.00
harmonic_centrality	7.69	84.62	2.00	3.00	17.00	15.00	19.00	3.00	5.00	8.00	37.00	1.00	2.00	9.00	7.00	30.00	7.00	22.00
bridges	0.00	9.09	1.00	0.00	44.00	9.00	16.00	0.00	3.00	1.00	5.00	0.00	0.00	42.00	40.00	23.00	28.57	29.92
isomophic_mapping	0.00	4.00	0.00	0.00	10.00	0.00	1.00	1.00	0.00	0.00	1.00	0.00	0.00	11.00	11.00	12.00	1.00	1.00
global_efficiency	4.76	71.43	0.00	1.00	10.00	3.00	1.00	0.00	0.00	2.00	11.00	0.00	0.00	4.00	4.00	2.00	1.00	3.00
maximal_independent_set	5.56	55.56	2.00	1.00	26.00	2.00	13.00	2.00	2.00	3.00	31.00	0.00	0.00	19.00	24.00	79.00	7.00	13.00
maximum_flow	0.00	80.95	5.00	2.00	8.00	10.00	7.00	3.00	6.00	1.10	12.00	3.30	5.49	9.00	7.00	9.00	4.00	10.00
wiener_index	0.00	73.68	0.00	1.00	14.00	6.00	8.00	0.00	4.00	4.00	22.00	0.00	0.00	6.00	3.00	13.00	7.00	7.00
hamiltonian_path	0.00	4.76	0.00	1.00	11.00	3.00	2.00	1.00	2.00	1.09	3.00	0.00	0.00	10.00	10.00	5.00	5.00	12.00
min_vertex_cover	13.04	60.87	1.00	3.00	22.00	8.00	21.00	3.00	9.00	6.00	21.00	0.00	0.00	16.00	18.00	42.00	10.00	21.00

F. Discussion on Reward Weighting

In Section 4.3, we analyze the factor of data mixture by introducing a model G1-Hard-3B trained exclusively on *Hard* and *Challenging* tasks. We observe that G1-Hard-3B effectively improves performance on hard tasks, while on easier tasks still lags behind G1-3B (Table 15).

In this section, we further explore a *soft* data mixture strategy that scales the reward for each task according to its difficulty. In detail, we fix the scaling factor s as 0.2, 0.4, 0.6, and 0.8 for *Easy*, *Medium*, *Hard* and *Challenging* tasks, respectively, and name the resulting model as G1-Soft-3B. As shown in Table 15, G1-Soft-3B achieves a balance between G1-3B and G1-Hard-3B. On easy tasks, G1-Soft-3B largely surpasses G1-Hard-3B and is on par with G1-3B which applies uniform scaling across all tasks. For hard tasks, G1-Soft-3B outperforms G1-3B (*e.g.*, 11.71% v.s 7.57% for *Challenging* tasks), but there is still a gap to G1-Hard-3B. The results show the soft scaling method take effects, but the RL optimization remains dominated by easy tasks. This suggests that further reducing the reward scaling factor for easy tasks or a dynamic weighting strategy could be beneficial—a direction we leave for future work.

Table 15: Test accuracy (%) on our benchmark. $\star$ denotes the tasks are excluded in model training. G1-Hard-3B is only RL-trained on Hard and Challenging tasks. G1-Soft-3B is trained on all tasks but with different reward scaling factors based on the task difficulty.

Category	Model	Easy	Medium	Hard	Challenging	Average
Base Model	Qwen2.5-3B-Instruct	45.71	30.18	9.44	1.29	22.72
Ours	Direct-SFT-3B	74.43	75.27	<u>43.69</u>	<u>14.43</u>	53.78
	G1-3B	<u>94.86</u>	**84.64**	41.25	7.57	<u>59.76</u>
	G1-Hard-3B	69.36*	70.64*	**48.50**	**17.43**	53.30
	G1-Soft-3B	**96.07**	<u>83.55</u>	40.88	11.71	**60.38**

G. Detailed Description of Erdős

G.1. Comparing Erdős with Other Graph Reasoning Benchmarks for LLMs

There is a growing interest in evaluating LLMs' graph reasoning abilities. NLGraph (Wang et al., 2023) evaluate LLMs on graph-theoretic tasks and discover preliminary yet brittle reasoning abilities in the face of spurious correlations and large graphs. Later, GraphArena (Tang et al., 2025) and GraCoRe (Yuan et al., 2025) include a broader task coverage and recently released LLMs, finding that even OpenAI o1-mini struggles a lot with complex tasks. Moreover, GraphEval2000 (Wu et al., 2024) and ProGraph (Li et al., 2024) emphasize code-oriented problem solving using library-based prompts, and GraphOmni (Xu et al., 2025) unify varying graph types, encodings, and prompt styles for a comprehensive evaluation. Overall, these benchmarks suggest that LLMs overall demonstrate moderate success on simple tasks but struggle with abstraction, generalization, and larger or more complex graph instances. Nevertheless, these datasets are either too small (e.g., thousands of examples) or not diverse enough (e.g., 8 tasks in NLGraph) for training general-purpose graph reasoners, which motivates the design of Erdős. We show the detailed comparison of existing graph reasoning benchmarks for LLM with our Erdősin Table 16.

Table 16: Comparison of existing graph-theoretic reasoning benchmarks for LLM with our Erdős.

Benchmark	#Tasks	# Q-A Samples	Graph Types	Node Size
NLGraph (Wang et al., 2023)	8	5,902	Synthetic	5 to 35
GraphWiz (Chen et al., 2024)	9	3,600	Synthetic	2 to 100
GraphArena (Tang et al., 2025)	10	10,000	Real-world	4 to 50
GraCoRe (Yuan et al., 2025)	19	5,140	Synthetic & Real-world	8 to 30
GraphOmni (Xu et al., 2025)	6	241,726	Synthetic	5 to 30
Erdős(ours)	50	100,000	Real-world	5 to 35

G.2. Full list of tasks in Erdős

Table 17: Benchmark exmaples

Task	Prompt	Answer
adamic adar index	The task is to determine the Adamic-Adar index of two nodes in a graph. The Adamic-Adar index is the sum of the inverse logarithm of the degrees of the common neighbors of the two nodes. The input graph is guaranteed to be undirected. Here is an undirected graph containing nodes from 1 to 9. The edges are: (1, 5), (1, 4), (1, 8), (1, 2), (1, 3), (1, 7), (5, 2), (5, 3), (5, 4), (5, 9), (5, 6), (4, 8), (4, 9), (4, 7), (8, 2), (8, 3), (8, 6), (8, 7), (8, 9), (2, 3), (2, 7), (2, 6), (3, 9), (3, 7), (7, 6), (7, 9). Question: What is the Adamic-Adar index between node 4 and node 6? You need to format your answer as a float number.	1.5859
avg neighbor degree	The task is to determine the average degree of the neighbors of a node in the graph. Here is an undirected graph containing nodes from 1 to 8. The edges are: (1, 7), (1, 8), (1, 4), (7, 8), (8, 5), (2, 3), (2, 6), (3, 5). Question: What is the average neighbor degree of node 2 in the graph? You need to format your answer as a float number.	1.5
barycenter	The task is to determine the barycenter of a graph. The barycenter of a graph is also called the median. It includes the node that minimizes the sum of shortest path lengths to all other nodes. The input graph is guaranteed to be connected. Here is an undirected graph containing nodes from 1 to 7. The edges are: (1, 2), (1, 6), (1, 5), (1, 7), (1, 4), (2, 6), (2, 5), (2, 7), (2, 4), (6, 4), (6, 5), (6, 7), (7, 3), (7, 4). Question: What is the barycenter of the graph? You need to format your answer as a list of nodes in ascending order, e.g., [node-1, node-2, ..., node-n].	[1, 2, 6, 7]
betweenness centrality	The task is to determine the betweenness centrality of a node in the graph. Betweenness centrality of a node *u* is the sum of the fraction of all-pairs shortest paths that pass through *u*. Here is an undirected graph containing nodes from 1 to 9. The edges are: (1, 6), (1, 4), (1, 8), (1, 9), (6, 2), (6, 7), (4, 7), (4, 5), (8, 3), (8, 5), (8, 7), (9, 3), (9, 5), (2, 7). Question: What is the betweenness centrality of node 5 in the graph? You need to format your answer as a float number.	0.0679
bfs	The task is to determine the breadth-first search (BFS) traversal order given a starting node. Stop when the BFS cannot be continued. Here is an undirected graph containing nodes from 1 to 7. The edges are: (1, 2), (1, 5), (2, 3), (2, 4), (5, 3), (5, 4), (3, 4), (4, 7), (7, 6). Question: What is the breadth-first search (BFS) traversal order for the starting node 1? You need to format your answer as a list of edges, e.g., [(u1, v1), (u2, v2), ..., (un, vn)].	[(1, 2), (1, 5), (2, 3), (2, 4), (4, 7), (7, 6)]
bipartite maximum matching	The task is to determine the maximal matching in a bipartite graph. The input graph is guaranteed to be a bipartite graph. Here is an undirected graph containing nodes from 1 to 4. The edges are: (1, 3), (1, 4), (2, 3), (2, 4). Question: What is the bipartite maximal matching of the bipartite graph? You need to format your answer as a list of edges in ascending dictionary order, e.g., [(u1, v1), (u2, v2), ..., (un, vn)].	[(1, 3), (2, 4)]

Continuing table 17

Task	Prompt	Answer
bridges	The task is to find all bridges of a graph. A bridge is an edge in a graph whose removal increases the number of connected components. The input graph is guaranteed to be undirected. Here is an undirected graph containing nodes from 1 to 5. The edges are: (1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (2, 5), (3, 4), (3, 5). Question: What are the bridges of the graph? You need to format your answer as a list of edges in ascending dictionary order, e.g., [(u1, v1), (u2, v2), ..., (un, vn)].	[]
center	The task is to determine the center of a graph. The center of a graph includes the node that minimizes the maximum distance to any other nodes in the graph. The input graph is guaranteed to be connected. Here is an undirected graph containing nodes from 1 to 6. The edges are: (1, 5), (5, 2), (2, 6), (6, 4), (3, 4). Question: What is the center of the graph? You need to format your answer as a list of nodes in ascending order, e.g., [node-1, node-2, ..., node-n].	[2, 6]
closeness centrality	The task is to determine the closeness centrality of a node in the graph. For a node *u*, closeness centrality is the reciprocal of the average shortest path distance to *u* over all *n-1* reachable nodes. For directed graphs, it computes the incoming distance to *u*. Here is an undirected graph containing nodes from 1 to 8. The edges are: (1, 3), (3, 6), (2, 8), (2, 6), (8, 6), (8, 7), (4, 7), (7, 5). Question: What is the closeness centrality of node 2 in the graph? You need to format your answer as a float number.	0.4667
clustering coefficient	The task is to compute the clustering coefficient for a given node. For unweighted graphs, the clustering of a node is the fraction of possible triangles through that node that exist. Here is an undirected graph containing nodes from 1 to 7. The edges are: (1, 4), (1, 5), (1, 3), (4, 2), (4, 3), (4, 5), (4, 6), (4, 7), (5, 2), (5, 3), (5, 6), (5, 7), (2, 6), (2, 7), (6, 7). Question: What is the clustering coefficient of node 6? You need to format your answer as a float number.	1.0
common neighbor	The task is to determine common neighbors between two nodes in the graph. The input graph is guaranteed to be undirected. Here is an undirected graph containing nodes from 1 to 7. The edges are: (1, 7), (1, 6), (1, 4), (1, 5), (7, 2), (7, 3), (6, 2), (4, 3), (5, 3). Question: What are the common neighbors between node 2 and node 3? You need to format your answer as a list of nodes in ascending order, e.g., [node-1, node-2, ..., node-n].	[7]
connected component number	The task is to determine the number of connected components in an undirected graph. A connected component is a subgraph where any two nodes are connected to each other by paths. Here is an undirected graph containing nodes from 1 to 10. The edges are: (1, 4), (1, 7), (1, 5), (1, 9), (1, 10), (1, 6), (1, 2), (4, 2), (4, 3), (4, 8), (4, 5), (4, 9), (4, 10), (7, 2), (7, 3), (7, 5), (7, 6), (7, 8), (7, 9), (5, 2), (5, 3), (5, 8), (5, 9), (5, 10), (9, 2), (9, 3), (9, 6), (9, 8), (9, 10), (10, 2), (10, 3), (10, 8), (6, 2), (6, 3), (6, 8), (2, 8), (2, 3). Question: How many connected components are there in the graph? Your answer should be an integer.	1

Continuing table 17

Task	Prompt	Answer
degree	The task is to determine the degree of a node in the graph. For the undirected graph, you should count the edge between two nodes only once. Here is an undirected graph containing nodes from 1 to 6. The edges are: (1, 6), (6, 5), (2, 3), (2, 4), (3, 5). Question: What is the degree of node 6 in the graph? Your answer should be an integer.	2
degree centrality	The task is to determine the degree centrality of a node in the graph. Degree centrality for a node is the fraction of nodes it is connected to. Here is an undirected graph containing nodes from 1 to 7. The edges are: (1, 2), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5), (2, 6), (4, 3), (4, 5), (4, 7), (5, 3). Question: What is the degree centrality of node 3 in the graph? You need to format your answer as a float number.	0.5
density	The task is to determine the density of the graph. Density is defined as the ratio of the number of edges in the graph to the number of possible edges. Here is an undirected graph containing nodes from 1 to 5. The edges are: (1, 2), (1, 3), (2, 3), (2, 4), (2, 5), (3, 4), (4, 5). Question: What is the density of the graph? You need to format your answer as a float number.	0.7
dfs	The task is to determine the depth-first search (DFS) traversal order given a starting node. Stop when the DFS cannot be continued. Here is an undirected graph containing nodes from 1 to 9. The edges are: (1, 2), (1, 3), (1, 6), (3, 9), (4, 8), (4, 5), (8, 7). Question: What is the depth-first search (DFS) traversal order for the starting node 1? You need to format your answer as a list of edges, e.g., [(u1, v1), (u2, v2), ..., (un, vn)].	[(1, 2), (1, 3), (3, 9), (1, 6)]
diameter	The task is to determine the diameter of a graph. The diameter of a graph is the longest shortest path between any two nodes in the graph. The input graph is guaranteed to be connected. Here is an undirected graph containing nodes from 1 to 7. The edges are: (1, 5), (1, 7), (1, 4), (5, 6), (2, 6), (2, 3). Question: What is the diameter of the graph? You need to format your answer as a float number.	5
dominating set	The task is to determine the dominating set of a graph. A dominating set is a subset of nodes such that every node in the graph is either in the set or adjacent to a node in the set. For directed graphs, any node not in the dominating set must be a successor of a node within the set. Here is an undirected graph containing nodes from 1 to 7. The edges are: (1, 2), (1, 5), (1, 6), (1, 7), (2, 3), (2, 4), (5, 6), (7, 3), (7, 4). Question: What is the dominating set of the graph? You need to format your answer as a list of nodes in ascending order, e.g., [node-1, node-2, ..., node-n].	[1, 3, 4]

Continuing table 17

Task	Prompt	Answer
edge existence	The task is to determine if there is an edge connecting two nodes. For an undirected graph, determine if there is an edge between nodes *u* and *v*. For a directed graph, determine if there is an edge from *u* to *v*. Here is an undirected graph containing nodes from 1 to 8. The edges are: (1, 2), (1, 6), (3, 8), (3, 4), (8, 4), (8, 5), (8, 7), (4, 7), (4, 5), (7, 5). Question: Is there an edge between node 5 and node 3? Your answer should be Yes or No.	No
edge number	The task is to determine the number of edges in the graph. For the undirected graph, you should count the edge between two nodes only once. Here is an undirected graph containing nodes from 1 to 10. The edges are: (1, 10), (1, 8), (10, 7), (8, 6), (2, 5), (2, 4), (2, 6), (5, 4), (5, 9), (4, 3), (4, 9), (3, 7). Question: How many edges are there in the graph? Your answer should be an integer.	12
global efficiency	The task is to determine the global efficiency of a graph. Global efficiency is the average efficiency of all pairs of nodes. The efficiency of a pair of nodes is the multiplicative inverse of the shortest path distance between the nodes. The input graph is guaranteed to be undirected. Here is an undirected graph containing nodes from 1 to 7. The edges are: (1, 5), (1, 4), (5, 2), (2, 7), (7, 3), (3, 6). Question: What is the global efficiency of the graph? You need to format your answer as a float number.	0.5310
hamiltonian path	The task is to return a Hamiltonian path in a directed graph. A Hamiltonian path is a path in a directed graph that visits each vertex exactly once. The input graph is guaranteed to be directed and tournable. Here is a directed graph containing nodes from 1 to 8. The edges are: (2, 1), (2, 4), (2, 5), (2, 6), (2, 7), (1, 3), (1, 4), (1, 7), (3, 2), (3, 7), (3, 8), (4, 3), (4, 5), (4, 7), (5, 1), (5, 3), (5, 8), (6, 1), (6, 3), (6, 4), (6, 5), (7, 5), (7, 6), (8, 1), (8, 2), (8, 4), (8, 6), (8, 7). Question: Return a Hamiltonian path in the graph. You need to format your answer as a list of nodes, e.g., [node-1, node-2, ..., node-n].	[2, 1, 4, 5, 3, 8, 7, 6]
harmonic centrality	The task is to determine the harmonic centrality of a node in the graph. Harmonic centrality of a node *u* is the sum of the reciprocal of the shortest path distances from all other nodes to u. Here is a directed graph containing nodes from 1 to 8. The edges are: (6, 2), (6, 1), (6, 4), (6, 5), (6, 3), (7, 8). Question: What is the harmonic centrality of node 3 in the graph? You need to format your answer as a float number.	1.0
has cycle	The task is to determine if the graph has a cycle. Here is an undirected graph containing nodes from 1 to 9. The edges are: (1, 2), (1, 4), (1, 5), (2, 4), (2, 5), (4, 9), (5, 3), (3, 6), (3, 8), (6, 8), (9, 7). Question: Does the graph have a cycle? Your answer should be Yes or No.	Yes
is bipartite	The task is to determine if the graph is bipartite. A bipartite graph is a graph whose nodes can be divided into two disjoint sets such that no two graph vertices within the same set are adjacent. Here is an undirected graph containing nodes from 1 to 6. The edges are: (1, 4), (4, 3), (2, 5), (2, 3), (5, 6), (3, 6). Question: Is the graph bipartite? Your answer should be Yes or No.	Yes

Continuing table 17

Task	Prompt	Answer
is eularian	The task is to determine if the graph is Eulerian. An Eulerian graph is a graph that contains an Eulerian circuit, which is a cycle that visits every edge exactly once. Here is an undirected graph containing nodes from 1 to 6. The edges are: (1, 5), (1, 3), (1, 2), (1, 4), (5, 2), (3, 2), (3, 4), (3, 6), (2, 4), (4, 6). Question: Is the graph Eulerian? Your answer should be Yes or No.	Yes
is regular	The task is to determine if the graph is regular. A regular graph is a graph where every node has the same degree. Here is an undirected graph containing nodes from 1 to 10. The edges are: (1, 5), (1, 7), (1, 10), (5, 2), (5, 10), (7, 8), (7, 10), (3, 9), (3, 8), (3, 4), (9, 4), (4, 6). Question: Is the graph regular? Your answer should be Yes or No.	No
is tournament	The task is to determine if the graph is a tournament. A tournament is a directed graph where every pair of nodes is connected by a single directed edge. The input graph is guaranteed to be directed. Here is a directed graph containing nodes from 1 to 10. The edges are: (1, 2), (2, 1), (2, 4), (4, 2), (4, 3), (3, 1), (5, 2), (5, 4), (6, 2), (6, 5), (7, 8), (8, 6), (9, 7), (10, 7). Question: Is the graph a tournament? Your answer should be Yes or No.	No
isomophic mapping	Given a pair of isomorphic graphs, determine the node correspondence between the two graphs. The first graph is: G describes an undirected graph among 0, 1, 2, 3, 4, 5, and 6. In this graph: Node 0 is connected to nodes 6, 3, 4. Node 1 is connected to nodes 4, 5, 6. Node 2 is connected to nodes 3, 4. Node 3 is connected to nodes 0, 2, 5. Node 4 is connected to nodes 0, 1, 2. Node 5 is connected to nodes 1, 3. Node 6 is connected to nodes 0, 1. The second graph is: G describes an undirected graph among 102, 106, 105, 101, 103, 100, and 104. In this graph: Node 100 is connected to nodes 106, 101. Node 101 is connected to nodes 102, 105, 100. Node 102 is connected to nodes 104, 101, 103. Node 103 is connected to nodes 102, 106, 105. Node 104 is connected to nodes 102, 106. Node 105 is connected to nodes 101, 103. Node 106 is connected to nodes 103, 100, 104. Provide a node matching dictionary such as {Graph1 #Node1: Graph2 #Node1, Graph1 #Node2: Graph2 #Node2, ...}	{0: 102, 3: 101, 2: 105, 4: 103, 1: 106, 5: 100, 6: 104}
jaccard coefficient	The task is to determine the Jaccard coefficient of two nodes in a graph. The Jaccard coefficient is the size of the intersection divided by the size of the union of the neighbors of the two nodes. The input graph is guaranteed to be undirected. Here is an undirected graph containing nodes from 1 to 5. The edges are: (1, 2), (1, 3), (2, 5), (2, 3), (3, 5), (5, 4). Question: What is the Jaccard coefficient between node 2 and node 4? You need to format your answer as a float number.	0.3333

Continuing table 17

Task	Prompt	Answer
local connectivity	The task is to determine the local connectivity of two nodes in the graph. Local connectivity is whether there exists at least one path between the two nodes. Here is a directed graph containing nodes from 1 to 7. The edges are: (1, 7), (7, 6), (3, 1), (4, 3), (5, 4), (6, 2). Question: What is the local connectivity between node 7 and node 4 in the graph? Your answer should be Yes or No.	No
max weight matching	The task is to determine the maximum weight matching of a graph. A matching is a set of edges without common vertices. A maximal matching cannot add more edges and still be a matching. The weight of a matching is the sum of the weights of its edges. If not specified, all edges have equal edge weights. The input graph is guaranteed to be undirected. Here is an undirected graph containing nodes from 1 to 7. The edges are: (1, 7), (7, 5), (2, 4), (2, 5), (4, 3), (3, 6). Question: What is the maximum weight matching of the graph? You need to format your answer as a list of edges in ascending dictionary order, e.g., [(u1, v1), (u2, v2), ..., (un, vn)].	[(2, 4), (5, 7), (6, 3)]
maximal independent set	The task is to determine the maximal independent set guaranteed to contain a given node in the graph. An independent set is a set of nodes such that the subgraph induced by these nodes contains no edges. A maximal independent set is an independent set such that it is not possible to add a new node and still get an independent set. The input graph is guaranteed to be undirected. Here is an undirected graph containing nodes from 1 to 6. The edges are: (1, 2), (1, 6), (1, 3), (2, 3), (2, 4), (2, 5), (3, 5), (4, 5). Question: What is the maximal independent set that includes node 4 of the graph? You need to format your answer as a list of nodes in ascending order, e.g., [node-1, node-2, ..., node-n].	[3, 4, 6]
maximum flow	The task is to determine the value of the maximum flow for the given source node and sink node. The maximum flow is the greatest amount of flow that can be sent from the source to the sink without violating capacity constraints. Here is a directed graph containing nodes from 1 to 5. The edges are: (2, 5, 8), (3, 1, 9), (3, 5, 3), (4, 2, 4). (u ,v, w) denotes the edge from node *u* to node *v* has a capacity of *w*. Question: What is the value of the maximum flow from node 3 to node 2? You need to format your answer as a float number.	0.0
min edge covering	The task is to determine the minimum edge covering of a graph. An edge cover is a set of edges such that every vertex in the graph is incident to at least one edge in the set. The minimum edge cover is the edge cover with the smallest number of edges. The input graph is guaranteed to be undirected. Here is an undirected graph containing nodes from 1 to 9. The edges are: (1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (2, 5), (3, 4), (3, 6), (3, 7), (3, 8), (3, 5), (4, 7), (4, 8), (5, 6), (5, 7), (6, 7), (6, 9), (7, 9). Question: What is the minimum edge covering of the graph? You need to format your answer as a list of edges in ascending dictionary order, e.g., [(u1, v1), (u2, v2), ..., (un, vn)].	[(2, 1), (5, 2), (7, 4), (8, 3), (9, 6)]

Continuing table 17

Task	Prompt	Answer
min vertex cover	The task is to determine the minimum vertex cover of a graph. A vertex cover is a set of nodes such that every edge in the graph is incident to at least one node in the set. Here is an undirected graph containing nodes from 1 to 5. The edges are: (1, 2), (2, 3), (3, 5), (5, 4). Question: What is the minimum vertex cover of the graph? You need to format your answer as a list of nodes in ascending order, e.g., [node-1, node-2, ..., node-n].	[2, 5]
minimum spanning tree	The task is to determine the minimum spanning tree of a graph. A minimum spanning tree is a subset of the edges that connects all vertices in the graph with the minimum possible total edge weight. If not specified, all edges have equal edge weights. The input graph is guaranteed to be undirected and connected. Here is an undirected graph containing nodes from 1 to 9. The edges are: (1, 2), (1, 8), (1, 5), (1, 6), (1, 4), (1, 7), (1, 9), (2, 5), (2, 6), (2, 4), (2, 7), (2, 3), (8, 3), (8, 4), (8, 6), (8, 7), (5, 3), (5, 4), (5, 6), (5, 7), (5, 9), (6, 3), (6, 4), (6, 7), (6, 9), (4, 3), (4, 7), (4, 9), (7, 9), (9, 3). Question: What is the minimum spanning tree of the graph? You need to format your answer as a list of edges in ascending dictionary order, e.g., [(u1, v1), (u2, v2), ..., (un, vn)].	[(1, 2), (1, 4), (1, 5), (1, 6), (1, 7), (1, 8), (1, 9), (2, 3)]
neighbor	The task is to determine the neighbors of a node in the graph. For directed graph, you should return the successors of the node. Here is an undirected graph containing nodes from 1 to 10. The edges are: (1, 3), (1, 9), (1, 6), (1, 7), (3, 2), (3, 8), (3, 9), (6, 7), (2, 10), (10, 8), (4, 5). Question: What are the neighbors of node 2 in the graph? You need to format your answer as a list of nodes in ascending order, e.g., [node-1, node-2, ..., node-n].	[3, 10]
node number	The task is to determine the number of nodes in the graph. Here is an undirected graph containing nodes from 1 to 10. The edges are: (1, 10), (1, 3), (10, 6), (10, 8), (3, 7), (3, 4), (2, 7), (2, 5), (2, 9), (5, 9), (5, 8), (9, 4), (8, 6). Question: How many nodes are there in the graph? Your answer should be an integer.	10
periphery	The task is to determine the periphery of a graph. The periphery of a graph is the set of nodes with the maximum eccentricity. The eccentricity of a node is the maximum distance from this node to all other nodes in the graph. The input graph is guaranteed to be connected. Here is an undirected graph containing nodes from 1 to 6. The edges are: (1, 3), (3, 2), (3, 4), (3, 5), (3, 6). Question: What is the periphery of the graph? You need to format your answer as a list of nodes in ascending order, e.g., [node-1, node-2, ..., node-n].	[1, 2, 4, 5, 6]
radius	The task is to determine the radius of a graph. The radius of a graph is the minimum eccentricity of any node in the graph. The eccentricity of a node is the maximum distance from this node to all other nodes in the graph. The input graph is guaranteed to be connected. Here is an undirected graph containing nodes from 1 to 5. The edges are: (1, 2), (2, 3), (3, 4), (3, 5), (4, 5). Question: What is the radius of the graph? You need to format your answer as a float number.	2

Continuing table 17

Task	Prompt	Answer
resource allocation index	The task is to determine the resource allocation index of two nodes in a graph. The resource allocation index of two nodes is the sum of the inverse of the degrees of the common neighbors of the two nodes. The input graph is guaranteed to be undirected. Here is an undirected graph containing nodes from 1 to 5. The edges are: (1, 2), (1, 3), (2, 3), (3, 4), (3, 5), (4, 5). Question: What is the resource allocation index between node 1 and node 4? You need to format your answer as a float number.	0.25
shortest path	The task is to determine the shortest path between two nodes. The input nodes are guaranteed to be connected. Here is an undirected graph containing nodes from 1 to 6. The edges are: (1, 2), (1, 3), (2, 4), (2, 3), (2, 5), (3, 4), (3, 5), (4, 6). Question: What is the shortest path between node 1 and node 6? You need to format your answer as a list of nodes, e.g., [node-1, node-2, ..., node-n].	[1, 2, 4, 6]
strongly connected number	The task is to determine the number of strongly connected components in a directed graph. A strongly connected component is a maximal subgraph where every node is reachable from every other node. Here is a directed graph containing nodes from 1 to 6. The edges are: (2, 5), (5, 1), (3, 4), (6, 2). Question: How many strongly connected components are there in the graph? Your answer should be an integer.	6
topological sort	The task is to determine the topological sort of a directed acyclic graph (DAG). Here is a directed graph containing nodes from 1 to 6. The edges are: (1, 6), (1, 5), (1, 4), (1, 3), (1, 2). Question: What is the topological sort of the directed acyclic graph (DAG)? You need to format your answer as a list of nodes, e.g., [node-1, node-2, ..., node-n].	[1, 6, 5, 4, 3, 2]
traveling salesman problem	The task is to determine the minimal cost of the Traveling Salesman Problem (TSP). The Traveling Salesman Problem asks for the shortest possible route that visits each vertex exactly once and returns to the starting vertex. The input graph is guaranteed to be a complete graph. Here is an undirected graph containing nodes from 1 to 8. The edges are: (1, 2, 9), (1, 3, 3), (1, 4, 6), (1, 5, 8), (1, 6, 7), (1, 7, 4), (1, 8, 9), (2, 3, 10), (2, 4, 11), (2, 5, 5), (2, 6, 11), (2, 7, 1), (2, 8, 9), (3, 4, 11), (3, 5, 1), (3, 6, 9), (3, 7, 2), (3, 8, 9), (4, 5, 8), (4, 6, 3), (4, 7, 4), (4, 8, 8), (5, 6, 3), (5, 7, 3), (5, 8, 10), (6, 7, 8), (6, 8, 1), (7, 8, 10). (u ,v, w) denotes the edge from node *u* to node *v* has a weight of *w*. Question: What is the minimal cost of the Traveling Salesman Problem on the graph? You need to format your answer as a float number.	27.0
triangles	The task is to find the number of triangles that include a specific node as one vertex. A triangle is a set of three nodes that are all connected to each other. The input graph is guaranteed to be undirected. Here is an undirected graph containing nodes from 1 to 8. The edges are: (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (1, 8), (2, 3), (2, 4), (2, 5), (2, 6), (2, 7), (2, 8), (3, 4), (3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8), (5, 6), (5, 7), (5, 8), (6, 7), (6, 8), (7, 8). Question: How many triangles include node 1 in the graph? Your answer should be an integer.	21

Continuing table 17

Task	Prompt	Answer
weighted minimum spanning tree	The task is to determine the minimum spanning tree of a weighted graph. A minimum spanning tree is a subset of the edges that connects all vertices in the graph with the minimum possible total edge weights. If not specified, all edges have equal edge weights. The input graph is guaranteed to be undirected and connected. Here is an undirected graph containing nodes from 1 to 5. The edges are: (1, 4, 5), (2, 4, 11), (2, 3, 10), (3, 4, 2), (3, 5, 2). (u ,v, w) denotes the edge from node *u* to node *v* has a weight of *w*. Question: What is the minimum spanning tree of the weighted graph? You need to format your answer as a list of edges in ascending dictionary order, e.g., [(u1, v1), (u2, v2), ..., (un, vn)].	[(1, 4), (2, 3), (3, 4), (3, 5)]
weighted shortest path	The task is to determine the shortest path between two nodes of a weighted graph. The input nodes are guaranteed to be connected. Here is a directed graph containing nodes from 1 to 8. The edges are: (1, 2, 5), (1, 4, 3), (1, 7, 9), (2, 3, 10), (2, 4, 10), (3, 1, 11), (3, 4, 2), (3, 5, 6), (4, 1, 1), (4, 2, 4), (4, 6, 8), (4, 8, 2), (5, 1, 7), (5, 2, 11), (5, 6, 2), (5, 7, 5), (5, 8, 11), (6, 1, 7), (6, 2, 11), (6, 3, 4), (6, 5, 1), (6, 8, 11), (7, 1, 3), (7, 2, 8), (7, 4, 7), (7, 6, 6), (7, 8, 3), (8, 1, 11), (8, 2, 7), (8, 4, 5), (8, 7, 5). (u ,v, w) denotes the edge from node *u* to node *v* has a weight of *w*. Question: What is the shortest path between node 1 and node 5? You need to format your answer as a list of nodes, e.g., [node-1, node-2, ..., node-n].	[1, 4, 6, 5]
wiener index	The task is to determine the Wiener index of a connected graph. The Wiener index of a graph is the sum of the shortest-path distances between each pair of reachable nodes. For pairs of nodes in undirected graphs, only one orientation of the pair is counted. In the input graph, all node pairs are guaranteed to be reachable. Here is an undirected graph containing nodes from 1 to 5. The edges are: (1, 2), (1, 4), (2, 3), (4, 5), (3, 5). Question: What is the Wiener index of the graph? You need to format your answer as a float number.	15.0