Confident or Seek Stronger: Exploring Uncertainty-Based Small LM Routing

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Abstract

Small language models (SLMs) are increasingly deployed on edge devices for personalized applications, offering low-latency inference and reduced energy consumption. However, they often struggle with complex queries, leading to unreliable responses. Uncertainty-based SLM routing addresses this by offloading low-confidence queries to stronger large language models (LLMs), following the principle "if uncertain, seek stronger support" to improve reliability. While leveraging LLMs enhances accuracy, it also incurs high invocation costs, making it crucial to balance efficiency and efficacy. In this paper, we conduct a comprehensive investigation into ***benchmarking of uncertainty-driven routing strategies from SLMs to LLMs over 5000+ settings***. Our findings highlight: *First*, uncertainty-correctness alignment in different uncertainty quantification (UQ) methods significantly impacts routing performance. The extracted uncertainty distribution is primarily influenced by the selected SLM and uncertainty quantification (UQ) method, showing minimal dependence on the downstream dataset. The source code is available at `https://anonymous.4open.science/r/quodlibeta`.

1 Introduction

Large language models (LLMs) have gained increasing attention for deployment on edge devices due to their potential for low-latency and privacy-preserving inference. However, given the computational and memory constraints of edge hardware, small language models (SLMs) (e.g., Phi2-mini [33], Llama3.2-3B [64]) are designed for resource-efficient deployment on smartphones, wearables, and similar devices. Their overarching goal is to democratize LM deployment, making it accessible and affordable across diverse settings [46, 78, 76]. Despite this, SLMs often lack the robustness and scalability of LLMs [8] (e.g., GPT-4o [2], Llama-3.1-405B), especially under diverse and complex queries in edge deployments, leading to degraded overall performance and motivating the need for improved response reliability.

To address this limitation, recent work proposes partial offloading of challenging queries from SLMs to stronger LLMs [10, 53, 30, 60], forming hybrid systems that intelligently route queries for more reliable and deterministic responses. However, these methods face significant challenges when encountering new downstream tasks, as the data fall outside the distribution of the existing training data, making them less practical for real-world scenarios, such as in personal edge device deployment, where adaptability to unseen conditions is crucial. While LLMs achieve superior performance, their deployment incurs high inference and infrastructure costs, e.g., a single NVIDIA A100 GPU can cost $\sim$ \$2,000 per month. Moreover, inaccurate routing by SLMs can unnecessarily increase LLM query traffic, requiring greater bandwidth and further raising operational expenses, particularly under continuous deployment scenarios. Therefore, developing an effective routing strategy is crucial for fully leveraging SLMs [53, 60, 10], as it enhances response reliability while reducing service and data transmission costs.

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Leveraging SLMs' self-uncertainty estimation emerges as a robust strategy for enhancing routing effectiveness [10, 15]. By relying on the self-assessed uncertainty, the system can better decide whether to handle a query locally or delegate it to a larger model without the aid of extra routers, ensuring that only queries deemed unreliable by the SLMs are routed to LLMs. As a result, the uncertainty-based routing approach not only generalizes well to new datasets, as only self-assessed information from SLM is needed, but it also reduces the high operational costs associated with accurately running LLMs. To this end, we aim to explore two open and nontrivial research questions for uncertainty-based SLM routing:

What is the best practice of uncertainty estimation for query routing from SLMs to LLMs? We benchmark the uncertainty-correctness alignment of each uncertainty quantification (UQ) method under its impact on SLM routing. A good alignment is a key factor for successful routing decisions, as any misalignment can cause unnecessary offloading with extra cost. However, SLMs may struggle to provide reliable uncertainty estimates [31, 14, 67], making them less effective as indicators for query routing. Thus, we benchmark the alignment between uncertainty and correctness, paving the insights for establishing more effective routing strategies[1]. Our contributions include as follows:

- **Comprehensive benchmarking and detailed analysis:** This benchmark evaluates 8 UQ methods across 15 datasets to examine the alignment between uncertainty and correctness in routing tasks. We incorporate 8 SLMs and 2 LLMs to emulate uncertainty-based SLM routing.
- **Uncertainty distribution characterization:** The extracted uncertainty distribution is determined by the choice of SLM and UQ method, exhibiting minimal dependence on the downstream dataset.

2 Benchmarking Uncertainty-based SLM Routing

In this section, we systematically evaluate 12 SLMs and 4 LLMs on 15 datasets using 8 UQ methods (see Table 2) for uncertainty-based SLM routing. This section details the datasets, models, and UQ methods, followed by several key findings and practical considerations. All experiments are conducted on four 80GB NVIDIA A100 GPUs.

2.1 Benchmark Coverage and Setup

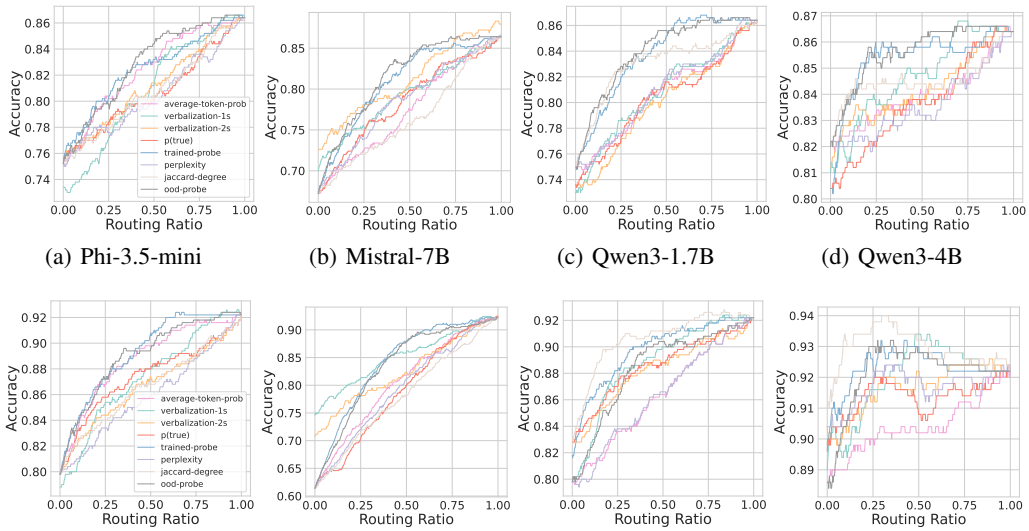
The Language Models. We evaluate 12 open-source SLMs, organized into three categories: non-reasoning LMs, reasoning LMs, and a recurrent neural network (RNN) model. The non-reasoning models are Llama-3.2-1B-Instruct [49], Llama-3.2-3B-Instruct [49], Phi-3.5-mini-instruct [1], Mistral-7B-Instruct-v0.3 [34], Qwen2.5-7B-Instruct [71], Llama-3.1-8B-Instruct [18], and Granite-3.1-8B-Instruct [24]. The reasoning models are Qwen3-0.6B, Qwen3-1.7B, Qwen3-4B, and Phi-4-mini-reasoning [70]. The RNN model is RWKV-7-2.9B [55]. We also include four LLMs: three open-source models—Llama-3.1-70B-Instruct [18], Qwen3-32B, and DeepSeek-R1 [27]—and one proprietary API model, GPT-4.1 mini [32]. Qwen3-32B and DeepSeek-R1 are reasoning LLMs, whereas Llama-3.1-70B and GPT-4.1 mini are non-reasoning.

Datasets and Experiment Settings. Experiments span 15 datasets from four domains: *(1) Mathematical Reasoning* (AQuA [44], GSM8K [12], MultiArith [57], SVAMP [54], MATH-500 [40]), *(2) Commonsense Reasoning* (CommonsenseQA [61], HellaSwag [73], OpenBookQA [51], PIQA [6], TruthfulQA [41], WinoGrande [58], BoolQ [11], Social IQa [59]), *(3) Conversational and Contextual Understanding* (CoQA [56]), and *(4) Problem Solving* (MMLU [28]). These cover free-form, multiple-choice, and True/False question answering and are available via Hugging Face. Appendix A provides further details about the settings of datasets and selected UQ methods.

2.2 Report Observations

In this section, we present our benchmarking results analyzing the impact of uncertainty-correctness alignment on routing tasks. More observations and experimental results on proxy routing and routing can be found in Appendix C.1.

[1]For the convenience of writing, we interchangeably use uncertainty and confidence, where low uncertainty refers to high confidence.



(a) Phi-3.5-mini (b) Mistral-7B (c) Qwen3-1.7B (d) Qwen3-4B

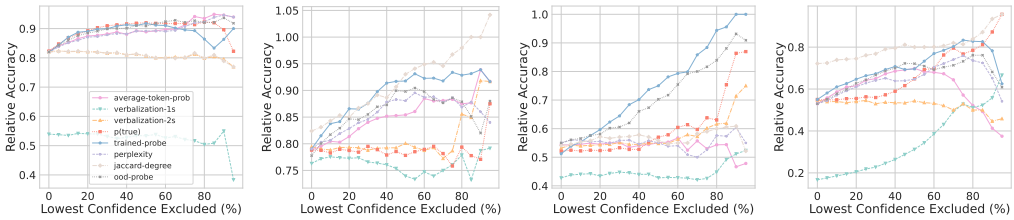
(e) Phi-3.5-mini (f) Mistral-7B (g) Qwen3-1.7B (h) Qwen3-4B

Figure 1: Overall accuracy vs. routing ratio with different UQ methods and SLMs. (a) and (b) show the results of routing to DeepSeek-R1 on the CommonsenseQA dataset; c and (d) show the results of routing to GPT4-mini on the Openbook QA dataset.

Observation ❶: Uncertainty estimation in SLMs may exhibit misalignment with prediction correctness. From the theoretical perspective, well-calibrated uncertainty scores do not necessarily imply a strong correlation with the correctness of the predictions [31, 10]. The predictions of models might be perfectly calibrated yet still display relatively low accuracy (i.e., confidently provide wrong answers). This phenomenon is also evident in our benchmark results (illustrated in Figure 6). We compute AUC scores to quantify the correlation between extracted uncertainty and prediction correctness, treating correctness as a binary ground truth and using confidence values as the ranking metric. The results show that not all UQ methods effectively exhibit a strong alignment between confidence and prediction correctness. Moreover, from Figure 6 and Figure 5, we can observe that the alignment may vary across datasets for the same SLM and UQ method. For instance, Perplexity [20] demonstrates strong alignment for Phi-3.5-mini on the MultiArith dataset but fails on the OpenBookQA dataset. On the other hand, OOD Probe, Trained Probe, and Perplexity obtain consistently decent alignment compared to other UQ methods across different SLMs and domains of datasets. Conversely, we notice that verbalization-based methods, namely verbalization-1s [63, 47], and verbalization-2s [63], consistently withhold low alignment between uncertainty and prediction correctness. More experimental results can be found in Appendix C.1.

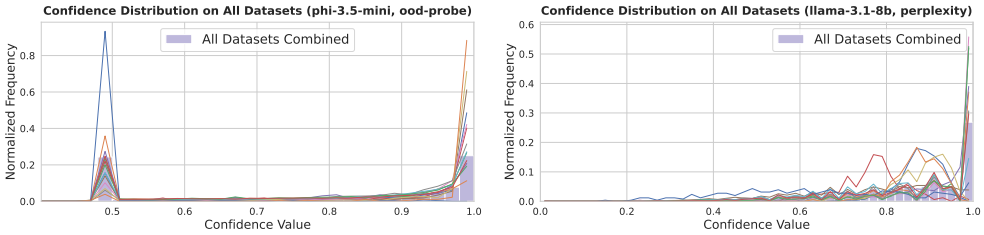
Observation ❷: Verbalization-based UQ methods struggle to extract uncertainty in SLMs for query routing. We find that verbalization methods like verbalization-2s [63] obtain poor alignment between confidence and prediction correctness, and this misalignment can lead to inferior routing performance in SLMs, where the conclusion can be found in Figure 1. Recent advancements [68, 72] also show that uncertainty scores derived from verbalization may exhibit good reflection on models' intrinsic uncertainty of prediction across multiple models and datasets. This discrepancy poses a significant challenge for establishing effective routing performance since queries that are actually correct may be unnecessarily routed from SLMs to LLMs, thereby increasing the overall cost of deploying routing systems.

Observation ❸: A good routing standard highly depends on UQ methods with good uncertainty-correctness alignment. A notable phenomenon occurs when UQ methods, such as Trained Probe [47], exhibit strong alignment, leading to significant improvements in routing performance. This is because the extracted uncertainty scores from these UQ methods more effectively indicate whether SLMs produce correct predictions. Among all UQ methods evaluated for routing tasks, we find that Trained Probe [47], OOD Probe [36, 47], and Perplexity [19] consistently rank as the top three methods for SLM routing. Therefore, a comprehensive analysis of UQ methods before deploying a routing system in SLMs is highly recommended to ensure efficient query routing.



(a) Granite-3.1-8B (b) Qwen3-0.6B (c) Llama-3.2-1B (d) RWKV-7-2.9B

Figure 2: Relative accuracy of SLMs vs. LLMs on top-$k\%$ confident queries. "Relative accuracy" is the ratio of SLM accuracy to LLM accuracy. The x-axis "Lowest Conf. Excluded" shows the percentage of low-confidence queries removed; for example, 80 means 80% of queries with the lowest confidence are excluded, leaving the top 20%. (a) and (b) compare SLMs to Llama-3.1-70B on GSM8K, while (c) and (d) compare SLMs to Qwen3-32B on BoolQ.



(a) Phi-3.5-mini (b) Llama-3.1-8B

Figure 3: Uncertainty score distributions across 15 datasets. The histogram depicts the aggregated distribution from all datasets, while each curve represents a single dataset. (a) Confidence of Phi-3.5-mini by OOD Probe; (b) Confidence of Llama-3.1-8B by Perplexity.

Observation ❹: SLMs can match LLM performance on high-confidence queries. Although SLMs generally underperform LLMs, we find that for queries where SLMs exhibit high confidence, their accuracy approaches that of LLMs. To illustrate, we progressively remove queries starting from those with the lowest SLM confidence and compute the ratio of SLM to LLM accuracy on the remaining top-$k\%$ queries (Figure 2). As more low-confidence queries are excluded, SLMs achieve comparable performance to LLMs. For instance, on GSM8K, Qwen3-0.6B achieves performance nearly equal to Llama-3.1-70B on the top 20% highest-confidence queries. Moreover, the effectiveness of this selection depends on the uncertainty quantification (UQ) method: approaches with stronger alignment (e.g., Trained Probe [47]) yield higher relative accuracy than weaker ones (e.g., verbalization-2s) across all query exclusion rates. Additional results appear in Appendix C.2.

Insights ❺: The extracted confidence distribution is predominantly determined by the chosen SLM and UQ method, with minimal dependence on the downstream dataset. As shown in Figure 3, the confidence scores aggregated from 15 different tasks maintain a consistent overall shape across diverse datasets, indicating that dataset-specific factors have limited influence. In contrast, the distribution varies substantially across different SLMs and UQ methods, highlighting their dominant role in shaping the resulting confidence profiles.

3 Conclusion

This paper investigates the routing accuracy of SLMs in estimating their uncertainty and establishing best practices for initiating effective routing strategies. Through comprehensive benchmarking of 15 SLMs, 4 LLMs, 8 UQ methods, and 15 datasets across 5000+ settings, we found that the alignment between uncertainty and correctness significantly impacts routing performance. Additionally, we also observe that extracted uncertainty is primarily influenced by the selected SLM and uncertainty quantification (UQ) method, showing minimal dependence on the downstream dataset. Future work will focus on leveraging these insights to develop high-quality routing datasets that enable efficient SLM deployment on edge devices without re-calculating the uncertainty scores for routing.

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A Benchmark Details

A.1 Details about Datasets

The details of the 15 datasets are further listed in Tabel 1. We applied the original dataset directly from the Huggingface dataset repositories without any further processing. A thorough examination of each dataset's attributes, size, and notable characteristics is provided below.

Table 1: Details of the 15 datasets used in our benchmark. FF: Free-form question answering (including numerical answers for math tasks); MCQ: Multiple-choice question answering; TF: True/False question answering.

Dataset	Type	Domain	# Train	# Test	Description
GSM8K	FF	Mathematical Reasoning	7473	1319	Grade school math word problems
AQuA	MCQ	Mathematical Reasoning	97467	254	Algebraic word problems
MultiArith	FF	Mathematical Reasoning	420	180	Algebraic word problems
SVAMP	FF	Mathematical Reasoning	700	300	Algebraic word problems
MATH-500	FF	Mathematical Reasoning	– –	500	Algebraic word problems
BoolQ	TF	Commonsense Reasoning	9427	3270	Commonsense and factual reasoning questions
CommonsenseQA	MCQ	Commonsense Reasoning	9741	1221	Questions assessing various types of commonsense knowledge
HellaSwag	MCQ	Commonsense Reasoning	39905	10042	Sentence completion based on narrative understanding
OpenBookQA	MCQ	Commonsense Reasoning	4957	500	Open-book science and commonsense questions
PIQA	MCQ	Commonsense Reasoning	16113	1838	Physical commonsense reasoning questions
Social IQa	MCQ	Commonsense Reasoning	33410	1954	Social commonsense intelligence questions
TruthfulQA	FF	Commonsense Reasoning	653	164	Assessing models' ability to prevent false information
WinoGrande	MCQ	Commonsense Reasoning	2558	1267	Pronoun ambiguity resolution with commonsense reasoning
CoQA	FF	Conversational & Contextual Understanding	7199	500	Conversational questions on text passages from diverse domains
MMLU	MCQ	Problem Solving	99842	14042	Problem solving across various subjects

A.2 Details about Uncertainty Quantification Methods

We evaluate 8 approaches from the four categories in Section B.1. *(1) Average token probability* uses the probability of the chosen option token (e.g., "A") for multiple-choice tasks or the mean probability of all generated tokens for free-form tasks. *(2) Perplexity* is computed for a sequence of N output tokens $\{y_i\}_{i=1}^N$ with probabilities $\{p(y_i)\}_{i=1}^N$ as $\exp\big(\frac{1}{N}\sum_{i=1}^N \ln p(y_i)\big)$, and its reciprocal serves as the confidence score. *(3) $p(\textit{True})$* is a method where the LM first outputs an answer, then evaluates the generated response using only "True" or "False." The probabilities for these two tokens are normalized to sum to 1, and the probability of "True" is used as confidence. *(4) Verbalized confidence in a single response* (denoted as verbalization-1s) prompts the model to output both the answer and numeric confidence in one step. *(5) Verbalized confidence in the second round* (denoted as verbalization-2s) obtains the confidence in a separate, follow-up query after the model has provided an answer. *(6) The degree matrix* (denoted as jaccard-degree) generates $m = 5$ samples (temperature 1.0) for one query, computes pairwise Jaccard similarities, and sets confidence to $\mathrm{trace}(mI - D)/m^2$, where D is the degree matrix. *(7) Trained probe* is a four-layer MLP with LeakyReLU activations, trained on a fixed subsample of the in-domain training set for each dataset, taking as input the hidden states from the eighth-to-last transformer layer. We train for 20 epochs (learning rate 5×10^{-4}). *(8) Trained probe on out-of-distribution data* (denoted as ood-probe) is identical in architecture but trained on all other datasets. e.g., if AQuA is evaluated, the ood-probe is trained on the remaining 15 datasets (20 epochs, learning rate 1×10^{-4}).

For verbalization-based methods, we discard queries when the model does not follow instructions to produce a confidence score. For free-form question answering, we use GPT-4.1 mini to evaluate whether a response is essentially equivalent to the ground truth answer [77].

B Related Work

B.1 Reviewing Different Schools of Uncertainty Quantification and LLM Routing

Uncertainty quantification methods estimate a model's confidence in its predictions [29]. For traditional classification and regression, uncertainty estimation is well-established [21]. However, for LLMs generating free-form responses to complex queries, estimating uncertainty is more challenging because the output space can grow exponentially with vocabulary size, and each sequence spans multiple tokens [19]. Existing uncertainty quantification approaches for LLMs can be grouped into the following four categories.

Via verbalizing uncertainty. This line of work prompts language models to report linguistic confidence [47, 50]. To enable LMs to verbalize confidence, researchers have proposed fine-tuning them to express uncertainty [42] or teaching them to verbalize confidence through in-context learning [16]. Verbalized confidence can take the form of linguistic expressions of uncertainty or numerical scores [22]. Multiple studies find that LLMs tend to be overconfident when reporting confidence [69, 63]. To mitigate this overconfidence, prompting strategies such as multi-step elicitation, top-k, and Chain-of-Thought [66] have been explored [63]. Sampling multiple response-confidence pairs and designing more effective aggregation strategies can also help mitigate overconfidence [69]. Moreover, [63] reports that verbalized confidence is typically better calibrated than the model's conditional probabilities.

Via analyzing token/sequence probabilities. This line of research derives confidence scores from model logits for output tokens [22, 31, 35]. The confidence of a generated sequence is computed by aggregating the log-probabilities of its tokens. Common aggregation strategies include arithmetic average, minimum, perplexity, and average entropy [19, 20, 65]. Because not all tokens in a sequence equally reflect semantic content, SAR reweights token likelihoods to emphasize more meaningful tokens [17]. However, different surface realizations of the same claim can yield different probabilities, implying that the calculated confidence reflects how a claim is articulated rather than the claim itself [47]. To combine LM self-assessment with token probabilities, $p(\text{True})$ is proposed: the model is asked whether its generated response is correct, and the probabilities of True/False tokens serve as the confidence score [36, 63].

Via gauging output consistency. This line of research (e.g., SelfCheckGPT [48]) assumes that high-confidence LLMs produce consistent outputs [47]. A typical approach samples m responses for a given input query, measures inter-response similarity, and calculates a confidence score from meaning diversity [19]. Common ways to measure pairwise similarity include Natural Language Inference (NLI) and Jaccard similarity [22]. Consistency is then assessed by analyzing the similarity matrix, for instance, by counting semantic sets, summing eigenvalues of the graph Laplacian or computing eccentricity [43]. Because different sentences can express the same meaning, semantic entropy [37] first clusters responses by semantic equivalence before measuring consistency.

Via training uncertainty probes. This approach trains classifiers to predict whether an LLM will arrive at the correct answer for a particular query, using predicted probabilities as confidence scores [22]. Training data is often obtained by sampling multiple answers per question at a fixed

Table 2: Uncertainty quantification (UQ) methods evaluated in our benchmark. "Model Access" specifies whether a method views the LM's weights/logits (white-box) or only its generated output (black-box). "Require Training?" indicates if additional training is needed. See Subsection B.1 for taxonomy details and Subsection 2.1 for method descriptions.

Uncertainty Quantification (UQ) Methods	Taxonomy	Model Access	Require Training?
Average Token Prob [47]	Token/sequence probabilities	White-box	No
$p(\text{True})$ [36]	Token/sequence probabilities	White-box	No
Perplexity [20]	Token/sequence probabilities	White-box	No
Jaccard Degree [43]	Output consistency	Black-box	No
Verbalization-1s [69, 63]	Verbalized uncertainty	Black-box	No
Verbalization-2s [63]	Verbalized uncertainty	Black-box	No
Trained Probe [4, 36, 47]	Uncertainty probe	White-box	Yes
OOD Probe [36, 47]	Uncertainty probe	White-box	Yes

temperature and labeling each for correctness [36]. A probe (commonly a multi-layer perceptron) then takes hidden states as inputs to predict correctness [4, 39]. Because in-domain training data is not always available, Contrast-Consistent Search trains probes unsupervisedly by maximizing representation distances between contradictory answers on Yes/No questions [7]. Furthermore, whether probes trained on out-of-distribution data remain effective is still under debate [36, 47, 37].

B.2 Small Language Models

Small Language Models (SLMs) are designed for deployment on resource-constrained devices like desktops, smartphones, and wearables. Specifically, we consider the Transformer-based SLMs in this work due to their state-of-the-art performance, like Phi-3-mini [1], TinyLlama [74], MobileLLM [45], and Qwen-1.5B [5], LiteLLaMa-460M, OPT-125M [75], BLOOMZ (560M, 1.1B, 1.7B, 3B) [38], SmolLM (135M, 360M, 1.7B) [3], OLMo (1B) [26], OLMoE (1B) [52], MobiLlama (0.5B, 1B) [62], MobileLLaMA (1.4B, 2.7B) [9], OpenLLaMA (3B) [23]. These models are designed with lightweight architectures to operate effectively within the computational and storage limitations of mobile devices and edge hardware.

Recurrent Neural Networks (RNNs), like RWKV (1B, 3B, 7B) [55], Mamba (1.4B, 6.9B) [13], and RecurrentGemma-2B [25], can provide promising solutions for on-device inference in resource-constrained environments. These models leverage the recurrent nature of RNNs to process sequential data efficiently without requiring a KV cache, which is suitable for resource-constrained on edge devices. Specifically, RWKV introduces a hybrid RNN-Transformer backbone to capture long-term dependencies while maintaining computational efficiency. Similarly, Mamba and RecurrentGemma design recurrent layers for low-power consumption and high throughput inference, which can significantly reduce memory and computational requirements, fostering low-latency applications directly on devices.

C Additional Benchmarking Results

In this section, we present additional experimental results on evaluating the impact of uncertainty-correctness alignment on small language model (SLM) routing. **Since our studies yield over 5,600 results, we here present a representative subset in the following section. The full set of results will be provided in the Github repository.** *For the experiment (Section C.1 and Section C.2)*, we provide the complete set of results, including the AUC measurements for uncertainty-correctness alignment and the performance of uncertainty-based routing. Each dataset referenced in the experiments is treated as a novel dataset for evaluation.

C.1 Evaluation on Uncertainty-correctness Alignment

Results of Alignment between uncertainty and correctness.

All the experiments shown on this page are conducted under AQUA, BoolQ, and CoQA datasets with all 8 UQ methods.

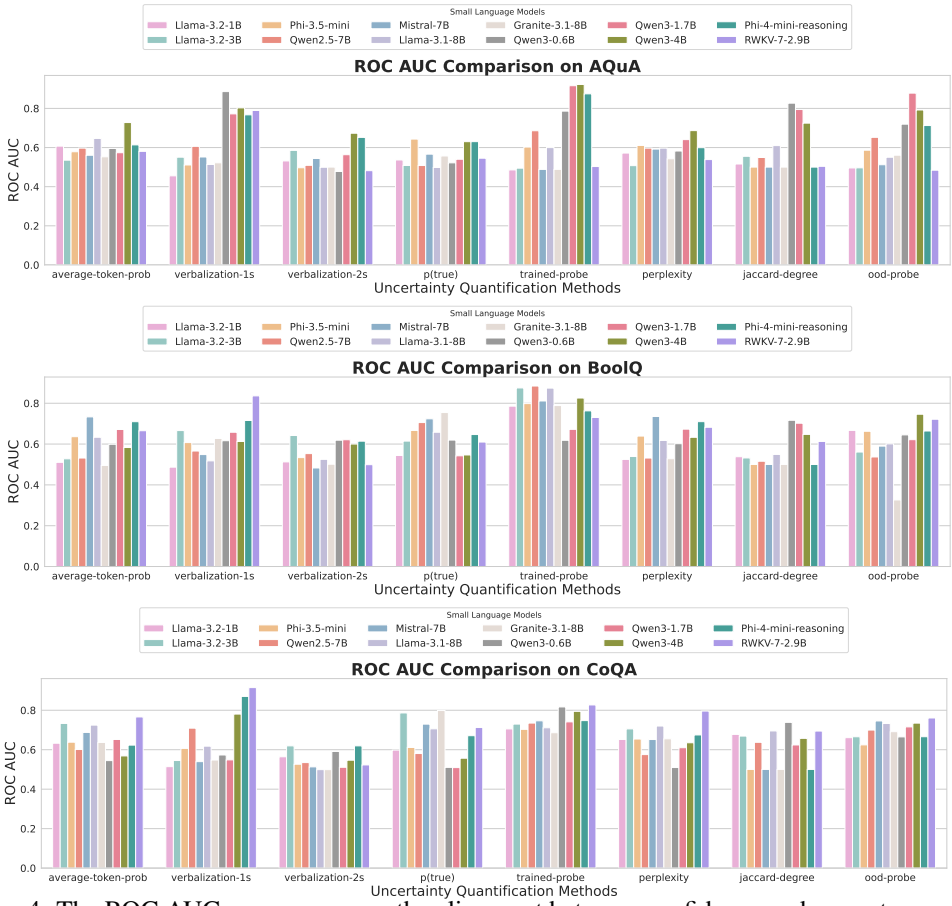

Figure 4: The ROC AUC scores measure the alignment between confidence and correctness across different SLMs and uncertainty quantification methods on AQuA, BoolQ, and CoQA. A higher ROC AUC indicates a stronger alignment.

Results of Alignment between uncertainty and correctness.

All the experiments shown on this page are conducted under GSM8K, HellaSwag, MMLU, and MultiArith datasets with all 8 UQ methods.

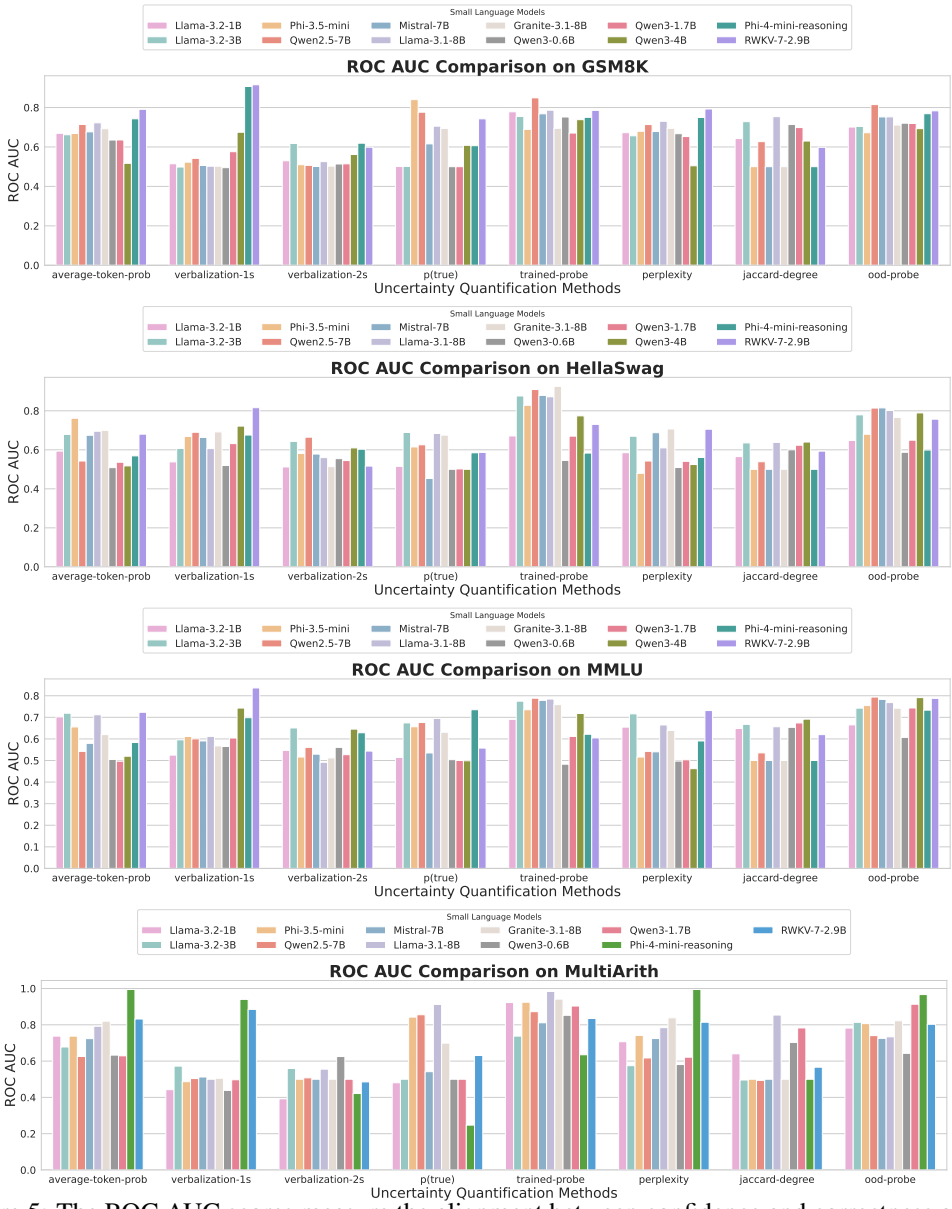

Figure 5: The ROC AUC scores measure the alignment between confidence and correctness across different SLMs and uncertainty quantification methods on GSM8K, HellaSwag, MMLU, and MultiArith. A higher ROC AUC indicates a stronger alignment.

Results of Alignment between uncertainty and correctness.

All the experiments shown on this page are conducted under OpenBookQA, PIQA, SocialIQA, and SVAMP datasets with all 8 UQ methods.

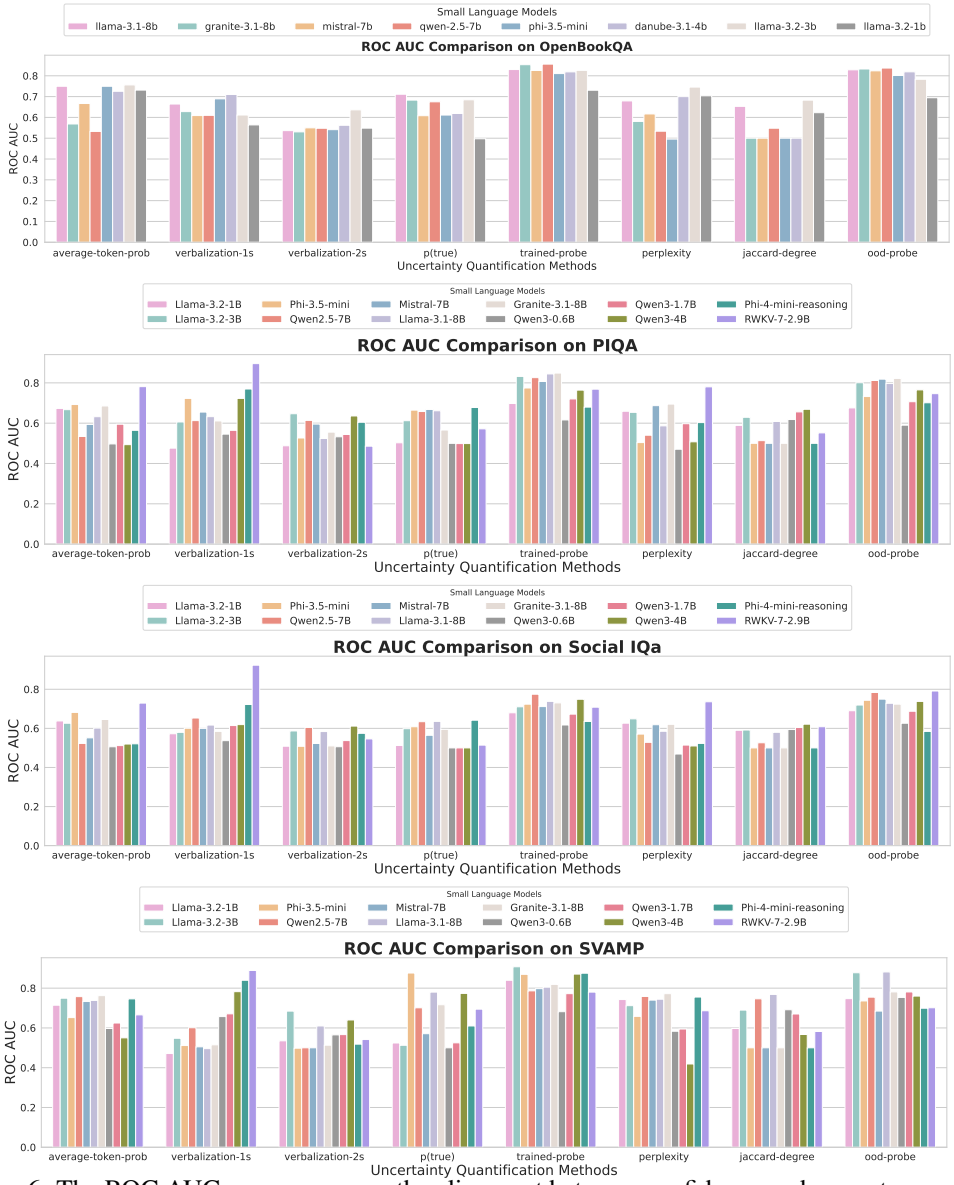

Figure 6: The ROC AUC scores measure the alignment between confidence and correctness across different SLMs and uncertainty quantification methods on OpenBookQA, PIQA, SocialIQA, and SVAMP datasets. A higher ROC AUC indicates a stronger alignment.

Results of Alignment between uncertainty and correctness.

All the experiments shown on this page are conducted under CommonsenseQA, SVAMP, TruthfulQA, WinoGrande, and Math500 dataset with all 8 UQ methods.

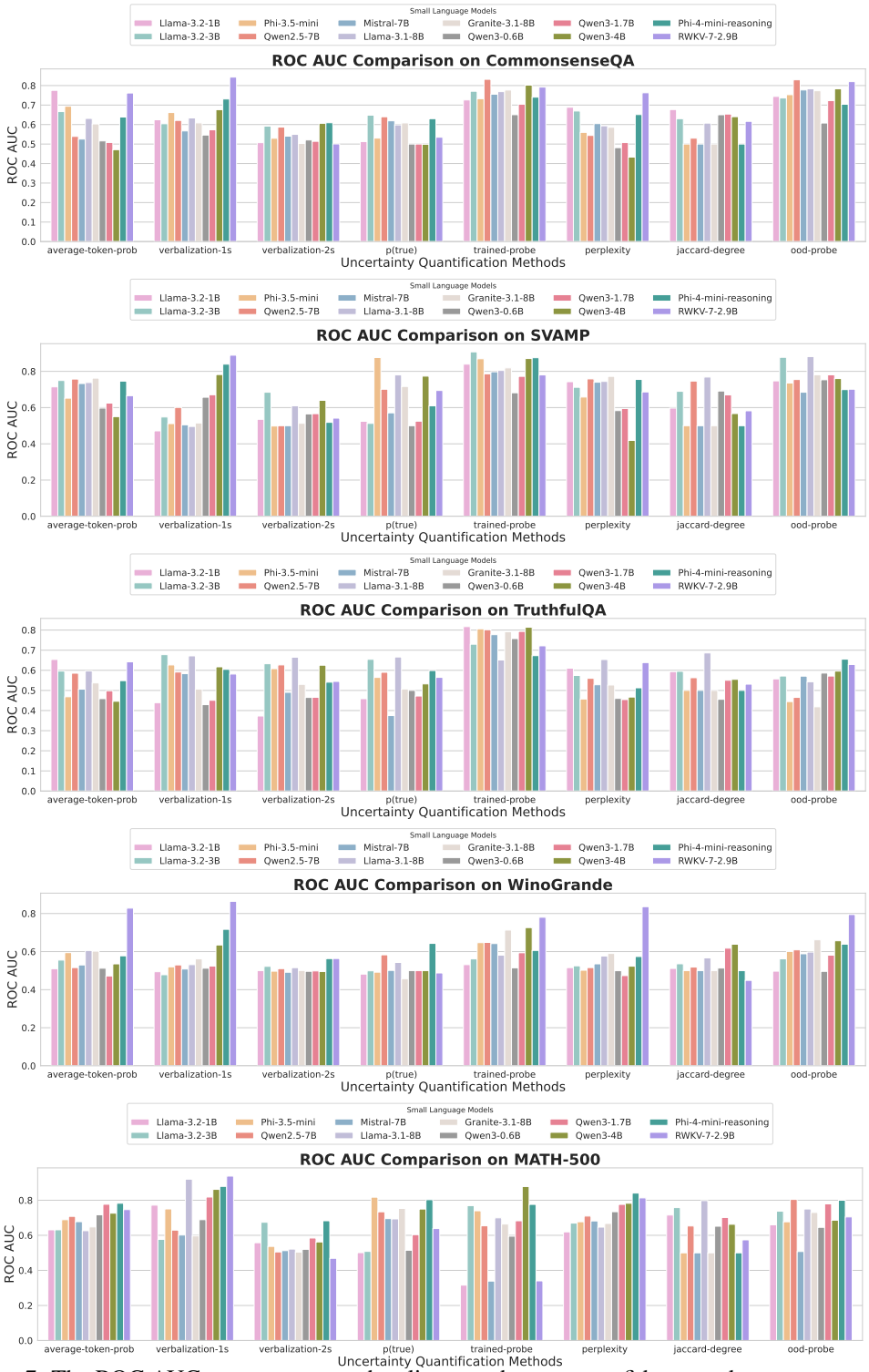

Figure 7: The ROC AUC scores measure the alignment between confidence and correctness across different SLMs and uncertainty quantification methods on CommonsenseQA. A higher ROC AUC indicates a stronger alignment.

C.2 Evaluation on Uncertainty-based Routing Approaches

Results of routing to GPT-4.1-Mini

All the experiments shown on this page are conducted under all benchmark datasets with selected SLMs. We only showcase partial of the experimental results.

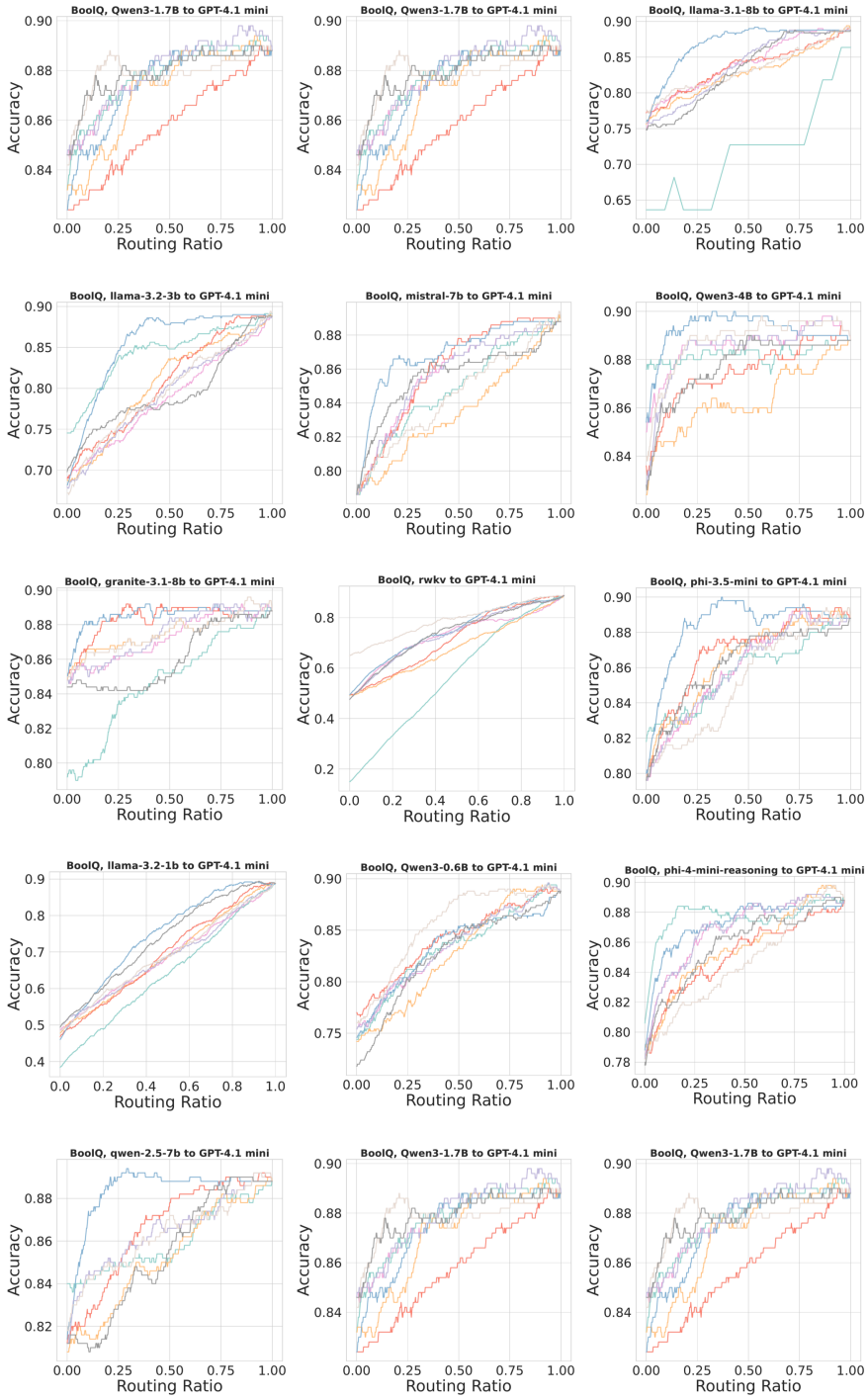

Figure 8: Overall accuracy vs. routing ratio with different UQ methods and SLMs.

Results of routing to DeepSeek-R1

All the experiments shown on this page are conducted under all benchmark datasets with selected SLMs. We only showcase partial of the experimental results.

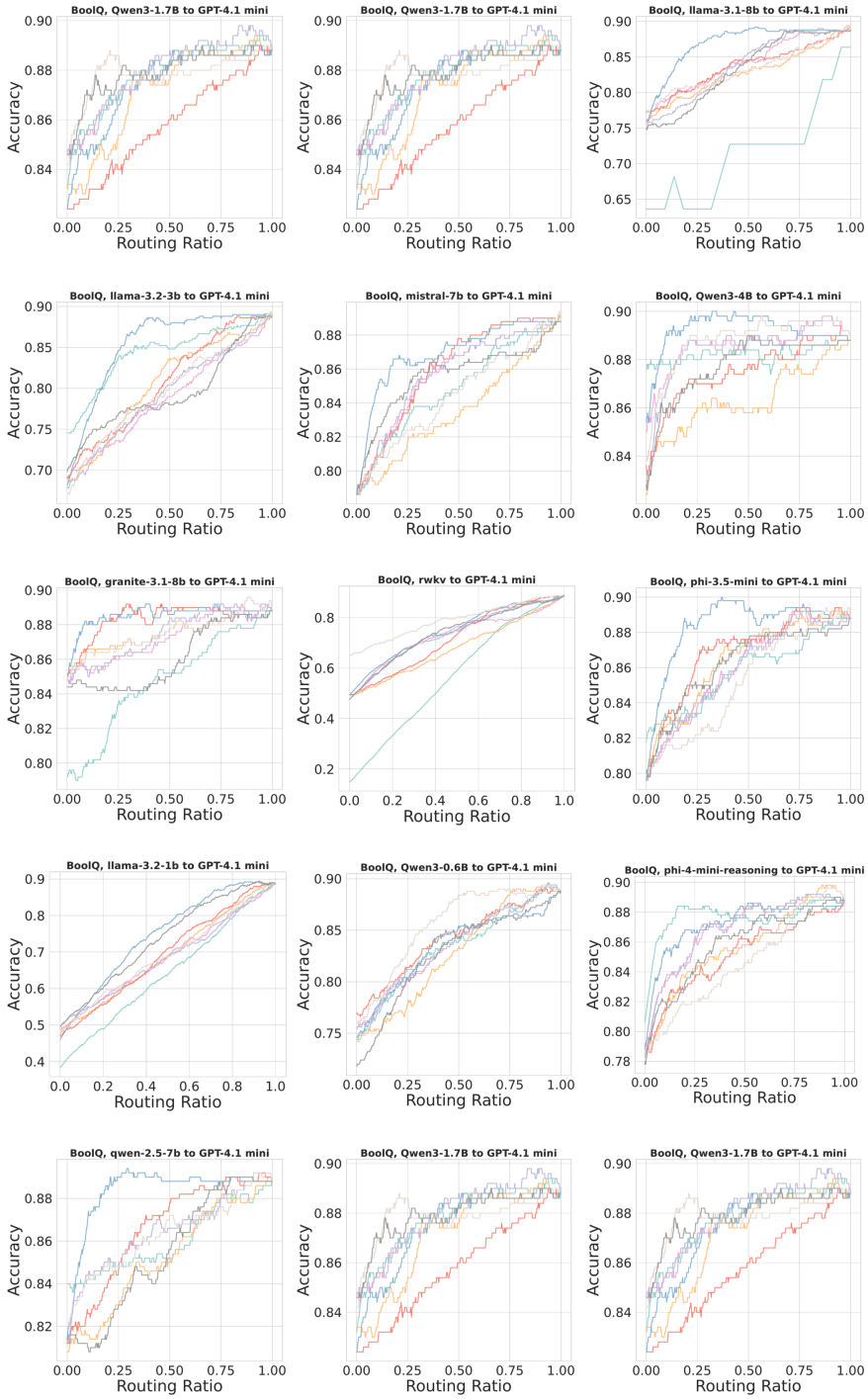

Figure 9: Overall accuracy vs. routing ratio with different UQ methods and SLMs.

Results of routing to Llama-3.1-70B-Instruct

All the experiments shown on this page are conducted under all benchmark datasets with selected SLMs. We only showcase partial of the experimental results.

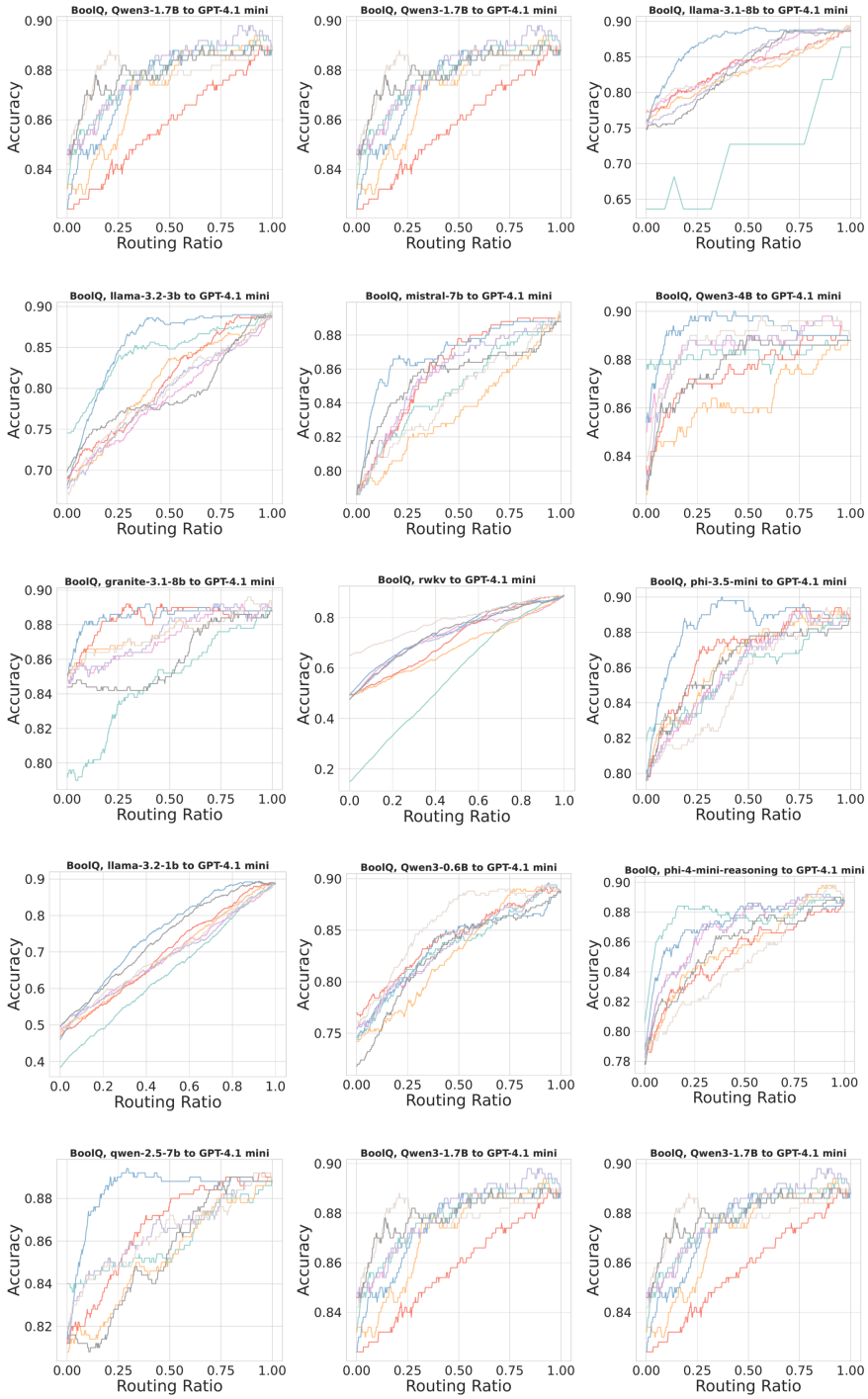

Figure 10: Overall accuracy vs. routing ratio with different UQ methods and SLMs.

Results of routing to Qwen3-32B

All the experiments shown on this page are conducted under all benchmark datasets with selected SLMs. We only showcase partial of the experimental results.

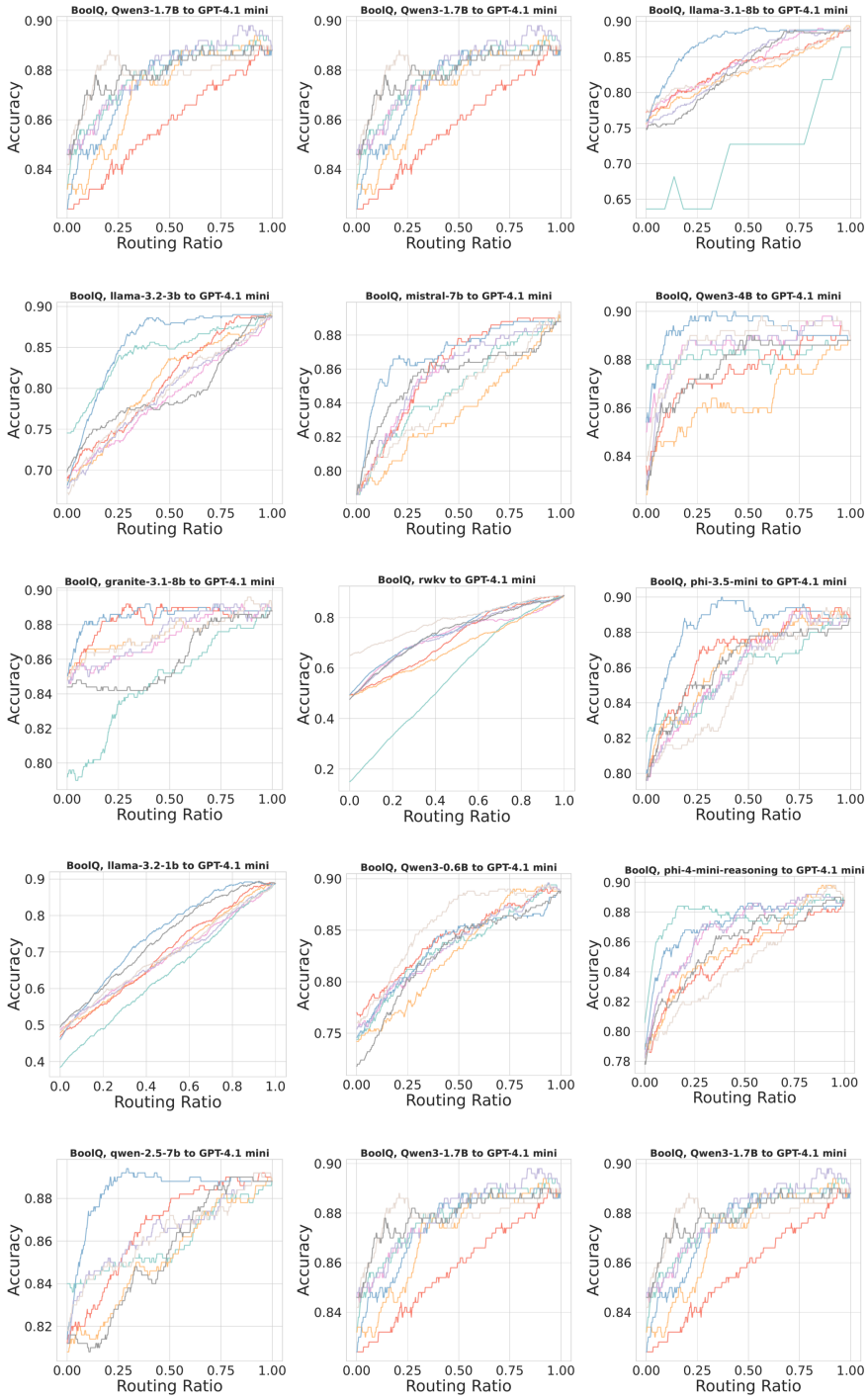

Figure 11: Overall accuracy vs. routing ratio with different UQ methods and SLMs.