# Magic Pyramid: Accelerating Inference with Early Exiting and Token Pruning

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### Abstract

Pre-training and then fine-tuning large language models is commonly used to achieve state-of-the-art performance in natural language processing (NLP) tasks. However, most 004 pre-trained models suffer from low inference speed. Deploying such large models to applica-007 tions with latency constraints is challenging. In this work, we focus on accelerating the inference via conditional computations. To achieve this, we propose a novel idea, Magic Pyramid (MP), to reduce both width-wise and depthwise computation via token pruning and early 012 exiting for Transformer-based models, partic-014 ularly BERT. The former manages to save the computation via removing non-salient tokens, while the latter can fulfill the computation reduction by terminating the inference early be-017 fore reaching the final layer, if the exiting condition is met. Our empirical studies demonstrate that compared to previous state of arts, MP is not only able to achieve a speed-adjustable inference, but also to surpass token pruning and early exiting by reducing up to 70% giga floating point operations (GFLOPs) with less than 0.5% accuracy drop. Token pruning and early exiting express distinctive preferences to sequences with different lengths. However, MP 027 is capable of achieving an average of 8.06x speedup on two popular text classification tasks, regardless of the sizes of the inputs.

#### 1 Introduction

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In the past few years, owing to the success of Transformer-based (Vaswani et al., 2017) pretrained models, such as BERT (Devlin et al., 2019), RoBERTa (Liu et al., 2019), GPT2 (Radford et al., 2019), *etc.*, we have experienced a performance breakthrough in natural language processing (NLP) tasks. With a small amount of fine-tuning, the pre-trained models can achieve state-of-the-art performance across different tasks (Devlin et al., 2019; Liu et al., 2019; Radford et al., 2019). Nevertheless, the outperforming models are evaluated in offline settings, and the inference latency is not assessed or considered as a quality factor. 043

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However, adapting and deploying such large pretrained models to production systems (e.g., online shopping services) is not straightforward due to the latency constraint and the large volume of incoming requests (e.g., millions of requests per second). Prior to this work, researchers have proposed to compress a large model via either model pruning (Michel et al., 2019; Fan et al., 2019; Hou et al., 2020) or token pruning (Wang et al., 2021; Goyal et al., 2020; Kim et al., 2021). In addition, compressing a large teacher model into a compact model via knowledge distillation has been studied extensively in the past (Sanh et al., 2019; Sun et al., 2019, 2020; Jiao et al., 2020; He et al., 2021). Finally, another line of work targets on plugging multiple sub-classifiers into deep neural networks to enable a flexible computation on demand, a.k.a., early exiting (Teerapittayanon et al., 2016; Kaya et al., 2019; Schwartz et al., 2020; Liu et al., 2020)

The token pruning concentrates on a width-wise computational reduction, whereas the early exiting succeeds in a depth-wise inference acceleration. Our study shows that for certain tasks where the input data is diverse (in terms of sequence length), these two latency reduction methods perform in the opposite direction. As illustrated in Figure 1 (a), speedup (Y-axis) achieved via early exiting (FastBERT) decreases for long input sizes (X-axis). However, token pruning (LTP) speedup rises as the input size increases. We believe these two approaches are orthogonal and can be combined into a single model to maintain the latency reduction gain across the variable input length. In this work, we present a novel approach, Magic Pyramid (MP), to encourage a speed-adjustable inference. The contribution of this paper includes:

• Our empirical study shows that token pruning and early exiting are potentially orthogonal. This motivates us to exploit the synergy be-



(b) AG news

Figure 1: Speedup of LTP (token pruning), FastBERT (early exiting) and MP (ours) under different sequence lengths on Yelp and AG news. X axis is sequence length, while Y axis is speedup. short: 1-35 tokens; middle: 35-70 tokens; long: >70 tokens

tween token pruning and early exiting and attain higher computational reduction from width and depth perspectives.

> · Compared to two strong baselines, our approach can significantly accelerate the inference time with an additional 0.5-2x speedup but less than 0.5% degradation on accuracy across five classification tasks.

#### 2 **Related Work**

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Large pre-trained models have demonstrated that increasing the model capacity can pave the way for the development of superior AI. However, as we have limited resources allocated for production systems, there has been a surge of interest in efficient inference. Previous works (Sanh et al., 2019; Sun et al., 2019, 2020; Jiao et al., 2020; He et al., 2021) have opened a window into an effective model compression via knowledge distillation (KD) (Hinton et al., 2015). The core of KD is to use a compact student model to mimic the behavior or structure of a large teacher model. As such, the performance of the student model is as accurate as its teacher,

but consuming less computation.

Other researchers approach efficient inference by manipulating the original model. One elegant solution is pruning, which can reduce the computation by removing non-essential components. These components can be either model parameters (model pruning) (Michel et al., 2019; Fan et al., 2019; Hou et al., 2020) or tokens (token pruning) (Wang et al., 2021; Goyal et al., 2020; Kim et al., 2021). In addition, one can boost the speed of the numerical operations of a model through quantization (Zafrir et al.; Wróbel et al., 2020; Shen et al., 2020).

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The aforementioned works lack flexibility in terms of the speedup, albeit some success. To satisfy varying demands, we have to train multiple models. Since deep neural networks can be considered as a stack of basic building blocks, a list of works introduces early exiting (Teerapittayanon et al., 2016; Kaya et al., 2019; Schwartz et al., 2020; Liu et al., 2020), which attaches a set of subclassifiers to these sub-networks to encourage an adjustable inference within a single model, when needed. As opposed to the prior works, which focus on the one-dimensional speedup, this work takes the first step to superimpose token pruning on early exiting. Our empirical studies confirm that these two approaches can accelerate the inference collaboratively and significantly.

#### 3 Methodology - Proposed Method

Prior to this work, token pruning and early exiting have been proven to be effective in accelerating the inference (Wang et al., 2021; Goyal et al., 2020; Schwartz et al., 2020; Liu et al., 2020; Kim et al., 2021). However, as shown in Figure 1, these approaches fall short of reducing the latency at two ends, *i.e.*, short sequences and long sequences. For example, Figure 1 (a) shows that LTP (tokenpruning) provides the highest speed-up for long input sequences. While FastBERT (early exiting) speedup drops as the input size increases from short to long. Therefore we propose a novel approach: Magic Pyramid (MP), which benefits from a combination of token pruning and early exiting. Figure 2 provides a schematic illustration of MP. First of all, MP enables to terminate an inference at any layer when needed. Second, with the increase of the depth of Transformer, redundant tokens can be expelled. The detailed designs are provided in the rest of this section.



As a visual treat , the film is almost perfect

Figure 2: Schematic illustration of magic pyramid for a sentiment analysis task

**Transformer** Owing to it outstanding performance, Transformer (Vaswani et al., 2017) has become a de facto model for NLP tasks, especially after the triumph of pre-trained language models (Devlin et al., 2019; Liu et al., 2019; Radford et al., 2019). A standard Transformer is comprised of *L* stacked Transformer blocks:  $\{B_l\}_{l=1}^L$ , where each block primarily consists of a multi-head attention module and a position-wise feedforward module<sup>1</sup>.

**Token pruning** The core of the Transformer block is the MultiHead module, which is responsible for a context-aware encoding of each token. Notably, since we compute pairwise importance among all tokens within the input via self-attention, the complexity of MultiHead quadratically scales *w.r.t* the sequence length.

Previous works (Wang et al., 2021; Kim et al., 2021) have shown that one can leverage the attention scores to remove non-salient tokens, such that the sequence length can be gradually reduced. Thus, we follow Kim et al. (2021), and superimpose a learnable token pruning on the Transformer model as depicted in Figure 2.

The pruning process is comprised of two stages. We first learn layer-wise threshold parameters  $\Delta$  to softly regulate the flow of hidden states from the current block to the subsequent one. Then, we freeze  $\Delta$  and enforce a hard gating between two consecutive blocks. Finally, the Transformer is optimized under a hard-pruning condition. Please refer to Appendix B and Kim et al. (2021) for the details.

**Early exiting** As shown in Figure 2, in addition to the Transformer backbone and a main classifier, one has to attach an individual sub-classifier module (subclassifier( $\cdot$ )) to the last layer of each

Transformer block  $B_i$ . As such, one can choose to terminate the computation at any layer, when a halt value  $\tau$  is reached.

Similar to Liu et al. (2020), we leverage a two-stage fine-tuning to enhance the performance of sub-classifiers via knowledge distillation. Specifically, we first train the Transformer backbone and the primary classifier through a standard cross entropy between the ground truth yand the predictions y'. Afterward, we freeze the backbone and the primary classifier, but train the L-1 subclassifier( $\cdot$ ) via a Kullback–Leibler divergence:

$$\mathcal{L}(p_{s_1}, ..., p_{s_{L-1}}, p_t) = \sum_{l=1}^{L-1} D_{KL}(p_{s_i}, p_t) \quad (1)$$

where  $p_s$  and  $p_t$  are the predicted probability distribution from the subclassifier( $\cdot$ ) and the main classifier respectively. Once all modules are welltrained, we can stitch them together to achieve a speed-adjustable inference via a halt value  $\tau$ . Appendix A details the early exiting at the inference time.

In summary, we first fine-tune the Transformer model. After that, token pruning and early exiting are applied to the model sequentially.

# 4 Experiments

To examine the effectiveness of the proposed approach, we use five language understanding tasks as the testbed.

Data	Train	Test	Task
AG news	120K	7.6K	topic
Yelp	560K	38K	sentiment
QQP	364K	40K	paraphrase
MRPC	3.7K	408	paraphrase
RTE	2.5K	277	language inference

Table 1: The statistics of datasets

**Datasets** The first two tasks are: *i*) AG news topic identification (Zhang et al., 2015), and *ii*) Yelp polarity sentiment classification (Zhang et al., 2015). The last three are: *i*) Quora Question Pairs (QQP) similarity detection dataset, *ii*) Microsoft Research Paraphrase Corpus (MRPC) dataset and *iii*) Recognizing Textual Entailment (RTE) dataset. The datasets are summarized in Table 1.

**Experimental setup** We compare our approach with four baselines: *i*) standard BERT (Devlin

<sup>&</sup>lt;sup>1</sup>We omit Residual module and Layernorm module in between for simplicity.

	AG news Yelp		Yelp	QQP		MRPC		RTE		
	Acc.	GFLOPs	Acc.	GFLOPs	Acc.	GFLOPs	Acc.	GFLOPs	Acc.	GFLOPs
BERT	94.3	9.0 (1.00x)	95.8	17.2 (1.00x)	91.3	5.1 (1.00x)	85.3	9.2 (1.00x)	68.6	11.2 (1.00x)
distilBERT	94.4	4.5 (2.00x)	95.7	8.6 (2.00x)	90.4	2.6 (2.00x)	84.6	4.6 (2.00x)	58.8	5.6 (2.00x)
LTP	94.3	5.3 (1.72x)	94.7	7.4 (2.32x)	90.6	3.2 (1.60x)	84.8	6.2 (1.48x)	67.8	7.5 (1.50x)
FastBERT	94.3	2.3 (3.97x)	94.8	2.8 (6.18x)	90.7	1.6 (3.20x)	84.3	4.3 (2.13x)	67.6	8.4 (1.33x)
MP (ours)	94.3	1.8 (4.95x)	94.5	2.1 (8.25x)	90.4	1.3 (4.03x)	83.8	3.3 (2.77x)	67.5	6.5 (1.72x)

Table 2: The accuracy and GFLOPs of BERT (Devlin et al., 2019), distilBERT (Sanh et al., 2019), LTP (learned token pruning) (Kim et al., 2021), FastBERT (Liu et al., 2020) and MP (ours) on different datasets. The numbers in parentheses are speedup.

et al., 2019), *ii*) distilBERT (Sanh et al., 2019), *iii*) learned token pruning (LTP) (Kim et al., 2021) and *iv*) FastBERT (Liu et al., 2020). Except distilBERT, all approaches are fine-tuned on uncased BERT-base model (12 layers). We consider accuracy for the classification performance and giga floating point operations (GFLOPs) for the speedup. The training details can be found in Appendix C.

		AG news	6	Yelp			
au	0.1	0.5	0.8	0.1	0.5	0.8	
FastBERT MP (ours)	3.97x 4.95x	10.30x 10.53x	11.95x 11.95x	3.15x 5.35x	6.18x 8.25x	8.84x 10.10x	

Table 3: Speedup of FastBERT and MP with different  $\tau$ .

For token pruning approach, previous works (Wang et al., 2021; Kim et al., 2021) have shown that there exits a trade-off between accuracy and speedup. Thus, we report the performance of models achieving smallest GFLOPs with at most 1% accuracy drop compared to the BERT baseline. Similarly, the speedup of FastBERT is also controlled by the halt value  $\tau$ . We select  $\tau$ obtaining a on-par accuracy with the token pruning competitors for the sake of a fair comparison. This selection criterion is applied to MP as well.

Table 2 demonstrates that all approaches experience loss in accuracy, when a fast inference is activated. Overall, FastBERT is superior to disilBERT and LTP in terms of both accuracy and GFLOPs. Under the similar accuracy, our approach manages to have a significantly faster inference than Fast-BERT, which leads to up to 2.13x extra speedup. We notice that the speedup and accuracy also correlate to the complexity of tasks and the number of training data. Specifically, for the sentence-pair classification tasks, since QQP has much more data (c.f., Table 1), it achieves 4.03x speedup with a loss of 1% accuracy. On the contrary, RTE and MRPC obtain at most 2.77x speedup with the same amount of accuracy degradation. Under the same magnitude of the training data, as AG news and

Yelp are simpler than QQP, they can gain up to 8.25x speedup after sacrificing 1% accuracy.

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**Gains over FastBERT** In section 3, we have claimed that MP can benefit from both token pruning and early exiting. Although this claim is evidenced in Table 2, we are interested in investigating whether such gains consistently hold, when tuning  $\tau$  to control the speed of the inference. According to Table 3, MP can drastically boost the speedup of FastBERT, except for an aggressive  $\tau$ , which will cause the computation to terminate at the first two layers.

**Speedup on sequences with different lengths** Intuitively, longer sentences tend to have more redundant tokens, which can confuse the lower subclassifiers. Consequently, longer sentences require more computation before reaching a lower uncertainty *u*. We bucket the Yelp and AG news dataset into three categories: *i*) short sequences (1-35 tokens), *ii*) middle sequences (35-70 tokens) and *iii*) long sequences (>70 tokens). Figure 1 indicates that LTP prefers long sequences, while FastBERT favors short sequences. Since MP combines the early exiting with the token pruning, it can significantly accelerate both short and long sequences, compared to the two baselines.

# 5 Conclusion

In this work, we introduce Magic Pyramid, which can maintain a trade-off between speedup and accuracy for BERT-based models. Since MP is powered by two outstanding efficiency-encouraging approaches, it can yield substantially faster inference over the baselines up to additional 2x speedup. We also found that token pruning and early exiting falls to efficiently handle sequences under certain length groups. In contrary, such limitations can be alleviated by MP, thereby our approach can indiscriminately accelerate inference for every input data (i.e, inference request) regardless of its length.

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$$s^{l}(x_{i}) = \frac{1}{N_{h}} \frac{1}{n} \sum_{h=1}^{N_{h}} \sum_{j=1}^{n} A_{l}^{h}(x_{i}, x_{j}) \qquad (4)$$



can prune the non-salient tokens to save the com-

putation. We formally define an importance score

of each token  $x_i$  at layer l as:

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Figure 3: An example of attention probability in a single head. Darker color suggests a higher attention score. The bottom heatmap is the column mean of the attention matrix.

Before this work, researchers have proposed two approaches to remove the unimportant tokens based on  $s^{l}(\cdot)$ : *i*) top-k based pruning and *ii*) thresholdbased pruning (Goyal et al., 2020; Wang et al., 2021; Kim et al., 2021). In this work, we follow (Kim et al., 2021), which leverages a layerwise learnable threshold to achieve a fast inference and is superior to other works (Wang et al., 2021; Goyal et al., 2020). We first fine-tune Transformer parameters  $\Theta$  on a downstream task. Then we introduce a two-stage pruning scheme to seek a suitable threshold  $\Delta$  and  $\Theta$ , which can accelerate the inference and maintain a decent accuracy. In the first pruning stage, we apply a gating function  $\sigma(\cdot)$ to weight the outputs  $H_l$  from the current layer l, before we pass them to the next layer l + 1 like this:

$$M^{l}(x_{i}) = \sigma(\frac{s^{l}(x_{i}) - \Delta^{l}}{T})$$
(5)

$$\hat{H}_l(x_i) = H_l(x_i) \otimes M^l(x_i) \tag{6}$$

where  $\sigma$  is a sigmoid function,  $\otimes$  is an elementwise multiplication, T is a temperature parameter and  $\Delta^l \in \mathbb{R}$  is a learnable threshold at layer l. If  $M^l(x_i)$  approaches zero,  $\hat{H}_l(x_i)$  will become zero

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# A Inference for Early Exiting

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To enable a speed-adjustable inference, at each layer l, we first obtain the hidden states  $H_l$  from the Transformer block  $B_l$ . Then a probability  $p_{s_l}$ can be computed from subclassifier( $H_l$ ). One can use  $p_{s_l}$  to calculate the uncertainty  $u_l$  via:

$$u_l = \frac{\sum p_{s_l} \log p_{s_l}}{\log \frac{1}{N}} \tag{2}$$

where  $u_l$  is bound to  $\{0,1\}$ . If  $u_l \leq \tau$ , we can terminate the computation. A larger  $\tau$  suggests a faster exit.

# **B** Learnable Token Pruning

The core of the Transformer block is the MultiHead module, which is responsible for a context-aware encoding of each token. Notably, we compute pairwise importance among all tokens within the input via self-attention. The attention score of each head h between  $x_i$  and  $x_j$  is obtained from:

$$A^{h}(x_{i}, x_{j}) = \operatorname{softmax}\left(\frac{H(x_{i})^{T} \mathbf{W}_{q}^{T} \mathbf{W}_{k} H(x_{j})}{\sqrt{d}}\right)$$
(3)

where  $H(x_i) \in \mathbb{R}^d$  and  $H(x_j) \in \mathbb{R}^d$  are the hid-442 den states of  $x_i$  and  $x_j$  respectively.  $\mathbf{W}_q \in \mathbb{R}^{d_h imes d}$ 443 and  $\mathbf{W}_k \in \mathbb{R}^{d_h imes d}$  are learnable parameters.  $d_h$ 444 is set to  $d/N_h$ , and  $N_h$  is the number of heads 445 used in the Transformer block B. Since we have 446 to conduct  $n^2$  such operation to acquire an atten-447 tion score matrix  $A \in \mathbb{R}^{n \times n}$ , the complexity of 448 MultiHead quadratically scales with the sequence 449 length. Therefore, we encounter a computational 450 bottleneck when working on long sequences. How-451 ever, if we take the average of A along the *i*th 452 column, we notice that the different tokens have 453 distinctive scores as shown in Figure 3. Tokens 454 with large scores tend to be more salient than oth-455 ers, as they receive more attention. As such, we 456

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as well. As such,  $\hat{H}_l(x_i)$  has no impact on the subsequent layers. At this stage, since  $\sigma(\cdot)$  allows the flow of the back-propagation, both  $\Theta$  and  $\Delta$ can be optimized. In addition, (Kim et al., 2021) also impose a L1 loss on M as a regularizer to encourage the pruning operation. Please refer to their paper for the details.

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In the second pruning stage, we binarize the mask values at the inference time via:

$$M^{l}(x_{i}) = \begin{cases} 1, & s^{l}(x_{i}) - \Delta^{l} > 0.5\\ 0, & \text{otherwise} \end{cases}$$
(7)

If  $s^{l}(x_{i})$  is below the threshold  $\Delta^{l}$ ,  $x_{i}$  is subject to the removal from layer l and will not contribute towards the final predictions. We freeze  $\Delta$  but update  $\Theta$ , such that the model can learn to accurately predict the labels merely conditioning on the retained tokens.

Datasets	BERT	DistilBERT	LTP	FastBERT	MP (ours)
AG news	3,-,-,-	3,-,-,-	3,1,2,-	3,-,-,2	3,1,2,2
Yelp	3,-,-,-	3,-,-,-	3,1,2,-	3,-,-,2	3,1,2,2
QQP	5,-,-,-,	5,-,-,-,	5,2,5,-	5,-,-,5	5,2,5,5
MRPC	10,-,-,-,	10,-,-,-,	10,10,5,-	10,-,-,5	10,10,5,5
RTE	10,-,-,-,	10,-,-,-,	10,10,5,-	10,-,-,5	10,10,5,5

Table 4: The number of epochs used for regular training, soft pruning, hard pruning, subclassifiers training on different datasets. "-" indicates the corresponding stage is inactive.

# C Training Details

For training, we use a batch size of 32 for QQP, MRPC, and RTE. We set this to 64 for AG news and Yelp. Since different approaches adopt different training strategies, we unify them as four steps:

- 1. Regular training: training a model  $\Theta$  without additional components;
- 2. Soft pruning: training a model  $\Theta$  and threshold  $\Delta$ ;
- 3. Hard pruning: training a model  $\Theta$  with the binarized mask values;
- 4. Sub-classifiers training: training subclassifiers. For MP, we also activate the pruning operations.

514 We report the number of the training epochs of 515 different steps for all approaches in Table 4. Similar 516 to Kim et al. (2021), we vary the threshold of the 517 final layer  $\Delta^L$  from 0.01 to 0.08, and the threshold for  $\Delta^l$  is set to  $\Delta^L l/L$ . We search the temperature518T in a search space of {1e-5,2e-5,5e-5} and vary519 $\lambda$  from 0.001 to 0.2. We use a learning rate of 2e-5520for all experiments.521