Hierarchy-aligned Language Modeling in Hyperbolic Space for mRNA Coding Sequences

Anonymous authors
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Abstract

Language models are increasingly applied to biological sequences such as proteins and mRNA, yet their default Euclidean geometry may mismatch the hierarchical structures inherent to biological data. While hyperbolic geometry provides a better alternative for accommodating hierarchical data, it has yet to find a way into language modeling for mRNA sequences. In this work, we introduce HyperHELM, a novel framework that implements masked language model pre-training in hyperbolic space for coding (CDS) regions of mRNA sequences. Using a hybrid design with hyperbolic layers atop a Euclidean backbone, HyperHELM aligns learned representations with the biological hierarchy defined by the relationship between mRNA and amino acids. Across multiple multi-species datasets, it outperforms Euclidean baselines on 9 out of 10 tasks involving property prediction, with 10% improvement on average, and excels in out-of-distribution generalization to long and low-GC content sequences; for antibody region annotation, it surpasses hierarchy-aware Euclidean models by 3% in annotation accuracy. Our results highlight hyperbolic geometry as an effective inductive bias for hierarchical language modeling of the CDS regions of mRNA sequences.

1 Introduction

Language models have been increasingly applied to biological sequence data, fueled by the growth of large-scale omics datasets (Lin et al., 2023; Celaj et al., 2023; Brixi et al., 2025). While originally designed for natural language, these models demonstrate promising performance in capturing dependencies within DNA (Zhou et al., 2024; Nguyen et al., 2024b;a; Brixi et al., 2025), RNA (Celaj et al., 2023; Prakash et al., 2024; Yazdani-Jahromi et al., 2025a;b), and protein sequences (Ferruz et al., 2022; Lin et al., 2023). Biological sequences, however, are structured differently from natural language, particularly in their hierarchical organization, where nucleotides or amino acids form motifs that can be nested within larger functional groups (Buhr et al., 2016). In this work, we focus on the rapidly expanding therapeutic domain of RNA, where the codon–amino acid hierarchy plays a key role in determining the biophysical properties of mRNA sequences and their expressed proteins (Clancy & Brown, 2008), and we encode this hierarchy directly into the representation space of a bio-language model by leveraging hyperbolic geometry.

While standard language models rely on Euclidean geometry, the number of concepts in hierarchies grows exponentially, outpacing the polynomial expansion of Euclidean volumes (Matoušek, 1996; 1999). This can severely limit the representation capacity of a model and hinder generalization (Liu et al., 2020). In contrast, the volume of hyperbolic space expands exponentially, maintaining well-separated representations across different branches of the hierarchy and reducing distortion in hierarchical relationships. The advantages of hyperbolic geometry are demonstrated in graph representation learning (Chami et al., 2019) and computer vision (Mettes et al., 2024), and are beginning to inform natural language modeling (He et al., 2024; 2025), though they have yet to be systematically applied to mRNA data.

In this work, we present Hyperbolic Hierarchical Encoding for mRNA Language Modeling (HyperHELM), a hyperbolic language-modeling framework for the CDS regions of mRNA sequences. In HyperHELM, we project token representations onto the Poincaré ball and pre-train a language model with the masked language modeling (MLM) objective directly in hyperbolic space (Figure 1). Rather than making the entire model

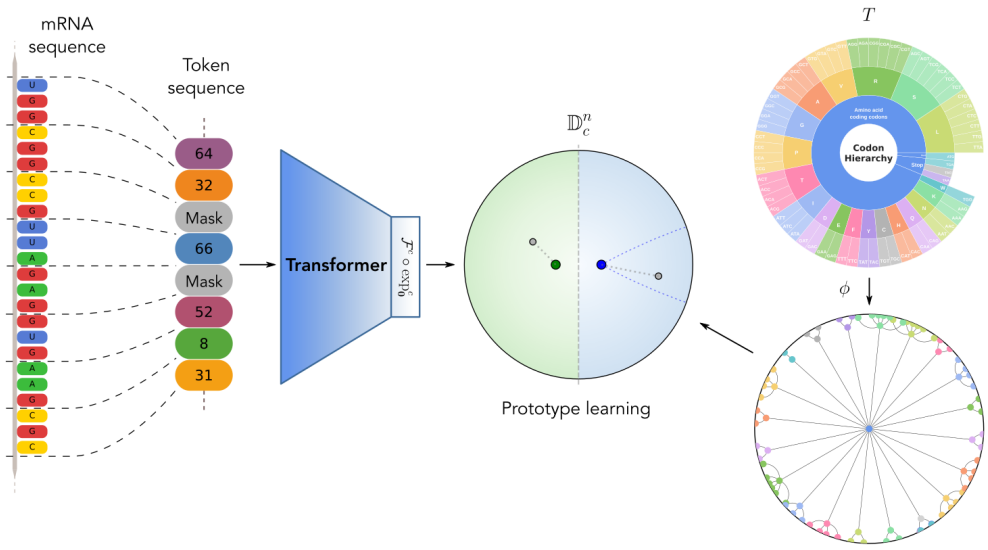

Figure 1: **High-level overview of the HyperHELM method** for MLM. The method consists of three main components: 1) the language modeling of mRNA, where a sequence transformer is used to obtain token representations, as shown on the *left*; 2) a hyperbolic embedding of the codon hierarchy (large version in Appendix A) is generated to serve as prototypes for guiding the language model during pre-training, shown on the *right*; and 3) hyperbolic hierarchical prototype learning, where the prototypes are used to predict the true label of masked tokens using either distances (*green*) or entailment cones (*blue*), visualized in the *center*.

hyperbolic, we keep the backbone Euclidean and project only the final-layer representations, thus retaining hardware efficiency while leveraging the hierarchical inductive bias of hyperbolic geometry.

For hyperbolic MLM pre-training, we mask a portion of input tokens and use a modular hyperbolic prediction head that scores candidates while respecting hierarchical relations. In particular, we instantiate two head options for hyperbolic learning: distance-to-prototype learning (Snell et al., 2017) and prototype classifiers based on hyperbolic entailment cones (Ganea et al., 2018a), both with prototypes derived from the predefined codon–amino acid hierarchy. While Ganea et al. (2018a) primarily introduce entailment cones as a means to model hierarchical relations, our work extends this concept further by exploring its use as a similarity function instead of hyperbolic distances, aiming to capture richer relational structures. The resulting hyperbolic latent space with hierarchy-aware MLM pre-training aligns representation geometry with the codon–amino acid structure, clustering synonymous codons under their amino acid parents and separating non-coding tokens (Figure 1). To our knowledge, HyperHELM is the first systematic development of hyperbolic language models for mRNA sequence data.

We conduct experiments to compare our HyperHELM to the standard Euclidean language modeling approach, the hierarchical Euclidean HELM (Yazdani-Jahromi et al., 2025a), and HELM's direct hyperbolic generalization. We keep the language model backbone architecture and pre-training dataset fixed for all models, to isolate the impact of our approach. We evaluate the pre-trained models on 10 diverse multi-species mRNA datasets for downstream property prediction and on a region annotation task. Across 9 out of 10 property prediction tasks, our approach consistently outperforms the baselines, even when these are hierarchy-aware or hyperbolic, achieving an average improvement of 10%. We also observe that in property prediction tasks, our hyperbolic language model generalizes exceptionally well to out-of-distribution data, maintaining strong performance even on long sequences with low GC-content, where standard bio-language models tend to struggle. Moreover, for the task of antibody region annotation, our HyperHELM surpasses the hierarchy-aware Euclidean baseline by 3%. Our experimental results suggest that, when paired with prototype learning, hyperbolic geometry can provide a powerful inductive bias for capturing hierarchical structures in CDS regions of mRNA sequences.

To sum up, we make the following contributions:

- We explore hierarchical learning for bio-language models through the lens of hyperbolic geometry, aiming to align the representation space with the hierarchical structure of CDS regions of mRNA.
- We propose, implement, and evaluate two hierarchy-guided hyperbolic learning methods for masked language pre-training of a language model on CDS regions of mRNA.
- We experimentally demonstrate the benefits of hyperbolic language models on downstream mRNA property prediction and antibody region annotation, where they outperform Euclidean models, and excel in out-of-distribution settings.

2 Related works

RNA and mRNA Models Several supervised models for RNA and mRNA modeling exist, such as RiboNN (Zheng et al., 2025), which uses a convolutional model for predicting the translation efficiency of mature mRNA sequences; or Optimus 5-Prime (Sample et al., 2019), which is a convolutional model aimed at predicting the regulatory activity of 5' UTRs of mRNA sequences. Our focus is on unsupervised pre-training, for which the common approach is language modeling. RNA and mRNA language models enable diverse downstream tasks in property prediction, annotation, and generation. These include foundation models trained for different RNA regions such as non-coding RNA (RNA-FM (Chen et al., 2022a), RINALMO (Penić et al., 2025), and AIDO.RNA-CDS (Zou et al., 2024) which is subsequently fine-tuned to CDS regions within mRNA), splice sites (SpliceBERT (Chen et al., 2023)) or UTRs (UTR-LM (Chu et al., 2024)), as well as methods using transfer learning from DNA and protein models (Prakash et al., 2024; Garau-Luis et al., 2024; Mollaysa et al., 2025) for mRNA-focused downstream tasks. For mRNA, codon-level models such as CodonBERT (Li et al., 2023) use codon tokenization with MLM to optimize coding-region embeddings. Others employ nucleotide-level tokenization, such as Orthrus (Fradkin et al., 2024), which is a Mamba-based RNA model that is pre-trained on mature RNA sequences; LoRNASH (Saberi et al., 2024), which is a Hyena-based RNA model pre-trained on pre-mRNA; or Helix-mRNA (Wood et al., 2025), which employs hybrid attention and state-space architectures for improved sequence resolution and generation. Several recent models incorporate domain priors. Equi-mRNA (Yazdani-Jahromi et al., 2025b) and HELM (Yazdani-Jahromi et al., 2025a) promote equivariance and hierarchical understanding in Euclidean space, respectively. Moskalev et al. (2024); Xu et al. (2025a;b) link sequence to structure. Despite these advances, all existing methods are confined to Euclidean spaces. To our knowledge, this is the first work to explore language model pre-training for RNA or mRNA in hyperbolic space.

Hyperbolic Learning The exponential growth of hyperbolic space makes it a suitable domain for learning on data with an inherent hierarchical structure (Sarkar, 2011; Chamberlain et al., 2017; Nickel & Kiela, 2017). This realization has led to a surge in the popularity of hyperbolic learning (Peng et al., 2021). Deep hyperbolic architectures have been developed (Ganea et al., 2018b; Shimizu et al., 2021; Chen et al., 2022b) alongside the algorithms for optimizing such networks (Bonnabel, 2013; Bécigneul & Ganea, 2019). As a result, hyperbolic geometry has seen successful applications across many areas of machine learning, such as in computer vision (Khrulkov et al., 2020; Liu et al., 2020; Long et al., 2020; Ghadimi Atigh et al., 2021; van Spengler et al., 2023a; Mettes et al., 2024), graph learning (Liu et al., 2019; Chami et al., 2019; Zhang et al., 2021; Yang et al., 2022), natural language processing (Dhingra et al., 2018; Tifrea et al., 2019) and multimodal learning (Desai et al., 2023; Pal et al., 2025). These have shown the potential of hyperbolic learning, particularly in scenarios where the data has a clear hierarchical structure. Recently, a first work has explored the application of fully hyperbolic convolutional networks for DNA modeling (Khan et al., 2025), finding that hyperbolic geometry improves genomic sequence understanding. While the structuring of mRNA is highly hierarchical in nature, existing mRNA language modeling approaches do not leverage hyperbolic geometry.

Prototype Learning The prototype learning setting (Snell et al., 2017) has become a commonly used approach for classification tasks, where each class is represented by a prototype, resembling in some way the perfect instance of its corresponding class. Within hyperbolic learning, prototype learning approaches are mostly distinguishable by their method of obtaining prototypes (Mettes et al., 2024). Many works follow

the original approach for generating prototypes based on labeled input data (Khrulkov et al., 2020; Gao et al., 2021; 2022; Guo et al., 2022). These typically create prototypes by aggregating features of labeled instances of the corresponding class using, for example, the Fréchet mean. Another approach is to use prior knowledge of the label set to generate prototypes. Examples include Ghadimi Atigh et al. (2021) and Long et al. (2020), which create prototypes using a known hierarchy over the labels, or Yu et al. (2022), which optimizes prototypes concurrently with their model through the use of known hierarchical relations. Concurrent work by Fonio et al. (2025) generates prototypes using maximal separation, not making use of any known hierarchies. While each of these works deals with an image classification setting, we instead focus on masked language modeling. Moreover, unlike our work, none of these works explores the use of recent low-distortion embedding methods for generating prototypes from hierarchies. Lastly, except for the concurrent work by Fonio et al. (2025), these works restrict the use of similarity functions to hyperbolic distances.

3 Background on Hyperbolic Space

In this paper, we make use of the n-dimensional Poincaré ball model $(\mathbb{D}_c^n, \mathfrak{g})$ of hyperbolic space with constant negative curvature $-c$ and Riemannian metric $\mathfrak{g}_c^n$, where

$$\mathbb{D}_c^n = \Big\{\mathbf{x} \in \mathbb{R}^n : ||\mathbf{x}||^2 < \frac{1}{c}\Big\}, \quad \mathfrak{g}_c^n = (\lambda_{\mathbf{x}}^c)^2 I_n, \quad \lambda_{\mathbf{x}}^c = \frac{2}{1 - c||\mathbf{x}||^2}, \tag{1}$$

with I_n being the n-dimensional identity matrix. For an extensive background on other isometric models and on hyperbolic geometry in general, we refer the reader to (Cannon et al., 1997; Anderson, 2006). Here, we introduce the operations that are used throughout the paper.

Using the Riemannian metric, one can compute the distances between any two points $\mathbf{x}, \mathbf{y} \in \mathbb{D}_c^n$ as

$$d_{\mathbb{D}}^c(\mathbf{x}, \mathbf{y}) = \frac{1}{\sqrt{c}} \cosh^{-1}\left(1 + 2c\frac{||\mathbf{x} - \mathbf{y}||^2}{(1 - c||\mathbf{x}||^2)(1 - c||\mathbf{y}||^2)}\right). \tag{2}$$

Using the Möbius addition operation (Ungar, 2022), defined as

$$\mathbf{x} \oplus_c \mathbf{y} = \frac{(1 + 2c\langle \mathbf{x}, \mathbf{y}\rangle + c||\mathbf{y}||^2)\mathbf{x} + (1 - c||\mathbf{x}||^2)\mathbf{y}}{1 + 2c\langle \mathbf{x}, \mathbf{y}\rangle + c^2||\mathbf{x}||^2||\mathbf{y}||^2}, \tag{3}$$

we can define the exponential and logarithmic maps (Ganea et al., 2018b)

$$\exp_{\mathbf{x}}^c : \mathcal{T}_{\mathbf{x}}\mathbb{D}_c^n \to \mathbb{D}_c^n, \quad \exp_{\mathbf{x}}^c(\mathbf{v}) = \mathbf{x} \oplus_c \left(\tanh\left(\frac{\sqrt{c}\lambda_{\mathbf{x}}^c||\mathbf{v}||}{2}\right)\frac{\mathbf{v}}{\sqrt{c}||\mathbf{v}||}\right), \tag{4}$$

$$\log_{\mathbf{x}}^c : \mathbb{D}_c^n \to \mathcal{T}_{\mathbf{x}}\mathbb{D}_c^n, \quad \log_{\mathbf{x}}^c(\mathbf{y}) = \frac{2}{\sqrt{c}\lambda_{\mathbf{x}}^c} \tanh^{-1}\left(\sqrt{c}||-\mathbf{x} \oplus_c \mathbf{y}||\right)\frac{-\mathbf{x} \oplus_c \mathbf{y}}{||-\mathbf{x} \oplus_c \mathbf{y}||}, \tag{5}$$

which are used to map tangent vectors from the tangent space $\mathcal{T}_{\mathbf{x}}\mathbb{D}_c^n$ at $\mathbf{x}$ onto $\mathbb{D}_c^n$ and vice versa, respectively.

Ganea et al. (2018b) have generalized multinomial logistic regression (MLR) to the Poincaré ball model by interpreting the MLR scores as signed distances to hyperplanes. The resulting hyperbolic MLR computes scores as

$$\ell_k(\mathbf{x}) = \frac{2}{\sqrt{c}}||\mathbf{z}_k|| \sinh^{-1}\left(\lambda_{\mathbf{x}}^c\left\langle \sqrt{c}\mathbf{x}, \frac{\mathbf{z}_k}{||\mathbf{z}_k||}\right\rangle \cosh(2\sqrt{c}r_k) - (\lambda_{\mathbf{x}}^c - 1)\sinh(2\sqrt{c}r_k)\right), \tag{6}$$

where $\mathbf{z}_k$ and r_k are the parameters corresponding to the k-th class. This MLR has been further extended into a hyperbolic fully connected layer $\mathcal{F}^c : \mathbb{D}_c^n \to \mathbb{D}_c^m$ by Shimizu et al. (2021), which is computed as

$$\mathcal{F}^c(\mathbf{x}; \mathbf{Z}, \mathbf{r}) = \frac{\mathbf{w}}{1 + \sqrt{1 + c||\mathbf{w}||^2}}, \quad \mathbf{w} = \Big(\frac{1}{\sqrt{c}}\sinh\left(\sqrt{c}\ell_k(\mathbf{x})\right)\Big)_{k=1}^m, \tag{7}$$

where $\mathbf{Z}$ and $\mathbf{r}$ contain the learnable parameters.

4 HyperHELM

The setting that we consider is the pre-training of a CDS region mRNA sequence model through masked language modeling (MLM) with the goal of obtaining a strong backbone for any downstream predictive task. For our approach, we take the HELM method—a language model for the hierarchical modeling of mRNA that operates fully in Euclidean space (Yazdani-Jahromi et al., 2025a)—as a starting point and replace the final representation space and classifier to help guide the backbone model more effectively. More specifically, we project token representations to hyperbolic space and we replace the Euclidean multinomial logistic regression classifier with a hyperbolic prototypical classifier, inspired by works such as (Snell et al., 2017; Yu et al., 2022). The prototypes are generated directly from the codon–amino acid hierarchy which is shown in Figure 1 and, more clearly, in Figure 4 in Appendix A. A high-level overview of our method is given in Figure 1. Each individual component will be discussed in detail in the following subsections.

4.1 Language Modeling of mRNA Sequences

Our goal is to train some sequence transformer model f of CDS regions of mRNA through MLM. Following recent works (Li et al., 2023; Yazdani-Jahromi et al., 2025a;b), we first apply codon-level tokenization to the mRNA sequences, where each triplet of nucleotides is represented as a single token, giving $4^3 = 64$ potential tokens, excluding special tokens. During MLM, we mask 15% of the tokens in sequences and feed these into model f, which outputs a representation in $\mathbb{R}^n$ for each individual token. Then, we use a classifier $g : \mathbb{R}^n \to [64]$ to predict the true label of the masked tokens. Following the HELM approach (Yazdani-Jahromi et al., 2025a), the hierarchical cross-entropy loss (Bertinetto et al., 2020) with respect to the codon hierarchy shown in Figure 1 is computed and used to update f and g (see Appendix I for an ablation on this choice of loss function).

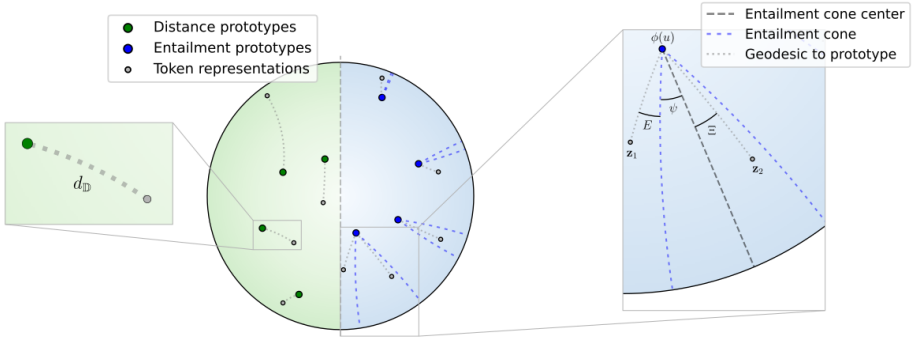

Figure 2: **Hyperbolic prototype learning.** The *center* part presents a Poincaré disk where either distances (green) or entailment cone energies (blue) are used to predict the label of embedded tokens. On the *left*, a close up of a masked token representation with its closest prototype, together with the geodesic between these is shown. The *right* part takes a closer look at one of the entailment cones, showing the geometric interpretation of Equations 11, 12 and 13.

4.2 Hyperbolic Embeddings of Hierarchies

The manner in which mRNA encodes for proteins can be understood through a hierarchy defined over the codons, visualized in Figure 1. Yazdani-Jahromi et al. (2025a) softly enforce this hierarchy in their model in Euclidean space by using the hierarchical cross-entropy loss. Here, we explicitly structure our token representation space by directly embedding the hierarchy. A hierarchy typically consists of a tree $T = (V, E)$, where the nodes V contain the relevant concepts and the edges E the relations between these. Moreover, we denote the leaf nodes of the tree by L. The tree metric d_T, resulting from T, defined as the length of the path between two nodes, contains the information of how strongly related any pair of concepts is. Therefore, the goal of embedding some hierarchy into a continuous space is to keep this tree metric intact. More formally, we want an embedding $\phi : V \to M$ into some connected Riemannian manifold M such that

ϕ is approximately an isometry onto $\phi(V)$, i.e.,

$$d_M\big(\phi(u), \phi(v)\big) \approx d_T(u, v). \tag{8}$$

The amount by which the metric is changed by the embedding is called the distortion. It can be shown that Euclidean spaces are unsuitable as targets for embedding trees (Sarkar, 2011), generally leading to highly distorted embeddings. Therefore, we opt to use hyperbolic space instead.

Several methods exist for embedding graphs or trees into hyperbolic space (Sarkar, 2011; Nickel & Kiela, 2017; Sala et al., 2018; van Spengler & Mettes, 2025). We embed the codon hierarchy using the HS-DTE method (van Spengler & Mettes, 2025), as it achieves the lowest distortion and thus most effectively preserves the underlying hierarchical structure, while also being fast. Empirically, we find that the model is quite insensitive to the choice of tree embedding method (see Appendix G). We use the embeddings of the leaf nodes obtained with HS-DTE, corresponding to individual codons, as prototypes within the classifier g. A 2-dimensional example embedding of the entire codon hierarchy obtained with HS-DTE is shown in Figure 1.

4.3 Prototype Learning in Hyperbolic Space

From the hierarchy embedding, we have a set of prototypes $\phi(L) \subset \mathbb{D}_c^{n_p}$ where each prototype corresponds to a particular codon and where n_p is the prototype dimension. Since the embedding ϕ respects the tree metric d_T, these prototypes structure the space according to the hierarchy, without having seen any sequence data. We want to define a classifier that uses these prototypes to generate token-level predictions. Since our backbone model f outputs representations in $\mathbb{R}^n$, these are first projected onto $\mathbb{D}_c^{n_p}$ through two steps: 1) the representations are projected into hyperbolic space $\mathbb{D}_c^n$ and 2) a hyperbolic linear layer is used to project to $\mathbb{D}_c^{n_p}$. Following the convention in hyperbolic learning (Mettes et al., 2024), the first step is performed by treating the representations as tangent vectors at the origin and applying the corresponding exponential map (see Appendix J for an ablation). The second step is performed using the hyperbolic linear layer $\mathcal{F}^c : \mathbb{D}_c^n \to \mathbb{D}_c^{n_p}$ from Equation 7. So, the projection can be written as

$$\mathbf{z}_i = \mathcal{F}^c\big(\exp_{\mathbf{0}}^c(\mathbf{h}_i)\big), \quad \mathbf{h}_i = f(\mathbf{t}^*)_i, \tag{9}$$

where $\mathbf{t}^*$ is the masked token sequence.

Generally, to generate token-level predictions using prototypes, softmaxed pairwise similarities between representations and prototypes are computed (Snell et al., 2017):

$$p(t_i = u|\mathbf{t}^*) = \frac{\exp\big(\beta \cdot s(\mathbf{z}_i, \phi(u))\big)}{\sum_{v \in L} \exp\big(\beta \cdot s(\mathbf{z}_i, \phi(v))\big)}, \tag{10}$$

where $\beta > 0$ is a temperature hyperparameter (set to 1.0), t_i is the true i-th token and $s : \mathbb{D}_c^{n_p} \times \mathbb{D}_c^{n_p} \to \mathbb{R}$ is some similarity function. Typically, negative distances $s = -d_{\mathbb{D}}$ are used as similarities, which leads the model to simply assign a token to its closest prototype. This approach is shown in Figure 2 *left*.

Alternatively, we can compute similarities using the hyperbolic entailment cone energy (Ganea et al., 2018a). Entailment cones are a geometric approach to defining hierarchical relationships in hyperbolic space. These are defined for any point $\mathbf{z} \in \mathbb{D}_c^{n_p}$ as the hyperbolic cone with $\mathbf{z}$ as its apex and with the axis of symmetry being the Euclidean straight line segment from $\mathbf{z}$ perpendicular onto the boundary of the manifold. The half aperture of the cone is

$$\psi(\mathbf{z}) = \sin^{-1}\left(\frac{K(1 - c||\mathbf{z}||^2)}{\sqrt{c}||\mathbf{z}||}\right), \tag{11}$$

where K is a hyperparameter which we set to $K = 0.1$. The hyperbolic entailment cone energy is then computed as

$$E(\mathbf{x}, \mathbf{y}) = \max(0, \Xi(\mathbf{x}, \mathbf{y}) - \eta\psi(\mathbf{x})), \tag{12}$$

where $\eta > 0$ is a threshold hyperparameter (Pal et al., 2025) (set to 1.05) and where

$$\Xi(\mathbf{x}, \mathbf{y}) = \cos^{-1}\left(\frac{\langle \mathbf{x}, \mathbf{y} \rangle (1 + c||\mathbf{x}||^2) - ||\mathbf{x}||^2(1 + c||\mathbf{y}||^2)}{||\mathbf{x}|| \cdot ||\mathbf{x} - \mathbf{y}|| \sqrt{1 + c^2||\mathbf{x}||^2||\mathbf{y}||^2 - 2c\langle x, y \rangle}}\right), \tag{13}$$

is the aperture required for $\mathbf{y}$ to be within the entailment cone at $\mathbf{x}$. In other words, the hyperbolic entailment cone energy is the angle by which $\mathbf{y}$ is removed from $\mathbf{x}$'s entailment cone. Examples of entailment cones and a visualization of the entailment cone energy are shown in Figure 2 *right*. The hyperbolic entailment cone energy has recently grown in popularity in areas such as vision-language learning (Desai et al., 2023; Pal et al., 2025) for encoding hierarchical relations. We propose to use both distance-based prototypes and entailment-based prototypes. For both approaches, we set the negative curvature to $c = 1.0$. We also present a sensitivity analysis for the key hyperparameters in Appendix H. Lastly, we experiment using both fixed and learnable prototypes, where the prototypes are considered learnable parameters of the model, which allows learning further data-driven refinements of the hierarchy embedding. For the optimization of the learnable hyperbolic prototypes we use Riemannian SGD Bonnabel (2013). Further details regarding the optimization can be found in Appendix B.

5 Experiments

In our experiments, we follow the pre-training guidelines established in HELM (Yazdani-Jahromi et al., 2025a), adopting codon-level tokenization and the masked language modeling (MLM) objective. We pre-train on the same curated OAS pre-training corpus (Olsen et al., 2022) consisting of CDS-only sequences with identified open reading frame, using a standard transformer backbone with the official HELM repository[1], ensuring full comparability (further details in Appendix B). The key differences lie in the representation space and the MLM head where we evaluate three hyperbolic variants: hyperbolic multinomial logistic regression, hyperbolic distance-based prototypes, and hyperbolic prototypes based on entailment cones discussed in Sections 3 and 4. We keep the rest of the method unchanged, allowing us to isolate the effect of learning the hierarchy in hyperbolic space for mRNA. For downstream tasks, we freeze the pre-trained backbone and probe the learned representations by training a TextCNN head (Kim, 2014), following standard practice (Harmalkar et al., 2023; Li et al., 2023; Yazdani-Jahromi et al., 2025a; Mollaysa et al., 2025; Yazdani-Jahromi et al., 2025b). Further experimental details are in Appendices B and D. Note that, since we only change the head of the model, the overall complexity is dominated by the backbone for each method. As a result, the difference in runtimes between the different methods is negligible (Appendix C).

Datasets and Evaluation Metrics We use 10 property prediction datasets of CDS-only mRNA sequences spanning diverse organisms and label types: Ab1 (662 antibody-encoding mRNAs) and Ab2 (2,672 antibody-encoding mRNA sequences) both with protein expression labels from (Prakash et al., 2024); mRFP (1,459 sequences with protein production levels) (Nieuwkoop et al., 2023); COVID-19 Vaccine (2,400 degradation-labeled sequences) (Wayment-Steele et al., 2022); *Drosophila melanogaster* (10,338 mRNA sequences) and *Saccharomyces cerevisiae* (4,937 mRNA sequences) with protein abundance labels, and *Pichia pastoris* (4,682 mRNA sequences) with transcript abundance from Outeiral & Deane (2024); Fungal (7,056 genome-derived sequences with expression labels) (Wint et al., 2022); *E. coli* (6,348 mRNAs labeled with low/medium/high protein expression) (Ding et al., 2022); and iCodon (65,357 sequences with thermostability profiles from humans, mice, frogs, and fish) (Diez et al., 2022). Except for the *E. coli* classification task, all datasets provide regression labels for evaluating property prediction. Following prior works (Li et al., 2023; Yazdani-Jahromi et al., 2025a;b), we use the available predefined train/val/test data splits and report Spearman rank correlation for regression and accuracy for classification tasks.

Baselines We evaluate the HyperHELM variants against three baselines: a standard Euclidean transformer model trained with the cross-entropy (XE) loss; the hierarchy aware HELM (Yazdani-Jahromi et al., 2025a) method, which trains a standard Euclidean transformer model using the hierarchical cross-entropy loss; and a hyperbolic version of the HELM method, where representations are projected into hyperbolic space and classified using hyperbolic MLR (Ganea et al., 2018b). To ensure a fair comparison, each variant is trained with the same 50M parameter backbone architecture, pre-training data, and tokenization strategy. Results for additional baselines and comparisons against large scale foundation models are shown and discussed in Appendix F.

[1] https://github.com/johnsonandjohnson/HELM

Table 1: Accuracy (for *E.coli*) and Spearman rank correlation (for all other datasets). Bold indicates the best performing model per dataset and underline indicates second best model.

	Euclidean baselines		Hyperbolic baseline	HyperHELM (Ours)			
Dataset	**Transformer XE**	**HELM**	**MLR**	**Proto Dist.**	**Proto Entail.**	**Proto Dist. Learnable**	**Proto Entail. Learnable**
Ab1	0.701	0.714	0.650	0.713	**0.751**	0.743	0.736
Ab2	0.507	0.548	0.532	0.575	0.569	**0.603**	0.589
mRFP	0.825	**0.848**	0.744	0.819	0.802	0.800	0.820
COVID-19	0.757	0.775	0.411	0.785	0.807	**0.822**	**0.822**
D. melanogaster	0.332	0.341	0.374	0.394	**0.450**	0.442	0.447
S. cerevisiae	0.354	0.398	0.465	0.434	0.397	0.424	**0.480**
P. pastoris	0.596	0.620	0.605	**0.676**	0.671	0.672	0.672
Fungal	0.690	0.702	0.712	0.735	0.741	0.742	**0.754**
E. coli	44.7	45.8	40.0	50.8	48.4	**53.0**	50.9
iCodon	0.503	0.525	0.517	0.535	0.539	**0.545**	0.536

Table 2: (a) Accuracy of antibody sequence region annotation, (b) Spearman rank correlation across sequence lengths for *P. pastoris*, (c) Spearman rank correlation across GC content for the COVID-19 dataset. Best performance is shown in bold.

Model	**Acc. (%)**
HELM	73.48
HyperHELM (Dist.)	**76.48**
HyperHELM (Entail.)	75.21

(a) Antibody annotation

Model	**Short**	**Med.**	**Long**
HELM	0.54	0.58	0.46
HyperHELM (Dist.)	**0.65**	**0.59**	0.65
HyperHELM (Entail.)	0.61	0.56	**0.70**

(b) Sequence length analysis

Model	**Low**	**Med.**	**High**
HELM	**0.78**	0.64	0.56
HyperHELM (Dist.)	0.77	0.62	0.54
HyperHELM (Entail.)	**0.78**	**0.73**	**0.62**

(c) GC content analysis

5.1 HyperHELM Improves Downstream mRNA Property Prediction Performance over Euclidean Models

Table 1 summarizes the performance of HyperHELM variants across 10 mRNA property prediction datasets. Of these, the four HyperHELM variants achieve the best performance on 9 out of 10 datasets and the best and second best performance on 8 out of 10 datasets, demonstrating the benefits of modeling hierarchical relationships in hyperbolic spaces for mRNA sequences. Compared to the non-hierarchical Transformer XE baseline, HyperHELM improves downstream performance by 2.8–35.6%, with the largest gains observed for *D. melanogaster* (35.5%) and *S. cerevisiae* (35.6%). When compared to HELM, performance improvements range up to 32%, with particularly strong improvements on *D. melanogaster* (32.0%) and *S. cerevisiae* (20.6%) datasets. Interestingly, simple hyperbolic MLR only performs well on the *S. cerevisiae* dataset while underperforming on all other tasks even relative to the Euclidean baselines, indicating that the proposed combination of hyperbolic geometry with the novel prototype-based approach is crucial for capturing hierarchical structure in mRNA embeddings (see Appendix E for additional details). Lastly, learnable prototypes yield the best performance in 6 out of 10 datasets and either the best or second best performance in 9 out of 10 datasets, which shows that the model benefits from the freedom to refine the hierarchical embeddings during pre-training.

5.2 Codon Usage Bias/Pattern is an Indicator for Hyperbolic Model Gains

We observed that HyperHELM's performance gains vary significantly across datasets (Table 1). Building on prior work that links gains from hierarchical learning to codon usage bias (Yazdani-Jahromi et al., 2025a), we investigated if this holds for models trained in hyperbolic spaces.

To this end, we measured each dataset's synonymous codon usage bias using the Effective Number of Codons (ENC) metric (Wright, 1990). This metric quantifies codon diversity: a low ENC value signifies high bias (a strong preference for specific codons for a given amino acid), while a high value indicates codons are used more uniformly. As shown in Figure 3, our results confirm the hypothesis: datasets with greater codon usage bias (lower ENC) consistently achieve larger gains with HyperHELM prototype based variants. Intuitively, this is because a strong codon bias creates a stronger learnable hierarchical pattern even among synonymous codons beyond the hierarchy defined by codons and amino acids. This additional hierarchy is naturally

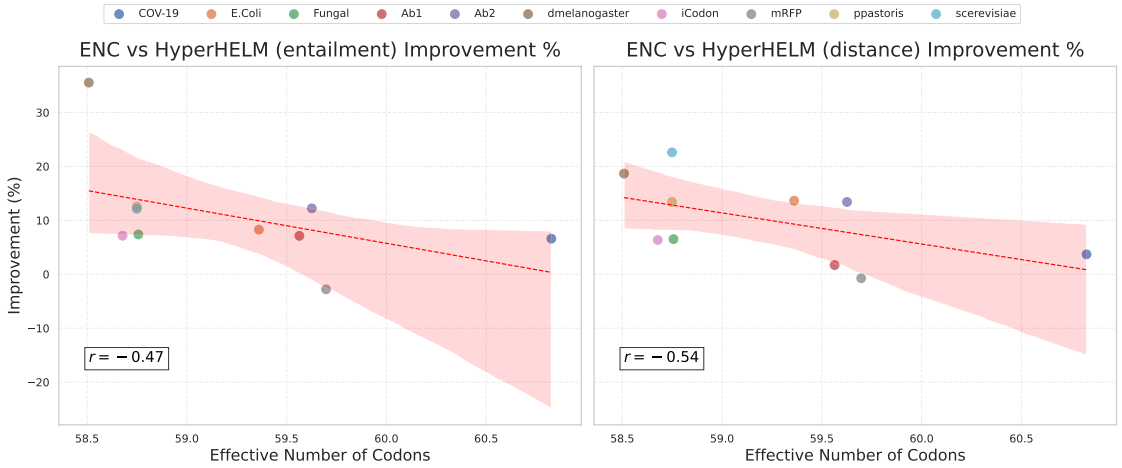

Figure 3: Relationship between codon usage metric (ENC) and HyperHELM performance gains. Hyperbolic gains are largest for sequences with higher codon usage bias indicated by lower ENC.

suited to the geometry of hyperbolic space, allowing HyperHELM to capture these dependencies from data more effectively than non-hierarchical models.

5.3 HyperHELM Improves Antibody Sequence Annotation

We further assess HyperHELM on the task of antibody (Ab) sequence region annotation, a benchmark introduced in prior work (Yazdani-Jahromi et al., 2025a), important for immunological studies (Briney & Burton, 2018). This task involves predicting the identity of nucleotides in Ab-coding mRNA into one of four biologically meaningful regions: signal peptides, V, DJ, or constant regions.

We use the same held-out test set of 2000 curated antibody sequences as used in Yazdani-Jahromi et al. (2025a) for this task and compare our prototype based HyperHELM models against the HELM baseline. As shown in Table 2(a), both HyperHELM variants outperform Euclidean HELM, with the prototype distance model achieving the best accuracy of 76.48%, and the prototype entailment variant being second best with an accuracy of 75.21%, compared to 73.48% achieved by HELM. The results highlight the advantage of hierarchy-aware learning in hyperbolic space to effectively capture the structure of antibody mRNA regions.

5.4 Impact of Sequence Length and GC Content on Model Performance

We examine model robustness across different biologically meaningful mRNA sequence characteristics by stratifying datasets according to sequence length and GC content. These factors are known to be relevant for mRNA engineering (Zhang et al., 2011; Courel et al., 2019; Jia & Qian, 2021) and have been linked to differences in model generalization (Castillo-Hair & Seelig, 2021; Szikszai et al., 2022; Qiu, 2023). Longer sequences often contain more complex dependencies and are underrepresented in training data, while extreme GC content alters secondary structure; both scenarios making it challenging for models to learn effectively.

Sequence Length Analysis We analyzed performance on the *Pichia pastoris* dataset by dividing sequences into three length categories: *short* (30–1000 nucleotides), *medium* (1000–2000 nucleotides), and *long* (2000–3000 nucleotides). Since the pre-training data consists of sequences around 1400 nucleotides (a typical range for mRNA vaccines (Gunter et al., 2023)), the long sequences represent an out-of-distribution (OOD) challenge.

As shown in Table 2(b), Euclidean HELM's performance degrades sharply with increasing length, consistent with prior findings (Yazdani-Jahromi et al., 2025a). In contrast, both HyperHELM variants reverse this trend, with performance improving on long sequences compared to medium ones. The entailment-based variant reached a Spearman correlation of 0.70 (a +0.24 absolute improvement over HELM), while the distance-based variant showed a +0.19 improvement. This indicates that HyperHELM's hyperbolic repre-

sentation space is beneficial even for out-of-distribution length shifts, a trend also reported for hyperbolic models in other domains (Ibrahimi et al., 2024; Kasarla et al., 2025).

GC Content Analysis For the COVID-19 dataset, we categorize sequences based on GC content into: *low* (GC $\leq 47\%$), *medium* ($47\% <$ GC $\leq 55\%$), and *high* (GC $> 55\%$). These thresholds align with widely used biological definitions, where GC content below 47% is considered low and above 55% is high (Brown, 2007; Courel et al., 2019).

Performance for both HELM and HyperHELM (shown in Table 2(c)) is reasonably high in the low GC range but diminishes for high GC content sequences due to their relative scarcity in the pre-training corpora. Notably, the entailment-based HyperHELM attains a Spearman rank correlation of 0.62 in the high GC category compared to HELM's 0.56, and achieves a strong Spearman rank correlation of 0.73 in the medium GC category, a gain of +0.09 over HELM.

6 Conclusion

The strong performance of our hyperbolic prototype-based models indicates that explicitly modeling hierarchical mRNA relationships in hyperbolic space is more effective than standard Euclidean approaches, even when the latter are made hierarchy-aware. Hyperbolic embeddings not only improve downstream property prediction but also offer a more faithful reflection of codon–amino acid relationships, particularly in sequences with strong codon usage bias. Results also demonstrate that hyperbolic hierarchy-aware modeling improves generalization to out-of-distribution settings such as modeling long sequences and low GC content. The observed improvements highlight the potential of hybrid language models for biological sequences, where Euclidean backbones are paired with hyperbolic heads, as a practical strategy to integrate hierarchical inductive biases without incurring the computational overhead of fully hyperbolic networks.

Impact Statement

Advances in modeling mRNA coding sequences can contribute to an improved understanding of gene function, protein behavior, and molecular interactions, which are foundational to biomedical research and therapeutic development. While this work focuses on representation learning rather than sequence generation or synthesis, its methods could be incorporated into broader pipelines that warrant careful oversight. We emphasize that responsible deployment should include domain-specific validation, ethical review, and adherence to existing biosafety and regulatory frameworks. Overall, this work highlights the importance of aligning methodological advances with responsible use in sensitive biological domains.

References

James W Anderson. *Hyperbolic geometry*. Springer Science & Business Media, 2006.

Gary Bécigneul and Octavian-Eugen Ganea. Riemannian adaptive optimization methods. *International Conference on Learning Representations*, 2019.

Luca Bertinetto, Romain Mueller, Konstantinos Tertikas, Sina Samangooei, and Nicholas A Lord. Making better mistakes: Leveraging class hierarchies with deep networks. In *Proceedings of the IEEE/CVF conference on computer vision and pattern recognition*, pp. 12506–12515, 2020.

Silvere Bonnabel. Stochastic gradient descent on riemannian manifolds. *IEEE Transactions on Automatic Control*, 58(9):2217–2229, 2013.

Bryan Briney and Dennis R Burton. Massively scalable genetic analysis of antibody repertoires. *BioRxiv*, pp. 447813, 2018.

Garyk Brixi, Matthew G Durrant, Jerome Ku, Michael Poli, Greg Brockman, Daniel Chang, Gabriel A Gonzalez, Samuel H King, David B Li, Aditi T Merchant, Mohsen Naghipourfar, Eric Nguyen, Chiara Ricci-Tam, David W Romero, Gwanggyu Sun, Ali Taghibakshi, Anton Vorontsov, Brandon Yang, Myra Deng,

Liv Gorton, Nam Nguyen, Nicholas K Wang, Etowah Adams, Stephen A Baccus, Steven Dillmann, Stefano Ermon, Daniel Guo, Rajesh Ilango, Ken Janik, Amy X Lu, Reshma Mehta, Mohammad R.K. Mofrad, Madelena Y Ng, Jaspreet Pannu, Christopher Re, Jonathan C Schmok, John St. John, Jeremy Sullivan, Kevin Zhu, Greg Zynda, Daniel Balsam, Patrick Collison, Anthony B. Costa, Tina Hernandez-Boussard, Eric Ho, Ming-Yu Liu, Tom McGrath, Kimberly Powell, Dave P. Burke, Hani Goodarzi, Patrick D Hsu, and Brian Hie. Genome modeling and design across all domains of life with evo 2. *bioRxiv*, 2025. doi: 10.1101/2025.02.18.638918. URL `https://www.biorxiv.org/content/early/2025/02/21/2025.02.18.638918`.

J. Brown. High g+c content of herpes simplex virus dna: Proposed role in protection against retrotransposon insertion. *Open Biochem J*, 1:33–42, 2007. doi: 10.2174/1874091X00701010033.

Florian Buhr, Sujata Jha, Michael Thommen, Joerg Mittelstaet, Felicitas Kutz, Harald Schwalbe, Marina V Rodnina, and Anton A Komar. Synonymous codons direct cotranslational folding toward different protein conformations. *Molecular cell*, 61(3):341–351, 2016.

James W Cannon, William J Floyd, Richard Kenyon, Walter R Parry, et al. Hyperbolic geometry. *Flavors of geometry*, 31(59-115):2, 1997.

Sebastian M Castillo-Hair and Georg Seelig. Machine learning for designing next-generation mrna therapeutics. *Accounts of chemical research*, 55(1):24–34, 2021.

Albi Celaj, Alice Jiexin Gao, Tammy TY Lau, Erle M Holgersen, Alston Lo, Varun Lodaya, Christopher B Cole, Robert E Denroche, Carl Spickett, Omar Wagih, et al. An rna foundation model enables discovery of disease mechanisms and candidate therapeutics. *bioRxiv*, pp. 2023–09, 2023.

Benjamin Paul Chamberlain, James Clough, and Marc Peter Deisenroth. Neural embeddings of graphs in hyperbolic space. In *CoRR*. MLG Workshop, 2017.

Ines Chami, Zhitao Ying, Christopher Ré, and Jure Leskovec. Hyperbolic graph convolutional neural networks. *Advances in neural information processing systems*, 32, 2019.

Bo Chen, Xingyi Cheng, Pan Li, Yangli-ao Geng, Jing Gong, Shen Li, Zhilei Bei, Xu Tan, Boyan Wang, Xin Zeng, et al. xtrimopglm: unified 100b-scale pre-trained transformer for deciphering the language of protein. *arXiv preprint arXiv:2401.06199*, 2024.

Jiayang Chen, Zhihang Hu, Siqi Sun, Qingxiong Tan, Yixuan Wang, Qinze Yu, Licheng Zong, Liang Hong, Jin Xiao, Tao Shen, et al. Interpretable rna foundation model from unannotated data for highly accurate rna structure and function predictions. *arXiv preprint arXiv:2204.00300*, 2022a.

Ken Chen, Yue Zhou, Maolin Ding, Yu Wang, Zhixiang Ren, and Yuedong Yang. Self-supervised learning on millions of pre-mrna sequences improves sequence-based rna splicing prediction. *bioRxiv*, pp. 2023–01, 2023.

Weize Chen, Xu Han, Yankai Lin, Hexu Zhao, Zhiyuan Liu, Peng Li, Maosong Sun, and Jie Zhou. Fully hyperbolic neural networks. In *Proceedings of the 60th Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*, pp. 5672–5686, 2022b.

Yanyi Chu, Dan Yu, Yupeng Li, Kaixuan Huang, Yue Shen, Le Cong, Jason Zhang, and Mengdi Wang. A 5' utr language model for decoding untranslated regions of mrna and function predictions. *Nature Machine Intelligence*, 6(4):449–460, 2024.

Suzanne Clancy and William Brown. Translation: Dna to mrna to protein. *Nature Education*, 1(1):101, 2008.

Maïté Courel, Yves Clément, Clémentine Bossevain, Dominika Foretek, Olivia Vidal Cruchez, Zhou Yi, Marianne Bénard, Marie-Noelle Benassy, Michel Kress, Caroline Vindry, et al. Gc content shapes mrna storage and decay in human cells. *elife*, 8:e49708, 2019.

Karan Desai, Maximilian Nickel, Tanmay Rajpurohit, Justin Johnson, and Shanmukha Ramakrishna Vedantam. Hyperbolic image-text representations. In *International Conference on Machine Learning*, pp. 7694–7731. PMLR, 2023.

Bhuwan Dhingra, Christopher Shallue, Mohammad Norouzi, Andrew Dai, and George Dahl. Embedding text in hyperbolic spaces. In *Proceedings of the Twelfth Workshop on Graph-Based Methods for Natural Language Processing*, pp. 59–69, 2018.

Michay Diez, Santiago Gerardo Medina-Muñoz, Luciana Andrea Castellano, Gabriel da Silva Pescador, Qiushuang Wu, and Ariel Alejandro Bazzini. icodon customizes gene expression based on the codon composition. *Scientific Reports*, 12(1):12126, 2022.

Zundan Ding, Feifei Guan, Guoshun Xu, Yuchen Wang, Yaru Yan, Wei Zhang, Ningfeng Wu, Bin Yao, Huoqing Huang, Tamir Tuller, et al. Mpepe, a predictive approach to improve protein expression in e. coli based on deep learning. *Computational and Structural Biotechnology Journal*, 20:1142–1153, 2022.

Noelia Ferruz, Steffen Schmidt, and Birte Höcker. Protgpt2 is a deep unsupervised language model for protein design. *Nature communications*, 13(1):4348, 2022.

Samuele Fonio, Roberto Esposito, Marco Aldinucci, et al. Hyperbolic prototypical entailment cones for image classification. In *Proceedings of The 28th International Conference on Artificial Intelligence and Statistics*, volume 258, pp. 3358–3366. Proceedings of Machine Learning Research, 2025.

Philip Fradkin, Ruian Shi, Taykhoom Dalal, Keren Isaev, Brendan J Frey, Leo J Lee, Quaid Morris, and Bo Wang. Orthrus: towards evolutionary and functional rna foundation models. *BioRxiv*, pp. 2024–10, 2024.

Octavian Ganea, Gary Bécigneul, and Thomas Hofmann. Hyperbolic entailment cones for learning hierarchical embeddings. In *International conference on machine learning*, pp. 1646–1655. PMLR, 2018a.

Octavian Ganea, Gary Bécigneul, and Thomas Hofmann. Hyperbolic neural networks. *Advances in neural information processing systems*, 31, 2018b.

Zhi Gao, Yuwei Wu, Yunde Jia, and Mehrtash Harandi. Curvature generation in curved spaces for few-shot learning. In *Proceedings of the IEEE/CVF international conference on computer vision*, pp. 8691–8700, 2021.

Zhi Gao, Yuwei Wu, Yunde Jia, and Mehrtash Harandi. Hyperbolic feature augmentation via distribution estimation and infinite sampling on manifolds. *Advances in neural information processing systems*, 35: 34421–34435, 2022.

Juan Jose Garau-Luis, Patrick Philippe Bordes, Liam Gonzalez, Maša Roller, Bernardo P de Almeida, Christopher F. Blum, Lorenz Hexemer, Stefan Laurent, Maren Lang, Thomas PIERROT, and Guillaume Richard. Multi-modal transfer learning between biological foundation models. In *The Thirty-eighth Annual Conference on Neural Information Processing Systems*, 2024. URL `https://openreview.net/forum?id=xImeJtdUiw`.

Mina Ghadimi Atigh, Martin Keller-Ressel, and Pascal Mettes. Hyperbolic busemann learning with ideal prototypes. *Advances in neural information processing systems*, 34:103–115, 2021.

Helen M Gunter, Senel Idrisoglu, Swati Singh, Dae Jong Han, Emily Ariens, Jonathan R Peters, Ted Wong, Seth W Cheetham, Jun Xu, Subash Kumar Rai, et al. mrna vaccine quality analysis using rna sequencing. *Nature Communications*, 14(1):5663, 2023.

Yunhui Guo, Xudong Wang, Yubei Chen, and Stella X Yu. Clipped hyperbolic classifiers are super-hyperbolic classifiers. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, pp. 11–20, 2022.

Ameya Harmalkar, Roshan Rao, Yuxuan Richard Xie, Jonas Honer, Wibke Deisting, Jonas Anlahr, Anja Hoenig, Julia Czwikla, Eva Sienz-Widmann, Doris Rau, et al. Toward generalizable prediction of antibody thermostability using machine learning on sequence and structure features. In *Mabs*, volume 15, pp. 2163584. Taylor & Francis, 2023.

Neil He, Rishabh Anand, Hiren Madhu, Ali Maatouk, Smita Krishnaswamy, Leandros Tassiulas, Menglin Yang, and Rex Ying. Helm: Hyperbolic large language models via mixture-of-curvature experts. *arXiv preprint arXiv:2505.24722*, 2025.

Yuan He, Moy Yuan, Jiaoyan Chen, and Ian Horrocks. Language models as hierarchy encoders. *Advances in Neural Information Processing Systems*, 37:14690–14711, 2024.

Sarah Ibrahimi, Mina Ghadimi Atigh, Nanne Van Noord, Pascal Mettes, and Marcel Worring. Intriguing properties of hyperbolic embeddings in vision-language models. *Transactions on Machine Learning Research*, 2024.

Longfei Jia and Shu-Bing Qian. Therapeutic mrna engineering from head to tail. *Accounts of Chemical Research*, 54(23):4272–4282, 2021.

Tejaswi Kasarla, Max van Spengler, and Pascal Mettes. Balanced hyperbolic embeddings are natural out-of-distribution detectors. *arXiv preprint arXiv:2506.10146*, 2025.

Raiyan R Khan, Philippe Chlenski, and Itsik Pe'er. Hyperbolic genome embeddings. In *International Conference on Learning Representations*, 2025.

Valentin Khrulkov, Leyla Mirvakhabova, Evgeniya Ustinova, Ivan Oseledets, and Victor Lempitsky. Hyperbolic image embeddings. In *Proceedings of the IEEE/CVF conference on computer vision and pattern recognition*, pp. 6418–6428, 2020.

Yoon Kim. Convolutional neural networks for sentence classification. In Alessandro Moschitti, Bo Pang, and Walter Daelemans (eds.), *Proceedings of the 2014 Conference on Empirical Methods in Natural Language Processing (EMNLP)*, pp. 1746–1751, Doha, Qatar, October 2014. Association for Computational Linguistics. doi: 10.3115/v1/D14-1181. URL https://aclanthology.org/D14-1181.

Sizhen Li, Saeed Moayedpour, Ruijiang Li, Michael Bailey, Saleh Riahi, Lorenzo Kogler-Anele, Milad Miladi, Jacob Miner, Dinghai Zheng, Jun Wang, et al. Codonbert: Large language models for mrna design and optimization. *bioRxiv*, pp. 2023–09, 2023.

Zeming Lin, Halil Akin, Roshan Rao, Brian Hie, Zhongkai Zhu, Wenting Lu, Nikita Smetanin, Robert Verkuil, Ori Kabeli, Yaniv Shmueli, et al. Evolutionary-scale prediction of atomic-level protein structure with a language model. *Science*, 379(6637):1123–1130, 2023.

Qi Liu, Maximilian Nickel, and Douwe Kiela. Hyperbolic graph neural networks. *Advances in neural information processing systems*, 32, 2019.

Shaoteng Liu, Jingjing Chen, Liangming Pan, Chong-Wah Ngo, Tat-Seng Chua, and Yu-Gang Jiang. Hyperbolic visual embedding learning for zero-shot recognition. In *Proceedings of the IEEE/CVF conference on computer vision and pattern recognition*, pp. 9273–9281, 2020.

Teng Long, Pascal Mettes, Heng Tao Shen, and Cees GM Snoek. Searching for actions on the hyperbole. In *Proceedings of the IEEE/CVF conference on computer vision and pattern recognition*, pp. 1141–1150, 2020.

Ilya Loshchilov and Frank Hutter. Decoupled weight decay regularization. In *International Conference on Learning Representations*, 2019.

Céline Marquet, Michael Heinzinger, Tobias Olenyi, Christian Dallago, Kyra Erckert, Michael Bernhofer, Dmitrii Nechaev, and Burkhard Rost. Embeddings from protein language models predict conservation and variant effects. *Human genetics*, 141(10):1629–1647, 2022.

Jiří Matoušek. On the distortion required for embedding finite metric spaces into normed spaces. *Israel Journal of Mathematics*, 93(1):333–344, 1996.

Jiří Matoušek. On embedding trees into uniformly convex banach spaces. *Israel Journal of Mathematics*, 114(1):221–237, 1999.

Pascal Mettes, Mina Ghadimi Atigh, Martin Keller-Ressel, Jeffrey Gu, and Serena Yeung. Hyperbolic deep learning in computer vision: A survey. *International Journal of Computer Vision*, 132(9):3484–3508, 2024.

Amina Mollaysa, Artem Moskalev, Pushpak Pati, Tommaso Mansi, Mangal Prakash, and Rui Liao. Biolangfusion: Multimodal fusion of dna, mrna, and protein language models. *arXiv preprint arXiv:2506.08936*, 2025.

Artem Moskalev, Mangal Prakash, Rui Liao, and Tommaso Mansi. Se (3)-hyena operator for scalable equivariant learning. In *Geometry-grounded Representation Learning and Generative Modeling Workshop (GRaM) at ICML 2024*, pp. 7–19. PMLR, 2024.

Eric Nguyen, Michael Poli, Matthew G Durrant, Armin W Thomas, Brian Kang, Jeremy Sullivan, Madelena Y Ng, Ashley Lewis, Aman Patel, Aaron Lou, et al. Sequence modeling and design from molecular to genome scale with evo. *BioRxiv*, pp. 2024–02, 2024a.

Eric Nguyen, Michael Poli, Marjan Faizi, Armin Thomas, Michael Wornow, Callum Birch-Sykes, Stefano Massaroli, Aman Patel, Clayton Rabideau, Yoshua Bengio, et al. Hyenadna: Long-range genomic sequence modeling at single nucleotide resolution. *Advances in neural information processing systems*, 36, 2024b.

Maximillian Nickel and Douwe Kiela. Poincaré embeddings for learning hierarchical representations. *Advances in neural information processing systems*, 30, 2017.

Thijs Nieuwkoop, Barbara R Terlouw, Katherine G Stevens, Richard A Scheltema, Dick De Ridder, John Van der Oost, and Nico J Claassens. Revealing determinants of translation efficiency via whole-gene codon randomization and machine learning. *Nucleic acids research*, 51(5):2363–2376, 2023.

Tobias H Olsen, Fergus Boyles, and Charlotte M Deane. Observed antibody space: A diverse database of cleaned, annotated, and translated unpaired and paired antibody sequences. *Protein Science*, 31(1): 141–146, 2022.

Carlos Outeiral and Charlotte M Deane. Codon language embeddings provide strong signals for use in protein engineering. *Nature Machine Intelligence*, 6(2):170–179, 2024.

Avik Pal, Max van Spengler, Guido Maria D'Amely di Melendugno, Alessandro Flaborea, Fabio Galasso, and Pascal Mettes. Compositional entailment learning for hyperbolic vision-language models. In *International Conference on Learning Representations*, 2025.

Wei Peng, Tuomas Varanka, Abdelrahman Mostafa, Henglin Shi, and Guoying Zhao. Hyperbolic deep neural networks: A survey. *IEEE Transactions on pattern analysis and machine intelligence*, 44(12):10023–10044, 2021.

Rafael Josip Penić, Tin Vlašić, Roland G Huber, Yue Wan, and Mile Šikić. Rinalmo: General-purpose rna language models can generalize well on structure prediction tasks. *Nature Communications*, 16(1):5671, 2025.

Mangal Prakash, Artem Moskalev, Peter DiMaggio Jr., Steven Combs, Tommaso Mansi, Justin Scheer, and Rui Liao. Bridging biomolecular modalities for knowledge transfer in bio-language models. In *Neurips 2024 Workshop Foundation Models for Science: Progress, Opportunities, and Challenges*, 2024. URL `https://openreview.net/forum?id=dicOSQVPLm`.

Xiangyun Qiu. Sequence similarity governs generalizability of de novo deep learning models for rna secondary structure prediction. *PLOS Computational Biology*, 19(4):e1011047, 2023.

Alec Radford, Jeffrey Wu, Rewon Child, David Luan, Dario Amodei, Ilya Sutskever, et al. Language models are unsupervised multitask learners. *OpenAI blog*, 1(8):9, 2019.

Ali Saberi, Benedict Choi, Simai Wang, Aldo Hernández-Corchado, Mohsen Naghipourfar, Arsham Mikaeili Namini, Vijay Ramani, Amin Emad, Hamed S Najafabadi, and Hani Goodarzi. A long-context rna foundation model for predicting transcriptome architecture. *BioRxiv*, pp. 2024–08, 2024.

Frederic Sala, Chris De Sa, Albert Gu, and Christopher Ré. Representation tradeoffs for hyperbolic embeddings. In *International conference on machine learning*, pp. 4460–4469. PMLR, 2018.

Paul J Sample, Ban Wang, David W Reid, Vlad Presnyak, Iain J McFadyen, David R Morris, and Georg Seelig. Human 5' utr design and variant effect prediction from a massively parallel translation assay. *Nature biotechnology*, 37(7):803–809, 2019.

Rik Sarkar. Low distortion delaunay embedding of trees in hyperbolic plane. In *International symposium on graph drawing*, pp. 355–366. Springer, 2011.

Ryohei Shimizu, Yusuke Mukuta, and Tatsuya Harada. Hyperbolic neural networks++. In *International Conference on Learning Representations*, 2021.

Jake Snell, Kevin Swersky, and Richard Zemel. Prototypical networks for few-shot learning. *Advances in neural information processing systems*, 30, 2017.

Martin Steinegger and Johannes Söding. Clustering huge protein sequence sets in linear time. *Nature communications*, 9(1):2542, 2018.

Marcell Szikszai, Michael Wise, Amitava Datta, Max Ward, and David H Mathews. Deep learning models for rna secondary structure prediction (probably) do not generalize across families. *Bioinformatics*, 38 (16):3892–3899, 2022.

Alexandru Tifrea, Gary Bécigneul, and Octavian-Eugen Ganea. Poincaré glove: Hyperbolic word embeddings. In *International Conference on Learning Representations*, 2019.

Abraham Ungar. *A gyrovector space approach to hyperbolic geometry*. Springer Nature, 2022.

Max van Spengler and Pascal Mettes. Low-distortion and gpu-compatible tree embeddings in hyperbolic space. In *International Conference on Machine Learning*, 2025.

Max van Spengler, Erwin Berkhout, and Pascal Mettes. Poincare resnet. In *Proceedings of the IEEE/CVF International Conference on Computer Vision*, pp. 5419–5428, 2023a.

Max van Spengler, Philipp Wirth, and Pascal Mettes. Hypll: The hyperbolic learning library. In *Proceedings of the 31st ACM International Conference on Multimedia*, pp. 9676–9679, 2023b.

Hannah K Wayment-Steele, Wipapat Kladwang, Andrew M Watkins, Do Soon Kim, Bojan Tunguz, Walter Reade, Maggie Demkin, Jonathan Romano, Roger Wellington-Oguri, John J Nicol, et al. Deep learning models for predicting rna degradation via dual crowdsourcing. *Nature Machine Intelligence*, 4(12):1174–1184, 2022.

Rhondene Wint, Asaf Salamov, and Igor V Grigoriev. Kingdom-wide analysis of fungal protein-coding and trna genes reveals conserved patterns of adaptive evolution. *Molecular biology and evolution*, 39(2): msab372, 2022.

Matthew Wood, Mathieu Klop, and Maxime Allard. Helix-mrna: A hybrid foundation model for full sequence mrna therapeutics. In *ICLR Workshop on Machine Learning for Genomics Explorations*, 2025.

Frank Wright. The 'effective number of codons' used in a gene. *Gene*, 87(1):23–29, 1990.

Junjie Xu, Artem Moskalev, Tommaso Mansi, Mangal Prakash, and Rui Liao. Beyond sequence: Impact of geometric context for RNA property prediction. In *The Thirteenth International Conference on Learning Representations*, 2025a. URL `https://openreview.net/forum?id=9htTvHkUhh`.

Junjie Xu, Artem Moskalev, Tommaso Mansi, Mangal Prakash, and Rui Liao. Harmony: A multi-representation framework for rna property prediction. In *ICLR Workshop on Machine Learning for Genomics Explorations*, 2025b.

Menglin Yang, Min Zhou, Zhihao Li, Jiahong Liu, Lujia Pan, Hui Xiong, and Irwin King. Hyperbolic graph neural networks: A review of methods and applications. *arXiv preprint arXiv:2202.13852*, 2022.

Mehdi Yazdani-Jahromi, Mangal Prakash, Tommaso Mansi, Artem Moskalev, and Rui Liao. HELM: Hierarchical encoding for mRNA language modeling. In *International Conference on Learning Representations*, 2025a.

Mehdi Yazdani-Jahromi, Ali Khodabandeh Yalabadi, and Ozlem Ozmen Garibay. Equi-mrna: Protein translation equivariant encoding for mrna language models. *arXiv preprint arXiv:2508.15103*, 2025b.

Zhen Yu, Toan Nguyen, Yaniv Gal, Lie Ju, Shekhar S Chandra, Lei Zhang, Paul Bonnington, Victoria Mar, Zhiyong Wang, and Zongyuan Ge. Skin lesion recognition with class-hierarchy regularized hyperbolic embeddings. In *International conference on medical image computing and computer-assisted intervention*, pp. 594–603. Springer, 2022.

Jing Zhang, CC Jay Kuo, and Liang Chen. Gc content around splice sites affects splicing through pre-mrna secondary structures. *BMC genomics*, 12(1):90, 2011.

Yiding Zhang, Xiao Wang, Chuan Shi, Xunqiang Jiang, and Yanfang Ye. Hyperbolic graph attention network. *IEEE Transactions on Big Data*, 8(6):1690–1701, 2021.

Dinghai Zheng, Logan Persyn, Jun Wang, Yue Liu, Fernando Ulloa-Montoya, Can Cenik, and Vikram Agarwal. Predicting the translation efficiency of messenger rna in mammalian cells. *Nature biotechnology*, pp. 1–14, 2025.

Zhihan Zhou, Yanrong Ji, Weijian Li, Pratik Dutta, Ramana Davuluri, and Han Liu. Dnabert-2: Efficient foundation model and benchmark for multi-species genome. In *International Conference on Learning Representations*, 2024.

Shuxian Zou, Tianhua Tao, Sazan Mahbub, Caleb N Ellington, Robin Algayres, Dian Li, Yonghao Zhuang, Hongyi Wang, Le Song, and Eric P Xing. A large-scale foundation model for rna function and structure prediction. *bioRxiv*, pp. 2024–11, 2024.

A Hierarchical Relationship of Codons and Amino Acids in mRNA

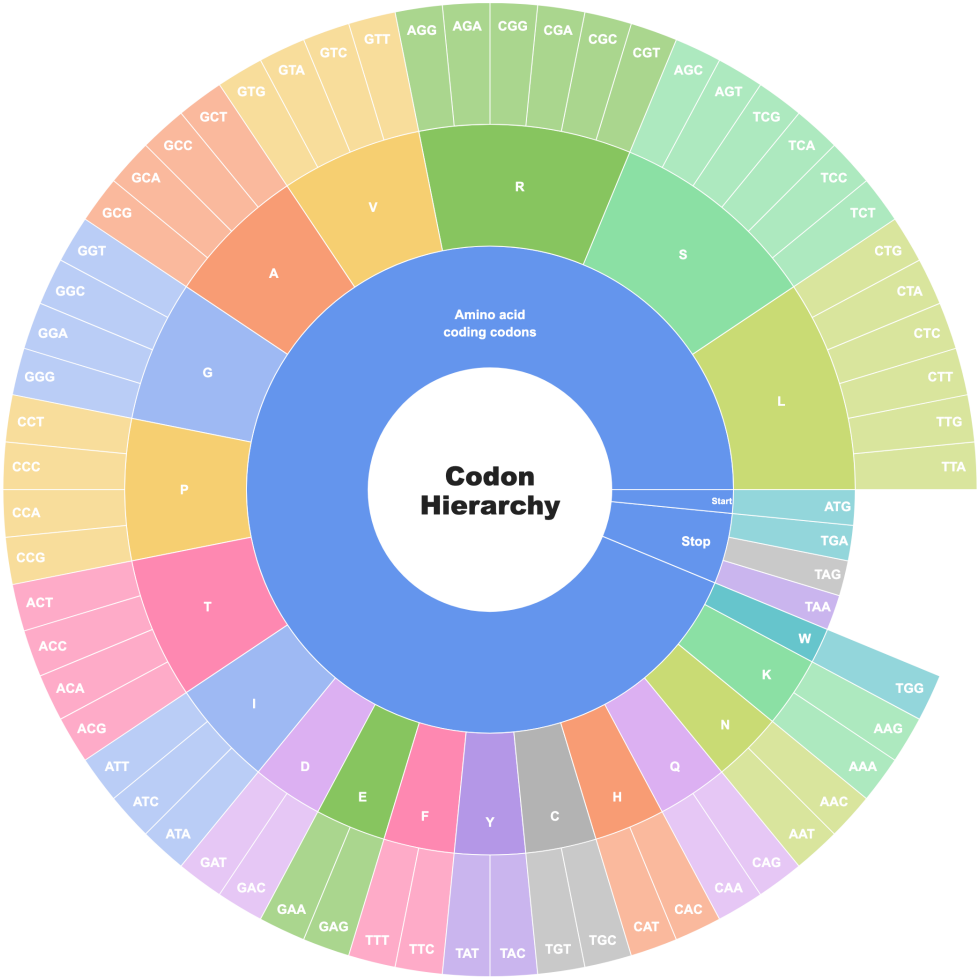

Figure 4: The codon hierarchy that is used for creating prototypes and structuring the representation space.

B Pre-training Details

Model and Training All our experiments were run with a transformer backbone, consisting of 10 transformer layers with an intermediate size of 2560 and a hidden size of 640, resulting in a total of ∼50M parameters. All models were pre-trained for 40 epochs with a batch size of 1024 spread across 8 Nvidia A100 GPUs using the hierarchical cross-entropy (HXE) loss with respect to the codon hierarchy shown in Figure 4 following (Yazdani-Jahromi et al., 2025a).

Sequences were tokenized using codon-level tokenization, resulting in a vocabulary size of 70, including special tokens. The maximum context-length was set to 444, which is enough to accommodate all sequences in the pre-training dataset. However, the positional embedding layer was configured to support up to 2048 tokens, since longer sequences can appear in certain downstream tasks. Positional embedding was applied following the strategy from GPT-2 (Radford et al., 2019).

Optimization was performed using the AdamW optimizer (Loshchilov & Hutter, 2019) with a weight decay of 1e-1. The learning rate was scheduled using linear warmup, followed by cosine decay, using an initial learning rate of 1e-4 which decayed to a minimum of 1e-5. Following (Yazdani-Jahromi et al., 2025a), the α of the HXE loss was set to 0.2.

For the prototype classifiers, we used a prototype embedding dimension of 128 and used a scaling factor $\tau = 2.0$ for the embedding with h-MDS (van Spengler & Mettes, 2025). A hyperbolic linear layer (Shimizu et al., 2021) was used to project to the representation space. The temperature β was set to 10. When the prototypes are made learnable, their optimization is performed using Riemannian SGD Bonnabel (2013), which performs updates as

$$\mathbf{p}_i^{(t+1)} = \exp^c_{\mathbf{p}_i^{(t)}}(\alpha \nabla_{\mathbf{p}_i^{(t)}} \mathcal{L}), \tag{14}$$

where $\mathbf{p}_i^{(t)}$ is the i-th prototype at t iterations, where $\nabla_{\mathbf{p}_i^{(t)}} \mathcal{L}$ is the gradient of the loss evaluated at $\mathbf{p}_i^{(t)}$ and where α is the learning rate. The learning rate is scheduled identically to the learning rate of the AdamW optimizer. The hyperbolic operations were implemented using the HypLL library van Spengler et al. (2023b).

Pre-training Corpus The pre-training corpus consists of the curated OAS database (Olsen et al., 2022) adopted from HELM (Yazdani-Jahromi et al., 2025a). For completeness and self-consistency, the curation procedure is summarized here.

The full OAS database contains more than two billion unpaired and around two million paired antibody sequences from various species, each with a known open reading frame. However, the raw database exhibits a high degree of sequence redundancy and includes a non-trivial fraction of functionally invalid sequences (e.g., sequences with frameshifts, truncations, or non-canonical residues). To obtain a high-quality pre-training corpus, the filtering strategy introduced in HELM is followed.

First, filtering based on the *ANARCI status* annotation provided in OAS is applied, excluding sequences with unusual residues, indels, truncations, or missing conserved cysteines, all of which are often indicative of problematic or non-functional sequences. Sequences with V and J gene identity below 0.7 are then discarded, ensuring a high degree of similarity to known reference germline genes. Only sequences labeled as *productive* and *complete vdj* are retained, indicating that the corresponding sequences are fully functional.

The corpus is subsequently restricted to human antibodies by applying a species filter. To reduce redundancy, sequence similarity clustering using Linclust (Steinegger & Söding, 2018) is performed independently on paired and unpaired sequences with a sequence identity threshold of 0.5, and only the cluster centroids are kept as representatives. Because paired sequences are much fewer in number than unpaired ones, paired antibodies are split into their heavy and light chains and further treated as unpaired. Finally, heavy chains are subsampled to approximately match the number of light chains while maintaining realistic gene frequency distributions.

This process yields a curated pre-training corpus of 15.3 million mRNA sequences, comprising 7.7 million heavy-chain and 7.6 million light-chain CDSs.

C Runtime Comparison of Pre-training Methods

Table 3 shows the runtime in minutes per epoch for each of the methods on 8×Nvidia A100 GPUs as obtained using the pre-training setting discussed in detail in Appendix B. As expected, the runtimes of each method are rather similar, due to the identical backbones dominating the computational complexity.

Table 3: Comparison of the runtime between the different methods that were used for pre-training.

	Transformer XE	**HELM**	**MLR**	**Proto Dist.**	**Proto Entail.**
Runtime (min/epoch)	73.2	71.1	71.7	72.2	73.1

D Downstream Tasks Details

For downstream evaluation, we used a TextCNN (Kim, 2014) for each downstream task, following (Marquet et al., 2022; Chen et al., 2024; Outeiral & Deane, 2024; Harmalkar et al., 2023; Yazdani-Jahromi et al., 2025a). Our downstream configuration exactly matches that of (Yazdani-Jahromi et al., 2025a). So, we use a hidden size of 640 and 100 channels in the convolutions. The pre-trained weights of the backbone are frozen during training. For each model we perform a hyperparameter search on the grid spanned by learning rates of 3e-4, 1e-4, 1e-5 and batch sizes 8, 16, 32, 64. The optimal hyperparameter configuration was chosen based on an unseen validation set. The final reported performance is determined on a separate test set. Each downstream dataset is split into 70% training, 15% validation and 15% test data.

E Performance of Hyperbolic MLR

As shown in the results in Table 1, hyperbolic MLR performs poorly even when compared to the Euclidean baselines, showcasing that simply replacing the geometry by hyperbolic geometry is not sufficient for improving performance. This poor performance is likely due to the numerical problems that occur near the boundary of the space. Training using MLR causes the representations of each token to be pushed towards the hyperplane corresponding to their class and then beyond it. As a result, as training progresses, the representations obtained by MLR rapidly grow in norm, causing these to end up in the region of numerical instability. HypLL van Spengler et al. (2023b) and other hyperbolic libraries deal with this potential numerical problem by clipping points to a region within which numerical issues will certainly not arise. However, this means that when training with MLR, representations often get clipped after a few iterations, destroying all the information stored in the norms. This effect can be seen in Figure 5. We suspect that this severely hinders further learning past this point, resulting in poor performance.

Note that both our proposed prototype methods do not suffer from this issue, as observed for the entailment method in Figure 5. For distance prototypes this is quite straightforward, since token representations are pushed towards their corresponding prototype, which itself has a relatively small norm. For entailment cones, once a token representation lies within the cone of its corresponding prototype, the embedding is no longer pushed away from the origin.

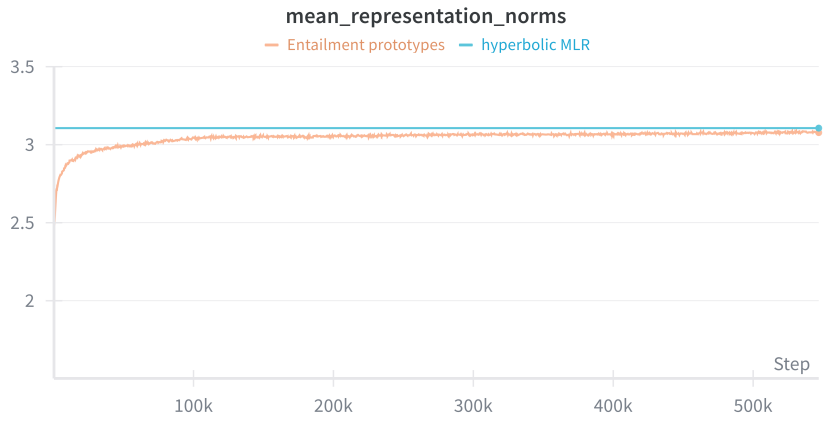

Figure 5: The mean hyperbolic norm of representations at each training step during pre-training using prototypes with entailment cones or hyperbolic MLR.

F Comparison Versus Additional Baselines and Large Scale Foundation Models

Table 4 contains results for 7 of the downstream prediction tasks with additional baselines: linear regression with 1-hot embeddings at the codon-level, and Euclidean and hyperbolic CNNs with codon-level to-

kenization. Moreover, additional relevant genomics foundation models are included in Table 4 as well: Helix-mRNA (Wood et al., 2025), mRNA-FM (Chen et al., 2022a), SpliceBERT (Chen et al., 2023), CodonBERT (Li et al., 2023), AIDO.RNA-CDS (Zou et al., 2024) and EVO 2 (7B) (Brixi et al., 2025). Because of the large sizes of the foundation models, not all experiments were feasible due to hardware constraints. Moreover, SpliceBERT and CodonBERT cannot be applied to several datasets due to the maximal sequence length and due to known implementation issues in the published repository, respectively. Our methods with fixed prototypes perform best in 5 out of 7 cases and best or second best in all cases, showcasing the strong performance of our proposed approach. Note that the foundation models were pre-trained on different corpora, making the comparison less relevant than the comparison shown in Table 1. Moreover, the backbones vary significantly across these foundation models, with CodonBERT, mRNA-FM, AIDO.RNA-CDS and EVO-2 (7B) containing 2x, 5x, 32x, and 140x more parameters than our models, respectively. The particularly poor performance of EVO 2 (7B) can be explained by it being a general model not specialized to mRNA.

Table 4: Accuracy (for *E.coli*) and Spearman rank correlation (for all other datasets) for additional baselines and foundation models. Bold indicates the best performing model per dataset and underline indicates second best model. The missing values indicate inability to perform an experiment due to hardware constraints, the presence of sequences past the maximum sequence length of a model or due to implementation issues of the corresponding repository.

Dataset	Linear regression	Euclidean CNN	Hyperbolic CNN	Helix-mRNA	mRNA-FM	SpliceBERT	CodonBERT	AIDO.RNA-CDS	EVO 2 (7B)	Proto Dist.	Proto Entail.
Ab1	0.582	0.421	0.518	0.535	0.656	0.652	0.686	0.663	0.129	<u>0.713</u>	**0.751**
Ab2	0.499	0.243	0.252	0.283	0.373	0.542	0.557	0.398	0.141	**0.575**	<u>0.569</u>
mRFP	0.687	0.474	0.193	0.432	0.739	0.596	0.770	0.787	0.239	**0.819**	<u>0.802</u>
COVID-19	0.545	0.602	0.480	0.643	0.762	0.757	0.780	<u>0.804</u>	0.386	0.785	**0.807**
Fungal	0.475	0.606	0.580	0.689	0.722	-	-	**0.747**	0.400	0.735	<u>0.741</u>
E. coli	37.7	40.0	40.0	40.0	**53.3**	-	-	50.7	40.0	<u>50.8</u>	48.4
iCodon	0.391	0.152	0.143	0.157	-	0.520	-	-	-	<u>0.535</u>	**0.539**

G Sensitivity Analysis with Respect to the Hyperbolic Tree Embedding Method

Table 5 shows results on several downstream datasets obtained when using fixed entailment prototypes generated using either Poincaré embeddings Nickel & Kiela (2017) or HS-DTE van Spengler & Mettes (2025). As can be seen, both approaches result in similar performance, showcasing that our method is insensitive to the quality of the embedding method.

Table 5: Spearman rank correlation for several datasets obtained using fixed entailment prototypes generated using either Poincaré embeddings or HS-DTE.

Dataset	Poincaré Embeddings	HS-DTE
Ab1	0.752	0.751
Ab2	0.546	0.569
mRFP	0.829	0.802
COVID-19	0.820	0.807
Fungal	0.728	0.741

H Sensitivity Analysis with Respect to Choice of Hyperparameters

To evaluate the robustness of our hyperbolic modeling approach, we performed a sensitivity analysis examining variations in curvature and threshold hyperparameters. The results, summarized in Table 6, indicate that the model's performance is relatively stable across the tested ranges.

Across most datasets, changes in hyperparameters lead to minor fluctuations in performance, demonstrating that the model does not rely heavily on precise hyperparameter tuning within this scope. For example, on COVID-19, Ab1, and Fungal, the performance varies by a few percentage points across different hyperparameter settings.

I Ablation on Choice of Loss Function

We want to verify that our method is not "double-dipping" on hierarchical information by using hierarchical cross-entropy loss, and potentially introducing redundancy or conflicting optimization signals. To evaluate

Table 6: Sensitivity of model performance to hyperparameter variations.

Dataset	c=0.20, η=1.05	c=0.50, η=1.05	c=1.00, η=1.1	c=1.00, η=1.2	c=1.00, η=1.05
COVID-19	0.779	0.816	0.800	0.806	0.807
Ab1	0.739	0.742	0.717	0.724	0.751
Ab2	0.593	0.584	0.578	0.583	0.569
Fungal	0.733	0.748	0.733	0.732	0.741
P. pastoris	0.667	0.650	0.678	0.680	0.671

this, we performed an ablation comparing our method when used with (i) standard cross-entropy (XE) loss and (ii) hierarchical cross-entropy (HXE) loss. We trained the same model architecture under both loss configurations, keeping all other training conditions identical. This allows us to isolate the effect of the loss function on performance and determine whether hierarchical information is being over-used or inconsistently exploited. The results show that combining our method with standard XE leads to lower performance in general. In contrast, pairing our method with HXE yields improved performance. This confirms that HXE provides a more coherent optimization signal and does not introduce conflicting gradients with our method. In other words, the hierarchical supervision is complementary rather than redundant.

Table 7: Comparison of Proto Distance under hierarchical cross-entropy (HXE) and standard cross-entropy (XE).

Dataset	**Proto Dist. (HXE)**	**Proto Dist. (XE)**
P. pastoris	**0.676**	0.666
S. cerevisiae	**0.434**	0.342
mRFP	**0.819**	0.752
E. coli	**50.8**	48.6
Fungal	0.735	**0.740**
COVID-19	**0.785**	0.775

J Effect of the Base Point of the Exponential Map

To examine whether centering the exponential map at the origin may introduce an information bottleneck, we conducted additional experiments in which the base point was made fully learnable. For a fair comparison, we fixed the entailment prototypes and directly compared this learnable–base model with the origin-centered mapping used in the main paper. Across nine datasets, the origin-centered model performs better in 4 out of 9 cases, is on par in 4 out of 9, and is worse in only 1 out of 9. These results indicate that learning the base point does not yield consistent improvements, and that the origin choice is not a bottleneck in practice. These findings support our choice of using the origin as the base point: it aligns naturally with the hierarchical geometry and performs as well as, or better than, a learnable alternative.

Table 8: Comparison between a learnable base point and the origin-centered exponential map.

Dataset	**Origin base**	**Learnable Base**
Ab1	**0.751**	0.701
Ab2	**0.569**	**0.570**
mRFP	**0.802**	**0.805**
COVID-19	**0.807**	0.783
D. melanogaster	**0.450**	**0.451**
S. cerevisiae	**0.397**	0.369
P. pastoris	**0.671**	**0.671**
Fungal	**0.741**	0.724
E. coli	48.4	**50.6**