

LYAPUNOV BARRIER POLICY OPTIMIZATION

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ABSTRACT

Deploying Reinforcement Learning (RL) agents in the real-world require that the agents satisfy safety constraints. Current RL agents explore the environment without considering these constraints, which can lead to damage to the hardware or even other agents in the environment. We propose a new method, LBPO, that uses a Lyapunov-based barrier function to restrict the policy update to a safe set for each training iteration. Our method also allows the user to control the conservativeness of the agent with respect to the constraints in the environment. LBPO significantly outperforms state-of-the-art baselines in terms of the number of constraint violations during training while being competitive in terms of performance. Further, our analysis reveals that baselines like CPO and SDDPG rely mostly on backtracking to ensure safety rather than safe projection, which provides insight into why previous methods might not have effectively limit the number of constraint violations.

1 INTRODUCTION

Current reinforcement learning methods are trained without any notion of safe behavior. As a result, these methods might cause damage to themselves, their environment, or even harm other agents in the scene. Ideally, an agent in a real-world setting should start with a conservative policy and iteratively refine it while maintaining safety constraints. For example, an agent that is learning to drive around other agents should start driving slowly and gradually learn to improve its performance by exploring carefully while avoiding accidents. In contrast, most deep reinforcement learning methods learn by trial and error without taking into consideration the safety-related consequences of their actions (Silver et al., 2016; Vinyals et al., 2019; Akkaya et al., 2019). In this work, we address the problem of learning control policies that optimize a reward function while satisfying some predefined constraints throughout the learning process.

As in previous work for safe reinforcement learning, we use human-defined constraints to specify safe behavior. A classical model for RL with constraints is the constrained Markov Decision Process (CMDP) (Altman, 1999), where an agent tries to maximize the standard RL objective of expected returns while satisfying constraints on expected costs. A number of previous works on CMDPs mainly focus on environments that have low dimensional action spaces, and they are difficult to scale up to more complex environments (Turchetta et al., 2019; Wachi & Sui, 2020). One popular approach to solve Constrained MDPs in large state and action spaces is to use the Lagrangian method (Altman, 1998; Ray et al., 2019). This method augments the original RL objective with a penalty on constraint violations and computes the saddle point of the constrained policy optimization via primal-dual methods (Altman, 1998). While safety is ensured asymptotically, no guarantees are made about safety during training. As we show, other methods which claim to maintain safety during training also lead to many safety violations during training in practice. In other recent work, Chow et al. (2018; 2019) use Lyapunov functions to explicitly model constraints in the CMDP framework to guarantee safe policy updates. We build on this idea, where the Lyapunov function allow us to convert trajectory-based constraints in the CMDP framework to state-based constraints which are much easier to deal with.

In this work, we present a new method called LBPO (Lyapunov Barrier Policy Optimization) for safe reinforcement learning in the CMDP framework. We formulate the policy update as an unconstrained update augmented by a barrier function which ensures that the policy lies in the set of policies induced by the Lyapunov function, thereby guaranteeing safety. We show that LBPO allows us to control the amount of risk-aversion of the agent by adjusting the barrier. We also analyze pre-

vious baselines that use a backtracking recovery rule and empirically show that their near-constraint satisfaction can be explained by their recovery rule; this approach leads to many constraint violations in practice. Finally, we demonstrate that LBPO outperforms state-of-the-art CMDP baselines in terms of the number of constraint violations while being competitive in performance.

2 BACKGROUND

We consider the Reinforcement Learning setting where an agent’s interaction with the environment is modeled as a Markov Decision Process (MDP). An MDP is a tuple $(\mathcal{S}, \mathcal{A}, P, r, s_0)$ with state-space \mathcal{S} , action-space \mathcal{A} , dynamics $P : \mathcal{S} \times \mathcal{A} \rightarrow \mathcal{S}$, reward function $r(s, a) : \mathcal{S} \times \mathcal{A} \rightarrow \mathbb{R}$ and initial state s_0 . $P(\cdot|s, a)$ is the transition probability distribution and $r(s, a) \in [0, R_{\max}]$. We focus on the special case of constrained Markov decision processes (CMDP) (Altman, 1999), which is an augmented version of MDP with additional costs and trajectory-based constraints. A CMDP is represented as a tuple $(\mathcal{S}, \mathcal{A}, P, r, c, s_0, d_0)$. The terms $\mathcal{S}, \mathcal{A}, P, r, s_0$ are the same as in the unconstrained MDP; the additional terms $c(s)$ is the immediate cost and $d_0 \in \mathcal{R}_{\geq 0}$ is the maximum allowed value for the expected cumulative cost of the policy. We define a generic version of the Bellman operator w.r.t policy π and a function $h(s, a) : \mathcal{S} \times \mathcal{A} \rightarrow \mathbb{R}$ as follows:

$$\mathcal{B}_{\pi, h}[V][s] = \sum_a \pi(a|s)[h(s, a) + \gamma \sum_{s' \in \mathcal{S}} P(s'|s, a)V(s')] \quad (1)$$

The function $h(s, a)$ can be instantiated to be the reward function or the cost function. When it is the reward function, this becomes the normal Bellman operator in RL. When $h(s, a)$ is replaced by the cost function $c(s)$, it becomes the Bellman operator over the cost objective, which will be used later in designing the Lyapunov function. We further define $J_\pi(s_0) = \mathbb{E}[\sum_{t=0}^{\infty} \gamma^t r(s_t, a_t)|s_0, \pi]$ to be the performance of the policy π , $D_\pi(s_0) = \mathbb{E}[\sum_{t=0}^{\infty} \gamma^t c(s_t, a_t)|s_0, \pi]$ to be the expected cumulative cost of the policy π , where π belongs to the set of stationary policies \mathcal{P} . Given a CMDP, we are interested in finding a solution to the following constrained optimization problem:

$$\max_{\pi \in \mathcal{P}} [J_\pi(s_0)] \text{ s.t } D_\pi(s_0) \leq d_0 \quad (2)$$

2.1 SAFE REINFORCEMENT LEARNING USING LYAPUNOV FUNCTIONS

We will build upon the Lyapunov framework introduced by Chow et al. (2018), also known as SDDPG. It proposes to use Lyapunov functions to derive a policy improvement procedure with safety guarantees. The basic idea is that, given a safe baseline policy π_B , it finds a set of safe policies based on π_B using Lyapunov functions. For each policy improvement step, it will then choose the policy with the best performance within this set.

The method works by first designing a Lyapunov function for a safe update around the current safe baseline policy π_B . A set of Lyapunov functions is defined as follows:

$$\mathcal{L}_{\pi_B}(s_0, d_0) = \{L : \mathcal{S} \rightarrow \mathcal{R}_{\geq 0} : \mathcal{B}_{\pi_B, c}[L](s) \leq L(s), \forall s \in \mathcal{S}; L(s_0) \leq d_0\} \quad (3)$$

The Lyapunov functions in this set are designed in a way to construct provably safe policy updates. Given any Lyapunov function within this set $L_{\pi_B} \in \mathcal{L}_{\pi_B}(s_0, d_0)$, we define the set of policies that are consistent with it to be the L_{π_B} -induced policies:

$$\mathcal{I}_{L_{\pi_B}} = \{\pi \in \mathcal{P} : \mathcal{B}_{\pi, c}[L_{\pi_B}](s) \leq L_{\pi_B}(s), \forall s \in \mathcal{S}\} \quad (4)$$

It can be shown that any policy π in the L_{π_B} -induced policy set is “safe”, i.e $D_\pi(s_0) \leq d_0$ (Chow et al., 2018).

The choice of L_{π_B} affects the L_{π_B} -induced policy set. We need to construct L_{π_B} such that the L_{π_B} -induced policy set contains the optimal policy π^* . Chow et al. (2018) show that one such Lyapunov function is $L_{\pi_B, \epsilon}(s) = \mathbb{E}[\sum_{t=0}^{\infty} \gamma^t (c(s_t) + \epsilon(s_t)) | \pi_B, s]$, where $\epsilon(s_t) \geq 0$. The function $L_{\pi_B, \epsilon}(s)$ can be thought of as a cost-value function for policy π_B augmented by an additional per-step cost $\epsilon(s_t)$. Accordingly, we can define the following state-action value function:

$$Q_{L_{\pi_B, \epsilon}}(s, a) = c(s) + \epsilon(s) + \gamma \sum_{s'} P(s'|s, a) L_{\pi_B, \epsilon}(s') \quad (5)$$

It was shown in Chow et al. (2018) that finding a state dependent function ϵ such that the optimal policy is inside the corresponding $L_{\pi_B, \epsilon}$ -induced set is generally not possible and requires knowing the optimal policy. As an approximation, they suggest to create the Lyapunov function with the largest auxiliary cost $\hat{\epsilon}$, such that $L_{\pi_B, \hat{\epsilon}}(s) \geq \mathcal{B}_{\pi_B, c}[L_{\pi_B, \hat{\epsilon}}](s)$ and $L_{\pi_B, \hat{\epsilon}}(s_0) \leq d_0$. A larger auxiliary cost ϵ per state ensures that we have a larger set of L -induced policies, making it more likely to include the optimal policy in the set. The authors show that the following $\hat{\epsilon}(s)$ in the form of a constant function satisfies the conditions described:

$$\hat{\epsilon}(s) = (1 - \gamma)(d_0 - D_{\pi_B}(s_0)) \quad (6)$$

Plugging this function $\hat{\epsilon}(s)$ and the definition of $Q_{L_{\pi_B, \epsilon}}(s, a)$ into the CMDP objective, the policy update under the set of policies that lie in the $L_{\pi_B, \hat{\epsilon}}$ -induced policy set, or equivalently the policies that are safe, is given by:

$$\pi_+(\cdot|s) = \max_{\pi \in \mathcal{P}} J_\pi(s_0), \text{ s.t. } \int_{a \in \mathcal{A}} (\pi(a|s) - \pi_B(a|s)) Q_{L_{\pi_B, \hat{\epsilon}}}(s, a) \leq \hat{\epsilon}(s) \quad (7)$$

In the case of a deterministic policy, the policy update becomes:

$$\pi_+(\cdot|s) = \max_{\pi \in \mathcal{P}} J_\pi(s_0), \text{ s.t. } Q_{L_{\pi_B, \hat{\epsilon}}}(s, \pi(s)) - Q_{L_{\pi_B, \hat{\epsilon}}}(s, \pi_B(s)) \leq \hat{\epsilon}(s) \quad (8)$$

We build upon this objective in our work. We include the proof of the Lyapunov approach for completeness in Appendix A.1, and we advise the reader to see previous work (Chow et al., 2018) for a more detailed derivation. Using the Lyapunov function, the trajectory-based constraint of the CMDP is converted to a per-state constraint (Eq. 7), which is often much easier to deal with.

3 METHOD

3.1 BARRIER FUNCTION FOR LYAPUNOV CONSTRAINT

We present Lyapunov Barrier Policy Optimization (LBPO) that aims to update policies inside the $L_{\pi_B, \hat{\epsilon}}$ -induced policy set. We work under the standard policy iteration framework which contains two steps: Q-value Evaluation and Safe Policy Improvement. We initialize LBPO with a safe baseline policy π_B . In practice, we can obtain safe initial policies using a simple (usually poorly performing) hand-designed control policy; in our experiments, we simplify this process and achieve safe initial policies by training on the safety objective. We assume that we have m different constraints, as LBPO naturally generalizes to more than one constraint.

3.1.1 Q EVALUATION

We use on-policy samples to evaluate the current policy. We compute a reward Q function Q^R , and cost Q functions Q^{C_i} corresponding to each cost constraint $i \in [1, 2, \dots, m]$. Each Q function is updated using TD(λ) (Sutton, 1988) which helps us more accurately estimate the Q functions. Furthermore, we use the on-policy samples to get the cumulative discounted cost $D_\pi^i(s_0)$ of the current policy, which allows us to set up the constraint budget for each constraint given by $\epsilon_i = (1 - \gamma)(d_0^i - D_\pi^i(s_0))$ as shown in Eq. 6.

3.1.2 REGULARIZED SAFE POLICY UPDATE

In this work, we focus on deterministic policies, where we have the following policy update under the L -induced set for each constraint as given in Eq. 8 :

$$\pi_+(\cdot|s) = \max_{\pi \in \mathcal{P}} J_\pi(s_0), \text{ s.t. } Q_{L_{\pi_B, \hat{\epsilon}}}^i(s, \pi(s)) - Q_{L_{\pi_B, \hat{\epsilon}}}^i(s, \pi_B(s)) \leq \hat{\epsilon}_i(s) \forall i \in [1, 2, \dots, m] \quad (9)$$

We can simplify this equation further by replacing $Q_{L_{\pi_B, \hat{\epsilon}}}^i$ with $Q_{\pi_B}^{C_i}$ which is the i^{th} cost Q-function under the policy π_B , when ϵ is a constant function (see Appendix A.1.1). To ensure that the Lyapunov constraints are satisfied, we construct an unconstrained objective using an indicator penalty

$I(Q_{\pi_B}^{C_i}(s, \pi_\theta(s)))$ for each constraint.

$$I(Q_{\pi_B}^{C_i}(s, \pi_\theta(s))) = \begin{cases} 0 & Q_{\pi_B}^{C_i}(s, \pi_\theta(s)) - Q_{\pi_B}^{C_i}(s, \pi_B(s)) \leq \hat{\epsilon}_i(s) \\ \infty & Q_{\pi_B}^{C_i}(s, \pi_\theta(s)) - Q_{\pi_B}^{C_i}(s, \pi_B(s)) > \hat{\epsilon}_i(s) \end{cases} \quad (10)$$

We will use a differentiable version of the indicator penalty called the logarithmic barrier function:

$$\psi(Q_{\pi_B}^{C_i}(s, \pi_\theta(s))) = -\beta \log \left(\hat{\epsilon}_i(s) - (Q_{\pi_B}^{C_i}(s, \pi_\theta(s)) - Q_{\pi_B}^{C_i}(s, \pi_B(s))) \right) \quad (11)$$

The function ψ is parameterized by θ and $Q_{\pi_B}^{C_i}(s, \pi_B(s))$ is a constant. Our policy update will use the gradient at $\pi_\theta = \pi_B$, ensuring that the logarithmic barrier function is well defined, since $\hat{\epsilon}_i(s) > 0 \forall s$.

Figure 1 captures the behavior of the function ψ for different β . The parameter β captures the amount of risk-aversion we desire from the agent. A high β will help avoid constraint violations occurring due to approximation errors in our learned Q-functions. We verify this empirically in Section 4.

We use the Deterministic Policy Gradient Theorem (Silver et al., 2014) for the policy update. For updating a θ -parameterized policy with respect to the expected return, the objective can be written as: $\operatorname{argmin}_\theta \mathbb{E}_{s \sim \rho^{\pi_B}} [(-Q_{\pi_B}^R(s, \pi_\theta(s)))]$, where ρ^{π_B} is the on-policy state distribution.

Since we rely on on-policy samples for Q-function evaluation, the Q-function estimation outside the on-policy state distribution can be arbitrarily bad. Similar to Schulman et al. (2015), we constrain our policy update using a hard KL constraint (i.e. a trust region) between the current policy and the updated policy under the presence of stochastic exploration noise. The trust region also allows us to make sure that our policy change is bounded, which allows us to ensure that with a small enough trust region, our first order approximation of the objective is valid.

Augmenting the on-policy update with the Lyapunov barrier and a KL regularization, we have the following LBPO policy update:

$$\theta_{k+1} = \operatorname{argmin}_\theta \mathbb{E}_{s \sim \rho^{\pi_B}} \left[-Q_{\pi_B}^R(s, \pi_\theta(s)) + \sum_{i=1}^m \psi(Q_{\pi_B}^{C_i}(s, \pi_\theta(s))) \right] \quad (12)$$

$$\text{subject to } \mathbb{E}_{s \sim \rho^{\pi_B}} [D_{\text{KL}}(\pi_\theta[\cdot|s] + \mathcal{N}(0, \delta) \| \pi_B[\cdot|s] + \mathcal{N}(0, \delta))] < \mu \quad (13)$$

where δ is the exploration noise, ρ^{π_B} is the state distribution induced by the current policy, μ is the expected KL constraint threshold and we set π_B to the safe policy at iteration k , as the update guarantees safe policies at each iteration. In practice, we expand our objective using a Taylor series expansion and solve to a leading order approximation around θ_k . Letting the gradient of the objective in Eq 12 be denoted by g and the Hessian of the KL divergence by H , our objective becomes:

$$\theta_{k+1} = \operatorname{argmin}_\theta g^T(\theta - \theta_k), \quad \text{subject to } \frac{1}{2}(\theta - \theta_k)^T H(\theta - \theta_k) \leq \mu \quad (14)$$

We solve this constrained optimization using the Fisher vector product with Conjugate gradient method similar to Schulman et al. (2015).

4 EXPERIMENTS

In this section, we evaluate LBPO and compare it to prior work. First, we benchmark our method against previous baselines to show that LBPO can achieve better constraint satisfaction while being competitive in performance. Second, we give empirical evidence that previous methods near constraint satisfaction can be explained by backtracking. Third, we show by a didactic example

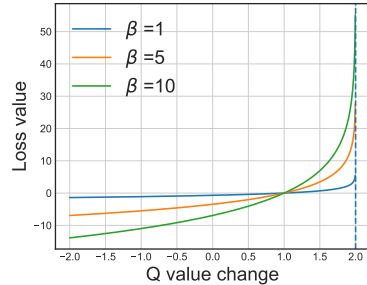


Figure 1: As the difference between Q values for action a and the baseline action reaches ϵ (in this case $\epsilon = 2$), the loss increases to ∞ .

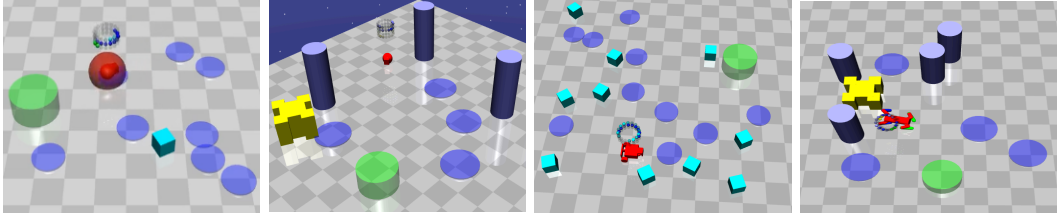


Figure 2: OpenAI Safety Environments: PointGoal1, PointPush2, CarGoal2, DoggoPush2

that LBPO is more robust than CPO and SDDPG to Q-function errors, hence making it a preferable alternative, especially when function approximation is used. Finally, we show that LBPO allows flexible tuning of the amount of risk-aversion for the agent.

Comparisons. For our experiments, we compare LBPO against a variety of baselines: PPO, PPO-lagrangian, SDDPG, CPO and BACKTRACK. PPO (Schulman et al., 2017) is an on-policy RL algorithm that updates in an approximate trust-region without considering any constraints. PPO-lagrangian belongs to a class of Lagrangian methods (Altman, 1998) for safe Reinforcement Learning, which transforms the constrained optimization problem to a penalty form $\max_{\pi \in \mathcal{P}} \min_{\lambda \geq 0} \mathbb{E}[\sum_{t=0}^{\infty} \gamma^t r(s_t, a_t) + \lambda(\sum_{t=0}^{\infty} \gamma^t c(s_t) - d_0)] \pi, s_0$. π and λ are jointly optimized to find a saddle point of the penalized objective. SDDPG (Chow et al., 2018; 2019) introduces the Lyapunov framework for safe-RL and proposes an action-projection method which in theory guarantees the update of the policy within a safe set. We evaluate the α -projection version of SDDPG (Chow et al., 2019). Since the original implementation for the method is unavailable, we re-implemented the method to the best of our abilities. CPO (Achiam et al., 2017) derives a trust-region update rule which guarantees the monotonic improvement of the policy while satisfying constraints. CPO also uses a backtracking recovery rule. We elaborate on the BACKTRACK baseline in Section 4.2.

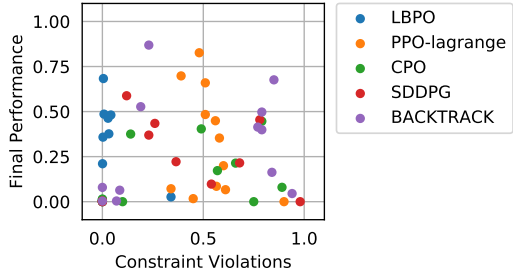


Figure 3: Each point corresponds to a particular safety method applied to a certain safety environment. The x-axis shows the fraction of constraint violations encountered by the behavior policy during training and y-axis shows the policy performance normalized by the corresponding environment’s PPO return.

Tasks. We evaluate these methods using the OpenAI Safety Gym (Ray et al., 2019), which consists of 12 continuous control MuJoCo tasks (Todorov et al., 2012). These tasks use 3 robots: Point, Car, and Doggo. Point is the simplest of three which can be commanded to move forward/backward or to turn. Car has two driven wheels which needs to be controlled together to obtain forward/backward and turning behavior. Doggo is a quadrupedal robot whose joint angles at hip, knee and torso can be commanded to obtain similar behavior. Each robot has 2 types of tasks (Goal, Push) with 2 difficulty levels (1, 2). In Goal tasks, the robot has to move to a series of goal locations, and in Push tasks, the robot has to push a box to a series of goal locations. There are mobile and immobile obstacles made up of a hazard region, vases and pillars which generate a cumulative penalty for the agent. Point has an observation space of 60 dimensions, Car has 72 dimensions, and Doggo has 104 dimensions, which constitute sensor readings, joint angles, and velocities. The environments are shown in Figure 2.

4.1 SAFE REINFORCEMENT LEARNING BENCHMARKS

We summarize the comparison of LBPO to all of the baselines (PPO, PPO-lagrangian, BACKTRACK, SDDPG and CPO) on the OpenAI safety benchmarks in Figure 3 and Tables 1 and 2. Additional training plots for policy return and policy cost can be found in Appendix A.2.1.

Constraint Satisfaction. Table 1 shows that in all the environments, LBPO actively avoids constraint violations, staying below the threshold in most cases. In the PointGoal2 environment, no

Method	PPO	PPO-lagrangian	CPO	SDDPG	BACKTRACK	LBPO
PointGoal1	1.00	0.48	0.79	0.78	0.85	0.04
PointGoal2	0.98	0.60	0.75	0.98	0.94	0.34
PointPush1	1.00	0.51	0.14	0.12	0.19	0.00
PointPush2	1.00	0.51	0.66	0.36	0.77	0.00
CarGoal1	1.00	0.56	0.89	0.54	0.79	0.03
CarGoal2	1.00	0.61	0.57	0.68	0.84	0.00
CarPush1	0.99	0.39	0.10	0.26	0.23	0.01
CarPush2	1.00	0.58	0.49	0.23	0.79	0.03
DoggoGoal1	1.00	0.90	0.00	0.00	0.07	0.00
DoggoGoal2	1.00	0.45	0.00	0.00	0.00	0.00
DoggoPush1	0.98	0.56	0.00	0.00	0.00	0.00
DoggoPush2	1.00	0.34	0.00	0.00	0.09	0.00

Table 1: We report the fraction of unsafe behavior policies encountered during training across different OpenAI safety environments for the policy updates across $2e^7$ training timesteps. LBPO obtains fewer constraint violations consistently across all environments.

Method	PPO	PPO-lagrangian	CPO	SDDPG	BACKTRACK	LBPO
PointGoal1	1.00	0.826	0.450	0.451	0.670	0.480
PointGoal2	1.00	0.200	0.000	0.000	0.045	0.026
PointPush1	1.00	0.659	0.375	0.587	0.527	0.683
PointPush2	1.00	0.483	0.213	0.221	0.413	0.358
CarGoal1	1.00	0.449	0.079	0.097	0.497	0.376
CarGoal2	1.00	0.066	0.172	0.215	0.162	0.210
CarPush1	1.00	0.697	0.000	0.434	0.868	0.485
CarPush2	1.00	0.353	0.403	0.369	0.399	0.430
DoggoGoal1	1.00	0.000	0.003	0.002	0.003	0.007
DoggoGoal2	1.00	0.016	0.002	0.003	0.003	0.003
DoggoPush1	1.00	0.080	0.014	0.001	0.079	0.012
DoggoPush2	1.00	0.071	0.000	0.000	0.063	0.000

Table 2: Cumulative return of the converged policy for each safety algorithm normalized by PPO’s return. Negative returns are clipped to zero. LBPO tradeoffs return for better constraint satisfaction. Bold numbers show the best performance obtained by a safety algorithm (thus excluding PPO).

method can achieve good constraint satisfaction which we attribute to the nature of the environment as it was found that safe policies were not obtained even when training only on the cost objective. In all the other cases we note that LBPO achieves near-zero constraint violations.

Like our method, SDDPG also builds upon the optimization problem from Equation 8 but solves this optimization using a projection onto a safe set instead of using a barrier function. We noticed the following practical issues with this approach: First, in SDDPG, each safe policy is composed of a projection layer, which itself relies on previous safe policies. This requires us to maintain all of the previous policies and thus the memory requirement grows linearly with the number of iterations. SDDG circumvents this issue by using a policy distillation scheme (Chow et al., 2018), which behavior clones the safe policy into a parameterized policy not requiring a projection layer. However, behavior cloning introduces errors in the policy leading to frequent constraint violations. Second, we will show in section 4.3 that SDDPG is more sensitive to Q-function errors. PPO-lagrangian produces policies that are only safe *asymptotically* and makes no guarantee of the safety of the behavior policy during each training iteration. In practice, we observe that it often violates constraints during training.

Behavior Policy Performance. OpenAI safety gym environment provide a natural tradeoff between reward and constraint. A better constraint satisfaction often necessitates a lower performance. We observe in Table 2 that LBPO achieves performance competitive to the baselines.

4.2 BACKTRACKING BASELINE

CPO (Achiam et al., 2017) and SDDPG (Chow et al., 2019) both use a recovery rule once the policy becomes unsafe, which is to train on the safety objective to minimize the cumulative cost until the

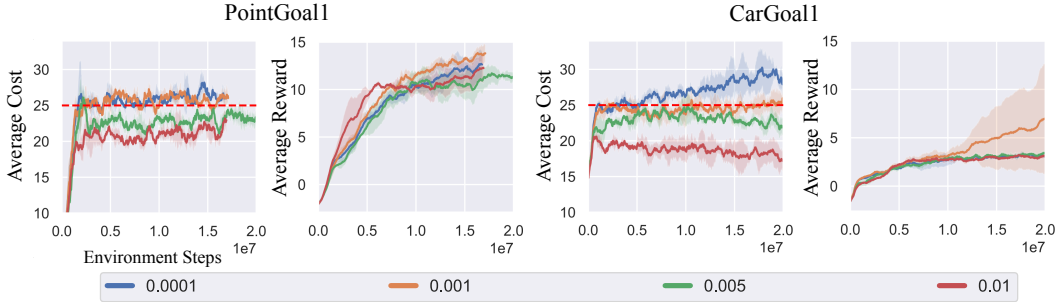


Figure 5: Increasing β parameter for the barrier increases the risk aversion of the agent as can be seen in the plots above.

policy becomes safe again. In this section, we test the hypothesis that CPO and SDDPG are unable to actively avoid constraint violation but their near constraint satisfaction behavior can be explained by the recovery rule. To this end, we introduce a simple baseline, BACKTRACK, which uses the following objective for policy optimization under a trust region (we use the same trust region as in LBPO):

$$\pi = \begin{cases} \max_{\pi \in \mathcal{P}} \mathbb{E}_{s \sim \rho^{\pi_B}} [Q_{\pi_B}^R(s, \pi(s))] & \text{if } \pi_B \text{ is safe} \\ \min_{\pi \in \mathcal{P}} \mathbb{E}_{s \sim \rho^{\pi_B}} [Q_{\pi_B}^C(s, \pi(s))] & \text{if } \pi_B \text{ is unsafe} \end{cases} \quad (15)$$

Thus, if the most recent policy π_B is evaluated to be safe, BACKTRACK exclusively optimizes the reward; however, if the most recent policy π_B is evaluated to be unsafe, BACKTRACK exclusively optimizes the safety constraint. Effectively, BACKTRACK relies only on the recovery behavior that is used in CPO and SDDPG, without incorporating their mechanisms for constrained policy updates. In Tables 1 and 2, we see that BACKTRACK is competitive to both CPO and SDDPG in terms of both constraint satisfaction and performance (maximizing reward), suggesting that the recovery behavior is itself sufficient to explain their performance. In Appendix A.2.2, we compare the number of backtracks performed by CPO, SDDPG and BACKTRACK.

4.3 ROBUSTNESS TO FINITE SAMPLE SIZES

We generally work in the function approximation setting to accommodate high dimensional observations and actions, and this makes it necessary to rely on safety methods that are robust to Q-function errors. To analyze how robust different methods are to such errors, we define a simple reinforcement learning problem: Consider an MDP with two dimensional state space given by $(x, y) \in \mathbb{R}^2$. The initial state is $(0,0)$. Actions are two dimensional, given by $(a_1, a_2) : a_1, a_2 \in [-0.2, 0.2]$. The horizon is 10 and the transition probability distribution is $(x', y') = (x, y) + (a_1, a_2) + \mathcal{N}(0, 0.1)$. The reward function is $r(x, y) = \sqrt{x^2 + y^2}$. The cost function is equal to the reward function for all states, and the constraint threshold is set to 2. We plot in Figure 4 the total constraint violations during 100 epochs of training with varying number of samples used to estimate the cost Q-function. We find that LBPO is more robust to Q-function errors due to limited data compared to CPO and SDDPG. In this experiment we use $\beta = 0.005$, similar to the value used for the benchmark experiments.

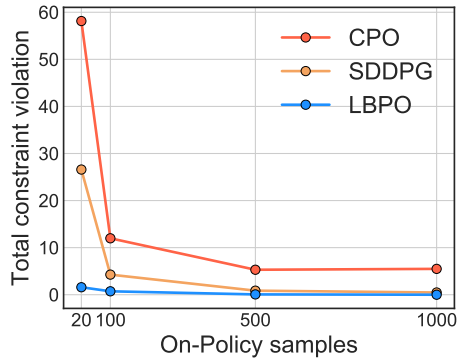


Figure 4: An analysis of the robustness of safe RL algorithms CPO, SDDPG, and LBPO to finite sample Q function errors for a simple didactic environment. Constraint violations in CPO and SDDPG increase quickly as the number of on-policy samples used to estimate the Q function decreases. Results are averaged over 5 seeds.

4.4 TUNING CONSERVATIVENESS WITH THE BARRIER

A strength of LBPO is the ability to tune the barrier to adjust the amount of risk-aversion of the agent. Specifically, β in Equation 11 can be tuned; a larger β leads to more conservative policies. In Figure 5, we empirically demonstrate the sensitivity of β to the conservativeness of the policy update. For our benchmark results, we do a hyperparameter search for β in the set (0.005, 0.008, 0.01, 0.02) and found that 0.005 works well across most environments.

5 RELATED WORK

Constrained Markov Decision Process CMDP’s (Altman, 1998) have been a popular framework for incorporating safety in the form of constraints. In CMDP’s the agent tries to maximize expected returns by satisfying constraints on expectation of costs. Altman (1999) demonstrated that for finite MDP with known models, CMDP’s can be solved by solving the dual LP program. For large state dimensions (or continuous), solving the LP becomes intractable. A common way to solve CMDP in large spaces is to use the Lagrangian Method (Altman, 1999; Geibel & Wysotzki, 2005; Chow et al., 2017). These methods augment the original RL objective with a penalty on constraint violation and computes the saddle point of the constrained policy optimization via primal-dual methods. These methods give no guarantees of safety during training and are only guaranteed asymptotically at convergence. CPO (Achiam et al., 2017) is another method for solving CMDP’s that derives an update rule in the trust region which guarantees monotonic policy improvement under constraint satisfaction, similar to TRPO (Schulman et al., 2015). Chow et al. (2018; 2019) presents another class of method that formulates safe policy update under a Lyapunov constraint. Perkins & Barto (2002) explored the relevance of Lyapunov functions in control and (Berkenkamp et al., 2017) used Lyapunov functions in RL to guarantee exploration such that the agent can return to a "region of attraction" in the model-based regime. In our work, we show that previous baselines rely on a backtracking recovery rule to ensure near constraint satisfaction and are sensitive to Q-function errors; we present a new method that uses a Lyapunov constraint with a barrier function to ensure a conservative policy update.

Other notions of safety. Recent works (Pham et al., 2018; Dalal et al., 2018) use a safety layer along with the policy, which ensures that all the unsafe actions suggested by the policy are projected in the safe set. Dalal et al. (2018) satisfies state-based costs rather than trajectory-based costs. Thananjeyan et al. (2020) utilizes demonstrations to ensure safety in the model based framework, and Zhang et al. (2020) learns the epistemic uncertainty of the environment by training the model in simulation for a distribution of environments, which is then used to cautiously adapt the policy while deploying on a new test environment. Another line of work focuses on optimizing policies that minimize an agent’s conditional value at risk (cVAR). cVAR (Rockafellar et al., 2000) is commonly used in quantitative finance, which aims to maximize returns in the worst $\alpha\%$ of cases. This allows the agent to ensure that it learns safe policies for deployment that achieve high reward under the aleatoric uncertainty of the MDP (Tang et al., 2019; Keramati et al., 2019; Tamar et al., 2014; Kalashnikov et al., 2018; Borkar & Jain, 2010; Chow & Ghavamzadeh, 2014).

6 CONCLUSION

In this work, we present a new method, LBPO, that formulates a safe policy update as an unconstrained policy optimization augmented by a barrier function derived from Lyapunov-based constraints. LBPO allows the agent to control the risk aversion of the RL agent and is empirically observed to be more robust to Q-function errors. We also present a simple baseline BACKTRACK to provide insight into previous method’s reliance on backtracking recovery behavior to achieve near constraint satisfaction. LBPO achieves fewer constraint violations, in most cases close to zero, on a number of challenging continuous control tasks and outperforms state-of-the-art safe RL baselines.

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