Towards Efficient Mixture of Experts: A Holis-Tic Study of Compression Techniques

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ABSTRACT

011 Scaling large language models has driven remarkable advancements across various 012 domains, yet the continual increase in model size presents significant challenges 013 for real-world deployment. The Mixture of Experts (MoE) architecture offers a 014 promising solution by dynamically selecting and activating only a subset of experts during inference, thus substantially reducing computational costs while preserving 015 high performance. Despite these benefits, MoE introduces new inefficiencies, such 016 as excessive parameters and communication overhead. In this work, we present 017 a holistic study on compression techniques of Mixture of Experts to enhance 018 both efficiency and scalability. While recent efforts have focused on reducing the 019 number of experts, these approaches still suffer from considerable communication 020 and computational costs. To address this, we propose more aggressive strategies, 021 such as Layer Drop, which removes entire MoE layers, and Block Drop, which 022 eliminates transformer blocks. Surprisingly, these aggressive structure pruning techniques not only preserve model performance but also substantially improve 024 efficiency. Additionally, beyond Expert Trimming, we also introduce Expert 025 Slimming, which compresses individual experts to further boost performance and 026 can be seamlessly integrated with Expert Trimming. Extensive experimental results demonstrate the effectiveness of our proposed methods — Layer Drop and Block 027 Drop — along with the comprehensive recipe that integrates Expert Slimming and 028 Expert Trimming, achieving a $6.05 \times$ speedup with 77.1% reduced memory usage 029 while maintaining over 92% of performance on Mixtral-8×7B. Our code will be made publicly available upon acceptance. 031

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033 1 INTRODUCTION

While scaling large language models has shown exceptional performance across various domains (Ramesh et al., 2021; OpenAI, 2024; Team, 2024a), the increasing model size poses significant challenges in real-world deployments (Sun et al., 2023; Frantar et al., 2022) due to excessive computational demands and associated costs. The Mixture of Experts (MoE) (Shazeer et al., 2017), which selectively activates a subset of parameters during inference, offers a promising solution to reduce these computational burdens. Additionally, integrating MoE with Large Language Models (LLMs) has been shown to enhance performance further (Jiang et al., 2024; Dai et al., 2024).

042 Despite these advances, MoE models still suffer from significant redundancies that increase deploy-043 ment costs. tandard MoE implementations replicate feed-forward layers across multiple experts, 044 resulting in models that are still heavily parameterized. For instance, Mixtral-8×7B (Jiang et al., 2024) contains 47B parameters, but only 13B parameters are activated per token, leading to the substantial GPU memory consumption and limited scalability. In addition, replicating experts often 046 introduces redundant experts. For example, He et al. (2023) observed that expert parameters could 047 be compressed through parameter sharing. Similarly, Lu et al. (2024) noted that not all experts are 048 essential, suggesting that some can be safely removed. These findings underscore the potential for compressing MoE models to improve efficiency without sacrificing effectiveness. 050

In this paper, we first investigate the Expert Trimming based compression techniques that reduce the number of experts to enhance the efficiency of MoE (Cheng et al., 2020; Liang et al., 2021).
The most prevalent approach for Expert Trimming is Expert Drop, which scores each expert and drops the less important ones (Lu et al., 2024; Muzio et al., 2024). While Expert Drop reduces

model size, it does not eliminate the costly computations within the MoE layer and the complex 055 communication among experts, leading to negligible improvements on the inference speed. To this 056 end, we propose aggressive Expert Trimming methods to enhance MoE efficiency. Specifically, 057 to mitigate communication and computation costs, we present Layer Drop that removes the entire 058 MoE layer. Additionally, given the computation-intensive nature of the attention mechanism within transformer blocks, we further propose Block Drop, which removes the whole transformer blocks. We use similarity-based metrics to demonstrate the feasibility of Layer Drop and Block Drop. 060 Surprisingly, these two coarse-grained methods outperform fine-grained Expert Drop by a large 061 margin in balancing performance and efficiency. Additionally, with small-scale post-finetuning, the 062 compressed models can be further optimized to achieve near-original performance. 063

Beyond removing experts, we also explore Expert Slimming, which focuses on compressing individual
experts. Techniques such as network pruning (Han et al., 2016; Zhu & Gupta, 2017) and quantization
(Jacob et al., 2017; Nagel et al., 2021) have proven effective for model compression, with quantization
being particularly well-suited for hardware acceleration. By integrating Expert Slimming with Expert
Trimming, we propose a unified framework for compressing MoE models that maximizes efficiency
gains while maintaining strong performance.

Our experimental results on two widely-used MoE models, Mixtral-8×7B (Jiang et al., 2024) and 071 DeepSeek-MoE-16B (Dai et al., 2024), demonstrate the effectiveness of our proposed methods. For Expert Trimming, Expert Drop significantly reduces the memory usage but it provides only 072 marginal improvements in inference speed. In contrast, Layer Drop and Block Drop significantly 073 accelerate inference and reduce memory usage while maintaining comparable performance to the 074 original models. The combined strategy of Expert Trimming and Expert Slimming results in a $6.05 \times$ 075 speedup with only 22.8% memory usage (20.0GB) while maintaining over 92% of the original 076 performance on Mixtral- $8 \times 7B$. The findings offer valuable insights for enhancing the efficiency 077 of MoE models. Additionally, post-finetuning allows compressed models to recover most of their 078 original performance, resulting in a minimal 0.6% performance gap compared to the uncompressed 079 DeepSeek-MoE-16Bmodel.

- In summary, by conducting a holistic study on compressing Mixture of Experts, our key contributions are as follows:
 - We extend Expert Trimming to a higher architectural level by introducing Layer Drop and Block Drop, significantly improving the efficiency while maintaining the model performance.
 - We introduce Expert Slimming, a method that compresses individual experts. By integrating Expert Slimming with Expert Trimming, we achieve further efficiency gains without compromising performance.
 - Extensive experimental results demonstrate the effectiveness of our proposed methods, achieving a $6.05 \times$ speedup and reducing memory usage to just 20.0 GB, all while maintaining over 92% of performance on Mixtral-8×7B.
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2 RELATED WORK

Mixture of Experts The Mixture of Experts (MoE) is a kind of neural network architecture with an 096 extended set of parameters (referred to as "experts") controlled by a router, which is first introduced 097 in the context of conditional computation (Jacobs et al., 1991; Jordan & Jacobs, 1994). The potential 098 of sparse activation in MoE is subsequently exploited by Shazeer et al. (2017) for efficient training and inference on pretrained models with special designs, opening the door for MoE in various vision 100 (Riquelme et al., 2021) and language (Lepikhin et al., 2020; Du et al., 2022; Fedus et al., 2022) 101 scenarios. Attributed to its exceptional efficiency, MoE has been adopted as a foundational framework 102 in the designs of large language models (LLMs) (Jiang et al., 2024; Dai et al., 2024; Xue et al., 2024a; 103 Zhu et al., 2024; Team, 2024b), achieving superior scaling laws at low computational costs (Clark 104 et al., 2022). Further investigations emerge in developing improved expert structures (Gururangan 105 et al., 2022; Rajbhandari et al., 2022; Dai et al., 2024), router designs (Lewis et al., 2021; Roller et al., 2021; Zhou et al., 2022), and training strategies (Shen et al., 2023; Chen et al., 2022), propelling the 106 continuous evolution on the representation capability and computational efficiency of MoE models. 107 Despite the success, MoE also suffers from efficiency issues. For instance, MoE replicates the experts,

108 significantly increasing the parameter budget (He et al., 2023). On the other hand, adopting multiple 109 experts to process input tokens introduces communication costs and enhances latency (Song et al., 110 2023; Xue et al., 2024b).

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112 **Compression Methods** The escalating size of large language models presents considerable hurdles 113 for their practical implementation. Consequently, a range of efficient methods has emerged to 114 address the implementation issues. Among them, model quantization (Frantar et al., 2022; Lin et al., 2024) and network pruning (Sun et al., 2023; Frantar & Alistarh, 2023) are widely utilized. Model 115 116 quantization reduces the precision of neural network weights to lower bits (Jacob et al., 2017), while network pruning (Han et al., 2016) removes redundant parameters or architectures. Although these 117 methods have shown promising results on dense models, they lack consideration for the inductive 118 bias inherent in MoE. To bridge this gap, Expert Drop, as proposed in studies like (Muzio et al., 119 2024; Lu et al., 2024), addresses the unique nature of MoE by removing unimportant experts. By 120 eliminating redundant experts, the MoE architecture becomes more compact and can be deployed at 121 a lower cost. However, while Expert Drop leads to a more compact architecture, it may also lead to 122 non-negligible performance drop and rely on post-training procedures for recovery. 123

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3 **PRELIMINARIES**

3.1 MIXTURE OF EXPERTS 127

128 A Mixture of Experts (MoE) layer consists of a collection of n experts, E_1, E_2, \ldots, E_n , each 129 associated with weights W_1, W_2, \ldots, W_n , and a router G that dynamically selects the most relevant 130 experts for a given input x. The router computes selection scores, $G(x) \in \mathbb{R}^n$, for all experts and 131 selects the top k experts, resulting in a sparse activation pattern. The input x is processed by the 132 selected experts, and their outputs are combined into a weighted sum based on the router's scores. 133 This process is mathematically expressed as:

$$\mathcal{K} = \text{TopK}(\text{Softmax}(\boldsymbol{G}(\boldsymbol{x})), k), \tag{1}$$

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 $oldsymbol{y} = \sum_{i \in \mathcal{K}} oldsymbol{G}(oldsymbol{x})_i \cdot oldsymbol{E}_i(oldsymbol{x} | oldsymbol{W}_i),$ where \mathcal{K} denotes the indices of selected experts, $G(x)_i$ represents the selection score for the *i*-th expert, and $E_i(x)$ is the output from the *i*-th expert. In transformer models, the MoE layer is often used as a replacement for the feed-forward network (FFN). In this context, each expert functions as an independent FFN module, enhancing the model's capacity without a proportional increase in the

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144 **Challenges** While MoE models have demonstrated strong performance across various tasks (Jiang et al., 2024; Dai et al., 2024), they also encounter significant deployment challenges. On one hand, 145 MoE models replicate multiple expert networks, inflating model size and memory usage. For instance, 146 Mixtral-8×7B has a total of 47B parameters, requiring 87.7GB of memory for deployment, though 147 only 13B parameters are activated per token. On the other hand, the communication required to 148 manage multiple expert networks increases latency and slows down inference speed, especially in 149 distributed environments (Song et al., 2023; Yu et al., 2024).

3.2 OVERVIEW OF PREVIOUS COMPRESSION METHODS

computational cost (Vaswani et al., 2017).

153 To address the efficiency challenges, we first review several mainstream and state-of-the-art compres-154 sion techniques for MoE models. 155

Pruning: Pruning reduces the number of active parameters by selectively disabling parts of the 156 model's weights. In an MoE layer with n experts $E_i i = 1^n$ and corresponding weights $W_i i = 1^n$, 157 pruning introduces binary masks $M_{i_{i=1}}^{n}$ to deactivate certain weights: 158

$$\hat{W}_i = M_i \odot W_i. \tag{3}$$

Pruning can be unstructured (Lee et al., 2021; Bai et al., 2022), semi-structured, or structured. 161 Unstructured sparsity tends to yield the best performance, semi-structured sparsity strikes a balance

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162 (b) Mixture of Experts (c) Expert Slimming $\hat{W} \leftarrow f(W)$ (a) Transformer Block 163 Pruning Quantization 164 (†)∢ Đ $f(W) = M \odot W$ f(W) = quant(W)165 E_1 E_2 E_3 E_N 166 Mixture of Experts 167 G_2 MoE Normalization (d) Expert Trimming $E \leftarrow \{E_i\}_{i \in \mathcal{T}'}$ 169 Expert Drop Layer Drop Block Drop 170 $G \leftarrow G_{i \in \mathcal{T}}$ \emptyset . MoENorm $\leftarrow \emptyset$ $\mathbf{MHA} \leftarrow \varnothing, \ \mathbf{MHANorm} \leftarrow \varnothing$ $\in \mathcal{T},$ 171 Multi-Head Attention E_3 E_1 172 173 MHA No G_1 G_3 X X 20 174 Granularity Scales Up 175

Figure 1: The Unified View of MoE Compression. The view integrates two complementary
 perspectives: Expert Slimming and Expert Trimming. Expert Slimming compresses individual
 experts, while Expert Trimming directly drops structured modules.

between efficiency and performance, and structured sparsity, while hardware-friendly, often results in lower performance.

Quantization: Unlike pruning, which involves masking out unimportant parameters, quantization
 reduces memory usage by converting model weights to lower-bit representations. For MoE layers,
 quantization is applied as follows:

$$\hat{\boldsymbol{W}}_i = \text{Quant}(\mathbf{W}_i),\tag{4}$$

where "Quant" denotes the quantization function. Quantization decreases memory consumption
 without reducing FLOPs or the total number of parameters, making it particularly advantageous for
 hardware acceleration.

189 **Expert Drop**: Different from fine-grained pruning and quantization, Expert Drop entails the removal 190 of expert networks, based on the observation that not all experts are equally important (Lu et al., 191 2024; Muzio et al., 2024). Given expert-wise importance scores S (e.g., the routing scores, $S(E_i) = G(x)_i$), Expert Drop retains only the experts with the highest n' scores:

$$\tau' = \operatorname{TopK}(\boldsymbol{S}(\{\boldsymbol{E}_i\}_{i=1}^n), n'),$$
(5)

$$\boldsymbol{E} \leftarrow \{\boldsymbol{E}_i\}_{i \in \mathcal{T}'}, \quad \boldsymbol{G} \leftarrow \boldsymbol{G}_{i \in \mathcal{T}'}. \tag{6}$$

Here, \mathcal{T}' denotes the subset of the original expert indices $\mathcal{T} = \{1, 2, ..., n\}$. Expert Drop reduces FLOPs conditionally: when \mathcal{T}' contains more than or equal to k indices, MoE still utilizes the top k experts for each input; otherwise, it uses all remaining experts. While this approach reduces communication between experts, the resulting speedup is usually insignificant when maintaining acceptable performance.

Other Compression Techniques: Other methods, such as low-rank decomposition (Li et al., 2024b;a), aim to compress model weights into smaller matrices, further reducing memory and computational costs. In this work, we primarily focus on the widely-used methods (pruning, quantization, and Expert Drop), leaving a more detailed exploration of these additional methods for future research.

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4 A HOLISTIC STUDY OF MOE COMPRESSION TECHINIQUES

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In this section, we propose a general framework that unifies various compression methods for MoE.
 This framework provides a comprehensive understanding of MoE model efficiency issues and helps
 identify new design spaces for further performance improvements.

212 4.1 OVERVIEW

Existing MoE compression methods primarily address two types of inefficiencies: structural re dundancies in the overall architecture and internal redundancies within individual experts. To address both issues, we categorize these methods into two complementary perspectives: Expert

216 Table 1: Summary of Compression Methods. "✓" means effective and "X" means ineffective, while 217 "O" represents conditionally effective, depending on specific settings and environments.

	Method	Formulation	Parameter	Memory	FLOPs	Speedup
Expert Trimming	Expert	$\mathcal{T} \leftarrow \mathcal{T}'$	1	 ✓ 	0	0
	Layer Block	$\mathcal{T} \leftarrow \varnothing$	<i>✓</i>	1	1	1
Even ant Slimmin a	Pruning	$oldsymbol{M}\odotoldsymbol{W}$	✓	О	1	О
Expert Similing	Quantization	Quant(W)	×	✓	×	1

Trimming focuses on removing structured components (e.g., experts, layers, or blocks), and Expert 225 Slimming that compresses individual experts through techniques like pruning or quantization. An 226 overview of these perspectives is illustrated in Figure 1. 227

228 Expert Trimming deals with compressing structured modules by selecting and retaining only a subset 229 of the experts, denoted as \mathcal{T}' . This is represented by the transformation $\mathcal{T} \leftarrow \mathcal{T}'$. Methods like Expert Drop, which selectively drops unimportant experts, are examples of this approach. On the 230 other hand, the compression of individual experts (Expert Slimming) focuses on the transformation 231 and reduction of expert weights, denoted as W. We utilize a transformation function f(W) to 232 represent this process. The transformation function f(W) can be understood as a general mapping 233 that applies various compression techniques to the weights of the model. For example, in pruning, 234 $f(\mathbf{W})$ could be a function that sets a subset of the weights to zeros. In quantization, $f(\mathbf{W})$ might 235 reduce the precision of the weights from 32-bit floats to 8-bit integers. By integrating these two 236 perspectives, we can derive a general form for efficient MoE models. The compression within and 237 across experts can be expressed as follows: 238

$$\boldsymbol{y} = \sum_{i \in \mathcal{T}'} \boldsymbol{G}_i \cdot \boldsymbol{E}_i(\boldsymbol{x} | f(\boldsymbol{W}_i)).$$
(7)

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In the following sections, we will elaborate on Expert Trimming and Expert Slimming, respectively.

4.2 EXPERT TRIMMING

The core operation of Expert Trimming involves updating the set of remaining experts denoted as 245 $\mathcal{T} \leftarrow \mathcal{T}'$, where \mathcal{T}' is a subset of the original expert indices \mathcal{T} . Specifically, Expert Drop updates the 246 experts and their corresponding routing weights as follows: $E \leftarrow \{E_i\}_{i \in \mathcal{T}'}$ and $G \leftarrow G_{i \in \mathcal{T}'}$. 247

However, Expert Drop carries the risk of collapsing feature transformation. The absence of certain 248 experts can lead to incorrect selections for given inputs, thereby degrading model performance 249 (Chen et al., 2022). Additionally, partially reducing experts can disrupt routing patterns, negatively 250 impacting the model's overall efficiency and effectiveness. Despite its benefits, Expert Drop still 251 retains the costly computation within each expert and the complex communication between experts. 252 These limitations highlight the need for further optimization of Expert Trimming to promote the 253 efficiency. By systematically analyzing the redundancies and inefficiencies inherent in MoE models, 254 we propose extending beyond expert-level optimizations to identify new design spaces for efficiency 255 improvements.

256 We propose two novel techniques: Layer Drop and 257 Block Drop. Layer Drop focuses on removing entire 258 MoE layers, which significantly reduces both computa-259 tion and communication overhead. Block Drop extends 260 this concept by eliminating entire blocks, including atten-261 tion layers and MoE layers, within transformer models. 262 These advanced techniques aim to streamline the model 263 architecture, improve performance, and enhance overall 264 efficiency. -



266 Layer Drop Inspired by Raposo et al. (2024); El- Figure 2: Illustration of Similarity Meahoushi et al. (2024), we consider a special scenario of Ex- surements in Layer Drop. Features for 267 pert Drop where all experts are dropped ($\mathcal{T} \leftarrow \mathcal{T}' = \emptyset$), calculating $S^{(M)}$ and $S^{(M)}$ are colored 268 effectively removing entire MoE layers. We refer to this with red and blue, respectively. 269 approach as Layer Drop. To perform Layer Drop, we use a similarity-based metric where high

270 similarity indicates high redundancy in transformation. One straightforward metric is the cosine 271 similarity between the input x and the output y = MoE(x): 272

$$S^{(M)} = \frac{\boldsymbol{x} \cdot \boldsymbol{y}}{||\boldsymbol{x}||_2 ||\boldsymbol{y}||_2}, \text{ where } \boldsymbol{y} = MoE(\boldsymbol{x}).$$
(8)

275 However, this metric alone does not adequately capture the impact of the MoE layer within the context 276 of a transformer block, which includes a layer normalization module ("Norm") (Ba et al., 2016) and residual connections (He et al., 2015). To address this, we propose concurrently removing both the 277 278 MoE and Norm layers. This approach ensures that the similarity metric more accurately reflects the combined functionality of these layers, allowing for a more precise identification of redundancy and 279 a streamlined model architecture, as illustrated in Figure 2. By considering the similarity between the 280 raw residual input and the aggregated output, we can better evaluate the necessity of the MoE layer in 281 the overall architecture: 282

$$\boldsymbol{S}^{(\mathrm{NM})} = \frac{\boldsymbol{x}' \cdot \boldsymbol{y}'}{||\boldsymbol{x}'||_2 ||\boldsymbol{y}'||_2}, \text{ where } \boldsymbol{y}' = \boldsymbol{x}' + \mathrm{MoE}(\mathrm{Norm}(\boldsymbol{x}')).$$
(9)

286 **Block Drop** Within a transformer block, Layer Drop removes the MoE layers but retains the 287 computation-costly attention layers (Ribar et al., 2024; Zhang et al., 2023). To address this issue, we further utilize the same similarity-based metrics to investigate whether the attention layer can 288 be dropped without a significant performance drop. If feasible, this allows us to drop the entire 289 block within MoE models, thus enhancing efficiency. We introduce Block Drop as an extension of 290 Layer Drop, which also removes the attention layers. Specifically, for the *i*-th block, we assess its importance score by evaluating the similarity between its inputs x_l and outputs y_l . Compared to 292 Expert Drop, both Layer Drop and Block Drop focus on structures beyond expert level, with the 293 potential to further enhance the efficiency of MoE models. 294

4.3 EXPERT SLIMMING

297 Given that employing multiple experts in MoE significantly escalates parameters and inference costs, 298 Expert Slimming, stemming from single-model compression techniques, targets the compression 299 of individual expert weights W exclusively. We denote any efficient transformation function as $f(\cdot)$, which encompasses pruning $M \odot W$ and quantization Quant(W). Through the application 300 of such functions, we reduce the redundancy within each expert and create several light-weighted 301 slim experts, thus improving their intrinsic efficiency. However, it is important to note that Expert 302 Slimming primarily focuses on compressing individual experts without addressing the redundancy 303 across multiple experts. For maximum efficiency gains, Expert Slimmingand Expert Trimming can 304 be integrated to compress both individual experts and structured components. We summarize the 305 efficiency contributions of all the discussed Expert Trimming and Expert Slimming methods in Table 306 1, highlighting the unique advantages of each approach. 307

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5 **EXPERIMENTS ON EXPERT TRIMMING**

n this section, we evaluate the effectiveness of Expert Trimmingtechniques, starting with Expert Drop, 311 and comparing it with our proposed methods, Layer Dropand Block Drop. Implementation details 312 are provided in Appendix A. 313

314 Expert Drop: Performance Degradation with Limited Efficiency Gains While experts are 315 specific structures in MoE, not all experts hold equal significance. Figure 11 visualizes the distribution 316 of expert-wise importance scores, highlighting this variability. To systematically drop experts at 317 varying proportions, we conduct experiments using both layer-wise and global dropping approaches 318 (see Appendix A.3). Given the importance of shared experts (Appendix E), we only dropped normal 319 experts for DeepSeek-MoE-16B. Under both settings, Expert Drop causes consistent performance 320 degradation. For example, dropping 25% of experts in Mixtral-8×7B results in a 23% performance 321 drop on the MMLU task. The efficiency improvement from Expert Drop is also marginal. For instance, 322 dropping 12.5% of experts results in less than a 1% speedup, despite significant performance losses. 323 More experimental results are available in Appendix F.



Figure 3: **Evaluation of Expert Drop.** We consider two strategies: layer-wise (dotted lines) and global (solid lines).

334 Layer Drop: Comparable Performance with Greater Efficiency To verify the feasibility of Layer Drop, we vi-335 sualize feature similarity across different modules in Figure 336 4. This visualization shows a high level of similarity for fea-337 tures across the the MoE normalization module (Norm) and 338 the MoE layer. In contrast, the low similarity for features across the MoE layer indicates the infeasibility of removing 340 only MoE layers. Results from Figure 5 show that Layer 341 Drop preserves performance within a wide range of com-342 pression ratio, e.g. 1% performance drop on MMLU when 343 dropping 8 layers for Mixtral- $8 \times 7B$, revealing significant 344 redundancy in the MoE layers. 345

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Layer-wise Similarity

Figure 4: Layer-Wise Similarity. We consider two scenarios, i.e., for "MoE" and "Norm + MoE".



Figure 5: **Evaluation of Layer Drop.** We show results on Mixtral- $8 \times 7B$ and DeepSeek-MoE-16B (solid lines), along with the baseline and random guess performances (dotted lines).



Figure 6: **Evaluation of Block Drop.** We show results on Mixtral-8×7B and DeepSeek-MoE-16B (solid lines), along with the baseline and random guess performances (dotted lines).

Block Drop: Further Optimizing Efficiency by Pruning Entire Transformer Blocks While
 Layer Drop maintains the performance of the original models, it still preserves the computation-costly
 attention layers. To address this, Block Dropextends Layer Dropby removing whole transformer
 blocks, including both MoE and attention layers, further reducing computational and memory
 costs. Figure 7 visualizes block-wise similarity, where both Mixtral-8×7B and DeepSeek-MoE demonstrate high similarity between specific blocks. Based on this observation, we conduct the
 empirical study by varying the number of dropped blocks.

Surprisingly, as shown in Figure 6, the Mixtral-8×7B maintains over 90% of the original performance even after removing 5 blocks (over 7 billion parameters). Similar observations are also found in DeepSeek-MoE-16B, where 4 blocks can be removed when maintaining 90% performance. Since Block Drop removes computationally expensive attention layers, it outperforms Layer Drop by a large margin in terms of both memory and inference cost, as illustrated in Figure 8.





Figure 7: **Normalized Block-Wise Similarity.** We measure the cosine similarity among hidden features between blocks.



Figure 8: Speedup Scaling Curves of Expert Trimming Methods. where we measure the averaged decoding speed during generation.

higher-level structures, Layer Drop and Block Drop achieve substantial efficiency improvements
 while maintaining acceptable performance levels.

MoE Layers are More Redundant than Dense 393 **Counterparts** Since Layer Drop and Block 394 Drop can also be applied to dense models, we 395 take Mistral-7B, the corresponding dense model 396 of Mixtral-8×7B for comparison. Both models 397 have the same depth and differ only in the FFN 398 implementation, so we remove the same number 399 of layers or blocks from each. When dropping 400 an equal number of blocks, both MoE and dense 401 models exhibit performance degradation. How-402 ever, the MoE model suffers less performance drop under the same compression setting. For ex-403 ample, when dropping 8 MoE layers, the Mistral-404 7B receives a performance drop of 24.3, while 405 Mixtral- $8 \times 7B$ only receives a drop of 7.0. This 406 interesting finding highlights the higher redun-407 dancy in MoE layers, and further validates the 408 effectiveness of applying Layer Drop and Block 409 Drop to MoE models. 410

Table 2: Comparison of Layer Drop and Block Drop on dense and MoE models. "-Ln/m", "-Bn/m" represents dropping *n* out of *m* corresponding modules with Layer Drop and Block Drop, respectively.

		Mistral-7	B (Dense)							
Method	ARC-C	HellaSwag	MMLU	OBQA	Average						
Baseline	61.5	83.7	62.5	43.8	<u>62.9</u>						
+ L4/32	53.2	77.7	61.7	40.0	<u>58.2</u> (-4.7)						
+ L8/32	36.7	33.6	53.3	30.6	38.6 (-24.3)						
+ B4/32	53.1	77.5	61.6	40.0	58.1 (-4.8)						
+ B8/32	40.0	63.9	60.0	30.6	<u>48.6</u> (-14.3)						
Mixtral-8×7B (MoE)											
		Mixtral-8	×7B (Mol	E)							
Method	ARC-C	Mixtral-8 HellaSwag	× 7B (Mol MMLU	E) OBQA	Average						
Method Baseline	ARC-C 59.4	Mixtral-8 HellaSwag 84.0	< 7B (Mol MMLU 67.9	E) OBQA 46.8	Average 64.6						
Method Baseline + L4/32	ARC-C 59.4 56.2	Mixtral-82 HellaSwag 84.0 81.3	<7B (Mol MMLU 67.9 67.6	E) OBQA 46.8 44.6	<u>Average</u> <u>64.6</u> 62.4 (-2.2)						
Method Baseline + L4/32 + L8/32	ARC-C 59.4 56.2 47.7	Mixtral-83 HellaSwag 84.0 81.3 75.2	<7B (Mol MMLU 67.9 67.6 67.3	E) OBQA 46.8 44.6 40.0	<u>Average</u> <u>64.6</u> <u>62.4</u> (-2.2) <u>57.6</u> (-7.0)						
Method Baseline + L4/32 + L8/32 + B4/32	ARC-C 59.4 56.2 47.7 -53.8	Mixtral-8> HellaSwag 84.0 81.3 	< 7B (Mol MMLU 67.9 67.6 67.3 - 67.3 - 67.9	E) OBQA 46.8 44.6 - 40.0 - 43.0	$\frac{\text{Average}}{\underline{64.6}}$ $\frac{\underline{62.4}}{\underline{57.6}} (-2.2)$ $-\underline{57.6} (-7.0)$ $-\underline{61.2} (-3.4)$						

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6 VISUALIZATION EXAMPLES OF LAYER DROP AND BLOCK DROP

In this section, we visualize the layer-wise similarity and the corresponding dropping order of MoE
 layers and blocks to investigate the varying levels of redundancy across different depths.

Since our similarity-based metrics depend on the hidden states of each block, the choice of data may influence feature similarity across layers. To investigate this, we conducted ablation studies on Mixtral-8×7B, examining both the number of samples and the types of datasets used for feature extraction. This analysis helps us understand how data selection affects decisions regarding the dropping of layers or blocks. The results are presented in Figure 9.

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Robustness to Calibration Datasets In Figure 9a, we note that feature similarity remains relatively
 stable across different layers as the sample size increases, indicating that Layer Drop and Block
 Drop maintain consistency regardless of sample quantity. This confirms that using 128 samples
 suffices for computing similarity, which is adopted for all our experiments. Similarly, Figure 9b shows
 that varying the datasets, from pretraining with C4 to instruction tuning with Lima and MetaMathQA,
 does not significantly alter feature similarity. This demonstrates the resilience of Layer Drop and
 Block Drop to variations in data distribution.

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Redundant Deeper Layers Figure 10 visualizes the remaining and dropped layers/blocks as the number of dropped modules increases. Both MoE architectures exhibit similar patterns in Layer Drop and Block Drop: initially, both models tend to drop the deeper layers, followed by the shallower



Figure 9: Influence of Data Choices on Feature Similarity. We measure the similarity among layers and blocks on Mixtral- $8 \times 7B$. (a) The similarity calculated using different number of samples from C4 Raffel et al. (2019). (b) The normalized similarity calculated using 1,024 samples from different datasets, i.e., C4, Lima Zhou et al. (2023) and MetaMathQA Yu et al. (2023).



Figure 10: Dropping Patterns for Layer Drop and Block Drop. We visualize of the remaining layers and blocks under different dropped numbers, where yellow areas represent the retained portions and red areas indicate the dropped layers/blocks.

ones. These findings are consistent with Xu et al. Men et al. (2024), which suggests that deeper layers tend to be more redundant.

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INTEGRATION OF EXPERT TRIMMING AND EXPERT SLIMMING 7

Beyond Expert Trimming, another avenue for MoE compression is Expert Slimming, which targets 466 the compression of individual experts. Techniques such as quantization and network pruning are among the most commonly employed methods. We provide a detailed comparison of network pruning and quantization in Appendix B, where quantization outperforms in both performance and efficiency.

Since Expert Trimmingand Expert Slimmingfocus on different aspects of compression, we further 470 explore their potential integration. Given the superior average performance and practical efficiency of 471 quantization, we use it for Expert Slimming. For Expert Trimming, we include all three methods to 472 offer a comprehensive comparison. The orders of applying these two compression techniques are 473 discussed in Appendix D. 474

475 Quantization Preserves the Performance of Expert Trimming As shown in Table 3, the integra-476 tion of Expert Slimming and Expert Trimming significantly enhances overall efficiency. Quantization 477 can be seamlessly combined with three different levels of dropping, achieving comparable perfor-478 mance. For instance, after quantization, the average performance of Layer Drop and Block Drop is 479 nearly the same, maintaining more than 90% of the performance of the original models. 480

- 481 **The Integration Significantly Enhances Efficiency** In Table 3, the integration of Expert Trim-482 ming and quantization promotes efficiency by a large margin. Different Expert Trimming strategies
- showcase different advantages. Specifically, Expert Drop contributes to reducing memory usage 483 but its speedup is marginal. Layer Drop and Block Drop excel in speedup as illustrated in Figure 484 8, with Block Drop demonstrating both higher performance and greater speedup. Considering all 485 settings, the combination of Block Drop and quantization offers the best efficiency with comparable

Table 3: Experimental Results of the Integration of Expert Trimming and Expert Slimming. "-En/m" denotes dropping n out of m experts per MoE layer on average. "-Ln/m", "-Bn/m" represents dropping n out of m layers/blocks with Layer Drop and Block Drop, respectively. The FLOPs are measured using an input with the 2,048 sequence length.

	Mixtral-8×7B											
Method	SpeedUp	FLOPs	Memory	ARC-C	BoolQ	HellaSwag	MMLU	OBQA	PIQA	RTE	WinoGrande	Avg
Baseline	-	54.4T	87.7GB	59.4	84.2	84.0	67.9	46.8	83.8	70.4	75.6	71.5
w/ĀŴQ	$5.08 \times$	54.4T	24.4GB	- 58.4	84.2	83.3	66.6	45.8	83.0	69.0	76.3	70.8
+ E2/8	$1.06 \times$	54.4T	66.7GB	53.2	77.7	80.5	52.2	46.2	81.7	55.6	76.8	65.5
w/ĀŴQ	$5.28 \times$	54.4T	20.1GB	50.7	79.1	78.9	52.4	44.2	81.2	55.6	75.9	64.8
+ L8/32	$1.19 \times$	42.9T	66.6GB	47.7	85.3	75.2	67.3	40.0	75.8	69.7	74.6	67.0
w/ĀŴQ	$\overline{6.05\times}$	42.9T	20.0GB	46.2	84.2	74.2	66.2	39.0	75.5	<u>69.3</u>	74.2	<u>66.1</u>
+ B5/32	$1.17 \times$	46.0T	74.1GB	51.3	85.3	78.7	67.9	42.0	79.3	69.7	74.3	68.6
w/AWQ	5.94×	46.0T	21.9GB	50.6	85.1	77.5	66.9	41.4	76.1	71.8	74.5	68.0
					DeepS	eek-MoE-16	6B					
Method	SpeedUp	FLOPs	Memory	ARC-C	BoolQ	HellaSwag	MMLU	OBQA	PIQA	RTE	WinoGrande	Avg.
Baseline	-	11.7T	30.8GB	48.1	72.4	77.3	37.9	44.0	80.4	63.9	70.3	61.8
w/ĀŴQ	$\overline{3.16\times}$	11.7T	9.8GB	46.8	71.2	76.6	36.4	- 43.6	- 80.1	62.1	70.1	<u>60.9</u>
+ E16/64	$1.06 \times$	11.7T	23.9GB	45.0	67.1	75.6	31.8	42.2	80.2	59.9	70.0	59.0
w/ĀŴQ	$\overline{3.34\times}$	11.7T	7.7GB	44.0	66.0	74.5	27.9	42.6	78.5	56.3	67.3	57.1
+ L4/28	$1.14 \times$	10.6T	26.6GB	39.5	70.2	67.6	35.2	40.4	75.8	48.4	65.7	55.3
w/ĀŴQ	$\overline{3.60\times}$	10.6T	8.5GB	42.1	72.0	69.2	33.7	39.8	75.1	47.7	66.5	<u>55.8</u>
+ B4/28	$1.16 \times$	10.1T	26.4GB	40.3	71.3	69.0	36.2	37.8	75.8	51.6	68.0	56.3
	0.05	10.10										

performance: a $6.05 \times$ speedup with only 20.0GB memory usage, while maintaining over 92% of the performance on Mixtral-8×7B, making it available to be deployed on a NVIDIA RTX 3090 GPU.

POST-FINETUNING RECOVERS THE PERFORMANCE

While the discussed compression techniques maintains most of the performance of the original models, we further conduct post-finetuning to recover the degraded performance. Specifically, for comparison, we full-finetune DeepSeek-MoE-16B and corresponding compressed models on the Alpaca-GPT4 dataset Peng et al. (2023) for 3 epochs using a learning rate of 8e-6 with 0.03 warmup ratio and cosine scheduling, where the global batch size is set to 32. As shown in Figure 4, the post-finetuning process significantly reduces the performance gap between the compressed models and the original models, e.g. narrowing it from 5.5% to 0.6% for the model following Block Drop.

Table 4: Performance of the DeepSeek-MoE-16B models finetuned after Expert Trimming.

	DeepSeek-MoE-16B												
Method	SpeedUp	FLOPs	Memory	ARC-C	BoolQ	HellaSwag	MMLU	OBQA	PIQA	RTE	WinoGrande	Avg.	
Baseline +SFT	_	11.7T	30.8GB	48.1 44.6	72.4 75.3	77.3 79.0	37.9 40.3	44.0 44.6	80.4 80.3	63.9 70.4	70.3 71.7	<u>61.8</u> <u>63.3</u>	
+ E16/64 +SFT	$1.06 \times$	11.7T	23.9GB	45.0 44.4	67.1 74.0	75.6 78.6	31.8 38.5	42.2 45.8	80.2 79.6	59.9 65.7	70.0 70.1	<u>59.0</u> 62.1	
+ L4/28 +SFT	$1.14 \times$	10.6T	26.6GB	39.5 42.1	70.2 78.9	67.6 75.2	35.2 40.8	40.4 43.4	75.8 77.6	48.4 71.1	65.7 69.5	<u>55.3</u> <u>62.3</u>	
+ B4/28 +SFT	$1.16 \times$	10.1T	26.4GB	40.3 43.2	71.3 78.2	69.0 75.0	36.2 40.4	37.8 43.8	75.8 76.8	51.6 74.0	68.0 70.2	<u>56.3</u> <u>62.7</u>	

CONCLUSION

In this paper, we conducted a holistic study of MoE compression techniques, facilitating a systematic understanding of the efficiency issue of MoE and identifying the new design space to improve the performance further. Based on this study, we propose a comprehensive recipe that integrates Expert Slimming and Expert Trimming to further enhance efficiency. Our proposed methods and insights not only address current challenges but also set the stage for future advancements in the field of MoE.

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A IMPLEMENTATION DETAILS

758 A.1 MODELS AND DATASETS 759

Models. For our experiments, we employed Mixtral-8×7B Jiang et al. (2024) and DeepSeek-MoE-16B Dai et al. (2024). Mixtral-8×7B utilizes 8 experts for MoE layers and activates the top two for
 each input token. In contrast, DeepSeek-MoE-16B employs an dense FFN in the first block and utilizes two shared experts with additional 64 experts within MoE layers in other blocks.

Datasets. For compression experiments, we used the C4 dataset Raffel et al. (2019), with 128 samples and an input sequence length of 2,048, following the setup in Sun et al. (2023); Lu et al. (2024); Lin et al. (2024); Frantar et al. (2022). To evaluate model performance, we report normalized zero-shot accuracy on the LM-harness benchmark, which includes multiple tasks: ARC-C Clark et al. (2018), BoolQ Clark et al. (2019), HellaSwag Zellers et al. (2019), MMLU Hendrycks et al. (2021), OBQA Mihaylov et al. (2018), PIQA Bisk et al. (2019), RTE Wang et al. (2019), and WinoGrande ai2 (2019). The evaluation code is based on EleutherAI LM Harness Gao et al. (2023).

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A.2 IMPLEMENTATION DETAILS OF EXPERT SLIMMING

774Both Expert Slimming methods (i.e., pruning and quantization) require calibration data to estimate775input statistics. To control this variable, we use 128 samples from the C4 dataset Raffel et al. (2019)776as the calibration dataset for pruning. For quantization, we follow the default settings of GPTQ 1777and AWQ 2, using 128 random samples from Alpaca Taori et al. (2023) and Pile Gao et al. (2020),778respectively. We use the default group size 128 for Mixtral-8×7B and 64 for DeepSeek-MoE-16B.

A.3 IMPLEMENTATION DETAILS OF EXPERT DROP

The Expert Drop compresses MoE by preserving only important experts $\{E_i\}_{i \in \mathcal{T}'}$ while removing others, where \mathcal{T}' is determined by the importance scores $\{S(E_i)\}_{i \in \mathcal{T}}$. Following Muzio *et al.* Muzio et al. (2024), we measure the importance scores through the averaged routing scores of a batched data \mathcal{X} , i.e., $\{S(E_i)\} = \frac{1}{|\mathcal{X}|} \sum_{x \in \mathcal{X}} G_i(x)$, and consider two dropping strategies for Expert Drop: layer-wise dropping and global dropping.

Layer-Wise dropping removes the same number of experts for each layer. Given the total number of experts $n = |\mathcal{T}|$ and the preserved number of experts $n' = |\mathcal{T}'| < n$ in layer *l*, the preserved expert set $\mathcal{T}'^{(l)}$ is obtained by:

$$\mathcal{T}^{(l)} = \{ E_t^{(l)} \}, \quad \text{where} \quad S(E_t^{(l)}) \in \text{TopK}(\{ S(E_i^{(l)}) \}_{i=1}^n, n').$$
(10)

Global dropping constrains the total number of preserved experts for the entire model. Given the total number of layers L in the model, the preserved expert set $\mathcal{T}^{\prime(l)}$ for layer l is obtained by:

$$\mathcal{T}^{\prime(l)} = \{ \boldsymbol{E}_t^{(l)} \}, \quad \text{where} \quad \boldsymbol{S}(\boldsymbol{E}_t^{(l)}) \in \text{TopK}\Big(\bigcup_{j=1}^m \{ \boldsymbol{S}(\boldsymbol{E}_i^{(j)}) \}_{i=1}^n, n'L \Big).$$
(11)

For the integration of Expert Slimming and Expert Trimming, we choose the global dropping as the strategy of Expert Drop, which shows competitive performance compared to the layer dropping for Mixtral-8×7B under low dropping ratios, as well as consistent better performance for DeepSeek-MoE-16B in Figure 13.

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¹https://github.com/AutoGPTQ/AutoGPTQ

²https://github.com/casper-hansen/AutoAWQ

810 B EXPERT SLIMMING

Pruning: Comparable Performance with Deployment Challenges In Table 5, we evaluate representative pruning algorithms (i.e., Wanda Sun et al. (2023), SparseGPT Frantar & Alistarh (2023)) on Mixtral-8×7B and DeepSeek-MoE-16B. Since DeepSeek-MoE-16B utilizes both shared experts and normal experts, we conduct an ablation study on whether to prune shared experts, as discussed in Appendix E. We find that unstructured pruning preserves more than 95% of performance. However, it is not compatible with existing hardware. Conversely, the hardware-friendly semi-structured pruning (i.e., 4:8 and 2:4 patterns) undergoes a significant performance drop. Nevertheless, according to Lu et al. Lu et al. (2024), semi-structured sparsity is ineffective in speeding up MoE models.

Table 5: **Performance of Pruning on MoE.** We consider two mainstream pruning methods (i.e., Wanda Sun et al. (2023) and SparseGPT Frantar & Alistarh (2023)) under 50% unstructured sparsity and 2:4 semi-structured sparsity.

	Mixtral-8×7B											
Method	Sparsity	ARC-C	BoolQ	HellaSwag	MMLU	OBQA	PIQA	RTE	WinoGrande	Avg.		
Baseline	0%	59.4	84.2	84.0	67.9	46.8	83.8	70.4	75.6	<u>71.5</u>		
Wanda SparseGPT Wanda SparseGPT	50%	56.1 - 56.4 - 51.4 49.2	85.8 85.7 79.4 81.0	81.7 81.5 77.8 77.6	64.3 - 64.6 - 60.3 59.2	46.4 45.0 44.0 44.0	82.2 82.4 - 80.7 80.6	65.0 66.8 65.3 63.9	76.0 75.8 74.1 74.8			
				DeepSeek	-MoE-16B	;						
Method	Sparsity	ARC-C	BoolQ	HellaSwag	MMLU	OBQA	PIQA	RTE	WinoGrande	Avg.		
Baseline	0%	48.1	72.4	77.3	37.9	44.0	80.4	63.9	70.3	<u>61.8</u>		
Wanda SparseGPT	50%	43.6 43.9	74.3 73.5	72.6 74.0	31.1 33.8	43.0 41.4	79.5 79.0	58.1 61.0	69.4 68.3	$\frac{59.0}{59.4}$		
Wanda	2:4	38.2	- 66.1 - 68.9	67.5 71.6	- 27.6 27.6	- 39.4 41.6	77.0	-53.8 57.4		<u>54.5</u> 56.9		

Quantization: Better Performance and Greater Efficiency In Table 6, we evaluate the impact of 4-bit quantization on MoE. Quantization offers two major benefits: it maintains the comparable performance of the original models and significantly reduces memory costs. Specifically, the quantized models achieve over 98% of the original performance while using less than 30% of the memory. Moreover, when quantized with AWQ Lin et al. (2024), Mixtral- $8 \times 7B$ and DeepSeek-MoE-16B achieve impressive speedups of $\times 5.08$ and $\times 3.16$, respectively. This demonstrates that 4-bit quantization is an effective technique for deploying MoE models in resource-constrained environments.

Table 6: **Performance of Quantization on MoE.** We utilize GPTQ Frantar et al. (2022) and AWQ Lin et al. (2024) as the quantization methods for 4-bit compression.

	Mixtral-8×7B											
Method	Bits	Memory	ARC-C	BoolQ	HellaSwag	MMLU	OBQA	PIQA	RTE	WinoGrande	Avg.	
Baseline GPTQ AWQ	$-\frac{16}{4}$	87.7GB 24.4GB	<u>59.4</u> 59.0 58.4	- <u>84.2</u> 84.4 84.2	<u>84.0</u> 83.4 83.3	67.9 - 67.1 66.6	- <u>46.8</u> 45.2 45.8	<u>83.8</u> - <u>83.1</u> - 83.0	$-\frac{70.4}{70.1}$ 69.0		$\frac{71.5}{70.9}$	
 DeepSeek-MoE-16B												
					DeepSeek-M	AoE-16B						
Method	Bits	Memory	ARC-C	BoolQ	DeepSeek-M HellaSwag	AoE-16B MMLU	OBQA	PIQA	RTE	WinoGrande	Avg.	

C ANALYSIS ON THE DROPPING PATTERNS OF EXPERT DROP

866 Score Distribution Directs Expert Drop. 867 The distribution of importance scores is infor-868 mative to determine the proportion of dropped experts. In Figure 11, we visualize the 870 score distribution of Expert Drop for Mixtral-8×7B and DeepSeek-MoE-16B, respectively. 871 872 DeepSeek-MoE-16B, which allocates more experts, shows a left-skewed distribution where 873 most experts have low scores. In contrast, 874 Mixtral-8×7B demonstrates a right-skewed 875 distribution, with only a few experts being 876 deemed unimportant. This distribution differ-877 ence results in different resistance capability 878 against Expert Drop, where DeepSeek-MoE-879 16B can drop much more experts than Mixtral-880 $8 \times 7B$ while maintaining competitive perfor-881 mance, as demonstrated in Table 3 and Figure 882 13.

Global Expert Drop Removes Experts Fine-884 Grainedly. We employed two different strate-885 gies for Expert Drop, namely layer-wise and 886 global. Layer-wise dropping treats each layer 887 equally by dropping the same number of experts, while global dropping results in different pro-889 portions of remaining experts across layers. We 890 visualize the distribution of remaining experts 891 after global dropping in Figure 12. We find 892 the global dropping shows a more fine-grained 893 pattern on dropping experts, where the bottom 894 layers are more vulnerable under lower dropping ratios (yellow part). 895



Figure 11: **Distribution of Normalized Importance Scores** *S* **for Expert Drop.** We highlight the density of scores under different drop ratios with different colors.



Figure 12: **Distribution of Dropped Experts for Expert Drop.** We visualize of the dropped experts under different drop ratios, where the dropped experts are colored from yellow to blue as the drop ratio increases.

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918 D ABLATION STUDY ON COMPRESSION ORDERS

In Section 7, we discussed the combination of Expert Trimming and Expert Slimming. Here we ablate on the orders of compression when combining these two techniques. Results in Table 7 show that the order of Expert Trimming and Expert Slimming doesn't have a significant influence on the performance, where applying Expert Slimming then Expert Trimming ("S+T") performs slightly better for Mixtral-8×7B (e.g. +0.5, +0.4 and +0.1 for Expert Drop, Layer Drop and Block Drop, respectively). To this end, we choose "S+T" as the final implementation in our experiments.

Table 7: Ablation results on different orders of Expert Slimming and Expert Trimming. "S+T" denotes first applying Expert Slimming then Expert Trimming, and "T+S" denotes the reversed order.

Mixtral-8×7B											
Method	ARC-C	BoolQ	HellaSwag	MMLU	OBQA	PIQA	RTE	WinoGrande	Avg.		
Baseline	59.4	84.2	84.0	67.9	46.8	83.8	70.4	75.6	71.5		
+ E2/8, AWQ (S+T)	50.7	79.1	78.9	52.4	44.2	81.2	55.6	75.9	64.8		
+ E2/8, AWQ (T+S)	50.8	79.9	78.7	49.2	44.4	80.9	55.2	75.4	<u>64.3</u>		
+ L8/32, AWQ (S+T)	46.2	84.2	74.2	66.2	39.0	75.5	69.3	74.2	66.1		
+ L8/32, AWQ (T+S)	46.8	84.4	74.0	65.3	39.8	75.0	66.8	73.2	<u>65.7</u>		
+ B5/32, AWQ (S+T)	50.6	85.1	77.5	66.9	41.4	76.1	71.8	74.5	68.0		
+ B5/32, AWQ (T+S)	50.3	84.7	77.4	65.8	42.0	78.8	70.4	74.0	<u>67.9</u>		
			DeepSeek-I	MoE-16E	3						
Method	ARC-C	BoolQ	HellaSwag	MMLU	OBQA	PIQA	RTE	WinoGrande	Avg.		
Baseline	48.1	72.4	77.3	37.9	44.0	80.4	63.9	70.3	<u>61.8</u>		
+ E16/64, AWQ (S+T)	44.0	66.0	74.5	27.9	42.6	78.5	56.3	67.3	57.1		
+ E16/64, AWQ (T+S)	44.7	64.1	74.0	29.0	42.6	79.9	54.2	68.4	<u>57.1</u>		
+ L4/28, $AWQ(S+T)$	42.1	72.0		33.7	39.8	75.1	47.7	66.5	55.8		
+ L4/28, AWQ (T+S)	42.4	71.7	69.1	33.4	40.1	74.8	47.6	66.2	<u>55.7</u>		
$+ \overline{B4/28}, \overline{AWQ}(\overline{S+T})$	40.1	70.2	68.6	36.1	38.4	76.2	51.6	66.4	56.0		
+ B4/28, AWQ (T+S)	41.6	69.4	69.1	35.8	38.6	76.2	50.9	67.0	56.1		

E ABLATION STUDY ON SHARED EXPERTS IN DEEPSEEK-MOE-16B

While most MoE models follow Equation 2 to implement the experts, models like DeepSeek-MoE-16B adopt a residual Rajbhandari et al. (2022) form of experts, which brings a special scenario to discuss. In the residual MoE, an extra set of m shared experts $\{\bar{E}_1, \bar{E}_2, \ldots, \bar{E}_m\}$ are always selected by the router G and activated for all inputs. Given an input x, the output can be represented as a degenerated form of Equation 2, where the scores of shared experts are fixed to 1:

$$\boldsymbol{y} = \sum_{i \in \mathcal{K}} \boldsymbol{G}(\boldsymbol{x})_i \cdot \boldsymbol{E}_i(\boldsymbol{x}) + \sum_{j=1}^m \bar{\boldsymbol{E}}_j(\boldsymbol{x}).$$
(12)

This special form of expert routing may bring a difference in the redundancy distribution of MoE. Here we discuss the influence of shared experts through pruning and present the results in Table 8. We find that pruning without the shared experts will boost the performance at a considerable scale, i.e., +3.6% and +1.5% of the averaged accuracy for unstructured pruning with Wanda and SparseGPT, respectively. This finding reveals a different pattern of the inner redundancy in that the shared experts are less compressible compared to the others in residual MoE models, which may inform future work.

Table 8: Ablation Study of Pruning Shared Experts on DeepSeek-MoE-16B. We consider two
scenarios, i.e., pruning both shared experts and normal experts ("w/Pruning Shared Experts") and
pruning normal experts only ("w/o Pruning Shared Experts"). We use two mainstream pruning
methods (i.e., Wanda Sun et al. (2023) and SparseGPT Frantar & Alistarh (2023)) under both
unstructured sparsity (50%) and semi-structured sparsity (2:4).

	DeepSeek-MoE-16B											
Method	Sparsity	ARC-C	BoolQ	HellaSwag	MMLU	OBQA	PIQA	RTE	WinoGrande	Avg.		
Baseline	0%	48.1	72.4	77.3	37.9	44.0	80.4	63.9	70.3	61.8		
w/ Pruning Shared Experts												
Wanda SparseGPT Wanda	50%	43.6 - 43.9 - -38.2 -	74.3 73.5 	72.6 74.0 -67.5	31.1 $-\frac{33.8}{27.6}$	43.0 41.4 -39.4	79.5 79.0 77.0	58.1 61.0 -53.8	69.4 $\frac{68.3}{66.7}$	$\frac{59.0}{59.4}$		
SparseGPT	2:4	43.1	68.9	71.6	27.6	41.6	78.3	57.4	66.6	56.9		
				w/o Pruning	Shared Exp	perts						
Wanda SparseGPT	50%	44.0 45.0	76.3 75.5	73.5 74.4	36.2 36.3	41.0 41.0	79.3 79.4	59.9 64.3	70.2 69.3	$\frac{60.0}{60.7}$		
Wanda SparseGPT	2:4	40.1 40.7	75.7 75.7	69.9 69.9	- 33.5 33.3	40.0 39.0	77.9 77.7	58.8 61.4		<u>58.1</u> 58.4		

1026 FULL EXPERIMENTAL RESULTS F 1027

We provide the full results of Expert Trimming, including Expert Drop, Layer Drop and Block Drop, 1029 in Figure 13, 14, and 15, respectively. 1030 Mixtral-8×7B 1031 ARC-C BoolQ HellaSwag MMLU ----1032 Layer-wise Global Random G Layer-wise Global Random Gues Layer-v Layer-v Global -#-Global Random Gues 1033 Random 1034 75 Accuracy(%) 70 60 1035 65 50 1036 3! 60 1037 41 55 1038 1039 20 Dropped Experts RTE Dropped Experts PIQA Dropped Experts OBQA Dropped Experts WinoGrande 1040 ---- Layer-wise Global ---- Random Gue Layer-wise Global Random Gue --- Layer-wise Global ---- Random Guess Layer-wi Global 1041 ----- Random Gues 1042 6 75 1043 70 1044 3 1045 5 60 1046 55 1047 2! 1048 Dropped Experts Dropped Experts Dropped Experts ² Dropped Experts 1049 DeepSeek-MoE-16B BoolQ ARC-C HellaSwag MMLU 1050 Layer-wise Global Layer-wise Global ----Layer-wise Global Layer-wise Global * - * ------ Random Gues ----- Random Gues 70 Random Gue Random Gue 1051 65 1052 34 6 Accuracy(%) 60 32 1053 55 51 1054 50 28 41 1055 45 2! 31 * 1056 24 1057 20 20 22 ¹⁶ ³² ⁴ Dropped Experts ¹⁶ ³² ⁴ Dropped Experts Dropped Experts 6 32 4 Dropped Experts 1058 OBQA PIQA RTE WinoGrande Layer-wise Global Random Gue Layer-wise Global Random Gu 65 Layer-wise Global Random Gue Layer-wise Global Random Gues -#-1059 62. 1060 75 60. 1061 70 57.5 1062 35 65 55. 60 1063 30 52. 55 1064 50. 1065 47.5 1066 Dropped Experts Dropped Experts Dropped Experts Dropped Experts



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Figure 15: Full Results for Block Drop. We show results on Mixtral-8×7B and DeepSeek-MoE-16B (solid lines), along with the baseline and random guess performances (dotted lines).