
CASNSC: A context-based approach for accurate pedestrian motion prediction at intersections

Anonymous Author(s)

Affiliation

Address

email

Abstract

1 Intention recognition of pedestrians is crucial to safe and reliable working of
2 autonomous vehicles, when serving as, for instance, indoor service robots or
3 self-driving cars in busy urban scenes. Previously, Chen et al. [2016] combined
4 Markovian-based and clustering-based approaches to learn motion primitives and
5 subsequently predict pedestrian trajectories by modeling the transition between
6 learned primitives as a Gaussian Process (GP). This work further develops their ap-
7 proach by incorporating semantic features from the environment (relative distance
8 to curbside and status of pedestrian traffic lights) for more confident prediction of
9 pedestrian trajectories at intersections. Adding the environmental context, when
10 available, not only makes prediction more robust but can also provide increased
11 flexibility of prediction in new environments. We test our algorithm on real data.
12 The results show 26% improvement in prediction *accuracy* as compared to previous
13 work, on incorporation of new features.

14 1 Introduction

15 Recent advances in sensor technologies and computing power have led to a surge in research on
16 autonomous driving to improve road safety (Fagnant and Kockelman [2015], Bagloee et al. [2016]),
17 reduce traffic congestion and improve vehicle utilization. For safe and efficient autonomous driving in
18 complex urban environments, a self-driving vehicle must be able to interact with other moving objects,
19 including pedestrians, cyclists, and, of course, cars. Pedestrian trajectory prediction is challenging as
20 compared to that of other cars and cyclists because of the absence of a regular flow, such as driving
21 within lanes and staying within road boundaries, that result from a fairly uniform set of predefined
22 “rules of the road” for cars (and to some extent cyclists). The complexity is increased further when
23 the urban environment includes pedestrian traffic lights or tightly packed sidewalks with numerous
24 pedestrian interactions.

25 Several papers have been written on short-term prediction of human motion (Kooij et al. [2014],
26 Bissacco and Soatto [2009]), but understanding goals or intent is needed to plan for longer timescales
27 (Karasev et al. [2016], Alahi et al. [2016]). Previous work has focused on two main approaches
28 (Lefèvre et al. [2014]) to modeling maneuvers of dynamic agents, including pedestrians: 1) prototype
29 trajectories-based and 2) maneuver intention estimation-based. In general, prototype trajectories-
30 based/clustering-based approaches are more robust to measurement noise when compared to maneuver
31 intention estimation-based approaches, which are mostly Markovian (Makris and Ellis [2002],
32 Vasquez et al. [2009]) and rely on the current state only for prediction. However, the prototype
33 trajectories-based approaches can be computationally quite expensive (Rasmussen and Ghahramani
34 [2002], Ferguson et al. [2015]) and hence slow in detecting changes in pedestrian intent. They are
35 also susceptible to issues like partial trajectories in the training dataset being grouped into a cluster
36 and learned as a trajectory prototype.

37 Chen et al. [2016] use a combination of prototype trajectory-based and Markovian-based methods to
 38 inherit the benefits of both techniques in developing a dictionary learning algorithm, called augmented
 39 semi nonnegative sparse coding (ASNNSC). Learning motion primitives instead of complete prototype
 40 trajectories addresses partial observability of trajectories caused by occlusions or a limited field of
 41 view of on-board perception sensors. ASNNSC creates a set of feasible trajectories as its prediction that
 42 are learned based on solely the spatial features of the training dataset (absolute x and y position and
 43 orientation of pedestrians), independent of the environment context that may influence a pedestrian’s
 44 intent.

45 The accuracy of these predictions could be im-
 46 proved by adding semantic features from the
 47 environment in the learning process. Incorpor-
 48 ating the environmental context can also pro-
 49 vide the flexibility of application of the learned
 50 model to prediction in new, but similar envi-
 51 ronments, unexplored earlier, which is in gen-
 52 eral difficult to achieve with clustering-based
 53 approaches (Lefèvre et al. [2014]). Fig. 1 shows
 54 an intersection scenario in which, when faced
 55 by a choice between two crosswalks, pedestrian
 56 traffic light status for each of those crosswalks
 57 influences pedestrian choice. Similarly, a com-
 58 parison of the relative distance to each curbside
 59 could be indicative of future direction of mo-
 60 tion. Most of the previous work on context-
 61 based pedestrian trajectory prediction is limited
 62 to a classification problem (Schulz and Stiefel-
 63 hagen [2015]). This work, in contrast, provides a continuous trajectory as its prediction output.
 64 Karasev et al. [2016] used jump-Markov process for long term prediction of pedestrian motion by
 65 incorporating traffic light and crosswalks as semantic features. The output of their prediction model
 66 is an *occupancy map* of feasible trajectory predictions. Our goal is to make prediction confident and
 67 output the *most likely trajectory* with increased accuracy.

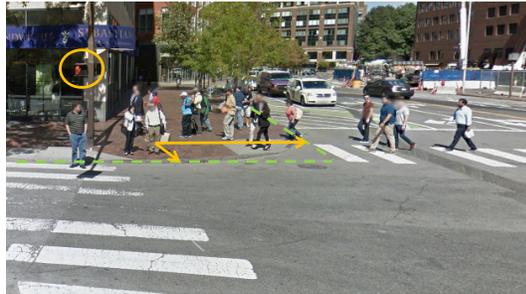


Figure 1: Example intersection scenario. Dotted green line denotes a rectangular approximation to the curbside in view. Orange arrows denote relative distance of a pedestrian from the two curbsides, which can indicate pedestrian intention. Pedestrian traffic light status is highlighted in orange, which influences pedestrian movement.

68 Our approach extends ASNNSC by incorporating semantic features from the environment. In order to
 69 meet these objectives, a dictionary of motion primitives is learned as in Chen et al. [2016]. However,
 70 the transition between these motion primitives is learned with respect to both spatial as well as
 71 additional environmental contexts. As illustrated in Fig. 2(c), the influence of pedestrian traffic light
 72 status on the probability of transition between two motion primitives is not captured in ASNNSC. Two
 73 main features are used to incorporate the environmental context in this work: pedestrian traffic light
 74 status and relative distance to curbside. Similar to the approach followed by Chen et al. [2016], GP
 75 models are used to learn the transition between motion primitives and subsequently predict pedestrian
 76 velocity. A squared exponential (SE) kernel function with automatic relevance determination (ARD)
 77 (Rasmussen and Williams [2006]) is used to determine the relevance of each of the individual features.
 78 The results show a 26% increase in the *accuracy* of pedestrian trajectory prediction.

79 2 Augmented semi nonnegative sparse coding

80 Given a training dataset of n samples, $\mathbf{Z} = [\mathbf{x}_1, \dots, \mathbf{x}_n]$, where \mathbf{x}_i is a column vector of length p , the
 81 objective is to learn a set of K dictionary atoms, $\mathbf{D} = [\mathbf{d}_1, \dots, \mathbf{d}_K]$, and the corresponding nonneg-
 82 ative sparse coefficients, $\mathbf{S} = [s_1, \dots, s_n]$. Mathematically, this can be formulated as a constrained
 83 optimization problem of the form (Chen et al. [2016])

$$\underset{\mathbf{D}, \mathbf{S}}{\operatorname{argmin}} \|\mathbf{Z} - \mathbf{D}\mathbf{S}\|_F^2 + \lambda \sum_{i=1}^n \|\mathbf{s}_i\|_1 \quad (1)$$

$$\text{s.t. } \mathbf{d}_k \in \mathbf{Q}, s_{ki} \geq 0 \forall k, i \quad (2)$$

84 where λ is a regularization parameter and \mathbf{Q} is the feasible set in which \mathbf{d}_k resides. Fig. 2(b) shows
 85 an example of dictionary atoms learned using ASNNSC. \mathbf{D} is used to segment the original training
 86 trajectories \mathbf{x}_i into clusters, where each cluster is best explained by one of the learned dictionary
 87 atoms.

88 2.1 Trajectory prediction using learned dictionary

89 A transition matrix, $\mathbf{T} \in \mathbb{Z}^{K \times K}$ is thus created, where $T(i, j)$ denotes the number of trajectories
 90 exhibiting a transition from i -th dictionary atom to j -th dictionary atom. A transition will, therefore,
 91 be mathematically represented as a concatenation of two dictionary atoms $\{\mathbf{d}_i, \mathbf{d}_j | T(i, j) > 0\}$. Each
 92 transition is modeled as a two-dimensional GP flow field (Joseph et al. [2011], Aoude et al. [2013]).
 93 In particular, two independent GPs, (GP_x, GP_y) , called GP motion patterns are used to learn a mapping
 94 from chosen features $\mathbf{X} \in \mathbb{R}^N$ to the x-y velocities. ASNSC uses $\mathbf{X} = \mathbf{X}_p = (x, y)^\top \in \mathbb{R}^2$ as the feature
 95 vector.

$$GP_x : \mathbf{X} \rightarrow v_x, \quad GP_y : \mathbf{X} \rightarrow v_y \quad (3)$$

96 The learned GP motion patterns, (GP_x, GP_y) , are used for generating a predicted path using (3) as
 97 well as for computing the likelihood of an observed trajectory, $\mathbf{t}' = \{(\mathbf{X}_1', \mathbf{v}_1'), \dots, (\mathbf{X}_l', \mathbf{v}_l')\}$ using

$$P(\mathbf{t}' | GP_x, GP_y) = \prod_{\mathbf{X}' \in \mathbf{t}'} \mathcal{N}(v_x; \mu_{GP_x}(\mathbf{X}'), \sigma_{GP_x}^2(\mathbf{X}')) \mathcal{N}(v_y; \mu_{GP_y}(\mathbf{X}'), \sigma_{GP_y}^2(\mathbf{X}')) \quad (4)$$

98 Trajectory prediction has two main steps. 1) Unitary GP motion patterns (GP_x^{uni}, GP_y^{uni}) are learned
 99 from training trajectories corresponding to $\mathbf{T}(i, j) \forall i = j$. The unitary GP motion pattern that most
 100 likely generated the observed trajectory \mathbf{t}' is determined using (4), which is equivalent to selecting the
 101 most likely initial dictionary atom \mathbf{d}_k (Algorithm 1, line 12). 2) The set of possible future dictionary
 102 atoms can be found as $\mathcal{D} = \{j | \mathbf{T}_{kj} > 0\}$ (Algorithm 1, line 13). Transitional GP motion patterns,
 103 $(GP_{x_{kj}}^{tran}, GP_{y_{kj}}^{tran}) \forall j \in \mathcal{D}$ are used for generating a set of predicted trajectories $\{\mathbf{s}_j\}$.

104 3 Context-based augmented semi nonnegative sparse coding

105 This work develops ASNSC by incorporating semantic features from the environment in the prediction
 106 phase (Algorithm 1, lines 5-14) and is motivated by situations in which the environmental context
 107 influences transition between learned dictionary atoms (see Fig. 2(c)). The proposed approach uses
 108 two sets of features: 1) *learning features*, \mathbf{X}_p , which are used for learning \mathbf{D} (Algorithm 1, lines 1-4);
 109 and 2) *prediction features*, \mathbf{X} , which are used for prediction using GP models (lines 5-14). ASNSC
 110 uses the same set of features, \mathbf{X}_p , for both learning and prediction. In contrast, CASNSC uses the
 111 same set of *learning features*, \mathbf{X}_p , as used in ASNSC, but an augmented set of features, $\mathbf{X} = (\mathbf{X}_p : \mathbf{X}_c)$,
 112 as *prediction features*. Here, \mathbf{X}_c denotes the set of additional *context features*.

$$GP_x : \mathbf{X} \rightarrow v_x, \quad GP_y : \mathbf{X} \rightarrow v_y \quad (5)$$

$$\text{where } \mathbf{X} \in \mathbb{R}^N \text{ s.t. } \mathbf{X} = (\mathbf{X}_p : \mathbf{X}_c), \mathbf{X}_p \in \mathbb{R}^2, \mathbf{X}_c \in \mathbb{R}^{N-2} \quad (6)$$

113 3.1 Context features

114 The relative distance of a pedestrian from the left/right curbside and pedestrian traffic lights' status
 115 have been used as additional *context features* in this work. However, the described framework is
 116 generalizable to any number and type of feature selection.

117 3.1.1 Distance to curbside

118 The relative distance of a pedestrian (treated as a point mass) to curbside can be computed using a
 119 map of the environment. When approaching an intersection, pedestrians are in the vicinity of two
 120 different curbsides, assumed to intersect at a point (see Fig. 2(a)). A two-dimensional vector, $(c_l, c_r)^\top$,
 121 is therefore used as the curbside feature. This particular feature influences pedestrian intention only
 122 when the observed trajectory starts on the sidewalk. As explained in Fig. 2(a), this aspect is captured
 123 by assigning a positive or negative sign to the distance computed when constructing the feature vector.

124 3.1.2 Pedestrian traffic light

125 A pedestrian's decision to go left or right is influenced by the status of two pedestrian traffic lights
 126 (T1, T2) in a four-way intersection scenario. However, in contrast to the curbside feature, a single-
 127 dimensional feature vector, (tr) , is sufficient to capture the environment context with respect to both
 128 the traffic lights as the change in status of (T1, T2) captures redundant information.

Algorithm 1: CASNSC

(Context-based augmented semi nonnegative sparse coding)

```

/* Learning Phase */
1 D  $\leftarrow$  0, S  $\leftarrow$  0;
2 while not converged do
3    $\{\mathbf{D}, \mathbf{S}\} = \text{ASN}(\mathbf{Z}, \mathbf{X}_p, \lambda)$ 
4   T  $\leftarrow$  Transition_Matrix(D, Z, S);
/* Prediction Phase */
5 X = (Xp : Xc);
6  $GP^{uni} \leftarrow \emptyset, GP^{tran} \leftarrow \emptyset$ ;
7 for  $\forall (i, j)$  s.t.  $\{\mathbf{T}(i, j) > 0\}$  do
8   if  $i == j$  then
9      $GP^{uni}.insert((GP_x(\mathbf{X}), GP_y(\mathbf{X})))$ 
10  else
11     $GP^{tran}.insert((GP_x(\mathbf{X}), GP_y(\mathbf{X})))$ 
12   $\hat{k} = \text{argmax}_k P(\mathbf{t}' | GP_{x_k}^{uni}, GP_{y_k}^{uni})$ 
13  for  $\forall j \in \mathcal{D} = \{j | \mathbf{T}_{\hat{k}j} > 0\}$  do
14     $\mathbf{s}_j \leftarrow \text{Predict}(\mathbf{t}', (GP_{x_{\hat{k}j}}^{tran}, GP_{y_{\hat{k}j}}^{tran}))$ 

```

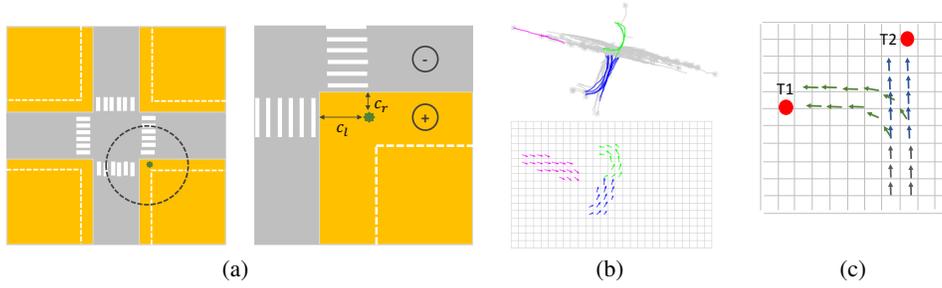


Figure 2: (a) A typical four-way intersection used to explain the curbside feature. The zoomed portion (right) shows a pedestrian location as a green star. c_l, c_r denote the distance to the two curbsides of interest. If the green star is in the orange region (curbside), a positive value is assigned to the curbside feature. Otherwise, a negative value is assigned. (b) From Chen et al. [2016]: examples of dictionary atoms learned using ASN; each color represents one dictionary atom (below). Training trajectory segments that agree with each of the dictionary atoms are also shown (top). (c) Pedestrian traffic light status influences transition between dictionary atoms. T1 and T2 denote two different traffic lights. Transition between dictionary atoms represented by black and blue has a higher probability than that between black and green for T1 = 0, T2 = 1.

129 3.2 Kernel function

130 A SE kernel function with ARD is used as it allows for the combination of features with different
 131 characteristics and scales each feature in accordance with its relevance. Mathematically, it is given by
 132 the following form (Rasmussen and Williams [2006]):

$$k(\mathbf{X}, \mathbf{X}') = \sigma_f^2 \exp\left(-\frac{1}{2}(\mathbf{X} - \mathbf{X}')\Lambda(\mathbf{X} - \mathbf{X}')\right) \quad (7)$$

$$\text{where } \theta = (\{\Lambda\}, \sigma_f^2)^\top \text{ and } \Lambda = \text{diag}(\mathbf{I}^{-2}) \quad (8)$$

133 Here, θ is a vector containing all hyperparameters and \mathbf{I} is a vector of positive values. Thus, for
 134 an N-dimensional feature vector, the number of hyperparameters needed to define the SE kernel
 135 function with ARD is $(N + 1)$. The l_1, \dots, l_N hyper parameters represent characteristic lengths of the
 136 individual features and aid in determining the relevance of each feature in the N-dimensional feature
 137 space. In this work, the *GPML Toolbox* has been used for learning hyperparameters.

138 In particular, for predictions using CASNSC with pedestrian traffic light status as an additional
 139 contextual feature, $\mathbf{X} = (x, y, tr)^\top$ in (7) and $\theta = (l_x, l_y, l_{tr}, \sigma_f)^\top$ in (8). For distance to curbside as
 140 an additional *context feature*, in order to account for its dependency on pedestrian position, a linear
 141 combination of two SE with ARD kernel functions is used as follows:

$$k(\mathbf{x}, \mathbf{x}') = \sigma_{f_1}^2 \exp\left(-\frac{1}{2}(\mathbf{X}_p - \mathbf{X}_{p'})\Lambda_p(\mathbf{X}_p - \mathbf{X}_{p'})\right) + \sigma_{f_2}^2 \exp\left(-\frac{1}{2}(\mathbf{X}_c - \mathbf{X}_{c'})\Lambda_c(\mathbf{X}_c - \mathbf{X}_{c'})\right) \quad (9)$$

$$\text{where } \theta = (l_x, l_y, l_{c_l}, l_{c_r}, \sigma_{f_1}, \sigma_{f_2})^\top \text{ and } \mathbf{X}_c = (c_l, c_r)^\top \quad (10)$$

142 4 Results

143 Our approach is tested on real pedestrian data collected by a Polarix GEM vehicle equipped with
 144 three Logitech C920 camera and a SICK LMS151 LIDAR (Miller and How [2017], Miller et al.
 145 [2016]). The dataset consists of 186 training trajectories and 32 test trajectories. An observation
 146 history of 2.5 seconds prior to the pedestrian entering the intersection is used to predict 5 seconds
 147 ahead in time. Fig. 3 provides a qualitative comparison of our approach with Chen et al.
 148 [2016] on the inclusion of distance to curbside as an additional feature. While ASNSC provides
 149 all feasible pedestrian trajectories, given the intersection geometry, CASNSC only picks those
 150 that are closest to the actual trajectory. Similarly, Fig. 5 compares prediction results in three different
 151 scenarios, on the inclusion of pedestrian traffic light status as an additional feature.

152 In the first scenario (trajectory 18), pedestrian traffic lights' status is given by $T1 = 0, T2 = 1$ and the
 153 pedestrian has already crossed the intersection and is entering the sidewalk. While ASNSC predicts a set
 154 of two trajectories, CASNSC provides a more confident prediction. In the second scenario (trajectory
 155 25), the traffic lights' status is the same but the pedestrian is now entering the intersection and is
 156 faced with a choice between two crosswalks. CASNSC picks the correct direction out of the set of
 157 feasible directions (as predicted by ASNSC) taking the pedestrian traffic light status into account.
 158 In the last scenario (trajectory 11), the traffic lights' status is given by $T1 = 1, T2 = 0$. While
 159 both ASNSC and CASNSC pick the correct direction, predictions using the latter follow the actual
 160 trajectory more closely since it can utilize the information that $T2 = 0$ to predict that the pedestrian
 161 will continue moving straight, with little or no probability of turning left.

162 Fig. 4 illustrates the metrics used for performance evaluation and Table 1 provides a quantitative
 163 comparison of ASNSC with CASNSC on the inclusion of pedestrian traffic light status as an additional
 164 *context feature*. As illustrated in Fig. 4, the *Area Under the Curve (AUC)* (Hand [2009]) is used as a
 165 metric for comparing the confidence level of predictions, such that a larger *AUC* corresponds to a
 166 lower confidence.

167 Table 1 indicates that *AUC* for predictions using CASNSC is lower than when using ASNSC,
 168 confirming that CASNSC provides a more confident prediction. Classification *accuracy* is also

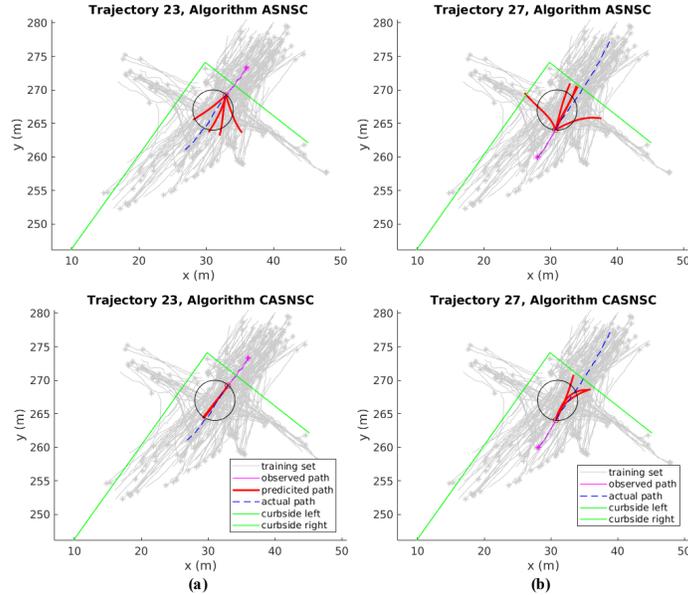


Figure 3: ASNSC vs. CASNSC on incorporation of relative distance to curbside as a *context feature*. Training trajectories are shown in gray and prediction is performed for test trajectories in the black circle.

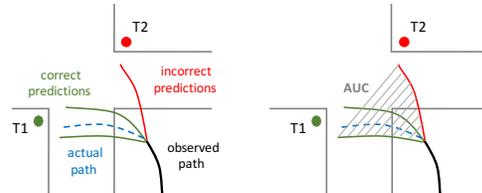


Figure 4: (Left) *Incorrect* and *correct* predictions at an intersection scenario. (Right) Use of *AUC* as a metric for variance in prediction.

Table 1: Performance evaluation comparison of CASNSC with ASNSC

Algorithm	Classification accuracy(%)	MHD(m)	AUC(m ²)	Computation time(s)
ASNSC	73.95	2.12	75.48	0.14
CASNSC	100	1.79	30.51	0.3

189 measured, which represents the fraction of *correct* predictions. For an correct representation of this
 190 metric, the likelihood of prediction of each trajectory is taken into account when computing the
 191 *accuracy*. For instance, if a set of n trajectories is predicted, $\{t_1, \dots, t_n\}$, with their likelihood of
 192 prediction given by $\{l_1, \dots, l_n\}$, and the *correct* predictions are identified as $\{t_i \mid \forall i \in C \subset \{1, \dots, n\}\}$,
 193 the classification *accuracy* is given by:

$$\text{Classification accuracy \%} = \frac{\sum_{i \in C} l_i}{\sum_{k=1}^n l_k} \times 100\%. \quad (11)$$

194 In addition to the illustrated metrics, the *modified Hausdorff distance (MHD)* (Dubuisson and Jain
 195 [1994]) is used to compare predicted pedestrian trajectories with the ground truth. Note that MHD
 196 is used to compare the *correct* predictions only. Table 1 shows an improvement in all the chosen
 197 metrics, with only a slight increase in computation time.

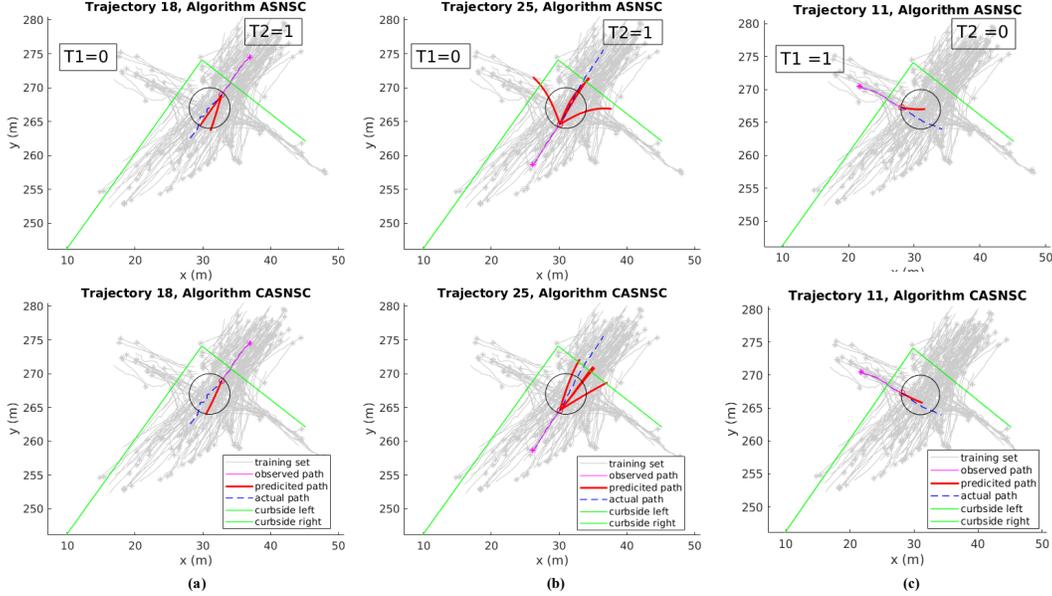


Figure 5: Comparison of prediction performance of ASNSC with CASNSC on addition of pedestrian traffic light status as a *context feature*.

198 5 Conclusion

199 We extend ASNSC by incorporating relative distance to curbside and pedestrian traffic light status
 200 as additional *context features* for more confident and accurate prediction. Our approach, CASNSC,
 201 shows a 26% improvement in *accuracy*, 15.5% improvement in *MHD* of *correct* predictions and
 202 reduces variance in prediction, as measured by *AUC*, by a factor of 2.5. There is scope for further
 203 improvement on incorporation of features (specific to intersections) that are constant in time (e.g.,
 204 the existence of crosswalks and areas of interest like subway stations). Furthermore, testing the
 205 learned prediction model on new but similar, four-way intersections and incorporating interactions
 206 between pedestrians would provide good insight into the flexibility and robustness of this approach
 207 respectively.

208 **References**

- 209 Alexandre Alahi, Kratarth Goel, Vignesh Ramanathan, Alexandre Robicquet, Li Fei-Fei, and Silvio
210 Savarese. Social lstm: Human trajectory prediction in crowded spaces. In *Proceedings of the IEEE*
211 *Conference on Computer Vision and Pattern Recognition*, pages 961–971, 2016.
- 212 Georges S Auode, Brandon D Luders, Joshua M Joseph, Nicholas Roy, and Jonathan P How.
213 Probabilistically safe motion planning to avoid dynamic obstacles with uncertain motion patterns.
214 *Autonomous Robots*, 35(1):51–76, 2013.
- 215 Saeed Asadi Bagloee, Madjid Tavana, Mohsen Asadi, and Tracey Oliver. Autonomous vehicles:
216 challenges, opportunities, and future implications for transportation policies. *Journal of Modern*
217 *Transportation*, 24(4):284–303, Dec 2016. ISSN 2196-0577. doi: 10.1007/s40534-016-0117-3.
218 URL <https://doi.org/10.1007/s40534-016-0117-3>.
- 219 Alessandro Bissacco and Stefano Soatto. Hybrid dynamical models of human motion for the
220 recognition of human gaits. *International journal of computer vision*, 85(1):101–114, 2009.
- 221 Yu Fan Chen, Miao Liu, and Jonathan P How. Augmented dictionary learning for motion prediction.
222 In *Robotics and Automation (ICRA), 2016 IEEE International Conference on*, pages 2527–2534.
223 IEEE, 2016.
- 224 M-P Dubuisson and Anil K Jain. A modified hausdorff distance for object matching. In *Pattern*
225 *Recognition, 1994. Vol. 1-Conference A: Computer Vision & Image Processing., Proceedings of*
226 *the 12th IAPR International Conference on*, volume 1, pages 566–568. IEEE, 1994.
- 227 Daniel J Fagnant and Kara Kockelman. Preparing a nation for autonomous vehicles: opportunities,
228 barriers and policy recommendations. *Transportation Research Part A: Policy and Practice*, 77:
229 167–181, 2015.
- 230 Sarah Ferguson, Brandon Luders, Robert C Grande, and Jonathan P How. Real-time predictive
231 modeling and robust avoidance of pedestrians with uncertain, changing intentions. In *Algorithmic*
232 *Foundations of Robotics XI*, pages 161–177. Springer, 2015.
- 233 David J Hand. Measuring classifier performance: a coherent alternative to the area under the roc
234 curve. *Machine learning*, 77(1):103–123, 2009.
- 235 Joshua Joseph, Finale Doshi-Velez, Albert S Huang, and Nicholas Roy. A bayesian nonparametric
236 approach to modeling motion patterns. *Autonomous Robots*, 31(4):383, 2011.
- 237 Vasily Karasev, Alper Ayvaci, Bernd Heisele, and Stefano Soatto. Intent-aware long-term prediction
238 of pedestrian motion. In *Robotics and Automation (ICRA), 2016 IEEE International Conference*
239 *on*, pages 2543–2549. IEEE, 2016.
- 240 Julian Francisco Pieter Kooij, Nicolas Schneider, Fabian Flohr, and Dariu M Gavrila. Context-based
241 pedestrian path prediction. In *European Conference on Computer Vision*, pages 618–633. Springer,
242 2014.
- 243 Stéphanie Lefèvre, Dizan Vasquez, and Christian Laugier. A survey on motion prediction and risk
244 assessment for intelligent vehicles. *Robomech Journal*, 1(1):1, 2014.
- 245 Dimitrios Makris and Tim Ellis. Spatial and probabilistic modelling of pedestrian behaviour. In
246 *British Machine Vision Conference 2002, vol. 2*. Citeseer, 2002.
- 247 Justin Miller and Jonathan P How. Predictive positioning and quality of service ridesharing for campus
248 mobility on demand systems. In *Robotics and Automation (ICRA), 2017 IEEE International*
249 *Conference on*, pages 1402–1408. IEEE, 2017.
- 250 Justin Miller, Andres Hasfura, Shih-Yuan Liu, and Jonathan P How. Dynamic arrival rate estimation
251 for campus mobility on demand network graphs. In *Intelligent Robots and Systems (IROS), 2016*
252 *IEEE/RSJ International Conference on*, pages 2285–2292. IEEE, 2016.
- 253 Carl E Rasmussen and Zoubin Ghahramani. Infinite mixtures of gaussian process experts. In
254 *Advances in neural information processing systems*, pages 881–888, 2002.

- 255 Carl Edward Rasmussen and Christopher KI Williams. *Gaussian processes for machine learning*,
256 volume 1. MIT press Cambridge, 2006.
- 257 Andreas Th Schulz and Rainer Stiefelhagen. Pedestrian intention recognition using latent-dynamic
258 conditional random fields. In *Intelligent Vehicles Symposium (IV), 2015 IEEE*, pages 622–627.
259 IEEE, 2015.
- 260 Dizan Vasquez, Thierry Fraichard, and Christian Laugier. Incremental learning of statistical motion
261 patterns with growing hidden markov models. *IEEE Transactions on Intelligent Transportation*
262 *Systems*, 10(3):403–416, 2009.