LIFELONG PERCEPTUAL PROGRAMMING BY EXAMPLE

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ABSTRACT

We introduce and develop solutions for the problem of *Lifelong Perceptual Programming By Example (LPPBE)*. The problem is to induce a series of programs that require understanding perceptual data like images or text. LPPBE systems learn from weak supervision (input-output examples) and incrementally construct a shared library of components that grows and improves as more tasks are solved. Methodologically, we extend differentiable interpreters to operate on perceptual data and to share components across tasks. Empirically we show that this leads to a lifelong learning system that transfers knowledge to new tasks more effectively than baselines, and the performance on earlier tasks continues to improve even as the system learns on new, different tasks.

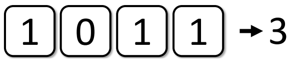
1 INTRODUCTION

A goal of artificial intelligence is to build a single large neural network model that can be trained in a *lifelong learning* setting; i.e., on a sequence of diverse tasks over a long period of time, and gain cumulative knowledge about different domains as it is presented with new tasks. The hope is that such systems will learn more accurately and from less data than existing systems, and that they will exhibit more flexible intelligence. However, despite some work showing promise towards multitask learning (training on many tasks at once) and transfer learning (using source tasks to improve learning in a later target task) (Caruana, 1997; Luong et al., 2015; Parisotto et al., 2015; Rusu et al., 2016), most successes of neural networks today come from training a single network on a single task, indicating that this goal is highly challenging to achieve.

We argue for two properties that such systems should have in addition to the ability to learn from a sequence of diverse tasks. First is the ability to learn from weak supervision. Gathering high-quality labeled datasets is expensive, and this effort is multiplied if all tasks require strong labelling. In this work, we focus on weak supervision in the form of pairs of input-output examples that come from executing simple programs with no labelling of intermediate states. Second is the ability to distill knowledge into subcomponents that can be shared across tasks. If we can learn models where the knowledge about shared subcomponents is disentangled from task-specific knowledge, then the sharing of knowledge across tasks will likely be more effective. Further, by isolating shared subcomponents, we expect that we could develop systems that exhibit reverse transfer (i.e., performance on earlier tasks automatically improves by improving the shared components in later tasks).

A key challenge in achieving these goals with neural models is the difficulty in interpreting weights inside a trained network. Most notably, with a purely neural model, subcomponents of knowledge gained after training on one task cannot be easily transferred to related tasks. Conversely, traditional computer programs naturally structure solutions to diverse problems in an interpretable, modular form allowing (1) re-use of subroutines in solutions to new tasks and (2) modification or error correction by humans. Inspired by this fact, we develop end-to-end trainable models that structure their solutions as a library of functions, some of which are represented as source code, and some of which are neural networks.

Methodologically, we start from recent work on programming by example (PBE) with differentiable interpreters, which shows that it is possible to use gradient descent to induce source code operating on basic data types (e.g. integers) from input-output examples (Gaunt et al., 2016; Riedel et al., 2016; Bunel et al., 2016). In this work we combine these differentiable interpreters with neural network classifiers in an end-to-end trainable system that learns programs that manipulate perceptual data.



```
T = 5; tape_length = 4; max_int = tape_length

@Runtime([max_int , 2], max_int)
def add(a, b): return (a + b) % max_int

@Runtime([tape_length], tape_length)
def inc(a): return (a + 1) % tape_length



tape = Input(2)[tape_length]
instr = Param(2)[T]
count = Var(max_int)[T + 1]
pos = Var(tape_length)[T + 1]


pos[0].set_to(0)
count[0].set_to(0)

for t in range(T):
    if instr[t] == 0:   # MOVE
        pos[t + 1] = inc(pos[t])
        count[t + 1].set_to(count[t])
    elif instr[t] == 1: # READ
        pos[t + 1].set_to(pos[t])
        with pos[t] as p:
            count[t + 1].set_to(
             add(count[t], tape[p]))

final_count = Output(max_int)
final_count.set_to(count[T - 1])
```

```
T = 5; tape_length = 4; max_int = tape_length

@Runtime([max_int , 2], max_int)
def add(a, b): return (a + b) % max_int

@Runtime([tape_length], tape_length)
def inc(p): return (p + 1) % tape_length

@Learn([Tensor(28,28)],2,hid_sizes=[256,256])
def is_dinosaur(image): pass

tape = InputTensor(28,28)[tape_length]
instr = Param(2)[T]
count = Var(max_int)[T + 1]
pos = Var(tape_length)[T + 1]
tmp = Var(2)[T + 1]

pos[0].set_to(0)
count[0].set_to(0)

for t in range(T):
    if instr[t] == 0:   # MOVE
        pos[t + 1] = inc(pos[t])
        count[t + 1].set_to(count[t])
    elif instr[t] == 1: # READ
        pos[t + 1].set_to(pos[t])
        with pos[t] as p:
            tmp[t].set_to(is_dinosaur(tape[p]))
            count[t + 1].set_to(
             add(count[t], tmp[p]))

final_count = Output(max_int)
final_count.set_to(count[T - 1])
```

Figure 1: (NEURAL) TERPRET programs for counting symbols on a tape, with input-output examples. Both programs describe an interpreter with instructions to MOVE on the tape and READ the tape according to source code parametrized by instr. (left) A TERPRET program that counts '1's. (right) A NEURAL TERPRET program that additionally learns a classifier is_dinosaur.

In addition, we make our interpreter modular, which allows *lifelong learning* on a sequence of related tasks: rather than inducing one fresh program per task, the system is able to incrementally build a library of (neural) functions that are shared across task-specific programs. To encapsulate the challenges embodied in this problem formulation, we name the problem *Lifelong Perceptual Programming By Example (LPPBE)*. Our extension of differentiable interpreters that allows perceptual data types, neural network function definitions, and lifelong learning is called NEURAL TERPRET *(NTPT)*.

Empirically, we show that a NTPT-based model learns to perform a sequence of tasks based on images of digits and mathematical operators. In early tasks, the model learns the concepts of digits and mathematical operators from a variety of weak supervision, then in a later task it learns to compute the results of variable-length mathematical expressions. The approach is resilient to catastrophic forgetting (McCloskey & Cohen, 1989; Ratcliff, 1990); on the contrary, results show that performance continues to improve on earlier tasks even when only training on later tasks. In total, the result is a method that can gather knowledge from a variety of weak supervision, distill it into a cumulative, re-usable library, and use the library within induced algorithms to exhibit strong generalization.

2 PERCEPTUAL PROGRAMMING BY EXAMPLE

We briefly review the TERPRET language (Gaunt et al., 2016) for constructing differentiable interpreters. To address LPPBE, we develop NEURAL TERPRET, an extension to support lifelong learning, perceptual data types, and neural network classifiers. We also define our tasks.

2.1 TERPRET

TERPRET programs describe differentiable interpreters by defining the relationship between Inputs and Outputs via a set of inferrable Params that define an executable program and Vars that store intermediate results. TERPRET requires all of these variables to be finite integers. To learn using gradient descent, the model is made differentiable by a compilation step that lifts the relationships

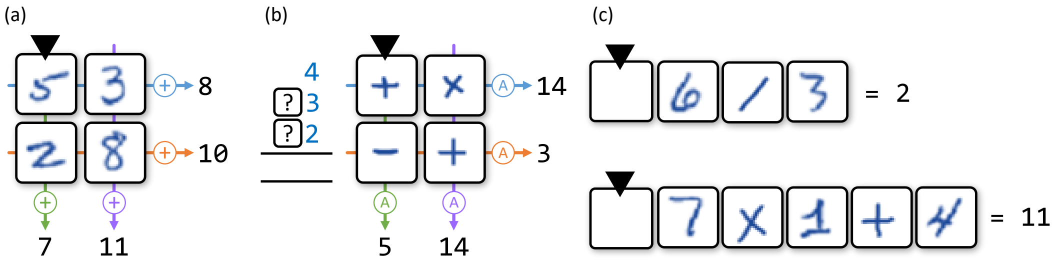


Figure 2: Overview of tasks in the (a) ADD2X2, (b) APPLY2X2 and (c) MATH scenarios. 'A' denotes the `APPLY` operator which replaces the ? tiles with the selected operators and executes the sum. We show two MATH examples of different length.

between integers specified by the TERPRET code to relationships between marginal distributions over integers in finite ranges. There are two key operations in this compilation process:

- **Function application**. The statement `z.set_to(foo(x, y))` is translated into $\mu_i^z = \sum_{jk} I_{ijk}\mu_j^x\mu_k^y$ where $\boldsymbol{\mu}^a$ represents the marginal distribution for the variable a and $\boldsymbol{I}$ is an indicator tensor $\mathbb{1}[i = \texttt{foo}(j,k)]$. This approach extends to all functions mapping any number of integer arguments to an integer output.
- **Conditional statements** The statements `if x == 0: z.set_to(a); elif x == 1: z.set_to(b)` are translated to $\boldsymbol{\mu}^z = \mu_0^x\boldsymbol{\mu}^a + \mu_1^x\boldsymbol{\mu}^b$. More complex statements follow a similar pattern, with details given in Gaunt et al. (2016).

This compilation process yields a TensorFlow (Abadi et al., 2016) computation graph containing many of these two operations, which can then be trained using standard methods.

2.2 NEURAL TERPRET

To handle perceptual data, we relax the restriction that all variables need to be finite integers. We introduce a new *tensor* type whose dimensions are fixed at declaration, and which is suitable to store perceptual data. Additionally, we introduce *learnable functions* that can process vector variables. A learnable function is declared using $\texttt{@Learn([}d_1,\ldots,d_D\texttt{],}\ d_{out}\texttt{,}\ \texttt{hid\_sizes=[}\ell_1,\ldots,\ell_L\texttt{])}$, where the first component specifies the dimensions $d_1,\ldots,d_D$ of the inputs (which can be finite integers or tensors) and the second the dimension of the output. NTPT compiles such functions into a fully-connected feed-forward neural network whose layout can be controlled by the `hid_sizes` component, which specifies the number of layers and neurons in each layer. The inputs of the function are simply concatenated. Vector output is generated by learning a mapping from the last hidden layer, and finite integer output is generated by a softmax layer producing a distribution over integers up to the declared bound. Learnable parameters for the generated network are shared across every use in the NTPT program, and as they naturally fit into the computation graph for the remaining TERPRET program, the whole system is trained end-to-end.

A simple TERPRET program counting bits on a tape, and a related NTPT program that counts up images of a particular class on a tape are displayed in Fig. 1.

2.3 TASKS

To demonstrate the benefits of our approach for combining neural networks with program-like architecture, we consider three toy scenarios consisting of several related tasks depicted in Fig. 2.

ADD2X2 scenario: The first scenario in Fig. 2(a) uses of a 2 × 2 grid of MNIST digits. We set 4 tasks based on this grid: compute the sum of the digits in the (1) top row, (2) left column, (3) bottom row, (4) right column. All tasks require classification of MNIST digits, but need different programs to compute the result. As training examples, we supply *only* a grid and the resulting sum. Thus, we *never* directly label an MNIST digit with its class.

APPLY2X2 scenario: The second scenario in Fig. 2(b) presents a 2 × 2 grid of of handwritten arithmetic operators. Providing three auxiliary random integers $\texttt{d}_1$, $\texttt{d}_2$, $\texttt{d}_3$, we again set 4 tasks

(a)

```
# initialization:
R0 = READ
# program:
R1 = MOVE_EAST
R2 = MOVE_SOUTH
R3 = SUM(R0, R1)
R4 = NOOP
return R3
```

(b)

```
# initialization:
R0 = InputInt[0]
R1 = InputInt[1]
R2 = InputInt[2]
R3 = READ
# program:
R4 = MOVE_EAST
R5 = MOVE_SOUTH
R6 = APPLY(R0, R1, R4)
R7 = APPLY(R6, R2, R5)
return R7
```

Figure 3: Example solutions for the tasks on the right columns of the (a) ADD2X2 and (b) APPLY2X2 scenarios. The read head is initialized `READ`ing the top left cell and any auxiliary `InputInt`s are loaded into memory. Instructions and arguments shown in black must be learned.

based on this grid, namely to evaluate the expression[1] $\texttt{d}_1$ $\texttt{op}_1$ $\texttt{d}_2$ $\texttt{op}_2$ $\texttt{d}_3$ where ($\texttt{op}_1$, $\texttt{op}_2$) are the operators represented in the (1) top row, (2) left column, (3) bottom row, (4) right column. In comparison to the first scenario, the dataset of operators is relatively small and consistent[2], making the perceptual task of classifying operators considerably easier. However, the algorithmic part is more difficult, requiring non-linear operations on the supplied integers.

MATH scenario: The final task in Fig. 2(c) requires combination of the knowledge gained from the weakly labeled data in the first two scenarios to execute a handwritten arithmetic expression.

3 MODELS

We design one NTPT model for each of the three scenarios outlined above. Knowledge transfer is achieved by defining a library of 2 neural networks shared across all tasks and scenarios. Training on each task should produce a task-specific source code solution (from scratch) and improve the overall usefulness of the shared networks. Below we outline the details of the specific models for each scenario along with baseline models.

3.1 SHARED COMPONENTS

We refer to the 2 networks in the shared library as `net_0` and `net_1`. Both networks have similar architectures: they take a 28×28 monochrome image as input and pass this sequentially through two fully connected layers each with 256 neurons and ReLU activations. The last hidden vector is passed through a fully connected layer and a softmax to produce a 10 dimensional output (`net_0`) or 4 dimensional output (`net_1`) to feed to the differentiable interpreter. Note that the output sizes are chosen to match the number of classes of MNIST digits and arithmetic operators respectively.

If we create an interpreter model which is allowed to make calls to N untrained networks, and part of the interpreter uses a parameter `net_choice = Param(N)` to deciding which network to apply, then the system effectively sees one large untrained network, which cannot usefully be split apart into the N components after training. To avoid this, we enforce that no more than one untrained network is introduced at a time (i.e. the first task has access to only `net_0`, and all other tasks have access to both nets). We find that this breaks the symmetry sufficiently to learn separate, useful classifiers.

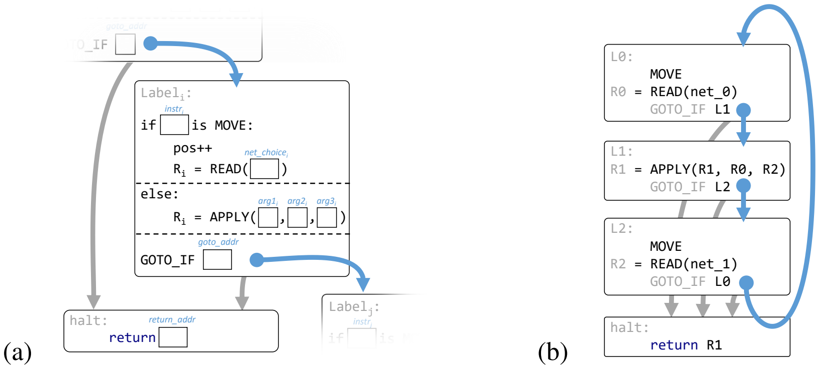
3.2 ADD2X2 MODEL

For the ADD2X2 scenario we build a model capable of writing short straight line algorithms with up to 4 instructions. The model consists of a read head containing `net_0` and `net_1` (with the exception of the very first task, which only has access to `net_0`, as discussed above) which are connected to a set of registers each capable of holding integers in the range $0, \ldots, M$, where $M = 18$. The head is initialized reading the top left cell of the 2×2 grid, and at each step in the program, one instruction can be executed from the following instruction set:

- `NOOP`: a trivial no-operation instruction

[1] Note that for simplicity, our toy system ignores operator precedence and executes operations from left to right - i.e. the sequence in the text is executed as $((\texttt{d}_1\ \texttt{op}_1\ \texttt{d}_2)\ \texttt{op}_2\ \texttt{d}_3)$.

[2] 200 handwritten examples of each operator were collected from a single author to produce a training set of 600 symbols and a test set of 200 symbols from which to construct random 2×2 grids.



```
GOTO_IF □

Label_i:
if □ is MOVE:
    pos++
    R_i = READ(□)
else:
    R_i = APPLY(□,□,□)
GOTO_IF □

halt:
    return □
```

```
L0:
     MOVE
R0 = READ(net_0)
     GOTO_IF L1
L1:
R1 = APPLY(R1, R0, R2)
     GOTO_IF L2
L2:
     MOVE
R2 = READ(net_1)
     GOTO_IF L0
halt:
     return R1
```

Figure 4: Overview of the MATH model. (a) The general form of a block in the model. Blue elements are learnable. (b) A loop-based solution to the task in the MATH scenario.

- MOVE_NORTH, MOVE_EAST, MOVE_SOUTH, MOVE_WEST: translate the head (if possible) and return the result of applying the neural network chosen by net_choice to the image in the new cell
- ADD(·,·): accepts two register addresses and returns the sum of their contents.

where the parameter net_choice is to be learned and decides which of net_0 and net_1 to apply.

To construct each line of code requires choosing an instruction and (in the case of SUM) addresses of arguments for that instruction. We follow Feser et al. (2016) and allow each line to store its result in a separate immutable register. Finally, we learn a parameter specifying which register to return after execution of the program. An example program in this model is shown in Fig. 3(a). Even this simple model permits $\sim 10^7$ syntactically distinct programs for the differentiable interpreter to search over.

3.3 APPLY2X2 MODEL

We adapt the ADD2X2 model to the APPLY2X2 scenario by initializing three immutable registers with the auxiliary integers supplied with each 2×2 operator grid [see Fig. 2(b)]. In addition, we swap the ADD(·,·) instruction for APPLY(·,·,·). The action of APPLY(a, b, op) is to interpret the integer stored at op as an arithmetic operator and to compute a op b. All operations are performed modulo $(M + 1)$ and division by zero returns M. In total, this model exposes a program space of size $\sim 10^{12}$ syntactically distinct programs.

3.4 MATH MODEL

We design the final scenario to investigate the synthesis of more complex control flow than straight line code. A natural solution to execute the expression on the tape is to build a loop with a body that alternates between moving the head and applying the operators [see Fig. 4(b)]. This loopy solution has the advantage that it generalizes to handle arbitrary length arithmetic expressions.

Fig. 4(a) shows the basic architecture of the interpreter used in this scenario. We provide a set of blocks each containing the instruction MOVE or APPLY. A MOVE instruction increments the position of the head and loads the new symbol into a block specific immutable register using either net_0 or net_1 as determined by a block specific net_choice. After executing the instruction, the interpreter executes a GOTO_IF statement which checks whether the head is over the end of the tape and if not then it passes control to the block specified by goto_addr, otherwise control passes to a halt block which returns a chosen register value and exits the program. This model describes a space of $\sim 10^6$ syntactically distinct programs.

4 BASELINES

NTPT aims to combine neural networks and differentiable interpreters for handling perceptual and algorithmic parts of a task respectively. A natural baseline is to replace the differentiable interpreter with a neural network to create a purely neural solution. In this spirit we define a *column* as the following architecture for handling the 2×2 tasks (see Fig. 5(a)):

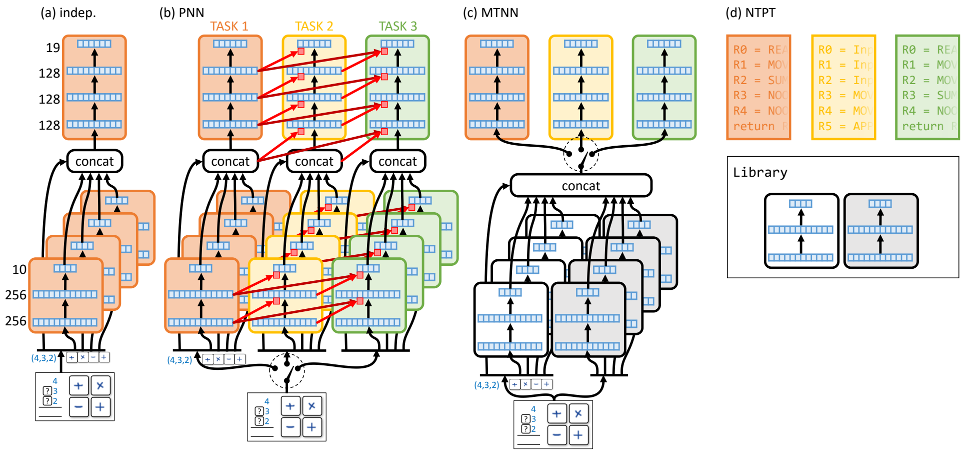


Figure 5: Cartoon illustration of all models used in the experiments. See text for details

- Each of the images in the 2 × 2 grid is passed through an embedding network with 2 layers of 256 neurons (c.f. `net_0/1`) to produce a 10-dimensional embedding. The weights of the embedding network are shared across all 4 images.
- These 4 embeddings are concatenated into a 40-dimensional vector and for the APPLY2X2 the auxiliary integers are represented as one-hot vectors and concatenated with this 40-dimensional vector.
- This is then passed through a network consisting of 3 hidden layers of 128 neurons to produce a 19-dimensional output

We construct 3 different neural baselines derived from this column architecture (see Fig. 5):

1. **Indep.**: Each task is handled by an independent column with no mechanism for transfer.
2. **Progressive Neural Network (PNN)**: We follow Rusu et al. (2016) and build lateral connections linking each task specific column to columns from tasks appearing earlier in the learning lifetime. Weights in all columns except the active task's column are frozen during a training update. Note that the number of layers in each column must be identical to allow lateral connections, meaning we cannot tune the architecture separately for each task.
3. **Multitask neural network (MTNN)**: We split the column into a shared perceptual part and a task specific part. The perceptual part consists of `net_0` and `net_1` embedding networks. In an ideal case the symmetry between these embedding networks will be broken and one will become specialized to handle handwritten digits while the other will handle handwritten operators. In order to encourage this symmetry breaking we zero out one of the networks when training on the first task (cf. the symmetry breaking technique mentioned in Sec. 3.1).

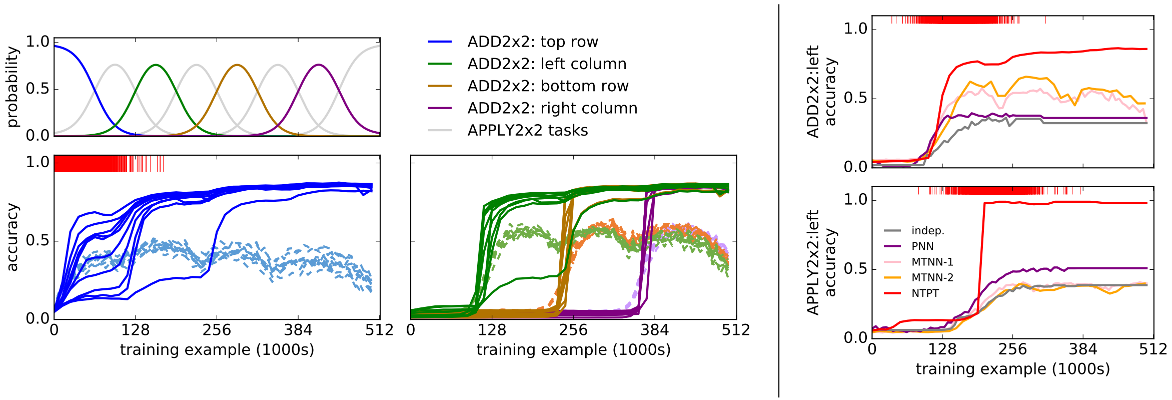
   The task-specific part consists of a neural network that maps the perceptual embeddings to a 19 dimensional output. Note that unlike PNNs, the precise architecture of the task specific part of the MTNN can be tuned for each individual task. We consider two MTNN architectures:

   (a) MTNN-1: All task-specific parts are 3 layer networks comparable to the PNN case.

   (b) MTNN-2: We manually tune the number of layers for each task and find best performance when the task specific part contains 1 hidden layer for the ADD2X2 tasks and 3 layers for the APPLY2X2 tasks.

For the MATH task, we build a purely neural baseline by replacing the task-specific part of the MTNN network with an LSTM. At each step, this network takes in the shared embeddings of the current symbol, updates an LSTM hidden state and then proceeds to the next symbol. We make a classification of the final answer using the last hidden states of the LSTM. We find that we achieve best performance with a 3 layer LSTM with 1024 elements in each hidden state and dropout between layers. In addition, we investigate a Neural GPU baseline based on Kaiser & Sutskever (2016)[3].

[3] We use the original authors' implementation available at `https://github.com/tensorflow/models/tree/master/neural_gpu`



(a) (b) (c)

Figure 6: Lifelong learning with NTPT. (a) top: the sequential learning schedule for all 8 tasks, bottom: performance of NTPT (solid) and the MTNN-2 baseline (dashed) on the first ADD2X2 task. (b) performance on the remaining ADD2X2 tasks. (c) Performance of all the baselines on the *:left tasks.

5 EXPERIMENTS

In this section we report results illustrating the key benefits of NTPT for the LPPBE problem in terms of knowledge transfer (Sec. 5.1) and generalization (Sec. 5.2).

First we create a data set in a regime which best demonstrates the LPPBE problem. The most convincing demonstration of LPPBE requires a series of tasks for which there is insufficient data to learn independent solutions to all tasks and instead, success requires transferring knowledge from one task to the next. Empirically, we find that training on any individual ADD2X2 task with only 1k distinct 2×2 examples produces low accuracies of around $40 \pm 20\%$ (measured on a held-out test set of 10k examples) for both the purely neural baselines and NTPT methods. Since none of our models can satisfactorily solve an ADD2X2 task independently in this regime, we work with this limited data set and argue that any success on these tasks during a lifetime of learning can be attributed to successful knowledge transfer. In addition, we check that in a data rich regime (e.g. $\geq$4k examples) all of the baseline models and NTPT can independently solve each task with $>$80% accuracy. This indicates that the models all have sufficient capacity to represent satisfactory solutions, and the challenge is to find these solutions during training.

5.1 LIFELONG LEARNING

To test knowledge transfer between tasks we train on batches of data drawn from a time-evolving probability distribution over all 8 tasks in the ADD2X2 and APPLY2X2 scenarios (see the top of Fig. 6(a)). During training, we observe the following key properties of the knowledge transfer achieved by NTPT:

Reverse transfer: Fig. 6(a) focuses on the performance of NTPT on the first task (ADD2X2:top). The red bars indicate times where the the system was presented with an example from this task. Note that even when we have stopped presenting examples, the performance on this task continues to increase as we train on later tasks - an example of *reverse transfer*. We verify that this is due to continuous improvement of net_0 in later tasks by observing that the accuracy on the ADD2X2:top task closely tracks measurements of the accuracy of net_0 directly on the digit classification task.

Avoidance of catastrophic forgetting: Fig. 6(b) shows the performance of the NTPT on the remaining ADD2X2 tasks. Both Fig. 6(a) and (b) include results for the MTNN-2 baseline (the best baseline for the ADD2X2 tasks). Note that whenever the dominant training task swaps from an ADD2X2 task to an APPLY2X2 task the baseline's performance on ADD2X2 tasks drops. This is because the shared perceptual network becomes corrupted by the change in task - an example of *catastrophic forgetting*. To try to limit

	task	indep	PNN	MTNN-1	MTNN-2	NTPT
ADD2X2	top	35%	35%	26%	24%	87%
	left	32%	36%	38%	47%	87%
	bottom	34%	33%	40%	56%	86%
	right	32%	35%	44%	60%	86%
APPLY2X2	top	38%	39%	40%	38%	98%
	left	39%	51%	41%	39%	100%
	bottom	39%	48%	41%	40%	100%
	right	39%	51%	42%	37%	100%

Figure 7: Final accuracies on all 2×2 tasks for all models at the end of lifelong learning

the extent of catastrophic forgetting and make the shared components more robust, we have a separate learning rate for the perceptual networks in both the MTNN baseline and NTPT which is 100 fold smaller than the learning rate for the task-specific parts. With this balance of learning rates we find empirically that NTPT does not display catastrophic forgetting.

Final performance: Fig. 6(b) focuses on the ADD2X2:left and ADD2X2:left tasks to illustrate the relative performance of the baselines described in Sec. 4. Note that although PNNs avoid catastrophic forgetting, there is no clear overall winner between the MTNN and PNN baselines. NTPT learns faster and to a higher accuracy than all baselines for all the tasks considered here. For clarity we only plot results for the *:left tasks: the other tasks show similar behavior and the accuracies for all tasks at the end of the lifetime of learning are presented in Fig. 7.

5.2 GENERALIZATION

In the final experiment we take net_0/1 from the end of the NTPT 2 × 2 training and start training on the MATH scenario. For the NTPT model we train on arithmetic expressions containing only 2 digits. The loopy structure of the MATH model introduces many local optima into the optimization landscape and only 2/100 random restarts converge on a correct program. We detect convergence to the correct program by a rapid increase in the accuracy on a validation set (typically occurring after around 30k training examples). Once the correct program is found, continuing to train the model model mainly leads to further improvement in the accuracy of net_0, which saturates at 97.5% on the digit classification task. The learned source code generalizes perfectly to expressions containing any number of digits, and the only limitation on the performance on long expressions comes from the repeated application of the imperfect net_0.

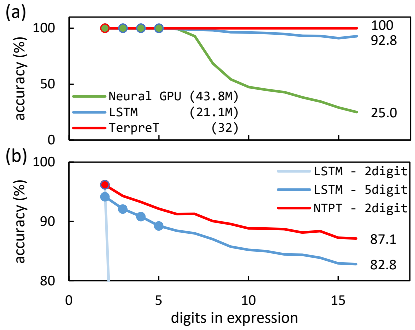


Figure 8: Generalization behavior on MATH expressions. Solid dots indicate expression lengths used in training. We show results on (a) a simpler non-perceptual MATH task (numbers in parentheses indicate parameter count in each model) and (b) the MATH task including perception.

To pick a strong baseline for the MATH problem, we first perform a preliminary experiment with two simplifications from the case above: (1) rather than expecting strong generalization from just 2-digit training examples, we train candidate baselines with supervision on examples up to 5 digits in length, and (2) we remove the perceptual component of the task, presenting the digits and operators as one-hot vectors rather than images. Fig. 8(a) shows the generalization performance of the LSTM and Neural GPU (512-filter) baselines in this simpler setting after training to convergence[4]. Based on these results, we restrict attention to the LSTM baseline and return to the full task including the perceptual component. In the full MATH task, we initialize the embedding networks of each model using net_0/1 from the end of the NTPT 2 × 2 training. Fig. 8(b) shows generalization of the NTPT and LSTM models on expressions of up to 16 digits after training to convergence. We find that even though the LSTM shows surprisingly effective generalization when supplied supervision up to 5 digits, NTPT trained on only 2-digit expressions still offers better results.

6 RELATED WORK

Lifelong Machine Learning. We operate in the paradigm of Lifelong Machine Learning (LML) (Thrun, 1994; 1995; Thrun & O'Sullivan, 1996; Silver et al., 2013; Chen et al., 2015), where a learner is presented a sequence of different tasks and the aim is to retain and re-use knowledge from earlier tasks to more efficiently and effectively learn new tasks. This is distinct from related paradigms of multitask learning (presentation of a finite set of tasks simultaneously rather than in sequence (Caruana, 1997; Kumar & Daume III, 2012; Luong et al., 2015; Rusu et al., 2016)), transfer learning (transfer of knowledge from a source to target domain without notion of knowledge retention (Pan & Yang, 2010)), and curriculum learning (training a single model for a single task of varying difficulty (Bengio et al., 2009)).

[4] Note that Price et al. (2016) find similarly poor generalization performance for a Neural GPU applied to the similar task of evaluating arithmetic expressions involving binary numbers.

The challenge for LML with neural networks is the problem of catastrophic forgetting: if the distribution of examples changes during training, then neural networks are prone to forget knowledge gathered from early examples. Solutions to this problem involve instantiating a knowledge repository (KR) either directly storing data from earlier tasks or storing (sub)networks trained on the earlier tasks with their weights frozen. This knowledge base allows either (1) rehearsal on historical examples (Robins, 1995), (2) rehearsal on virtual examples generated by the frozen networks (Silver & Mercer, 2002; Silver & Poirier, 2006) or (3) creation of new networks containing frozen sub networks from the historical tasks (Rusu et al., 2016; Shultz & Rivest, 2001)

To frame our approach in these terms, our KR contains partially-trained neural network classifiers which we call from learned source code. Crucially, we never freeze the weights of the networks in the KR: all parts of the KR can be updated during the training of all tasks - this allows us to improve performance on earlier tasks by continuing training on later tasks (so-called reverse transfer). Reverse transfer has been demonstrated previously in systems which assume that each task can be solved by a model parametrized by an (uninterpretable) task-specific linear combination of shared basis weights (Ruvolo & Eaton, 2013). The representation of task-specific knowledge as source code, learning from weak supervision, and shared knowledge as a deep neural networks distinguishes this work from the linear model used in Ruvolo & Eaton (2013).

Neural Networks Learning Algorithms. Recently, extensions of neural networks with primitives such as memory and discrete computation units have been studied to learn algorithms from input-output data (Graves et al., 2014; Weston et al., 2014; Joulin & Mikolov, 2015; Grefenstette et al., 2015; Kurach et al., 2015; Kaiser & Sutskever, 2016; Reed & de Freitas, 2016; Bunel et al., 2016; Andrychowicz & Kurach, 2016; Zaremba et al., 2016; Graves et al., 2016; Riedel et al., 2016; Gaunt et al., 2016; Feser et al., 2016). Whereas many of these works use a neural network controller managing a differentiable computer architecture, we flip this relationship. In our approach, a differentiable interpreter is the controller which makes calls to a library of neural network components.

The methods above, with the exception of Reed & de Freitas (2016) and Graves et al. (2016), operate on inputs of (arrays of) integers. However, Reed & de Freitas (2016) requires extremely strong supervision, where the learner is shown all intermediate steps to solving a problem; our learner only observes input-output examples. Reed & de Freitas (2016) also show the performance of their system in a multitask setting. In some cases, additional tasks harm performance of their model and they freeze parts of their model when adding to their library of functions. Only Bunel et al. (2016), Riedel et al. (2016) and Gaunt et al. (2016) aim to consume and produce source code that can be provided by a human (e.g. as sketch of a solution) to or returned to a human (to potentially provide feedback).

7 Discussion

We have presented Neural TerpreT, a framework for building end-to-end trainable models that structure their solution as a library of functions represented as source code or neural networks. Experimental results show that these models can successfully be trained in a lifelong learning context, and they are resistant to catastrophic forgetting; in fact, they show that even after instances of earlier tasks are no longer presented to the model, performance still continues to improve.

Learning neural network models within differentiable interpreters has several benefits. First, learning programs imposes a bias that favors learning models that exhibit strong generalization, as illustrated by many works on program-like neural networks. Second, the source code components are interpretable by humans, allowing incorporation of domain knowledge describing the shape of the problem through the source code structure. Third, source code components can be inspected, and the neural network components can be queried with specific instances to inspect whether the shared classifiers have learned the expected mappings. A final benefit is that the differentiable interpreter can be seen as *focusing the supervision*. If a component is un-needed for a given task, then the differentiable interpreter can choose not to use the component, which shuts off any gradients from flowing to the component. We speculate that this could be a reason for the models being resistant to catastrophic forgetting, as the model either chooses to use a classifier, or ignores it (which leaves the component unchanged).

It is known that differentiable interpreters are difficult to train (Kurach et al., 2015; Neelakantan et al., 2016; Gaunt et al., 2016), and being dependent on differentiable interpreters is the primary limitation of this work. However, if progress can be made on more robust training of differentiable interpreters (perhaps extending ideas in Neelakantan et al. (2016); Feser et al. (2016)), then we believe there to be great promise in using the models we have presented here to build large lifelong neural networks.

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