Time Stepping with Space and Time Errors and Stability of The Material Point Method

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ABSTRACT

The challenge of choosing the best time step for the Material Point Method (MPM) is often addressed by using a simple stability criterion, such as the speed of sound. While in many instances this works well it is important to understand how this relates to the overall errors present in the method. This is particularly true as the spatial error appears to dominate the temporal one and makes the use of high-order time stepping methods challenging [3].

Recently there have been several advances in understanding the stability of MPM. These range from Non-linear stability analysis, [1] through to Von Neumann type approaches [3]. While it is has been long observed that the spatial errors dominate [4] at the same time recent work has made more precise the forms of the different MPM errors [5].

Both these advances now make it possible to understand how the different errors and the stability analysis are connected. At the same time this also requires simple computable estimates of the different errors in the material point method.

Local estimation of the temporal errors is relatively straightforward but theses errors may not be the most significant ones.

Of the other errors perhaps the most significant are those in mapping from particles to nodes to calculate forces and the differentiation process needed to calculate spatial derivatives at particles. By using simple estimates of these errors an attempt will be made to reconcile the errors introduced with the stability criteria used. A number of simple computational experiments will be used to illustrate the theoretical results.

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