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# Rethinking Mixture-of-Agents: Is Mixing Different Large Language Models Beneficial?

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Anonymous Author(s)

Affiliation

Address

email

## Abstract

1       Ensembling outputs from diverse sources is a straightforward yet effective approach  
2       to boost performance. Mixture-of-Agents (MoA) is one such popular ensemble  
3       method that aggregates outputs from multiple *different* Large Language Models  
4       (LLMs). This paper raises the question in the context of language models: is mixing  
5       different LLMs truly beneficial? We propose Self-MoA — an ensemble method  
6       that aggregates outputs from only the *single* top-performing LLM. Our extensive  
7       experiments reveal that, surprisingly, Self-MoA outperforms standard MoA that  
8       mixes different LLMs in a large number of scenarios: Self-MoA achieves 6.6%  
9       improvement over MoA on the AlpacaEval 2.0 benchmark, and an average of 3.8%  
10      improvement across various benchmarks, including MMLU, CRUX, and MATH.  
11      Applying Self-MoA to one of the top-ranking models in AlpacaEval 2.0 directly  
12      achieves the new state-of-the-art performance ranking 1<sup>st</sup> on the leaderboard. To  
13      understand the effectiveness of Self-MoA, we systematically investigate the trade-  
14      off between diversity and quality of outputs under various MoA settings. We  
15      confirm that the MoA performance is rather sensitive to the quality, and mixing  
16      different LLMs often lowers the average quality of the models. To complement  
17      the study, we identify the scenarios where mixing different LLMs could be helpful.  
18      This paper further introduces a sequential version of self-MoA, that is capable of  
19      aggregating a large number of LLM outputs on-the-fly over multiple rounds, and is  
20      as effective as aggregating all outputs at once.

## 21   1 Introduction

22   Large language models, like GPT [Achiam et al., 2023], Gemini [Team et al., 2023], and Claude [An-  
23   thropic, 2023], have significantly advanced performance across various domains. Efforts have focused  
24   on increasing model size and training data to enhance capabilities, but this approach incurs high costs.  
25   Meanwhile, scaling computation during inference remains relatively underexplored.

26   A straightforward way to leverage test-time computation is through ensembling, which combines  
27   outputs from multiple LLMs [Wang et al., 2024a, Lin et al., 2024, Jiang et al., 2023a]. One promising  
28   approach is Mixture-of-Agents (MoA)[Wang et al., 2024a], which has shown strong performance in  
29   tasks like instruction following, summarization, data extraction[OpenPipe, 2024], and resolving real-  
30   world code issues [Zhang et al., 2024b]. MoA works by first querying several LLMs (proposers) to  
31   generate responses, which are then synthesized into a high-quality response by an LLM (aggregator).

32   Previous research highlights the significance of model diversity within the proposers for optimizing  
33   the performance of MoA, primarily focusing on strategies for ensembling a diverse set of individual  
34   models. We consider **cross-model diversity** as the variation among different models. However,  
35   pursuing cross-model diversity may inadvertently include low-quality models, resulting in a quality-  
36   diversity trade-off. While previous studies mainly concentrate on achieving a high cross-model

37 diversity [Wang et al., 2024a, Zhang et al., 2024b], we adopt a holistic perspective on model diversity  
38 by considering **in-model diversity**, which arises from the variability of multiple responses generated  
39 by the same model. In-model diversity enables us to aggregate multiple outputs from an individual  
40 model. Intuitively, leveraging outputs from the best-performing individual model can more effectively  
41 navigate the quality-diversity trade-off by creating a higher-quality proposer mixture. Thus, we  
42 propose Self-MoA as depicted in Figure 2b, which utilizes the same prompting template as MoA  
43 but aggregates outputs that are repeatedly sampled from the same model, rather than from a set of  
44 different models. To distinguish, we use Mixed-MoA to refer to MoA configurations that combine  
45 different individual models when necessary.

46 Surprisingly, we find that Mixed-MoA is usually sub-optimal compared with Self-MoA, especially  
47 when there exist significant quality differences among the proposers. Specifically, we revisit the  
48 same experiment setting of MoA with six open-source instruction fine-tuned models as Wang et al.  
49 [2024a]. Compared with Mixed-MoA which aggregates all six models, Self-MoA on the strongest  
50 model surpasses its mixed counterpart with merely half of the forward passes on the AlpacaEval 2.0  
51 benchmark, showing a case of when intra-model diversity is more effective. Moreover, Self-MoA  
52 combined with two best-performed models on AlpacaEval 2.0 consistently achieves a 2-3 point gain  
53 and secures the top position on the leaderboard, which further confirms the effectiveness of Self-MoA  
54 in this evaluation task.

55 To explore the limits of model diversity for MoA, we extend our experiments to a setting with  
56 three specialized models, each excelling in a specific task. Specifically, we utilize Qwen2-7B-  
57 Instruct [Bai et al., 2023] for common sense QA (MMLU-redux [Gema et al., 2024]), Qwen2-Math-  
58 7B-Instruct [Bai et al., 2023] for mathematics (MATH [Hendrycks et al., 2020]), and DeepSeek-  
59 Coder-V2-Lite-Instruct for coding (CRUX [Gu et al., 2024]). We compare Self-MoA against a range  
60 of Mixed-MoA strategies, evaluating 13 combinations of individual models based on their average  
61 performance across the three tasks. Our findings indicate that, even in this promising scenario for  
62 Mixed-MoA where each individual model excels in a specific subtask, only two Mixed-MoA strategies  
63 slightly outperform Self-MoA by 0.17% and 0.35%. Furthermore, if we have prior knowledge of the  
64 tasks and employ task-specific models as proposers for Self-MoA such as DeepSeek-Coder-V2-Lite-  
65 Instruct on CRUX or Qwen2-Math-7B-Instruct on MATH, Self-MoA can significantly outperform  
66 the best Mixed-MoA.

67 To better understand Self-MoA’s effectiveness, we conducted a comprehensive analysis of the quality-  
68 diversity trade-off in MoA through over 200 experiments. We used the Vendi Score [Dan Friedman  
69 and Dieng, 2023] to assess diversity among proposers’ outputs and measured quality by their average  
70 performance. In Section 3, we confirm that MoA performance has a positive correlation with both  
71 quality and diversity. Additionally, we reveal a clear trade-off along the Pareto front between these two  
72 factors. Notably, we find that MoA is highly sensitive to quality variations, with optimal performance  
73 typically occurring in regions with high quality and relatively low diversity. This explains Self-MoA’s  
74 effectiveness, as it leverages the strongest model, ensuring consistently high-quality outputs.

75 Finally, we assess Self-MoA’s performance under increasing computational budgets. As the number  
76 of outputs increases, its scalability is limited by the aggregator’s context length. To overcome this,  
77 we introduce Self-MoA-Seq (Figure 2c), a sequential version that processes outputs with a sliding  
78 window, enabling it to handle any number of model outputs. Our results show that Self-MoA-Seq  
79 performs at least as well as Self-MoA, allowing scalable ensembling for LLMs with shorter context  
80 lengths without sacrificing performance.

81 Overall, our contributions are three-fold:

- 82 • We propose Self-MoA, which leverages in-model diversity by synthesizing multiple outputs  
83 from the same model. Surprisingly, it outperforms existing Mixed-MoA approaches that  
84 focus on cross-model diversity across a variety of benchmarks.
- 85 • Through systematic experiments and statistical analysis, we uncover a core trade-off between  
86 diversity and quality among the proposers, emphasizing that MoA is highly sensitive to  
87 proposer quality. This finding also explains the success of Self-MoA, which leverages  
88 outputs from the highest-performing model, ensuring superior overall quality.
- 89 • We extend Self-MoA to its sequential version Self-MoA-Seq, which iteratively aggregates a  
90 small amount of outputs step by step. Self-MoA-Seq unlocks LLMs that are constrained by  
91 the context length and enables computation scaling during inference.

Table 1: Comparison of Self-MoA and Mixed-MoA on AlpacaEval 2.0 leaderboard. We use Qwen1.5-110B-Chat as the aggregator.

|            | Model Configuration              | LC Win Rate | # Forward Passes |
|------------|----------------------------------|-------------|------------------|
| Individual | WizardLM-2-8x22B                 | 53.1        | 1                |
|            | Qwen1.5-110B-Chat                | 43.9        | 1                |
|            | LLaMA-3-70B-Instruct             | 34.4        | 1                |
|            | Qwen1.5-72B-Chat                 | 36.6        | 1                |
|            | Mixtral-8x22B-Instruct-v0.1      | 30.2        | 1                |
|            | dbrx-instruct                    | 25.4        | 1                |
| Mixed-MoA  | MoA-Lite [Wang et al., 2024a]    | 59.1        | 7                |
|            | 3-Layer MoA [Wang et al., 2024a] | 65.4        | 13               |
| Self-MoA   | Self-MoA + WizardLM-2-8x22B      | <b>65.7</b> | 7                |

## 92 2 Is Ensembling Different LLMs Beneficial?

93 As introduced in Section 1, previous research primarily emphasizes **cross-model diversity**, which  
 94 can inadvertently include low-quality proposers. In this work, we introduce Self-MoA (Figure 2),  
 95 which uses a single top-performing model to generate multiple outputs and aggregate them to produce  
 96 the final result. Self-MoA leverages **in-model diversity** as repeated sampling often produces varied  
 97 outputs. We propose our research question as follows:

98 *Does the benefit of MoA stem from cross-model diversity?*  
 99 *Can we build a stronger MoA by utilizing in-model diversity?*

100 We adopt the same experiment setting as Wang et al. [2024a] in AlpacaEval 2.0 benchmark (Ap-  
 101 pendix B.2) and compare the performance of MoA and Self-MoA<sup>1</sup>. Following Wang et al. [2024a],  
 102 we construct MoA based on six individual models: Qwen1.5-110B-Chat [Bai et al., 2023], Qwen1.5-  
 103 72B-Chat [Bai et al., 2023], WizardLM-8x22B [Xu et al., 2023], LLaMA-3-70B-Instruct [Touvron  
 104 et al., 2023], Mixtral-8x22B-Instruct-v0.1 [Jiang et al., 2024a], and dbrx-instruct [Team et al., 2024b].  
 105 Each model is sampled with a temperature of 0.7, following the default in [Wang et al., 2024a].  
 106 For Self-MoA, we aggregate six outputs sampled from WizardLM-2-8x22B, as it consistently out-  
 107 performs the other models. In line with Wang et al. [2024a], we use Qwen1.5-110B-Chat as the  
 108 aggregator for both MoA and Self-MoA.

109 We present the LC win rate for each model configuration in Table 1. For individual models, we  
 110 report the higher value between the leaderboard results and our reproduction. Additionally, we  
 111 include the total number of forward passes, where one forward pass is counted each time a proposer  
 112 model generates an output or an aggregator synthesizes a result. Notably, Self-MoA demonstrates  
 113 remarkable effectiveness in this task, outperforming the strongest MoA baseline with only half the  
 114 forward passes. This suggests that, while using multiple models intuitively offers greater diversity,  
 115 ensembling multiple outputs from a single model is more effective.

116 To further validate the effectiveness of Self-MoA, we apply it to the two top-performing models  
 117 on AlpacaEval 2.0, and find Self-MoA consistently achieves a 2-3 point gain and secures the top  
 118 position on the leaderboard during submission. We also extend experiments to more diverse tasks  
 119 and specialized models, observing promising results of aggregating outputs from only the single  
 120 top-performing LLM. More details are deferred to Appendix C.1 and Appendix C.2.

## 121 3 The Quality-Diversity Trade-off

122 We investigate factors that contribute to the strong performance of Self-MoA through careful ex-  
 123 periments. Previous studies have mainly focused on increasing model diversity within the group

<sup>1</sup>We note that this experiment is similar to the “single-proposer” setting in Wang et al. [2024a], however our reproduced result is different. We conjecture that such a major difference is due to different choices of the proposer model, which is not mentioned in Wang et al. [2024a]. As we shall see later in Section 3, ensembling performance is more sensitive to quality rather than diversity. Therefore, a worse proposer model will lead to suboptimal performance of Self-MoA.

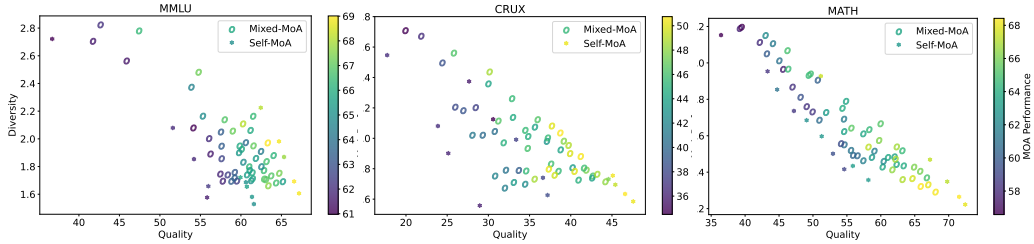


Figure 1: The diversity-quality trade-off: Mixed-MoA incorporates different individual models as proposers, while Self-MoA uses the same individual model for this role. Quality is assessed based on the average performance of each proposer, and diversity is computed with the Vendi Score [Dan Friedman and Dieng, 2023] of outputs generated by proposers on the same prompts. A zoomed version is provided in Appendix D.

124 (cross-model diversity) [Wang et al., 2024a, Jiang et al., 2023a, Zhang et al., 2024b]. However,  
 125 searching for diverse models can sometimes lead to including poorly performed models, resulting  
 126 in a trade-off between diversity and quality, where quality refers to how well each individual model  
 127 performs in the group.

128 Therefore, we aim to identify the existence of a general relationship between MoA’s performance  
 129 and quality as well as diversity. Following Section 2, we evaluate MoA’s performance on MMLU,  
 130 CRUX, and MATH, which cover tasks requiring a wide range of capabilities. We vary the quality  
 131 and diversity with two orders of freedom: 1) combinations of individual models in proposers from  
 132 Section C.2; and 2) sampling temperature. i.e., 0.5, 0.7, 1.0, 1.1, and 1.2. This results in a total of  
 133 over 70 unique MoA proposer mixtures. We measure the quality and diversity with Vendi Score  
 134 (Appendix B.4) and average accuracy.

135 **Results.** We plot MoA’s performance with corresponding diversity and quality for each mixture of  
 136 proposers in Figure 1. We summarize key observations as follows:

- 137 • The trends among MMLU, CRUX, and MATH are consistently aligned.
- 138 • When the quality is fixed, increasing diversity can enhance MoA’s performance.
- 139 • When the diversity is fixed, improving quality can also boost MoA’s performance.
- 140 • There exists a trade-off in the achievable Pareto front between diversity and quality.
- 141 • Notably, the best performance of MoA is typically observed in the bottom right of each  
 142 subplot, indicating a strong sensitivity to quality.

143 Previous work on ensembles [Wang et al., 2024a, Jiang et al., 2023a, Zhang et al., 2024b] primarily  
 144 focuses on increasing the diversity of models within the proposer mixture. However, as shown in  
 145 Figure 1, compared to Self-MoA on the best-performing model, simply aiming for greater diversity  
 146 in the proposer mixture can result in lower overall quality, which may negatively impact MoA’s  
 147 performance. This trade-off between diversity and quality helps to explain why Self-MoA achieves  
 148 superior performance across various benchmarks.

149 With statistical analysis conducted in Appendix C.3, we further confirm the positive correlation  
 150 between MoA performance and both quality and diversity, while prioritizing quality over diversity.

## 151 4 Conclusion

152 In this paper, we introduce Self-MoA, an innovative approach that utilizes in-model diversity to en-  
 153 hance the performance of large language models during inference. Our experiments demonstrate that  
 154 Self-MoA outperforms traditional Mixed-MoA strategies in many popular benchmarks, particularly  
 155 when the proposer model quality varies. By aggregating outputs from a single high-performing model,  
 156 Self-MoA effectively addresses the quality-diversity trade-off. We further identify the scenarios where  
 157 mixing LLM can be potentially beneficial (deferred to Appendix C.4) and extend Self-MoA to the  
 158 constrained context length setting (deferred to Appendix C.5). These findings highlight the potential  
 159 of in-model diversity in optimizing LLM performance and pave the way for further advancements in  
 160 ensemble methods.

161 **References**

162 J. Achiam, S. Adler, S. Agarwal, L. Ahmad, I. Akkaya, F. L. Aleman, D. Almeida, J. Altenschmidt,  
163 S. Altman, S. Anadkat, et al. Gpt-4 technical report. *arXiv preprint arXiv:2303.08774*, 2023.

164 A. Anthropic. Introducing claude, 2023.

165 H. J. Arnold. Introduction to the practice of statistics. *Technometrics*, 32:347–348, 1990. URL  
166 <https://api.semanticscholar.org/CorpusID:122891525>.

167 J. Bai, S. Bai, Y. Chu, Z. Cui, K. Dang, X. Deng, Y. Fan, W. Ge, Y. Han, F. Huang, et al. Qwen  
168 technical report. *arXiv preprint arXiv:2309.16609*, 2023.

169 B. Brown, J. Juravsky, R. Ehrlich, R. Clark, Q. V. Le, C. Ré, and A. Mirhoseini. Large language  
170 monkeys: Scaling inference compute with repeated sampling. *arXiv preprint arXiv:2407.21787*,  
171 2024.

172 J. C.-Y. Chen, S. Saha, and M. Bansal. Reconcile: Round-table conference improves reasoning via  
173 consensus among diverse llms. *arXiv preprint arXiv:2309.13007*, 2023a.

174 M. Chen, J. Tworek, H. Jun, Q. Yuan, H. P. D. O. Pinto, J. Kaplan, H. Edwards, Y. Burda,  
175 N. Joseph, G. Brockman, et al. Evaluating large language models trained on code. *arXiv preprint*  
176 *arXiv:2107.03374*, 2021.

177 S. Chen, L. Zeng, A. Raghunathan, F. Huang, and T. C. Kim. Moa is all you need: Building llm  
178 research team using mixture of agents. *arXiv preprint arXiv:2409.07487*, 2024.

179 X. Chen, R. Aksitov, U. Alon, J. Ren, K. Xiao, P. Yin, S. Prakash, C. Sutton, X. Wang, and D. Zhou.  
180 Universal self-consistency for large language model generation. *arXiv preprint arXiv:2311.17311*,  
181 2023b.

182 D. Dan Friedman and A. B. Dieng. The vendi score: A diversity evaluation metric for machine  
183 learning. *Transactions on machine learning research*, 2023.

184 Y. Du, S. Li, A. Torralba, J. B. Tenenbaum, and I. Mordatch. Improving factuality and reasoning in  
185 language models through multiagent debate. *arXiv preprint arXiv:2305.14325*, 2023.

186 Y. Dubois, B. Galambosi, P. Liang, and T. B. Hashimoto. Length-controlled alpacaeval: A simple  
187 way to debias automatic evaluators. *arXiv preprint arXiv:2404.04475*, 2024.

188 A. P. Gema, J. O. J. Leang, G. Hong, A. Devoto, A. C. M. Mancino, R. Saxena, X. He, Y. Zhao,  
189 X. Du, M. R. G. Madani, et al. Are we done with mmlu? *arXiv preprint arXiv:2406.04127*, 2024.

190 A. Gu, B. Rozière, H. Leather, A. Solar-Lezama, G. Synnaeve, and S. I. Wang. Cruxeval: A  
191 benchmark for code reasoning, understanding and execution. *arXiv preprint arXiv:2401.03065*,  
192 2024.

193 L. Gui, C. Gârbasea, and V. Veitch. Bonbon alignment for large language models and the sweetness  
194 of best-of-n sampling. *arXiv preprint arXiv:2406.00832*, 2024.

195 D. Hendrycks, C. Burns, S. Basart, A. Zou, M. Mazeika, D. Song, and J. Steinhardt. Measuring  
196 massive multitask language understanding. *arXiv preprint arXiv:2009.03300*, 2020.

197 D. Hendrycks, C. Burns, S. Kadavath, A. Arora, S. Basart, E. Tang, D. Song, and J. Steinhardt.  
198 Measuring mathematical problem solving with the math dataset. *arXiv preprint arXiv:2103.03874*,  
199 2021.

200 A. Q. Jiang, A. Sablayrolles, A. Roux, A. Mensch, B. Savary, C. Bamford, D. S. Chaplot, D. d. l.  
201 Casas, E. B. Hanna, F. Bressand, et al. Mixtral of experts. *arXiv preprint arXiv:2401.04088*,  
202 2024a.

203 A. Q. Jiang, A. Sablayrolles, A. Roux, A. Mensch, B. Savary, C. Bamford, D. S. Chaplot, D. de las  
204 Casas, E. B. Hanna, F. Bressand, G. Lengyel, G. Bour, G. Lample, L. R. Lavaud, L. Saulnier, M.-A.  
205 Lachaux, P. Stock, S. Subramanian, S. Yang, S. Antoniak, T. L. Scao, T. Gervet, T. Lavril, T. Wang,  
206 T. Lacroix, and W. E. Sayed. Mixtral of experts, 2024b. URL [https://arxiv.org/abs/2401.](https://arxiv.org/abs/2401.04088)  
207 04088.

- 208 D. Jiang, X. Ren, and B. Y. Lin. Llm-blender: Ensembling large language models with pairwise  
209 ranking and generative fusion. *arXiv preprint arXiv:2306.02561*, 2023a.
- 210 D. Jiang, X. Ren, and B. Y. Lin. Llm-blender: Ensembling large language models with pairwise  
211 ranking and generative fusion, 2023b. URL <https://arxiv.org/abs/2306.02561>.
- 212 J. Li, Q. Zhang, Y. Yu, Q. Fu, and D. Ye. More agents is all you need, 2024. URL <https://arxiv.org/abs/2402.05120>.  
213
- 214 Y. Li, D. Choi, J. Chung, N. Kushman, J. Schrittwieser, R. Leblond, T. Eccles, J. Keeling, F. Gimeno,  
215 A. Dal Lago, et al. Competition-level code generation with alphacode. *Science*, 378(6624):  
216 1092–1097, 2022.
- 217 T. Liang, Z. He, W. Jiao, X. Wang, Y. Wang, R. Wang, Y. Yang, Z. Tu, and S. Shi. Encourag-  
218 ing divergent thinking in large language models through multi-agent debate. *arXiv preprint*  
219 *arXiv:2305.19118*, 2023.
- 220 Y. Lin, H. Lin, W. Xiong, S. Diao, J. Liu, J. Zhang, R. Pan, H. Wang, W. Hu, H. Zhang, H. Dong,  
221 R. Pi, H. Zhao, N. Jiang, H. Ji, Y. Yao, and T. Zhang. Mitigating the alignment tax of rlhf, 2024.  
222 URL <https://arxiv.org/abs/2309.06256>.
- 223 K. Lu, H. Yuan, R. Lin, J. Lin, Z. Yuan, C. Zhou, and J. Zhou. Routing to the expert: Efficient reward-  
224 guided ensemble of large language models, 2023. URL <https://arxiv.org/abs/2311.08692>.
- 225 A. Madaan, N. Tandon, P. Gupta, S. Hallinan, L. Gao, S. Wiegrefe, U. Alon, N. Dziri, S. Prabhunoye,  
226 Y. Yang, et al. Self-refine: Iterative refinement with self-feedback. *Advances in Neural Information*  
227 *Processing Systems*, 36, 2024.
- 228 Y. Meng, M. Xia, and D. Chen. SimPO: Simple preference optimization with a reference-free reward.  
229 *arXiv preprint arXiv:2405.14734*, 2024.
- 230 OpenPipe. Openpipe mixture of agents: Outperform gpt-4 at 1/25th the cost, 2024. URL <https://openpipe.ai/blog/mixture-of-agents>.  
231
- 232 A. Ramé, J. Ferret, N. Vieillard, R. Dadashi, L. Hussenot, P.-L. Cedo, P. G. Sessa, S. Girgin,  
233 A. Douillard, and O. Bachem. Warp: On the benefits of weight averaged rewarded policies, 2024.  
234 URL <https://arxiv.org/abs/2406.16768>.
- 235 B. Roziere, J. Gehring, F. Gloeckle, S. Sootla, I. Gat, X. E. Tan, Y. Adi, J. Liu, R. Sauvestre, T. Remez,  
236 et al. Code llama: Open foundation models for code. *arXiv preprint arXiv:2308.12950*, 2023.
- 237 K. Sarjana, L. Hayati, and W. Wahidaturrahmi. Mathematical modelling and verbal abilities: How  
238 they determine students’ ability to solve mathematical word problems? *Beta: Jurnal Tadris*  
239 *Matematika*, 13(2):117–129, 2020.
- 240 C. Snell, J. Lee, K. Xu, and A. Kumar. Scaling llm test-time compute optimally can be more effective  
241 than scaling model parameters, 2024. URL <https://arxiv.org/abs/2408.03314>.
- 242 K. Stechly, M. Marquez, and S. Kambhampati. Gpt-4 doesn’t know it’s wrong: An analysis of  
243 iterative prompting for reasoning problems. *arXiv preprint arXiv:2310.12397*, 2023.
- 244 G. Team, R. Anil, S. Borgeaud, Y. Wu, J.-B. Alayrac, J. Yu, R. Soricut, J. Schalkwyk, A. M.  
245 Dai, A. Hauth, et al. Gemini: a family of highly capable multimodal models. *arXiv preprint*  
246 *arXiv:2312.11805*, 2023.
- 247 G. Team, M. Riviere, S. Pathak, P. G. Sessa, C. Hardin, S. Bhupatiraju, L. Hussenot, T. Mesnard,  
248 B. Shahriari, A. Ramé, J. Ferret, P. Liu, P. Tafti, A. Friesen, M. Casbon, S. Ramos, R. Kumar,  
249 C. L. Lan, S. Jerome, A. Tsitsulin, N. Vieillard, P. Stanczyk, S. Girgin, N. Momchev, M. Hoffman,  
250 S. Thakoor, J.-B. Grill, B. Neyshabur, O. Bachem, A. Walton, A. Severyn, A. Parrish, A. Ahmad,  
251 A. Hutchison, A. Abdagic, A. Carl, A. Shen, A. Brock, A. Coenen, A. Laforge, A. Paterson,  
252 B. Bastian, B. Piot, B. Wu, B. Royal, C. Chen, C. Kumar, C. Perry, C. Welty, C. A. Choquette-Choo,  
253 D. Sinopalnikov, D. Weinberger, D. Vijaykumar, D. Rogozińska, D. Herbison, E. Bandy, E. Wang,  
254 E. Noland, E. Moreira, E. Senter, E. Eltyshv, F. Visin, G. Rasskin, G. Wei, G. Cameron, G. Martins,  
255 H. Hashemi, H. Klimczak-Plucińska, H. Batra, H. Dhand, I. Nardini, J. Mein, J. Zhou, J. Svensson,

256 J. Stanway, J. Chan, J. P. Zhou, J. Carrasqueira, J. Iljazi, J. Becker, J. Fernandez, J. van Amersfoort,  
257 J. Gordon, J. Lipschultz, J. Newlan, J. yeong Ji, K. Mohamed, K. Badola, K. Black, K. Millican,  
258 K. McDonnell, K. Nguyen, K. Sodhia, K. Greene, L. L. Sjoesund, L. Usui, L. Sifre, L. Heuermann,  
259 L. Lago, L. McNealus, L. B. Soares, L. Kilpatrick, L. Dixon, L. Martins, M. Reid, M. Singh,  
260 M. Iverson, M. Görner, M. Velloso, M. Wirth, M. Davidow, M. Miller, M. Rahtz, M. Watson,  
261 M. Risdal, M. Kazemi, M. Moynihan, M. Zhang, M. Kahng, M. Park, M. Rahman, M. Khatwani,  
262 N. Dao, N. Bardoliwalla, N. Devanathan, N. Dumai, N. Chauhan, O. Wahltinez, P. Botarda,  
263 P. Barnes, P. Barham, P. Michel, P. Jin, P. Georgiev, P. Culliton, P. Kuppala, R. Comanescu,  
264 R. Merhej, R. Jana, R. A. Rokni, R. Agarwal, R. Mullins, S. Saadat, S. M. Carthy, S. Perrin,  
265 S. M. R. Arnold, S. Krause, S. Dai, S. Garg, S. Sheth, S. Ronstrom, S. Chan, T. Jordan, T. Yu,  
266 T. Eccles, T. Hennigan, T. Kocisky, T. Doshi, V. Jain, V. Yadav, V. Meshram, V. Dharmadhikari,  
267 W. Barkley, W. Wei, W. Ye, W. Han, W. Kwon, X. Xu, Z. Shen, Z. Gong, Z. Wei, V. Cotruta,  
268 P. Kirk, A. Rao, M. Giang, L. Peran, T. Warkentin, E. Collins, J. Barral, Z. Ghahramani, R. Hadsell,  
269 D. Sculley, J. Banks, A. Dragan, S. Petrov, O. Vinyals, J. Dean, D. Hassabis, K. Kavukcuoglu,  
270 C. Farabet, E. Buchatskaya, S. Borgeaud, N. Fiedel, A. Joulin, K. Kenealy, R. Dadashi, and  
271 A. Andreev. Gemma 2: Improving open language models at a practical size, 2024a. URL  
272 <https://arxiv.org/abs/2408.00118>.

273 M. R. Team et al. Introducing dbrx: A new state-of-the-art open llm, 2024. URL [https://www.  
274 databricks.com/blog/introducing-dbrx-new-state-art-open-llm](https://www.databricks.com/blog/introducing-dbrx-new-state-art-open-llm). Accessed on April, 26, 2024b.

275 H. Touvron, L. Martin, K. Stone, P. Albert, A. Almahairi, Y. Babaei, N. Bashlykov, S. Batra,  
276 P. Bhargava, S. Bhosale, et al. Llama 2: Open foundation and fine-tuned chat models. *arXiv  
277 preprint arXiv:2307.09288*, 2023.

278 K. Valmeekam, M. Marquez, and S. Kambhampati. Can large language models really improve by  
279 self-critiquing their own plans? *arXiv preprint arXiv:2310.08118*, 2023.

280 J. Wang, J. Wang, B. Athiwaratkun, C. Zhang, and J. Zou. Mixture-of-agents enhances large language  
281 model capabilities. *arXiv preprint arXiv:2406.04692*, 2024a.

282 Q. Wang, Z. Wang, Y. Su, H. Tong, and Y. Song. Rethinking the bounds of llm reasoning: Are  
283 multi-agent discussions the key? *arXiv preprint arXiv:2402.18272*, 2024b.

284 X. Wang, J. Wei, D. Schuurmans, Q. Le, E. Chi, S. Narang, A. Chowdhery, and D. Zhou.  
285 Self-consistency improves chain of thought reasoning in language models. *arXiv preprint  
286 arXiv:2203.11171*, 2022.

287 Y. Wu, Z. Sun, S. Li, S. Welleck, and Y. Yang. An empirical analysis of compute-optimal inference for  
288 problem-solving with language models, 2024. URL <https://arxiv.org/abs/2408.00724>.

289 C. Xu, Q. Sun, K. Zheng, X. Geng, P. Zhao, J. Feng, C. Tao, and D. Jiang. Wizardlm: Empowering  
290 large language models to follow complex instructions. *arXiv preprint arXiv:2304.12244*, 2023.

291 K. Zhang, B. Qi, and B. Zhou. Towards building specialized generalist ai with system 1 and system 2  
292 fusion. *arXiv preprint arXiv:2407.08642*, 2024a.

293 K. Zhang, W. Yao, Z. Liu, Y. Feng, Z. Liu, R. Murthy, T. Lan, L. Li, R. Lou, J. Xu, et al. Diversity  
294 empowers intelligence: Integrating expertise of software engineering agents. *arXiv preprint  
295 arXiv:2408.07060*, 2024b.

296 W. Zhou, R. Agrawal, S. Zhang, S. R. Indurthi, S. Zhao, K. Song, S. Xu, and C. Zhu. Wpo: Enhancing  
297 rlhf with weighted preference optimization. *arXiv preprint arXiv:2406.11827*, 2024.

## 298 A Related Work

299 **Ensembles of LLMs.** Model ensembling aims to combine strengths from multiple models. Previous  
300 studies have explored various methods to leverage a diverse set of models, including but not limited to  
301 prompting [Wang et al., 2024a], weight averaging [Lin et al., 2024, Ramé et al., 2024], routing [Jiang  
302 et al., 2024b, Lu et al., 2023], training a generative fusion model [Jiang et al., 2023b], and so on.  
303 Zhang et al. [2024a] argues that the fusion of specialized models with certain general abilities could  
304 be a promising direction toward Artificial General Intelligence. Mixture-of-Agents (MoA, Wang et al.  
305 [2024a]) first queries multiple LLMs to generate responses, then iteratively aggregates these samples  
306 through several rounds of synthesis. MoA shows promising results on several benchmarks, and its  
307 variants achieve superior performance on the AlpacaEval 2.0 leaderboard. Our method is inspired  
308 by the prompt pipeline proposed in MoA. However, while existing MoA focuses on unleashing the  
309 strength from multiple different models [Wang et al., 2024a, Jiang et al., 2023b, Zhang et al., 2024b],  
310 we demonstrate the trade-off between diversity and quality within the proposers, highlighting that  
311 focusing solely on diversity may compromise overall quality and final performance.

312 **LLM Inference with Repeated Sampling.** Previous studies have shown that combining model  
313 outputs from repeated sampling can yield a better response in various domains. In tasks with  
314 automatic verifiers available, such as math [Hendrycks et al., 2021] and code [Chen et al., 2021],  
315 simply sampling LLMs multiple times can significantly improve the pass@k metric and hence boost  
316 the success rate of solving the tasks [Roziere et al., 2023, Li et al., 2022, Brown et al., 2024]. In  
317 more general tasks without verification tools, we can conduct techniques like majority vote, self-  
318 consistency, and best-of-n to choose the most promising one from candidate responses [Wang et al.,  
319 2022, Chen et al., 2023b, Gui et al., 2024, Li et al., 2024]. Therefore, repeated sampling is recently  
320 regarded as one approach of scaling compute during inference time [Brown et al., 2024]. In this work,  
321 we identify the surprising effectiveness of repeated sampling in the context of MoA. Unlike majority  
322 vote or best-of-N, Self-MoA asks LLMs to synthesize outputs generated from repeated sampling,  
323 hence can further improve over each individual output.

324 **Collaborative Agents** There is a surge of interest in building agent systems based on verification,  
325 critique, discussion, and refinement. For example, Stechly et al. [2023], Valmeekam et al. [2023], and  
326 Madaan et al. [2024] use self-critique to iteratively refine outputs through a chain structure. Madaan  
327 et al. [2024], Chen et al. [2024], and Wang et al. [2024a] explore the incorporation of multiple models  
328 to create a stronger agent that outperform each individual model. Du et al. [2023] incorporates  
329 multiple LLMs that propose and debate their individual responses over several rounds to reach a  
330 common final answer. Liang et al. [2023] proposes Multi-Agent Debate, which encourages divergent  
331 thinking during LLM debates to arrive at more informative conclusions and avoid rushing to incorrect  
332 answers. Chen et al. [2023a] introduces RECONCILE, which adopts a confidence-weighted voting  
333 mechanism for better consensus among LLM discussions. Interestingly, Wang et al. [2024b] shows  
334 that a single model with carefully designed prompts can sometimes match the performance of agent  
335 discussions. Moreover, agent discussions mainly outperform a single LLM when the prompts are  
336 insufficient.

## 337 B Supplements

### 338 B.1 Visual Illustrations of Our Proposed Methods

339 Please check Figure 2 for an illustration of MoA, Self-MoA, and Self-MoA-Seq.

### 340 B.2 Evaluation Benchmarks

341 **AlpacaEval 2.0 [Dubois et al., 2024]** is a widely used benchmark for assessing the instruction-  
342 following abilities of LLMs. It offers a set of real-world instructions and employs a GPT-4-based  
343 annotator to compare the model’s responses against reference answers generated by GPT-4. To address  
344 length bias inherent in model-based evaluation, Dubois et al. [2024] introduced the length-controlled  
345 (LC) win rate as a more robust evaluation metric.

346 **MMLU [Hendrycks et al., 2020]** is a multiple-choice dataset designed to assess a model’s multitask  
347 accuracy. MMLU is widely used to evaluate both the breadth and depth of language understanding



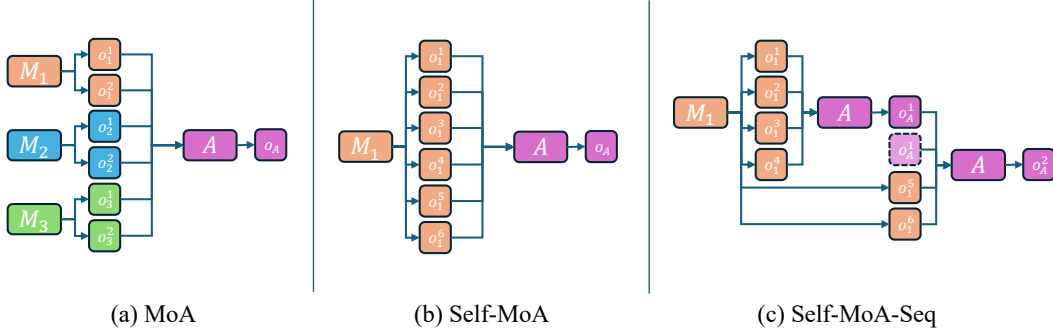


Figure 2: Comparison of MoA, Self-MoA, and Self-MoA-Seq. (a) In MoA, multiple models respond to a query, followed by an aggregator synthesizing their outputs. (b) Self-MoA simplifies this by repeatedly sampling from a single model. (c) Self-MoA-Seq extends Self-MoA by applying a sliding window to combine the best output so far with candidate outputs. At each timestep, the synthesized output is repeated to bias the aggregator towards it, reducing the context length requirements and expanding the method’s applicability. Note that MoA can extend to multiple rounds of aggregation (Appendix B.3), while Self-MoA and Self-MoA-Seq can extend to more outputs, but we omit them here for clarity.

348 capabilities of current LLMs across a diverse array of subjects, including mathematics, history,  
 349 computer science, logic, and law. We adopt MMLU-redux [Gema et al., 2024] for evaluation, which  
 350 is a subset of MMLU with 3,000 samples fixing the errors in the dataset through human re-annotating.

351 **CRUX** [Gu et al., 2024] consists of 800 Python code functions, each containing 3 to 13 lines  
 352 along with an input-output pair. Based on this dataset, Gu et al. [2024] constructs two tasks: input  
 353 prediction and output prediction. To successfully complete these tasks, the LLM must demonstrate  
 354 code reasoning abilities.

355 **MATH** [Hendrycks et al., 2021] comprises 12,500 challenging competition-level mathematics  
 356 problems. For our analysis, we utilize the testing subset of MATH, which consists of 5,000 samples.

### 357 B.3 Multi-Layer MoA

358 MoA can be extended to multiple layers. For MoA with  $l$  layers and  $n$  LLMs  $\{A_{i,j}\}_{j=1}^n$  in each layer  
 359  $i$ , we can formulate it as follows:

$$y_i = \bigoplus_{j=1}^n [A_{i,j}(x_i)] + x_1, \quad x_{i+1} = y_i,$$

360 where each LLM  $A_i^j$  generates a response for the query  $x_i$ , which is further concatenated with the  
 361 original query by the aggregator’s prompt  $\bigoplus$ .

### 362 B.4 Vendi Score

363 The Vendi Score (VS) is a metric designed to evaluate diversity in machine learning. It takes as input  
 364 a collection of samples along with a pairwise similarity function, and it outputs a single value that  
 365 represents the effective number of unique elements within the sample set.

366 The score is computed using a positive semi-definite similarity matrix  $K \in \mathbb{R}^{n \times n}$  as follows:

$$VS(K) = \exp \left( -\text{tr} \left( \frac{K}{n} \log \left( \frac{K}{n} \right) \right) \right) = \exp \left( -\sum_{i=1}^n \lambda_i \log(\lambda_i) \right)$$

367 Here,  $\lambda_i$  are the eigenvalues of the normalized matrix  $\frac{K}{n}$ , and  $0 \log 0 = 0$ . Essentially, the Vendi  
 368 Score is the exponential of the von Neumann entropy of  $\frac{K}{n}$ , which reflects the Shannon entropy of

369 its eigenvalues, also referred to as the effective rank. This metric provides a quantitative measure of  
370 diversity based on the distribution of similarity scores among the samples.

### 371 **B.5 Normalization of Inputs**

372 Given a sequence of inputs  $x_1, \dots, x_n$ . Let  $x'$  denote the normalized  $x$ . We have

$$x' = \frac{x_i - \bar{x}}{\text{std}(x)}, \text{ where } \bar{x} = \frac{1}{n} \sum_{i=1}^n x_i, \text{ and } \text{std}(x) = \sqrt{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2}$$

### 373 **B.6 Implication of R-square**

374 The implications of R-squared are presented in Table 2, illustrating the degree of influence between  
the independent and dependent variables. [Sarjana et al., 2020].

Table 2: The interpretation of R-square

| R-square   | Level       |
|------------|-------------|
| [0, 0.2)   | Very weak   |
| [0.2, 0.4) | Weak        |
| [0.4, 0.6) | Median      |
| [0.6, 0.8) | Strong      |
| [0.8, 1.0] | Very Strong |

375

## 376 **C Additional Results**

### 377 **C.1 Applying Self-MoA on AlpacaEval 2.0**

378 To further validate the effectiveness of Self-MoA, we apply it to the two top-performing models on  
379 AlpacaEval 2.0: gemma-2-9b-it-WPO-HB [Zhou et al., 2024] and gemma-2-9b-it-SimPO [Meng  
380 et al., 2024]. We use each model as both the proposer and the aggregator<sup>2</sup>, with a temperature of  
381 0.7 for all the generations. Due to the context length constraint of Gemma 2 [Team et al., 2024a],  
382 the aggregator can only take four samples as the input. As shown in Table 3, Self-MoA consistently  
383 achieves a 2-3 point gain and secures the top position on the leaderboard during submission.

### 384 **C.2 Experiments on Multiple Datasets with Specialized Models**

385 In this section, we explore different ensembling methods on a diverse set of benchmarks using  
386 specialized models.

387 **Evaluation datasets.** We conduct evaluations across a diverse set of benchmarks: MMLU, CRUX,  
388 and MATH. Please check Appendix B.2 for more details.

389 **Models.** To ensure sufficient diversity, we select three LLMs with specialized strengths: Qwen2-7B-  
390 Instruct<sup>3</sup>, DeepSeek-Coder-V2-Lite-Instruct<sup>4</sup>, and Qwen2-Math-7B-Instruct<sup>5</sup>. We fix the number of  
391 proposers to six and sweep various combinations of these three models. For convenience, we denote  
392 Qwen2-7B-Instruct as *i*, DeepSeek-Coder-V2-Lite-Instruct as *d*, and Qwen2-Math-7B-Instruct as *m*.  
393 The evaluation results in Table 4 show that Qwen2-7B-Instruct, DeepSeek-Coder-V2-Lite-Instruct,  
394 and Qwen2-Math-7B-Instruct excel on MMLU, CRUX, and MATH, respectively. We use the short

<sup>2</sup>Qwen1.5-110B-Chat is not used as the aggregator since the two top models significantly outperform it.

<sup>3</sup><https://huggingface.co/Qwen/Qwen2-7B-Instruct>

<sup>4</sup><https://huggingface.co/deepseek-ai/DeepSeek-Coder-V2-Lite-Instruct>

<sup>5</sup><https://huggingface.co/Qwen/Qwen2-Math-7B-Instruct>

Table 3: Self-MoA achieves state-of-the-art performance on the AlpacaEval 2.0 leaderboard when using top-performing models as both proposers and aggregators. We only ensemble 4 outputs due to context window constraints.

|            | Model Configuration             | LC Win Rate           |
|------------|---------------------------------|-----------------------|
| Individual | gemma-2-9b-it-WPO-HB            | 76.7                  |
|            | gemma-2-9b-it-SimPO             | 72.4                  |
| Self-MoA   | Self-MoA + gemma-2-9b-it-WPO-HB | <b>78.5 (rank #1)</b> |
|            | Self-MoA + gemma-2-9b-it-SimPO  | 75.0                  |

Table 4: Comparison of Self-MoA and Mixed-MoA in MMLU, CRUX, and MATH. Mixed-MoA models with top two average performances are highlighted by underline. The labels i, m, and d refer to Qwen2-7B-Instruct, DeepSeek-Coder-V2-Lite-Instruct, and Qwen2-Math-7B-Instruct, respectively. The average performance represents the mean accuracy across MMLU, CRUX, and MATH. TaskBest indicates that we use the strongest model for each task as both proposer and aggregator. For instance, in the case of CRUX, TaskBest refers to DeepSeek-Coder-V2-Lite-Instruct (d).

|            | Aggregator | Proposer   | MMLU         | CRUX         | MATH                     | Average      |
|------------|------------|------------|--------------|--------------|--------------------------|--------------|
| Individual | -          | i          | 66.16        | 36.25        | 53.81                    | 52.07        |
|            | -          | d          | 60.91        | 49.51        | 53.82                    | 54.74        |
|            | -          | m          | 54.36        | 27.88        | 69.57 <sup>6</sup>       | 50.60        |
| Mixed-MoA  | i          | iimddd     | 67.89        | 42.88        | 64.38                    | 58.38        |
|            |            | imdddd     | 67.42        | 44.50        | 63.90                    | 58.61        |
|            |            | iiiiid     | 68.90        | 41.25        | 63.00                    | 57.72        |
|            |            | immmmd     | 66.63        | 42.75        | 66.02                    | 58.47        |
|            |            | iimmmm     | 66.23        | 39.25        | 66.10                    | 57.19        |
|            |            | iiimmm     | 67.49        | 38.25        | 64.16                    | 56.63        |
|            |            | iiiiim     | 68.00        | 37.00        | 62.92                    | 55.97        |
|            |            | iidddd     | 68.21        | 45.50        | 62.56                    | 58.76        |
|            |            | iiiddd     | 68.21        | 42.88        | 62.38                    | 57.82        |
|            |            | iiiiid     | 68.47        | 40.75        | 61.24                    | 56.82        |
|            |            | mmdddd     | 66.34        | 46.75        | 66.48                    | <u>59.86</u> |
| mmmddd     | 65.80      | 47.00      | 67.32        | <u>60.04</u> |                          |              |
| mmmmdd     | 65.44      | 42.50      | 67.62        | 58.52        |                          |              |
| Self-MoA   | i          | dddddd     | 65.23        | 50.75        | 63.08                    | 59.69        |
|            | i          | 6×TaskBest | <b>69.01</b> | 50.75        | 68.42                    | 62.73        |
|            | TaskBest   | 6×TaskBest | <b>69.01</b> | <b>52.62</b> | <b>69.80<sup>6</sup></b> | <b>63.81</b> |

395 name for the mixture of proposers. For example, iidmmm indicates the inclusion of two samples from  
396 each model respectively. When a model is represented multiple times in the proposer mixture, we  
397 ensure that two samples are generated with different random seeds. We set the temperature of each  
398 model to be 0.7 for the individual model, and use temperature 0 for the aggregator. We mainly use  
399 Qwen2-7B-Instruct as the aggregator but also try different models as the aggregator. We explore  
400 various MoA configurations, including individual models, combinations of two or three models as  
401 proposers, and using a single model as the proposer (Self-MoA).

402 **Results.** The results are shown in Table 4. When considering i as the aggregator, among 11 tested  
403 combinations of proposers for MoA, only two combinations slightly outperformed Self-MoA with  
404 ddddd. Specifically, the combinations mmdddd and mmmddd outperformed ddddd by 0.17% and  
405 0.35%, respectively. The performance of the remaining MoA models was inferior to that of ddddd.

<sup>6</sup>As Qwen2-Math-7B-Instruct only supports context length of 4096, for these two data points, we sample the proposer with a reduced token length of 1024, and only aggregates three outputs from the proposer.

Table 5: Linear regression (Equation 1) of MoA’s performance  $t$  on diversity  $d$  and quality  $q$ .

| Dataset | $\alpha$          |           | $\beta$           |           | R-square |
|---------|-------------------|-----------|-------------------|-----------|----------|
|         | Coefficient       | P-value   | Coefficient       | P-value   |          |
| MMLU    | $2.558 \pm 0.176$ | $< 0.001$ | $1.841 \pm 0.176$ | $< 0.001$ | 0.771    |
| CRUX    | $4.548 \pm 0.459$ | $< 0.001$ | $1.421 \pm 0.459$ | $< 0.001$ | 0.685    |
| MATH    | $4.719 \pm 0.416$ | $< 0.001$ | $2.839 \pm 0.416$ | $< 0.001$ | 0.760    |

406 Adding model diversity does not necessarily enhance performance. For instance, MoA with `iimddd`  
 407 performs worse than `mmddd` in terms of average accuracy. Although model `i` is the strongest on  
 408 MMLU among individual models, its inclusion in the proposers does not improve overall performance  
 409 on the mixed datasets, i.e., `mmddd` has 60.04% overall performance while `iimddd` only has 58.38%.

410 The performance of Self-MoA can be significantly improved when we are allowed to select the  
 411 strongest model for each task. This is particularly beneficial when we have prior knowledge of the  
 412 task we wish to address. As shown in Table 4, when we use Qwen2-7B-Instruct as the aggregator,  
 413 Self-MoA achieves a performance of 62.73% by selecting the appropriate proposer for each task.  
 414 Additionally, employing a task-specific aggregator further boosts overall performance to 63.81%. We  
 415 postpone more discussion to Section C.4.

### 416 C.3 Statistical Analysis

417 To further understand the numerical correlation between MoA’s performance and diversity as well  
 418 as quality, we conduct linear regression for MoA’s performance  $t$  on diversity  $d$  and quality  $q$ .  
 419 Specifically, we fit the following equation for each dataset:

$$t = \alpha \times q + \beta \times d + \gamma, \tag{1}$$

420 where  $\alpha, \beta, \gamma \in \mathbb{R}$  are real-valued coefficients to be determined. For each dataset, we collect around  
 421 70 data points from Figure 1 to construct the set  $\{q^i, d^i, t^i\}_{i=1}^N$ . The coefficients  $\alpha$ ,  $\beta$ , and  $\gamma$  are then  
 422 derived by solving a linear regression on  $\{q^i, d^i, t^i\}_{i=1}^N$ . To make coefficients  $\alpha$  and  $\beta$  comparable,  
 423 we normalize  $q$  and  $d$  by subtracting their means and dividing by their standard deviations (detailed  
 424 in Appendix B.5), respectively. The results are presented in Table 5. We observe that the p-values for  
 425 both  $\alpha$  and  $\beta$  are less than 0.001, indicating a significant correlation between MoA’s performance  
 426 and both quality and diversity [Arnold, 1990]. The R-squared values from the linear regression  
 427 across three datasets are approximately around 0.7, indicating that the linear model based on quality  
 428 and diversity explains 70% MoA’s performance and hence a strong correlation between inputs and  
 429 outputs, according to Appendix B.6. In later parts, we show that using a more fine-grained quality  
 430 calculation can further increase the R-square value.

431 **Comparing the effect strength of quality and diversity.** From Table 5, we observe that  $\alpha$  is  
 432 greater than  $\beta$  across all three datasets. In particular, for CRUX and MATH, the gap between  
 433 these two measures is even more pronounced. These results suggest that MoA’s performance is  
 434 particularly sensitive to variations in quality, highlighting the importance of prioritizing quality within  
 435 the proposer mixture. This finding is also consistent with our observation that MoA achieves its best  
 436 performance in the bottom right of the plot in Figure 1, further supporting the effectiveness of our  
 437 proposed Self-MoA approach.

438 **Alternative quality measurements.** We use the averaged accuracy of each individual model to  
 439 measure quality in the previous analysis. In this section, we explore alternative methods for assessing  
 440 the quality of proposers. Recall that  $q_1, \dots, q_6$  denote the accuracy of each individual model among  
 441 proposers, and without loss of generality, we assume  $q_1 \geq q_2 \geq \dots \geq q_6$ . It is reasonable to assume  
 442 that the aggregator can select the correct answer from the proposers, particularly when the responses  
 443 of individual models are inconsistent. In such cases, the aggregator would rely more heavily on  
 444 models with better individual performance, meaning the weight of  $q_1$  would be greater than that of  
 445  $q_6$ .

446 Therefore, we compare the following methods to calculate quality:

- 447 • **Average:**  $\frac{1}{6} \sum_{i=1}^6 q_i$ .

Table 6: The R-square of the linear regression when we use different quality measurement methods. We find using Centered-1/K-Norm with K=2 can achieve good performance among all these three datasets.

| Dataset | Method            | Average (K=1) | K=2          | K=3   | K=4   |
|---------|-------------------|---------------|--------------|-------|-------|
| MMLU    | K-Norm            | 0.771         | 0.809        | 0.832 | 0.845 |
|         | Centered-1/K-Norm | 0.771         | <b>0.881</b> | 0.902 | 0.903 |
| CRUX    | K-Norm            | 0.685         | 0.736        | 0.765 | 0.779 |
|         | Centered-1/K-Norm | 0.685         | <b>0.753</b> | 0.758 | 0.753 |
| MATH    | K-Norm            | 0.760         | 0.720        | 0.692 | 0.672 |
|         | Centered-1/K-Norm | 0.760         | <b>0.720</b> | 0.692 | 0.672 |

448 • **K-Norm:**  $\left(\frac{1}{6} \sum_{i=1}^6 q_i^K\right)^{1/K}$ , where a larger  $K$  places more emphasis on stronger individual  
449 models.

450 • **Centered-1/K-Norm:**  $q_1 - \left(\frac{1}{6} \sum_{i=1}^6 (q_1 - q_i)^{1/K}\right)^K$ . In this formulation, we first compute  
451 the difference between  $q_i$  and the best model’s  $q_1$ . The  $1/K$  norm emphasizes the weights  
452 of models whose performance is closer to  $q_1$ .

453 All three methods are the same when  $K = 1$ . For each quality measurement, we fit a linear regression  
454 to assess the relationship between MoA’s performance and the quality and diversity metrics, reporting  
455 the R-squared values in Table 6. Our analysis shows that in MMLU and CRUX, applying a larger  
456 weight to better-performing individual models tends to increase the R-squared values. However, this  
457 trend is inconsistent for MATH. We conjecture that this inconsistency arises because the aggregator  
458 Qwen2-7B-Instruct is relatively weak on MATH compared to the strongest individual model, Qwen2-  
459 Math-7B-Instruct. This limitation constrains the performance of MoA, leading to an inconsistent  
460 trend in the linear regression results. In contrast, on MMLU, where Qwen2-7B-Instruct is the  
461 strongest individual model, we find that the R-squared value can exceed 0.9 with  $K = 2$  using the  
462 Centered-1/K-Norm. This indicates a very strong linear relationship between MoA performance and  
463 the quality and diversity metrics. Overall, we conclude that employing Centered-1/K-Norm with  
464  $K = 2$  (marked in blue) achieves strong performance across all three datasets.

#### 465 C.4 When Mixed-MoA Outperforms Self-MoA?

466 According to the quality-diversity trade-off illustrated in Figure 1, we conjecture that increasing  
467 diversity can enhance MoA’s performance when the quality is controlled.

468 Typically, Mixed-MoA exhibits greater diversity than Self-MoA. Therefore, conditioned on similar  
469 quality, Mixed-MoA can outperform Self-MoA. This scenario arises when individual models demon-  
470 strate similar performance while still exhibiting significant cross-model diversity. For instance, if  
471 we combine three tasks of MMLU, CRUX, and MATH, the average performances of the individual  
472 models are 52.07%, 54.74%, and 50.60%, respectively (Table 4). In this combined task, each model  
473 specializes in different parts, with i performing best on MMLU, d on CRUX, and m on MATH.

474 From the “Average” column of Table 4, we observe that Mixed-MoA indeed outperforms Self-MoA of  
475 dddd, which is aggregating samples from the individual model with the best average performance.  
476 Specifically, Mixed-MoA of mddd and mddd achieves the average performance of 59.86% and  
477 60.04%, improves upon Self-MoA of dddd by 0.35%. Given the reported small margin, we argue  
478 that Self-MoA is still a very competitive baseline under this setting, not to mention the dominant  
479 performance of Self-MoA over Mixed-MoA when focusing on one single task.

480 We further consider another single-task case on MMLU, involving two individual models: Llama-  
481 3.1-8B-Instruct and Qwen2-7B-Instruct, with Qwen2-7B-Instruct serving as the aggregator. We  
482 choose Llama-3.1-8B-Instruct because it performs similarly to Qwen2-7B-Instruct as an individual  
483 model. Table 7 demonstrates that even when the performance of two individual models is close,  
484 Self-MoA—utilizing six Llama-3.1-8B-Instruct proposers (denoted as llllll)—still outperforms  
485 the Mixed-MoA configuration (denoted as iiii).

Table 7: MoA of Llama-3.1-8B-Instruct and Qwen2-7B-Instruct. 1 is short for Llama-3.1-8B-Instruct and i is short for Qwen2-7B-Instruct.

|            | Aggregator | Proposer | MMLU  |
|------------|------------|----------|-------|
| Individual | -          | i        | 66.16 |
|            | -          | 1        | 66.40 |
| Mixed-MoA  | i          | ii1111   | 70.73 |
| Self-MoA   | i          | iiiii    | 69.01 |
|            | i          | 111111   | 71.27 |

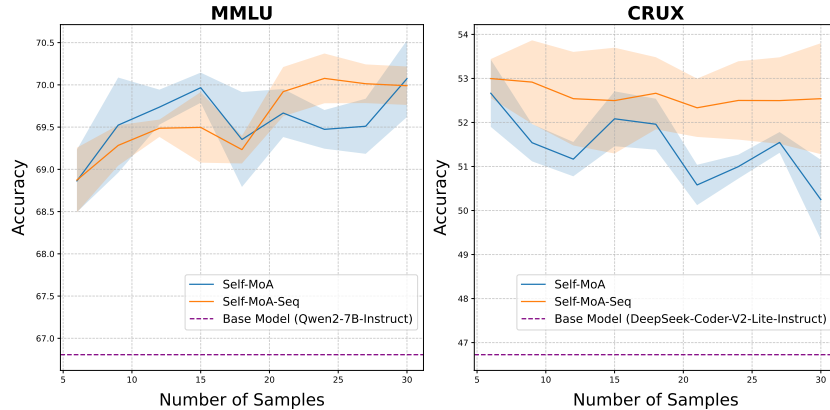


Figure 3: The performance of Self-MoA and Self-MoA-Seq with a growing number of samples. Dashed lines indicate the performance of a single forward pass with the base model.

## 486 C.5 Scaling Inference Compute with Self-MoA

487 In previous sections, we have provided evidence that Self-MoA over one strong model is straightforward but effective. As the community is becoming more aware of scaling inference time computing [Brown et al., 2024, Snell et al., 2024, Wu et al., 2024], one natural question to ask is:

490 *Given a strong model, does Self-MoA’s performance scale with the number of repeated samples?*

491 Intuitively, Self-MoA cannot scale indefinitely by simply increasing the computation budget for at least three reasons:

- 493 • As more responses are sampled from a single model, the diversity among those samples tends to plateau.
- 495 • Aggregating information from many samples is more challenging for LLMs compared to handling a smaller number of samples.
- 497 • Every LLM has a context length limit (e.g., 8192 tokens for Gemma 2), which restricts the number of responses an aggregator can process at once.

499 While the first limitation is inherent to repeated sampling, we address the latter two by introducing Self-MoA-Seq, a sequential variant designed to manage large numbers of responses without overwhelming the aggregator. Self-MoA-Seq uses a sliding window to aggregate a fixed number of responses at a time, allowing it to handle an unlimited number of responses, regardless of context length constraints. 503 A visual illustration is provided in Figure 2.

504 We evaluate the performance of Self-MoA and Self-MoA-Seq with increasing sample sizes on the MMLU and CRUX benchmarks to study their scaling behavior. For each benchmark, we use the best-performing model as both the proposer and aggregator (Qwen2-7B-Instruct for MMLU and DeepSeek-Coder-V2-Lite-Instruct for CRUX), with a sampling temperature of 0.7. In Self-MoA-Seq, the window size is set to six, with the first three slots reserved for the current synthesized output. We

509 vary the number of samples from 6 to 30 and plot the accuracy curves from three runs with different  
510 seeds in Figure 3. Our key observations are as follows:

- 511 • Both Self-MoA and Self-MoA-Seq significantly improve performance over the individual  
512 base model.
- 513 • Adding more samples can have both positive and negative effects, meaning there is no  
514 universal compute-optimal solution.
- 515 • Self-MoA-Seq delivers performance that is comparable to, or slightly better than, Self-MoA.

516 These findings suggest that Self-MoA-Seq can extend the effectiveness of Self-MoA to LLMs with  
517 shorter context lengths, without sacrificing performance. Following Section C.4, we explore whether  
518 introducing a second model can enhance performance in the sequential setting. Given that Llama-3.1-  
519 8B-Instruct performs similarly to Qwen2-7B-Instruct on the MMLU task, we compare the impact of  
520 adding Llama-3.1-8B-Instruct and DeepSeek-Coder-V2-Lite-Instruct (which underperforms Qwen2-  
521 7B-Instruct by 5%) after aggregating 30 samples from Qwen2-7B-Instruct in Self-MoA-Seq. We find  
522 that incorporating Llama-3.1-8B-Instruct boosts accuracy by around 2%, whereas adding DeepSeek-  
523 Coder-V2-Lite-Instruct reduces accuracy by more than 1.5%. This result provides another example  
524 of cross-model diversity benefiting MoA, and shows the potential of Self-MoA-Seq with increasing  
525 computation budget.

## 526 **D Zoomed Figures**

527 Figure 4 is a zoomed version of Figure 1.

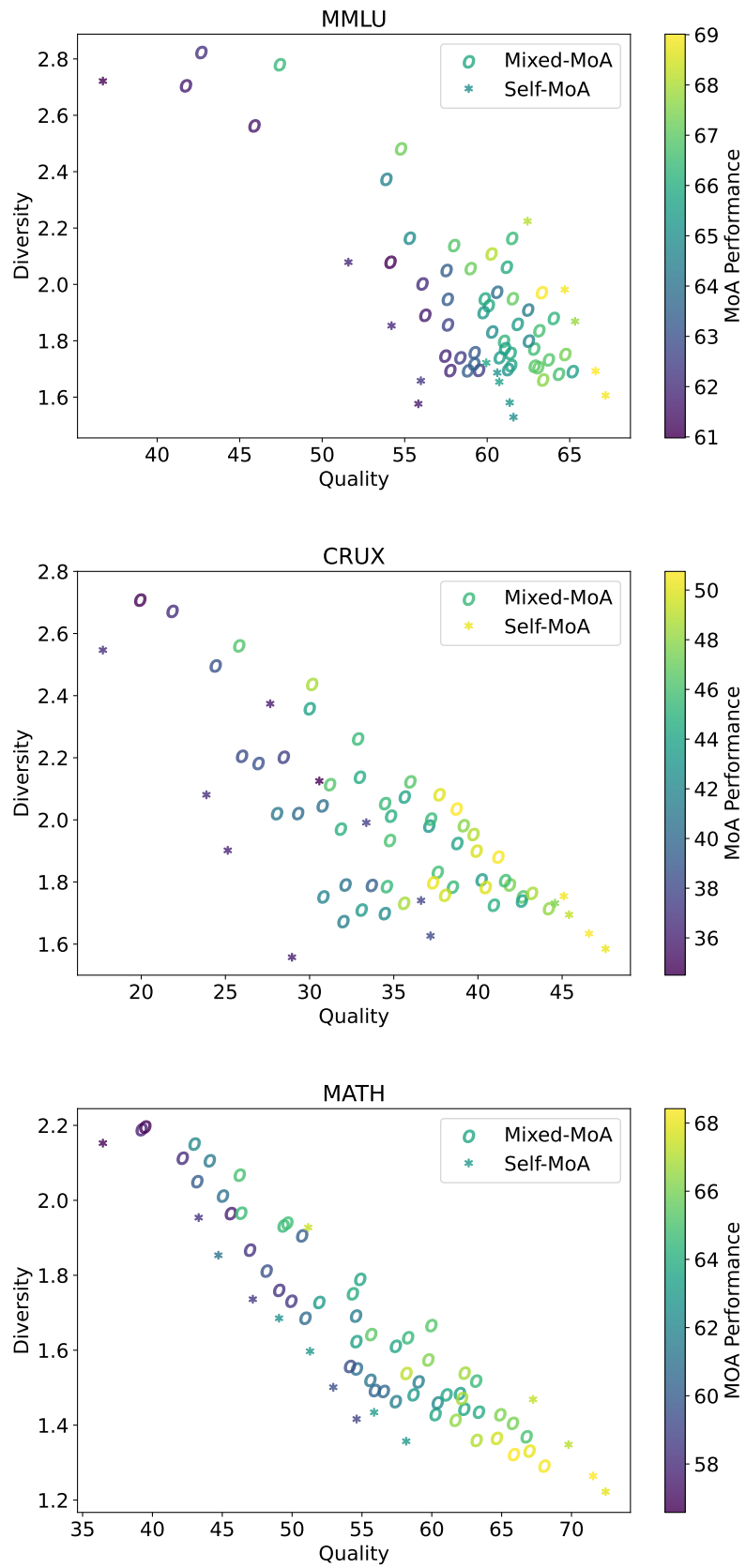


Figure 4: A zoomed version of Figure 1.