# SP-LORA: SPARSITY-PRESERVED LOW-RANK ADAP TATION FOR SPARSE LARGE LANGUAGE MODEL

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#### Abstract

Large Language Models (LLMs) excel in various natural language processing tasks but face significant hardware resource demands and inference latency due to their large parameter counts. To address these challenges, post-training pruning techniques like SparseGPT, Wanda, and RIA have been developed to reduce parameters. However, these methods often result in performance gaps, particularly for smaller models, and lack efficient fine-tuning strategies that preserve sparsity.

This paper presents SP-LoRA, a novel approach that integrates the advantages of low-rank adaptation (LoRA) with the efficiency of sparse models. Our method preserves sparsity when merging LoRA adapters with sparse matrices by introducing a mask matrix,  $\mathcal{M}$ . Additionally, to address the significant memory overhead associated with maintaining sparsity, we propose a hybrid technique that combines gradient checkpointing and memory reuse. This approach effectively reduces GPU memory usage during fine-tuning while achieving comparable efficiency to standard LoRA. Through extensive evaluations on sparse LLMs pruned by Wanda or SparseGPT, followed by fine-tuning with SP-LoRA, we demonstrate its effectiveness in both zero-shot scenarios and domain-specific tasks.



Figure 1: Memory and time usage of LoRA, SP-LoRA, and SPP, with GC denoting gradient checkpointing and NO representing no optimization (See Section 4.2 for details). Our approach SP-LoRA performs close to LoRA and outperforms the existing method SPP while preserving the sparsity.

#### 1 INTRODUCTION

Large language models (LLMs) have exhibited exceptional performance across various natural language processing tasks, leading to their growing adoption. However, their extensive number of parameters demands substantial hardware resources for deployment, which limits accessibility. Additionally, the sheer scale of these models can slow down inference speed, posing challenges in applications where low latency is critical.

Various post-training unstructured pruning methods, such as SparseGPT (Frantar & Alistarh, 2023),
Wanda (Sun et al., 2024), and RIA (Zhang et al., 2024), have been proposed to reduce model parameters and tackle the challenges mentioned earlier. These techniques require only a small number of
samples and can transform a dense model into an unstructured or semi-structured sparse model in
just a few minutes. While efficient and user-friendly, there remains a performance gap between the
original dense model and the pruned sparse model, particularly for small- and medium-sized models
under 2:4 semi-structured sparsity (Mishra et al., 2021). This gap hinders the practical application
of these pruned sparse methods.

To utilize these models effectively, continuous pre-training is essential to compensate for the performance decline in sparse models. However, achieving desired performance through continuous pre-training can be quite costly. Therefore, there is an urgent need for efficient and low-resource tuning methods for sparse LLMs that preserve their sparsity. Unfortunately, current research has primarily concentrated on pruning strategies, with insufficient focus on the tuning of sparse models.

Contrasted with sparse language models, low-rank adaptation (LoRA; Hu et al., 2021) and other
parameter-efficient fine-tuning (PEFT) techniques have been developed for dense language models
to alleviate the computational burdens associated with various training phases. These methodologies
facilitate the fine-tuning of dense LLMs with reduced resource requirements, thereby prompting the
question: Can LoRA be effectively utilized for the fine-tuning of sparse LLMs?

072 In addressing this query, we introduce SP-LoRA, a simple yet effective method for preserving spar-073 sity while performing low-rank adaptation on sparse LLMs. The primary challenge in applying 074 LoRA to sparse LLMs lies in the fact that integrating LoRA's adapter with the weight matrix results 075 in the loss of sparsity. To address this issue, we introduce an additional mask matrix  $\mathcal{M}$ , derived 076 from the pruned weight matrix, as an extra weight term in LoRA. This mask delineates the locations 077 of non-zero elements within the weight matrix  $\mathcal{W}$ , ensuring that sparsity is maintained throughout the training process. However, the introduction of this mask leads to an increased number of activa-078 tions being tracked in the computational graph, consequently imposing a significantly higher GPU 079 memory overhead for SP-LoRA compared to LoRA (See Section 3.2.1 for a detailed analysis). To 080 address this issue, we propose a hybrid approach that combines gradient checkpointing (Chen et al., 081 2016) with memory reutilization techniques for SP-LoRA. This strategy minimizes unnecessary 082 GPU memory allocation, making SP-LoRA as efficient as LoRA. Specifically, during each forward 083 pass, we first compute the mask and generate the new weight matrix by merging the adapter, mask, 084 and initial weight matrix. This process reuses the weight matrix to directly store the new weight 085 matrix. In the backward pass, we recompute the mask, and then calculate the gradients of the input 086 activations and adapters. Finally, we restore the initial weight matrix from the updated one for use 087 in the next iteration's computation (see Section 3.2.2 for a detailed implementation).

We evaluate the proposed SP-LoRA on various LLMs. First, an LLM is pruned using a post-training pruning method, specifically Wanda or SparseGPT. Next, SP-LoRA is employed to fine-tune the pruned models using a portion of the collected pre-training and instruction data. We then directly assess the zero-shot performance of the tuned sparse LLM across a range of well-known text tasks. Additionally, we use SP-LoRA to fine-tune the sparse models on task-specific datasets, particularly for well-known challenging tasks, including math and code. This aims to explore the domain adaptation capabilities of SP-LoRA when addressing difficult problems.

The main contributions of this paper are summarized in the following:

(1) We propose SP-LoRA, a parameter-efficient fine-tuning method for sparse LLMs that preserves
 model sparsity during the fine-tuning process. This approach employs a hybrid technique that com bines gradient checkpointing and memory reuse, effectively reducing the GPU memory overhead
 typically associated with fine-tuning sparse LLMs.

(2) Extensive experiments on sparse LLMs with various sparsity patterns and ratios demonstrate the
 effectiveness of SP-LoRA. As illustrated in Figure 1, SP-LoRA achieves comparable performance
 to LoRA—despite not preserving sparsity—in terms of memory and time usage. It significantly
 outperforms the sparsity-preserved SPP (Lu et al., 2024), especially regarding memory efficiency.

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Figure 2: The workflow of SP-LoRA with memory optimization. We introduce an additional mask  $\mathcal{M}$  into the LoRA framework to preserve the sparsity of the model. Meanwhile, the memory overhead of SP-LoRA is optimized by reutilizing the memory of  $\tilde{\mathcal{W}}$  to store weight matrix  $\tilde{\mathcal{W}}^{(t)}$  and by recomputing the mask  $\mathcal{M}$ .

2 RELATED WORK

2.1 PRUNING

136 Pruning (Han et al., 2016) is a promising technique for compressing neural networks by removing unimportant weights. From the perspective of sparse structure, pruning methods can be categorized 137 into structured (Ashkboos et al., 2024; Chen et al., 2024; Hu et al., 2024; Liu et al., 2024; Men et al., 138 2024; Muralidharan et al., 2024) and unstructured pruning (Frantar & Alistarh, 2023; Sun et al., 139 2024; Zhang et al., 2024). Structured pruning achieves compression by selectively eliminating en-140 tire structural units such as channels, filters, attention heads, or layers from the neural network. 141 Conversely, unstructured pruning achieves compression by removing individual unimportant ele-142 ments from the weight matrices, effectively transforming dense matrices into sparse ones. And 143 thanks to hardware developments, models obtained with unstructured pruning can also be efficiently 144 accelerated when using a specific sparse structure, such as 2:4 sparsity (Mishra et al., 2021). 145

From the perspective of optimization methods, pruning techniques can be further classified into 146 training-based and post-training pruning. Training-based pruning (Louizos et al., 2018; Sanh et al., 147 2020) progressively thins out a dense model during the training phase. This approach typically 148 involves introducing masks into the model and controlling its sparsity through an additional reg-149 ularization loss computed based on these masks. Although widely applicable to smaller models, 150 training-based pruning is challenging to implement for larger models due to the substantial increase 151 in GPU memory overhead and the requirement for extensive training data. Consequently, there has 152 been a growing interest in post-training methods (Frantar & Alistarh, 2023; Sun et al., 2024; Zhang 153 et al., 2024) that enable pruning with a small number of calibration data, particularly for large LLMs.

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2.2 PARAMETER-EFFICIENT FINE-TUNING

PEFT methods are designed to fine-tune pre-trained models with minimal trainable parameters. Typically these methods freeze the original model and insert a series of trainable adapters, including but
not limited to prefix tokens (Liu et al., 2022), side networks (Zhang et al., 2020), parallel and serial
adapters (Houlsby et al., 2019; Hu et al., 2023). These techniques are particularly advantageous
when working with large pre-trained models, as full fine-tuning of all parameters can be both computationally prohibitive and data-intensive. Among these methods, LoRA and its variants (Hu et al.,

162 2021; Zhang et al., 2023; Zhao et al., 2024) are the most widely adopted PEFT approaches, offering 163 the benefit of merging the adapter's parameters with the model weights post-training. However, for 164 sparse LLMs, this merging process can transform the sparse model into a dense one, thereby under-165 mining the benefits of sparsity. In this work, we aim to enhance LoRA to make it compatible with 166 sparse LLMs.

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#### 2.3 SPARSITY PRESERVED TRAINING

170 Contrary to pruning, which transforms a dense model into a sparse one, some approaches aim to train a sparse model from scratch or an existing sparse model. We refer to these techniques as sparsity-171 preserved training methods, which include STE (Zhou et al., 2021), RigL (Evci et al., 2021), and 172 others (Huang et al., 2024; Kurtic et al., 2023). These methods can produce sparse models that 173 perform comparably to dense models; however, they require the training of all the parameters of 174 the model and even require more GPU memory than the training of dense models, thereby posing 175 challenges for application to LLMs. Recent work SPP (Lu et al., 2024), has proposed to reduce the 176 training cost of sparse models by combining PEFT methods with sparsity-preserved training. SPP 177 can be viewed as a variant of LoRA, using a special form of matrices as adapters and introducing 178 additional weight terms in LoRA. SPP in the forward pass requires the construction of a matrix with 179 the same size as the weight matrix and recording it in the computational graph. Therefore, despite requiring only a limited number of trainable parameters, SPP still encounters the issue of high GPU 181 memory overhead. This work will address the high GPU overhead issue for sparsity-preserved 182 training.

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#### 3 METHOD

186 In this section, we first review unstructured pruning and low-rank adaptation (Section 3.1), then 187 introduce our proposed method, SP-LoRA (Section 3.2). We subsequently discuss the challenges 188 of training sparse LLMs while preserving sparsity (Section 3.2.1) and explain how our approach 189 addresses these challenges (Section 3.2.2). 190

3.1 PRELIMINARY

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Unstructured Pruning Unstructured pruning methods are employed to transform the dense weight matrices of LLMs into sparse matrices. Let  $\mathcal{W}$  denote a weight matrix of an LLM. The objective of unstructured pruning is to determine a mask  $\mathcal{M}$  and weight updates  $\Delta \mathcal{W}$ , such that the dense matrix can be transformed into a sparse matrix  $\hat{W}$ . Mathematically, this transformation is expressed as:  $\tilde{\mathcal{W}} = \mathcal{M} \odot (\mathcal{W} + \Delta \mathcal{W})$ , where  $\mathcal{W} \in \mathbb{R}^{R \times C}$ ,  $\mathcal{M} \in \{0, 1\}^{R \times C}$ , and  $\Delta \mathcal{W} \in \mathbb{R}^{R \times C}$ . R and C represent the number of rows and columns of the weight matrix, respectively. 199

200 LoRA LoRA is a method for adapting LLMs to specific tasks or domains by training only a small 201 number of parameters. Its mathematical formulation is given by:  $\mathcal{W}^{(t)} = \mathcal{W} + \mathcal{A}^{(t)} \times \mathcal{B}^{(t)}$ , where  $\mathcal{W}$ 202 denotes the initial weight matrix,  $\mathcal{W}^{(t)}$  represents the weight matrix at the t-th iteration of training, 203 and  $\mathcal{A}$  and  $\mathcal{B}$  are the introduced trainable adapters,  $\mathcal{A}^{(t)}$  and  $\mathcal{B}^{(t)}$  represent the adapters at the *t*-th iteration of training. Here,  $\mathcal{W} \in \mathbb{R}^{R \times C}$ ,  $\mathcal{A} \in \mathbb{R}^{R \times r}$ ,  $\mathcal{B} \in \mathbb{R}^{r \times C}$ , and *r* is much smaller than *R* and 204 205 C. During training, all parameters except A and B remain frozen.

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3.2 SP-LORA

209 To preserve the sparsity of the model, we adopt a simple approach by introducing a mask as an 210 additional weighting term in the LoRA framework. Let us consider a sparse LLM with a weight 211 matrix  $\mathcal{W}$  and its corresponding mask  $\mathcal{M}$ . Based on LoRA, we first introduce adapters  $\mathcal{A}$  and  $\mathcal{B}$  for 212 the weight matrix  $\mathcal{W}$ . Then, we incorporate the mask to ensure the sparsity of the weight matrix at 213 each training iteration t: 214

$$\tilde{\mathcal{W}}^{(t)} = \tilde{\mathcal{W}} + \mathcal{M} \odot (\mathcal{A}^{(t)} \times \mathcal{B}^{(t)}).$$
(1)

We refer to this LoRA variant as SP-LoRA, which stands for Sparsity Preserved Low-Rank Adaptation. However, the introduction of the mask while ensuring the sparsity of the weights, alters the computational graph of LoRA, thus incurring significant GPU memory overhead, posing practical challenges for its implementation. Consequently, we will first analyze the cause of this high GPU memory overhead and propose a solution to address this issue.

#### 222 3.2.1 MEMORY COMPLEXITY

Assuming that the current iteration is the *t*-th training step, let the input to the weight matrix be denoted as  $X \in \mathbb{R}^{C \times L}$ . For LoRA, the output can be represented as

$$Y = \tilde{\mathcal{W}}X + \mathcal{A}^{(t)}\mathcal{B}^{(t)}X.$$
(2)

This formulation corresponds to the following computational steps:

$$I_{a}^{1} = \tilde{\mathcal{W}}X, \quad I_{a}^{2} = \mathcal{B}^{(t)}X, \quad I_{a}^{3} = \mathcal{A}^{(t)}I_{a}^{2}, \quad Y = I_{a}^{1} + I_{a}^{3},$$
(3)

where  $I_a^1 \in \mathbb{R}^{R \times L}$ ,  $I_a^2 \in \mathbb{R}^{r \times L}$ , and  $I_a^3 \in \mathbb{R}^{R \times L}$  represent the intermediate activations. In the context of back-propagation, the gradients for the parameters  $\mathcal{A}^{(t)}$ ,  $\mathcal{B}^{(t)}$ , and X must be computed. Given the gradient of Y as dY, the gradients can be formulated as follows:

$$d\mathcal{A}^{(t)} = dY I_a^{2\top}, \quad dI_a^2 = \mathcal{A}^{(t)\top} dY, \quad d\mathcal{B}^{(t)} = dI_a^2 X^{\top}, \quad dX = \tilde{\mathcal{W}}^{\top} dY + \mathcal{B}^{(t)\top} dI_a^2.$$
(4)

Consequently, during the forward pass, GPU memory must be allocated for the intermediate activations  $I_a^1$ ,  $I_a^2$ , and  $I_a^3$ , along with the output activation Y, encompassing a total of rL + 3RLparameters. Additionally, the input activation X and the intermediate activation  $I_a^2$  are retained for back-propagation, involving rL + CL parameters. During the backward pass, GPU memory allocation is required for the gradients  $d\mathcal{A}^{(t)}$ ,  $dI_a^2$ ,  $d\mathcal{B}^{(t)}$ , and dX, totaling rR + rL + rC + CLparameters.

Then, considering the proposed method SP-LoRA, the mathematical expression for the output can be written as

$$Y = \{ \tilde{\mathcal{W}} + \mathcal{M} \odot (\mathcal{A}^{(t)} \times \mathcal{B}^{(t)}) \} X.$$
(5)

Compared to LoRA, which first multiply X with  $\mathcal{B}^{(t)}$  and then with  $\mathcal{A}^{(t)}$ , SP-LoRA needs to compute  $\mathcal{M} \odot (\mathcal{A}^{(t)} \times \mathcal{B}^{(t)})$  first, corresponding to the following computational steps:

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$$I_w^1 = \mathcal{A}^{(t)} \mathcal{B}^{(t)}, \quad \mathcal{M} = [\tilde{\mathcal{W}} \neq 0], \quad I_w^2 = \mathcal{M} \odot I_w^1, \quad I_w^3 = \tilde{\mathcal{W}} + I_w^2, \quad Y = I_w^3 X, \tag{6}$$

where  $I_w^1, I_w^2, I_w^3 \in \mathbb{R}^{R \times C}$  represent the intermediate weights. The corresponding back-propagation process is outlined as follows:

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$$dI_w^3 = dYX^{\top}, \ dX = I_w^{3\top}dY, \ dI_w^1 = dI_w^3 \odot \mathcal{M}, \ d\mathcal{A}^{(t)} = dI_w^1 \mathcal{B}^{(t)\top}, \ d\mathcal{B}^{(t)} = \mathcal{A}^{(t)\top}dI_w^1.$$
(7)

Hence, for SP-LoRA, during the forward pass, GPU memory allocation is necessary for the intermediate weights  $\mathcal{M}$ ,  $I_w^1$ ,  $I_w^2$ ,  $I_w^3$ , and the output activation Y, encompassing a total of 4RC + RLparameters (> rL+3RL). Additionally, the input activation X, the intermediate weights  $\mathcal{M}$ , and  $I_w^3$ must be retained for the back-propagation process, involving 2RC + CL parameters (> rL + CL). In the backward pass, GPU memory must be allocated for the gradients  $dI_w^1$ ,  $dI_w^3$ , dX,  $d\mathcal{A}^{(t)}$ , and  $d\mathcal{B}^{(t)}$ , summing to 2RC + CL + rR + rC parameters (> rR + rL + rC + CL).

Comparing the number of parameters retained for back-propagation by SP-LoRA and LoRA, it
 becomes evident that including masks significantly increases GPU memory overhead, despite not
 increasing the number of trainable parameters. In addition, SP-LoRA also allocates more tempo rary GPU memory than LoRA for both forward and backward, thus increasing the time overhead.
 Consequently, optimizing the GPU memory usage of SP-LoRA is imperative.

270 Algorithm 1: SP-LoRA Forward Pass 271 **Input:** Activation X, Sparse weight matrix  $\tilde{\mathcal{W}}$ , SP-LoRA adapters  $\mathcal{A}^{(t)}, \mathcal{B}^{(t)}$ . 272 **Output:** Activation Y 273 274 1 Compute mask:  $\mathcal{M} = [\mathcal{W} \neq 0];$ 275 <sup>2</sup> Update  $\tilde{\mathcal{W}}$  to  $\tilde{\mathcal{W}}^{(t)}$  in-place:  $\tilde{\mathcal{W}}^{(t)} = \tilde{\mathcal{W}}$ .addmm\_( $\mathcal{A}^{(t)}, \mathcal{B}^{(t)}$ ).mul\_( $\mathcal{M}$ ); 276 <sup>3</sup> Save X into context for backward; 277 4 Compute Y:  $Y = \tilde{\mathcal{W}}^{(t)}X$ ; 278 279 281 282 Algorithm 2: SP-LoRA Backward Pass 283 284 Input: Gradient dY, Activation X, Sparse weight matrix  $\tilde{\mathcal{W}}^{(t)}$ , SP-LoRA adapters  $\mathcal{A}^{(t)}, \mathcal{B}^{(t)}$ . **Output:** Gradients  $d\mathcal{A}^{(t)}$ ,  $d\mathcal{B}^{(t)}$ , and dX1 Compute mask:  $\mathcal{M} = [\tilde{\mathcal{W}}^{(t)} \neq 0];$ 287 <sup>2</sup> Compute gradient of X:  $dX = \tilde{\mathcal{W}}^{(t)\top} dY$ ; 288 <sup>3</sup> Compute gradient of  $\tilde{\mathcal{W}}^{(t)}$ :  $d\tilde{\mathcal{W}}^{(t)} = (dYX^{\top}).mul_{-}(\mathcal{M})$ ; 289 4 Compute gradient of  $\mathcal{A}^{(t)}$ :  $d\mathcal{A}^{(t)} = d\tilde{\mathcal{W}}^{(t)} \mathcal{B}^{(t)\top}$ ; 290 s Compute gradient of  $\mathcal{B}^{(t)}$ :  $d\mathcal{B}^{(t)} = \mathcal{A}^{(t)\top} d\tilde{\mathcal{W}}^{(t)}$ : 291 292 6 Update  $\tilde{\mathcal{W}}^{(t)}$  to  $\tilde{\mathcal{W}}$  in-place:  $\tilde{\mathcal{W}} = \tilde{\mathcal{W}}^{(t)}$ .addmm\_ $(-\mathcal{A}^{(t)}, \mathcal{B}^{(t)})$ .mul\_ $(\mathcal{M})$ ; 293 295 296 297 298 MEMORY OPTIMIZATION 3.2.2 299 300 301 We propose a hybrid gradient checkpointing and memory reutilizing approach to optimize memory usage. During the forward propagation phase of SP-LoRA, memory allocation is required for inter-302 mediate weights denoted as  $\mathcal{M}, I_w^1, I_w^2$ , and  $I_w^3$ . Despite their substantial demand on GPU memory, 303 these intermediate weights entail minimal computational effort. Therefore, instead of providing 304 extra memory for storing these intermediate weights, we can either recompute them during back-305 propagation or reuse existing memory to store them. Algorithm 1 and 2 provide the pseudo-code<sup>1</sup> 306 detailing the forward and backward passes of SP-LoRA, respectively. Specifically, in the forward 307 pass, we compute the weight matrix  $\tilde{\mathcal{W}}^{(t)}$  and leverage the existing memory footprint of  $\tilde{\mathcal{W}}$  to store 308 it (Algorithm 1 Line 2). Upon transitioning to the backward propagation phase, we first recompute 309 the mask  $\mathcal{M}$  (Algorithm 2 Line 1), then the gradients of the weight matrices  $\mathcal{A}^{(t)}$  and  $\mathcal{B}^{(t)}$ , alongside 310 the input activation X, are computed (Algorithm 2 Line 2, 3, 4 and 5). Subsequently, we restore  $\hat{W}$ 311 from  $\mathcal{W}^{(t)}$  (Algorithm 2 Line 6). The operational workflow of the optimized SP-LoRA is illustrated 312 in Figure 2. 313 Refer to the Formula 6 and 7, after memory optimization, the requisite GPU memory allocation is 314 confined to the parameters  $\mathcal{M}$  and Y, encompassing RC + RL parameters (a reduction from the 315 initial 4RC + RL). Similarly, only the input activation X, comprising CL parameters (a decrease 316

from the original 2RC + CL), needs to be retained for the back-propagation process. During the backward pass, memory allocation is necessary for the gradients dX,  $d\tilde{W}^{(t)}$ ,  $dA^{(t)}$ , and  $dB^{(t)}$ , along with the mask  $\mathcal{M}$ , totaling 2RC + CL + rR + rC parameters, consistent with the memory requirements before optimization.

While this optimization incurs an additional computational cost of rR + rC + 2RC FLOPs (Algorithm 2 Line 6), this increment is relatively insignificant against the total computational FLOPs ( $\approx RCL$ ). As shown in Figure 1, the optimized SP-LoRA achieves similar time and memory overheads with LoRA, thereby ensuring its practical viability.

4	Model	Mehtod	Sparsity	ARC-c	ARC-e	BoolQ	Hellaswag	OBQA	RTE	Winogrande	Average
5 Lla	uma-2-7B	None	None	43.52	76.35	77.74	57.14	31.40	62.82	69.06	59.72
6		SparseGPT	2:4	31.31	63.93	68.90	43.54	24.60	63.18	65.90	51.62
		SparseGPT+SPP	2:4	34.30	67.38	68.29	50.54	27.00	64.26	66.93	54.10
		SparseGPT+LoRA	None	35.58	68.86	66.76	50.92	27.00	66.79	66.61	54.65
		SparseGPT+SP-LoRA	2:4	34.98	68.27	66.61	50.79	27.00	63.18	66.77	53.94
		Wanda	2:4	30.03	61.95	68.32	41.21	24.20	53.07	62.35	48.73
		Wanda+SPP	2:4	34.81	68.39	70.03	49.56	26.60	57.40	65.43	53.17
		Wanda+LoRA	2:4	36.01	69.19	71.71	50.61	27.00	58.84	64.72	54.01
		Wanda+SP-LoRA	2:4	35.75	70.29	70.43	50.33	27.60	60.29	64.48	54.16
Lla	ma-2-13B	None	None	48.38	79.42	80.55	60.04	35.20	65.34	72.30	63.03
	-	SparseGPT	2:4	37.29	69.07	79.05	48.00	25.80	58.84	69.14	55.31
		SparseGPT+SPP	2:4	40.78	72.43	76.82	55.23	29.20	59.21	68.75	57.49
		SparseGPT+LoRA	None	39.76	72.81	76.54	55.51	31.20	66.79	69.61	58.89
		SparseGPT+SP-LoRA	2:4	39.85	72.90	76.30	55.65	30.00	67.51	69.38	58.80
		Wanda	2:4	34.47	68.48	75.72	46.39	24.40	57.04	66.69	53.31
		Wanda+SPP	2:4	40.02	71.51	75.72	54.55	29.40	62.09	69.61	55.56
		Wanda+LoRA	None	41.38	72.35	76.24	55.12	29.60	63.18	68.75	58.09
		Wanda+SP-LoRA	2:4	40.44	72.39	75.66	55.05	30.40	59.93	67.56	57.35
Lla	uma-3-8B	None	None	50.26	80.09	81.35	60.18	34.80	69.31	72.38	64.05
		SparseGPT	2:4	32.00	62.67	73.70	43.19	22.20	53.79	65.75	50.47
		SparseGPT+SPP	2:4	39.42	69.95	71.93	51.67	25.80	63.18	68.27	55.75
		SparseGPT+LoRA	None	38.74	70.03	75.54	52.24	28.80	59.93	67.01	56.04
		SparseGPT+SP-LoRA	2:4	38.14	70.29	75.87	52.35	26.80	63.90	67.56	56.42
		Wanda	2:4	26.45	55.93	66.18	37.51	18.60	52.71	60.06	45.35
		Wanda+SPP	2:4	36.77	67.39	72.97	49.49	25.80	59.21	64.88	53.79
		Wanda+LoRA	None	37.12	69.11	73.61	50.94	27.60	59.21	66.38	54.85
		Wanda+SP-LoRA	2:4	38.31	69.53	71.56	50.83	28.00	54.87	66.30	54.20

Table 1: Zero-shot evaluation results of 7 tasks from EleutherAI LM Harness with models trained on a subset of the SlimPajama dataset with 0.5B tokens.

		Sparse	GPT	Wanda			
	SPP	LoRA	SP-LoRA	SPP	LoRA	SP-LoRA	
SlimPajama-0.5B	7.33	7.09	7.10	7.39	7.12	7.13	
Stanford Alpaca	8.19	9.73	9.34	8.42	9.83	10.16	

Table 2: Perplexity of pruned Llama-2-7B on wikitext2 after fine-tuning through SlimPajama-0.5B and Alpaca datasets respectively.

## 4 EXPERIMENTS

In this section, we will illustrate the effectiveness of SP-LoRA in training sparse LLMs through experiments.

Experiment Setup We conducted our experiments using the Llama-2-7B, Llama-2-13B, Llama-3-8B and Llama-3.1-8B-instruct models (Touvron et al., 2023a;b; Dubey et al., 2024). Initially, we applied post-training pruning techniques, specifically SparseGPT and Wanda, with the 2:4 sparsity type. Subsequently, the pruned models were fine-tuned using three distinct datasets: pre-training, instruction, and domain-specific. During fine-tuning, adapters were added to all sparse weight matrices within the model.

<sup>&</sup>lt;sup>1</sup>addmm\_ and mul\_ are APIs in PyTorch for implementing in-place matrix multiplication and element-wise multiplication.

378	Model	Mehtod	Sparsity	ARC-c	ARC-e	BoolQ	Hellaswag	OBQA	RTE	Winogrande	Average
379	Llama-2-7B	None	None	43.52	76.35	77.74	57.14	31.40	62.82	69.06	59.72
380		SparseGPT	2:4	31.31	63.93	68.90	43.54	24.60	63.18	65.90	51.62
381		SparseGPT+SPP	2:4	36.86	69.15	72.91	50.67	28.80	62.45	66.30	55.31
382		SparseGPT+LoRA	None	35.67	63.13	70.73	51.19	26.40	70.40	64.09	54.52
383		SparseGPT+SP-LoRA	2:4	36.01	64.35	72.17	51.84	29.60	59.93	63.61	53.93
384		Wanda	2:4	30.03	61.95	68.32	41.21	24.20	53.07	62.35	48.73
385		Wanda+SPP	2:4	36.26	69.44	72.02	49.64	27.80	55.96	63.77	53.56
386		Wanda+LoRA	None	35.32	64.18	71.99	50.60	28.40	60.65	63.14	53.47
387		Wanda+SP-LoRA	2:4	35.41	65.03	72.39	50.18	30.00	60.29	62.67	53.71
388	Llama-2-13B	None	2:4	48.38	79.42	80.55	60.04	35.20	65.34	72.30	63.03
389		SparseGPT	2:4	37.29	69.07	79.05	48.00	25.80	58.84	69.14	55.31
390		SparseGPT+SPP	2:4	42.06	73.32	78.62	55.02	29.40	65.70	69.77	59.13
391		SparseGPT+LoRA	None	40.78	67.93	76.48	54.68	29.40	71.12	69.38	58.54
392		SparseGPT+SP-LoRA	2:4	43.00	70.37	76.88	55.91	31.60	68.95	70.17	59.55
393		Wanda	2:4	34.47	68.48	75.72	46.39	24.40	57.04	66.69	53.31
394		Wanda+SPP	2:4	41.89	72.73	77.37	54.84	30.40	65.34	68.27	58.69
395		Wanda+LoRA	None	40.02	68.35	76.09	54.17	29.80	64.98	66.93	57.19
396		Wanda+SP-LoRA	2:4	39.42	69.40	78.01	55.16	30.00	72.20	67.80	58.86
307	Llama-3-8B	None	2:4	50.26	80.09	81.35	60.18	34.80	69.31	72.38	64.05
202		SparseGPT	2:4	32.00	62.67	73.70	43.19	22.20	53.79	65.75	50.47
200		SparseGPT+SPP	2:4	40.78	71.09	75.35	52.01	26.40	59.93	67.88	56.21
399		SparseGPT+LoRA	2:4	38.31	65.45	76.79	50.51	28.20	54.51	62.98	53.82
400		SparseGPT+SP-LoRA	2:4	38.05	64.02	73.27	48.89	25.20	60.65	62.12	53.17
401		Wanda	2:4	26.45	55.93	66.18	37.51	18.60	52.71	60.06	45.35
402		Wanda+SPP	2:4	38.48	68.64	74.77	49.53	25.20	58.48	64.64	54.25
403		Wanda+LoRA	2:4	38.05	64.02	73.27	48.89	25.20	60.65	62.12	53.17
404		Wanda+SP-LoRA	2:4	37.46	65.07	73.36	49.48	26.00	63.18	62.75	53.90
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Table 3: Zero-shot evaluation results of 7 tasks from EleutherAI LM Harness with models trained on the Alpaca dataset.

Model	Sparsity	ARC-c	ARC-e	BoolQ	Hellaswag	OBQA	RTE	Winogrande	Average
Llama-3.1-8B-instruct	None	51.71	81.86	84.07	59.10	33.80	67.87	73.95	64.62
+SparseGPT	2:4	34.30	65.45	77.74	43.56	22.20	61.73	66.30	53.04
+SP-LoRA									
+FineWeb-Edu-5B	2:4	43.60	77.90	76.36	54.19	32.40	64.62	69.85	59.85
+FineWeb-Edu-5B & Alpaca	2:4	44.80	74.54	77.98	55.86	34.80	67.87	70.01	60.83

Table 4: Zero-shot evaluation results of 7 tasks from EleutherAI LM Harness with Llama-3.1-8B-instruct model trained on the FineWeb-edu-5B and Alpaca dataset.

• For the pre-training data, we utilized a subset of the SlimPajama dataset (Penedo et al., 2023), consisting of 0.5B tokens. After continual pre-train the model, we tested the model's zero-shot performance on seven datasets selected from EleutherAI LM Harness (Gao et al., 2024), including ARC-c, ARC-e (Clark et al., 2018), BoolQ (Clark et al., 2019), Hellaswag (Zellers et al., 2019), OBQA (Mihaylov et al., 2018), RTE, and Winogrande (Sakaguchi et al., 2019). During the training, the rank of adapters is set to 16, the batch size is set to 256k tokens, and the learning rate is set to  $1 \times 10^{-3}$ .

• For the instruction data, we use the Stanford-Alpaca dataset (Taori et al., 2023). After fine-tuning the model, we tested the model's zero-shot performance as above. During the training, the rank of adapters is set to 16, the batch size is set to 32 samples, and the learning rate is set to  $1 \times 10^{-3}$ .

For the domain-specific dataset, we consider three domains: chat, math, and code. Specially, we used a 52k subset of WizardLM (Xu et al., 2023) for chat, a 100k subset of MetaMathQA (Yu et al., 2024) for math, and a 100k subset of Code-Feedback (Zheng et al., 2024) for code. Before the fine-

432	Method	Sparsity	MT-Bench	GSM8k (0-shot)	Human-eval (Pass@5)
433	LoRA	None	7.58	80.21	79.4
434 435	SparseGPT & LoRA	None	6.11	67.93	51.8
436	SparseGPT & SP-LoRA	2:4	5.91	67.85	49.4

Table 5: Evaluation results of pruned Llama-3.1-8B-instruct model that continually pre-trained on the FineWeb-edu-5B and fine-tuned on Meta-Math, CodeFeedback, and WizardLM.

tuning, we first continually pre-train the model on a subset of FineWeb-edu dataset (Penedo et al., 2024) with 5B tokens and Stanford Alpaca dataset. Then, we fine-tune the model on three datasets WizardLM, MetaMathQA, and Code-Feedback, respectively. Finally, we tested the model's performance in each domain on the benchmarks MT-Bench (Zheng et al., 2023), GSM8K (Cobbe et al., 2021), and Human-eval (Chen et al., 2021) respectively. During the training, the rank of adapters is set to 128, the batch size is set to 256k tokens for the FineWeb-edu dataset and 32 samples for domain-specific data and the Stanford Alpaca dataset, and the learning rate is set to  $2 \times 10^{-4}$ .

All the training and testing processes are conducted on Nvidia A800-80G GPU and Nvidia A6000-48G GPU.

Baselines We evaluated models trained using SP-LoRA against both the original dense models
and those pruned by SparseGPT and Wanda. We also compared SP-LoRA with LoRA, a wellknown parameter-efficient tuning method for LLMs, and SPP, an existing sparsity-preserving tuning
method for sparse LLMs. Beyond evaluating model performance, we also measured each approach's
training time and memory overhead.

458 4.1 MAIN RESULTS

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Table 1 and Table 3 illustrate the zero-shot performance of the Llama-2-7B, Llama-2-13B, and Llama-3-8B models, along with their respective versions that were pruned and fine-tuned using the SlimPajama-0.5B and Stanford Alpaca datasets.

The experimental outcomes indicate that SP-LoRA enhances the performance of sparse models, 463 demonstrating an improvement ranging from 2% to 9% over sparse models derived through post-464 training pruning techniques. Furthermore, SP-LoRA performs similarly to established methodolo-465 gies such as LoRA and SPP. Notably, while LoRA effectively improves the performance of pruned 466 LLMs, this approach diminishes practical usability due to the resultant dense model. Conversely, 467 SPP relies on tensor parallelism (Shoeybi et al., 2020) to mitigate the high memory footprint asso-468 ciated with sparse LLMs training, limiting its applicability in resource-constrained environments. 469 At the same time, it may also introduce additional communication overheads when considering sce-470 narios of parallel training through multiple GPUs. A detailed comparative analysis between SPP and SP-LoRA is provided in Appendix A. It is important to acknowledge that our training involved 471 a constrained dataset; hence, augmenting the volume of training data would likely yield further 472 enhancements in model performance, as evidenced in Table 4. 473

474 Tables 1 and 2 indicate that we utilized the SlimPajama (pre-training data) and Stanford Alpaca 475 (instruction data) datasets for fine-tuning, observing that the resulting models exhibit comparable 476 performance. However, the perplexity scores on the wikitext2 dataset, as shown in Table 2, reveal a 477 significant discrepancy. Fine-tuning with the pre-training data results in lower perplexity compared to fine-tuning with the instruction data. This suggests that instruction fine-tuning data may be more 478 effective in enhancing performance on downstream tasks than pre-training data. While existing 479 methods, such as SPP, evaluate sparse models trained on instruction fine-tuned datasets against the 480 base model, our findings suggest that utilizing pre-trained data for comparisons might provide a 481 more equitable assessment. 482

To evaluate the domain adaptation capabilities of SP-LoRA, we conducted experiments using the Llama-3.1-8B-instruct model. Initially, the model was pruned using SparseGPT. Subsequently, to restore the model's performance, we employed SP-LoRA for fine-tuning alongside the FineWebedu-5B and Alpaca datasets. The evaluation results of the fine-tuned sparse model are presented in

486 Table 4. Furthermore, we fine-tuned both the dense and sparse models using LoRA and SP-LoRA 487 on the WizardLM, MetaMathQA, and Codefeedback datasets, respectively. The models were then 488 evaluated on the MT-bench, GSM-8k, and Huam-Eval benchmarks, as summarized in Table 5. Our 489 results indicate that the fine-tuned sparse model achieves approximately 78% of the performance 490 level of the dense model on chat tasks, 85% of the performance level on mathematical tasks, and 65% of the performance level on coding tasks. At the same time, SP-LoRA has a competitive 491 performance compared to LoRA in fine-tuning sparse model. In terms of code-related task Human-492 Eval, SP-LoRA exhibits poorer performance. A potential reason for this could be the lack of code 493 data during continuous pre-training. We posit that the performance of the sparse model could be 494 further enhanced by supplementing additional code data. 495

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#### 4.2 TIME AND MEMORY OVERHEAD

498 In addition to model performance, we also evaluate the time and memory overhead of fine-tuning 499 the sparse LLM using different methods, including LoRA, SP-LoRA with our proposed mem-500 ory optimization (SP-LoRA), SP-LoRA with gradient checkpointing optimization (SP-LoRA(GC)), 501 SP-LoRA with no optimization (SP-LoRA(NO)), SPP with gradient checkpointing optimization 502 (SPP(GC)), and SPP with no optimization (SPP(NO)). The implementation details of these methods are presented in Appendix B. We performed our experiments on a single Nvidia A6000 GPU with 504 the batch size set to 1 and the sequence length set to 2048. The experimental results are shown 505 in Figure 1. It can be seen that SP-LoRA outperforms SPP(GC) and SPP(NO) in terms of speed and memory overhead, where SPP(NO) leads to out-of-memory error, and gradient checkpointing 506 significantly reduces SPP(GC)'s training speed. Also, SP-LoRA is faster and uses less memory 507 than SP-LoRA(GC), while significantly reducing memory usage compared to the SP-LoRA(NO). 508 Finally, compared to LoRA, SP-LoRA has similar time and memory overheads. All these results 509 demonstrate the effectiveness of our approach. 510

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#### 5 CONCLUSION AND FUTURE WORKS

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514 In this paper, we introduce the SP-LoRA method, which is a parameter-efficient and memory-515 efficient approach for training sparse models while preserving the sparsity. Our approach addresses 516 the challenges of domain adaptation and performance restoration for sparse LLMs. Specifically, we introduce additional masks in the LoRA framework, thus preserving the sparsity of the LLM dur-517 ing training, and achieve memory efficiency by using a hybrid gradient checkpointing and memory 518 reutilizing approach. Experiments on the Llama family show that SP-LoRA can effectively recover 519 the performance of pruned LLMs and has comparable performance to LoRA on domain migration 520 tasks. 521

Currently, in the SP-LoRA framework, we only consider static masks, and at the same time, we
 do not use LoRA variants to further improve the performance of SP-LoRA. Therefore, looking
 ahead, we will try to use different improved versions of LoRA combined with dynamic mask tuning
 methods for better performance.

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# A COMPARISON BETWEEN SPP AND SP-LORA

SPP (Lu et al., 2024) is also a parameter-efficient and sparsity-preserving fine-tuning methodology.
 The formulation of SPP can be mathematically described as follows:

$$\tilde{\mathcal{W}}^{(t)} = \tilde{\mathcal{W}} + \tilde{\mathcal{W}} \odot \operatorname{Repeat}_{1}(\mathcal{A}^{(t)}, \frac{C}{r}) \odot \operatorname{Repeat}_{0}(\mathcal{B}^{(t)}, R),$$
(8)

where  $\tilde{\mathcal{W}} \in \mathbb{R}^{R \times C}$  denotes the updated weight matrix,  $\mathcal{A} \in \mathbb{R}^{R \times r}$  and  $\mathcal{B} \in \mathbb{R}^{1 \times C}$  represent the learnable parameter matrices, and  $\operatorname{Repeat}_i(x, n)$  means repeating the tensor x along axis i for ntimes. The adjustment to the weight matrix, denoted by  $\tilde{\mathcal{W}} \odot \operatorname{Repeat}_1(\mathcal{A}^{(t)}, \frac{C}{r}) \odot \operatorname{Repeat}_0(\mathcal{B}^{(t)}, R)$ , is formulated as the Hadamard product of these three matrices, thereby maintaining the sparsity structure inherent in the matrices involved. Furthermore, the parameters  $\mathcal{A}^{(t)}$  and  $\mathcal{B}^{(t)}$  are the only ones subject to training, which significantly reduces the parameters compared to that of  $\tilde{\mathcal{W}}$ , thus exemplifying the parameter efficiency of this approach.

It is observed that SPP can be conceptualized as a variant of LoRA. To illustrate this perspective, consider partitioning each sequence of r consecutive elements within  $\mathcal{B}$  into segments, such that:

$$\mathcal{B} = [\mathcal{B}_1, \mathcal{B}_2, \dots, \mathcal{B}_{\underline{C}}],\tag{9}$$

where each segment  $\mathcal{B}_i$  is a vector of length r. Subsequently, we define a block-diagonal matrix  $\hat{\mathcal{B}}$  constructed from these segments:

$$\hat{\mathcal{B}} = [\operatorname{diag}(\mathcal{B}_1), \operatorname{diag}(\mathcal{B}_2), \dots, \operatorname{diag}(\mathcal{B}_{\underline{C}})].$$
(10)

836 With this definition, the update rule for the weight matrix  $\hat{W}$  can be rewritten as:

$$\tilde{\mathcal{W}}^{(t)} = \tilde{\mathcal{W}} + \tilde{\mathcal{W}} \odot (\mathcal{A}^{(t)} \times \hat{\mathcal{B}}^{(t)}).$$
(11)

Therefore, SPP can be interpreted as a LoRA variant that employs a specialized matrix  $\hat{\mathcal{B}}$ , augmented with the initial weight matrix  $\tilde{\mathcal{W}}$  as a weight term, to achieve its parameter-efficient and sparsitypreserving properties.

Recalling the mathematical form of the SP-LoRA,

$$\tilde{\mathcal{W}}^{(t)} = \tilde{\mathcal{W}} + \mathcal{M} \odot (\mathcal{A}^{(t)} \times \mathcal{B}^{(t)}).$$
(12)

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The distinctions between SPP and SP-LoRA can be delineated as follows:

- SPP employs a composite weight matrix  $\hat{\mathcal{B}}$  formed by stitching together multiple diagonal matrices, whereas SP-LoRA utilizes a standard matrix  $\hat{\mathcal{B}}$  as its weight matrix.
- SPP incorporates the initial weight matrix  $\tilde{W}$  as an additional weight term, while SP-LoRA leverages a mask matrix M as an additional weight term.

854 Incorporating the initial weight matrix  $\hat{W}$  as an additional weight term endows SPP with certain advantages in instruction fine-tuning. However, this approach precludes SPP from benefiting from 855 the proposed memory reuse technique and poses the challenge of high GPU memory overhead. 856 To solve the problem of high GPU memory usage, SPP uses tensor parallelism, where the weight 857 matrices are sliced and stored separately within different GPUs. However, this optimization requires 858 multiple GPUs to implement and thus cannot be applied to low-resource fine-tuning scenarios with 859 only a single GPU. Also, in multi-GPU parallel training scenarios, SPP enforcing the use of tensor 860 parallelism may reduce the training speed due to the increased communication overhead. 861

Conversely, the proposed method, SP-LoRA, achieves comparable time and memory overheads to
 those of LoRA through optimized memory usage, while simultaneously maintaining equivalent per formance levels as SPP.

## **B** IMPLEMENTATION OF SPP AND SP-LORA VARIANTS

```
def forward_adapter(x, W, A, B):
    n, m = W.shape
    r = A.shape[1]
    A = torch.repeat_interleave(weight, m // r, dim=1)
    B = torch.repeat_interleave(weight, n, dim=0)
    W_adapter = W * A * B
    return F.linear(x, W_adapter)
def forward_spp(x, W, A, B):
    y1 = F.linear(x, W)
    y2 = forward_adapter(x, W, A, B)
    return y1 + y2
Listing 1: Implementation of SPD(NO)
```

Listing 1: Implementation of SPP(NO)

```
def forward_adapter(x, W, A, B):
    n, m = W.shape
    r = A.shape[1]
    A = torch.repeat_interleave(weight, m // r, dim=1)
    B = torch.repeat_interleave(weight, n, dim=0)
    W_adapter = W * A * B
    return F.linear(x, W_adapter)
def forward_spp(x, W, A, B):
    y1 = F.linear(x, W)
    # gradient checkpointing
    y2 = checkpoint(forward_adapter, x, W, A, B)
    return y1 + y2
```

#### Listing 2: Implementation of SPP(GC)

```
def forward_adapter(W, A, B):
    M = (W != 0)
    return W + M * (A @ B)
def forward_sp_lora(x, W, A, B):
    W_new = forward_adapter(W, A, B)
    return F.linear(x, W_new)
```

#### Listing 3: Implementation of SP-LoRA(NO)

```
def forward_adapter(W, A, B):
    M = (W != 0)
    return W + M * (A @ B)
def forward_sp_lora(x, W, A, B):
    # gradient checkpointing
    W_new = checkpoint(forward_adapter, W, A, B)
    return F.linear(x, W_new)
Listing 4. Implementation of SDL = DA(CC)
```

Listing 4: Implementation of SP-LoRA(GC)

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