

Optimistic Exploration in Reinforcement Learning Using Symbolic Model Estimates

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Abstract

There has been an increasing interest in using symbolic models along with reinforcement learning (RL) problems, where these coarser abstract models are used as a way to provide RL agents with higher level guidance. However, most of these works are inherently limited by their assumption of having an access to a symbolic approximation of the underlying problem. To address this issue, we introduce a new method for learning optimistic symbolic approximations of the underlying world model. We will see how these representations, coupled with fast diverse planners developed by the automated planning community, provide us with a new paradigm for optimistic exploration in sparse reward settings. We investigate the possibility of speeding up the learning process by generalizing learned model dynamics across similar actions with minimal human input. Finally, we evaluate the method, by testing it on multiple benchmark domains and compare it with other RL strategies.

1 Introduction

A popular trend in recent years is using symbolic planning models with reinforcement learning (RL) algorithms. Works have shown how these models could be used to provide guidance to RL agents (Yang et al. 2018; Lee et al. 2022; Gehring et al. 2022), to provide explanations (Sreedharan et al. 2022), and as an interface to receive guidance and advice from humans (Kambhampati et al. 2022). Coupled with the fact that advances in automated planning has made available a number of robust tools that RL researchers could adapt directly to their problems (cf. (Francés, Ramirez, and Collaborators 2018; Muise et al. 2022; Silver and Chitnis 2020)), these methods have the potential to help addressing many problems faced by state-of-the-art RL methods. However, a major hurdle to using these methods is the need to access a complete and correct symbolic model of the underlying sequential-decision problems. While there have been efforts from the planning community to learn such models (Juba and Stern 2022; Yang, Wu, and Jiang 2007), most of those methods have focused on cases where the models are synthesized from a set of plan traces, hence corresponding to the traditional offline reinforcement learning setting. Interestingly, very few works have been done in synthesizing such models in the arguably more prominent RL paradigm, namely, online RL.

To fill this gap, in this paper we propose a novel algorithm to learn relevant fragments of symbolic models in an online fashion. We show how it could be used to address one of the central problems within RL, namely effective exploration. We show how our method allows us to perform goal-directed optimistic exploration, while providing rigorous theoretical guarantees. The exploration mechanism leverages two distinct components: (a) a representation that captures the most optimistic model that is consistent with the set of observations received, and (b) the use of a fast and suboptimal diverse planner that generates multiple possible exploration paths, which are still goal-directed.

The idea of optimistic exploration is not new within the context of RL. The most prominent method being the RMax algorithm (Brafman and Tenenholz 2002). RMax modifies the reward function to develop agents that are optimistic under uncertainty. Our use of symbolic models, however, allows us to maintain an optimistic hypothesis regarding the underlying transition function. Coupled with a goal-directed planner, this lets us perform directed exploration in sparse reward settings, where we have a clear specification of the goal state but no intermediate rewards. As we show in this work, for a finite state deterministic MDP our method is guaranteed to generate a goal-reaching policy.

Additionally, we investigate the use of a structured form of generalization rule that leverages a very simple intuition, namely the effects of an action don't depend on specific object labels but only on object types. Commonly referred to as lifted representation in planning literature, we show this rule to speed up learning with minimal human input.

The rest of the paper is structured as follows. We start with related work in Section 2. Section 3 provides a formal definition of the exact problem we are investigating and Section 5 shows the empirical evaluation of our method against a set of baselines. Finally, Section 6 concludes the paper with a discussion of the methods and possible future directions.

2 Related Work

As mentioned earlier, one of the foundational works in optimistic exploration in the context of reinforcement learning is R-max (Brafman and Tenenholz 2002). Even before the formulation in its current popular form, the idea of optimism under uncertainty has found several uses within the RL literature (cf. (Kaelbling, Littman, and Moore 1996)).

R-max can be seen as an instance of a larger class of intrinsic reward based learning (Aubret, Matignon, and Has-sas 2019), but one where the reward is tied to state novelty. Other forms of intrinsic rewards incentivizes the agent to learn potentially useful skills and new knowledge. A context where model simplification has been used in areas related to RL is in the context of stochasticity, where methods like certainty equivalence and hindsight optimization has been applied (Bertsekas 2021; Yoon et al. 2008). In Section 6, we will see how we can also apply our methods directly in settings with stochastic dynamics. In regards to the user of symbolic models, the most common use is in the context of hierarchical reinforcement learning. Many works (Lee et al. 2022; Illanes et al. 2020; Yang et al. 2018; Lyu et al. 2019), have investigated the possibility of using the symbolic model to generate potential options and then using a meta-controller to learn policies over such options. While most of these work assume that the model is in some way an approximation of the true model, all inferences performed at the symbolic level is performed over the original model provided as part of the problem. While in this work, we focused on cases where the symbolic model could in theory exactly capture the underlying model, the same techniques can also be applied to cases where the planning model may represent some abstraction of the true model. Another popular use of symbolic model is as source of reward shaping information (cf. (Gehring et al. 2022)). In this context, works have also looked at symbolic models as a vehicle to precisely specify their objective (Icarte et al. 2018; Giacomo et al. 2019).

In terms of learning symbolic model, interestingly the work has mostly focused on learning plan or execution traces (Yang, Wu, and Jiang 2007; Juba and Stern 2022; Callanan et al. 2022; Cresswell, McCluskey, and West 2013). In most of these works, the theoretical guarantee you are aiming for is to generate more pessimistic models that are always guaranteed to work, but may overlook plausible plans. This is completely antithetical to considerations one must employ when performing explorations in common online RL settings, where the agent is either operating in a safe environment or interacting with a simulator. To the best of our knowledge, all existing online methods to acquire symbolic models (Carbonell and Gil 1990; Lamanna et al. 2021), focuses on extracting an exact representation of the true underlying model. Since our primary motivation for learning this model is to drive the exploration process, we do not have that limitation. Instead, we focus on learning a (more permissive) optimistic approximation. Also it is worth noting that, the assumption that the system will be provided action arguments (something we will leverage in Section 4.3) is one commonly made by most of these works. There are also some works that are trying to automatically acquire abstract symbolic models from an underlying MDP (including potential symbols) like that by (Konidaris, Kaelbling, and Lozano-Pérez 2018). This direction is orthogonal to our work, as symbols produced by them may be meaningless to the human and we are explicitly trying to leverage human’s intuitions about the problem.

3 Problem Setting

3.1 Deterministic MDP

The central problem we are interested in addressing is that of an RL agent trying to come up with a policy for a deterministic MDP with sparse reward. We specifically chose a setting, which forefronts the problems related to exploration, while placing less emphasis on other aspects of an RL problem (though in Section 6 how one could easily apply this approach to other settings). In particular, the underlying model (which is unknown) is assumed to be of the form $\mathcal{M} = \langle \mathcal{S}; T; A; I; R \rangle$. Under this definition, \mathcal{S} is a finite set that represents the set of possible states that the agent could find itself in. We will assume that this set includes a special absorbing state $g \in \mathcal{S}$, allowing to capture both abnormal and normal trace termination. Additionally, there is a subset of states $S^G \subseteq \mathcal{S}$, which correspond to ‘goal’ states, which are desirable states for the agent. A is the set of possible actions and given the deterministic nature of the problem, the transition function is specified as $T : \mathcal{S} \times A \rightarrow \mathcal{S} \cup \{f\}$; $0g$. We will refer to the action that transitions from states in S^G to the absorbing state g as the goal action (a^G). Given that we are interested in sparse reward setting, we will define the reward function as follows

$$R(s; a^G; s^j) = \begin{cases} 1; & \text{if } s \in S^G \text{ and } s^j = g \\ 0 & \text{otherwise.} \end{cases}$$

We will refer to the transition to g through a non goal action, as a failure transition. Now, $I \in \mathcal{S}$ captures the initial state from which the agent starts. To enumerate the implications of the design choices we made in picking this model, consider the fact that reward is zero everywhere except for the goal. This means that any policy that can help reach the goal from the initial state, would be optimal for the agent (since the agent always starts from the initial state). Coupled with the fact that the transition function is deterministic, once the agent identifies a goal state, constructing an optimal policy is relatively easy as they can just reuse the path taken by the exploration strategy. Now, this setting also renders most existing methods that may use intermediate value bootstrapping or generalization mostly ineffective as there are no intermediate values to use. So it makes sense to focus on tabular methods as the RL baseline. In fact, possibly the only effective methods in the mainstream RL repertoire we can use are curiosity driven or intrinsic reward based methods and we will use such a method as a baseline. One of the central components we will leverage are state action traces we can sample from the underlying model. In particular, we say that a trace $\langle s_0; a_1; \dots; a_i; s_i; \dots; a_{k+1}; s_{k+1} \rangle$ is valid or equivalently goal-reaching if $s_0 = I; s_k \in S^G; s_{k+1} = g$, and for every $0 \leq i < k$ we have $T(s_i; a_{i+1}; s_{i+1}) > 0$. The action a_{k+1} is the goal action.

In the course of discussion, we will use the word ‘original model’ to refer to this true but unknown underlying MDP. The agent itself is expected to be either interacting with a generative simulator that encodes this MDP or is acting in the true environment provided that they can reset to the initial state at the end of each episode.

3.2 Symbolic Planning Models

For the symbolic model, we will be using an a representation scheme called Planning Domain Definition Language or PDDL. In particular, we will consider a version that will ignore object types (Helmert 2009). Here, a planning task is defined in relational terms, i.e., states are described in terms of objects and relationships between objects and each action is described in terms of the objects involved in that action and how they may affect or be affected by the relationships between these objects. Such a model is usually defined by the tuple $\mathcal{M}^S = \langle L; O; I; G \rangle$, where L is a first-order language, O a finite set of action schemas, I and G are specifications of the initial state and the goal, respectively. The first order language describes the objects and the relationships between these objects (captured as *predicates*). In the blocksworld example, the object consists of the various blocks and $\text{on}(\text{red}, \text{green})$ is a predicate that captures the fact that the red block is on top of the green block. Additionally, first order language allows specifying predicates over variables as well as actual objects. Formally, the first order language is specified as $L = \langle B; V; P \rangle$, where B is the set of all objects, V are the variable names and P are the predicates. Each predicate $p \in P$, will take a fixed number of arguments. For the purpose of discussion in this paper, we will either have cases where the arguments consist of only variables or only objects. We will refer to the former case as being the lifted representation of the predicate and the latter as a grounded instance of the predicate (or ground predicate). In general, however, predicates can be partially grounded, with some of the arguments being actual objects while others being variables. States of the model correspond to truth assignments to ground predicates. Each ground predicate can take either a true or a false value. Each possible state for a given model is captured by a specific instantiation of all ground predicates. Thus, possible states can be uniquely represented by sets of ground predicates that are true (assuming the rest to be false under the closed world assumption). In this representation, I denotes a set of ground predicates, capturing the unique initial state. For our purposes, G will be captured by a subset of ground predicates, denoting all states where all of these ground predicates (and possibly some more) are true. All such states will be considered valid goal states.

Each action schema $o \in O$ provides the basic structure shared by a set of ground actions that can be actually executed by the agent. Each action schema is defined as $o = \langle \text{params}(o); \text{pre}(o); \text{add}(o); \text{del}(o) \rangle$, where $\text{params}(o)$ indicates the parameters of the action schema (variables and objects), the preconditions $\text{pre}(o)$, the add-effects $\text{add}(o)$, and the delete-effects $\text{del}(o)$. The latter three are first-order formula over the language L , specifying the conditions that must be satisfied for the action to be executable in a state, as well as the change in a state resulting from executing the action. Ground actions are obtained from the action schema by assigning objects to variables in the parameters. The agents are executing the ground actions and therefore it is common to describe the semantics of ground actions, henceforth simply referred to as actions. In this work we restrict ourselves to preconditions in disjunctive normal form. For ground ac-

tions these would be disjunctions of conjunctions of ground predicates. All states in which the formula holds, the action is applicable. The add/delete effects are conjunctions of ground predicates, making these predicates true respectively false in the state resulting in successful action execution.

For a given action schema o , we will denote the grounded instance obtained by replacing the parameter with an object list using the function symbol $\gamma(o; \cdot)$. We can also define an inverse mapping function $\gamma^{-1}(o^\#; \cdot)$ that retrieves the lifted model given a grounded instance (we can do this by replacing all instances of an object with a variable). This lifting function γ^{-1} is well defined in all cases where we don't have a repeating object in \cdot . In this particular work, we will only focus on applying such lifting functions in cases where they are well defined. Overloading the notations a bit, we will also apply the functions γ and γ^{-1} , to create grounded and lifted instances of predicates as well. Each planning problem can be represented equivalently in a grounded form as $\mathcal{M}_\# = \langle F_\#; A_\#; I; G \rangle$, where $F_\#$ consists of all grounded predicates and $A_\#$ grounded actions. At most this model may have $2^{|F_\#|}$ states. A solution for a planning model is a plan $\pi = \langle a_1; \dots; a_n \rangle$, which is a sequence of action whose execution in initial state will lead to a goal state, i.e., $\gamma^{-1}(I) \subseteq G$ (where $\gamma^{-1}(I) = \gamma^{-1}(\gamma(a_1; \dots; a_n(I)))$).

3.3 Connecting the Symbolic Model to the MDP

For any given deterministic MDP \mathcal{M} of the form defined in Section 3.1, there must exist a symbolic model that can exactly capture the MDP. In particular, there is a surjective function (many-to-one) mapping the (ground) actions in the symbolic model to MDP actions. Every plan under the symbolic model maps by this mapping to a valid trace of the MDP. Section 7.2, shows by construction how such a model will always exist. However, rather than creating an arbitrary mapping to a symbolic model, we are interested in creating one that leverages the expertise of a human domain expert to creating a potentially more effective representation of the problem. In particular, we start by taking human input to learn how to symbolically represent the states of the MDP. In particular, we expect the human to specify a set of predicates and objects that they might associate with the given problem. We use the symbol $F_\#^C$ to capture the set of all ground predicate possible under this specification. Similar to previous explanatory works (Sreedharan et al. 2022; Sreedharan, Kulkarni, and Kambhampati 2022; Kambhampati et al. 2022) that have tried to learn symbolic representations of RL problems, we use this to learn binary classifier that test whether a ground predicate may be true in a given MDP state. We can learn such classifiers by collecting positive and negative examples for each ground concept. Once the classifiers are available, we can construct the symbolic state corresponding to each MDP state, by testing each classifier on any given MDP state. We use the function $C : S \rightarrow 2^{F_\#^C}$ as a way to capture the mapping between the states. For potential actions, we assume that every symbolic ground action corresponds to exactly one action in the MDP. Overloading the notations a bit, we use $C^{-1}(a)$ to represent the MDP action corresponding to the symbolic action a . As we will see

in Section 4.3, the agent can also potentially leverage the human’s intuitions about how they structure actions to further improve the effectiveness of our method. Finally, we expect the human to provide a specification of the goal states specified in terms of the ground predicates in $F_{\#}^C$. We denote this goal specification by G^C . Additionally, we require that the initial state for the symbolic model corresponds to $\mathcal{C}(I)$ and for any goal state $s \geq S_G$, $\mathcal{C}(s)$ satisfies G^C (or, equivalently, there is a symbolic goal action whose precondition meets this requirement).

4 Our Approach

The basis of our approach is an observation that every deterministic MDP has a *precise* symbolic representation. By precise representation, we mean that for the specific setting we consider here, there exists a symbolic model that can exactly simulate the MDP: any transition possible under the symbolic model must correspond to non-zero probability transition possible under the MDP and vice versa. However, as discussed, our objective is not to learn such a precise representation but rather only to learn an optimistic approximation. We start from a trivially optimistic representation of the underlying model, which we iteratively refine towards the true representation. At each iteration, the current symbolic representation is used to generate potential plans to the goal. These plans are then tested out in the environment and the observed outcomes of the execution of such action sequences are then used to refine our estimate towards the true representation. At every point of our model refinement process, we ensure that every subsequent model estimate generated is an optimistic one. By maintaining the optimistic nature of the representation, we ensure that no potential valid solution is overlooked at any point in the learning process. So we will start the discussion of our approach by providing a rigorous definition of what we mean by an optimistic representation. In particular, we are interested in creating symbolic representations that allow all valid traces that are possible under the original MDP to be possible under the new representations. Formally, we can define this requirement as

Definition 1 For an MDP model M , a symbolic model M^C defined over a symbol mapping $\mathcal{C}(\cdot)$ is said to be an **optimistic representation**, if for every action sequence $ha_1; \dots; a_k i$ such that there exists a valid trace (i.e. it reaches goal), there exists a valid plan in M^C of the form $ha_1^o; \dots; a_k^o i$, such that $\mathcal{C}^{-1}(a_i^o) = a_i$.

For the given set of grounded actions $A_{\#}^C$ and a grounded set of predicates $F_{\#}^C$, we can create a symbolic model that is guaranteed to be optimistic for any MDP whose action set is isomorphic to $A_{\#}^C$ and the state space can be represented using $F_{\#}^C$. In particular, the model will have empty preconditions and delete effect and the add effects would correspond to the set of all ground predicates. This means that every action is executable in every state and an execution of any action will satisfy the goal. We will denote this model as $M_0^C = hF_{\#}^C; A_{\#}^C; I^C; G^C i$. More formally, every action $a \geq A_{\#}^C$ will be defined as follows: $a = hpre_0^a; add_0^a; del_0^a i$,

where $pre_0^a = del_0^a = \cdot$; and $add_0^a = F_{\#}^C$. The fact that its an optimistic representation for any MDP possible in this context can be trivially proved (discussed in Section 7.2).

4.1 Refining the Model

Now, of course, while all valid traces for the original model correspond to a plan in M_0^C , the symbolic model may also support plans that may not correspond to any valid trace in the original model. Our basic strategy would be to use this model as a starting point to sample potential plans, simulate/execute them in the environment or simulator and use the outcomes (both successful and failed executions) to refine the current the current estimate. We will continue this process until we find a plan that leads us to the goal. Keeping this general approach in mind, the next step would be to define our model update rule. In particular, let us assume that we receive the following observation from the environment $hs; a; s^o i$, such that $s^o \notin \mathcal{?}$. Now we know this corresponds to the symbolic observation $h\mathcal{C}(s); \mathcal{C}(a); \mathcal{C}(s^o) i$. Given this observation, we know that any changes made in the state must be the result of the action. We will use this information to update action’s effects. For add effects, if the estimate previously had hypothesized the action making a predicate true, which doesn’t hold in $\mathcal{C}(s^o)$ then it can be removed from the add effects. Similarly, if there was a predicate that is made false in $\mathcal{C}(s^o)$ but was not part of the delete effects, it can be added to the set of delete effects. Formally, we can set the new estimate of the action as follows $a = hpre_{i+1}^a; add_{i+1}^a; del_{i+1}^a i$, where $pre_{i+1}^a = f j \geq pre_i^a$ and $\mathcal{C}(s)g$ and for effects we have $add_{i+1}^a = add_i^a \cap (F_{\#}^C \cap \mathcal{C}(s^o))$ and $del_{i+1}^a = del_i^a \cup [(\mathcal{C}(s^o) \cap \mathcal{C}(s))]$. If the sampled transition corresponds to a failure ($hs; a; ? i$), we will only update the precondition. Specifically, we will remove any precondition clause that satisfies the state and replace it with a set of preconditions that includes one of the predicates that was false in the model (this follows from the fact that the action failed because some predicate part of the true underlying precondition wasn’t true in the given state). More formally, for any $j \geq pre_i^a$, such that $\mathcal{C}(s)$, we remove and add $= f [f j f \geq (F_{\#}^C \cap \mathcal{C}(s))g$. The proof for why this update rules result in optimistic representations are provided as part of Proposition 7.2 in Section 7.2.

4.2 Overall Algorithm

Algorithm 1 presents the overall iterative algorithm we will be using to identify the action sequence that can lead to a goal state. The algorithm starts with the initial estimate of the model. It iteratively generates plans for the model estimate, which will then be used to progressively refining the model until we get a plan that corresponds to a path to a goal state. These plans are derived using a diverse planner that identifies a set of plans that are diverse in terms of the actions used. This is represented by the procedure *DiversePlanner* that takes the number of diverse plans to be generated as an argument (\cdot). Readers can check (Katz and Sohrabi 2020) for a more detailed discussion of diverse planners. These plans are first tested on the underlying environment/simulator to check whether they lead to the goal from the initial

Algorithm 1: The main procedure that iteratively refines the model, until a goal reaching trace is found

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Iterative-Model-Refinement
Input:  $\mathcal{M}_0^c$ ;
Output: An action sequence  $ha_1; \dots; a_k$  that will lead to the goal
Procedure:
 $\mathcal{M}_{curr} \leftarrow \mathcal{M}_0^c$ 
execution_statistics  $\leftarrow \hat{f}g$ , solvability_flag  $\leftarrow$  True
while solvability_flag is True do
     $\hat{\mathcal{M}}_{curr} \leftarrow$  PruneModel( $\mathcal{M}_{curr}$ , execution_statistics)
     $\hat{\Pi} \leftarrow$  DiversePlanner( $\hat{\mathcal{M}}_{curr}; \cdot$ )
    if  $\hat{\Pi} > 0$  then
        for  $\hat{\Pi} \geq 2$  do
            if  $\hat{\Pi}$  leads to goal in the environment then
                return  $\hat{\Pi}$ 
            else
                 $\mathcal{M}_{curr}; \text{execution\_statistics}$ 
                UpdateModel( $\mathcal{M}_{curr}; \hat{\Pi}; \text{execution\_statistics}$ )
            end if
        end for
    else
        solvability_flag  $\leftarrow$  False
    end if
end while
return No policy with non-zero Value

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state and if not the experiences sampled from their execution are used to refine the current model. Note that, given the optimistic nature of the model estimate, the planner would generally try to use actions that haven't been previously executed successfully. However, each future use of the action would become progressively harder due to the growing precondition set. With that said, one could further improve the planner behavior by being more careful about the actions being used as part of plans. If an action has been tested quite frequently, it would be better to de-prioritize its usage until no better alternative has been found. Note that this is quite similar to the kind of exploration performed in the context of multi-armed bandits (Lattimore and Szepesvári 2020). In fact, one could directly apply methods like UCB (Auer 2002) to select the action sets to be considered by the planner. This part of the algorithm is captured by the procedure *PruneModel*. To keep our implementation of the approach simple, we will use a simple queue based system to identify the actions to be included. The exact procedure we use to control the selection of actions is described in Section 7.3. The variable *execution_statistics* keeps track of previous action trials and the frequency of success per action. The procedure *UpdateModel* uses the rules described in Section 4.1 to use the sampled traces to update the given model estimate. One could also further improve the efficiency of the search by always testing all possible actions in every new state that is identified as part of the procedure.

Theorem 1 *Algorithm 1 will (a) terminate in a finite number of steps and (b) identify a path to a goal (provided one exists); as long as the diverse planner used is complete (i.e., it will return a non-empty plan set as long as there exists a valid plan).*

The proof for the theorem is provided in Section 7.2.

Leveraging the Exploration Strategy in the Context of RL algorithms:

In the context of the overall RL learning process, this exploration method will be used as a way to update the Q values (and depending on the algorithm, structures like replay buffers). Specifically, we will first run this exploration procedure to find a valid trace to the goal. Once such a trace is found, we can update the Q values of all the states that are part of that trace to a more informed value. Once updated, we can employ traditional RL algorithms to identify optimal policies. One could also leverage the proposed method in conjunction with other exploration strategies, during the learning process. It is important to note that any consecutive use of our approach for generating goal directed paths would be much more efficient, as the method will start from a more refined estimate of the model.

4.3 Leveraging Lifted Representation

The algorithm described above tests each of the available actions to learn a symbolic model corresponding to the observed behavior. However, one of the important points to note here is the fact that this means that the testing and by extension learning of the model occurs at the level of ground actions. As we had discussed earlier, a very common assumption made throughout symbolic models is that of the existence of a lifted representation of actions. Namely, the fact that the nature of actions could be described independently of the exact objects it may be interacting with. This is a very natural outcome that comes out of relational representations of tasks, where the state is represented in terms of objects and relationship between objects. After all, it is very easy to see that the outcome of picking up the red block should be quite similar to the case of picking up the green block. For example, if we observe that the execution of the action 'pick-up red block' results in the agent holding the red block in it's gripper; then it would be quite natural to assume that the execution of 'pick-up green block' should result in the agent holding the green block. We will leverage such symmetry by asking the human to provide some additional information about each action. Specifically, the human can provide us a basic annotation over what actions could share a lifted structure and what objects each actions might interact with. Note that we are not asking the users to specify what the lifted structure may be, but just a grouping of actions and an ordered list of relevant objects. The order may reflect the different roles played by the objects participating in the action. For example, when an object is being placed on top of another, the annotation may list the destination object first and then the object being placed on top of it. The exact ordering wouldn't matter provided they remain consistent through the annotations. Additionally, even if the grouping provided by the human may be a subset of the true possible grouping and the human provides a superset of the objects relevant to any given action, our generalization approach remains valid. The set of objects associated with each action could also be automatically extracted from natural language descriptions of actions, as performed by works

like that by (Feng, Zhuo, and Kambhampati 2018).

For a given set of actions that are marked as being grounded instances of the same lifted action, we will ensure that learned effects of all actions comply with the most refined action in the set. As discussed earlier, the effects of an action comprises of add and delete effects and for each component we can select the most refined set independent of each other. From the set of effect descriptions, we select the add effect set containing the minimum number of elements and the delete effect set containing the maximum number of elements. For each such set, we can create the lifted description using the lift function described earlier. Let min_add be the lifted description corresponding to the smallest set of adds and max_del be the largest set of deletes for a given set of actions corresponding to the same lifted action. Then we can simply replace the effect of every action with a grounding of these lifted actions. This will still result in an optimistic model description, as we can show that the min_add and max_del are still optimistic estimates

Proposition 1 *Let $A = \{a_1, \dots, a_m\}$ be a set of actions marked as being instances of a single lifted action a . Then min_add must be a superset of add effects of a and max_del a superset of deletes of a , where min_add and max_del are calculated for A*

The validity of this proposition is discussed in Section 7.2. This proposition now means that, once max_del and min_add are identified, then for every action a in the set of possible groundings we can replace add effects and delete effects with the corresponding grounding of the lifted effects, i.e., $\text{add}^a = \text{lift}(\text{min_add}; a)$ and $\text{del}^a = \text{lift}(\text{max_del}; a)$, where a is the object list corresponding to the action a .

One can follow similar lines of reasoning to show that the lifted description of the maximal precondition description is also guaranteed to entail the true underlying preconditions.

Lifted Representation as a Basis for Curriculum Learning. While the above discussion focused on leveraging lifted representation to speed up learning within a given planning problem, one could also use lifted representations as basis of transferring model information from one problem instance to another. Within classical planning problems, it is very common to separate the domain information represented in lifted terms from specific problem instances. Each problem instance could differ in terms of the number and identities of objects involved, the initial and goal state. However, actions applicable in every instance share the same lifted definition. Even within RL, benchmarks consisting of various instances of the same problem domain are becoming more popular (Minigrid (Chevalier-Boisvert, Willems, and Pal 2018) being a popular example). When such a suite of problem instances are available, one could further speed up learning by using the smaller instance to learn as much of the lifted model as possible. Once such a partial lifted model is learned, it can then be used to refine the optimistic model in the target problem, where the normal learning process then takes over. Note that the access to a smaller problem instance doesn't obviate the need to perform learning in the true underlying model. After all, in the smaller problem there may

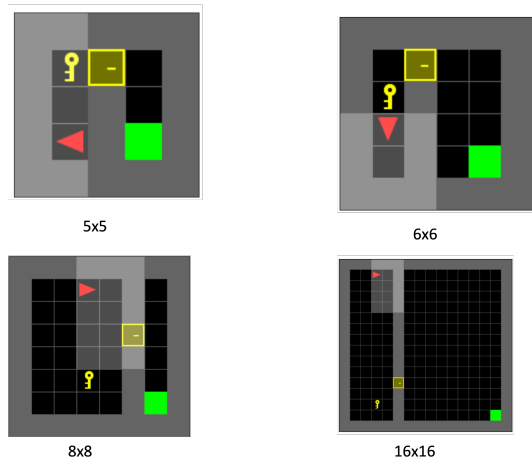


Figure 1: The visualization of the four minigrid maps considered for measuring the overall learning performance and for curriculum learning.

be lifted actions that may not be executable in any of the reachable states, but needs to be used in the target problem to reach the goal.

5 Evaluation

We perform our evaluation in four different domains. Three of these correspond to traditional planning domains and one a more traditional reinforcement learning benchmark. The planning benchmarks include blocksworld, a simple gridworld type domain involving robot picking up objects and a domain where the agent has to control elevator schedules. For the RL domain, we looked at two variants of minigrid problem. One was the version introduced by Lee et al. (2022) (henceforth referred to as Minigrid-Parl) and the other being a simplified version of the original minigrid testbed (Chevalier-Boisvert, Willems, and Pal 2018). We chose to use the Minigrid-Parl, since it provided us with annotations that allowed us to use hierarchical RL methods. For the latter variant, we dropped the turn action and introduced directional movement, pick up, drop and toggle actions. This allowed us to use simpler PDDL formalisms to capture the domain. For each planning domain, we selected five different problems (the sizes are approximately listed in the tables in terms of the number of grounded predicates) and two problems for the minigrid domain. We created a simulator wrapper around pddl models for each of the problems, as it allowed us easy access to the annotation information for lifting. For the minigrid problems, we auto generated pddl problem files from the simulator code for each specific environment.

5.1 Reaching Goal States

As a first step, we are interested in identifying how our proposed method stacks up against standard exploration strategies used as part of RL algorithms. In particular, we were interested in comparing our method against three baselines. First off, we were interested in see how it stacks up against

vanilla ϵ -greedy exploration (as implemented by the SimpleRL framework (Abel 2019), as part of the Q learning agent). Second, we compare to an R-max based exploration strategy (again taken from the SimpleRL framework), which as we discussed is a form of intrinsic reward. Finally, we compare to a hierarchical RL method that learns a policy over SMDP using PPO (as implemented by (Lee et al. 2022)) on instances from Minigrid-PARL.

Our interest is not only to see how well the current method performs, but also to see how much is contributed by the action-level generalization provided by lifted representations. Our primary metrics of evaluation are going to be, (a) do the methods consistently reach the goals, (b) the number of samples collected from the environment as part of reaching the goal, and (c) the time taken by the method to reach the goal. This third aspect is an important one to consider to make sure that the RL based exploration is given a fair chance when compared against planning based methods. After all, the planning methods reason over environment model, allowing them to perform less interactions with the environment. However, this adds a computational overhead, that might not be required for other method, such as vanilla RL methods. We capture that tradeoff of one computation for another by measuring the time to reaching the goal. Additionally, we set a time limit on the exploration step, as for some of these problems the exploration might not be completed in a reasonable amount of time. For all planning based instances we set the time limit to 10 minutes, while for the minigrid instances we extended the time limit to 30 minutes. Every experiment is run five times, averaging the results to account for possible randomness in the learning process. All seed values were randomly assigned and kept constant through the all five runs.

As the underlying diverse planner, we used FI (IBM 2022), generating ten different plans at every step. Table 1 presents the comparison of our method against Q learning for the planning benchmarks. Both R-max and SMDP time out on all tested instances, so we will skip reporting their values in the table. SMDP took 188416.8 and 106821.4 samples each for the two minigrid problems. We see that apart from Blocksworld and minigrid domain, our vanilla method is able to solve more problems and our method equipped with the application of lifting rule outperforms both by a wide margin. Neither R-Max or SMDP visited any of the goal state in the given time limit.

5.2 Overall Learning Performance

With the initial results collected on how well our method is able reach goal. The next question we wanted to answer was how well an RL algorithm augmented with our new exploration strategy is able to perform. For this question we focused on tabular Q learning and the minigrid environment. Specifically, we compared an instance of Q learning algorithm where Q values were initialized using the plan generated through our exploration process and a vanilla one that started with no such information (Figure 1 provides a visualization of the maps considered for these experiments). Columns two and three of Table 2 presents the time and samples taken to get such a plan for each of the problem

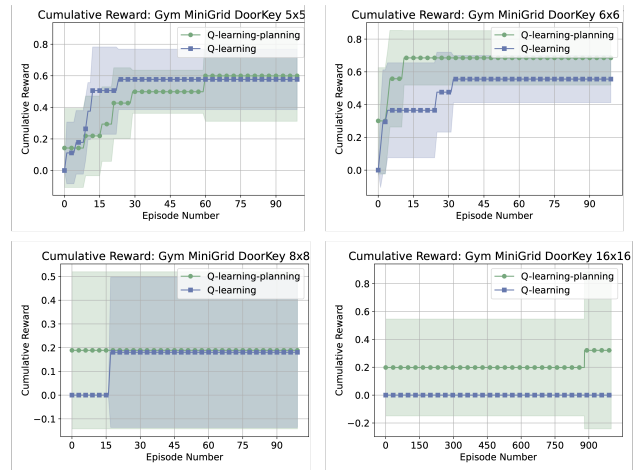


Figure 2: The above image presents the cumulative rewards obtained per episode by a vanilla RL process versus one that was initialized with the plans generated through our method. The value plotted along with their 95% confidence interval.

instance considered. Figure 2, presents how the total value per episode changes over episodes. As expected, the access to a valid plan (which isn't necessarily optimal) ensures that our method starts from a higher value. For smaller instances, we see that the vanilla method eventually catches up or at least gets closer. However for the largest problem instance, even after 1000 episodes the RL agent is still unable to get a positive reward since it never reaches a goal state.

5.3 Curriculum Learning

The final question we were interested in investigating was whether having access to a smaller problem instance had the potential of speeding up our learning process. We again went back to the minigrid problems and tested whether first learning a partial lifted model using the a problem of grid size 5 5, help speed up learning in larger problems. In particular, we stop learning in the smaller instance, as soon as we find a single solvable plan. We then use the lifted model information learned from the smaller instance to create a more informed model of the target problem. Then we follow the same procedure as before, until all the plans returned by the diverse planner is valid. One of the things we noticed was that bootstrapped models did significantly worse off under the original *PruneModel* method we used. We noticed that using a strategy based on failure count resulted in useful actions getting tested early and getting removed from consideration. So for this set of experiments we considered a different strategy that re-introduces actions more frequently. Last four columns of 2, presents the results for the minigrid problems, where the problem were bootstrapped with a lifted model learned from a 5x5 grid. The method without bootstrapping timed out for the largest problem without finding a solution after 90 mins. Except for the smallest problem we see the bootstrapping giving a definite advantage.

Problem Instance		greedy			Our method (w/o lifting)			Our method (with lifting)		
Name	Size	solved	time	no of Samples	solved	time	no of Samples	solved	time	no of Samples
Blocks	25	3/5	0.89	9164.2	3/5	26.7	19262	5/5	5.59	592
	25		11.35	115136.8		399.35	168859.6		31.96	56404.8
	25		1.99	20702		46.86	18901.8		9.28	4432.4
	36		-	2966996.4		-	96152.8		32.99	191451.2
	36		-	3135384		-	142605.4		33.93	138203.4
Elevator	20	0/5	-	2045316.4	2/5	408.79	3394856.8	5/5	36.94	88108
	20		-	2087898.2		-	3839855.6		26.73	66507
	20		-	2062441.2		401.85	3053364.2		21.25	88835
	20		-	2089477.6		-	3099277.6		36.12	83296
	20		-	2081322		-	4117586.6		36.57	87747.2
Gripper	25	1/5	5.56	53929.8	2/5	73.52	77450.2	5/5	15.9	16523
	25		-	1511944.6		328.74	252954.4		23.17	308598.6
	25		-	2663695		-	83551.8		35.93	309964.8
	36		-	2395190.2		-	55209.8		43.31	308598.6
	36		-	2112899.6		-	37136.8		61.99	624489.6
Minigrd-PARL	94	0/2	-	2743179.4	0/2	-	310032	1/2	86.41	342981.8
	593		-	334535.2		-	2458.2		-	8217319.4

Table 1: Comparison of our method with and without lifting generalization against Q learning. All times are listed in seconds and we only report the average time and number of samples (full data is provided in the supplementary package).

Problem	with original <i>PruneModel</i>		with bootstrapping		w/o bootstrapping	
	Time	Number of Samples	Time	Number of Samples	Time	Number of Samples
DoorKey-5x5	35.39	23977.2	N/A	N/A	N/A	N/A
DoorKey-6x6	34.45	24418.4	42.14	27666.0	35.91	21531.8
DoorKey-8x8	54.63	260839.8	52.1	143223.4	92.77756954	367749.2
DoorKey-16x16	1488.077	6705823.2	2353.63	11088198	-	-

Table 2: The results from our method (with lifting) applied to the second Minigrd variant. Includes the performance under our original *PruneModel* strategy and the results on how bootstrapping helps learning under the new action selection strategy.

6 Conclusions and Discussion

The effectiveness of our proposed method depends on three crucial factors; (a) the possibility of performing systematic refinements of our models while ensuring desirable properties, (b) availability of fast diverse planners, and (c) the ability to leverage human intuition about the task. The latter is of crucial importance: even if there were other model classes and planners we could exploit, the ability to tap into the human knowledge gives us a significant advantage. Importantly, the same knowledge has been used by many of the other state of the art methods. Further, it only represents a small subset of the information usually provided as part of a complete symbolic planning model. One of the aspects not discussed in the paper was the fact that instead of starting with an empty model, we could have started with a partially complete model. In such cases, the human could just provide whatever they know about the task and the RL agent can fill in the rest. We expect such settings to provide even more advantage to our method. For future work, a promising direction is to support stochastic transitions. We can directly use the current method in the stochastic setting by considering a different copy of an action for each possible transition (this is similar to the methods used by many probabilistic

planners (Yoon, Fern, and Givan 2007)). A more interesting extension would be to consider how these methods could be combined with RL methods that use function approximation. We also plan to investigate, how one could restrict the planning to a high level abstraction of the true task and look at how we can combine our current method with other exploration mechanisms like the one based on planning width (Lipovetzky and Geffner 2012).

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7 Appendix

7.1 Overview

In this appendix, Section 7.2 will cover the formal statements and proof sketches for various theoretical results, Section 7.3 will cover the implementation details including hyperparameters, and Section ?? will provide a list of files included in the supplementary package.

7.2 Theoretical Results

First result we are interested in establishing is the fact that for any given MDP of the form described in Section 3.1, there exists a corresponding symbolic model that meets the criteria discussed in Section 3.3.

Proposition 2 *For an MDP of the form $M = \langle S; T; A; I; R \rangle$, there exists a grounded symbolic model $M_{\#} = \langle hF_{\#}; A_{\#}; I; G \rangle$, such that there exists*

1. a mapping C from the state S of M to the states of $M_{\#}$,
2. a mapping C^{-1} from the actions $A_{\#}$ of $M_{\#}$ to the actions A of M , and
3. a mapping from valid traces in M to valid plans in $M_{\#}$ and vice-versa.

Proof Sketch: We can build such a model by adding one grounded predicate (each corresponding to a unique lifted predicate of arity 0) for each state other than $?$ into $F_{\#}$. Now a state in S maps (defined by C) exactly to the symbolic state where the corresponding predicate is true and none of the other fluents are true. Now for the action, we will start with an action definition that includes conditional effect, and then convert it to a form assumed by the work. A conditional effect captures cases where an effect of an action only fires if the state meets certain criteria. Now we create one symbolic action for each MDP action. For each possible transition between states (other than $?$), we will add a corresponding conditional effect that takes the predicate corresponding to source state as condition and as effect the predicate corresponding to the target state. We will keep as precondition of the actions a disjunctive list of all possible states where it will not fail. For the goal action, we will have corresponding symbolic goal action whose precondition corresponds to potential states in S_G and the effect is a goal predicate. The initial state consists of only the predicate corresponding to the state I and the goal corresponds to the goal predicate. Now we can convert the actions with conditional effects to ones with no conditional effect (cf. (Nebel 2000)). Now the action mapping C^{-1} , will map each of these new actions to the original MDP action from which it was defined. Now a plan is only valid in this model, if there exists a sequence of transitions from initial state to goal with non-zero probability. Similarly, for every valid trace there must exist a valid plan where each MDP action could be replaced by one of the potential symbolic actions that maps to it.

Next we will talk about the optimism of the initial model estimate

Proposition 3 $M_0^C = \langle hF_{\#}^C; A_{\#}^C; I^C; G^C \rangle$. More formally, every action $a \in A_{\#}^C$ will be defined as follows: $a = \langle hpre_0^a; add_0^a; del_0^a \rangle$, where $pre_0^a = del_0^a = ;$ and $add_0^a =$

$F_{\#}^C$ is optimistic for any MDP model such that there exists a mapping C from MDP state to symbolic states and a function C^{-1} mapping symbolic actions to MDP actions.

This can be easily shown by the fact that every possible action sequence is a possible plan here.

Moving onto the update rule.

Proposition 4 *Update rule as presented in Section 4.1, will only result in an optimistic representation.*

Proof Sketch: The important point to note is that at any point, the update rule is only applied to an optimistic representation. So, in order for it to result in a non-optimistic model, it must have removed a plan corresponding to a valid trace. Given our initial construction of M_0^C , we always ensure that in M_0^C the execution of an action a at a state $C(s)$ will result in a symbolic state that is a superset of $C(s^0)$, where $T(s; a; s^0) = 1$. Note that an application of an update rule will only extend the precondition if the corresponding MDP action fails and the preconditions are extended to exclude only the current state (though the list of excluded state, action pairs grows as the number of failed samples grows). Additionally, the effect is changed only to disallow impossible transitions. Since the transitions are deterministic, only one sample is needed to determine that no other transitions are possible from that state and action. This means that the above property (the fact that the resultant symbolic state will be superset) will be preserved through updates. Which in turn means that any plan that previously corresponded to a valid trace can become invalid.

Now coming to the theorem

Theorem 1 *Algorithm 1 will (a) terminate in a finite number of steps and (b) identify a path to a goal (provided one exists); as long as the diverse planner used is complete (i.e., it will return a non-empty plan set as long as there exists a valid plan).*

Proof Sketch The validity of this theorem follows from the fact that the update rule will remove any plan that doesn't correspond to a valid trace from consideration again. If the planner is complete then it will effectively iterate over all possible plans. Eventually finding one that corresponds to a path that goes to the goal. This is guaranteed to exit in finite steps, as the set of non-redundant plans is guaranteed to be finite when the state space is finite.

Now revisiting Proposition 1

Proposition 1 *Let $A = \langle a_1; \dots; a_m \rangle$ be a set of actions marked as being instances of a single lifted action a . Then min_add must be a superset of add effects of a and max_del a superset of deletes of a , where min_add and max_del are calculated for A*

Proof Sketch The validity is trivial. The update rule makes sure that every effect estimate will be an optimistic estimate of the true ground action effects. In the case of add effects this estimate will be a superset and for delete effects it will be a subset. Thus, the lifted representation of each

