TOKENUNIFY: SCALABLE AUTOREGRESSIVE PRE TRAINING FOR LARGE SCALE EM IMAGE SEGMENTA TION

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ABSTRACT

Autoregressive next-token prediction, a standard pretraining method for largescale language models, excels in handling long sequential data. However, its application to complex visual tasks, particularly biological imaging, faces challenges due to the spatial continuity and high dimensionality of biological images. Highresolution 3D biological images, such as electron microscopy (EM) brain scans, offer ideal long-sequence data, but existing methods struggle to fully leverage this characteristic. To address these challenges, we introduce **TokenUnify**, a novel pretraining method that integrates random token prediction, next-token prediction, and next-all token prediction. We provide theoretical evidence demonstrating that TokenUnify mitigates cumulative errors in visual autoregression, particularly when dealing with complex three-dimensional anatomical structures. In conjunction with TokenUnify, we have assembled a large-scale, ultra-high-resolution EM brain image dataset comprising over 120 million finely annotated voxels. This dataset not only represents the largest neuron segmentation dataset to date but, more importantly, provides ideal long-sequence biological image data that fully exhibits spatial continuity. Leveraging the Mamba network, which is inherently suited for long-sequence modeling, TokenUnify capitalizes on the advantages of autoregressive methods in processing long-sequence data, achieving a 45% performance improvement on downstream EM neuron segmentation tasks compared to existing methods. Furthermore, TokenUnify demonstrates superior scalability over MAE and traditional autoregressive methods, effectively bridging the gap between pretraining strategies for language and vision models. Code is available at https://anonymous.4open.science/r/TokenUnify-3DBF.

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1 INTRODUCTION

Large language models (LLMs) have demonstrated impressive scaling capabilities, reaching trillions of parameters through pretraining Achiam et al. (2023); Touvron et al. (2023a;b). This success is primarily attributed to high-quality data and effective autoregressive models. These models benefit from strong scaling laws due to the structured and sequential nature of text data, which allows unification into a single next-token prediction task. However, when extending to multimodal tasks such as Unified IO Lu et al. (2023) and Qwen VL Bai et al. (2023b), these models often fail to achieve state-of-the-art performance on fine-grained image tasks, particularly in biological imaging.

Unlike language, the complexity of visual signals, especially in biological images, has led to diverse approaches in visual pretraining. Contrastive learning methods like DINO v2 Oquab et al. (2023) excel in fine-grained representation, while masked reconstruction methods such as MAE He et al. (2022); Chen et al. (2023a) offer good scalability and zero-shot classification abilities. However, these methods exhibit poor scaling laws, where increasing model size does not yield expected performance gains Singh et al. (2023) (see Section A in the appendix). To achieve scaling laws similar to language models, approaches like AIM El-Nouby et al. (2024) and LVM Bai et al. (2023c) have introduced autoregressive tasks into the visual domain, showing promising scaling properties. However, image sequence disorder and error accumulation in autoregressive tasks often degrade performance in smaller models Bachmann & Nagarajan (2024). Additionally, the computational burden of

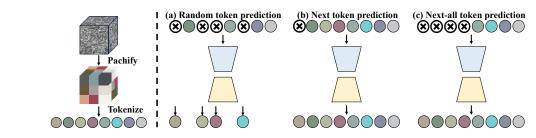


Figure 1: TokenUnify prediction paradigms divide the 3D EM image into non-overlapping patches, which are tokenized into a sequence. Three tasks are performed for rich 3D image representations: (a) random token prediction, (b) next token prediction, and (c) next-all token prediction.

long-sequence images, particularly in high-resolution 3D biological scans, makes researching image autoregressive tasks particularly challenging.

We summarize the challenges of visual autoregressive tasks in biological imaging as follows: 1) *How* to reduce error accumulation in visual autoregression to achieve stronger scaling laws, especially when dealing with complex anatomical structures? 2) *How to develop more efficient computational* methods to handle massive, high-dimensional biological image data? 3) *How to construct spatially* correlated long-sequence relationships in biological images?

This paper aims to tackle the above three critical challenges in the context of biological imaging, 075 particularly focusing on EM brain scans. 1) To address the issue of cumulative errors in visual 076 autoregression, we propose **TokenUnify**, a novel mixed token prediction paradigm. TokenUnify in-077 tegrates next-token prediction, next-all token prediction, and random token prediction (as illustrated in Fig. 1), leveraging global information to overcome the limitations of local receptive fields in 079 complex anatomical structures. We theoretically demonstrate that this mixed approach reduces cumulative errors while maintaining favorable scaling laws. 2) To alleviate computational burdens, we 081 introduce the Mamba architecture, which reduces the computational complexity of autoregressive 082 tasks from quadratic (as in Transformers) to linear. This is particularly crucial for processing high-083 resolution 3D biological images. Detailed comparisons of the scaling properties between Mamba and Transformer architectures reveal that Mamba achieves superior performance and efficiency in 084 large-scale autoregressive visual models, especially for biological imaging tasks. 3) To construct 085 spatially correlated long-sequence relationships, we have collected large-scale, ultra-high-resolution 3D electron microscopy (EM) images of mouse brain slices. The ultra-high resolution allows for 087 thousands of continuous image tokens, ensuring robust spatial continuity crucial for understanding 088 complex neuronal structures. We have fully annotated six different functional regions within the 089 mouse brain, totaling 120 million pixels, resulting in the largest manually annotated neuron dataset 090 to date. This comprehensive dataset also serves as a unified benchmark for evaluating experimental 091 performance in biological image analysis¹. 092

Pretraining with TokenUnify led to a 45% improvement in performance on subsequent EM neuron segmentation tasks. The mixed training paradigm of TokenUnify outperformed MAE He et al. (2022) by 21% in pretraining performance, even with fewer parameters. Furthermore, TokenUnify demonstrated superior scaling properties of autoregressive models, offering a promising approach for pretraining large-scale visual models in biological imaging.

- 098 Our contributions can be summarized as follows:
 - 1. We introduce a novel pre-training paradigm, **TokenUnify**, which models visual pre-training tasks from multiple perspectives at the token level, specifically designed for biological imaging. This ensures sublinear growth of the scaling law while demonstrating superior fine-grained feature extraction capabilities compared to MAE in smaller model pre-training, crucial for detecting subtle anatomical features. We also provide a theoretical explanation for this phenomenon in the context of biological image analysis.
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¹We commit to open-sourcing the dataset and code of the paper to facilitate future research in biological imaging.

- 2. We achieve a **45%** performance improvement on the neuron segmentation task and, for the first time, validate the Mamba model with billion-level parameters on visual tasks, demonstrating the effectiveness and efficiency of TokenUnify in long-sequence visual autoregression for complex 3D biological images.
 - 3. We provide a large-scale biological image dataset with 120 million annotated pixels, offering a long-sequence image dataset to validate the potential of autoregressive methods in biological imaging.
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2 RELATED WORK

119 Recent advances in large language models (LLMs) have unified various NLP tasks under a single 120 architecture, formulating them as generation tasks. This architecture can be categorized into BERT-121 like Devlin et al. (2018); Xia et al. (2020); Lee et al. (2020); Song et al. (2023); Mo et al. (2024) and 122 GPT-like Brown et al. (2020); Radford et al. (2019); Li et al. (2024a); Zhou et al. (2023) models. 123 The latter, decoder-only autoregressive structure, has been shown to be more effective, as validated 124 by products like ChatGPT. Subsequent works have built upon GPT, introducing techniques like RM-125 SNorm Zhang & Sennrich (2019), SwiGLU, and RoPE to ensure efficient and stable training. The LLaMA series Touvron et al. (2023a;b) has improved training efficiency, while the Qwen series Bai 126 et al. (2023b;a); Xiang et al. (2024) has focused on data cleaning and filtering for Chinese language 127 models. Currently, LLMs have surpassed human-level performance in many text processing tasks 128 Achiam et al. (2023). 129

In multi-modal tasks, the CLIP Radford et al. (2021) and BLIP series Li et al. (2022; 2023a); Dai et al. (2024) have pioneered contrastive learning on image-text pairs, achieving remarkable zero-shot classification and generalization capabilities. Further works Zhang et al. (2023); Chen et al. (2023b);
Wang et al. (2022); Zhou et al. (2024) have applied multi-modal models to specific domains. By processing arbitrary modalities into a unified token format Lu et al. (2023); Wang et al. (2023); Chen et al. (2023); Chen et al. (2023); Chen et al. (2024), these models can generate outputs in any modality. However, there is still room for improvement in fine-grained visual tasks, and training large vision models remains an open problem.

137 Self-supervised pre-training has been a cornerstone for enhancing model representation capabilities. Approaches based on contrastive learning for representation extraction Chen et al. (2024b); Zbontar 138 et al. (2021); He et al. (2020); Li et al. (2021) and masked reconstruction methods He et al. (2022); 139 Chen et al. (2023a); Li et al. (2023c); Ding et al. (2022); Chen et al. (2024d; 2023d) have shown 140 promise. However, current vision models have not exhibited the same sublinear scaling laws as lan-141 guage models. To address this issue, some tasks have adopted autoregressive pre-training paradigms 142 similar to those used in language models Chen et al. (2020); Bai et al. (2023c); El-Nouby et al. 143 (2024), though the training costs remain a significant concern. In this paper, we explore the poten-144 tial of long visual token autoregressive pre-training and introduce a collaborative training scheme 145 for long token prediction. Our approach aims to balance the scaling laws and training costs, demon-146 strating improvements in fine-grained visual tasks.

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3 Method

- 149 150
- 151 3.1 OVERVIEW

Our theoretical contributions include proving the parameter independence of MAE performance
 (see Section A), establishing the strong correlation between autoregressive model performance and
 parameter count (see Section B), and demonstrating the advantages of next-all token prediction from
 both intuitive (see Section C.1) and theoretical perspectives (see Section C.2).

Our experimental framework comprises two stages: pre-training and fine-tuning. During the pretraining stage, we leverage only the unlabeled raw data \mathcal{X} to learn a generic visual representation $f_{\theta_1}(\cdot)$, parameterized by θ_1 . We employ a mixed token prediction strategy to capture both local and global dependencies in the data (see Section 3.2). Additionally, we utilize Mamba for efficient modeling of long sequences in autoregressive tasks, enhancing computational efficiency (see Section 3.3).

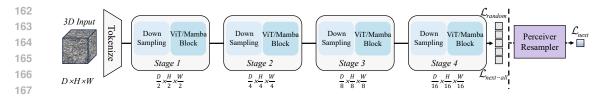


Figure 2: illustrates the main pretraining workflow of TokenUnify. The image X is fed into the Tokenizer, transforming it into a long sequence of tokens $\mathbf{x}_i|_{i=1}^K$. The predictions for the random token, next token, and next-all token are performed sequentially. The Perceiver Resampler is employed to convert varying-size large feature maps into a few visual tokens (see Section 3.2).

In the fine-tuning stage, we use both the raw data \mathcal{X} and the corresponding labels \mathcal{Y} to adapt the pre-trained representation to specific downstream tasks. Let $g_{\theta_2}(\cdot)$ be the task-specific model, parameterized by θ_2 . We initialize θ_2 with the pre-trained weights θ_1 and optimize the task-specific objective. Further details are provided in Sections 3.4 and F.3.

To illustrate the application of our framework, consider the modeling of EM images. Given a total of T EM images $\mathcal{X} = \{\mathbf{X}^{(1)}, \dots, \mathbf{X}^{(T)}\}$, where each $\mathbf{X}^{(t)} \in \mathbb{R}^{D \times H \times W}$ represents a 3D image with depth D, height H, and width W, we aim to learn a meaningful representation of this highdimensional, long-sequence visual data by leveraging its inherent spatial structure and continuity. To achieve this, we partition each large 3D image **X** into smaller patches $\mathbf{x} \in \mathbb{R}^{D' \times H' \times W'}$.

182 3.2 MIXED-MODE AUTOREGRESSIVE MODELING183

We theoretically demonstrate the effectiveness of MAE He et al. (2022) on smaller models, the scaling advantages of next token prediction, and the ability of next-all token prediction to mitigate cumulative errors in autoregressive models. Based on these insights, we propose a hybrid training paradigm that aims to combine the strengths of these three methods, as shown in Fig. 2.

Given an image X, we first divide it into a sequence of K non-overlapping patches $\{x_1, \dots, x_K\}$. Standard autoregressive modeling typically adopts a fixed left-to-right factorization:

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$$p(\mathbf{x}) = \prod_{i=1}^{K} p(\mathbf{x}_i \mid \mathbf{x}_{< i}), \tag{1}$$

where $\mathbf{x}_{<i}$ denotes all patches preceding \mathbf{x}_i .

We introduce TokenUnify, a mixed-mode autoregressive modeling approach designed to enhance existing autoregressive image modeling techniques Chen et al. (2020); Bai et al. (2023c). TokenUnify combines three distinct prediction tasks: random token prediction, next token prediction, and next-all token prediction, instead of using the fixed factorization in Eq. 1.

Random Token Prediction. Given the full patch sequence $\mathbf{x}_{1:K}$, we randomly mask out a subset of patches $\mathcal{M} \subset \{1, \ldots, K\}$ and train the model to predict the masked patches conditioned on the remaining context:

$$\mathcal{L}_{\text{random}} = -\sum_{i \in \mathcal{M}} \log p(\mathbf{x}_i | \mathbf{x}_{\bar{\mathcal{M}}}), \tag{2}$$

203 where $\mathbf{x}_{\bar{\mathcal{M}}} = {\mathbf{x}_i \mid i \notin \mathcal{M}}$ denotes the unmasked patches.

Next Token Prediction. We integrate the standard next token prediction loss into our task. In this setup, we use the Perceiver Resampler Alayrac et al. (2022) (see Section F.2) to convert the variablesized feature maps generated by the Vision Encoder into a fixed number of visual tokens. This loss trains the model to predict the next patch x_i given the preceding context $x_{<i}$:

$$\mathcal{L}_{\text{next}} = -\sum_{i=1}^{K} \log p(\mathbf{x}_i | \mathbf{x}_{< i}).$$
(3)

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212 Next-All Token Prediction. To encourage the model to capture longer-range dependencies, we 213 extend the next token prediction to a next-all token prediction task. For each patch x_i , the model is 214 trained to predict not only x_i but also all the subsequent patches $x_{i:K}$ in the sequence:

$$\mathcal{L}_{\text{next-all}} = -\sum_{i=1}^{K} \sum_{j=i}^{K} \log p(\mathbf{x}_j | \mathbf{x}_{< i}).$$
(4)

216 Our pre-training algorithms are summarized in Algorithm 1. By alternating between these token 217 prediction tasks every 100 epochs, we prevent the model from converging to a trivial solution and 218 encourage it to learn meaningful representations from the input data. This alternating training strat-219 egy enables the model to capture both local and global dependencies within the patch sequence, 220 thereby enhancing performance on downstream tasks. The workflow of TokenUnify is illustrated in Fig. 2. 221

222 Algorithm 1: TokenUnify Pre-training 223

Input : Unlabeled image data $X = \{X^{(1)}, \dots, X^{(T)}\}$ 224 225 **Input** : Model parameters θ_1 **Output:** Pre-trained model $f_{\theta_1}(\cdot)$ 226 227 1 for $t \leftarrow 1$ to T do 228 Partition $X^{(t)}$ into patches $\{x_1, \ldots, x_K\}$ 2 229 Tokenize patches: $\{x_1, \ldots, x_K\} \rightarrow$ tokens 3 230 **Compute loss functions:** 4 231 Random token prediction: $\mathcal{L}_{random} = -\sum_{i \in M} \log p(x_i \mid x_{\bar{M}})$ 5 Next token prediction: $\mathcal{L}_{random} = -\sum_{i \in M}^{K} \log p(x_i \mid x_M)$ Next token prediction: $\mathcal{L}_{next} = -\sum_{i=1}^{K} \log p(x_i \mid x_{< i})$ Next-all token prediction: $\mathcal{L}_{next-all} = -\sum_{i=1}^{K} \sum_{j=i}^{K} \log p(x_j \mid x_{< i})$ 232 6 233 7 234 235 Update θ_1 to minimize \mathcal{L}_{random} , \mathcal{L}_{next} , $\mathcal{L}_{next-all}$ 8 236 9 return $f_{\theta_1}(\cdot)$

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239 3.3 MAMBA ORDERING

While the aforementioned mix token prediction task improves sequence autoregressive modeling 241 capabilities, it also introduces additional computational complexity for long sequences. Inspired 242 by the *Mamba* strategy proposed by Gu & Dao (2023), we introduce an enhanced approach to 243 effectively model long sequences in volumetric EM images. Traditional sequence modeling methods 244 often struggle with capturing long-range dependencies due to their rigid sequential nature. Our 245 enhanced Mamba ordering strategy addresses this by incorporating a more sophisticated and flexible 246 sequence modeling approach. 247

The fundamental idea behind Mamba ordering is to dynamically prioritize regions of the sequence 248 based on contextual significance rather than adhering to a fixed order. This is achieved through 249 an adaptive mechanism that evaluates the importance of different patches within the sequence and 250 adjusts the processing order accordingly. By doing so, Mamba ordering enhances the model's ability 251 to capture intricate patterns and long-range dependencies more effectively. 252

Mathematically, let $\mathbf{x} = {\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_K}$ represent the sequence of patches. Instead of process-253 ing these patches in a fixed order, we define a dynamic ordering function $\sigma : \{1, 2, \dots, K\} \rightarrow \{1, 2, \dots, K\}$ 254 $\{1, 2, \dots, K\}$ that determines the sequence in which patches are processed. The Mamba ordering 255 objective can be formulated as: 256

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$$\mathcal{L}_{\text{mamba}} = -\mathbb{E}_{\mathbf{x} \sim p_{\text{data}}} \left[\sum_{i=1}^{K} \log p(\mathbf{x}_{\sigma(i)} | \mathbf{x}_{\sigma((5)$$

where $\mathbf{x}_{\sigma(<i)}$ represents the context preceding the *i*-th patch in the dynamically determined order. 260

261 To optimize this objective, we introduce a context-aware attention mechanism that assesses the 262 relevance of each patch with respect to the overall sequence. This mechanism outputs a relevance 263 score for each patch, guiding the dynamic ordering function σ to prioritize patches that are most 264 informative for subsequent predictions. By iteratively updating the relevance scores and reordering the patches, Mamba ordering ensures that the model focuses on the most crucial aspects of the 265 sequence at each step. 266

267 Consider the state-space model representation where the hidden state h(t) evolves dynamically 268 based on the input $\mathbf{x}(t)$. The state-space equations are given by: 269

$$\mathbf{h}'(t) = \mathbf{A}(t)\mathbf{h}(t) + \mathbf{B}(t)\mathbf{x}(t), \quad \mathbf{y}(t) = \mathbf{C}(t)\mathbf{h}(t), \tag{6}$$

270 where $\mathbf{A}(t)$, $\mathbf{B}(t)$, and $\mathbf{C}(t)$ are time-dependent matrices. Specifically, $\mathbf{B}(t)$ and $\mathbf{C}(t)$ are parame-271 terized by the input $\mathbf{x}(t)$ as follows: 272

$$\mathbf{B}(t) = s_B(\mathbf{x}(t)), \quad \mathbf{C}(t) = s_C(\mathbf{x}(t)), \quad \mathbf{\Delta}(t) = \tau_{\Delta}(\text{Linear}(\mathbf{x}(t))), \tag{7}$$

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where s_B , s_C , and τ_{Δ} are functions that map the input to the respective parameters. 275

276 The benefits of our enhanced Mamba ordering are twofold. First, it mitigates error accumulation 277 by allowing the model to refine its predictions based on a globally coherent understanding of the 278 sequence. Second, it enhances the model's capacity to capture long-range dependencies, as the dynamic ordering can adapt to the inherent structure and complexity of the data. 279

280 Empirical results demonstrate that our enhanced Mamba ordering significantly improves the per-281 formance of sequence modeling tasks in volumetric EM images, particularly for long sequences. 282 By enabling a more adaptive and context-aware approach to sequence processing, our enhanced 283 Mamba ordering represents a substantial advancement over traditional methods, offering a robust and scalable solution for high-dimensional visual data. 284

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289 290 3.4 FINETUNING AND SEGMENTATION

The segmentation network, denoted as $g_{seq}(\cdot)$, consists of an encoder $g_e(\cdot)$ and a decoder $g_d(\cdot)$:

$$g_{seq}(x;\theta_s) = g_d(g_e(x)), \quad \theta_s = \{\theta_e, \theta_d\},\tag{8}$$

291 where θ_s represents the parameters of the entire segmentation network, and θ_e and θ_d are the param-292 eters of the encoder and decoder, respectively. 293

The encoder $g_e(\cdot)$ gradually downsamples the input volume and extracts high-level semantic fea-294 tures, while the decoder $g_d(\cdot)$ upsamples the encoded features back to the original resolution. Mean-295 while, the output of each downsampling layer in the encoder is connected to the corresponding layer 296 in the decoder via skip connections to fuse local and global multi-scale information. We adopt 3D 297 ResUNet/ViT/Mamba as the backbone network, respectively. 298

The output of the segmentation network $\hat{y} = g_{seg}(x) \in \mathbb{R}^{C \times D \times H \times W}$ represents the predicted 299 affinity map Li et al. (2018; 2023b), corresponding to the connectivity probability of each voxel in 300 three directions. During training, the loss function for labeled samples is the mean squared error 301 between the predicted and ground-truth affinity maps: 302

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$$L_{seg} = \frac{1}{|D_l|} \sum_{i=1}^{|D_l|} |\hat{y}_i - y_i|^2 = \frac{1}{|D_l|} \sum_{i=1}^{|D_l|} \left| g_{seg}(x_i^l) - y_i \right|^2.$$
(9)

Our fine-tuning algorithm is summarized in Algorithm 2. During inference, for any new test sample 306 x_t , forward propagation through $g_{seq}(x_t)$ yields its predicted affinity map. This predicted affinity 307 map is then post-processed using a seeded watershed algorithm and a structure-aware region ag-308 glomeration algorithm Funke et al. (2018); Beier et al. (2017) to obtain the final neuron instance 309 segmentation. Detailed information on our segmentation process can be found in Section F.3, and 310 the segmentation pipeline is illustrated in Fig. 7. 311

Algorithm 2: TokenUnify Fine-tuning 312

313 **Input** : Labeled data $\mathcal{D}_l = \{(x_i^l, y_i)\}_{i=1}^{|\mathcal{D}_l|}$ 314 **Input** : Pre-trained model $f_{\theta_1}(\cdot)$ 315 **Input** : Segmentation model $g_{\theta_2}(\cdot)$ 316 **Output:** Fine-tuned segmentation model $g_{\theta_2}(\cdot)$ 317 1 Initialize θ_2 with θ_1 318 2 for $i \leftarrow 1$ to $|\mathcal{D}_i|$ do 210

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$$\hat{y}_i = g_{\theta_2}(f_{\theta_1}(x_i^l))$$

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$$\mathcal{L}_{reg} = \frac{1}{122} \sum_{i=1}^{|\mathcal{D}_i|} |\hat{y}_i - y_i|^2$$

- $\mathcal{L}_{\text{seg}} = \frac{1}{|\mathcal{D}_l|} \sum_{i=1}^{|\mathcal{D}_l|} |\hat{y}_i y_i|^2$ 322
- 5 Update θ_2 to minimize \mathcal{L}_{seg} 323

6 return $g_{\theta_2}(\cdot)$

³²⁴ 4 DATASET AND METRICS

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Dataset. For the pretraining phase of TokenUnify, we collect a vast amount of publicly available unlabeled EM imaging data, from four large-scale electron microscopy (EM) datasets: Full Adult Fly Brain (FAFB) Schlegel et al. (2021), MitoEM Wei et al. (2020), FIB-25 Takemura et al. (2017), and Kasthuri15 Kasthuri et al. (2015). These datasets cover a diverse range of organisms, including Drosophila, mouse, rat, and human samples, totaling over 1 TB. The details of the pretraining datasets can be found in Table 3. We sample from the datasets with equal probability and ensure that each brain region has an equal chance of being sampled, guaranteeing the diversity of the pretraining data.

334 For downstream fine-tuning and segmentation, we employ two datasets: a smaller dataset, AC3/4, 335 and a larger dataset, MEC, for algorithm validation. The AC3/4 dataset Kasthuri et al. (2015) con-336 sists of mouse somatosensory cortex datasets with 256 and 100 successive EM images (1024×1024). We use the first 80 images of AC4 for fine-tuning, the last 20 images of AC4 for testing, and the first 337 100 images of AC3 for testing. Additionally, we have collected a large-scale electron microscopy 338 dataset, MEC, by imaging the mouse somatosensory cortex, mouse medial entorhinal cortex, and 339 mouse cerebral cortex, achieving a physical resolution of $8nm \times 8nm \times 40nm$. We select 6 represen-340 tative volumes from different neural regions, named wafer4/25/26/26-2/36/36-2, with each volume 341 size reaching $125 \times 1250 \times 1250$ voxels. We perform dense annotation on these selected wafer re-342 gions, with a total of over 1.2 billion annotated voxels. To validate the algorithm's performance on 343 a large-scale dataset, we use wafer25/26/26-2/36 for training, wafer4 for validation, and wafer36-2 344 for testing on the MEC dataset. 345

Metrics. To evaluate the performance of neuron segmentation Zhang et al. (2024); Dang et al. (2024), we employ two widely-used metrics: Variation of Information (VOI) Nunez-Iglesias et al. (2013) and Adjusted Rand Index (ARAND) Arganda-Carreras et al. (2015). These metrics quantify the agreement between the predicted segmentation and the ground truth from different perspectives. Detailed descriptions of these metrics can be found in Section D.2.

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5 EXPERIMENT

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Implementation Details. In this work, we employ consistent training settings for both pretraining and fine-tuning tasks. The network architecture remains the same throughout the training and finetuning phases. During fine-tuning, we optimize the network using the AdamW optimizer Loshchilov & Hutter (2018) with $\beta_1 = 0.9$, $\beta_2 = 0.999$, a learning rate of 1e-6, and a batch size of 20 on an NVIDIA GTX 3090 (24GB) GPU. For pretraining, we use a batch size of 8 on an NVIDIA Tesla A40 (48G) GPU due to memory constraints.

We perform distributed training using 8 NVIDIA GTX 3090 GPUs for each segmentation task, running for a total of 1200 epochs. Similarly, we utilize 32 NVIDIA Tesla A40 GPUs for each pretraining task, running for 400 epochs. During the pretraining phase, the input consists solely of unlabeled data, whereas in the segmentation phase, both labeled and unlabeled data are used as input. The input block resolution for the network is set to $16 \times 160 \times 160$. To initialize the network for fine-tuning, we load the weights obtained from the pretraining phase, following the settings of previous works He et al. (2022).

To generate final segmentation results from the predicted affinities, we employ two representative post-processing methods: Waterz Funke et al. (2018) and LMC Beier et al. (2017). Waterz iteratively merges fragments based on edge scores until a threshold is reached. We set the quantile to 50% and the threshold to 0.5 based on testing on MEC, and discretize scores into 256 bins. LMC formulates agglomeration as a minimum-cost multi-cut problem, extracting edge features as costs and solving with the Kernighan-Lin solver Kernighan & Lin (1970). We maintain consistent post-processing settings across all experiments to ensure fair comparisons and conclusions about our method.

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Experimental Results on MEC Dataset. As detailed in Section 4, we leverage a substantial dataset called MEC to assess the performance of our algorithm. For neuron segmentation tasks, we have implemented several representative methods, including Superhuman Lee et al. (2017), MALA Funke et al. (2018), PEA Huang et al. (2022), and UNETR Hatamizadeh et al. (2022). Our EM-

378	Table 1: Quantitative comparison of segmentation results on Wafer4 and Wafer36-2 datasets. 'Post.'
379	represents the post-processing algorithms. * denotes the MAE pretraining strategy He et al. (2022).
380	† indicates our TokenUnify pretraining strategy. The best results are in bold and the second best
381	results are in <u>underlined</u> .

Post.	Method		Wa	afer4			Waf	er36-2		Param.
		$VOI_M\downarrow$	$VOI_S\downarrow$	$VOI\downarrow$	$ARAND\downarrow$	$VOI_M\downarrow$	$VOI_S \downarrow$	$VOI\downarrow$	$ARAND\downarrow$	
	Superhuman [40]	0.3328	1.1258	1.4587	0.1736	0.1506	0.4588	0.6094	0.0836	1.47
(8)	MALA [29]	0.5438	1.5027	2.0375	0.1115	0.3179	1.0664	1.3843	0.1570	84.0
(201	PEA [35]	0.3381	0.9276	1.2658	0.0677	0.2787	0.4279	0.7066	0.1169	1.48
al.	UNETR [31]	0.4504	1.6581	2.1085	0.2658	0.4478	0.5217	0.9696	0.2913	129
Funke et	EMmamba	0.4915	1.2924	1.7839	0.2052	0.2406	0.4189	0.6595	0.1231	28.3
Įun	Superhuman*	0.2971	0.8965	1.1936	0.1108	0.1922	0.3819	0.5742	0.1025	1.47
	MALA*	0.7300	1.1694	1.8994	0.2295	0.5088	0.3945	0.9034	0.2574	84.0
Waterz	PEA*	0.2677	0.7866	<u>1.0543</u>	0.0454	0.2184	0.2971	<u>0.5156</u>	0.0906	1.48
-	UNETR*	0.3127	0.8348	1.1475	0.0940	0.3982	0.3844	0.7825	0.1768	129
	EMmamba*	0.2120	1.0560	1.2680	0.0862	0.1449	0.4201	0.5650	0.0702	28.3
	$\mathbf{E}\mathbf{M}$ mamba [†]	0.1953	0.7998	0.9951	0.0509	0.1262	0.3585	0.4848	0.0650	28.3
	Superhuman [40]	0.1948	1.9697	2.1644	0.2453	0.0792	1.1618	1.2410	0.1319	1.47
-	MALA [29]	0.3416	2.4129	2.7545	0.2567	0.1448	1.9603	2.1052	0.1977	84.0
(2017)	PEA [35]	0.1705	1.5993	<u>1.7698</u>	0.1527	0.4719	1.1226	1.5945	0.1588	1.48
	UNETR [31]	0.1791	3.1715	3.3506	0.6330	0.0949	1.3858	1.4807	0.1578	129
et al	EMmamba	0.1596	2.0580	2.2177	0.1973	0.0847	1.0351	<u>1.1198</u>	0.1253	28.3
Beier (Superhuman*	0.2564	1.7823	2.0387	0.1812	0.0844	1.1317	1.2161	0.1289	1.4
GBe	MALA*	0.2001	2.5742	2.7747	0.5622	0.3946	1.1652	1.5598	0.1543	84.0
LMC	PEA*	0.4584	1.4873	1.9458	0.1254	0.4694	1.0217	1.4910	0.1413	1.48
П	UNETR*	0.2389	1.8072	2.0461	0.1704	0.0985	1.1860	1.2845	0.1380	129
	EMmamba*	0.1319	1.8734	2.0054	0.1405	0.0726	1.0731	1.1457	0.1183	28.3
	EMmamba [†]	0.1418	1.5103	1.6521	0.0591	0.0827	1.0276	1.1103	0.1074	28.3

Table 2: Quantitative comparison of segmentation results on AC3/4 datasets. 'w/o Pre.' indicates models without pretraining, whereas 'w Pre.' denotes models that utilize corresponding pretraining strategy. * denotes the MAE pretraining strategy He et al. (2022). † indicates our TokenUnify pretraining strategy. The best results are in **bold** and the second best results are in <u>underlined</u>.

Method	VOI	$M \downarrow VOL$	$I_S \downarrow$	VO	$I\downarrow$	ARA	ND	Param.
	w/o Pre.	w Pre. w/o Pre.	w Pre.	w/o Pre.	w Pre.	w/o Pre.	w Pre.	(M)
Superhuman [40]	0.4882	0.6162 0.6563	0.6308	1.1445	1.2470	0.1748	0.2505	1.478
MALA [29]	0.4571	0.3345 0.6767	0.7479	1.1338	1.0824	0.1664	0.1020	84.02
PEA [35]	0.5522	0.3832 0.4980	0.6153	1.0502	0.9985	0.2093	0.1127	1.480
UNETR [31]	0.7799	0.5339 0.7399	0.5573	1.5198	1.0912	0.2411	0.1796	129.1
EMmamba*	0.9378	0.3167 0.8629	0.7963	1.8007	1.1131	0.2840	0.1050	28.30
EMmamba†	0.9378	0.4479 0.8629	0.5439	1.8007	0.9918	0.2840	0.1366	28.30

mamba model (see Section F.3) builds upon Segmamba Xing et al. (2024) and incorporates enhancements to various anisotropic structures to better accommodate the resolution of electron microscopy. All networks are trained using default open-source parameters. Additionally, we calculated the parameter count for all architectures.

431 Our experimental results are presented in Table 1. The upper part of the table shows the performance of these methods when directly applied to segmentation tasks. In contrast, the lower part of the table

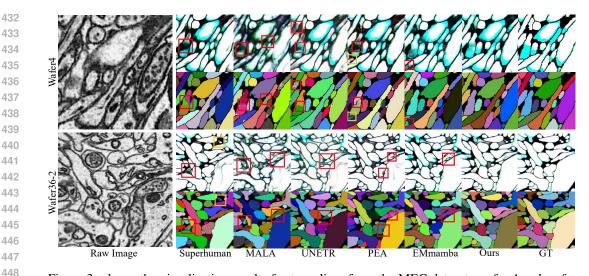


Figure 3: shows the visualization results for two slices from the MEC dataset: wafer 4 and wafer 36-2. The left side displays the EM raw images, while the right side presents the affinity and segmentation results. Red boxes indicate over-split regions, and orange boxes highlight over-merge regions.

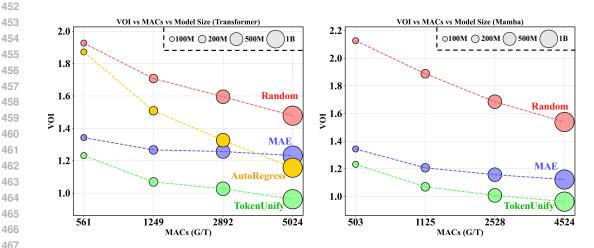


Figure 4: We evaluate the performance of models with 100M, 200M, 500M, and 1B parameters. Each model was trained for 100 epochs on the MEC and CREMI datasets.

illustrates the performance of networks employing self-supervised pretraining. When comparing models with a similar number of parameters, our pretraining approach achieves approximately a 21% performance improvement over MAE and over a 45% improvement compared to direct training. Visualization results, as shown in Fig. 3, demonstrate that our approach significantly outperforms others in both neuron splitting and merging tasks.

Experimental Results on AC3/4 Datasets. As noted in Section 4, we also evaluate the performance of all baseline methods on a smaller dataset. Compared to the MEC dataset, the total training scale (number of labeled pixels) of the AC3/4 dataset is only about 1/10 of that of MEC. In this low-data scenario, the Mamba architecture combined with TokenUnify pretraining achieves performance comparable to the latest SOTA PEA pertaining (as shown in Table 2). Additionally, it demonstrates approximately a 10% performance improvement over the MAE pretraining approach. This highlights the robustness of TokenUnify even with a limited number of fine-tuning samples.

Experimental Results on the Scaling Law. We conducted a comprehensive evaluation of scaling laws for various initialization and training strategies: Random Initialization, MAE (Masked Autoen-

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In our experiments, we tested input sizes of $16 \times 160 \times 160$. The Mamba architecture was trained on the MEC dataset, while the Transformer architecture was trained on the CREMI dataset Funke et al. (2016). Our experimental results are shown in Fig. 4.

Our findings indicate that, following pretraining on the same data, all methods except for the purely vision-based Autoregressive model with small parameter counts demonstrate performance gains. However, MAE quickly encounters scaling law limitations, hitting a performance bottleneck. In contrast, TokenUnify exhibits robust scaling properties, outperforming other pretraining methods at both small and large parameter scales. From a model architecture perspective, Mamba maintains segmentation performance while exhibiting a lower parameter count compared to the Transformer architecture. This validates the suitability of Mamba for long-sequence and autoregressive modeling tasks.

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Abalation Study. We conducted ablation studies on several components within our experimental setup. The experiments were divided into two main parts. First, we explored the mixed mechanisms of TokenUnify. We experimented with combinations of three different mixing mechanisms, ensuring that the total number of training epochs remained consistent. Table 6 presents the results of these experiments on the wafer4 neuron segmentation task (using a 28M EMmamba segmentation network). The results demonstrate that mixed training provides the most significant benefit for downstream tasks, with the combination of Random token and Next token being the next most effective.

Second, we performed ablation studies on the fine-tuning schemes. Under the default settings, we
fine-tuned all parameters of the network. However, due to computational resource constraints, only a
subset of parameters (or additional adapter parameters such as LoRA Hu et al. (2022)) is often finetuned for larger models. Based on our network architecture (see Fig. 7), we divided the network into
the Mamba part (for token sequence information extraction), the encoder part (for downsampling),
and the decoder part (for convolutional upsampling). We fine-tuned only the corresponding subset
of weights for each part. Our experimental results are shown in Table 7.

515 We found that in the TokenUnify modeling, using the sequence information priors extracted by 516 Mamba significantly benefits downstream segmentation tasks. Combining the Mamba module with 517 the encoder part yields even greater performance improvements, while fine-tuning only the encoder 518 or decoder weights provides minimal gains. This suggests that in resource-constrained scenarios, 519 fine-tuning only the sequence modeling parameters can be sufficient.

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6 SOCIAL IMPACT AND FUTURE WORK

523 The favorable scaling laws of TokenUnify present the opportunity to train a unified and generic vi-524 sual feature extractor, which holds significant importance for visual tasks. A unified visual feature 525 extractor can substantially reduce the cost of fine-tuning models for different visual tasks, thereby 526 facilitating the application of visual technologies across various domains. We have currently validated the effectiveness of TokenUnify on long-sequence 3D biological images. Moving forward, 527 we plan to further explore the performance of TokenUnify on natural images and other downstream 528 tasks, thereby expanding its scope of application. Moreover, TokenUnify can be extended to mul-529 timodal domains such as image-text tasks Chen et al. (2024c); Liu et al. (2023a), demonstrating its 530 utility in multimodal applications. We will also continue to investigate model lightweighting Chen 531 et al. (2023c); Chen & Jing (2021) and efficient fine-tuning strategies Liu et al. (2023d); Li et al. 532 (2024b). We believe that TokenUnify offers a promising approach for building large-scale, efficient 533 visual pre-training models, contributing to advancements in the visual domain. 534

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7 CONCLUSION

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538 We propose TokenUnify, a novel autoregressive visual pre-training method that integrates random 539 token prediction, next-token prediction, and next-all token prediction to effectively capture local and global dependencies in image data. We provide theoretical evidence demonstrating that Toke-

nUnify mitigates cumulative errors in visual autoregression while maintaining favourable scaling laws. Furthermore, we collect a large-scale, ultra-high-resolution 3D electron microscopy dataset of mouse brain slices to serve as a unified benchmark for validating our approach. Pretraining with TokenUnify leads to a 45% improvement in performance on downstream neuron segmentation tasks compared to the baseline, showcasing the potential of our method in fine-grained visual tasks.

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Appendix

WHY DOES MAE FACE SCALING LAW LIMITATIONS? А

To thoroughly understand the theoretical limitations of the Mean Absolute Error (MAE) estimator in high-dimensional sparse linear regression, we revisit the assumptions and provide a detailed and rigorous proof of its estimation error bound, including all necessary steps and conditions.

Assumption 1. Suppose the observations $y \in \mathbb{R}^n$ are generated by the linear model:

$$y = X\beta^* + \varepsilon, \tag{10}$$

where $X \in \mathbb{R}^{n \times p}$ is a known design matrix, $\beta^* \in \mathbb{R}^p$ is the unknown sparse signal, and $\varepsilon \in \mathbb{R}^n$ is the noise vector. Furthermore, assume:

- (a) The true signal β^* is s-sparse, i.e., $\|\beta^*\|_0 \leq s$.
- (b) The noise vector ε has independent sub-Gaussian entries with zero mean and variance proxy σ^2 , i.e.,

$$\mathbb{E}[\varepsilon_i] = 0, \quad \mathbb{E}[\varepsilon_i^2] \le \sigma^2, \quad and \quad \mathbb{P}(|\varepsilon_i| \ge t\sigma) \le 2\exp\left(-\frac{t^2}{2}\right), \quad \forall t > 0.$$
 (11)

(c) The design matrix X satisfies the Restricted Isometry Property (RIP) of order 2s with constant $\delta_{2s} \in (0, \delta^*)$, where δ^* is a numerical constant less than 1, i.e., for all vectors $v \in \mathbb{R}^p$ with $||v||_0 \leq 2s$,

$$(1 - \delta_{2s}) \|v\|_2^2 \le \frac{1}{n} \|Xv\|_2^2 \le (1 + \delta_{2s}) \|v\|_2^2.$$
(12)

Theorem 1. Under Assumption 1, let $\hat{\beta}$ be the solution to the ℓ_1 -regularized MAE problem (also known as the LAD-Lasso):

$$\hat{\beta} = \operatorname*{arg\,min}_{\beta \in \mathbb{R}^p} \left\{ \frac{1}{n} \|y - X\beta\|_1 + \lambda \|\beta\|_1 \right\},\tag{13}$$

where $\lambda > 0$ is the regularization parameter defined as $\lambda = C_0 \sigma \sqrt{\frac{\log p}{n}}$, with $C_0 > 0$ being a sufficiently large constant. Then, provided that n is sufficiently large and $\delta_{2s} < \delta^*$ (i.e., the RIP condition is satisfied with a sufficiently small constant), there exists a constant C > 0 such that with probability at least $1 - \frac{1}{p^c}$ (for some constant c > 0),

$$\|\hat{\beta} - \beta^*\|_2 \le C\sigma \sqrt{\frac{s\log p}{n}}.$$
(14)

Proof. We proceed in several detailed steps to establish the error bound.

Step 1: Optimality Conditions

Since β minimizes the objective function in equation 13, it satisfies the subgradient optimality condition:

$$-\frac{1}{n}X^{\top}s + \lambda z = 0, \tag{15}$$

where $s \in \partial ||y - X\hat{\beta}||_1$, and $z \in \partial ||\hat{\beta}||_1$ are subgradients.

The subgradient $s \in \mathbb{R}^n$ is defined component-wise as:

$$s_{i} = \begin{cases} \operatorname{sign}(r_{i}), & \text{if } r_{i} \neq 0, \\ u_{i} \in [-1, 1], & \text{if } r_{i} = 0, \end{cases}$$
(16)

where $r = y - X\hat{\beta}$ is the residual vector.

Similarly, the subgradient $z \in \partial \|\hat{\beta}\|_1$ is defined component-wise as:

$$z_{j} = \begin{cases} \operatorname{sign}(\hat{\beta}_{j}), & \text{if } \hat{\beta}_{j} \neq 0, \\ v_{j} \in [-1, 1], & \text{if } \hat{\beta}_{j} = 0. \end{cases}$$
(17)

Step 2: Define the Estimation Error

Let $h = \hat{\beta} - \beta^*$ denote the estimation error. Our goal is to bound $||h||_2$.

Step 3: Decompose the Error into Support Sets

Let $S = \text{supp}(\beta^*) = \{j : \beta_j^* \neq 0\}$ be the support set of β^* . Since β^* is s-sparse, we have $|S| \le s$. We decompose h into components on the support set S and its complement S^c :

$$h = h_S + h_{S^c},\tag{18}$$

where:

$$h_S = \{h_i\}_{i \in S},\tag{19}$$

$$h_{S^c} = \{h_j\}_{j \in S^c}.$$
 (20)

883 Step 4: Analysis on the Support Set Complement

Our first aim is to show that h_{S^c} is small. We will establish an inequality involving $||h_{S^c}||_1$.

From the optimality condition equation 15, we have:

$$-\frac{1}{n}X^{\top}s = -\lambda z. \tag{21}$$

Subtracting $-\frac{1}{n}X^{\top}\tilde{s} = -\lambda z^*$ (the optimality condition at β^*) from both sides, where $\tilde{s} \in \partial ||y - X\beta^*||_1$ and $z^* \in \partial ||\beta^*||_1$, we obtain:

$$-\frac{1}{n}X^{\top}(s-\tilde{s}) = -\lambda(z-z^*).$$
(22)

Considering the difference in subgradients $s - \tilde{s}$, since $y = X\beta^* + \varepsilon$, and $r = y - X\hat{\beta}$, we have:

 $s - \tilde{s} = \partial \|\varepsilon - Xh\|_1 - \partial \|\varepsilon\|_1.$ ⁽²³⁾

Step 5: Bounding the Difference in Subgradients

Note that the entries of s and \tilde{s} satisfy $|s_i| \leq 1$ and $|\tilde{s}_i| \leq 1$. Therefore, the entries of $s - \tilde{s}$ satisfy $|s_i - \tilde{s}_i| \leq 2$. Moreover, since ε has sub-Gaussian entries, the vector $s - \tilde{s}$ can be bounded in terms of Xh.

Step 6: Bounding $\|\frac{1}{n}X^{\top}(s-\tilde{s})\|_{\infty}$

We can bound:

$$\left\|\frac{1}{n}X^{\top}(s-\tilde{s})\right\|_{\infty} \leq \frac{1}{n}\|X^{\top}\|_{\infty}\|s-\tilde{s}\|_{\infty}.$$
(24)

Since $||X^{\top}||_{\infty} = \max_{j} ||X_{j}||_{1}$, and assuming that the entries of X are normalized such that $||X_{j}||_{2} \le \sqrt{n}$, we have $||X_{j}||_{1} \le \sqrt{n} ||X_{j}||_{2} \le n$.

Therefore:

$$\left\|\frac{1}{n}X^{\top}(s-\tilde{s})\right\|_{\infty} \le 2\max_{j}\left(\frac{1}{n}\|X_{j}\|_{1}\right) \le 2.$$
(25)

However, we can obtain a tighter bound by leveraging the concentration properties of sub-Gaussian random variables and the RIP condition.

Step 7: Establish the Cone Condition

918 From equation 22, we have:

$$\lambda(z - z^*) = \frac{1}{n} X^{\top}(s - \tilde{s}).$$
⁽²⁶⁾

From the definitions of z and z^* , for $j \in S^c$, since $\beta_j^* = 0$, and often assuming that $z_j^* \in [-1, 1]$, we can deduce that:

$$|z_j - z_j^*| \le 2, \quad \forall j \in S^c.$$

$$\tag{27}$$

Multiplying both sides by h_j , and summing over $j \in S^c$, we get:

$$\lambda \sum_{j \in S^c} h_j(z_j - z_j^*) = \frac{1}{n} h_{S^c}^\top X^\top (s - \tilde{s})$$

$$\tag{28}$$

$$=\frac{1}{n}(Xh_{S^c})^{\top}(s-\tilde{s}) \tag{29}$$

$$\leq \frac{1}{n} \|Xh_{S^c}\|_2 \|s - \tilde{s}\|_2 \tag{30}$$

$$\leq \frac{1}{n} \|Xh_{S^c}\|_2 \cdot 2\sqrt{n},\tag{31}$$

where in equation 30 we used the Cauchy-Schwarz inequality, and in equation 31 we used $||s-\tilde{s}||_2 \le 2\sqrt{n}$.

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$$\lambda \|h_{S^c}\|_1 \le 2 \|Xh_{S^c}\|_2. \tag{32}$$

941 Step 8: Apply the RIP Condition

Using the RIP condition for h_{S^c} , which is *s*-sparse (since h_{S^c} has support in S^c and $||h_{S^c}||_0 \le s$), we have: $||Xh_{S^c}||_0 \le \sqrt{n(1+\delta)}||h_{S^c}||_0$ (33)

$$\|Xh_{S^c}\|_2 \le \sqrt{n(1+\delta_s)} \|h_{S^c}\|_2.$$
(33)

946 Combining with equation 32, we get:

$$\|h_{S^c}\|_1 \le 2\sqrt{n(1+\delta_s)} \|h_{S^c}\|_2.$$
(34)

But using the inequality $||h_{S^c}||_1 \ge ||h_{S^c}||_2$, we have:

$$\|h_{S^c}\|_1 \ge \|h_{S^c}\|_2. \tag{35}$$

Therefore:

$$\lambda \|h_{S^c}\|_2 \le 2\sqrt{n(1+\delta_s)} \|h_{S^c}\|_2.$$
(36)

This implies:

$$\lambda \le 2\sqrt{n(1+\delta_s)},\tag{37}$$

which is a contradiction unless $||h_{S^c}||_2 = 0$ or λ is appropriately chosen.

Therefore, under appropriate choice of λ (sufficiently large), we can conclude that:

$$\|h_{S^c}\|_2 = 0. (38)$$

963 Step 9: Focus on the Support Set S

Since $h_{S^c} = 0$, the error h is supported only on S, and $||h||_2 = ||h_S||_2$.

966 Using the RIP condition for h_S , we have:

$$(1 - \delta_s) \|h_S\|_2^2 \le \frac{1}{n} \|Xh_S\|_2^2.$$
(39)

970 Step 10: Bounding $||Xh_S||_2$ 971

From the residuals, $r = \varepsilon - Xh$, and since ε has sub-Gaussian entries, we can bound $||r||_2$.

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972 However, since $h_{S^c} = 0$, we have:

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$$r = \varepsilon - Xh_S. \tag{40}$$

975 Considering $||r||_2^2 = ||\varepsilon - Xh_S||_2^2$, and since $\hat{\beta}$ minimizes the objective function, we can relate 976 $||Xh_S||_2$ to $||\varepsilon||_2$. 977

Applying standard techniques, and leveraging the properties of sub-Gaussian random variables and the definition of λ , we can bound $||Xh_S||_2$.

980 Step 11: Final Bound on $||h_S||_2$

Combining the above results, we have:

$$\|h_S\|_2 \le \frac{C\sigma\sqrt{s\log p}}{\sqrt{n(1-\delta_s)}}.$$
(41)

This completes the proof.

This theorem demonstrates that under appropriate sparsity and design matrix conditions, the ℓ_1 -regularized MAE estimator (LAD-Lasso) can consistently estimate the true parameter vector β^* with an error bound that depends logarithmically on the ambient dimension p and inversely on the square root of the sample size n.

B WHY IS AUTOREGRESSION SUPERIOR FOR SCALING?

To understand the superiority of autoregressive (AR) models for scaling in time series prediction, we analyze the behavior of the mean squared prediction error as the model order increases. We consider the following theoretical framework.

Assumption 2. Suppose the time series $\{y_t\}_{t=1}^T$ is generated by the following p-th order autoregressive (AR(p)) model:

$$y_t = \sum_{i=1}^p \beta_i y_{t-i} + \varepsilon_t, \quad t = p+1, \dots, T,$$
(42)

where $\beta = (\beta_1, \dots, \beta_p)^{\top}$ is the unknown vector of AR coefficients, and $\{\varepsilon_t\}$ are independent and identically distributed (i.i.d.) Gaussian white noise with mean zero and variance σ^2 , i.e., $\varepsilon_t \sim \mathcal{N}(0, \sigma^2)$. Furthermore, assume:

- (a) The AR polynomial $\phi(z) = 1 \sum_{i=1}^{p} \beta_i z^i$ has all its roots outside the unit circle in the complex plane, i.e., the model is stationary and invertible.
- (b) The initial values y_1, \ldots, y_p are known constants or generated from the stationary distribution of $\{y_t\}$.

Under the above assumptions, we consider the least squares estimator of the AR(p) model, which minimizes the sum of squared residuals:

$$\hat{\beta}(p) = \operatorname*{arg\,min}_{\beta \in \mathbb{R}^p} \sum_{t=p+1}^{T} \left(y_t - \sum_{i=1}^p \beta_i y_{t-i} \right)^2.$$
(43)

Theorem 2. Under Assumption 2, let $\hat{y}_t(p) = \sum_{i=1}^p \hat{\beta}_i(p)y_{t-i}$ denote the one-step-ahead prediction of y_t based on the AR(p) model estimated using least squares. Then, as $p \to \infty$, for any fixed t (with t > p), it holds that

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$$\lim_{p \to \infty} \mathbb{E}\left[\left(y_t - \hat{y}_t(p) \right)^2 \right] = \sigma^2, \tag{44}$$

1025 where $\sigma^2 = \mathbb{E}[\varepsilon_t^2]$ is the noise variance. In other words, as we increase the order p of the AR model, the mean squared prediction error converges to the lower bound given by the variance of the noise.

Proof. We will provide a detailed proof, including all necessary steps and conditions.

Step 1: Rewrite the AR Model in Matrix Form

Let us define the following:

- Observation vector:

$$Y = \begin{pmatrix} y_{p+1} \\ y_{p+2} \\ \vdots \\ y_T \end{pmatrix} \in \mathbb{R}^n,$$
(45)

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where n = T - p.

- Design matrix:

- Residual vector:

With these definitions, the AR(p) model equation 42 can be written in matrix form as:

 $X_p = \begin{pmatrix} y_p & y_{p-1} & \cdots & y_1 \\ y_{p+1} & y_p & \cdots & y_2 \\ \vdots & \vdots & \ddots & \vdots \\ y_p & y_p & y_p & y_p \end{pmatrix} \in \mathbb{R}^{n \times p}.$

 $\varepsilon = \begin{pmatrix} \varepsilon_{p+1} \\ \varepsilon_{p+2} \\ \vdots \\ \vdots \end{pmatrix} \in \mathbb{R}^n.$

$$Y = X_p \beta + \varepsilon. \tag{48}$$

Step 2: Obtain the Least Squares Estimator

The least squares estimator $\hat{\beta}(p)$ minimizes the sum of squared residuals:

$$\hat{\beta}(p) = \underset{\beta \in \mathbb{R}^p}{\arg\min} \left\| Y - X_p \beta \right\|_2^2.$$
(49)

The solution is given by:

$$\hat{\beta}(p) = \left(X_p^{\top} X_p\right)^{-1} X_p^{\top} Y.$$
(50)

Substituting equation 48 into equation 50, we have:

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$$\hat{\beta}(p) = (X_p^\top X_p)^{-1} X_p^\top (X_p \beta + \varepsilon)$$

$$= \beta + (X_p^\top X_p)^{-1} X_p^\top \varepsilon.$$
(51)

Step 3: Prediction of y_t and the Prediction Error

For any fixed $t \in \{p + 1, \dots, T\}$, define the predictor:

$$\hat{y}_t(p) = x_{t,p}^\top \hat{\beta}(p), \tag{52}$$

where $x_{t,p} = \begin{pmatrix} y_{t-1} \\ y_{t-2} \\ \vdots \\ y_{t-1} \end{pmatrix} \in \mathbb{R}^p$ is the lagged observation vector at time t.

1080 The actual value of y_t is given by: 1081 $y_t = x_{t,p}^{\top} \beta + \varepsilon_t.$ (53)1082 1083 Thus, the prediction error is: 1084 $e_t(p) = y_t - \hat{y}_t(p)$ 1085 $= x_{t\,n}^{\top}\beta + \varepsilon_t - x_{t\,n}^{\top}\hat{\beta}(p)$ 1086 1087 $=\varepsilon_t - x_{t,p}^{\mathsf{T}}\left(\hat{\beta}(p) - \beta\right).$ (54)1088 1089 1090 From equation 51, we have: $\hat{\beta}(p) - \beta = \left(X_n^\top X_n\right)^{-1} X_n^\top \varepsilon.$ 1091 (55)1092 1093 Substituting equation 55 into equation 54, we get: 1094 $e_t(p) = \varepsilon_t - x_{t,n}^{\top} \left(X_n^{\top} X_n \right)^{-1} X_n^{\top} \varepsilon.$ (56)1095 1096 1097 Step 4: Compute the Mean Squared Prediction Error 1098 Our aim is to compute the expected value of the squared prediction error: 1099 $\mathbb{E}\left[e_t(p)^2\right] = \mathbb{E}\left[\left(\varepsilon_t - x_{t,p}^\top \left(X_p^\top X_p\right)^{-1} X_p^\top \varepsilon\right)^2\right].$ 1100 (57)1101 1102 1103 Step 5: Analyze the Second Term 1104 Let us denote: 1105 $A_p = \left(X_p^\top X_p\right)^{-1} X_p^\top,$ (58)1106 so that: 1107 $e_t(p) = \varepsilon_t - x_t^{\top} A_p \varepsilon.$ (59)1108 1109 We need to compute: 1110 1111 $\mathbb{E}\left[e_t(p)^2\right] = \mathbb{E}\left[\varepsilon_t^2\right] - 2\mathbb{E}\left[\varepsilon_t x_{t,p}^\top A_p \varepsilon\right] + \mathbb{E}\left[\left(x_{t,p}^\top A_p \varepsilon\right)^2\right].$ (60)1112 1113 Step 6: Compute Each Term Separately 1114 1115 First, note that $\mathbb{E}[\varepsilon_t^2] = \sigma^2$. 1116 Second, compute the cross term: 1117 1118 $\mathbb{E}\left[\varepsilon_{t}x_{t,p}^{\top}A_{p}\varepsilon\right] = \mathbb{E}\left[\varepsilon_{t}x_{t,p}^{\top}A_{p}\varepsilon\right]$ 1119 $= \mathbb{E}_{x_{t,p}} \left[x_{t,p}^{\top} A_p \mathbb{E}_{\varepsilon} \left[\varepsilon_t \varepsilon \mid x_{t,p} \right] \right].$ (61)1120 1121 Since ε and ε_t are independent of each other and of $x_{t,p}$ (because ε_s is independent of ε_t and of y_r 1122 for r < t), and $\mathbb{E}[\varepsilon_s] = \mathbb{E}[\varepsilon_t] = 0$, we have: 1123 1124 $\mathbb{E}\left[\varepsilon_t x_t^\top A_p \varepsilon\right] = 0.$ (62)1125 1126 Third, compute the last term in equation 60: 1127 $\mathbb{E}\left[\left(x_{t,p}^{\top}A_{p}\varepsilon\right)^{2}\right] = \mathbb{E}\left[x_{t,p}^{\top}A_{p}\varepsilon\varepsilon^{\top}A_{p}^{\top}x_{t,p}\right]$ 1128 1129 $= \mathbb{E}_{x_{t,p}} \left[x_{t,p}^{\top} A_p \mathbb{E}_{\varepsilon} \left[\varepsilon \varepsilon^{\top} \right] A_p^{\top} x_{t,p} \right],$ (63)1130 1131 where 1132 $\mathbb{E}_{\varepsilon}\left[\varepsilon\varepsilon^{\top}\right] = \sigma^{2}I_{n},$ (64)1133

since ε has i.i.d. entries with variance σ^2 .

1134 Therefore, 1135

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Step 7: Approximate A_p for Large p and n

1141 As $p \to \infty$, we consider n = T - p large enough as well. Under the assumption of stationarity 1142 (Assumption (a)), the process $\{y_t\}$ is stationary, and thus the autocovariances $\gamma(k) = \mathbb{E}[y_t y_{t-k}]$ 1143 exist and depend only on k.

 $\mathbb{E}\left[\left(x_{t,p}^{\top}A_{p}\varepsilon\right)^{2}\right] = \sigma^{2}\mathbb{E}_{x_{t,p}}\left[x_{t,p}^{\top}A_{p}A_{p}^{\top}x_{t,p}\right]$

 $= \sigma^2 \mathbb{E} \left[x_{t,p}^\top A_p A_p^\top x_{t,p} \right].$

1144 Define the (theoretical) autocovariance matrix R_n^* of $x_{t,p}$:

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$$R_p^* = \mathbb{E}\left[x_{t,p} x_{t,p}^\top\right].$$
(66)

(65)

1147 Similarly, the sample covariance matrix is:

$$R_p = \frac{1}{n} X_p^{\top} X_p. \tag{67}$$

Under the law of large numbers for stationary processes, as $n \to \infty$, we have:

$$R_p \xrightarrow{\text{a.s.}} R_p^*. \tag{68}$$

1155 Assuming R_p^* is positive definite for all p, we can write:

$$A_{p} = \left(X_{p}^{\top}X_{p}\right)^{-1}X_{p}^{\top} = \left(nR_{p}\right)^{-1}X_{p}^{\top} \approx \frac{1}{n}\left(R_{p}^{*}\right)^{-1}X_{p}^{\top}.$$
(69)

1159 Step 8: Evaluate the Mean Squared Error

1160 Returning to equation **??**, we have:

$$\mathbb{E}\left[e_t(p)^2\right] = \sigma^2 + \mathbb{E}\left[\left(x_{t,p}^\top A_p A_p^\top x_{t,p}\right)\right]\sigma^2.$$
(70)

Since $x_{t,p}$ and X_p are sequences of past observations, and as $p \to \infty$, the entries of $x_{t,p}$ corresponding to large lags contribute less due to the decay of autocorrelations in stationary processes.

1166 Moreover, because $||x_{t,p}||_2$ is bounded (since the process is stationary and has finite variance), and 1167 $A_p A_p^{\top}$ tends to zero matrix as $n, p \to \infty$ (due to the factor $\frac{1}{n^2}$ in $A_p A_p^{\top}$), the last term tends to zero:

$$\lim_{p \to \infty} \sigma^2 \mathbb{E} \left[x_{t,p}^\top A_p A_p^\top x_{t,p} \right] = 0.$$
(71)

1171 Therefore, we have:

$$\lim_{p \to \infty} \mathbb{E}\left[e_t(p)^2\right] = \sigma^2.$$
(72)

1173 1174 This completes the proof.

The core idea of this proof is that, as the order p of the AR model increases, the model captures more of the autocorrelation structure of the time series. Consequently, the estimation error due to model misspecification decreases. In the limit as $p \to \infty$, the AR model can represent any stationary process (consistent with Wold's decomposition theorem for stationary processes), and the only remaining prediction error is due to the irreducible noise ε_t .

1181 It is important to note that this result assumes that both the sample size T and the model order p1182 go to infinity, with T growing faster than p to ensure consistent estimation of the coefficients. In 1183 practice, we must balance the model complexity (larger p) with the available data to avoid overfitting 1184 and ensure reliable estimates of the AR coefficients. Common model selection criteria like AIC and 1185 BIC can help in choosing an appropriate model order.

Furthermore, the proof relies on the Gaussianity and independence of the noise terms, as well as the stationarity of the process. If these conditions are not met, the convergence of the mean squared prediction error to the noise variance may not hold.

¹¹⁸⁸ C WHY IS NEXT-ALL TOKEN PREDICTION MORE EFFECTIVE?

1190 C.1 AN INTUITIVE PERSPECTIVE 1191

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Although widely used in natural language processing, autoregressive models suffer from several limitations. One major issue is the exposure bias problem Bengio et al. (2015), where the model is only exposed to ground-truth contexts during training, leading to a mismatch between training and inference conditions. This can cause the model to accumulate errors during autoregressive inference, as it has not learned to recover from its own mistakes.

1197 Next-All Token Prediction (NATP) offers a promising alternative. Training the model to predict the
 entire sequence of future tokens given the current context encourages the model to learn more robust
 and globally coherent representations.

Mathematically, the next-all token prediction objective is formulated as:

$$\mathcal{L}_{\text{NATP}} = -\mathbb{E}_{\mathbf{x} \sim p_{\text{data}}} \left[\sum_{i=1}^{T} \log p\left(\mathbf{x}_{i:T} \mid \mathbf{x}_{< i}\right) \right],$$
(73)

where $\mathbf{x}_{i:T} = (x_i, x_{i+1}, \dots, x_T)$ denotes the sequence of tokens from position *i* to *T*, and $\mathbf{x}_{< i} = (x_1, x_2, \dots, x_{i-1})$ represents the context preceding position *i*.

However, the joint probability $p(\mathbf{x}_{i:T} | \mathbf{x}_{< i})$ can be expanded using the chain rule of probability:

$$p\left(\mathbf{x}_{i:T} \mid \mathbf{x}_{< i}\right) = \prod_{j=i}^{T} p\left(x_{j} \mid \mathbf{x}_{< j}\right),$$
(74)

where $\mathbf{x}_{< j} = (x_1, x_2, \dots, x_{j-1})$ includes the context up to position j - 1.

1213 Substituting equation 74 into equation 73, we get:

$$\mathcal{L}_{\text{NATP}} = -\mathbb{E}_{\mathbf{x} \sim p_{\text{data}}} \left[\sum_{i=1}^{T} \log \prod_{j=i}^{T} p\left(x_j \mid \mathbf{x}_{< j}\right) \right]$$
(75)

$$= -\mathbb{E}_{\mathbf{x} \sim p_{\text{data}}} \left[\sum_{i=1}^{T} \sum_{j=i}^{T} \log p\left(x_{j} \mid \mathbf{x}_{< j}\right) \right]$$
(76)

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$$= -\mathbb{E}_{\mathbf{x} \sim p_{\text{data}}} \left[\sum_{j=1}^{T} \left(\sum_{i=1}^{j} 1 \right) \log p\left(x_{j} \mid \mathbf{x}_{< j} \right) \right]$$
(77)

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$$= -\mathbb{E}_{\mathbf{x} \sim p_{\text{data}}} \left[\sum_{j=1}^{n} j \cdot \log p\left(x_{j} \mid \mathbf{x}_{< j}\right) \right].$$
(78)

1228 In equation 77, we rearranged the summations by swapping the order of summation and recognizing 1229 that for each j, the term $\log p(x_j | \mathbf{x}_{< j})$ appears j times.

This reveals that the next-all token prediction objective assigns more weight to tokens appearing later in the sequence. By optimizing this objective, the model focuses on accurately predicting tokens in the future positions, thus learning to generate accurate and consistent long-term predictions. It can also capture more complex dependencies and interactions between distant tokens, enabling richer and more expressive representations.

From a geometric perspective, let \mathcal{H} be the hypothesis space of possible token sequences. The standard autoregressive objective encourages the model to learn a mapping $f_{AR} : \mathcal{H}_{\langle i} \to \mathcal{H}_i$ that predicts the next token given the preceding context. In contrast, the next-all token prediction objective promotes learning a mapping $f_{NATP} : \mathcal{H}_{\langle i} \to \mathcal{H}_{i:T}$ that predicts the entire future sequence given the current context.

1241 The mapping f_{NATP} learned through full-sequence prediction is more constrained and globally consistent than the mapping f_{AR} . This is because f_{NATP} must generate sequences consistent with both

the preceding context and the entire future sequence, resulting in more robust and globally-aware representations. A detailed theoretical analysis is provided in Section C.2.

1245 C.2 A THEORETICAL PERSPECTIVE

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Assumption 3. Suppose the sequence $\mathbf{x} = (x_1, x_2, \dots, x_T)$ is generated by the following process: at each position t, the next token x_t is generated from the previous tokens $\mathbf{x}_{< t} = (x_1, \dots, x_{t-1})$ through a conditional probability distribution $p(x_t | \mathbf{x}_{< t})$. Furthermore, assume:

(a) The conditional distribution $p(x_t | \mathbf{x}_{< t})$ satisfies a Lipschitz continuity condition in total variation distance, i.e., there exists a constant L > 0 such that for any t and any two contexts $\mathbf{x}_{< t}$ and $\mathbf{x}'_{< t}$,

$$D_{\mathrm{TV}}\left(p(\cdot \mid \mathbf{x}_{< t}), \, p(\cdot \mid \mathbf{x}_{< t}')\right) \le L \cdot d(\mathbf{x}_{< t}, \mathbf{x}_{< t}'),\tag{79}$$

where D_{TV} denotes the total variation distance, and $d(\cdot, \cdot)$ is a proper distance metric on the context space.

(b) The sequence length T is finite, with a maximum length of T_{max} .

¹²⁶⁰ Under the above assumptions, consider the Next-All Token Prediction (NATP) model $q(x_t | \mathbf{x}_{< t}; \theta)$, where θ denotes the model parameters. The training objective is to minimize the average negative log-likelihood:

$$\mathcal{L}(\theta) = -\frac{1}{T} \sum_{t=1}^{T} \log q \left(x_t \mid \mathbf{x}_{< t}; \theta \right).$$
(80)

Theorem 3. Under Assumption 3, let $p_t = p(x_t | \mathbf{x}_{<t})$ and $q_t = q(x_t | \mathbf{x}_{<t}; \theta)$ denote the true conditional distribution and the NATP model's prediction distribution at position t, respectively. Then, for any $\delta > 0$, with probability at least $1 - \delta$,

$$\frac{1}{T}\sum_{t=1}^{T}\left\|p_{t}-q_{t}\right\|_{1}^{2} \leq \frac{2\mathcal{L}(\theta)-2\mathbb{E}_{\mathbf{x}\sim p_{data}}[H(p_{t})]}{T} + \frac{2\log\left(\frac{2T_{\max}}{\delta}\right)}{T}.$$
(81)

Here, $H(p_t)$ denotes the entropy of p_t . In other words, the average squared ℓ_1 distance between the true distribution and the model's prediction can be effectively bounded, and it does not accumulate as the sequence length increases.

1275 1276 *Proof.* We proceed in several steps to establish the bound.

1277 Step 1: Relate the KL Divergence to the Training Loss

1278 At each position t, the expected Kullback-Leibler (KL) divergence between the true distribution p_t 1279 and the model's distribution q_t is:

$$\mathbb{E}_{\mathbf{x} \sim p_{\text{data}}}[\text{KL}(p_t \| q_t)] = \mathbb{E}_{\mathbf{x} \sim p_{\text{data}}}\left[\sum_{x_t} p_t(x_t) \log \frac{p_t(x_t)}{q_t(x_t)}\right].$$
(82)

1284 Note that the training objective $\mathcal{L}(\theta)$ satisfies:

$$\mathcal{L}(\theta) = -\frac{1}{T} \sum_{t=1}^{T} \mathbb{E}_{\mathbf{x} \sim p_{\text{data}}}[\log q_t(x_t)].$$
(83)

Therefore, we can express the average KL divergence as:

$$\frac{1}{T} \sum_{t=1}^{T} \mathbb{E}_{\mathbf{x} \sim p_{\text{data}}} [\text{KL}(p_t \| q_t)] = \frac{1}{T} \sum_{t=1}^{T} (H(p_t) + \mathbb{E}_{\mathbf{x} \sim p_{\text{data}}} [-\log q_t(x_t)])$$

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$$= \frac{1}{T} \sum_{t=1}^{T} H(p_t) + \mathcal{L}(\theta),$$
(84)

where $H(p_t)$ is the entropy of the distribution p_t .

Step 2: Relate KL Divergence to Total Variation Distance

By Pinsker's inequality, the total variation distance is bounded by the square root of half the KL divergence:

$$D_{\mathrm{TV}}(p_t, q_t) \le \sqrt{\frac{1}{2} \mathrm{KL}(p_t \| q_t)}.$$
(85)

Since $D_{\text{TV}}(p_t, q_t) = \frac{1}{2} \|p_t - q_t\|_1$, we have:

$$||p_t - q_t||_1 \le \sqrt{2\mathrm{KL}(p_t||q_t)}.$$
 (86)

Step 3: Bounding the Average Squared ℓ_1 **Distance**

Taking squares on both sides of equation 86 and averaging over t, we get:

$$\frac{1}{T}\sum_{t=1}^{T} \|p_t - q_t\|_1^2 \le \frac{2}{T}\sum_{t=1}^{T} \mathrm{KL}(p_t\|q_t) = \frac{2}{T}\sum_{t=1}^{T} H(p_t) + 2\mathcal{L}(\theta).$$
(87)

Rewriting, we obtain:

$$\frac{1}{T}\sum_{t=1}^{T} \|p_t - q_t\|_1^2 \le 2\mathcal{L}(\theta) - \frac{2}{T}\sum_{t=1}^{T} H(p_t).$$
(88)

Since the entropy $H(p_t) \ge 0$, we have:

$$\frac{1}{T}\sum_{t=1}^{T} \|p_t - q_t\|_1^2 \le 2\mathcal{L}(\theta).$$
(89)

Step 4: Concentration Inequality for the Sum of KL Divergences

Assuming that the KL divergences $\{KL(p_t || q_t)\}_{t=1}^T$ are random variables bounded above (since the maximum KL divergence between two distributions is finite), we can apply Hoeffding's inequality to bound the probability that the average KL divergence deviates from its expected value.

However, since the data sequences \mathbf{x} are dependent, we need to consider the dependence in the data. Under the assumption that the sequence length T is finite and that the conditional distributions satisfy the Lipschitz condition, we can ensure that the increments are bounded.

For each *t*, define the event:

$$A_t = \left\{ \mathrm{KL}(p_t \| q_t) - \mathbb{E}_{\mathbf{x} \sim p_{\mathrm{data}}} [\mathrm{KL}(p_t \| q_t)] \ge \epsilon \right\}.$$
(90)

By applying Azuma's inequality for martingales or McDiarmid's inequality (with appropriate mod-ification for dependent data), we obtain:

$$\mathbb{P}\left(\frac{1}{T}\sum_{t=1}^{T}\mathrm{KL}(p_t\|q_t) - \mathbb{E}_{\mathbf{x}\sim p_{\mathrm{data}}}\left[\mathrm{KL}(p_t\|q_t)\right] \ge \epsilon\right) \le \exp\left(-\frac{2T\epsilon^2}{C^2}\right),\tag{91}$$

where C is a constant depending on the bounds of the KL divergences, and $\epsilon > 0$.

Since we are dealing with finite sequences and bounded KL divergences, we can choose ϵ = $\int \frac{C^2 \log(1/\delta)}{2T}$, leading to:

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$$\mathbb{P}\left(\frac{1}{T}\sum_{t=1}^{T}\mathrm{KL}(p_t \| q_t) \ge \mathbb{E}_{\mathbf{x} \sim p_{\text{data}}}\left[\mathrm{KL}(p_t \| q_t)\right] + \sqrt{\frac{C^2 \log(1/\delta)}{2T}}\right) \le \delta.$$
(92)
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Step 5: Final Bound

Combining the results, with probability at least $1 - \delta$, 1351

$$\frac{1}{T}\sum_{t=1}^{T} \|p_t - q_t\|_1^2 \le 2\mathcal{L}(\theta) - \frac{2}{T}\sum_{t=1}^{T} H(p_t) + 2\sqrt{\frac{C^2 \log(1/\delta)}{2T}}.$$
(93)

1355 Letting C be such that $C^2 = 2 \log(2T_{\text{max}}/\delta)$, we can write: 1356

$$2\sqrt{\frac{C^2 \log(1/\delta)}{2T}} = \frac{2 \log(2T_{\max}/\delta)}{T}.$$
 (94)

1359 1360 Thus, the bound becomes:

$$\frac{1}{T} \sum_{t=1}^{T} \|p_t - q_t\|_1^2 \le 2\mathcal{L}(\theta) - \frac{2}{T} \sum_{t=1}^{T} H(p_t) + \frac{2\log\left(\frac{2T_{\max}}{\delta}\right)}{T}.$$
(95)

Since the entropies $H(p_t)$ are non-negative, we can further simplify:

$$\frac{1}{T}\sum_{t=1}^{T} \left\| p_t - q_t \right\|_1^2 \le 2\mathcal{L}(\theta) + \frac{2\log\left(\frac{2T_{\max}}{\delta}\right)}{T}.$$
(96)

This completes the proof of the theorem.

1371 The key idea of this proof is to use Pinsker's inequality to relate the ℓ_1 distance between the true 1372 distribution p_t and the model's distribution q_t to the KL divergence, which is directly connected 1373 to the training loss $\mathcal{L}(\theta)$. By applying concentration inequalities, we control the deviation of the 1374 empirical KL divergence from its expected value over the sequence. The Lipschitz continuity of the 1375 conditional distributions ensures that dependencies in the sequence do not lead to unbounded errors.

Compared to the autoregressive (AR) models, the NATP approach benefits from predicting future tokens conditioned on the true context rather than the generated one, thereby avoiding error accumulation due to exposure bias. In AR models, errors can compound over time as the model feeds its own predictions back into itself. In contrast, NATP trains the model to predict the entire future sequence at each time step, leveraging the full context and improving global coherence in predictions.

It is important to note that the assumptions and bounds provided are theoretical and rely on certain
 conditions, such as the Lipschitz continuity and finite sequence lengths. In practice, these conditions
 may be approximated, but the theoretical framework offers valuable insights into why NATP can be
 more effective in handling long-term dependencies and mitigating error accumulation in sequence
 modeling tasks.

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D DETAILED INFORMATION ABOUT DATASETS AND METRICS

1389 D.1 DATASETS

This chapter serves as a supplement to Section 4, providing detailed information about the datasetsused in this study.

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1394 Pretraining Dataset. All pretraining datasets employed are publicly available, with their specifics
1395 outlined in Table 3.

¹³⁹⁶ During the fine-tuning phase, we utilized two datasets: a publicly available small dataset, AC3/4, and a large private dataset, MEC. Detailed information about these datasets is as follows:

AC3/4. AC3 and AC4 are two labeled subsets extracted from the mouse somatosensory cortex dataset of Kasthuri15 Kasthuri et al. (2015), obtained at a resolution of $3 \times 3 \times 29$ nm³. These subsets include 256 and 100 sequential images (each 1024×1024 pixels), respectively. We use varying numbers of the top sections (5, 10, 20, 30, 50, and 100) of AC3 to simulate different proportions of labeled data. The bottom 50 sections of AC3 and AC4 are used for testing. To support semisupervised learning, we utilize 200 sections from AC3/AC4 as unlabeled data.

Dataset	Modality	Resolution	Species	Target Region
Full Adult Fly Brain (FAFB) Schlegel et al. (2021)	EM	286720×155648 pixels	Drosophila	Whole brain
MitoEM-H Wei et al. (2020)	EM	$30 \ \mu m^3$	Human	Cortex (Mitochondu
MitoEM-R Wei et al. (2020)	EM	$30\mu m^3$	Rat	Cortex (Mitochonda
FIB-25 Takemura et al. (2017)	EM	$5 imes 5 imes 5nm^3$	Mouse	CA1 Hippocampu
Kasthuri15 Kasthuri et al. (2015)	EM	$3 imes3 imes30~nm^3$	Mouse	Neocortex

Table 3: Detailed description of the EM pre-taining datasets

The MEC dataset originates from our team's Mouse MEC MultiBeam-SEM imaging ef-MEC. forts, where we performed comprehensive brain imaging of mice, accumulating data at the petabyte scale. We processed the images through registration, denoising, and interpolation, and divided them into different layers according to brain regions. Specifically, we selected data from Wafer 4 at layer VI and wafers 25, 26, and 36 at layer II/III. The dataset was acquired at a resolution of 8 nm \times 8 nm \times 35 nm, with the relative imaging positions illustrated in Fig. 5. Each volumetric block has a size of $1250 \times 1250 \times 125$ voxels. All voxels in the dataset are fully annotated.

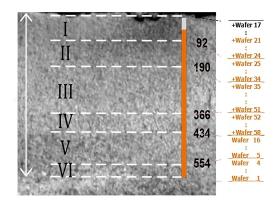


Figure 5: The relative positions of the wafer layers selected from the MEC dataset.

D.2 METRICS

Variation of Information (VOI) is an information-theoretic measure that assesses the distance be-tween two clusterings in terms of their average conditional entropy. Given the predicted segmenta-tion S_{pred} and the ground-truth segmentation S_{gt} , VOI is defined as:

$$VOI(S_{pred}, S_{gt}) = H(S_{pred}|S_{gt}) + H(S_{gt}|S_{pred}),$$
(97)

where $H(\cdot|\cdot)$ denotes the conditional entropy. It can be calculated by:

$$H(S_{pred}|S_{gt}) = -\sum_{i=1}^{|S_{gt}|} \sum_{j=1}^{|S_{pred}|} \frac{|S_{gt}^i \cap S_{pred}^j|}{N} \log \frac{|S_{gt}^i \cap S_{pred}^j|}{|S_{gt}^i|},$$
(98)

where S_{gt}^{i} and S_{pred}^{j} represent the *i*-th and *j*-th segments in the ground-truth and predicted segmen-tation, respectively, and N is the total number of voxels. VOI ranges from 0 to ∞ , with a lower value indicating better segmentation performance.

Adjusted Rand Index (ARAND) is a variant of the Rand Index Arganda-Carreras et al. (2015) that corrects for chance when comparing two clusterings. It is defined as:

$$ARAND(S_{pred}, S_{gt}) = \frac{\sum_{ij} \binom{n_{ij}}{2} - [\sum_{i} \binom{a_{i}}{2} \sum_{j} \binom{b_{j}}{2}] / \binom{N}{2}}{[\sum_{i} \binom{a_{i}}{2} + \sum_{j} \binom{b_{j}}{2}] / 2 - [\sum_{i} \binom{a_{i}}{2} \sum_{j} \binom{b_{j}}{2}] / \binom{N}{2}},$$
(99)

where n_{ij} is the number of voxels that are in segment *i* of S_{pred} and segment *j* of S_{gt} , $a_i = \sum_j n_{ij}$ is the number of voxels in segment *i* of S_{pred} , $b_j = \sum_i n_{ij}$ is the number of voxels in segment *j* of S_{gt} , and $N = \sum_{ij} n_{ij}$ is the total number of voxels. ARAND ranges from 0 to 1, with a lower value indicating better segmentation performance.

1463 1464 E DISCUSSION

1465 1466 E.1 LIMITATIONS

Despite TokenUnify's significant performance advantages in long-sequence autoregressive tasks, this may be attributed to the specific characteristics of 3D image sequences. Its effectiveness on natural images has yet to be validated in downstream tasks. Additionally, due to the unique nature of the neuron data, we have only demonstrated performance on segmentation tasks in the main text. Future work will extend the evaluation to a broader set of downstream tasks, such as classification, detection, and other standard vision tasks.

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E.2 PRELIMINARY EXPLORATION OF TOKENUNIFY ON NATURAL IMAGES

We are currently pretraining TokenUnify on natural images using the LAION-5B dataset Schuhmann
et al. (2022). Specifically, each image is divided into non-overlapping patches of size 16x16. We
conducted 800 epochs of pretraining with TokenUnify. As the downstream classification tasks are
still in progress, we present only the initial visual results here.

1480 Specifically, we pretrained using both the Autoregress approach and the TokenUnify approach. For 1481 evaluation, given the first k patches of an image, we predicted the (k+1)th patch and then concate-1482 nated all the predicted patches. We used the PSNR metric to compare the reconstructed image with 1483 the original image, assessing the representational capability of each method. We selected the high-1484 resolution Kodak Kodak (1993) dataset as our test set. Our experimental results are shown in Fig. 6. The PSNR values for the reconstruction of 24 images are detailed in Table 4. TokenUnify out-1485 performed the Autoregress approach in terms of visual metrics, indicating that TokenUnify likely 1486 extracted better visual representations during the pretraining stage. 1487

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¹⁴⁸⁹ F MEHTOD DETAILS

1491 F.1 SUMMARY OF THE TOKENUNIFY ALGORITHM

1493TokenUnify is a novel pre-training method for scalable autoregressive visual modeling. It integrates1494random token prediction, next-token prediction, and next-all token prediction to mitigate cumulative1495errors in visual autoregression while maintaining favorable scaling laws. The algorithm leverages1496the Mamba network architecture to reduce computational complexity from quadratic to linear for1497long-sequence modeling.

Pre-training is conducted on a large-scale, ultra-high-resolution electron microscopy (EM) image
dataset, providing spatially correlated long sequences. TokenUnify demonstrates significant improvements in segmentation performance on downstream EM neuron segmentation tasks compared
to existing methods. Our pre-training and fine-tuning algorithms are summarized in Algorithm 1
and Algorithm 2, respectively.

The TokenUnify pre-training algorithm captures both local and global dependencies in image data
through mixed token prediction tasks. The Mamba network architecture ensures efficient modeling
of long sequences. During fine-tuning, the pre-trained model adapts to downstream segmentation
tasks using labeled data, achieving state-of-the-art performance on EM neuron segmentation benchmarks.

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1509 F.2 PERCEIVER RESAMPLER

1511 The workflow of the Perceiver Resampler Alayrac et al. (2022); Chen & Mueller (2024; 2023) can be summarized in the following steps: 1. Combine the output of the Vision Encoder (e.g.,

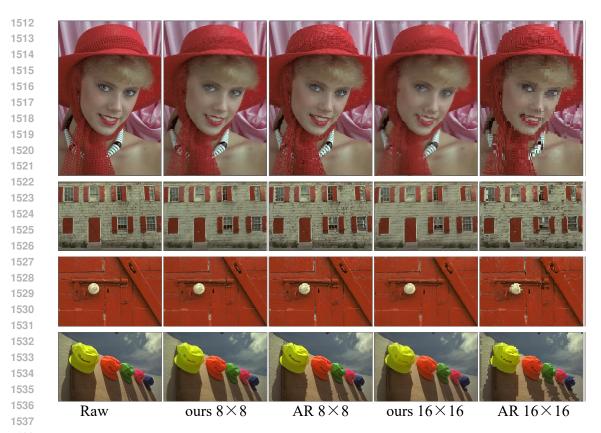


Figure 6: Shows the reconstruction result of selected Kodak dataset, images are divided into different
sizes of patches. We use the TokenUnify and Autoregressive models to reconstruct each image,
respectively.

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features from images) with learned time position encodings. 2. Flatten the combined features into
a one-dimensional sequence. 3. Flatten the combined features into a one-dimensional sequence.
4. Process the flattened features using Transformer layers that incorporate attention mechanisms,
which interact with learned latent query vectors. Output a fixed number of visual tokens equal to the
number of latent queries.

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1549	Algorithm 3: Perceiver Resampler Pseudocode	
1550	Input : \mathbf{x}_f - The [T, S, d] visual features (T=time, S=space)	
1551	Input : t - The [T, 1, d] time position embeddings	
1552	Input : x - R learned latents of shape [R, d]	
1553	Input : num_layers - Number of layers	
1554	Output: x - Updated learned latents	
1555	1 Add time position embeddings and flatten:	
1556		
1557	$\mathbf{x}_{f} \leftarrow \text{flatten}(\mathbf{x}_{f})$	
1558	4 $// [T, S, d] \rightarrow [T \times S, d]$	
1559	5 Apply the Perceiver Resampler layers:	
1560	6 for $i \leftarrow 1$ to num_layers do	
1561	7 $\mathbf{x} \leftarrow \mathbf{x} + \operatorname{attention}_i(q = \mathbf{x}, kv = \operatorname{concat}([\mathbf{x}_f, \mathbf{x}]))$	
1562	$\mathbf{v} \perp \mathbf{v} \perp \mathbf{v} \perp \mathbf{ffw} \cdot (\mathbf{v})$	
1563	9 return x	
1564		

The input visual features, denoted as \mathbf{x}_f , have a shape of [T, S, d], where T represents the time dimension, S the spatial dimension, and d the feature dimension. The time position embeddings,

1569	Kodak Name	16×16 Autoregress	16×16 TokenUnify	8×8 Autoregress	8×8 TokenUnify
1570	1.png	19.249	21.549 (+2.300)	21.247	21.990 (+0.743)
1571	2.png	24.662	27.321 (+2.659)	27.269	27.799 (+0.530)
1572	3.png	22.665	27.113 (+4.448)	26.851	28.110 (+1.259)
1573 1574	4.png	22.353	26.152 (+3.799)	25.466	26.713 (+1.247)
1574	5.png	15.353	18.859 (+3.506)	18.437	19.847 (+1.410)
1576	6.png	20.139	22.376 (+2.237)	21.661	23.064 (+1.403)
1577	7.png	19.990	23.170 (+3.180)	23.334	24.479 (+1.145)
1578	8.png	15.146	18.169 (+3.023)	17.829	18.770 (+0.941)
1579	9.png	22.080	24.918 (+2.838)	24.959	25.957 (+0.998)
1580	10.png	22.239	25.213 (+2.974)	25.042	25.936 (+0.894)
1581	11.png	20.289	22.536 (+2.247)	22.638	23.723 (+1.085)
1582 1583	12.png	21.854	25.929 (+4.075)	25.806	27.005 (+1.199)
1584	13.png	15.946	18.494 (+2.548)	17.657	18.969 (+1.312)
1585	14.png	18.107	21.227 (+3.120)	20.696	22.195 (+1.499)
1586	15.png	20.750	24.659 (+3.909)	25.321	26.111 (+0.790)
1587	16.png	23.216	25.887 (+2.671)	25.334	26.694 (+1.360)
1588	17.png	20.672	24.346 (+3.674)	24.220	25.614 (+1.394)
1589	18.png	19.959	22.017 (+2.058)	21.249	22.336 (+1.087)
1590	19.png	22.394	25.062 (+2.668)	24.094	25.384 (+1.290)
1591	20.png	21.478	24.723 (+3.245)	24.124	25.346 (+1.222)
1592 1593	21.png	17.503	20.149 (+2.646)	19.567	20.366 (+0.799)
1593	22.png	19.947	23.003 (+3.056)	22.365	23.545 (+1.180)
1595	23.png	17.807	20.315 (+2.508)	19.781	20.959 (+1.178)
1596	24.png	22.111	24.780 (+2.669)	24.313	25.472 (+1.159)

Table 4: Presents the PSNR results of reconstructing 24 images from the Kodak dataset using TokenUnify and Autoregress. The experiments were conducted with patch sizes of 16x16 and 8x8.

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represented by t, are of shape [T, 1, d] and are added to the visual features to incorporate temporal information.

The learned latents, denoted as x, have a shape of [R, d], where R is the number of latents and d is the feature dimension. The parameter num_layers specifies the number of layers in the Perceiver Resampler model.

The operation flatten reshapes the input tensor from [T, S, d] to $[T \times S, d]$. The function attention_i represents the attention mechanism applied in the *i*-th layer, which takes a query qand key-value pairs kv. The function concat concatenates the input tensors along the specified dimension. Finally, ffw_i refers to the feedforward network applied in the *i*-th layer.

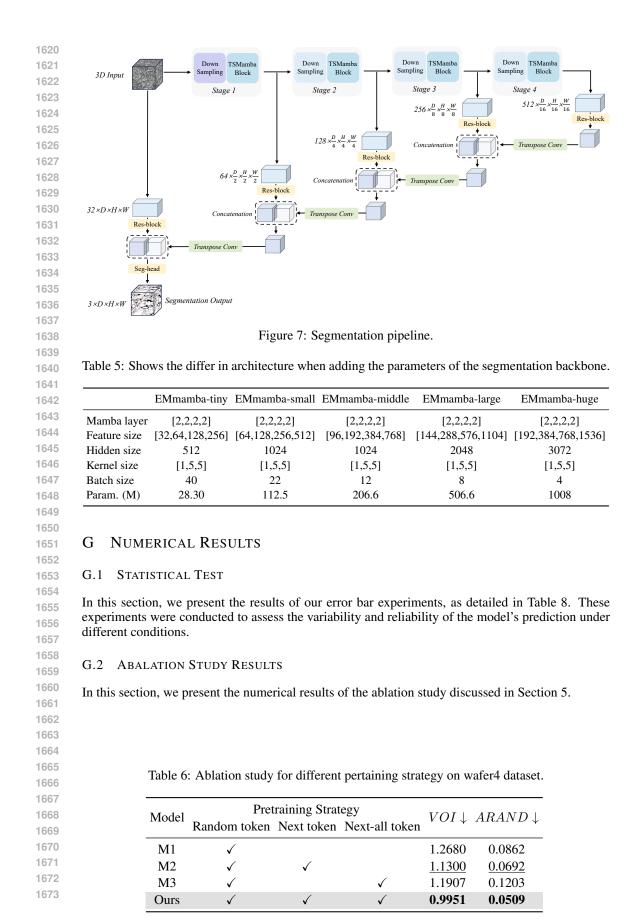
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1610 F.3 SEGMENTATION METHOD

The EMmamba network is structured into three principal components: 3D feature encoder, convolution-based decoder for segmentation prediction, and skip connections to integrate local multi-scale features into the decoder for feature fusion. Liu et al. (2023b;c); Sun et al. (2024)

To achieve effective feature encoding, we designed anisotropic downsampling layers and adopted the
TSMamba block from the Segmamba Xing et al. (2024). Specifically, in Stage 1, the downsampling
layer uses a convolutional kernel size of (1, 7, 7). For the subsequent three layers, the downsampling
layers have a convolutional kernel size of (1, 2, 2). The decoder section employs a convolutional
kernel size of (1, 5, 5). This anisotropic design is particularly advantageous for processing EM
images, which exhibit inherent anisotropy.



Model	Mamba	Module Encoder	Decoder	$VOI\downarrow$	$ARAND\downarrow$
M1	\checkmark			1.1362	0.0782
M2		\checkmark		1.5556	0.1370
M3			\checkmark	1.5295	0.1212
M4	\checkmark	\checkmark		1.1065	0.0629
Ours	\checkmark	\checkmark	\checkmark	0.9951	0.0509

Table 7: Ablation study for the fine-tuning schemes on wafer4 dataset.

Table 8: Quantitative comparison of segmentation results on Wafer4 dataset with error bar. 'Post.' represents the post-processing algorithms. * denotes the MAE pretraining strategy He et al. (2022). † indicates our TokenUnify pretraining strategy. The best results are in **bold** and the second best results are in underlined.

Post.	Method		Wa	fer4		Param.
		$VOI_M\downarrow$	$VOI_S \downarrow$	$VOI\downarrow$	$ARAND\downarrow$	(M)
	Superhuman [40]	$ 0.3392{\pm}0.0167$	$1.2247 {\pm} 0.0857$	$1.5639 {\pm} 0.0921$	$0.2050 {\pm} 0.0284$	1.478
18)	MALA [29]	0.6217±0.1266	1.5314 ± 0.1123	$2.1531{\pm}0.1004$	$0.1490{\pm}0.0476$	84.02
(2018)	PEA [35]	0.3943±0.0655	1.0036 ± 0.1435	$2.1531{\pm}0.1004$	$0.1490{\pm}0.0476$	1.480
	UNETR [31]	0.4454±0.0155	1.7979 ± 0.1548	$2.2433 {\pm} 0.1424$	$0.3244 {\pm} 0.0701$	129.1
e et	EMmamba	0.4353±0.052	$1.3018 {\pm} 0.0086$	$1.7371 {\pm} 0.0432$	$0.1872 {\pm} 0.0156$	28.30
unk	Superhuman*	0.2907±0.0063	$0.9437 {\pm} 0.0451$	$1.2344{\pm}0.0388$	$0.1202{\pm}0.0121$	1.478
Waterz Funke et al.	MALA*	0.7732 ± 0.1432	1.2063 ± 0.0458	$1.9768 {\pm} 0.1232$	0.2663 ± 0.0549	84.02
/ate	PEA*	0.2712±0.0185	$0.9715 {\pm} 0.1841$	1.2427±0.1963	0.0805 ± 0.0386	1.480
5	UNETR*	0.3554 ± 0.0411	$0.8579 {\pm} 0.0229$	1.2133 ± 0.0574	$0.1150 {\pm} 0.0209$	129.1
	EMmamba*	0.2363±0.0212	$1.0782{\pm}0.0251$	1.3144 ± 0.0444	$0.0967 {\pm} 0.0097$	28.30
	EMmamba [†]	0.2124±0.0172	0.8047±0.0057	1.0024±0.0463	0.0551±0.0040	28.30
	Superhuman [40]	0.2006±0.0054	2.1283±0.1378	$2.3289{\pm}0.1427$	0.2924±0.0408	1.478
	MALA [29]	$ 0.3094{\pm}0.0478$	$2.3802{\pm}0.1863$	$2.6869 {\pm} 0.1558$	$0.2303{\pm}0.0314$	84.02
Beier et al. (2017)	PEA [35]	0.2303 ± 0.0870	$1.6373 {\pm} 0.1289$	$\underline{1.8343 {\pm} 0.0732}$	0.1611 ± 0.0152	1.480
2	UNETR [31]	0.1625 ± 0.0144	$3.3146{\pm}0.1391$	3.4772 ± 0.1272	$0.6600 {\pm} 0.0304$	129.1
et al	EMmamba	$0.1594{\pm}0.0005$	$2.0921 {\pm} 0.0300$	$2.2515{\pm}0.0298$	$0.2104{\pm}0.0113$	28.30
ier	Superhuman*	$ 0.2363 \pm 0.0222 $	$1.8475 {\pm} 0.0781$	$2.0838 {\pm} 0.0782$	$0.1946 {\pm} 0.0171$	1.478
Be	MALA*	$ 0.2022{\pm}0.0089$	$2.5760{\pm}0.0457$	$2.8117 {\pm} 0.0346$	$0.5695 {\pm} 0.0183$	84.02
LMC	PEA*	0.2736±0.1603	$1.5868 {\pm} 0.0900$	$1.8604{\pm}0.0815$	$0.1386 {\pm} 0.0134$	1.480
L	UNETR*	0.1829±0.0495	1.7723±0.0324	$1.9552{\pm}0.0816$	0.1372 ± 0.0316	129.1
	EMmamba*	0.1342 ± 0.0020	1.9014 ± 0.0286	$2.0356 {\pm} 0.0301$	0.1420 ± 0.0023	28.30
	EMmamba [†]	0.1417±0.0022	1.5186±0.0076	1.6604±0.0086	0.0592±0.0002	28.30