Effectively Using Public Data in Privacy Preserving Machine Learning

Milad Nasr [1] **Saeed Mahloujifar** [2] **Xinyu Tang** [2] **Prateek Mittal** [2] **Amir Houmansadr** [3]

Abstract

Differentially private (DP) machine learning techniques are notorious for their degradation of model utility (e.g., they degrade classification accuracy). A recent line of work has demonstrated that leveraging *public data* can improve the trade-off between privacy and utility when training models with DP guarantee. In this work, we further explore the potential of using public data in DP models, showing that utility gains can in fact be significantly higher than what shown in prior works. Specifically, we introduce DOPE-SGD, a modified DP-SGD algorithm that leverages public data during its training. DOPE-SGD uses public data in two complementary ways: (1) it uses advance augmentation techniques that leverages public data to generate synthetic data that is effectively embedded in multiple steps of the training pipeline; (2) it uses a modified gradient clipping mechanism in DP-SGD to change the *origin* of gradient vectors using the information inferred from available public data, therefore boosting utility. We also introduce a technique to ensemble intermediate DP models by leveraging the post processing property of differential privacy to further improve the accuracy of the predictions. Our experimental results demonstrate the effectiveness of our approach in improving the state-of-the-art in DP machine learning across multiple datasets, network architectures, and application domains. For instance, assuming access to $2,000$ public images, and for a privacy budget of $\varepsilon = 2, \delta = 10^{-5}$, our technique achieves an accuracy of 75.1% on CIFAR10, significantly higher than 68.1% achieved by the state of the art.

1. Introduction

Machine learning is becoming an essential part of many technological advancements in various fields. One major concern with using machine learning is the privacy of individuals whose data is used to develop the machine learning models. To tackle this concern, recent works (De et al., 2022; Kurakin et al., 2022; Abadi et al., 2016; Yu et al., 2021; Amid et al., 2022; Li et al., 2022a) suggest to train ML models with *differential privacy (DP)* ((Dwork et al., 2014)) guarantees. However, existing DP techniques for ML impose large degradation to the utility of the trained models in comparison to non-private models. Recent works have used various techniques to improve the utility-privacy trade-off of such private ML techniques (e.g., by scaling the hyper-parameters ((De et al., 2022))); however, there still exists a huge gap between the accuracy of DP-guaranteeing ML mechanisms and their non-private alternatives, e.g., the SOTA technique of (De et al., 2022) achieves an accuracy of 65% on CIFAR10 (for $\varepsilon = 2.0$ and $\delta = 10^{-5}$) compared to the $> 90\%$ accuracy of non-private models.

In this work, we explore an emerging approach to close the utility gap between private and non-private models. Specifically, recent works ((De et al., 2022; Kurakin et al., 2022; Abadi et al., 2016; Yu et al., 2021; Li et al., 2022b)) show that leveraging *publicly available (therefore, non-private) data* can enhance the utility of DP-trained models without impacting their privacy guarantees. In such works, the public data is used to *pre-train* the model, and then the pre-trained model is fine-tuned with the private data while applying DP protections.

In this work, we show that public data can be leveraged significantly more effectively in boosting the utility of DP models. Specifically, we design a generic method to utilize public data in differentially private machine learning, an approach we call *Differentially Private Origin Estimation Stochastic Gradient Descent (DOPE-SGD)*. DOPE-SGD uses two complementary techniques to enhance the utility of differentially private models using public data. First, DOPE-SGD uses advanced data augmentation techniques to enhance the quality of the data used for training, therefore reducing overfitting to the public data and improving generalization. Second, it improves the quality of the noisy gradients by taking advantage of the available (augmented)

[1] Google Deepmind [2] Princeton University [3] University of Massachusetts Amherst. Correspondence to: Milad Nasr <srxzr@google.com>.

Proceedings of the 40^{th} International Conference on Machine Learning, Honolulu, Hawaii, USA. PMLR 202, 2023. Copyright 2023 by the author(s).

public (non-private) data to modify the clipping approach used in DP-SGD. This helps by reducing the variance of the noise added to the gradients in the DP model, therefore better preserving the information in the original gradient vector (Section 3).

In this work, we also take advantage of intermediate models in the DP training as an ensemble to improve the accuracy of the final model. Ensemble of intermediate models leverages the post-processing property of differential privacy which does not impact the privacy guarantees of the trained models. Specifically, we introduce two ensemble approaches (1) taking the average of the model parameters (2) aggregation of the prediction vectors by majority voting.

Through extensive experiments we show that DOPE-SGD's use of public data along with data augmentation improves the privacy-utility trade-off of private models by large margins. For instance, we show improvements up to 12.3% accuracy over DP-SGD models on the CIFAR10 test dataset, pre-trained with the same public data. We also show improvements on language models both on training from scratch (from 221 to 198 in perplexity on a small BERT model) and fine-tuning (from 21.23 to 19.09 perplexity using GPT-2) with $\varepsilon = 1.0$ and $\delta = 10^{-5}$.

2. Background

Differential privacy ((Dwork, 2011; Dwork et al., 2014)) is the gold standard for data privacy. It is formally defined as below:

Definition 1 (Differential Privacy)**.** *A randomized mechanism $\mathcal{M}$ with domain $\mathcal{D}$ and range $\mathcal{R}$ preserves $(\varepsilon, \delta)-$differential privacy iff for any two neighboring datasets $D, D' \in \mathcal{D}$ and for any subset $S \subseteq \mathcal{R}$ we have:*

$$\Pr[\mathcal{M}(D) \in S] \leq e^{\varepsilon} \Pr[\mathcal{M}(D') \in S] + \delta \qquad (1)$$

where ε is the privacy budget *and δ is the* failure probability.

Several works have used differential privacy in traditional machine learning algorithms to protect the privacy of the training data ((Li et al., 2014; Chaudhuri et al., 2011; Feldman et al., 2018; Zhang et al., 2016; Bassily et al., 2014)). Many of these works ((Feldman et al., 2018; Bassily et al., 2014; Chaudhuri et al., 2011)) use properties such as convexity or smoothness for their privacy analysis, which is not necessarily true in deep learning, and therefore, one cannot use many of such methods in practice. (Abadi et al., 2016) designed a deep learning training algorithm, DP-SGD, where they used gradient clipping to limit the sensitivity of the learning algorithm, and then add noise to a clipped model gradient proportional to its sensitivity. As we know, training a deep learning model is an iterative process. The main approach to analyze the privacy cost of private deep learning is to compute the privacy cost of the single step of the learning algorithm and then use composition method to calculate the overall privacy cost which is commonly done in RDP ((Mironov, 2017)) instead of (ε, δ)-DP. One of the important features of differential privacy is the post-processing ((Mironov, 2017; Dwork et al., 2014)) which we will utilize in this work. DP-SGD is now commonly used to train differentially private deep learning models.

One common technique to improve the utility of the private training is to use public datasets and pre-train the model on the public data. Recent works (Amid et al., 2022; Ferrando et al., 2021; Li et al., 2022a; Wang & Zhou, 2020; Alon et al., 2019) showed it is possible to utilize public data more than just using it for pre-training. With respect to private deep learning, recent work (Amid et al., 2022; Li et al., 2022a) and our work both use public data to estimate the geometry of the gradient field. All of the recent work used a different approach to approximate the geometry estimation using the public data. Mirror gradient descent (Amid et al., 2022) used the gradient of public data for first order approximation of the gradient geometry. In the case of DP-SGD, they modified its update step to:

$$\theta_{t+1} = \theta_t - \eta_t(\alpha_t(g_t^{\hat{pr}} + \mathcal{N}) + (1 - \alpha_t)(g_t^{pub})) \qquad (2)$$

where θ_t is the model parameters at step t, η_t is the learning rate, α_t is the weighting between the public and private gradients, $g_t^{\hat{pr}}$ is the gradients of the private data where gradient of each data point is clipped (as required in DP-SGD algorithm (Abadi et al., 2016)), $\mathcal{N}$ is the noise added by the Gaussian mechanism and g_t^{pub} is the gradient of the public data. Essentially this approach uses a weighted mean between the privatized gradients on the private dataset and the public gradient to update the model parameters. Li et al. (2022a) uses public data to re-scale individual gradients of private data before clipping. We compared our approach to both techniques in Section 4.1. Additionally, there are multiple studies (Andrew et al., 2021; Varshney et al., 2022) that investigate adjusting the clipping threshold in DP-SGD. These studies are orthogonal to our work and could potentially be combined with our approach to further boost utility. A previous study (Ye & Shokri, 2022) demonstrated that by projecting the gradient into a smaller subspace and perturbing the gradients, they could achieve improvements over the baseline. This approach could also be applied to our work to further enhance the results.

De et al.(De et al., 2022) demonstrated that by utilizing Exponential Moving Average (EMA), one can further improve the performance of differentially private training. In our work, we extend this approach to other averaging schemes and demonstrate improvements compared to the existing method. Concurrently, Shejwalkar et al. (2022) also examined the effects of employing averaging mechanisms on the performance of private training.

3. DOPE-SGD: Our Improved DPSGD alternative

One can improve the utility-privacy trade-off for differentially private ML algorithms in three phases: (1) use *pre-training* to improve the initial point of private training, (2) *enhance the algorithms* for differentially private training, (3) do a *post processing* on the private models. Previous works showed the effect of using pretraining ((Abadi et al., 2016; Kurakin et al., 2022; De et al., 2022)), so in this work we mainly focus on the two latter phases of private training.

3.1. DP-SGD with Adaptive Origin

Two of the main steps of DP-SGD are to clip the gradient of each instance and add noise to the gradient of each batch. The magnitude of noise is chosen based on the clipping threshold. Clipping the gradient results in a bias in the optimization process which can hurt the convergence. One way to prevent this is to use larger clipping values to ensure the bias is minimized. However, this will lead to a larger noise in order to obtain the desired privacy. The main idea of this work is to *clip the gradients around an estimate of the gradient* instead of clipping the gradient around the origin, as shown in Figure 1. As a result, we can potentially clip aggressively (i.e., use small clipping values) around the carefully chosen centers, and enjoy less bias in the optimization, compared to DP-SGD, and therefore obtain better accuracy while maintaining privacy.

We first introduce a general algorithm called DP-SGDA (Algorithm 1) that uses adaptive origin selection. We use a function $\mathcal{G}$ that takes the history of the protocol and also some auxiliary information as input, and outputs a point $\hat{g}$ that will be used as the origin for the clipping operation. The following proposition states that any instantiation of this algorithm will satisfy DP guarantee.

Proposition 2. *For any function $\mathcal{G}$, DP-SGDA (Algorithm 1) obtains the same DP and RDP guarantees as DP-SGD ((Abadi et al., 2016)), when the clipping threshold C, sub-sampling rate q, and noise parameter σ are equal in both mechanisms. See Appendix G for a proof.*

Theoretical Justification for adaptive origin selection To show the benefit of our Algorithm 1, we consider a setting of the Lipschitz loss function with concentrated gradients.

Definition 3 (Lipschitz and gradient-concentrated loss function)**.** *A loss function ℓ defined on a model space Θ and input space X is $\mathcal{L}$-lipschitz if for all $x \in X$ and $\theta \in \Theta$, $\|\frac{\partial \ell}{\partial \theta}(\theta, x)\|_2 \leq \mathcal{L}$. The loss function is r-concentrated if for all $\theta \in \Theta$, there exists a point c_θ in the gradient space such that we have*

$$\forall x \in X; \|\frac{\partial \ell}{\partial \theta}(\theta, x) - c_\theta\|_2 \leq r.$$

Note that any $\mathcal{L}$-lipschitz loss function is $\mathcal{L}$-concentrated. We call c_θ the concentration point for θ. We call an oracle function $C(\cdot)$ a concentration point oracle if given a model θ, it returns $C(\theta) = c_\theta$.

Algorithm 1 DP-SGD with Adaptive Origin (DP-SGDA)

```
Require: training dataset $D$, adaptive origin function $\mathcal{G}$, batch size $n$, learning rate $\eta$, noise scale $\sigma$, gradient norm clip $C$, loss function $l$, auxiliary information $aux$, $T$ number of training iterations
Initiate $\theta$ randomly
for $t \in \{T\}$ do
    $B_t \leftarrow$ sample $n$ instances from dataset $D$
    $\nabla_\theta^G[t] \leftarrow \vec{0}$
    $\hat{g} \leftarrow \mathcal{G}(\widetilde{\nabla_\theta^G[0]}, \dots, \widetilde{\nabla_\theta^G[t-1]}, aux)$.
    for all $(x, y) \in B_t$ do
        $\nabla_\theta^{(x,y)} \leftarrow$ gradient of $l(x, y)$
        $\overline{\nabla_\theta^{(x,y)}} \leftarrow \hat{g} + \frac{(\nabla_\theta^{(x,y)} - \hat{g}) \times C}{\max(C, \|\nabla_\theta^{(x,y)} - \hat{g})\|_2)}$
        $\nabla_\theta^G[t] \leftarrow \nabla_\theta^G[t] + \overline{\nabla_\theta^{(x,y)}}$
    end for
    $\widetilde{\nabla_\theta^G[t]} \leftarrow \nabla_\theta^G[t] + \mathcal{N}(0, \sigma^2 C^2 \mathbb{I})$
    $\theta \leftarrow \theta - \eta \widetilde{\nabla_\theta^G[t]}$.
end for
Return output $\theta$
```

Now we state the following proposition about the utility of our algorithm in comparison with that of DP-SGD. Note that our proposition below is based on the concentration assumptions, which might not hold true in practice of deep learning. Despite this, we believe it provides insight on why adaptive origin selection can lead to less privacy cost, while maintaining the same level of "bias" in optimization (as shown in Section 4.1). Additionally, it is crucial to emphasize that the assumption is only for utility analysis and our privacy analysis outlined in Proposition 2 holds regardless of the validity of the concentration assumptions.

Proposition 4. *Let X and Θ be example and model spaces and let ℓ be a $\mathcal{L}$-lipschitz and r-concentrated loss function for X and Θ. Optimizing this loss function with clipping threshold r in Algorithm 1 with concentration point oracle used as the adaptive origin function, will induce the same output (model) distribution as training with DP-SGD with clipping threshold $\mathcal{L}$, as long as all examples in the training set are in X. Algorithm 1 achieves $(c\frac{rq}{\sigma}\sqrt{T\ln(1/\delta)\ln(T/\delta)}), \delta)$-DP, where as DP-SGD achieves $(c\frac{\mathcal{L}q}{\sigma}\sqrt{T\ln(1/\delta)\ln(T/\delta)}), \delta)$-DP, for a constant c, sampling rate q, and number of iterations T and sufficiently large σ (See Appendix G for detailed proof).*

Proposition 4 shows that our algorithm can reduce the sensitivity of the gradient update at each iteration from $\mathcal{L}$ to r.

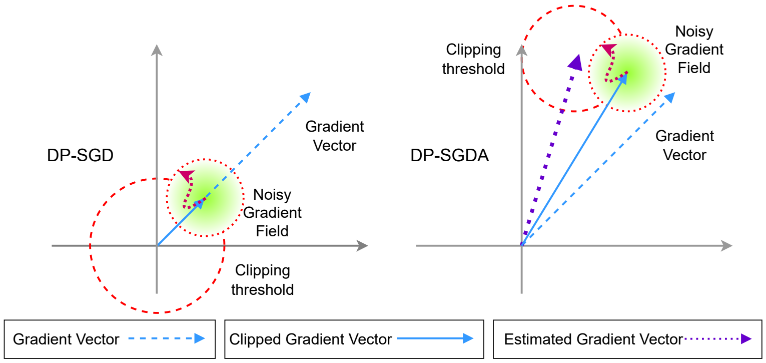


Figure 1: Clipping the gradients around an estimation of the origin (purple vector) can increase the ratio of the clipped gradient vector to the added noise. As a result the final privatized gradient vector is closer to original gradient vector compared to clipping around zero.

Given that $r < \mathcal{L}$, our algorithm would obtain better privacy guarantees when the gradients are concentrated. Although these bounds are stated based on the advanced composition theorem for approximate differential privacy, the privacy benefit of our algorithm holds for alternative notions of privacy, such as RDP, because of the reduced sensitivity in our algorithm.

Instantiation with (augmented) public data. To obtain the estimate of gradient vectors, we use the idea of *(augmented) public data*. Public data has been used previously in many traditional differentially private tasks. In this work, we show using public data to estimate gradients can improve deep learning with differential privacy significantly. Algorithm 2 summarizes the design of our approach. It is important to note that in Algorithm 2, gradient concentration is not a requirement for privacy guarantees. Even in the worst-case scenario, where our estimation of the gradient origin is significantly different from the actual gradient, our approach still offers the same level of privacy protection. Therefore, while gradient concentration can be useful for optimizing the convergence rate of the algorithm, it is not an essential component for ensuring privacy. We also take advantage of advanced augmentation techniques (such as diffusion models) to utilize the public data further. The exact data augmentation mechanism heavily depends on the specific task and type of the dataset. We can use several techniques for data augmentation, which can range from using a basic shifting of images to designing a synthesizer ((Mood, 1950)) using complex generative models ((Nichol & Dhariwal, 2021)).

3.2. Ensemble of Private Models

We analyze DP-SGD (at the time of writing this paper) by assuming every step of the iterative training is public and an adversary can use all of the intermediate steps for an attack. This approach facilitates the necessary DP analysis of the overall mechanism. On the other hand, we often only use the last model for making predictions, raising the question whether we are overestimating the privacy cost ((Nasr et al., 2021). Here, we ask the "dual" of this question; can we improve utility by leveraging all of the intermediate models?). One example of taking advantage of the intermediate models is by using the "Exponential Momentum Averages (EMA)" approach in differential private optimization ((De et al., 2022)). Here, we propose an alternative ensemble approach that uses an ensemble of the intermediate models and can further improve the accuracy of the predictions. We use two approaches to ensemble the models. First, we use the

Algorithm 2 Differentially Private Origin Estimation-SGD (DOPE-SGD)

```
Require: private training dataset $D$, non-private data $D_s$, non-private batch size $n_s$, learning rate $\eta$, private batch size $n$, noise scale $\sigma$, gradient norm clip $C$, loss function $l$, $T$ number of training iterations
Initiate $\theta$ randomly
for $t \in \{T\}$ do
    $B_t \leftarrow$ sample $n$ instances from dataset $D$
    $B_s \leftarrow$ sample $n_s$ instances from dataset $D_s$
    $\nabla_\theta^s \leftarrow \nabla L(B_s)$  ▷ Instantiate $\hat{g}$ in Algorithm 1 with $B_s$
    $\nabla_\theta^G[t] \leftarrow \vec{0}$
    for all $(x, y) \in B_t$ do
        $\nabla_\theta^{(x,y)} \leftarrow$ gradient of $l(x, y)$
        $\overline{\nabla_\theta^{(x,y)}} \leftarrow \nabla_\theta^s + \frac{((\nabla_\theta^{(x,y)} - \nabla_\theta^s)}{\max(C, \|\nabla_\theta^{(x,y)} - \nabla_\theta^s\|_2))} \times C$
        $\nabla_\theta^G[t] \leftarrow \nabla_\theta^G[t] + \overline{\nabla_\theta^{(x,y)}}$
    end for
    $\widetilde{\nabla_\theta^G[t]} \leftarrow \nabla_\theta^G[t] + \mathcal{N}(0, \sigma^2 C^2 \mathbb{I})$
    $\theta \leftarrow \theta - \eta \widetilde{\nabla_\theta^G[t]}$
end for
Return output $\theta$
```

idea of majority voting in which the ensemble model will query all sub-models and receive a label for the given input and output the majority. We use the last n models during the path of optimization and use them in this majority based ensemble architecture. In the case that we are interested in the value of logits, such as language models, we take the average of logits among the last n models . Second, we show the effectiveness of taking the average of the last n models, and evaluating the inputs on the average instead of the final model.

In the main body of our work, we use both of these ensemble techniques and only report the maximum accuracy. However, as shown in Appendix F we see that both methods have very similar performance and using majority voting achieves slightly higher performance.

We also note that our ensemble techniques can also be used in settings where we do not have access to any public datasets. Since the main focus of our work is on public dataset, we briefly explore this setting in Appendix F.

Why do ensemble methods work? Stochastic gradient descent with averaging has been studied as a way to cope with the excessive variance coming from stochasticity of SGD (Bach, 2014). One would expect the reduction of variance by averaging the resulting parameters of the last few models, assuming that the variance is caused by independent noise. Bach (2014) shows faster convergence on convex models by doing the simple averaging operation. In DP-SGD and its variants, a large portion of variance in the model comes from the added noise, which is independently sampled and added to the model after each iteration. This makes DP-SGD suitable for averaging methods as the variance is mostly the result of independent noise. It is important to note that in privacy analysis of DP-SGD and its variants, we always "pay" the privacy cost of all the intermediate models and using them as an ensemble will not violate the privacy guarantees.

We also note that several recent studies have tried to understand the privacy benefits of having hidden states during the course of training and not leaking the intermediate models (Feldman et al., 2018; Ye & Shokri, 2022). They conjecture that a portion of privacy cost spent for publishing the intermediate models in DP-SGD might be wasted, leading to sub-optimal trade-off between privacy and accuracy. The success of our ensemble methods can be seen as a verification of this conjecture and shows the possibility of better ways to use the privacy budget.

4. Experiments

To evaluate the suggested techniques we consider two main scenarios, where public data is coming from a similar distribution as the training dataset and also where the data is from a slightly different distribution. In the main body of this work, we focus on the CIFAR and WikiText datasets as they represent a majority of the applications of the deep learning models. In Appendix B, we show the effectiveness of our approach for other architectures and tasks.

CIFAR10 dataset. (De et al., 2022) showed that using a WideResNet architecture for the CIFAR10 dataset can improve the state-of-the-art differentially private model significantly. Therefore, we also use similar architecture for the majority of our experiments. In Appendix B, we also evaluate a smaller convolution neural network. Due to the computation limitation we use WideResNet16-4 as explained in (De et al., 2022) (Augmentation Multiplicity of 16) since the gain of using the larger model is not very significant.[1] We also use CIFAR100 as out-of-distribution public data.

WikiText-2 dataset. Our dataset setup mainly follows the previous work ((Amid et al., 2022)). The texts are tokenized by the top 8k most frequent words and a special token for the remaining words. The dataset is then constructed as length-35 sequences and we have 59,675 data points in the training set. We use 4% of the training set as in-distribution public data and WikiText-3 as imperfect out-of-distribution public data (we removed the overlapping part of WikiText-2 from WikiText-3). We use a BERT-based (Devlin et al., 2019) model with two blocks.

Settings: We show the results in several settings.

- **Warm/Cold**: in our experiments we use the term "*warm*" to describe a setting where we first train a model without any privacy on the non-private data (as opposed to "*cold*" where we do not pre-train).
- **Warm-Aug**: given a non-private dataset, we can augment this dataset without incurring a privacy cost and use the augmented non-private dataset to first pretrain a model.
- **Extended**: in addition to using a non-private dataset to pretrain a model, we can further improve the utility of the private models by including the public data in the training dataset. This way, with a fixed batch size, we can use a smaller sub-sampling rate in our privacy analysis. We use "*extended*" to note a case where the training dataset includes the private dataset and the (augmented) non-private dataset. Please note that we also do pretraining on the public dataset in this setting (therefore "*Extended*" is equal to "*warm-ext.*").[2]

[1] Training WRN40-4 on eight A100 in our setting takes more than 96 hours.

[2] We found that while many of the public instances have zero loss after the pretraining, it is still useful to include the public dataset in the training dataset. We explore this in our experiments

"*cold-ext.*" refers to the setting when we do not do pretraining.

In our experiments, we first evaluated the effect of each individual setting and then in cases where we did not specify the setting, the results represent the extended settings[3]. Please note that we did hyper-parameter tuning for each setting (as detailed in Appendix A). In the main body of this paper we focus on in-distribution results for CIFAR10 and WikiText-2 datasets. We evaluate the other datasets and out-of-distribution datasets in Appendices B and E.

4.1. Results

Using the in-distribution public dataset can improve the utility of differentially private machine learning significantly. However, one of the downsides of requiring an in-distribution public dataset is the cost of acquiring such data. As a result, requiring access to a large in-distribution public data can be a strong assumption. Therefore, we only assume that we have access to very limited in-distribution public data in this section. In order to address the challenge of small public dataset and prevent overfitting models on the public dataset, we use various augmentation techniques on the public dataset. In this section, we use a small portion of the target's training dataset as the public dataset.

Main in-distribution Results. In addition to traditional augmentation techniques, we train a generative model on the public dataset which can be seen as an ideal augmentation technique. This way, at each iteration, we can generate fresh samples from the generative model and calculate the average gradient over those samples. We used DDPM generative models (Nichol & Dhariwal, 2021) for image datasets (a sample of the generated images are iluustrated in Appendix D). DDPM models are currently slow in generating examples, therefore, in our implementation we generate a large set of data points from the generative models and use a random subsample of that for training the target model.

To understand the effect of each method described in this work, first we do ablation study when using different methods and algorithms in Table 1 for CIFAR10 dataset. One of the main approaches of leveraging public data is to pretrain the target model first on the public data which does not have any privacy cost and then do the private training. Since the public data for the in-distribution dataset is limited (while we are still using traditional augmentation methods) there is only a 3.5% increase in the accuracy. However, we apply our idea of using generative models to augment the public data and we see further improvements. One important step that to the best of our knowledge has not been used before is to combine the public dataset (or the generated dataset on public data) with the private data that is used during the private training. As shown in Table 1 only by including the public data in the training dataset we can increase the accuracy (1.7%).

When we combine all of the methods (use of generative models, origin estimation, and ensemble) for CIFAR10 we increase the accuracy of classification by 7.6 points when using only 4% of the data compared to the case when we pretrain on the training dataset which is the common practice. We also evaluated our results for different privacy budgets as shown in Table 2. Our results suggest that the common practice of just using the public dataset to pretrain the models is an ineffective way of using the public data and by properly using this additional information we can improve the utility of the private training significantly and we can achieve higher accuracies even at more private regions. When we increase the privacy budget, the gap between our approach and DP-SGD reduces. This behavior is expected because in the higher privacy regimes we are adding less noise, and the model has an easier time learning from the private data and does not rely on the public data as much. We show in Appendix B that our methods are also effective in smaller models and different datasets. One of the major downsides of our method is the increase in the computation cost which we will discuss in Appendix C.

Comparison. To compare our approach to the most recent works (Amid et al., 2022; Li et al., 2022a), we evaluate the accuracy of different approaches on the CIFAR10 dataset. Unfortunately, previous works all use different architectures to evaluate their method and none of them used the current state-of-the-art architecture, therefore, we can not directly compare the accuracy numbers. However, we did our best to train and do a hyper-parameter tuning of both of the previous works for a fair comparison. We used the same public dataset in all of the training runs.

Table 3 summarizes the results of our comparison. Original Setting refers to a case where we use the method from the previous works and replace the neural network with the current state-of-the-art method and do hyper-parameter tuning. As shown in Table 3 since the previous works do not use our data augmentation techniques there is a huge gap between our approach and previous works. Moreover, we showed that our augmentation techniques can also be used in any public data training approach and can improve the accuracy of the existing mechanism as well. Finally, we see that using all of the techniques our approach achieves higher accuracy compared to the existing methods. In our comparison, using mirror gradient descent approaches achieves very high accuracy as well when we add all of our techniques on top. Surprisingly the Gradient Scaling approach does not scale as well as Mirror Gradient Descent when we add our

in detail.

[3] When compared to other methods, we also evaluate in the above setting to make a fair comparison.

Table 1: An ablation study on the effect of different techniques using training data for models trained on CIFAR-10 under $(2, 10^{-5})$-DP. We use WRN16-4 with all of the augmentation and optimization techniques as detailed in (De et al., 2022). We use 2,000 images of the CIFAR10 training dataset (4% of the whole dataset) as the public data which is sampled uniformly from each class. The public dataset is augmented to 40,000 instances.

Setting	Test Acc $(\%)$
Baseline (WRN16-4 (De et al., 2022)) (cold)	64.9
+ Pretraining on the public data (warm)	68.1
+ Pretraining on the generated data using public data (warm-aug)	72.0
+ Including the generated data in the training dataset (extended)	73.7
+ Using DOPE-SGD (Algorithm 2)	74.8
+ Using ensemble models	75.1

Table 2: Test accuracy/perplexity for models trained with differential privacy with $\delta = 10^{-5}$ and using 4% of training of CIFAR10/WikiText-2 dataset as the public data. The accuracy of the model trained on only augmented public dataset (i.e, $\varepsilon = 0$) is 69.4% on CIFAR10 and perplexity is 240 on Wikitext-2 dataset.

	CIFAR10 (Test Acc)				WikiText-2 (Test Ppl)		
ε	DPSGD (cold)	DPSGD (warm)	DPSGD (warm-aug)	DOPE-SGD	DPSGD (cold)	DPSGD (warm)	DOPE-SGD
1.0	56.8%	60.1%	70.0%	**72.1**%	240	221	**198**
2.0	64.9%	68.1%	72.0%	**75.1**%	220	206	**184**
4.0	71.9%	72.4%	76.0%	**77.9**%	210	183	**177**
6.0	77.0%	77.1%	78.7%	**80.0**%	190	167	**156**

techniques. One issue might be that the Gradient Scaling approach is not optimized for the cases where the size of public data is large.

Effect of Size of the Public Dataset. The size of the public dataset can affect the utility of learning significantly. We studied the effect of different amounts of public data in Table 4. As expected by having a larger public dataset, the model can achieve better utility.

Effect of Size of Generated Data. One of the interesting trade-offs is to see how much the utility of a private model changes by increasing the number of augmentation that we generate for a given public dataset. Table 5 shows the results of this study and as we can see adding more augmentations of the public data in the training dataset increases the utility of the training. However, one of the main limiting factors of using more augmentations is the computation cost of the augmentation. Specially, the current DDPM models are costly in both training and generating different augmentations.

Pretrained Large Language Models. All of the state-of-the-art models in language tasks currently use a pretrained large language model and fine-tune them on their specific tasks. While this means that they are using a much larger training dataset, it is still very important to show we can see the same improvements as in previous experiments in this setting. To this aim, we used a pretrained GPT-2 (Radford et al., 2019) model (which is not trained on WikiText) and fine-tuned it using both DP-SGD and DOPE-SGD using 4% public data and when we also first train it on the public data without any privacy cost and then use private training. Table 6 summarizes our results, as we can see DOPE-SGD also outperforms DP-SGD in this setting even at small epsilons.

Extending to Federated Learning with User-Level Privacy. Another example of a setting where we can use non-private data is federated learning. In many federated applications, users can indicate that they do not require any privacy protections. While only a small number of the users fall in this category, the learning algorithms can benefit significantly from such users. (Amid et al., 2022) suggested using such users to first pretrain the model, however, this is not a practical approach since the data of these users will change during the training of a federated learning task. Pretraining the model only on the non-private users will slow down the federated learning process and it won't be as effective. Therefore, our evaluations of federated learning settings focuses on the cases where we do not pretrain on the data (cold setting).

For this setting, we used the EMNIST dataset which has 70,000 users and we uniformly divide the instances between the users and we assume a fixed 2% of the users are non-private. We use a WideResNet model for this task and

Table 3: Test accuracy of models trained with differential privacy with $\delta = 10^{-5}$ and using 4% of training of CIFAR10 dataset as the public data for different training algorithms.

		Original	Ours	
ε	Method	Setting	Augmentation	DOPE-SGD
$\varepsilon = 2$	Mirror Gradient Descent (Amid et al., 2022)	68.7%	70.5%	75.1%
	Gradient Scaling (Li et al., 2022a)	68.7%	69.1%	
$\varepsilon = 4$	Mirror Gradient Descent (Amid et al., 2022)	73.1%	74.5%	77.9%
	Gradient Scaling (Li et al., 2022a)	73.5%	74.1%	
$\varepsilon = 6$	Mirror Gradient Descent (Amid et al., 2022)	77.2%	78.2%	80.0%
	Gradient Scaling (Li et al., 2022a)	77.9%	78.1%	

Table 4: Test accuracy of models trained with differential privacy using different amounts of public data that is augmented to 40,000 instances on CIFAR10.

Public Data Size	$\varepsilon = 2$	$\varepsilon = 4$	$\varepsilon = 6$
50(0.1%)	66.6%	72.0%	75.4%
100(0.2%)	67.3%	72.1%	76.4%
500 (1%)	68.9%	72.1%	77.1%
1,000 (2%)	73.5%	74.9%	78.4%
2,000 (4%)	75.1%	77.9%	80.0%

Table 5: Test accuracy of models trained with differential privacy using different amounts of augmented data from $2,000(4\%)$ public data on CIFAR10.

Augmented Data Size	$\varepsilon = 2$	$\varepsilon = 4$	$\varepsilon = 6$
5K	69.5%	73.8%	78.4%
10K	70.1%	75.3%	79.3%
20K	70.9%	77.4%	79.9%
40K	75.1%	77.9%	80.0%

Table 6: Perplexity of finetuning WikiText-2 on pretrained large language models using 4% of the training dataset as public data.

	WikiText-2	
ε	DPSGD (warm)	Ours
0.25	21.30	19.16
0.5	21.29	19.14
1.0	21.23	19.09
∞	15.40	

Table 7: Test accuracy models trained with user-level differential privacy with $\delta = 10^{-5}$ on EMNIST using 2% users as public users.

ε	DP-SGD (all)	DP-SGD (only-private)	DOPE-SGD
0.5	82.2%	83.1%	84.7%
1.0	83.4%	84.6%	85.2%
2.0	85.0%	85.5%	86.0%
4.0	86.2%	86.4%	87.1%

Fed-SGD (McMahan et al., 2017) to train this model. In each round of the training we select a subset of users to update the model. We use the *user-level* central differential privacy definition to analyze the privacy cost in this setting.

Table 7 summarizes the results and the comparison with the existing approaches that can be applied in the federated learning setting. We compare our approach to using DP-SGD for this setting and show that by leveraging the public data we can achieve better results. One alternative to DP-SGD is to not apply differential privacy on the public user and only apply it on the private users. We compared to this baseline and showed superior performance. This result also shows the benefit of the Algorithm 2 even when we do not use the other techniques described in this work.

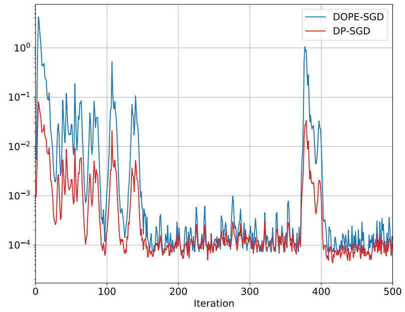


Figure 2: Comparison of DOPE-SGD and non-private's dot product with DP-SGD and non-private's dot product. Using DOPE-SGD will reduce the effect of the noise added by the Gaussian mechanism which can lead to a better performance (all experiments are in warm-aug setting).

Why DOPE-SGD Works? To answer this question we compute the inner product of the gradients of each batch when using DOPE-SGD with non-private gradients of the same batch and compare it with inner product of the DP-SGD and non-private gradients (average between 5 independent runs and after pretraining on public dataset). Figure 2 shows this comparison, as we see the dot product of DOPE-SGD is larger compared to DP-SGD especially in earlier iterations. As a result, we can achieve better convergence rates and privacy-utility trade-off.

Limitations In many real-world applications, in-distribution public data is leveraged for improving machine learning models. These applications often train models on user data, such as next word prediction for keyboards, embeddings and classifiers on user-captioned images, or spam detection systems based on user ratings. Typically, the public data is collected through explicit user permissions aimed at improving the product. data. Our main work is focused on cases where we have in-distribution public data. However, it's important to acknowledge situations where acquiring such data, for example in the case of medical datasets, might incur a high collection cost or be impossible.

To understand the limitations of our work, in Appendix E, we show that when the public data is not from the same distribution, we see smaller improvements compared to in-distribution data when directly applying our approaches. We introduce new modifications in our approaches to improve the privacy-utility trade-offs in this setting.

5. Conclusion

It is well recognized that deep learning models leak information about their training datasets. New privacy regulations will require the models to protect the privacy of the users whose data is being used to train large deep learning models. In response, differentially-private machine learning has emerged as the gold standard to train large models. However, the main limiting factor of using differentially-private machine learning is the degradation of the utility in many scenarios. To improve the utility-privacy trade-off, the research community has started to use non-private/public data. The current practice of using public data is to simply use it to pre-train the model and then use private learning on a private dataset. In this work, our main goal is to show this approach is not the optimal technique to take advantage of non-private data. We summarize our work in three practical steps that can be applied in any setting that uses public data. First, we should **use advanced augmentation techniques** (which include data synthesis approaches such as GAN, DDPM) on the public data to synthesize more non-private data points. Second, we should consider **including the non-private dataset in the private data during** the private training and finally, by **using more advanced private learning algorithms** (e.g, DOPE-SGD, (Amid et al., 2022), (Li et al., 2022a)) which are designed to leverage public data to achieve better trade-offs. Using these methods we showed that we can improve the state-of-the-art in private learning.

Acknowledgements

This work was supported in part by the NSF grant 2131910, CNS-2131938, the ARL's Army Artificial Intel- ligence Innovation Institute (A2I2), Schmidt DataX award, and Princeton E-ffiliates Award.

References

Abadi, M., Chu, A., Goodfellow, I., McMahan, H. B., Mironov, I., Talwar, K., and Zhang, L. Deep learning with differential privacy. In *Proceedings of the 2016 ACM SIGSAC Conference on Computer and Communications Security*, pp. 308–318. ACM, 2016.

Alon, N., Bassily, R., and Moran, S. Limits of private learning with access to public data. *Advances in neural information processing systems*, 32, 2019.

Amid, E., Ganesh, A., Mathews, R., Ramaswamy, S., Song, S., Steinke, T., Steinke, T., Suriyakumar, V. M., Thakkar, O., and Thakurta, A. Public data-assisted mirror descent for private model training. In *Proceedings of the 39th International Conference on Machine Learning*, pp. 517–535. PMLR, 2022.

Andrew, G., Thakkar, O., McMahan, B., and Ramaswamy, S. Differentially private learning with adaptive clipping. *Advances in Neural Information Processing Systems*, 34: 17455–17466, 2021.

Bach, F. Adaptivity of averaged stochastic gradient descent to local strong convexity for logistic regression. *The Journal of Machine Learning Research*, 15(1):595–627, 2014.

Bassily, R., Smith, A., and Thakurta, A. Private empirical risk minimization: Efficient algorithms and tight error bounds. In *2014 IEEE 55th Annual Symposium on Foundations of Computer Science*, pp. 464–473. IEEE, 2014.

Bradbury, J., Frostig, R., Hawkins, P., Johnson, M. J., Leary, C., Maclaurin, D., Necula, G., Paszke, A., VanderPlas, J., Wanderman-Milne, S., and Zhang, Q. JAX: composable transformations of Python+NumPy programs, 2018. URL `http://github.com/google/jax`.

Chaudhuri, K., Monteleoni, C., and Sarwate, A. D. Differentially private empirical risk minimization. *Journal of Machine Learning Research*, 12(3), 2011.

De, S., Berrada, L., Hayes, J., Smith, S. L., and Balle, B. Unlocking high-accuracy differentially private image classification through scale. *arXiv preprint arXiv:2204.13650*, 2022.

Devlin, J., Chang, M.-W., Lee, K., and Toutanova, K. BERT: Pre-training of deep bidirectional transformers for language understanding. In *Proceedings of the 2019 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies, Volume 1 (Long and Short Papers)*. Association for Computational Linguistics, 2019.

Dwork, C. Differential privacy. *Encyclopedia of Cryptography and Security*, pp. 338–340, 2011.

Dwork, C., Roth, A., et al. The algorithmic foundations of differential privacy. *Foundations and Trends in Theoretical Computer Science*, 9(3-4):211–407, 2014.

Feldman, V., Mironov, I., Talwar, K., and Thakurta, A. Privacy amplification by iteration. In *2018 IEEE 59th Annual Symposium on Foundations of Computer Science (FOCS)*, pp. 521–532. IEEE, 2018.

Ferrando, C., Gillenwater, J., and Kulesza, A. Combining public and private data. *CoRR*, abs/2111.00115, 2021. URL `https://arxiv.org/abs/2111.00115`.

Kurakin, A., Chien, S., Song, S., Geambasu, R., Terzis, A., and Thakurta, A. Toward training at imagenet scale with differential privacy. *arXiv preprint arXiv:2201.12328*, 2022.

Li, H., Xiong, L., Ohno-Machado, L., and Jiang, X. Privacy preserving rbf kernel support vector machine. *BioMed research international*, 2014, 2014.

Li, T., Zaheer, M., Reddi, S., and Smith, V. Private adaptive optimization with side information. In *Proceedings of the 39th International Conference on Machine Learning*, pp. 13086–13105. PMLR, 2022a.

Li, X., Tramer, F., Liang, P., and Hashimoto, T. Large language models can be strong differentially private learners. In *International Conference on Learning Representations*, 2022b.

McMahan, B., Moore, E., Ramage, D., Hampson, S., and y Arcas, B. A. Communication-efficient learning of deep networks from decentralized data. In *Artificial intelligence and statistics*, pp. 1273–1282. PMLR, 2017.

Mironov, I. Rényi differential privacy. In *2017 IEEE 30th Computer Security Foundations Symposium (CSF)*, pp. 263–275. IEEE, 2017.

Mood, A. M. Introduction to the theory of statistics. 1950.

Nasr, M., Shokri, R., and Houmansadr, A. Machine learning with membership privacy using adversarial regularization. In *Proceedings of the 2018 ACM SIGSAC Conference on Computer and Communications Security*, pp. 634–646, 2018.

Nasr, M., Song, S., Thakurta, A., Papernot, N., and Carlin, N. Adversary instantiation: Lower bounds for differentially private machine learning. In *2021 IEEE Symposium on Security and Privacy (SP)*, pp. 866–882. IEEE, 2021.

Nichol, A. Q. and Dhariwal, P. Improved denoising diffusion probabilistic models. In *International Conference on Machine Learning*, pp. 8162–8171. PMLR, 2021.

Panda, A., Tang, X., Sehwag, V., Mahloujifar, S., and Mittal, P. Dp-raft: A differentially private recipe for accelerated fine-tuning. *arXiv preprint arXiv:2212.04486*, 2022.

Radford, A., Wu, J., Child, R., Luan, D., Amodei, D., Sutskever, I., et al. Language models are unsupervised multitask learners. *OpenAI blog*, 1(8):9, 2019.

Shejwalkar, V., Ganesh, A., Mathews, R., Thakkar, O., and Thakurta, A. Recycling scraps: Improving private learning by leveraging intermediate checkpoints. *arXiv preprint arXiv:2210.01864*, 2022.

Shokri, R., Stronati, M., Song, C., and Shmatikov, V. Membership inference attacks against machine learning models. In *2017 IEEE Symposium on Security and Privacy (SP)*, pp. 3–18. IEEE, 2017.

Varshney, P., Thakurta, A., and Jain, P. (nearly) optimal private linear regression via adaptive clipping. *arXiv preprint arXiv:2207.04686*, 2022.

Wang, J. and Zhou, Z.-H. Differentially private learning with small public data. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 34, pp. 6219–6226, 2020.

Ye, J. and Shokri, R. Differentially private learning needs hidden state (or much faster convergence). *arXiv preprint arXiv:2203.05363*, 2022.

Yousefpour, A., Shilov, I., Sablayrolles, A., Testuggine, D., Prasad, K., Malek, M., Nguyen, J., Ghosh, S., Bharadwaj, A., Zhao, J., Cormode, G., and Mironov, I. Opacus: User-friendly differential privacy library in PyTorch. *arXiv preprint arXiv:2109.12298*, 2021.

Yu, D., Naik, S., Backurs, A., Gopi, S., Inan, H. A., Kamath, G., Kulkarni, J., Lee, Y. T., Manoel, A., Wutschitz, L., et al. Differentially private fine-tuning of language models. *arXiv preprint arXiv:2110.06500*, 2021.

Zhang, Z., Rubinstein, B. I., and Dimitrakakis, C. On the differential privacy of bayesian inference. In *Thirtieth AAAI Conference on Artificial Intelligence*, 2016.

A. Implementation Details

We implemented Algorithm 2 and the related works in JAX ((Bradbury et al., 2018)) and we implemented Algorithm 2 in Opacus ((Yousefpour et al., 2021) and private-transformers library (Li et al., 2022b)). Experiments in this work are the average between 5 independent runs. For each setting we did a grid hyper-parameters search as mentioned in Table 8 and picked one with the highest average. While doing hyper-parameter training will increase the privacy cost, we didn't consider this in our privacy calculation similar to the previous works ((Abadi et al., 2016; Amid et al., 2022; De et al., 2022; Kurakin et al., 2022)).

Table 8: Set of hyper-parameters used in the hyper-tuning phase.

Parameter	Values
Learning rate	[1,2,3,4,5,5.5,6]
Noise multiplier	[1,2,3,4,5,8]
Public data sample size	[80,160,640,1280,2560]
Clipping norm	[0.5,0.8,1.0,1.5]
Batch size	[512,1024,2048,4096]

We summarize different settings in the paper in the followings:

Cold: This refers to a setting where we do not pre-train on the available non-private dataset.

Warm: We call a setting warm where we first the model on all available non-private data (both augmented and original public data) and then train only on the private dataset.

Warm-aug: Similar to the previous setting we first train the model on the non-private dataset and then train it on both non-private and private dataset.

Extended: We refer to a setting where we include the non-private (which includes the augmented dataset) dataset in the private training dataset.

B. Evaluating different architectures and datasets.

To show that our techniques can also be applied to a simpler model, we used a ConvNetnetwork with three layers on CIFAR10 dataset with batch size of 256. Table 9 presents the accuracy of this model in different settings as used in Table 2. Similar to the results in Table 2 we see impressive improvement over DP-SGD.

We showed the effectiveness of our approach to the popular vision and language modeling tasks. However, many practical applications use categorical data. To show the effectiveness of our approach in such tasks, we focused on the Purchase dataset which is the shopping records for several thousand individuals. Each data record corresponds to one customer and has 600 binary features, we use Purchase10 which has 10 classes. Similar to previous works (Shokri et al., 2017; Nasr et al., 2018), we use a 4 layers fully connected network. We used 90,000 instances as the private training dataset, 5,000 instances as the public data and the public dataset is augmented 10,000 instances. Table 10 summarizes the results of our experiments. As we can see, the overall accuracy of this task is much lower than the previous datasets. However, we still see a significant gain when using public-data and using Algorithm 2.

C. Computation Complexity

The main downside of our approach is the additional computation cost compared to the DP-SGD. There are two main components to these additional costs. First, the costs of the augmentation. As we know, augmentation techniques are used widely in training deep learning models. In this work we try using diffusion models as our augmentation technique which have high computation costs. This can be the main source of the additional computation cost for our work. As a reference, training a diffusion model (Nichol & Dhariwal, 2021) on 2,000 CIFAR10 on a single V100 takes about 48 hours. Also generating 40,000 images using this model takes about a week of computation. The second component of the cost, is the additional computation cost since we have a larger non-private dataset which can increase the computation cost linearly. Moreover, Algorithm 2 uses an additional gradient computation call on the public data which will increase the computation cost. Training a model using our approach is about $2-3$ times slower than using the same model using DP-SGD without augmented images (without considering the time to generate the augmentations.).

D. Quality of the generated examples

A critical component of our work involves the utilization of advanced augmentation techniques and the employment of generative models as powerful augmentations. To demon-

Table 9: Comparison of the results on CIFAR10 dataset in different settings for both DP-SGD and DOPE-SGD using ConvNet architecture (* the setting that does not utilize the public data). Public data in these experiments are 2,000 images from the dataset which are augmented to 40,000 images.

	DP-SGD ((Abadi et al., 2016))				DOPE-SGD (Alg.2)			
ε	cold*	warm	cold-ext.	warm-ext.	cold	warm	cold-ext.	warm-ext.
1.0	46.7%	61.4%	48.0%	66.6%	67.2%	62.7%	67.8%	**70.3%**
2.0	50.1%	64.2%	51.9%	68.0%	70.1%	70.4%	71.8%	**73.0%**
4.0	54.7%	66.4%	57.4%	68.1%	75.7%	74.6%	**77.0%**	73.2%
6.0	58.4%	68.4%	59.4%	68.7%	76.2%	75.1%	**78.1%**	76.2%

Table 10: Comparison of the results on Purchase10 dataset in different settings for both DP-SGD and DOPE-SGD using Fully-Connected architecture. Public data in these experiments are 10,000 records.

	DP-SGD ((Abadi et al., 2016))				DOPE-SGD (Alg.2)			
ε	cold*	warm	cold-ext.	warm-ext.	cold	warm	cold-ext.	warm-ext.
1.0	30.2%	32.6%	34.3%	36.8%	33.2%	35.3%	40.7%	**42.1%**
2.0	34.1%	35.7%	37.9%	39.2%	35.1%	37.4%	44.6%	**45.6%**
4.0	44.4%	45.9%	47.7%	49.2%	45.2%	48.5%	50.2%	**52.3%**

strate the effectiveness of our approach, we present sample images from the output of our generative models when trained on just a small subset of the dataset in Figure 3. Remarkably, even with a limited number of examples, the generated images retain high quality. This underscores the potential of our proposed methodology in situations with limited access to training data.

E. Imperfect public data

Many real-world applications today operate in a practical setting where they have access to in-distribution public data. For example, there are applications that train models using data directly from users, such as next word prediction for keyboards. This data comes from a wide range of sources and can be extremely valuable for enhancing the accuracy and utility of machine learning models. Another common use case involves training embeddings and classifiers using captioned images from users. Similarly, user ratings can be instrumental in the development and improvement of spam detection systems.

Typically, industry deployments work with a set of users who have explicitly given permission for their data to be used in model training, usually with the aim of improving the product. It's crucial to note that this data can be categorized as in-distribution public data, providing a strong justification for the focus of our work. Currently, public data is mostly used for pre-training models. Our work aims to explore and demonstrate additional potential benefits of effectively leveraging in-distribution public data. However, there exists such as medical datasets or settings where the cost of the cost of collecting such a dataset is high. As a result many applications are starting to use other public datasets which are not exactly from the same data distribution for differentially private deep neural networks. We call such data sets imperfect data distributions which can be from a slight different distribution or have noise data or noisy labels.

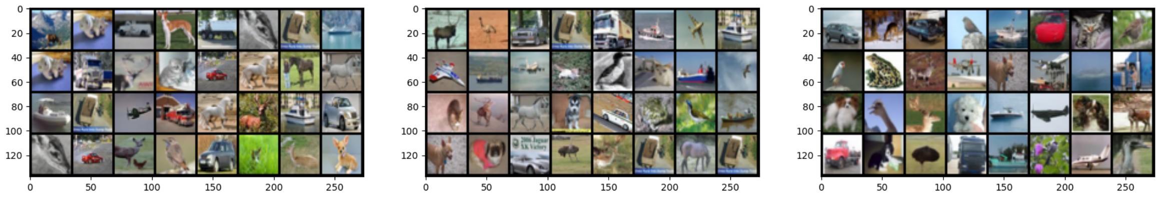
While many public data algorithms require the data to be in-distribution, the existing works showed that it is still helpful to pretrain on the out-of-distribution public datasets (Amid et al., 2022; Li et al., 2022a). In many cases naively using the imperfect public dataset can negatively affect the utility of the learning performance (see Figure 4).

This is because the main assumption in Algorithm 2 is that the gradient on the public dataset is a very close estimation of the private data. However, this is not true for the out-of-distribution dataset. In order to fix this assumption, we rewrite Algorithm 2 by modifying line 9 to the following two lines which allow us to limit the effect of the public data:

$$\nabla_\theta^{s'} = \nabla_\theta^{s} \times \frac{\lambda}{\max(\lambda, ||\nabla_\theta^{s}||_2)}$$

$$\overline{\nabla_\theta^{(x,y)}}[t] = \nabla_\theta^{s'} + (\nabla_\theta^{(x,y)}[t] - \nabla_\theta^{s'}) \tag{3}$$

$$\times \frac{C}{\max(C, \|\nabla_\theta^{(x,y)}[t] - \nabla_\theta^{s'}\|_2)}) \tag{4}$$



(a) 500 public Image (FID=107)

(b) 1000 public Image (FID=74.6)

(c) 2000 public Image (FID=12.8)

Figure 3: A sample of the generated images when only using the public images to train the diffusion models.

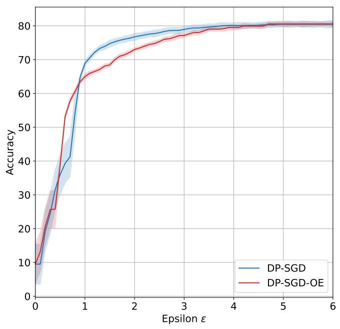


Figure 4: The effect of out-of-distribution data on Algorithm 2 without clipping the public gradient (Equation 3).

To show the effect of imperfect data in the training, we evaluate the effect of other datasets from different but similar distributions as public data (different distribution) and noisy public dataset. We used the CIFAR100 dataset as the public data dataset for CIFAR10, and WikiText-3 for WikiText-2. While many of the previous works showed that pretraining on the dataset from a slightly different distribution (such as CIFAR100 for CIFAR10) can help improve the accuracy of the original model, we also show our additional techniques can also boost the accuracy. In particular, we observe that if we include all of the training dataset in the private training it will reduce the overall test accuracy, however, by including a random subset of the public dataset in the private training part can improve the privacy-utility trade-off. In our experiments for CIFAR10 using 10% of the CIFAR100 gave us the highest boost in accuracy. For WikiText-2 dataset since the public dataset (Wikitext-3) is much more similar to the private dataset we do not see any downside of including all of the public data in the private training.

In Table 11, we study the effect of each technique on when the public data does not come from the same distribution as the private dataset. As we can see from the results, similar to the in-distribution dataset, using DOPE-SGD improves the utility further. Finally, in Table 12, we show the results for different dataset and privacy budgets. As we see, in all settings our method outperforms current approaches.

Although the CIFAR100 dataset is not typically considered in-distribution for CIFAR10, there is considerable similarity among many of the images. This raises the question of what would happen if the public data differed more substantially from the private dataset. A recent study (Panda et al., 2022) investigated the impact of different public data sources on the performance of private training. They found that as the disparity between the public and private datasets increased, the performance improvements diminished. To assess whether the same principle holds true in our case, we evaluated our approach using the PathMNIST dataset, which is a subset of the MedMNIST biomedical images dataset. Using the PathMNIST dataset, we were able to achieve accuracies of 57.9%, 68.9%, 75.6%, and 79.3% for epsilon values of 1, 2, 4, and 6, respectively. These results represent improvements of 0.8, 1.2, 1.5, and 1.8 over the respective baselines. However, when compared to our previous results, these improvements are significantly less pronounced. This suggests that the effectiveness of our approach may be constrained when the public dataset is considerably different from the private dataset.

F. Ensemble of private models without additional data

As mentioned in Section 3.2, we can also apply the idea of ensemble of the private models to improve the utility of the final model without additional privacy cost. In the main body of this work, we show the effectiveness of this approach when we have public data. However, this approach can also be used in cases when we do not have public data. In Figure 5, we compare the accuracy of different ensemble approaches. As we see using majority voting can achieve higher accuracy compared to the other approaches. We

Table 11: An ablation study on the effect of different techniques using imperfect public data on CIFAR10 models trained under $(2, 10^{-5})$-DP. We used CIFAR100 as public data for the CIFAR10 dataset.

Settings	Test Acc (%)
Baseline (WRN16-4 pretrained on CIFAR100)	75.9%
+ Including the 10% of public data in the training dataset (extended)	76.4%
+ Using DOPE-SGD (Algorithm 2) (warm)	77.1%
+ Using Ensemble models	77.3%

Table 12: Test accuracy/perplexity models trained with differential privacy with $\delta = 10^{-5}$ and imperfect data (CIFAR100 for CIFAR10 and WikiText-3 for WikiText-2).

	CIFAR10 (Test Acc)		WikiText-2 (Test Ppl)	
ε	DPSGD (warm)	DOPE-SGD	DPSGD (warm)	DOPE-SGD
1.0	68.9%	**76.3%**	79	77
2.0	76.4%	**77.3%**	78	68
4.0	79.2%	**81.5%**	76	65
6.0	82.5%	**84.9%**	75	62

also evaluate the effect of the number of models we use in the ensemble in Figure 6. In our experiments we observed that we need at least 50 models in an ensemble to have a noticeable gap between the final model and the ensemble model.

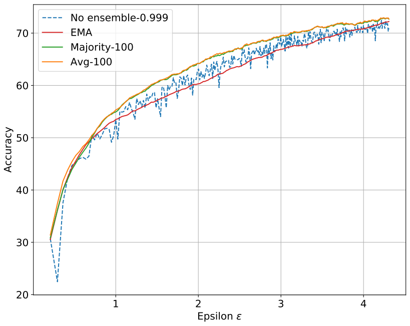


Figure 5: Comparison of the ensemble of the last 100 models, the exponential moving average (EMA with decay rate 0.999) and the model without using any ensemble techniques for DP-SGD and any additional data on test accuracy of CIFAR10.

G. Proofs

Proof of Proposition 2. We start by analyzing the sensitivity of the gradient update rule. The update before adding noise and public gradient is equal to

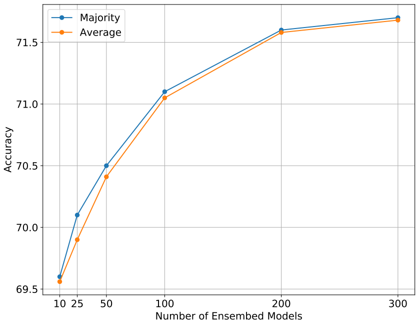


Figure 6: Comparison of the number of models in the ensemble on the accuracy for WRN-4-16 on CIFAR10 dataset with $\varepsilon = 4$, in our implementation of (De et al., 2022) we observed a slight difference in the accuracy and the reported accuracy in their work. (De et al., 2022) reported 71.4% for CIFAR10 dataset with $\varepsilon = 4$ but we were only able to achieve 71.1%.

$\nabla[t] = |B_t| \cdot \hat{g} + \sum_{(x,y)\in B_t} \frac{(\nabla^{(x,y)}-\hat{g})\times C}{\max(C,\|\nabla^{(x,y)}-\hat{g})\|_2)}$. Note that $\hat{g}$ is data independent and comes from public data, therefore we only need to understand the sensitivity of the sum of clipped gradients. Since each example (x, y) only affects one of the clipped gradients $\frac{(\nabla^{(x,y)}-\hat{g})\times C}{\max(C,\|\nabla^{(x,y)}-\hat{g})\|_2)}$ and each of these vectors have a norm bounded by C, therefore the sensitivity of the sum is C.

Now, since we are adding Gaussian noise, each iteration is an instantiation of the sub-sampled Gaussian mechanism with sensitivity C, sampling rate $\frac{|B_t|}{|D|}$ and noise σ. Therefore, all the existing analysis for general DP-SGD (without additional assumptions) would also apply to Algorithm 1. □

Proof of Proposition 4. As stated in proof of Proposition 2, each iteration of Algorithm 1 is a Gaussian mechanism with sampling rate $\frac{|B_t|}{|D|}$ and noise multiplier σ/r. Therefore, each iteration will be $(\frac{r\cdot\sqrt{2\ln(1.25/\delta')}}{\sigma}, \delta')$-DP. Assuming $\varepsilon' = \frac{r\cdot\sqrt{2\ln(1.25/\delta')}}{\sigma} < 1$, the sub-sampled mechanism will be $(2q \cdot \varepsilon', q\delta')$-DP. Then, by using advanced composition theorem for DP, we have that the composition of all T steps is $(4Tq^2 \cdot \varepsilon'^2 + 2q \cdot \varepsilon' \cdot \sqrt{2T \cdot \log(1/\hat{\delta})}, Tq\delta' + \hat{\delta})$-DP. Assuming $\varepsilon' < \sqrt{\frac{\ln(1/\hat{\delta})}{2q^2T}}$, the composition of T mechanisms is $(4q\varepsilon'\sqrt{2T\ln(1/\hat{\delta})}, T\delta' + \hat{\delta})$-DP. Now setting $\hat{\delta} = \delta/2$ and $\delta' = \delta/2n$, the entire mechanism is $(4\frac{qr}{\sigma}\sqrt{2T\ln(2.5 \cdot T/\delta)\ln(2/\delta)}, \delta)$-DP which in turn implies $(4\sqrt{10}\frac{qr}{\sigma}\sqrt{T\ln(T/\delta)\ln(1/\delta)}, \delta)$-DP. In order for the assumptions to be correct, we need $\sigma > \max(2r \cdot \sqrt{\frac{q^2T\ln(2.5\cdot T/\delta)}{\ln(2/\delta)}}, r \cdot \sqrt{2\ln(2.5 \cdot T/\delta)})$. The privacy analysis for DP-SGD follows similarly, the only thing that changes is that we should use the clipping threshold $\mathcal{L}$ instead of r.

We would now prove that the output of Algorithm 1 is the same as DP-SGD. Note that since the loss function is $\mathcal{L}$-lipschitz, the clipping operation for DP-SGD is non-operational. Similarly, for DP-SGDA, since we have r-concentrated gradients, the clipping operation is non-operational. Given that the clipping operation is the only difference between DP-SGD and DP-SGDA, and they both are non-operational, the output distributions of the two algorithms are exactly the same. □