

DISSECTING ATTENTION AND MLP ROLES: A STUDY OF DOMAIN SPECIALIZATION IN LLMs

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006 Paper under double-blind review

ABSTRACT

011 Large language models (LLMs) perform well across diverse domains such as
 012 programming, medicine, and law, yet it remains unclear how domain information is
 013 represented and distributed within their internal mechanisms. A key open question
 014 is the *division of labor* between the Transformer’s core components: self-attention
 015 and MLP layers. We address this question through a mechanistic study that
 016 dissects their roles by integrating three complementary analyses: representation
 017 separability via probes, parameter change under adaptation, and causal effects
 018 from activation swaps. **Across six domains and multiple models, we find that**
 019 **both Attention and MLP layers encode domain information, but in systematically**
 020 **different ways.** We find that attention layers concentrate domain information in
 021 localized ‘hotspots’ (high variance across depth), while MLP layers distribute it
 022 uniformly. During fine-tuning, MLPs absorb 2-3x larger parameter updates, yet
 023 causal interventions reveal that specific mid-depth attention layers (e.g., layers
 024 13-15) directionally steer domain predictions, while MLP interventions disrupt
 025 computation without directional control. These three lenses jointly support a
 026 coherent functional picture: MLP layers serve as the primary workbenches for
 027 domain-specific computation, while a small subset of attention layers act as high-
 028 gain steering points that route domain identity. Finally, we show a proof-of-concept
 029 parameter-efficient adaptation setup where tuning only the layers highlighted by
 030 our analysis matches full-model fine-tuning on domain benchmarks, illustrating
 031 the practical potential of mechanistically informed PEFT.

1 INTRODUCTION

032 Large Language Models (LLMs) master diverse domains, yet the internal mechanisms governing
 033 this domain representation remain an open question. *What is the division of labor between the*
 034 *Transformer’s core components – the self-attention and the MLP layers?* In this paper, we address
 035 these questions through a causal, layer-level analysis and propose a functional specialization that
 036 holds across models and domains.

037 The field of mechanistic interpretability has developed powerful methods for such analysis, pro-
 038 gressing from correlational analysis to causal interventions. Initial probing (Alain & Bengio, 2018;
 039 Tenney et al., 2019) analyses used simple neural classifiers to differentiate the outputs of a layer for
 040 varied inputs. A highly separable representation of domain identity, for example, would imply that a
 041 component contains domain-specific information. The contribution can be quantified by calculating
 042 the level of separation in the higher-dimensional space through separability scores, like v-usable
 043 information (Ethayarajh et al., 2022), Xu et al. (2020), Ju et al. (2024b) Fisher separability Fisher
 044 (1936), maximum mean discrepancy Gretton et al. (2008) etc. Although these methods can show
 045 where information separates, but not if or how the model uses it for downstream tasks.

046 Subsequently, the focus shifted towards establishing causality by reverse-engineering the circuits
 047 (Elhage et al., 2021) for specific behaviors, through methods like activation patching (Meng et al.,
 048 2023a) Wang et al. (2022) and zero-out testing (Dai et al., 2021). This research has yielded an
 049 important result: MLP layers have been characterized as the primary locus of holding factual
 050 knowledge. Concurrently, attention mechanisms are understood as routers, moving and aligning
 051 information throughout the context, enabling capabilities like in-context learning (Olsson et al.,
 052 2022).

054 A parallel line of evidence comes from studying parameter adaptation. Research on techniques
 055 like Low-Rank Adaptation (LoRA) has shown that model behavior can be improved for targeted
 056 adaptation by modifying only a small subset of weights (Hu et al., 2022) Zhang et al. (2023). However,
 057 a critical gap remains. These three powerful lenses—representational, causal, and adaptational—have
 058 largely been applied in isolation and to micro-scale tasks (e.g., factual recall, syntactic phenomena).
 059 It is unknown whether the “attention-as-router, MLP-as-compute” principle scales to govern how
 060 models handle high-level, abstract domains like programming or medicine. Furthermore, no existing
 061 framework exists to synthesize these three orthogonal sources of evidence into a single, coherent map
 062 of a model’s functional architecture.

063 **Our perspective.** We study *domain handling* in LLMs by jointly applying three lenses to the
 064 same models and domains. Concretely, we ask the following questions, at the level of individual
 065 Transformer blocks:

- 067 1. **Where is domain identity represented?** We measure how separable domain labels are
 068 from layer activations using Fisher ratio and kernel MMD.
- 069 2. **Where does domain adaptation write new computation?** We quantify layer-wise parame-
 070 ter changes under domain-specific LoRA fine-tuning.
- 071 3. **Which layers can steer domain-sensitive behavior under intervention?** We perform acti-
 072 vation swaps between matched domain prompts and measure both disruption and directional
 073 shifts in predictions.

074 Here, we qualify that domain control is complex and distributed; no single component type exclusively
 075 handles all aspects. Our analysis reveals relative differences in component contributions rather than
 076 absolute divisions. In summary, our contributions are:

- 078 • **Representational separability is shared, but distributed differently.** After excluding trivially
 079 input-driven and logit-driven layers, both Attention and MLP components broadly encode domain
 080 identity. However, separability across depth has markedly higher variance in Attention than in
 081 MLP: a few attention layers form sharp “hotspots” of domain-specific separability, whereas MLP
 082 layers encode domain information more uniformly.
- 083 • **Evidence for a Scaled Division of Labor in Abstract Tasks:** We provide direct evidence that the
 084 “attention as router, MLP as memory” principle, previously observed in low-level factual tasks, also
 085 governs how models handle high-level, abstract capabilities like domain control. This suggests it is
 086 a fundamental organizing principle of the Transformer architecture.
- 087 • **Demonstration of Mechanistically-Informed, Parameter-Efficient Fine-Tuning as proof of
 088 concept:** We show that our mechanistic map can be used to have direct practical utility. The
 089 models’ performance matches full-model fine-tuning on our domain benchmarks, illustrating the
 090 practical potential of mechanistically guided PEFT.

092 2 PROPOSED METHODOLOGY

094 Our work examines the roles of attention and MLP components across layers through/via three
 095 perspectives: representational patterns (Probing analysis), parameter changes (Fine tuning analysis),
 096 and causal interventions.

098 2.1 PROBING ANALYSIS

100 The objective of this experiment is to identify which layers contain the most linearly separable
 101 information about domain identity. Classical classification accuracy saturated at around 100% across
 102 all layers, providing insufficient discriminative power to determine where domain information is most
 103 concentrated. We instead quantify the degree of separability using distributional metrics. A high
 104 degree of separability indicates that a layer’s activations serve as a strong signal for the domain, a
 105 necessary condition for a component involved in routing or high-level control.

106 To quantify where domain identity is explicitly represented, we compute pairwise separability between
 107 domains for each layer and component using two complementary statistics: a scalar Fisher ratio
 Fisher (1936) and RBF-kernel Maximum Mean Discrepancy (MMD) Gretton et al. (2008). Let

108 $X \in \mathbb{R}^{N \times D}$ be pooled activations for a given (layer, component) and $y \in \{1, \dots, K\}^N$ the domain
 109 labels. Denote by X_i the rows of X with label i , $N_i = |X_i|$, and $\mu_i = \frac{1}{N_i} \sum_{x \in X_i} x$.
 110

111 **Fisher** : We report the scalar Fisher score between domains i and j :

$$112 \quad \text{Fisher}_{ij} = \frac{\|\mu_i - \mu_j\|^2}{\sum_{d=1}^D \text{Var}(X_{i,d}) + \sum_{d=1}^D \text{Var}(X_{j,d}) + \varepsilon},$$

$$113$$

$$114$$

115 with $\varepsilon = 10^{-6}$ for numerical stability. This ratio is high when domain means are well-separated
 116 relative to within-domain variance, indicating linear discriminability.

117 **MMD (RBF)**. Using an RBF kernel $k_\gamma(x, x') = \exp(-\gamma\|x - x'\|^2)$ we compute

$$119 \quad \text{MMD}_{k_\gamma}^2(X_i, X_j) = \frac{1}{N_i^2} \sum_{a,b \in X_i} k_\gamma + \frac{1}{N_j^2} \sum_{a,b \in X_j} k_\gamma - \frac{2}{N_i N_j} \sum_{a \in X_i} \sum_{b \in X_j} k_\gamma,$$

$$120$$

$$121$$

122 and report $\text{MMD}_{ij} = \sqrt{\max(0, \text{MMD}^2)}$. The kernel bandwidth γ is set by the median heuristic on
 123 pairwise distances.
 124

125 Activations are extracted by registered forward hooks at two probe points per block: post-attention
 126 and post-MLP (before residual addition). (for details on pipeline see Appendix A.2). We display only
 127 Fisher and MMD scores because they capture complementary linear (mean-vs-variance) and nonlinear
 128 (higher-moment) distributional differences and provide the clearest layer-wise differentiation in our
 129 experiments. Rather than exhaustively reporting all $\binom{K}{2}$ pairwise scores, we compute a **1-vs-all**
 130 statistic for each domain. For a domain D_i , activations from D_i are compared against the pooled
 131 activations from all other domains $\bigcup_{j \neq i} D_j$. This yields a per-layer, per-component separability
 132 score $S_{i,\ell}$ indicating how well layer ℓ distinguishes D_i from the rest of the corpus. To compare
 133 components on the same scale, we z-normalize scores across layers for each domain. **We observed**
 134 **strong separability in the first and last layers, but further analysis (Appendix E.1) shows that these**
 135 **peaks largely reflect input-distribution differences and logit effects, rather than internal steering.** To
 136 **avoid these confounds and focus on internal specialization, we exclude these layers from summary**
 137 **statistics such as variance and maximums in the main text.**

138 2.2 FINE-TUNING ANALYSIS

140 Probing identifies where domain identity is *separated* in activations; the complementary question
 141 is where parameters undergo adaptation. We answer this by measuring per-layer parameter updates
 142 under fine-tuning and by testing whether the layers that change most are also the layers that suffice
 143 for adaptation.

144 We use LoRA-style fine-tuning for targeted, parameter-efficient adaptation. For a dense weight
 145 $W \in \mathbb{R}^{n \times m}$ at layer ℓ the adapted weight is $\tilde{W} + \Delta W_\ell$ with $\Delta W_\ell = \frac{\alpha}{r} B_\ell A_\ell$ where $A_\ell \in \mathbb{R}^{r \times m}$,
 146 $B_\ell \in \mathbb{R}^{n \times r}$, r is the adapter rank and α is a scalar scaling. We summarize a layer's adaptivity by the
 147 Frobenius norm of the effective update

$$148 \quad S_\ell = \|\Delta W_\ell\|_F,$$

$$149$$

150 and aggregate multiple adapter tensors that belong to the same Transformer block by summation:
 151 $S_\ell^{\text{block}} = \sum_{t \in T_\ell} \|\Delta W_t\|_F$. A high S_ℓ indicates that the parameters in ℓ layer are a primary site for
 152 storing new, domain-specific computation learned during adaptation Gupta et al. (2025).

153 We run three fine-tuning regimes: (i) full-model fine-tuning (baseline), (ii) LoRA targeted only
 154 to attention projection matrices (e.g., q, k, v, o per block), and (iii) LoRA targeted only to MLP
 155 projection matrices (e.g., gate/up/down). **To validate the utility of our layer map, we additionally fine-**
 156 **tune only the top-3 and bottom-3 layers ranked by separability.** Crucially, to ensure robust adaptation
 157 and prevent catastrophic forgetting, we augment the training data with a set of generic prompts and
 158 apply loss masking so that gradients are backpropagated only from the model's generated responses,
 159 not the input instructions. For domain perplexity evaluation, we additionally fine-tune only the top 1
 160 and top 3 layers under each of these regimes. All fine-tune runs use fixed hyperparameters (epochs,
 161 learning rate, batch size, LoRA rank) and multiple random seeds to enable statistical comparison.
 (See Appendix D.1)

162 2.3 CAUSAL ACTIVATION SWAPPING
163164 Probing and fine-tuning establish where domain information is present and where the optimizer writes
165 it; to show that a layer’s activations actually *cause* domain-directed generation, we perform activation
166 swapping. The experiment asks: if we transplant the hidden state from a donor prompt in domain D_b
167 into a recipient prompt in domain D_a , does the model’s next-token distribution shift toward D_b ?168 To rigorously isolate domain routing from generation complexity, we replace the open-ended code
169 generation task with a controlled **Domain Classification** task. We construct matched prompt pairs
170 using the following template:
171172 Below are two sets of keywords that you need to classify into two domains.
173 (A) : [List of n representative tokens from domain A]
174 (B) : [List of n representative tokens from domain B]
175 Which set is domain X? Answer: Option (177 From this template, we define the recipient input x_a as the “correct” prompt where the queried domain
178 X corresponds to list (A). Conversely, the donor input x_b is a “conflicting” prompt where the queried
179 domain X corresponds to list (B). Ideally, the model predicts “A” for x_a . Our goal is to determine if
180 injecting activations from x_b steers the model to predict “B”.181 We focus our causal analysis on **multiple domain pairs** (e.g., C++/Python, Medicine/Finance) to ensure
182 **generalizability**. This setup offers several methodological advantages: (1) structural prompt similarity
183 enables precise matched comparisons, (2) **the output space is restricted to binary classification labels**
184 (**A** vs **B**), **providing a clean directional signal**, and (3) computational complexity is identical across
185 samples, isolating domain identity from task difficulty. For a chosen layer ℓ and the **final prompt**
186 **position** t^* , we:187 1. run a forward pass on the donor (**conflicting**) input x_b and save donor activations $a_\ell^{\text{donor}}(t^*)$;
188 2. run a forward pass on the recipient (**correct**) input x_a but, at layer ℓ and position t^* ,
189 replace the recipient activation with $a_\ell^{\text{donor}}(t^*)$ and continue inference to obtain the patched
190 distribution $p_{\text{swap}(\ell)}(\cdot | x_a)$;
191 3. repeat across many donor–recipient pairs and average metrics (see E.5).
192193 **Metrics.** We quantify the effect of a swap with two complementary statistics that capture magnitude
194 and directionality.196 (1) *KL Divergence.* For a donor input x_b and recipient input x_a we define

198
$$\text{KL}_{\text{swap}} = \mathbb{E}_{x_a} [\text{KL}(p(\cdot | x_a) \| p_{\text{swap}(\ell)}(\cdot | x_a))],$$

200 where $p(\cdot | x_a)$ is the original next-token distribution and $p_{\text{swap}(\ell)}(\cdot | x_a)$ is the patched distribution.
201 $\text{KL}_{\text{swap}}^{\ell}$ measures how strongly the swap perturbs the model’s predictive distribution at the
202 intervention point.203 (2) *Delta bias.* We define the target token sets $S_A = \{\text{“A”}\}$ and $S_B = \{\text{“B”}\}$ corresponding to the
204 options in the prompt. For a prompt x_a , let $P(S|x)$ be the probability mass on tokens S . Bias toward
205 the donor outcome is $\text{Bias}(x) = P(S_B|x) - P(S_A|x)$. We measure the change due to intervention as

207
$$\Delta \text{Bias}(D_a \xleftarrow{\ell} D_b) = \mathbb{E} [\text{Bias}_{\text{swap}}(x_a \xleftarrow{\ell} x_b) - \text{Bias}_{\text{base}}(x_a)].$$

208 Positive values indicate a shift toward the donor label, since bias is computed as the preference of the
209 donor option over the recipient option. For complete details, see Appendix E.4211 **KL** captures whether an intervention meaningfully alters the model’s beliefs; the domain-token Shift
212 tests whether the alteration is *directionally* consistent with the donor domain. Together they provide
213 strong, local causal evidence that activations at layer ℓ not only correlate with domain identity but
214 can drive domain-appropriate generation when transplanted into another context. The experimental
215 conditions ensure that trivial scale differences do not drive observed effects. For more implementation
details, see Experimental Setup E.5

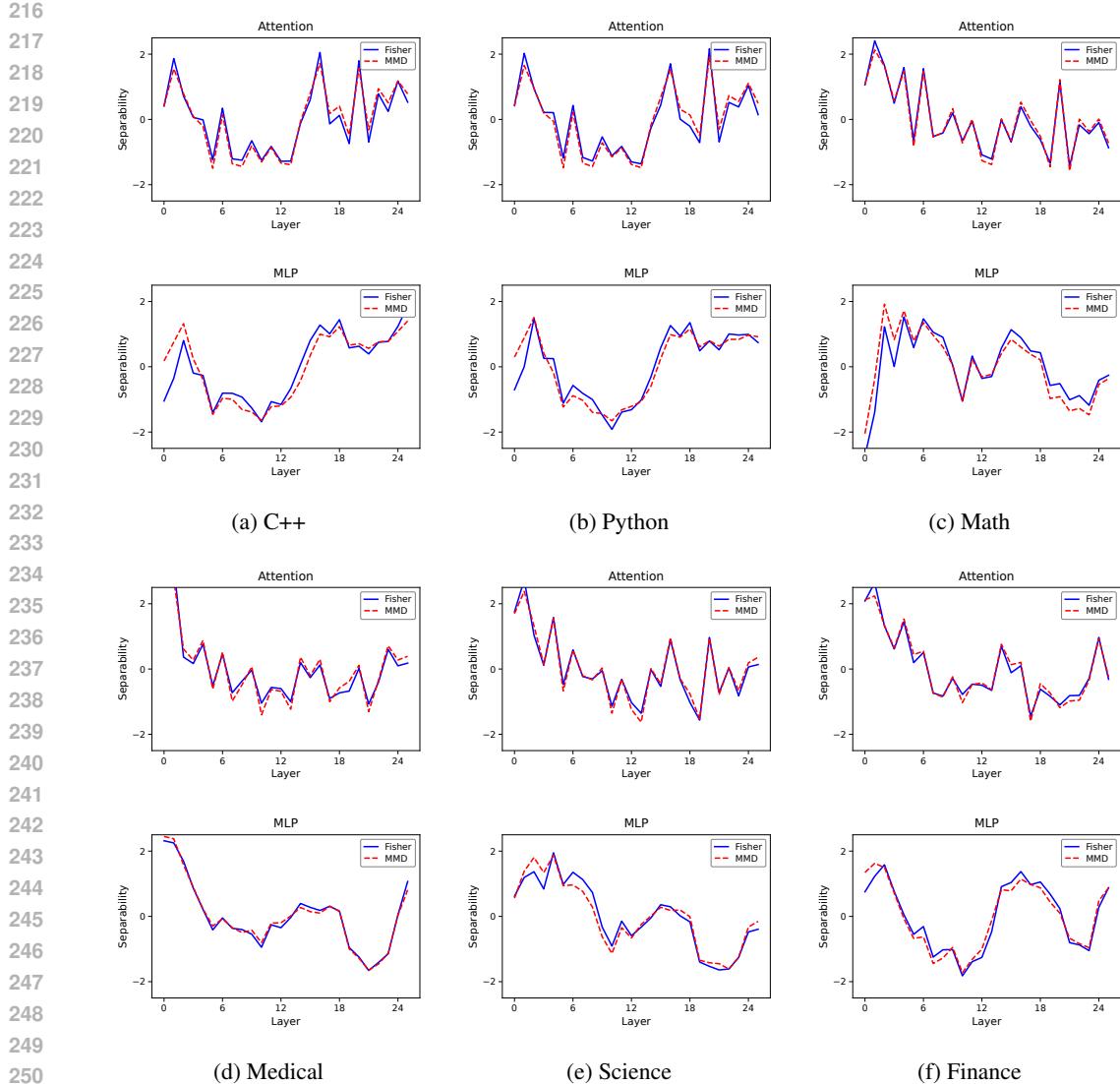


Figure 1: Separability scores across six domains. Each column displays Attention (top) and MLP (bottom) blocks for one domain.

3 RESULTS

Our investigation spans six domains: Medicine, Finance, Science, Mathematics, C++, and Python, and on four LLMs: Llama 3.2 3B, Llama 1B, Gemma 3 4B, and Gemma 3 1B (Grattafiori et al., 2024; Team et al., 2025). For more details on datasets used, see Appendix B. The following discussion is for the Llama 3.2 3B model, which consists of 28 layers, each with an MLP head and an attention mechanism. **Consistent with our analysis of the ‘Hydra effect’ (Appendix E.1), we focus the following quantitative results on the internal layers, excluding the immediate embedding and final output layers where representations are dominated by input/output constraints rather than internal processing.** For results on other models, see Appendix A.

3.1 WHERE DOMAIN KNOWLEDGE IS SEPARATED?

Figure 1 shows the 1-vs-all Fisher and MMD separability traces across layers for six domains, z-normalized to highlight relative variation in depth. Both Attention and MLP components exhibit non-uniform separability: some layers carry markedly stronger domain identity than others. While the

Domain	Attention				MLP			
	Fisher		MMD		Fisher		MMD	
	Max	Std	Max	Std	Max	Std	Max	Std
CPP	1.386	0.212	0.617	0.049	1.236	0.093	0.598	0.019
Python	1.359	0.202	0.615	0.048	1.038	0.067	0.592	0.020
Medical	1.532	0.218	0.657	0.037	1.392	0.110	0.664	0.017
Science	1.281	0.169	0.630	0.033	1.049	0.071	0.606	0.013
Math	1.356	0.191	0.639	0.035	1.062	0.060	0.613	0.011
Finance	2.307	0.323	0.717	0.031	1.987	0.133	0.714	0.010

Table 1: **Maximum Value and standard deviation** of 1-vs-all separability scores for Attention and MLP layers across six domains. Higher values indicate greater domain specificity **and localization** for that component.

overall trends are similar, the precise peaks do not fully coincide between Attention and MLP. This suggests that both components participate in domain representation, but their strongest contributions arise at slightly different depths.

After z-score normalization, Fisher and MMD traces nearly completely overlap across layers. This indicates that both linear mean-based separation (Fisher) and higher-moment distributional divergence (MMD) identify the same loci of domain information. Thus, the observed peaks are not artifacts of a particular separability metric, but reflect genuine structural patterns in the residual stream.

To compare components, Table 1 reports the **maximum and standard deviation** of 1-vs-all separability scores across layers. A clear pattern emerges. While both components possess **domain information**, the **distribution of this information differs fundamentally**. MLP layers exhibit consistently low standard deviation across all domains (e.g., C++ Fisher Std = 0.093), implying that domain-specific features are distributed relatively uniformly across depth. In contrast, Attention layers show significantly higher variance (e.g., C++ Fisher Std = 0.212), indicating that domain identity is not uniform but highly concentrated at specific ‘hotspot’ layers. The maximum separability reinforces this distinction. For all 6 domains, the maximum Fisher and MMD scores are higher for Attention layers than for MLP layers. For the C++ domain, the peak Attention separability (1.386) exceeds the peak MLP separability (1.236), but more importantly, the variance is over 2× higher in Attention. This indicates that while domain information is generally available, it becomes highly concentrated at specific bottleneck layers within the Attention mechanism.

3.2 ADAPTATIONAL ANALYSIS POINTS TO MLP LAYERS

While probing analysis suggests concentrated signals in attention layers, adaptational analysis reveals a different picture. Figure 2 plots the average normalized weight change ($\|\Delta W\|/\|W\|$) per layer for three LoRA fine-tuning regimes: targeting the full model, only MLP components, or only attention components.

The magnitude of weight change in MLP-only fine-tuning is substantially and consistently higher than in attention-only fine-tuning. This indicates that MLP layers are the primary locus where new, domain-specific computation is written during adaptation to a specific domain. The results are unambiguous across all six domains, **persisting even when using loss masking and generic prompt augmentation to prevent overfitting**. This implies that while attention layers had concentrated signals due to higher peaks of separability in specific layers, adapting to a new dataset **consistently relies on modifying** the MLP layers more, proposing that domain-specific knowledge is *stored* in the latter.

3.3 VALIDATING THE PROPOSED LAYER MAP VIA TARGETED FINE-TUNING

Before performing causal interventions, we first seek to validate the practical utility of our proposed layer map. If the layers, either those with the largest parameter deltas (primarily MLPs) or those with the most separable representations (peak attention layers), are indeed the most important for adaptation, then fine-tuning only these layers should achieve satisfactory results in comparison to fine-tuning the entire model. We test this hypothesis by fine-tuning only the **top-3 and bottom-3** layers

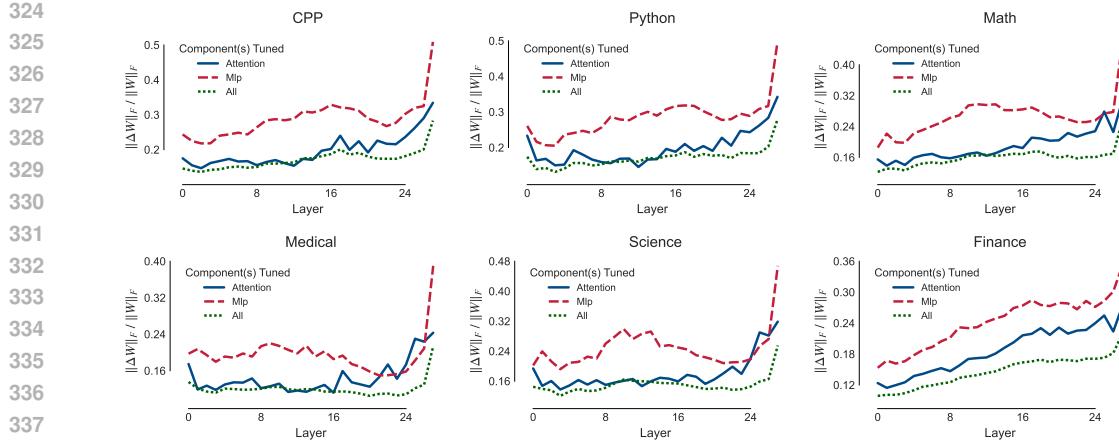


Figure 2: Change in the weights on Lora-based fine-tuning, separately on (1) Entire model, (2) Only Attention Layers, and (3) Only MLP Layers

(for both MLP and Attention) **ranked by their** separability scores and comparing their performance on the respective domain’s specific perplexity task against fine-tuning the full model. For every domain, the domain perplexity was devised using a benchmark evaluation method, normalized between 0 and 1. Details of domain-specific evaluation are mentioned in Appendix C.2.

Interestingly, the results in Table 2 are even better than expected. Targeted fine-tuning of just the **top-3** layers achieves domain-specific performance that is comparable to, and in some cases exceeds, that of fine-tuning the entire model, despite using a fraction of the parameters. For more insights, refer to Appendix C.1. The dataset used for fine-tuning had around 5000-7000 samples, as discussed in B.

It is important to note that due to the small scale of the models and limited fine-tuning data, fine-tuning can suffer from some forgetting of general capabilities. However, the relative performance gain across all fine-tuned results demonstrates that our layer importance map successfully identifies the most critical components for specialization. **We emphasize that we do not claim targeted PEFT is superior to full-model tuning. Instead, these results serve as a proof-of-concept indicating that mechanistically-guided selection of layers can enable efficient adaptation, highlighting the potential of interpretability to inform practical fine-tuning strategies.**

	PT	Full Fine Tuning	Bottom-3 MLP	Bottom-3 Attn	Top-3 MLP	Top-3 Attn
Math	0.07	0.02	0.08	0.07	0.12	0.03
Science	0.88	0.88	0.87	0.88	0.88	0.86
CPP	0.31	0.41	0.30	0.34	0.39	0.41
Python	0.73	0.69	0.68	0.65	0.71	0.73
Finance	0.94	0.95	0.91	0.91	0.93	0.95
Medical	0.67	0.68	0.66	0.67	0.67	0.68

Table 2: Performance of Llama-3.2-3B across domains on that domain-perplexity metric (normalized between 0 and 1). PT stands for pre-trained model. All the other column names resemble the components fine-tuned during adaptation.

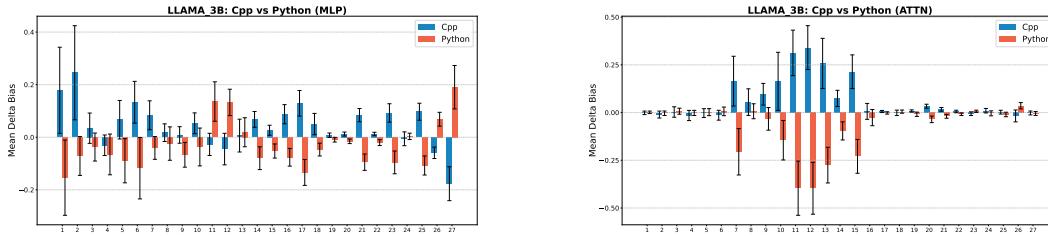


Figure 3: **Causal intervention results across all layers for Llama-3.2-3B.** Delta Bias when swapping activations between C++ and Python prompts using our domain-classification task (Section 2.3). X-axis: layer index (0-27). Y-axis: Delta bias. Swapping attention activations produces large, positive shifts at specific mid-depth layers (e.g., 13-15, 23-25), indicating sparse routing hotspots. Error bands show standard deviation over 200 prompt pairs. Full results for all domain pairs in Appendix E.5.

3.4 CAUSAL SWAPPING REVEALS ATTENTION AS DOMAIN ROUTER

Probing identified separable domain representations, and adaptation revealed MLPs as the primary locus of parameter change. To test which components actually *cause* domain-directed behavior, we use activation swapping in our **Domain Classification task**.

We measure the overall magnitude of the intervention’s effect using KL divergence as shown in figure 3. For components in layers with high Fisher separability, swapping activations from either an attention block or an MLP block induces a significant relative perturbation in the next-token distribution, resulting in high KL divergence. This confirms that both components in these layers are computationally active and influential on the final output. Conversely, interventions on components in low-Fisher layers produce a negligible KL divergence, confirming that the effect is localized to the information-rich parts of the network. Early layers occasionally exhibit high Fisher but low causal effect (e.g., Attention layer-2), suggesting the occurrence of “hydra” effect McGrath et al. (2023) (Discussion E.1) here.

However, a disruptive effect does not imply directional control. To test if a layer steers the output towards a specific domain, we measure the shift in probability mass towards the **donor option token** (i.e., the label corresponding to the conflicting domain). Crucially, we observe that this steering capability is highly specialized, and can be seen as distinct routing hotspots within specific mid-depth layers. In these concentrated points, swapping activations from a conflicting donor into a correct prompt reliably shifts the prediction toward the conflicting label. This provides direct causal evidence that these specific attention layers are not just active, but are providing a sparse, specific steering signal for domain identity. In contrast, swapping the output of a high-Fisher MLP layer does not produce a consistent directional shift. While the intervention is disruptive (high KL), the effect on the target label probability (Delta Bias) is centered around zero **across all layers**, albeit with high variance. This suggests that while the MLP is performing critical, domain-relevant computations, it is not the source of the high-level control signal that dictates “**the answer is Option B.**”

On bringing together these observations, we can conclude that the MLP layers change most during fine-tuning because they are the computational workbenches where domain-specific knowledge is implemented. Intervening on them is disruptive because it interrupts this computation. However, it is the **sparse set of** peak attention layers that act as the causal routers. Their activations, though less plastic during fine-tuning, carry the high-level steering signal that *directs* the downstream computational machinery of the MLPs.

4 DISCUSSION

Our investigation began with a foundational question: how does a monolithic network manage distinct domains? By analyzing the three lenses as proposed, we have moved beyond simple observation to a causal, mechanistic explanation. Our results resolve the apparent contradiction between representational and adaptational analyses, revealing a clear and consistent division of labor between the Transformer’s core components. Here, we synthesize these findings, discuss their implications for the field, and outline the limitations of our work to chart a path for future research.

432 **Transferability across models.** Our findings are not confined to a single checkpoint. We executed
 433 all analyses on LLaMA-1B, LLaMA-3.2B, and Gemma 3-1B/4B (See A). **We further validated our**
 434 **primary causal claims on the larger Llama-2-7B model to ensure scalability.** The overall pattern
 435 holds: **specific** attention layers exhibit localized, high-separability peaks that act as causal routers,
 436 while MLP layers accumulate the bulk of adaptation updates. Interestingly, Gemma models display
 437 sharper, more localized separability in causal swap experiments, with a single attention layer causing
 438 large directional effects. This acute localization of causal influence suggests a more specialized,
 439 hub-like routing mechanism within Gemma’s architecture, suggesting that architectural choices, such
 440 as logit soft-capping or normalization, may influence the concentration of domain representation.
 441 These findings highlight the need to explore how such architectural decisions affect causal control
 442 and domain adaptation, offering a promising direction for future research.
 443

444 **A coherent mechanistic picture.** Taken together, our three experiments point to a consistent
 445 proposition. **First, probing reveals a structural distinction:** while both components encode domain
 446 identity, MLP layers exhibit low variance across depth, implying a distributed representation. In
 447 contrast, Attention layers exhibit high variance with sharp hotspots of separability. **Second,** adaptation
 448 analysis confirms that MLPs function as the primary workbenches; even under robust fine-tuning
 449 conditions preventing overfitting, they absorb the vast majority of parameter updates. **Third,** causal
 450 interventions explain this structure. Swapping activations in the high-variance Attention layers
 451 provides a clean *steering signal*, reliably flipping the model’s decision in classification tasks. In
 452 contrast, swapping MLP activations causes *disruption* without directional steering. In the domain
 453 level of abstraction, attention acts as the router, steering domain identity, while MLPs implement the
 454 downstream computations that realize domain-specific behavior.
 455

456 **Implications.** This proposal has two important implications. **First,** it provides a layer-level map of
 457 where to look for domain control in Transformers, guiding mechanistic interpretability beyond micro-
 458 circuits to higher-level behaviors. **Second,** it has practical value: **our targeted fine-tuning experiments**
 459 **serve as a proof-of-concept, demonstrating that** a small set of components identified by our map
 460 suffices to replicate full-model domain tuning. **This highlights the potential for mechanistically-
 461 grounded strategies to enable more efficient model adaptation.**
 462

463 **Limitations and caveats.** Our study has several limitations. (i) **To isolate steering from generation,**
 464 **we relied on a controlled Classification task; however, complex open-ended generation may involve**
 465 **more distributed control signals that single-point swaps cannot fully capture.** (ii) We adopt a 1-vs-all
 466 separability framework, which simplifies analysis but may collapse informative pairwise distinctions
 467 between domains. (iii) Our models are relatively small and fine-tuned on modest datasets; **while we**
 468 **validated causal effects on 7B models, emergent behaviors in 70B+ scale models remain an open**
 469 **question.** (iv) Early-layer separability peaks (e.g., A2) did not always yield causal effects, consistent
 470 with the hydra effect, where distributed signals do not translate into single-point steering handles.
 471 (v) Finally, our causal swaps measure immediate next-token shifts; long-horizon effects and global
 472 coherence remain to be tested.
 473

474 **Future directions.** These caveats suggest clear paths forward. Future work should extend our work to
 475 even larger and more diverse models, refine domain prompts beyond code pairs, and analyze per-head
 476 specialization within the identified router layers. A natural next step is to connect layer-level maps to
 477 explicit circuit motifs, integrating coarse-grained and fine-grained mechanistic interpretability. On
 478 the practical side, our study could be used to guide efficient domain adaptation or controlled editing,
 479 narrowing the intervention space to the components that matter most. **Finally, a critical direction is to**
 480 **investigate how this implicit division of labor maps onto architectures with explicit routing, such as**
 481 **Mixture-of-Experts (MoE).**
 482

483 5 RELATED WORK

484 **Representation analysis :** The use of simple linear classifiers, or probes, to correlate internal
 485 activations with linguistic properties marked an early effort to map knowledge in neural networks
 486 (Alain & Bengio, 2018; Tenney et al., 2019). This method was quickly refined in response to critiques
 487 that high accuracy does not guarantee task-relevance, leading to the development of control methods
 488 and more sophisticated layer-wise analyses of information gain (Hewitt & Liang, 2019; Ravichander
 489 et al., 2020; Kunz & Kuhlmann, 2022). Applied to contemporary LLMs, these refined techniques
 490

486 have revealed clear knowledge hierarchies: the “Concept Depth” hypothesis posits that complex
 487 concepts are processed in deeper layers (Jin et al., 2024), while abstract traits like personality are
 488 localized to the middle-to-upper layers (Ju et al., 2024a). The search for greater precision has led to
 489 techniques like sparse probing for isolating the specific neurons responsible for a concept (Gurnee
 490 et al., 2023), and has connected analysis to action by using probe results to guide targeted edits on
 491 model behavior (Li et al., 2024).

492 **Critically, targeted fine-tuning of top-3 layers often matches or exceeds full-model performance**,
 493 suggesting that selective adaptation to mechanistically-identified layers can mitigate overfitting by
 494 constraining the parameter space while preserving domain-relevant updates. However, we emphasize
 495 that these results serve primarily as proof-of-concept for the utility of mechanistic layer selection,
 496 not as a claim that our current PEFT approach is superior to full-model training at scale. **Causal**
 497 **interventions**: To move from correlation to causation, a central method is activation patching: a
 498 family of techniques that swap activations between inputs to measure their causal effect (Vig et al.,
 499 2020; Geiger et al., 2021; Heimersheim & Nanda, 2024). Its application to model editing began with
 500 locating and updating single facts via ROME (Meng et al., 2022), a process later scaled to thousands
 501 of facts with MEMIT (Meng et al., 2023b) and made more efficient by SaLEM (Mishra et al., 2024).
 502 The scope of such causal analysis has since expanded beyond discrete facts, used to map the locality
 503 of categorical knowledge (Burger et al., 2024) and to reverse-engineer entire computational circuits
 504 ‘in the wild’ (Wang et al., 2022).

505 **Functional Specialization of Transformer Components** : Causal analysis reveals a functional
 506 specialization between a transformer’s primary sub-layers. MLP layers are established as key-value
 507 memories that store factual knowledge (Geva et al., 2021), a view substantiated by causal editing
 508 (Meng et al., 2022) and shown to hold in multilingual contexts (Fierro et al., 2023). Conversely,
 509 attention mechanisms act as dynamic routers, moving information through the residual stream (Elhage
 510 et al., 2021; Olsson et al., 2022). This simple dichotomy has evolved into a more nuanced view of
 511 integrated knowledge circuits, with work formalizing how attention filters information for MLPs to
 512 store (Xu & Chen, 2023) and detailing direct Attention-MLP interactions (Yao et al., 2024; Neo et al.,
 513 2024).

514 **Parameter-Efficient Fine-Tuning as a Locus of Knowledge**: A parallel line of research frames
 515 Parameter-Efficient Fine-Tuning (PEFT) as a mechanistic diagnostic. While foundational methods
 516 like Adapter-tuning (Houlsby et al., 2019) and LoRA (Hu et al., 2022) were developed for engineering
 517 efficiency, why and where they work has deep mechanistic implications. Analyses suggest LoRA
 518 learns low-rank updates that mimic full fine-tuning (Zhang et al., 2023), and critically, that the efficacy
 519 of these updates is highly dependent on their layer-wise placement (An et al., 2024; He et al., 2022).
 520 This localization principle is further exemplified by methods like LoFiT, which use interpretability to
 521 identify and then fine-tune only a sparse subset of task-critical attention heads (Yin et al., 2024).

522 6 CONCLUSION

523 We demonstrate a clear division of labor in Transformers at the high-level scale of complex, real-
 524 world domains: attention layers route domain identity, while MLP layers store domain-specific
 525 knowledge. This work establishes that the “router-compute” principle—previously observed in
 526 low-level tasks—organizes high-level domain specialization across programming, medicine, and
 527 other complex domains. By triangulating probing, adaptation, and causal interventions, we provide a
 528 definitive functional map: attention layers serve as domain routers that causally steer model behavior,
 529 while MLP layers act as domain-specific computational units. This architectural insight provides a
 530 blueprint for more interpretable and efficient model adaptation, advancing our understanding of how
 531 large language models master diverse capabilities.

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APPENDIX

A RESULTS ON OTHER MODELS

A.1 FINE TUNING ANALYSIS

Stage 1: Comprehensive Adaptational Mapping. The initial stage conducted a broad, component-wise analysis for each of the six domains independently. To map the division of labor between Transformer components, we applied LoRA adapters under three distinct regimes:

- **Attention-Only:** LoRA was applied exclusively to the attention projection matrices (`q_proj`, `k_proj`, `v_proj`, `o_proj`) in every layer.
- **MLP-Only:** LoRA was applied exclusively to the MLP projection matrices (`gate_proj`, `up_proj`, `down_proj`) in every layer.
- **Full Model (All):** LoRA was applied to all attention and MLP components simultaneously, establishing a baseline for unconstrained, full-model adaptation.

The primary objective of this stage was to quantify the magnitude of parameter updates for each component $c \in \{\text{Attn, MLP}\}$ at each layer ℓ , measured by the Frobenius norm of the effective weight change, $S_{\ell,c} = \|\Delta W_{\ell,c}\|_F$. The results from this analysis provide the data for the adaptational plots in the main paper (Figure 2) and this appendix.

STAGE 1 RESULTS FOR OTHER MODELS

The adaptational patterns observed in the Llama 3.2 3B model hold consistently across other model families and sizes, as shown below.

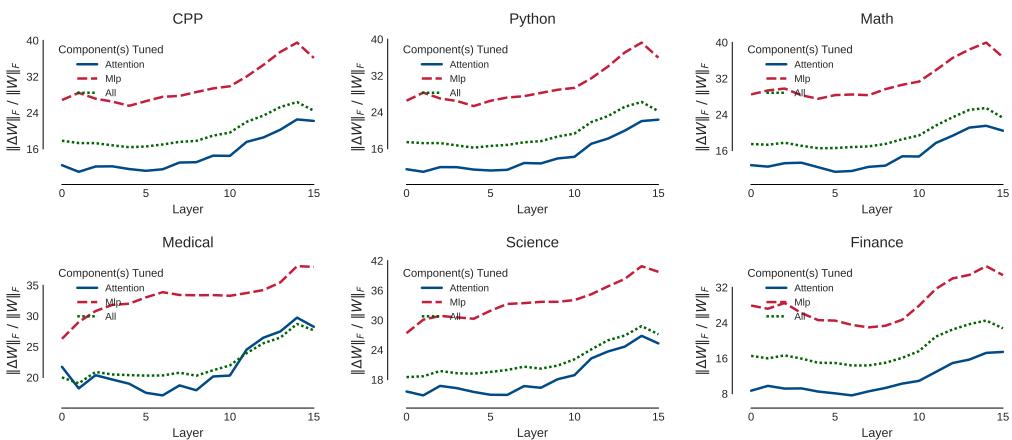


Figure 4: Layer-wise magnitude of parameter updates (S_ℓ) for **Llama 3.2 3B** under three LoRA fine-tuning regimes across six domains.

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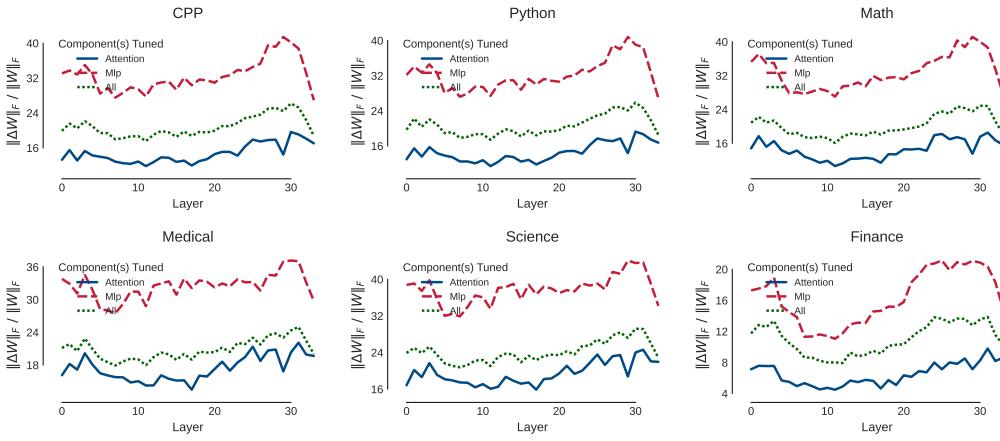
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831 Figure 5: Layer-wise magnitude of parameter updates (S_ℓ) for **Gemma 3 4B** under three LoRA
832 fine-tuning regimes across six domains.

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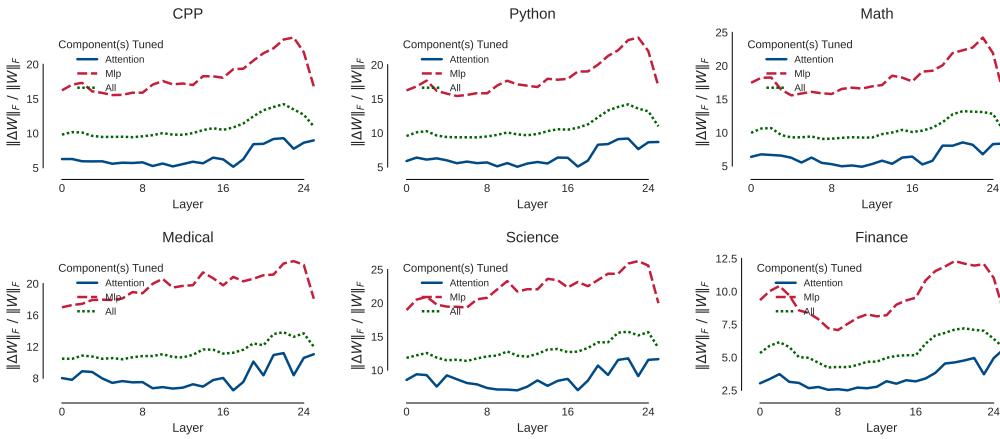
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858 Figure 6: Layer-wise magnitude of parameter updates (S_ℓ) for **Gemma 3 1B** under three LoRA
859 fine-tuning regimes across six domains.

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864 ADAPTATIONAL NORM ANALYSIS
865866 To dissect the dynamics of targeted adaptation, we compare the norms of LoRA weight updates
867 ($\|\Delta W_\ell\|_F$) for the top-3 most-adapted layers across three analytical contexts. The summary tables
868 aggregate these norms to reveal overarching patterns.
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- **Avg. Full Run Norm:** The average norm of a component group (e.g., Top-3 MLPs) from
871 the Stage 1 "Full Model" regime, where all layers were adapted on a single domain. This
872 represents the baseline update magnitude in an unconstrained setting.
- **Avg. Ensemble Norm:** The average norm of a component group from a Stage 2 "Ensemble
874 Tuning" run, where *only* those specific components (e.g., only the Top-3 MLP layers) were
875 adapted. This measures the update magnitude under targeted, multi-component fine-tuning.
876
- **Top Solo Run Norm:** The norm of the single highest-ranking component from a Stage 2
877 "Soloist Tuning" run, where it was the *only* component adapted in the entire model. This
878 quantifies a component's adaptational capacity in complete isolation.
879

880881 Table 3: Aggregated LoRA weight update norms for the Llama 3.2 3B model across all domains.
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Domain	Component Group	Avg. Full Run Norm	Avg. Ensemble Norm	Top Solo Run Norm
CPP	Top-3 MLP Components (Avg.)	1.019×10^2	1.287×10^2	1.651×10^2
	Top-3 Attn Components (Avg.)	7.042×10^1	9.357×10^1	1.149×10^2
Finance	Top-3 MLP Components (Avg.)	9.463×10^1	6.990×10^1	9.945×10^1
	Top-3 Attn Components (Avg.)	5.464×10^1	4.687×10^1	6.056×10^1
Math	Top-3 MLP Components (Avg.)	1.007×10^2	1.360×10^2	1.676×10^2
	Top-3 Attn Components (Avg.)	6.580×10^1	8.377×10^1	9.766×10^1
Medical	Top-3 MLP Components (Avg.)	9.545×10^1	1.239×10^2	1.560×10^2
	Top-3 Attn Components (Avg.)	9.134×10^1	9.702×10^1	1.181×10^2
Python	Top-3 MLP Components (Avg.)	1.010×10^2	1.311×10^2	1.744×10^2
	Top-3 Attn Components (Avg.)	6.978×10^1	9.599×10^1	1.250×10^2
Science	Top-3 MLP Components (Avg.)	1.019×10^2	1.343×10^2	1.660×10^2
	Top-3 Attn Components (Avg.)	8.013×10^1	9.999×10^1	1.145×10^2

890
901 Table 4: Aggregated LoRA weight update norms for the Llama 3.2 1B model across all domains.
902903

Domain	Component Group	Avg. Full Run Norm	Avg. Ensemble Norm	Top Solo Run Norm
CPP	Top-3 MLP Components (Avg.)	1.131×10^2	1.508×10^2	1.944×10^2
	Top-3 Attn Components (Avg.)	8.660×10^1	1.150×10^2	1.389×10^2
Finance	Top-3 MLP Components (Avg.)	1.063×10^2	8.506×10^1	1.201×10^2
	Top-3 Attn Components (Avg.)	6.711×10^1	5.694×10^1	7.221×10^1
Math	Top-3 MLP Components (Avg.)	1.151×10^2	1.607×10^2	2.025×10^2
	Top-3 Attn Components (Avg.)	8.405×10^1	1.062×10^2	1.209×10^2
Medical	Top-3 MLP Components (Avg.)	1.116×10^2	1.457×10^2	1.798×10^2
	Top-3 Attn Components (Avg.)	1.139×10^2	1.196×10^2	1.402×10^2
Python	Top-3 MLP Components (Avg.)	1.123×10^2	1.535×10^2	2.042×10^2
	Top-3 Attn Components (Avg.)	8.578×10^1	1.173×10^2	1.493×10^2
Science	Top-3 MLP Components (Avg.)	1.189×10^2	1.610×10^2	2.009×10^2
	Top-3 Attn Components (Avg.)	1.024×10^2	1.278×10^2	1.446×10^2

918 Table 5: Aggregated LoRA weight update norms for the Gemma-3 4B model across all domains.
919

920 921 Domain	922 923 Component Group	924 925 Avg. Full Run Norm	926 927 Avg. Ensemble Norm	928 929 Top Solo Run Norm
930 931 CPP	932 933 Top-3 MLP Components (Avg.)	934 935 7.481×10^1	936 937 8.510×10^1	938 939 9.509×10^1
	940 941 Top-3 Attn Components (Avg.)	942 943 4.523×10^1	944 945 5.179×10^1	946 947 6.092×10^1
948 949 Finance	950 951 Top-3 MLP Components (Avg.)	952 953 3.211×10^1	954 955 3.883×10^1	956 957 4.720×10^1
	958 959 Top-3 Attn Components (Avg.)	960 961 2.398×10^1	962 963 2.806×10^1	964 965 3.566×10^1
966 967 Math	968 969 Top-3 MLP Components (Avg.)	970 971 6.152×10^1	972 973 7.033×10^1	974 975 7.748×10^1
	976 977 Top-3 Attn Components (Avg.)	978 979 3.345×10^1	980 981 3.862×10^1	982 983 4.418×10^1
984 985 Medical	986 987 Top-3 MLP Components (Avg.)	988 989 7.913×10^1	990 991 8.882×10^1	992 993 1.060×10^2
	994 995 Top-3 Attn Components (Avg.)	996 997 4.881×10^1	998 999 5.361×10^1	999 999 6.759×10^1
999 999 Python	999 999 Top-3 MLP Components (Avg.)	999 999 7.612×10^1	999 999 8.496×10^1	999 999 9.706×10^1
	999 999 Top-3 Attn Components (Avg.)	999 999 4.755×10^1	999 999 5.305×10^1	999 999 6.187×10^1
999 999 Science	999 999 Top-3 MLP Components (Avg.)	999 999 8.339×10^1	999 999 9.547×10^1	999 999 1.049×10^2
	999 999 Top-3 Attn Components (Avg.)	999 999 4.698×10^1	999 999 5.223×10^1	999 999 5.652×10^1

937 Table 6: Aggregated LoRA weight update norms for the Gemma-3 1B model across all domains.
938

939 940 Domain	941 942 Component Group	943 944 Avg. Full Run Norm	945 946 Avg. Ensemble Norm	947 948 Top Solo Run Norm
949 950 CPP	951 952 Top-3 MLP Components (Avg.)	953 954 4.315×10^1	955 956 5.039×10^1	957 958 6.484×10^1
	959 960 Top-3 Attn Components (Avg.)	961 962 2.451×10^1	963 964 2.822×10^1	965 966 3.337×10^1
967 968 Finance	969 970 Top-3 MLP Components (Avg.)	971 972 2.478×10^1	973 974 2.891×10^1	975 976 3.953×10^1
	977 978 Top-3 Attn Components (Avg.)	979 980 1.691×10^1	981 982 1.956×10^1	983 984 3.240×10^1
985 986 Math	987 988 Top-3 MLP Components (Avg.)	989 990 4.022×10^1	991 992 4.570×10^1	993 994 5.823×10^1
	995 996 Top-3 Attn Components (Avg.)	997 998 2.003×10^1	999 999 2.292×10^1	999 999 2.922×10^1
999 999 Medical	999 999 Top-3 MLP Components (Avg.)	999 999 4.811×10^1	999 999 5.544×10^1	999 999 7.106×10^1
	999 999 Top-3 Attn Components (Avg.)	999 999 2.955×10^1	999 999 3.401×10^1	999 999 4.053×10^1
999 999 Python	999 999 Top-3 MLP Components (Avg.)	999 999 4.297×10^1	999 999 4.926×10^1	999 999 6.502×10^1
	999 999 Top-3 Attn Components (Avg.)	999 999 2.501×10^1	999 999 2.846×10^1	999 999 3.237×10^1
999 999 Science	999 999 Top-3 MLP Components (Avg.)	999 999 4.973×10^1	999 999 5.627×10^1	999 999 6.923×10^1
	999 999 Top-3 Attn Components (Avg.)	999 999 2.516×10^1	999 999 2.830×10^1	999 999 3.364×10^1

972 A.2 PROBING ANALYSIS
973974 The process of calculating separability scores between each pair of datasets, layer-wise, consists of 2
975 main components:
976977 1) Hooking to get activations
978 2) Using these activations to get the Separability Scores979 **Hook placement and construction of per-sample representations.** When analyzing representations
980 inside transformer layers, forward hooks are placed on sub-modules corresponding to the **Attention**
981 **block, MLP block, and Residual stream activations.** Each hook captures the output tensor of shape
982 $[B, S, D]$, where B is the batch size (examples per forward pass), S is the sequence length (tokens
983 per example), and D is the hidden dimension of the representation. To simplify, the token dimension
984 is mean-pooled, giving a $[B, D]$ embedding for each batch. These embeddings are concatenated
985 across multiple forward passes to construct a design matrix $X \in \mathbb{R}^{N \times D}$, where N is the total number
986 of collected samples. Alongside, a label vector $y \in \{0, \dots, C - 1\}^N$ is created so that each row X_r
987 corresponds to its class label y_r .988 To compute **Fisher separability** between two classes i and j , we first isolate the subsets of X
989 belonging to those labels, giving matrices $X_i \in \mathbb{R}^{n_i \times D}$ and $X_j \in \mathbb{R}^{n_j \times D}$. The mean representation
990 of each class (μ_i, μ_j) is calculated across their samples, and the variance within each class ($\text{var}_i, \text{var}_j$)
991 is also estimated. Fisher's score is then defined as the squared distance between the two class means,
992 normalized by the sum of their variances. Intuitively, if the means are far apart relative to how spread
993 out the classes are internally, the score is high, indicating that the two classes are well separated in
994 the representation space.995 For the **Maximum Mean Discrepancy (MMD)**, the same class-specific subsets X_i and X_j are
996 compared using a kernel function, typically a Gaussian RBF kernel. Pairwise distances between
997 samples are used to determine the kernel bandwidth γ , and kernel similarity matrices are constructed:
998 within-class (K_{ii}, K_{jj}) and cross-class (K_{ij}). The MMD score is then computed as the difference
999 between average within-class similarities and average cross-class similarities. A larger MMD value
1000 means the two distributions X_i and X_j are more dissimilar, capturing not just differences in means
1001 but also higher-order mismatches in distributional shape.1002 EXPERIMENT PARAMETERS
10031004
1005

Samples per domain (forward pass)	MLP hook	Attention hook	Batch size
1000	<i>up_proj</i>	<i>o_proj</i>	8

1006 Parameters used for all models: Llama 3.2 3B, Llama 3.2 1B, Gemma 3 4B, and Gemma 3 1B.
1007

1014 B DATASETS

1015 **C++, Python** For our coding datasets, we have used the Open Coder LLM Annealing Corpus
1016 (Huang et al. (2024)) which contains functional code snippets on various coding questions. This
1017 dataset aligns with our Human Benchmark Evaluation tests since it uses the same formatting. Each
1018 data point has a top level comment describing the task followed by a function that implements the
1019 task. The original dataset also contains inline comments inside the function body but these have been
1020 striped for conciseness. Listing 1 and Listing 10 showcase examples from our dataset on C++ and
1021 Python snippets.1022 **Science** We have used the SciQ dataset (Johannes Welbl, 2017) which contains crowd-sourced
1023 questions on Physics, Chemistry and Biology. The questions are in multiple-choice format with 4
1024 answer options each. For our purposes we have formatted the data-points into Context, Question and
1025 Answer.

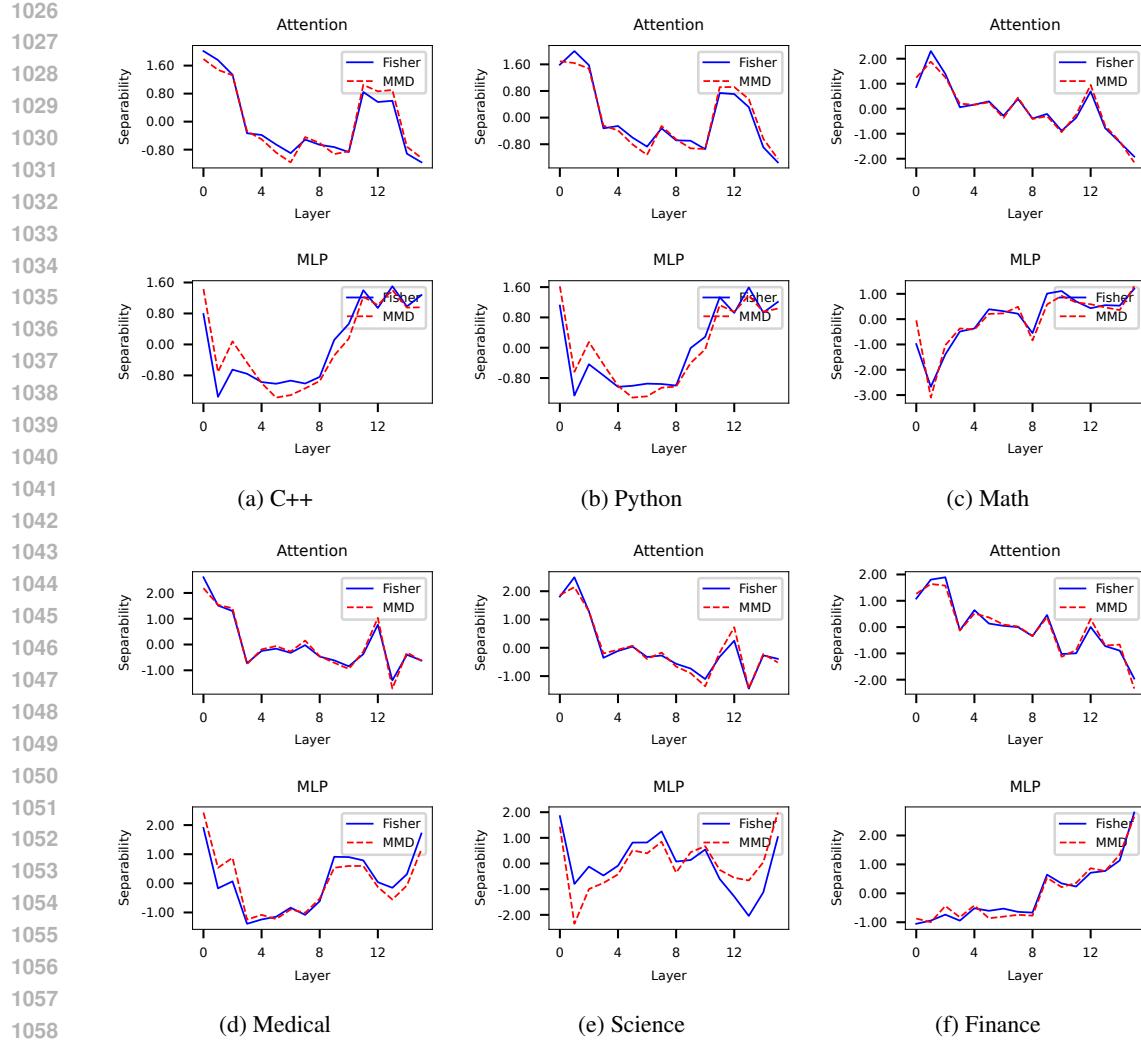


Figure 7: probe separability results for Llama 1B Model

Context: Enzymes are critical to the body's healthy functioning. They assist, for example, with the breakdown of food and its conversion to energy. In fact, most of the chemical reactions in the body are facilitated by enzymes.

Question: Most of the chemical reactions in the body are facilitated by what?

Options: A. proteins B. enzymes C. vitamins D. carbohydrates
Answer: B

Mathematics The Math dataset is GSM8K (Cobbe et al., 2021a) which is a dataset of 8.5k high quality math word problems. The dataset contains question answering on basic mathematical problems that require multi-step reasoning. The datapoints are also similarly formatted into Question, Answer and Final Answer.

Question: Natalia sold clips to 48 of her friends in April, and then she sold half as many clips in May. How many clips did Natalia sell altogether in April and May?

Answer: Natalia sold $48/2 = <<48/2=24>>24$ clips in May. Natalia sold $48+24 = <<48+24=72>>72$ clips altogether in April and May. #### 72
Final Answer: 72.

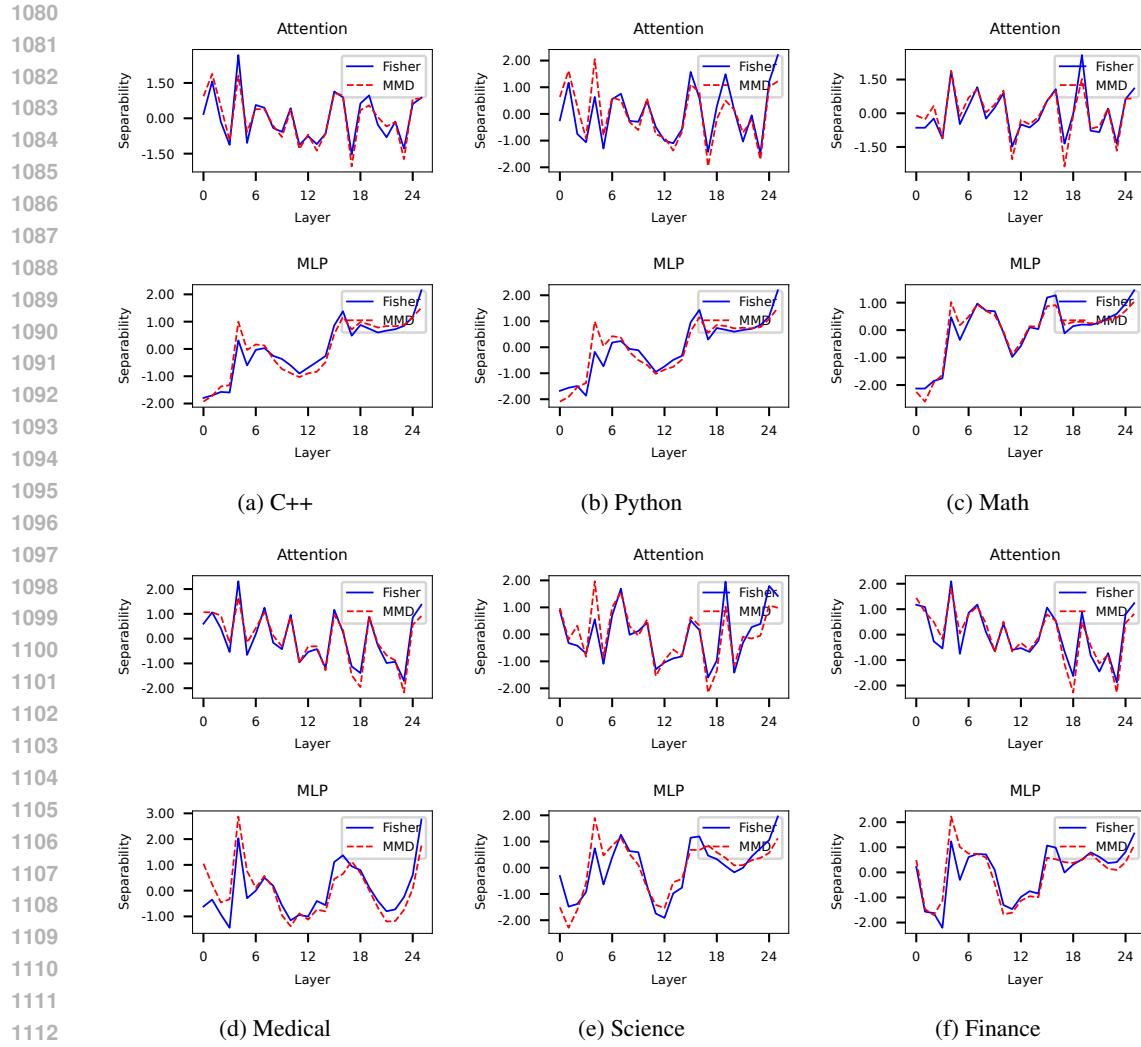


Figure 8: probe separability results for Gemma 1B Model

Finance The Finance dataset (Mateega et al., 2025) is a set of financial question and answer pairs extracted from company annual reports, balance sheets, and financial statements. The datapoints contain context with some financial values and the model is questioned upon some value that is dependant on this information. A similar formatting technique is used where we explicitly state the context, question and answer.

Context: Liabilities: 8,537.39 Total Capital And Liabilities: 13,410.53 ASSETS: nan NON-CURRENT ASSETS: nan Tangible Assets: 74.2 Intangible Assets: 4.16 Capital Work-In-Progress: 0 Other Assets: 0 Fixed Assets: 98.73 Non-Current Investments: 0 Deferred Tax Assets [Net]: 0 Long Term Loans And Advances: 0 Other Non-Current Assets: 15.61 Total Non-Current Assets: nan Question: What is the total value of assets of the company? Answer: The total value of assets of the company is \$13,410.53. Final Answer: 13410.53.

Medical We use the ReasonMed dataset (link lingshu-medical-mllm/ReasonMed) which is an open-source synthetic medical reasoning dataset containing multi-step chain-of-thought (CoT) rationales and concise summaries of LLMs such as Qwen-2.5-72B, DeepSeek-R1-Distill-Llama-70B, and HuatuoGPT-o1-70B on medical questions.

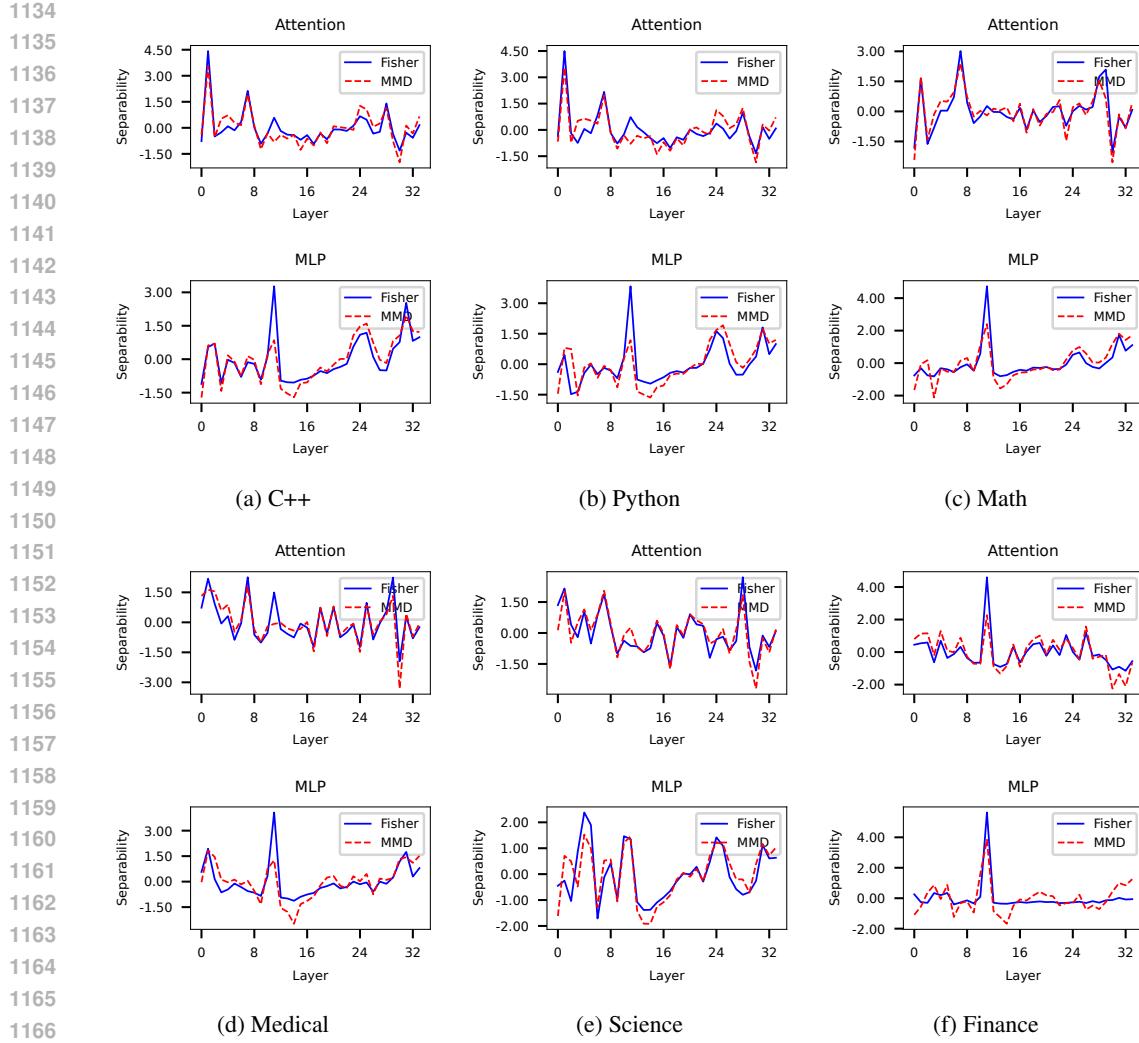


Figure 9: probe separability results for Gemma 4B Model

The question presents a radiographic scenario: a PA (posteroanterior) ulnar deviation view of the wrist, asking for the most likely diagnosis among the following options: Osteomyelitis, De Quervain tenosynovitis, Hypertrophic osteoarthropathy, and Rheumatoid arthritis. The correct answer is De Quervain tenosynovitis. To comprehensively understand and justify this answer, it's essential to dissect each component... (truncated)

C EVALUATION

C.1 EVALUATION RESULTS

Our evaluation of domain-specific performance uses two accuracy metrics tailored to the task type. For the Math, Science, Finance, and Medical domains, we report standard classification accuracy, defined as:

$$\text{Accuracy} = \frac{\text{Number of Correct Predictions}}{\text{Total Number of Samples}}$$

For the programming domains (C++ and Python), we evaluate code generation correctness using the `pass@k` metric. Specifically, we use `pass@10`, where the model generates 10 candidate solutions for

```

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1200
1201 def find_pivot_index(nums: list[int]) -> int:
1202     Finds the pivot index of a list of numbers.
1203
1204     Args:
1205         nums: A list of integers.
1206
1207     Returns:
1208         The pivot index if one exists, otherwise -1.
1209
1210     Examples:
1211         >>> find_pivot_index([1, 7, 3, 6, 5, 6])
1212         3
1213         >>> find_pivot_index([1, 2, 3])
1214         -1
1215         >>> find_pivot_index([2, 1, -1])
1216         0
1217
1218     total_sum = sum(nums)
1219     left_sum = 0
1220
1221     for i, num in enumerate(nums):
1222         if left_sum == total_sum - left_sum - num:
1223             return i
1224         left_sum += num
1225
1226     return -1

```

Listing 1: A Python snippet from our dataset illustrating a simple coding problem with a docstring that explicitly describes the working of the function

```

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1240
1241

```

```

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1251
1252
1253 // This function takes a positive integer as input and returns a
1254 // list of its prime factors,
1255 // which are the prime numbers that multiply together to equal the
1256 // original number.
1257 // The prime factors are returned in ascending order.
1258 // Parameters:
1259 // * number: A positive integer to be factorized into its prime
1260 //   factors.
1261 // Examples:
1262 // * find_prime_factors(8) => [2, 2, 2]
1263 // * find_prime_factors(25) => [5, 5]
1264 // * find_prime_factors(70) => [2, 5, 7]
1265 std::vector<int> find_prime_factors(int number) {
1266     std::vector<int> prime_factors;
1267
1268     while (number % 2 == 0) {
1269         prime_factors.push_back(2);
1270         number /= 2;
1271     }
1272
1273     for (int i = 3; i <= std::sqrt(number) + 1; i += 2) {
1274         while (number % i == 0) {
1275             prime_factors.push_back(i);
1276             number /= i;
1277         }
1278     }
1279
1280     if (number > 2) {
1281         prime_factors.push_back(number);
1282     }
1283
1284     return prime_factors;
1285 }

```

Figure 10: A C++ snippet from our dataset featuring a prime factorization problem. Each example contains a descriptive comment above the function body and clear naming conventions for the function itself.

```

1286
1287
1288
1289
1290
1291
1292
1293
1294
1295

```

1296 each problem. A problem is considered solved if at least one of these candidates passes all unit tests.
 1297 The accuracy is therefore calculated as:

$$1299 \text{pass@10} = \frac{\text{Number of Problems with at least one passing solution}}{1300 \text{Total Number of Problems}}$$

1301 It is important to note that the results presented, particularly for the smaller 1B models, may exhibit
 1302 some noise. These models operate with fewer parameters, making performance sensitive to minor
 1303 variations in fine-tuning, which can affect the robustness of the generated outputs.

	PT	MLP	Attn	Both	Top-1 MLP	Top-1 Attn	Top-3 MLP	Top-3 Attn
Math	0.040	0.070	0.050	0.020	0.050	0.040	0.030	0.040
Science	0.395	0.390	0.535	0.475	0.385	0.290	0.310	0.325
CPP	0.120	0.020	0.020	0.000	0.050	0.040	0.130	0.040
Python	0.440	0.020	0.180	0.040	0.040	0.350	0.160	0.290
Finance	0.180	0.020	0.010	0.000	0.060	0.040	0.020	0.070
Medical	0.847	0.687	0.787	0.813	0.904	0.424	0.916	0.864

Llama-3.2-1B

	PT	MLP	Attn	Both	Top-1 MLP	Top-1 Attn	Top-3 MLP	Top-3 Attn
Math	0.100	0.030	0.060	0.030	0.140	0.060	0.040	0.040
Science	0.625	0.755	0.700	0.650	0.610	0.600	0.425	0.425
CPP	0.320	0.000	0.030	0.000	0.000	0.286	0.000	0.150
Python	0.470	0.040	0.300	0.050	0.286	0.371	0.220	0.340
Finance	0.080	0.020	0.050	0.040	0.025	0.000	0.030	0.030
Medical	0.900	0.713	0.912	0.512	0.880	0.880	0.912	0.888

Gemma-3-1B

	PT	MLP	Attn	Both	Top-1 MLP	Top-1 Attn	Top-3 MLP	Top-3 Attn
Math	0.080	0.080	0.060	0.080	0.200	0.133	0.400	0.267
Science	0.715	0.780	0.780	0.760	0.840	0.820	0.760	0.700
CPP	0.833	0.033	0.000	0.000	0.028	0.457	0.000	0.286
Python	0.300	0.233	0.333	0.033	0.371	0.343	0.286	0.343
Finance	0.040	0.000	0.000	0.000	0.000	0.000	0.025	0.025
Medical	0.925	0.950	0.950	0.300	0.875	0.950	0.725	0.850

Gemma-3-4B

1338 As an alternative performance metric, we measured the asymptotic validation loss for different component
 1339 combinations. The results aligned with our separability analysis: layers identified as having
 1340 high activation separability consistently outperformed those with lower separability, converging to a
 1341 significantly lower validation loss.

C.2 DOMAIN EVALUATION

C.2.1 MATH

1347 **Dataset chosen:** *GSM8K* (Grade School Math 8K) introduced by Cobbe et al. (2021b) is a collection
 1348 of grade-school level math word problems designed to evaluate multi-step arithmetic and reasoning
 1349 ability. The dataset emphasizes chain-of-thought style reasoning where intermediate steps are useful
 to arrive at the correct numeric result.

1350
 1351 GSM8K is used here as it's a widely used benchmark for studying reasoning behavior in language
 1352 models and for evaluating self-consistency / majority-vote sampling methods. Also, it is not too
 1353 difficult, hence used for evaluation on the small models considered.
 1354

1355 **Prompt–Output Illustration:**

1356 # <prefix text (8-shot demos)
 1357 provided for context>
 1358
 1359 Q: John has 3 apples.
 1360 He buys 2 more.
 1361 How many apples does he have
 1362 now?
 1363
 1364 A: Let's reason step by step.
 1365 At the end, give the final
 1366 numeric
 1367 answer on its own line in
 1368 this exact format:
 1369 #### <number>
 1370
 1371 Answer:

1372
 1373 # Example reasoning and
 1374 output
 1375
 1376 Step 1: John starts **with** 3
 1377 apples.
 1378 Step 2: He buys 2 more.
 1379 Step 3: Total apples = 3 + 2
 1380 = 5.
 1381
 1382 #### 5
 1383 Answer:

1384 Illustration of the prompt (left) and an example of the expected LLM output (right).

Evaluation Samples	Sampling Amount Per Sample	Max Generation Tokens	Temperature	Top-p
100	10	1024	0.7	0.90

1385 (Hyper-Parameters used during Model inference For Evaluation(Self-Consistency))

1386 C.2.2 FINANCE

1387 **Dataset chosen:** *FinanceQA* introduced by Mateega et al. (2025) is a curated set of financial
 1388 question–answer pairs extracted from company filings (annual reports, balance sheets, and reports). It
 1389 supplies queries, short factual answers, and the supporting context passage from the source document
 1390 (e.g., a few sentences or table rows). Focus is on **numerical output comparison and extraction**.

1391 FinanceQA is used for evaluation as it provides a domain-specific “**finance + math**” evaluation
 1392 setting, requiring both factual retrieval and quantitative reasoning.

1393 **Prompt–Output Illustration:**

1394 # FinanceQA prompt builder
 1395
 1396 (context + query)
 1397
 1398 Context:
 1399 <supporting passage from
 1400 financial filings>
 1401
 1402 Question:
 1403 <query here>
 1404
 1405 Answer: The final answer is
 1406
 1407 Final Answer:

1408
 1409 # Example reasoning and
 1410 output
 1411
 1412 Step 1: From the context, the
 1413 net profit
 1414 margin in 2021 is explicitly
 1415 given.
 1416 Step 2: The reported margin
 1417 is 11.04%.
 1418
 1419 Final Answer: 11.04%

1404 Illustration of the FinanceQA prompt template (left) and an example expected LLM output (right).
 1405
 1406

Evaluation Samples	Sampling Amount Per Sample	Max Generation Tokens	Temperature	Top-p
100	10	512	0.7	0.95

1411 (Hyper-Parameters used during Model inference For Evaluation(Self-Consistency))
 1412
 1413

1414 C.2.3 MEDICAL

1415 **Dataset chosen:** *PubMedQA* introduced by Jin et al. (2019) is a dataset of biomedical research
 1416 questions paired with contexts and a short (yes/no) final decision derived from biomedical articles.
 1417 Each sample often contains an abstract or supporting passage and a question about the clinical finding;
 1418 the ground truth is typically a binary decision. Sometimes if LLM is highly undecisive the output of
 1419 LLM is assumed 'None'

1420 We use PubMedQA because it is a widely-used , biomedical QA benchmark for evaluating concise,
 1421 high-precision yes/no answers in the clinical/research domain.
 1422

1423 Prompt–Output Illustration:

```
1426 # PubMedQA prompt builder (   

1427   question + context)   

1428   

1429 Context:   

1430 <concatenated context sentences  

1431   or abstract>   

1432   

1433 Question: <question here>   

1434   

1435 Based on the context above,  

1436   answer the question  

1437 with exactly 'yes' or 'no' (   

1438   lowercase),  

1439 and do NOT provide any  

1440   explanation.  

1441 Answer:
```

1441 Illustration prompt template used Sample output is simply Yes/No , In case Bad output Then None is interpreted
 1442
 1443

Evaluation Samples	Sampling Amount Per Sample	Max Generation Tokens	Temperature	Top.p
250	1	512	0.0	1.00

1448 (Hyper-Parameters used during Model inference For Evaluation (Greedy))
 1449
 1450

1451 C.2.4 SCIENCE

1452 **Dataset chosen:** *SciQ* introduced by Welbl et al. (2017) is a data set of multiple choice science questions that
 1453 contains short grade-level science questions with four answer options (A–D) and optional supporting facts. Each
 1454 example includes a question, four candidate answers, and (sometimes) a support passage.

1455 SciQ is used because it provides well-formed multiple-choice prompts suitable for evaluation,it is easy for a
 1456 small LLM hence it is used.
 1457

1458 Prompt–Output Illustration:

```

1458
1459 // SciQ prompt builder (
1460   question + options)
1461
1462 Question:
1463 <question text>
1464
1465 Options:
1466 A. <option A>
1467 B. <option B>
1468 C. <option C>
1469 D. <option D>
1470
1471 Answer with the letter of the
1472   correct option only (A, B,
1473   C, or D).
1474 Do NOT provide any explanation.
1475 Answer:
1476
1477
1478
1479
1480
1481
1482
1483
1484

```

Answer:B

1475 Illustration: left = prompt template used for SciQ , model output is a single letter A/B/C/D.

Evaluation Samples	Sampling Amount Per Sample	Max Generation Tokens	Temperature	Top.p
200	1	256	0.0	1.00

1489 (Hyper-Parameters used during Model inference For Evaluation(Greedy))

1500 C.2.5 PYTHON

1501
1502 **Dataset chosen:** *HumanEvalPack (multilingual / Python subset)* Introduced by Chen et al. (2021) is a collection
of programming problems with formal problem descriptions, expected function signatures, and test harnesses.

1503
1504
1505 Inputs in the form of coding questions are provided, and the model is expected to output corresponding code
1506 which is executed against test cases. The accuracy used for evaluation is **pass@k**, a standard metric for
1507 code-generation tasks, rather than simple string-matching accuracy.

1508 HumanEvalPack is used here because it provides language-specific (C++/Python/etc.) prompts with a standard
1509 "declaration + examples + tests" scheme. The problems are relatively simple, making this dataset ideal for
1510 comparing small models on code generation and correctness.

1511

Prompt–Output Illustration:

```

1512
1513 # Problem:
1514 <prompt_or_instruction>
1515
1516 # Signature:
1517 <signature>
1518
1519 # Docstring:
1520 <docstring>
1521
1522 # Examples:
1523 <example_test>
1524
1525 Write the complete Python
1526     function
1527 implementation only.
1528 Output only valid Python code
1529     for the
1530 function (no explanation, no
1531     tests,
1532 no surrounding markdown).
1533 Make sure the function name and
1534     signature match the signature
1535     above.
1536
1537 Implementation:
1538
1539
1540
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1542
1543
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1547
1548
1549
1550
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1561
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1563
1564
1565

```

Illustration of the Python prompt template (left) and an example expected LLM output (right).

Evaluation Samples	Sampling Amount Per Sample	Max Generation Tokens	Temperature	Top-p
100	10	1024	0.7	0.95

(Hyper-Parameters used during Model inference For Evaluation(Self-Consistency))

C.2.6 CPP

Dataset chosen: *HumanEvalPack (multilingual / C++ subset)* Introduced by Chen et al. (2021) is a collection of programming problems with formal problem descriptions, expected function declarations/signatures, and test harnesses. Inputs in the form of coding questions are provided, and the model is expected to output corresponding code which is compiled against test cases.

HumanEvalPack is used here because it provides language-specific (C++/Python/etc.) prompts with a standard “declaration + examples + tests” scheme. The problems are relatively simple, making this dataset ideal for comparing small models on code generation and correctness.

Prompt–Output Illustration:

```

1566
1567 // Problem:
1568 <prompt_or_instruction>
1569
1570 // Declaration:
1571 <declaration>
1572
1573 // Docstring / Notes:
1574 <docstring>
1575
1576 // Examples:
1577 <example_test>
1578
1579 Write the C++ implementation
1580 only
1581 (no explanation, no tests, no
1582 surrounding markdown).
1583 Include necessary #include
1584 lines if needed.
1585 Ensure function name and
1586 signature match the
1587 declaration above.
1588
1589 Implementation:
1590
1591
1592
1593
1594
1595
1596
1597
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1600
1601
1602
1603
1604
1605
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1607
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1609
1610
1611
1612
1613
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1617
1618
1619

```

```

#include <bits/stdc++.h>
using namespace std;

// Example implementation for:
int add(int a, int b) {
    // simple implementation
    return a + b;
}

```

Illustration of the C++ prompt template (left) and an example expected LLM output (right).

Evaluation Samples	Sampling Amount Per Sample	Max Generation Tokens	Temperature	Top-p
100	10	1024	0.7	0.95

(Hyper-Parameters used during Model inference For Evaluation(Self-Consistency))

D EXPERIMENTAL SETUP

D.1 FINE TUNING

1618 All experiments were run on NVIDIA H100 GPUs, using PyTorch and the Hugging Face ‘transformers’ and
1619 ‘peft’ libraries. To maximize computational throughput, the model was JIT-compiled using ‘torch.compile()’. A
fixed set of hyperparameters, detailed in Table 7, was used across all experiments to ensure fair comparison.

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Table 7: Common hyperparameters for all fine-tuning experiments.

Parameter	Value
Training Configuration	
Optimizer	AdamW
Learning Rate	1×10^{-3}
Batch Size	8
Epochs (Stage 1 Mapping)	10
Epochs (Stage 2 Validation)	3
Seed	42
Precision	‘bf16’
LoRA Configuration	
Rank (r)	16
Alpha (α)	$32 (2 \times r)$
Dropout	0.05
Target Modules (Attn)	q_proj, k_proj, v_proj, o_proj
Target Modules (MLP)	gate_proj, up_proj, down_proj

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D.2 PROBING ANALYSIS

1646 In addition to Fisher Separability and Maximum Mean Discrepancy (MMD), we also evaluated probing separability using other metrics such as classification probing accuracy, cosine similarity, and V-bits. However, 1647 for high-level abstraction tasks such as *Domain Separability*, the results across layers were not clearly 1648 distinguishable. This arises because, in such tasks, the points in the activation hyperspace are widely dispersed. 1649 Consequently, strong metrics such as V-bits or probing classification accuracy can easily separate these spread- 1650 out representations, making them less informative for fine-grained layer-wise analysis. In contrast, weaker 1651 metrics such as Fisher separability and MMD are more useful in these cases, as they provide more sensitive 1652 distinctions when the data is already well separated.

1653 On the other hand, for low-level abstraction tasks such as *Concept-level Separability*, the points in the activation 1654 hyperspace are closely packed. In these scenarios, strong metrics such as V-bits prove more effective, yielding 1655 clearly distinguishable results across layers. This observation is consistent with findings reported in Ju et al. 1656 (2024b).

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E EXTENDED DISCUSSION1662
E.1 HYDRA EFFECT

1663 The Hydra Effect describes a form of self-repair capability present in LLMs. As described by McGrath et al. 1664 (2023), it refers to the mismatch between a layer’s apparent contribution (measured by projecting its activations 1665 through the unembedding mechanism, $\Delta_{unembed}$) and its functional importance (measured by ablating the layer, 1666 Δ_{ablate}). We expect the ablation to reduce the model’s confidence proportionally to its apparent contribution, 1667 but downstream layers reconstruct the corrupted signal so that

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$$\Delta_{ablate,l} < \Delta_{unembed,l}$$

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1672 During interventions, the KL divergence is lower for early layers with high fisher score due to this reason since
1673 the intervention done is reverted to some extent by downstream layers.

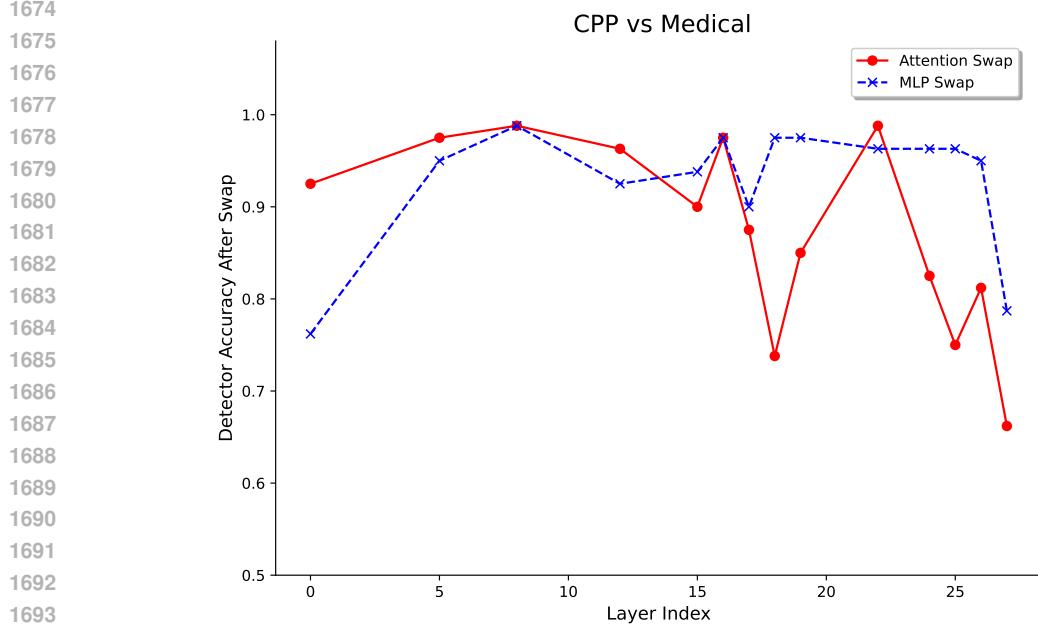


Figure 11: **Causal Impact of Attention and MLP Swaps on Domain Classification.** The plot shows the results of our last-token Causal Signal Probe for the CPP vs. Medical domains. Accuracy reflects the model’s ability to correctly classify the original text after a single-token activation swap. A lower accuracy indicates a more effective causal intervention (i.e., the swap successfully steered the model’s output).

To directly test the causal role of different components and investigate this self-repair phenomenon, we performed a Causal Signal Probe. As shown in Figure 11 for the CPP vs. Medical pair, we found that swapping the last-token activations from the Attention stream had a dramatically larger impact on the final classification than swapping MLP activations, particularly in the latter half of the network. While interventions in early layers were largely ignored by the model (accuracy ≈ 1.0), consistent with the Hydra effect, swapping Attention activations in layers 15-20 caused a significant drop in accuracy. This demonstrates that these late-stage attention blocks possess a powerful causal steering handle over the model’s final domain representation, whereas the influence of individual MLP blocks is less critical.

E.2 METHODOLOGY FOR THE LAST-TOKEN CAUSAL SIGNAL PROBE

E.2.1 OBJECTIVE

The Causal Signal Probe was designed to move beyond correlational measures and establish a direct causal link between the activations of specific model components and the model’s final domain classification. The primary goal was to answer the question: “If we forcibly inject information from Domain B into the processing of a text from Domain A at a specific layer L, does the model’s final thought flip to Domain B?” This experiment allows us to identify which layers and components act as influential steering handles for domain representation.

E.2.2 EXPERIMENTAL DESIGN

The probe consists of three main stages: (1) training a reliable Truth Detector to classify the model’s final hidden state, (2) performing a targeted causal intervention via activation swapping, and (3) measuring the effect of this intervention using the Truth Detector.

Stage 1: Training the Truth Detector To create an objective arbiter of the model’s final representation, we first trained a simple linear probe, which we term the Truth Detector.

- **Purpose:** The detector’s sole job is to look at the final hidden-state vector of the model and classify which of the two domains it belongs to.

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- **Data Generation:** We passed 500 text samples from the `cpp` domain and 500 from the `medical` domain through the frozen Llama-3.2-3B model. We collected the final hidden-state activation vector for each sample.
- **Architecture:** The detector is a simple linear model that maps to 2 output logits (one for each domain).
- **Training:** The collected activations were split into an 80/20 train/validation set. The detector was trained for 7 epochs using an Adam optimizer and Cross-Entropy loss. For the CPP vs. Medical pair, the detector rapidly achieved 100.0% validation accuracy, confirming it as a highly reliable ground truth for our experiment.

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1739 **Stage 2: The Causal Intervention (Last-Token Activation Swap)** This is the core of the causal
1740 experiment. We performed a precise activation swap for each layer and component under investigation.

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- **Procedure:** For each trial, a `base_text` (true label) and a `donor_text` (opposing domain) are
1742 randomly selected.
- The `donor_text` is passed through the model up to a target layer L . We cache the activation vector
1744 of its very last token for a specific component (e.g., the output of the attention block).
- The `base_text` is then passed through the model. Using PyTorch forward hooks, we intercept
1746 the computation at layer L , right after the target component (Attention or MLP) has finished its
1747 computation.
- The hook replaces the last-token activation vector of the `base_text` with the cached vector from the
1749 `donor_text`. All other token activations remain unchanged.
- The forward pass resumes, processing this patched sequence representation, and the final hidden-state
1750 vector is collected.

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1753 This last-token methodology is crucial as it is a clean, minimal intervention that directly targets the same vector
1754 representation our Truth Detector was trained on, avoiding confounding issues related to variable sequence
1755 lengths.

1757 E.2.3 DETAILED INTERPRETATION OF THE CPP VS. MEDICAL GRAPH

1759 The provided graph (Figure 11) plots the Detector Accuracy After Swap for the CPP vs. Medical domain pair.

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- **Overall Trend:** The most striking feature is the growing divergence between the Attention and MLP
1761 swaps. While both start with high accuracy, the Attention swap becomes significantly more impactful
1762 (lower accuracy) in the later layers.
- **Early Layers (0-8):** Interventions here have minimal effect (Accuracy 76%-99%), indicating that the
1764 initial distributed domain signal overpowers the single-token swap, and downstream layers repair the
1765 representation.
- **Mid-Layers (8-18):** A crucial divergence begins. MLP swap impact remains low, while Attention
1767 swap accuracy starts to dip, suggesting attention mechanisms here begin to refine the domain-specific
1768 representation.
- **Late Layers (18-27):** This region shows the strongest causal effect. The Attention swap becomes
1770 highly volatile and effective. This confirms that some attention layers act as powerful steering handles,
1771 whereas the MLP influence is secondary and less decisive.

1773 E.3 CHARACTERISTIC TOKENS

1775 The process of selecting characteristic tokens is derived from the same causal intervention process done in
1776 reverse. Instead of finding layers that do the most change to specific tokens, we find tokens that are most sensitive
1777 to interventions on all layers. This process is coined as the reverse causal intervention on a model.

1778 When we do an intervention on a single layer from one domain to another, the tokens of the new domain are
1779 shifted up in probability. The overall shift across the vocabulary is averaged across all layers and the Top-k
1780 “promoted” tokens are saved in a list for the intervening dataset. For example, we have found when intervening
1781 C++ prompts with Python activations, tokens such as `def`, `import` and `python` are promoted. These form the
characteristic token set for Python and this set is used in our causal intervention experiments further on.

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E.4 DELTA BIAS

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17861787 Let V be the entire vocabulary of the model. We denote the probability associated with a subset of vocabulary
1788 $S \subset V$ as $P(S|x) = \sum_{i \in S} p(i|x)$ with a prompt x . Suppose we perform the intervention $x_A \xleftarrow{l} x_B$
1789 where activations of prompt of domain B are inserted into the forward pass of A at layer l . Before intervention,
1790 $P_{base}(S_A|x_A)$ and $P_{base}(S_B|x_A)$ denote the probabilities of characteristic tokens of A and B before intervention,
1791 and $P_{swap}(S_A|x_A \xleftarrow{l} x_B)$ and $P_{swap}(S_B|x_A \xleftarrow{l} x_B)$ as the probabilities of the set of characteristic tokens of A
1792 and B after intervention. The *Bias* present in the probability distribution is defined as $\text{Bias} = P(S_B) - P(S_A)$.
1793 This represents the model's preference on predicting the intervening subset of tokens.1794
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$$\text{Bias}_{base}(x_A) = P_{base}(S_B|x_A) - P_{base}(S_A|x_A)$$

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$$\text{Bias}_{swap}(x_A \xleftarrow{l} x_B) = P_{swap}(S_B|x_A \xleftarrow{l} x_B) - P_{swap}(S_A|x_A \xleftarrow{l} x_B)$$

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$$\Delta \text{Bias}(A \xleftarrow{l} B) = \mathbb{E}_{x_A \sim A, x_B \sim B} [\text{Bias}_{swap}(x_A \xleftarrow{l} x_B) - \text{Bias}_{base}(x_A)]$$

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1816 In our results, we use the convention for when $A \xleftarrow{l} B$ is done, we plot bias with a positive sign, and when
1817 we do intervention $B \xleftarrow{l} A$, we plot bias with a negative sign to preserve perspective with respect to the set of
1818 characteristic tokens B. So, all bias computations are visualized as the shift in preference of B over A.
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18281829 E.5 CAUSAL INTERVENTION VARIATIONS
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18331834 We extend our causal intervention study to other domain pairs beyond the C++/Python case discussed in the main
1835 text. Below we present comprehensive results for all tested domain pairs across all four models: Llama-3.2-3B,
Llama-3.2-1B, Gemma-3-4B, and Gemma-3-1B.

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E.5.1 LLAMA-3.2-3B: ALL DOMAIN PAIRS

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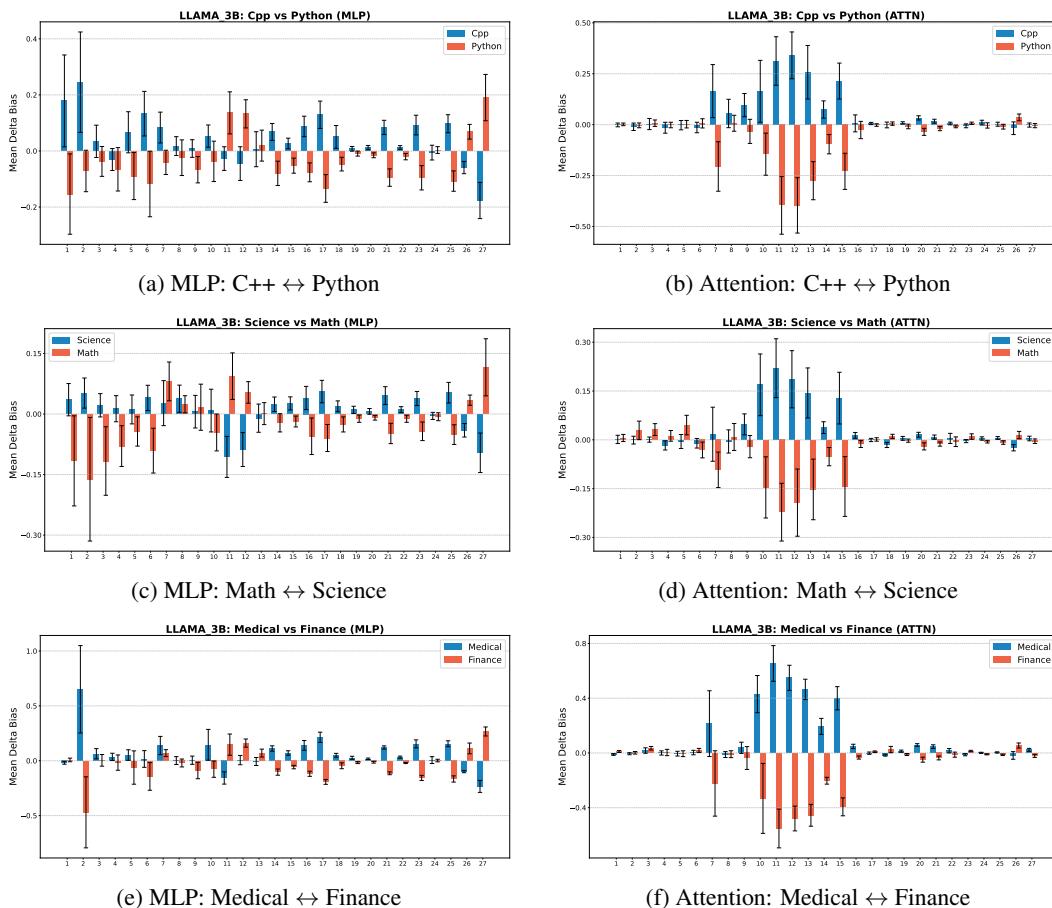
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Figure 12: **LLAMA-3.2-3B causal intervention results across all domain pairs.** Each row shows a different domain pair. Left column: MLP activations exhibit flat, high-variance profiles centered near zero (disruption without directional control). Right column: Attention activations show sharp, localized peaks at mid-depth layers (e.g., 13-15, 23-25), indicating sparse routing hotspots. The pattern is consistent across all three domain pairs, supporting domain-agnostic routing mechanisms.

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E.5.2 LLAMA-3.2-1B: ALL DOMAIN PAIRS

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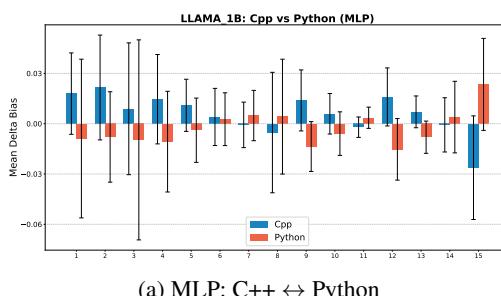
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(a) MLP: C++ ↔ Python

(b) Attention: C++ ↔ Python

(c) MLP: Math ↔ Science

(d) Attention: Math ↔ Science

(e) MLP: Medical ↔ Finance

(f) Attention: Medical ↔ Finance

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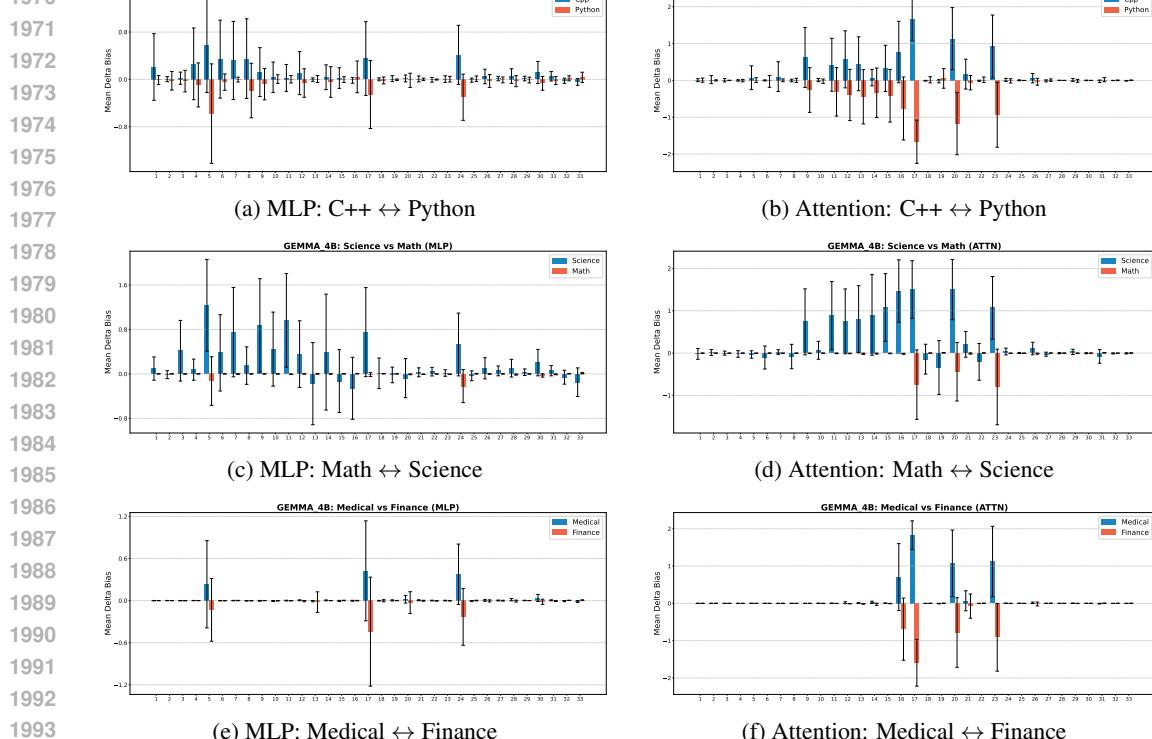
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Figure 13: **LLAMA-3.2-1B causal intervention results across all domain pairs.** Despite the smaller model size (1B parameters, 16 layers), the same qualitative pattern emerges: MLP swaps produce non-directional disruption, while attention swaps yield localized steering peaks. The reduced depth results in fewer but proportionally similar routing layers.

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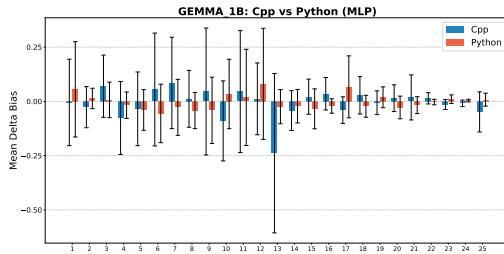
E.5.3 GEMMA-3-4B: ALL DOMAIN PAIRS



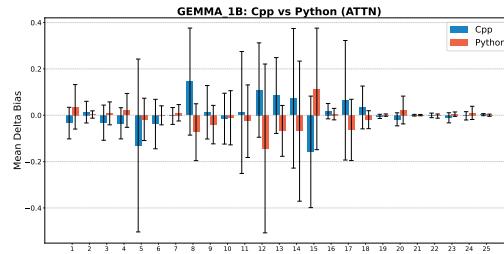
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Figure 14: **GEMMA-3-4B causal intervention results across all domain pairs.** Gemma models exhibit sharper, more concentrated attention peaks compared to Llama models, suggesting a more specialized hub-like routing architecture. This acute localization is particularly visible in the Medical/Finance pair (bottom right), where a single attention layer dominates the steering signal.

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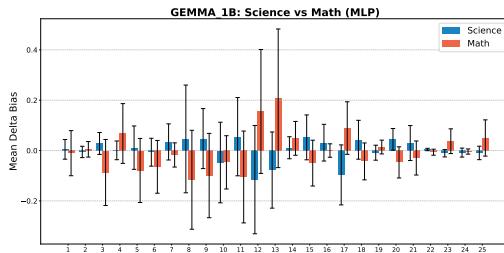
E.5.4 GEMMA-3-1B: ALL DOMAIN PAIRS

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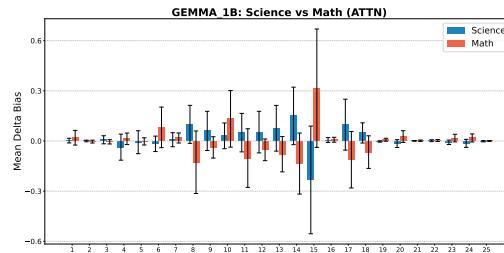
(a) MLP: C++ ↔ Python



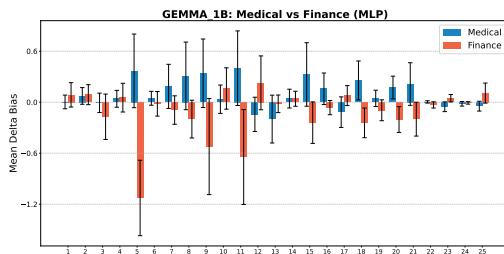
(b) Attention: C++ ↔ Python



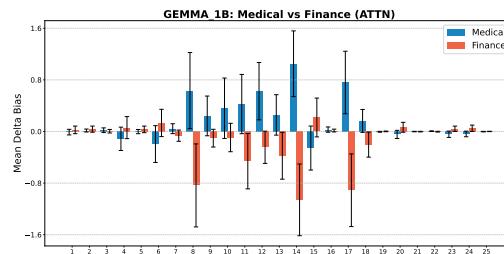
(c) MLP: Math ↔ Science



(d) Attention: Math ↔ Science



(e) MLP: Medical ↔ Finance



(f) Attention: Medical ↔ Finance

Figure 15: **Gemma-3-1B causal intervention results across all domain pairs.** Even at the smallest scale tested (1B parameters, 18 layers), the functional division persists: attention provides sparse routing, MLP provides distributed computation. The consistency across all four models (1B-4B) and three domain pairs provides robust evidence for this architectural organizing principle.

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We extend our intervention study to a larger language models, such as Llama-7B, where we observe that individual layers are not highly influential in the final prediction of the token. In these layers, we observe that the earliest layers have the most impact since small changes in these layers snowball into larger changes in the final output.

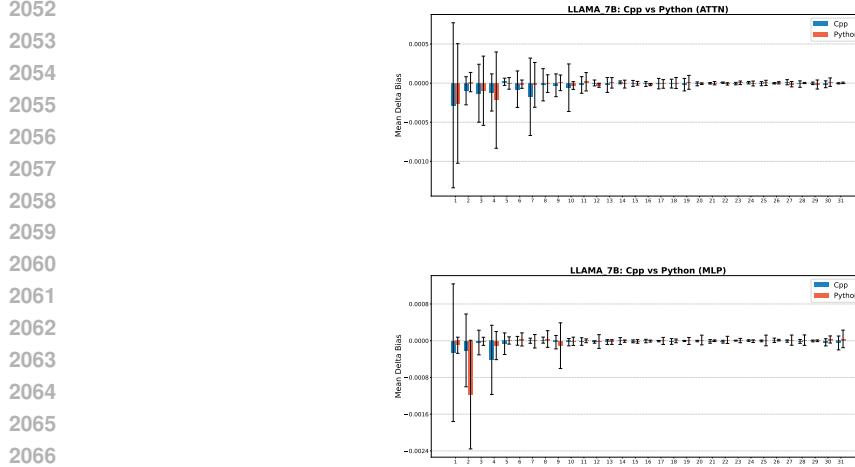


Figure 16: Results for Llama-7B showing influence of Attention vs MLP layers.

We investigated other methods of finding representative tokens from fine-tuning datasets to check if more frequent tokens are representative of a specific domain. We find that this is not the case since even though the relative effect of the causal shift in layer swapping is still the same, the absolute magnitude in shift is reduced by %.

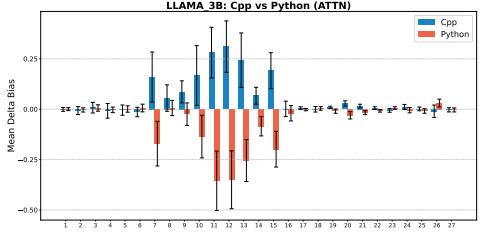
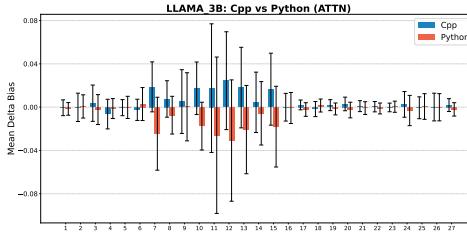


Figure 17: Comparison of Attention layer shifts for different token selection methods.

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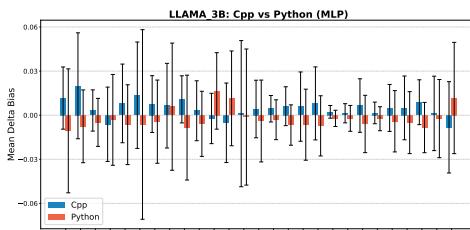
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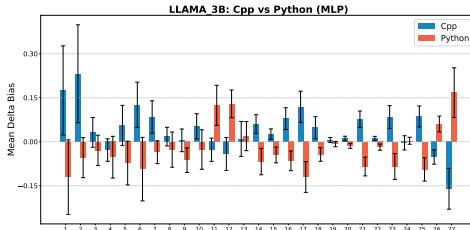
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(a) Representative tokens using frequency analysis (MLP)



(b) LLM Generated Representative Tokens (MLP)

Figure 18: Comparison of MLP layer shifts for different token selection methods. We observe in the top graph the high variance and low quality of domain separability.

LLM-generated tokens have been explicitly optimized for domain differentiation, whereas frequency-based tokens lack the semantic depth required for the model to distinguish between lists. This information deficit prevents effective separation, directly resulting in the higher variance observed in the top graphs.

E.5.5 ROBUSTNESS TO TOKEN COUNT (n)

We tested the sensitivity of our causal results to the number of domain-representative tokens (n) included in each prompt list. Using the domain-classification task (Section 2.3), we varied $n \in \{5, 10, 15, 20\}$ and measured Delta Bias at the peak attention layers (13-15, 23-25).

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