Similarity and complementarity in generalized random geometric graphs

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Extended Abstract

Many real-world networks are shaped by a tension between homophilic *similarity*—entities connect when they are geometrically or semantically close—and heterophilic *complementarity*, where links form between dissimilar yet synergistic nodes. While similarity produces triangle-rich structures [1–4], complementarity favors quadrangles and local bipartivity [5–7]. Most existing latent-space models capture only one of these principles.

We introduce the *generalized random geometric graph* (GRGG), a flexible latent-space formalism on compact isotropic manifolds (e.g. the d-sphere). In GRGGs, each node i is assigned a random position $x_i \in S^d$ and edges are governed by distance-based edge energies. Two common and useful energies capture the similarity-complementarity dichotomy:

$$\varepsilon_{ij}^{s} = f(g_{ij}), \qquad \varepsilon_{ij}^{c} = f(g_{\max} - g_{ij}),$$

where g_{ij} is the geodesic distance, $g_{\max}$ the diameter of the space, and $f(\cdot)$ a non-decreasing function (e.g. log). The first energy recovers the standard random geometric graph assumption that nearby nodes connect, while the second encodes complementarity by linking antipodal (maximally distant) points. Energies are mapped to edge couplings $\Theta(\varepsilon_{ij})$ via a sigmoidal, Fermi-Dirac form, yielding a maximum-entropy ensemble in which (μ, β) control expected degree and sharpness of transition. The resulting edge probability takes the form

$$p_{ij} = \frac{1}{1 + e^{\Theta(\varepsilon_{ij})}},$$

yielding a maximum-entropy ensemble. For instance, for $\Theta(\varepsilon_{ij}) = \beta\left(\varepsilon_{ij} - \mu\right)$ this recovers the familiar Fermi-Dirac distribution arising as the edge probability function in the degree-homogeneous soft RGG model:

$$p_{ij} = \frac{1}{1 + e^{\beta\left(\varepsilon_{ij} - \mu\right)}}$$

In the GRGG framework, multiple energies (layers) can be combined, with the final edge probability given by a noisy-OR across layers:

$$p_{ij} = 1 - \prod_{\ell=1}^{L}\left(1 - p_{ij}^{(\ell)}\right).$$

This construction generalizes many classical models: Erdős-Rényi graphs, hard and soft random geometric graphs, and hyperbolic graphs with fitness all arise as special cases (see Fig. 1a).

To illustrate the usefulness of the GRGG framework above, we define a new similarity-complementarity random geometric graph (SCRGG) model, which combines two GRGG layers: one driven by similarity (connecting nearby nodes) and one driven by complementarity (connecting nearly antipodal nodes). The relative average degree contributed by each layer controls whether the resulting network is dominated by triangle-rich or quadrangle-rich motifs (see Fig. 1b). Further introducing hidden-variable fitness terms yields scale-free networks in the hyperbolic limit [4].

We study the properties of the SCRGG and its behavior as the relative density of similarity-versus complementarity-driven layers changes, effectively interpolating between triangle-rich and quadrangle-rich relational patterns. Additionally, we use the SCRGG in a series of synthetic link-prediction experiments that show regimes where homophily-based algorithms perform well (when edges from the similarity layers dominate), while complementarity-aware rules outperform when quadrangles are prevalent. In summary, the GRGG formalism and its SCRGG instantiation provide a tractable, maximum-entropy framework that interpolates between similarity and complementarity. This offers both theoretical insight into motif-level patterns in graphs and a tunable benchmark for evaluating algorithms in regimes where real-world networks often lie.

References

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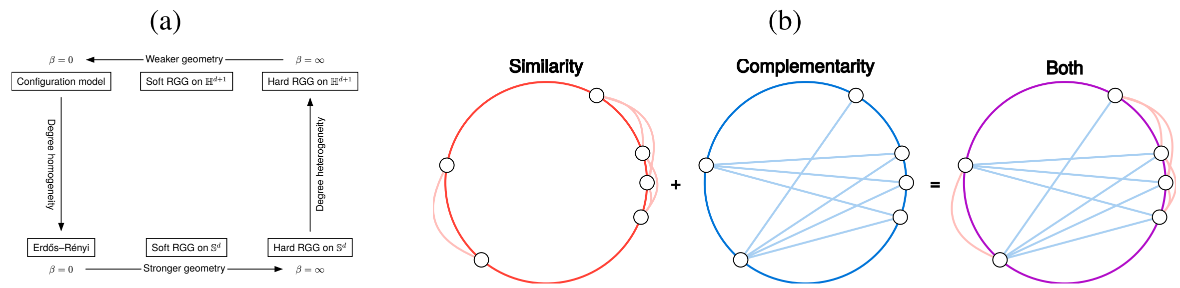

Figure 1: **Schematic summary of the GRGG model. (a)** Space of possible models that can be seen as special cases in the GRGG framework. **(b)** Geometric logic of the similarity and complementarity layers.