STARJOB: DATASET FOR LLM-DRIVEN JOB SHOP SCHEDULING

Anonymous authors
Paper under double-blind review

ABSTRACT

Large Language Models (LLMs) have shown remarkable capabilities across various domains, but their potential for solving combinatorial optimization problems remains largely unexplored. In this paper, we investigate the applicability of LLMs to the Job Shop Scheduling Problem (JSSP), a classic NP-hard challenge. We introduce Starjob, the first large-scale supervised dataset for JSSP, comprising 130,000 instances with natural language representations designed specifically for training LLMs. Leveraging this dataset, we fine-tune a Llama-3.1 8B model using the resource-efficient RsLoRA method to create an end-to-end scheduler. Our evaluation on standard benchmarks demonstrates that this LLM-based method surpasses traditional Priority Dispatching Rules (PDRs) and achieves significant performance gains over foundational neural approaches like L2D and RASCL, with an average gap improvement of 17.36% on DMU and 7.85% on Taillard benchmarks relative to L2D. These results highlight the untapped potential of fine-tuned LLMs in combinatorial optimization, establishing a new direction for developing interactive and high-performance scheduling systems.

1 INTRODUCTION

Despite their success in natural language processing, Large Language Models (LLMs) have not been traditionally considered strong candidates for solving computationally intensive problems. Their applicability to NP-hard combinatorial optimization problems is often viewed as limited, a perception reinforced by the scarcity of empirical evidence showing LLMs outperforming specialized methods like reinforcement learning in these domains. Furthermore, the propensity of LLMs to "hallucinate" can lead to infeasible solutions, making their direct application unreliable. Consequently, the systematic exploration of fine-tuned LLMs for hard combinatorial problems has remained limited.

In this paper, we challenge this prevailing view by demonstrating that representation is the key to unlocking the scheduling capabilities of LLMs. We present the first fine-tuned LLM for the Job Shop Scheduling Problem (JSSP). Our results show that when trained on a properly structured, text-based representation of the problem, an LLM can not only generate feasible schedules but also outperform classic Priority Dispatching Rules (PDRs) and foundational dedicated neural methods that first surpassed them (e.g., L2D Zhang et al. (2020) and RASCLIklassov et al. (2023)). These findings suggest that with appropriate data representation and fine-tuning, LLMs can become a competitive new paradigm for combinatorial optimization, complementing existing specialized solvers.

JSSP is a fundamental optimization problem with critical applications in manufacturing and logistics, where jobs must be scheduled on machines to minimize metrics like makespan (C_{max}). While traditional methods face scalability challenges, modern AI techniques, particularly reinforcement learning and Graph Neural Networks (GNNs), have offered promising data-driven alternatives Zhang et al. (2020); Corsini et al. (2024). Concurrently, LLMs have been explored for tasks involving structured reasoning, such as graph analysis Huang et al. (2022); Chen et al. (2024b) and planning Valmeekam et al. (2022). However, their application to the explicit, constraint-heavy domain of scheduling remains largely unexplored.

This work bridges that gap. We are the first to employ a fine-tuned LLM for end-to-end JSSP scheduling. To enable this, we introduce **Starjob**[1], a novel supervised dataset where JSSP instances

[1] https://github.com/starjob42/Starjob

and their solutions are framed in natural language. By fine-tuning a Llama model with the RsLoRA method Kalajdzievski (2023) on this dataset, we demonstrate on the well-known Taillard Taillard (1993) and DMU Demirkol et al. (1998) benchmarks that our approach finds high-quality solutions, surpassing both classic PDRs and the L2D neural baseline.

Our contributions are:

- We introduce **Starjob**, the first supervised dataset with 130,000 instances designed to train LLMs for JSSP using a structured natural language format.
- We present the first end-to-end JSSP scheduler based on a fine-tuned LLM, demonstrating its ability to reason over complex constraints using the Starjob dataset and the RsLoRA method.
- We conduct a rigorous evaluation against four PDRs and neural methods L2D and RASCL, showing our model's superior performance and generalization, particularly on large-scale instances with up to 1000 operations.
- Our LLM-based approach unlocks a new modality of interaction: users can query the scheduler in natural language to understand scheduling constraints or solution characteristics, significantly enhancing transparency and usability, as presented in Listing 3.

It is important to note that while our approach demonstrates competitive performance against several established baselines, we do not claim to have developed the absolute best scheduler for JSSP. Rather, this work represents the first systematic application of LLMs to end-to-end large JSSP instances, establishing a foundation for future research at the intersection of natural language processing and combinatorial optimization.

2 Related Work

JSSP with more than two machines is proven to be NP-hard Garey et al. (1976). As a result, finding exact solutions for JSSP is generally infeasible, leading to the widespread use of heuristic and approximate methods for practical efficiency Cebi et al. (2020). Traditional approaches to solving JSSP have primarily relied on search and inference techniques developed by the constraint programming community Beck et al. (2010). These techniques effectively leverage constraints to define the relationships and limitations between jobs and resources, enabling efficient exploration of feasible solution spaces and the identification of optimal or near-optimal schedules Nowicki & Smutnicki (2005). A widely used heuristic method in real-world scheduling systems is the Priority Dispatching Rule (PDR) Zahmani et al. (2015). PDRs are simple and effective, although designing an efficient PDR is time-consuming and requires extensive domain knowledge.

Recently, approaches utilizing Deep Learning and Neural Networks have gained attention for finding promising solutions to the JSSP Bonetta et al. (2023); Zhang et al. (2020); Corsini et al. (2024). These methods can be broadly categorized into supervised learning and reinforcement learning (RL). Current research in deep reinforcement learning (DRL) is actively focused on developing advanced methods to tackle JSSP. Existing DRL methods typically represent JSSP as a Markov Decision Process (MDP) and learn a policy network based on DRL techniquesZhang et al. (2020).

Large language models (LLMs) are now being applied to a wider range of tasks beyond language processing, in areas like robotics and planning Huang et al. (2022). While there are currently no papers that directly address the scheduling of Job Shop Scheduling Problems (JSSP) using LLMs, some notable works explore the potential of LLMs in mathematical reasoning and programming Chen et al. (2023); Wei et al. (2022); Ahn et al. (2024); Yang et al. (2023). Optimization using LLMs has gained significant interest in recent years, with several works exploring their capabilities across various domains Yang et al. (2023). The ability of LLMs to understand and generate natural language has opened new possibilities for optimization tasks that were traditionally solved using derivative-based algorithms or heuristic methods Yang et al. (2023). Chen et al. (2023) evaluated LLMs' performance in mathematical problem-solving and introduced "Program of Thoughts" (PoT) prompting. Unlike Chain of Thoughts (CoT) Wei et al. (2022), which combines reasoning and computation, PoT generates reasoning as code statements and delegates computation to an interpreter. Ahn et al. (2024) surveys mathematical problems and datasets studied with LLMs, analyzing their

strengths and weaknesses. Frieder et al. (2024) examines LLMs' impact on mathematicians, exploring their role in research, education, problem-solving, and proof generation, offering a balanced view of their capabilities. Recent works Yang et al. (2023) explore LLMs as optimizers, using prompts to refine solutions iteratively. Case studies on linear regression and the traveling salesman problem show LLMs can produce high-quality solutions, sometimes matching heuristic algorithms in small-scale scenarios. Explorations into using LLMs for graph learning tasks have yielded notable approaches. Huang et al. (2022) noted that LLMs exhibit some initial graph reasoning capabilities, but their performance decreases with problem complexity, Huang et al. (2022) introduced prompting strategies to improve LLMs graph reasoning. Valmeekam et al. (2022) developed a benchmark for assessing the planning and reasoning abilities of LLMs. More recently, Chen et al. (2024b) examined the use of LLMs for graph node classification tasks. Chen et al. (2024a) presents LLMs as enhancers for GNNs and as direct predictors from graph structures. Zhao et al. (2024) proposed GRAPHTEXT, which translates graphs to natural language for training-free reasoning, often rivaling GNNs. While LLMs show promise in graph tasks, their use in scheduling is still largely unexplored.

3 PRELIMINARIES

We consider the classical JSSP, which is defined as follows. Given N_J jobs and N_M machines, each job J_i is comprised of an ordered sequence of operations $(O_{i1}, \ldots, O_{in_i})$. Each operation O_{ij} must be processed on a designated machine m_{ij} for a processing time p_{ij}. The scheduling variables S_{ij} denote the start time of operation O_{ij}.

The JSSP is governed by two principal constraints: (i) *Precedence constraints* require that each operation starts only after the completion of its predecessor within the same job, i.e., $S_{i,j+1} \geq S_{ij} + p_{ij}$; and (ii) *Resource constraints* ensure that no two operations assigned to the same machine overlap in time, i.e., for any pair O_{ij}, O_{kl} such that $m_{ij} = m_{kl}$,

$$[S_{ij}, S_{ij} + p_{ij}) \cap [S_{kl}, S_{kl} + p_{kl}) = \emptyset.$$

The objective is to minimize the makespan, defined as the maximum completion time across all operations:

$$C_{\max} = \max_{i,j}\{S_{ij} + p_{ij}\}.$$

4 DATASET CONSTRUCTION AND REPRESENTATION

The use of LLMs for combinatorial optimization requires translating traditional mathematical representations into natural language encodings that maintain the problem structure for language-based processing. We present a methodology for representing JSSP instances as structured natural language, mapping the conventional matrix-based form (see Listing 1) into explicit descriptions of all constraints and requirements. This mapping is defined as a deterministic, bijective transformation $\mathcal{T} : P \to \mathcal{L}$, where P denotes the space of standard JSSP instances and $\mathcal{L}$ the corresponding space of natural language descriptions.

Listing 1: Example: Natural language encoding of a JSSP instance with $N_J = 3$ and $N_M = 3$.

```
Optimize the schedule for 3 jobs (J0, J1, J2) across 3 machines (
    M0, M1, M2) to minimize the makespan. Each machine can process
     only one job at a time and jobs are non-preemptive.

J0: M0:105, M1:29, M2:213
J1: M0:193, M1:18, M2:213
J2: M0:78,  M1:74, M2:221
```

For a JSSP instance with N_J jobs and N_M machines, we construct a natural language encoding that systematically specifies:

1. The problem dimensions ($N_J \times N_M$)

2. The operational constraints (non-preemption, machine exclusivity)
3. The sequential processing requirements for each job
4. The corresponding processing durations

This encoding establishes a bijective mapping between mathematical and linguistic representations, preserving all information required for solution generation while rendering the problem interpretable to language models. As illustrated in Listing 1, the natural language encoding presents the problem parameters in a clear, structured format that delineates job requirements across machines.

4.1 Corpus Generation for Model Training

To facilitate effective learning of the mapping between problem instances and their solutions, we constructed a comprehensive corpus of JSSP instances and their corresponding optimal or near-optimal solutions. The corpus encompasses approximately 130,000 random JSSP instances spanning dimensions from 2×2 to 20×20, supplemented by $\sim$1,000 larger and asymmetric instances to enhance generalization capabilities across problem scales. Operation durations were sampled from a uniform distribution ranging from 5 to 500 time units, ensuring comprehensive coverage of the solution space and robustness to varying temporal constraints. The testing dataset is out of distribution dataset from the trainng dataset. We conduct evaluations on the TAI Taillard (1993) and DMU Demirkol et al. (1998) benchmark sets, which are entirely held out from the training phase.

For solution generation, each instance was processed using Google's OR-Tools optimization framework with parameters configured to balance computational efficiency and solution quality. The solver was allocated a 300-second time limit with 42 parallel workers utilizing the `AUTOMATIC_SEARCH` strategy, providing near-optimal solutions even for larger problem instances. For problems exceeding 10×10 dimensions, we acknowledge potential suboptimality due to computational constraints while maintaining solution feasibility.

The solution encoding adopts a structured natural language format, specifically designed to guide the autoregressive nature of the LLMs. Each entry in the solution sequence (see Listing 2) details a job-machine assignment along with its explicit start time, duration, and resulting completion time. Notably, the use of summation notation (e.g., "J2-M0: 0+78 $\rightarrow$ 78") forces the model to compute the current makespan incrementally, based on the start time and duration, while taking into account the completion times of all previously scheduled operations.

This stepwise representation leverages the LLM's autoregressive generation process, requiring it to "think" about the current scheduling decision by explicitly calculating and verifying the timing constraints before proceeding to the next operation. The format ensures that each scheduling step is conditioned on the already constructed partial schedule, thus embedding temporal dependencies and constraint satisfaction directly into the generation process.

Listing 2: Consistent schedule with correct job precedence and operation durations for a JSSP instance with $N_J = 3$ jobs and $N_M = 3$ machines. The values after "-¿" denote operation completion times. The makespan is the maximum of these, i.e., 488.

```
Solution:
J2-M0: 0+78 -> 78,
J1-M2: 0+193 -> 193,
J0-M0: 78+105 -> 183,
J0-M1: 183+29 -> 212,
J2-M2: 193+74 -> 267,
J1-M1: 212+18 -> 230,
J1-M0: 230+213 -> 443,
J2-M1: 267+221 -> 488,
J0-M2: 267+213 -> 480
```

Empirical results (see Table 3) show that this explicit, computation-driven format significantly enhances the feasibility of generated solutions compared to formats omitting intermediate calculations. By prompting the model to perform and record intermediate makespan computations, the approach

enables real-time constraint checking and more effective optimization, reducing the frequency of infeasible schedules.

5 METHODOLOGY

We propose a novel method for solving JSSP by fine-tuning large language models with natural language representations of scheduling problems and solutions. Our framework, based on Meta-Llama-3.1-8B-Instruct (4-bit quantized), reframes JSSP as a sequence generation task and operates in two phases: (1) fine-tuning the model on problem-solution pairs using rsLoRA, and (2) generating and selecting optimal schedules for new instances. By expressing both problems and solutions in natural language, our approach leverages pre-trained knowledge and learns scheduling-specific patterns efficiently.

5.1 TRAINING METHODOLOGY

We fine-tune the model using rsLoRA Hu et al. (2022), an approach that replaces the standard scaling factor $\frac{\alpha}{r}$ with a stabilized $\sqrt{\frac{\alpha}{r}}$, which enables the use of higher ranks without causing gradient collapse and ensures more stable training dynamics. The model is initialized with pre-trained weights θ_0, which remain frozen throughout the process, while only the low-rank adaptation matrices U and V are updated to minimize the negative log-likelihood loss on tokenized problem-solution pairs. Training is conducted over 2 epoch with a learning rate of 2×10^{-4}, LoRA rank $r = 64$, scaling factor $\alpha = 64$, and a batch size of 16, utilizing a single Nvidia RTX A6000 GPU (48GB memory), with the training process taking approximately 70 hours and utilizing around 30GB of GPU RAM, highlighting the resource requirements for this procedure. Alpaca training template format is used during the training as described in Appendix Listings 1.

Algorithm 1: LLM Fine-Tuning for JSSP with rsLoRA

Input: Problem instance $\mathcal{L}_p$ in natural language, Fine-tuned LLM with parameters $\theta = \theta_0 + \gamma_r U V^\top$, Number of candidate solutions S

Initialize low-rank matrices $U, V \in \mathbb{R}^{d \times r}$
Define rank-stabilized factor $\gamma_r = \frac{\alpha}{\sqrt{r}}$
for *epoch* $= 1$ **to** E **do**
  for *each batch* $\{(\mathcal{L}_p^{(i)}, s^{(i)})\}_{i=1}^{B} \subset \mathcal{D}$ **do**
    Tokenize each problem $\mathcal{L}_p^{(i)}$ and solution $s^{(i)}$
    Construct inputs with problems $\mathcal{L}_p^{(i)}$ as context
    Set targets as tokenized solutions $s^{(i)} = \{w_1^{(i)}, \ldots, w_{T_i}^{(i)}\}$
    Compute effective parameters: $\theta = \theta_0 + \gamma_r U V^\top$
    Forward pass: Compute probabilities $p(w_t | w_{<t}, \mathcal{L}_p^{(i)}; \theta)$
    Compute NLL loss: $\mathcal{L} = -\sum_{i=1}^{B} \sum_{t=1}^{T_i} \log p(w_t^{(i)} | w_{<t}^{(i)}, \mathcal{L}_p^{(i)}; \theta)$
    Compute gradients $\nabla_U \mathcal{L}$ and $\nabla_V \mathcal{L}$
    Update $U \leftarrow U - \eta \nabla_U \mathcal{L}$
    Update $V \leftarrow V - \eta \nabla_V \mathcal{L}$
  Evaluate model performance on validation set

Result: Fine-tuned model parameters $\theta = \theta_0 + \gamma_r U V^\top$

5.2 INFERENCE AND SOLUTION SELECTION

At inference time (Algorithm 2), we employ a generate-and-select strategy. For each problem instance, the model produces multiple candidate solutions through temperature-controlled sampling. Each candidate undergoes rigorous feasibility checking to ensure satisfaction of all JSSP constraints, including job precedence, machine exclusivity, and non-preemption requirements. From the set of feasible solutions, we select the one with the minimum makespan.

The feasibility check validates that: (1) each job's operations are scheduled in the correct sequence, (2) no machine processes multiple jobs simultaneously, (3) each operation has the correct processing time, and (4) all operations are scheduled exactly once. This comprehensive validation ensures that all solutions adhere to the fundamental constraints of the JSSP problem domain.

Algorithm 2: LLM-Based JSSP Solution Generation and Selection

Input: Problem instance $\mathcal{L}_p$ in natural language, Fine-tuned LLM with parameters $\theta = \theta_0 + \gamma_r U V^\top$,Number of candidate solutions S, temperature τ

Initialize empty set of feasible solutions $\mathcal{S}_p^f \leftarrow \emptyset$
for $i = 1$ **to** S **do**
  Generate candidate solution $s_i \sim \text{LLM}_\theta(\mathcal{L}_p, \tau)$
  Parse solution s_i to extract job-machine assignments and timings
  $\text{valid}_{\text{precedence}} \leftarrow$ Check job operation precedence constraints
  $\text{valid}_{\text{exclusivity}} \leftarrow$ Check machine exclusivity constraints
  $\text{valid}_{\text{timing}} \leftarrow$ Check correct processing times
  $\text{valid}_{\text{completeness}} \leftarrow$ Check all operations are scheduled once
  if $\textit{valid}_{\textit{precedence}} \wedge \textit{valid}_{\textit{exclusivity}} \wedge \textit{valid}_{\textit{timing}} \wedge \textit{valid}_{\textit{completeness}}$ **then**
    Compute makespan $M(s_i)$
    Add to feasible set: $\mathcal{S}_p^f \leftarrow \mathcal{S}_p^f \cup \{s_i\}$
if $\mathcal{S}_p^f \neq \emptyset$ **then**
  return $s^* = \arg\min_{s \in \mathcal{S}_p^f} M(s)$ `// Best solution by makespan`
else
  return "No feasible solution found"

6 Validation, Baseline Methods, and Empirical Analysis

We evaluated our end-to-end LLM-based job shop scheduler using the standard Taillard Taillard (1993) and DMU Demirkol et al. (1998) benchmarks, comparing it to both traditional heuristics and state-of-the-art learning-based methods. As the first application of LLMs for end-to-end JSSP solution generation, we benchmarked our model against L2D Zhang et al. (2020)—an early neural approach that outperforms classic priority dispatching rules (PDRs) such as SPT, MWKR, MOPNR, and FDD/MWKR. L2D leverages a graph neural network and PPO Schulman et al. (2017) for generalization. Additionally, we compared our method with RASCLB Iklassov et al. (2023), a state-of-the-art reinforcement learning approach designed for cross-instance generalization. Here, "B" denotes the "base" learning method in Iklassov et al. (2023), which combines an RL-based method with rLSTM and set2set modules. RASCLB is trained on larger instances (30x20) with a sample size of 20. Its reverse LSTM Hochreiter & Schmidhuber (1997) component receives static, multidimensional embeddings for all operations in a job J_i, propagating information backward from the last operation to the current one. For all experiments, inference was performed with a context length of 40,000 tokens (the maximum number of tokens the model can process in a single input sequence) using default sampling settings and $S = 20$ samples per instance, with default temperature parameter of 1. Both training and inference used `4-bit` quantization for memory efficiency, requiring about 30GB of GPU memory on an NVIDIA A6000. Our largest evaluated instance (23,000 tokens) fits comfortably within this window. For faster inference, we converted the model to the `llama.cpp` format Gerganov (2023), achieving 102.22 tokens/sec on an RTX A6000 (48GB), as reported by Dai et al. Dai (2024). Notably, inference time scales with token sequence length rather than problem complexity; processing our largest instance (22,224 tokens) within the 40,000-token window took about 217 seconds per sample, regardless of task type.

6.1 Performance Metrics and Comparative Results

Performance on each benchmark was evaluated using the *Percentage Gap* (PG), defined as:

$$\text{PG} \;=\; 100 \times \left(\frac{M_{\text{alg}}}{M_{\text{ub}}} \;-\; 1 \right),$$

where M_{alg} is the makespan produced by the algorithm, and M_{ub} is the best-known or optimal makespan. Lower PG values correspond to solutions closer to the best-known objective. Tables 1 and 2 present the gap percentages of the fine-tuned Llama model compared to L2D and other methods (SPT, MWKR, FDD/WKR, MOPNR) on Tai and DMU datasets. Benchmark solutions are available at[2] and[3]. Across instances ranging from 15 jobs and 15 machines to 50 jobs and 20 machines, the fine-tuned Llama 3.1 consistently outperforms all other methods. On larger instances (50 jobs, 20 machines, 1000 nodes), Llama achieves a 24.32% gap, surpassing L2D (26.40%). The average gap for fine-tuned Llama (21.69%) is significantly lower than SPT (60.57%), MWKR (55.29%), FDD/WKR (46.77%), MOPNR (42.99%), and L2D (29.54%). On the DMU benchmark with 50 jobs and 15 machines, fine-tuned Llama (22.14%) again outperforms all methods, including L2D (37.50%). SPT consistently has the highest gap (often above 60%), reflecting its inability to handle complex constraints. MWKR and FDD/WKR, while more sophisticated, remain outperformed by learning-based approaches due to their myopic decision processes. Notably, our method shows average improvements over the dedicated model RASCLB of 13.41% on the Taillard dataset (see Table 1) and comparableresult on the DMU dataset (see Table 2).

6.2 Ablation Study

To understand the impact of key components, we conducted an ablation study focusing on our **Starjob** data representation and the model size. As shown in Table 3, the our proposed summation format significantly increases the percentage of feasible solutions produced by the Llama 8B model compared to the standard matrix format, not involving summation operations, which is familiar to the LLM from pretraining Bordt et al. (2024). Despite this familiarity, Starjob yields markedly better feasibility. The Llama 1B model, in contrast, failed on this task, highlighting the necessity of larger models for high-quality scheduling.

Listing 3: An interactive session where the LLM explains the primary scheduling bottleneck.

Example: Interactive Query with the Starjob-LLM

User:
Looking at the 3x3 problem instance, what was the main bottleneck that made it hard to solve?

Starjob-LLM:
The primary bottleneck was **Machine 2 (M2)**. All three jobs required this machine for long processing times, creating a highly contended resource.
The sequential processing on M2 (J1-M2 → J2-M2 → J0-M2) formed the critical path, which ultimately determined the final makespan of 488. Any improvement to the schedule would need to resolve this contention on Machine 2.

7 Conclusion

This work served as a test to see whether LLMs could generate any feasible solutions for NP-hard combinatorial optimization, rather than to claim the best scheduler. We showed that, with the right data representation, LLMs can indeed act as effective schedulers. To this end, we introduced **Starjob**, a large supervised dataset for the Job Shop Scheduling Problem (JSSP) in structured natural language, and fine-tuned a Llama 8B model using resource-efficient methods. While our goal was not to surpass specialized schedulers, our LLM-based approach still outperformed traditional Priority Dispatching Rules and even some dedicated neural baselines on standard benchmarks. These findings highlight the potential of LLMs for combinatorial optimization, and suggest promising directions for more interpretable and interactive scheduling systems in the future.

[2] http://optimizizer.com/TA.php
[3] http://jobshop.jjvh.nl/

8 LIMITATIONS AND FUTURE WORK

While our model's performance is dependent on the Starjob dataset and constrained by the LLM's context window, it provides high-quality heuristic solutions. Future work will focus on developing hybrid solvers that combine our approach with traditional optimization methods for further refinement. We also plan to explicitly integrate Graph Neural Networks (GNNs) into the LLM latent space via cross-attention to better capture relational structure, and to investigate the impact of full fine-tuning and larger LLMs for additional performance gains.

Table 1: Comparison of different methods on the **TAI** dataset (sampling budget = 20). Lower values indicate schedules closer to the optimal solution, representing better performance. * indicates the best result according to the Percentage Gap. Classic JSSP heuristics (FDD/WKR, MOPNR, MWKR, SPT) are described in Appendix D. *L2D*, *RASCLB*, and *LLM-FT-Ours* are neural methods.

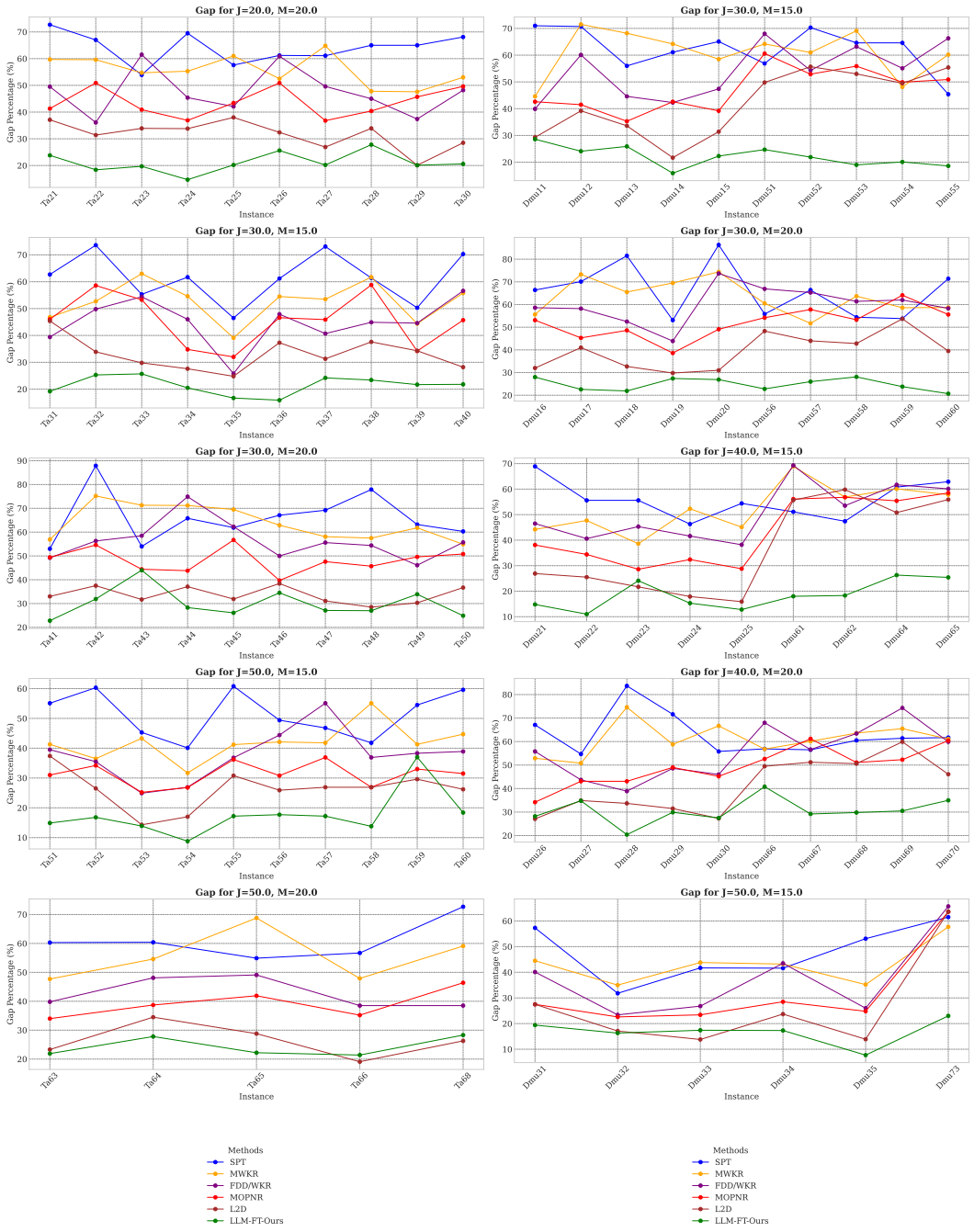
Method	15x15	20x15	20x20	30x15	30x20	50x15	50x20	Average
FDD/WKR	47.45	50.57	47.57	45.01	56.30	37.72	42.80	46.77
MOPNR	44.98	47.97	43.68	45.59	48.23	31.25	39.24	42.99
MWKR	56.74	60.65	55.60	52.61	63.93	41.90	55.62	55.29
SPT	54.64	65.24	64.11	61.61	66.03	51.37	61.00	60.57
L2D	25.95 ± 3.37	30.03 ± 3.90	31.60 ± 4.11	33.02 ± 4.29	33.62 ± 4.37	26.15 ± 3.40	26.40 ± 3.43	29.54 ± 3.84
RASCLB	20.59 ± 2.47	25.31 ± 3.04	25.47 ± 3.06	27.27 ± 3.27	30.40 ± 3.65	20.69 ± 2.48	26.40 ± 3.17	25.16 ± 3.02
LLM-FT-Ours	**19.34 ± 1.93***	**18.00 ± 1.80***	**21.11 ± 2.11***	**21.44 ± 2.14***	**30.05 ± 3.00***	**17.57 ± 1.76***	**24.32 ± 2.43***	**21.69 ± 2.17***

Table 2: Comparison of different methods on the **DMU** dataset (sampling budget = 60). Lower values indicate schedules closer to the optimal solution, representing better performance. * indicates the best result according to the Percentage Gap. Classic JSSP heuristics (FDD/WKR, MOPNR, MWKR, SPT) are described in Appendix D. *L2D*, *RASCLB*, and *LLM-FT-Ours* are neural methods.

Method	20x15	20x20	30x15	30x20	40x15	40x20	50x15	Average
FDD/WKR	53.58	52.51	54.12	60.08	50.76	55.52	37.58	52.02
MOPNR	49.17	45.18	47.14	51.97	43.23	49.22	31.73	45.38
MWKR	62.14	58.16	60.96	63.15	52.40	61.09	43.23	57.30
SPT	64.12	64.55	62.57	65.92	55.89	62.99	47.83	60.55
L2D	38.95 ± 5.06	37.74 ± 4.91	41.86 ± 5.44	39.48 ± 5.13	36.68 ± 4.77	41.18 ± 5.35	26.60 ± 3.46	37.50 ± 4.88
RASCLB	19.66 ± 2.36	**15.98 ± 1.92**	**16.35 ± 1.96**	23.00 ± 2.76	17.89 ± 2.15	26.42 ± 3.17	21.84 ± 2.62	20.16 ± 2.42
LLM-FT-Ours	**19.20 ± 1.92***	20.16 ± 2.02	22.11 ± 2.21	**21.82 ± 2.18***	**17.24 ± 1.72***	**23.61 ± 2.36***	**16.85 ± 1.69***	**20.14 ± 2.01***

Table 3: Ablation study comparing our **Starjob** representation against the standard **Matrix** format, and the Llama 8B model against Llama 1B model. Average Feasibility (%) indicates the percentage of valid solutions generated, where **higher values are better**. Our proposed format dramatically increases feasibility. The complete failure of the Llama 1B model highlights the task's complexity, while the Llama 8B model using Starjob consistently produces high-quality schedules. Feasibility and time for Llama 1B model are marked as N/A where not available.

Problem Size	**Without** Summation (%)		**With** Summation (%)		**Avg. Time (s) (8B)**
	1B	8B	1B	8B	
5×5	N/A	∼8.0	N/A	∼9.5	6.1
8×8	N/A	∼8.0	N/A	∼10.5	7.2
10×10	N/A	∼1.0	N/A	∼12.1	2.6
12×12	N/A	∼4.0	N/A	∼39.6	14.3
15×15	N/A	∼1.0	N/A	∼14.4	22.5
20×20	N/A	∼1.5	N/A	∼17.4	15.1
30×30	N/A	N/A	N/A	∼30.5	18.3
50×20	N/A	∼1.0	N/A	∼14.6	22.0



(a) TAI benchmark (b) DMU benchmark

Figure 1: Comparison of different methods on TAI Taillard (1993) and DMU Demirkol et al. (1998).

REFERENCES

Janice Ahn, Rishu Verma, Renze Lou, Di Liu, Rui Zhang, and Wenpeng Yin. Large language models for mathematical reasoning: Progresses and challenges. In Neele Falk, Sara Papi, and Mike Zhang (eds.), *Proceedings of the 18th Conference of the European Chapter of the Association for Computational Linguistics: Student Research Workshop*, pp. 225–237, St. Julian's, Malta, March 2024. Association for Computational Linguistics. URL https://aclanthology.org/2024.eacl-srw.17.

Meta AI. Llama 3 model card, 2024a. URL https://github.com/meta-llama/llama3/blob/main/MODEL_CARD.md. Accessed: 2024-08-10.

Unsloth AI. Unsloth: Accelerated fine-tuning for large language models, 2024b. URL https://github.com/unslothai/unsloth. Accessed: 2024-11-19.

J. Christopher Beck, T. K. Feng, and Jean-Paul Watson. Combining constraint programming and local search for job-shop scheduling. *INFORMS Journal on Computing*, 23(1):1–14, 2010.

Giovanni Bonetta, Davide Zago, Rossella Cancelliere, and Andrea Grosso. Job shop scheduling via deep reinforcement learning: a sequence to sequence approach. *Not Specified*, Aug 2023.

Sebastian Bordt, Harsha Nori, Vanessa Rodrigues, Besmira Nushi, and Rich Caruana. Elephants never forget: Memorization and learning of tabular data in large language models, 2024. URL https://arxiv.org/abs/2404.06209.

Ceren Cebi, Enes Atac, and Ozgur Koray Sahingoz. Job shop scheduling problem and solution algorithms: A review. In *2020 11th International Conference on Computing, Communication and Networking Technologies (ICCCNT)*, pp. 1–7, 2020. doi: 10.1109/ICCCNT49239.2020.9225581.

Wenhu Chen, Xueguang Ma, Xinyi Wang, and William W. Cohen. Program of thoughts prompting: Disentangling computation from reasoning for numerical reasoning tasks. *Transactions on Machine Learning Research*, 2023. ISSN 2835-8856. URL https://openreview.net/forum?id=YfZ4ZPt8zd.

Zhikai Chen, Haitao Mao, Hang Li, Wei Jin, Hongzhi Wen, Xiaochi Wei, Shuaiqiang Wang, Dawei Yin, Wenqi Fan, Hui Liu, and Jiliang Tang. Exploring the potential of large language models (llms) in learning on graphs, 2024a.

Zhikai Chen, Haitao Mao, Hang Li, Wei Jin, Hongzhi Wen, Xiaochi Wei, Shuaiqiang Wang, Dawei Yin, Wenqi Fan, Hui Liu, and Jiliang Tang. Exploring the potential of large language models (llms) in learning on graphs, 2024b. URL https://arxiv.org/abs/2307.03393.

Andrea Corsini, Angelo Porrello, Simone Calderara, and Mauro Dell'Amico. Self-labeling the job shop scheduling problem. In *Self-Labeling the Job Shop Scheduling Problem*. Arxiv, 2024.

Xiongjie Dai. Gpu-benchmarks-on-llm-inference. https://github.com/XiongjieDai/GPU-Benchmarks-on-LLM-Inference, 2024. Accessed: 2024-11-26.

Ebru Demirkol, Sanjay Mehta, and Reha Uzsoy. Benchmarks for shop scheduling problems. *European Journal of Operational Research*, 109(1):137–141, 1998.

Simon Frieder, Julius Berner, Philipp Petersen, and Thomas Lukasiewicz. Large language models for mathematicians, 2024.

Michael R Garey, David S Johnson, and Ravi Sethi. The complexity of flowshop and jobshop scheduling. *Mathematics of Operations Research*, 1(2):117–129, 1976.

Georgi Gerganov. llama.cpp: Llm inference in c/c++. https://github.com/ggerganov/llama.cpp, 2023. Accessed: 2024-11-26.

Sepp Hochreiter and Jürgen Schmidhuber. Long short-term memory. *Neural Computation*, 9(8): 1735–1780, 1997. doi: 10.1162/neco.1997.9.8.1735.

Edward J Hu, yelong shen, Phillip Wallis, Zeyuan Allen-Zhu, Yuanzhi Li, Shean Wang, Lu Wang, and Weizhu Chen. LoRA: Low-rank adaptation of large language models. In *International Conference on Learning Representations*, 2022. URL https://openreview.net/forum?id=nZeVKeeFYf9.

Wenlong Huang, Pieter Abbeel, Deepak Pathak, and Igor Mordatch. Language models as zero-shot planners: Extracting actionable knowledge for embodied agents. In *Proceedings of the International Conference on Machine Learning*. PMLR, 2022. *equal advising.

Zangir Iklassov, Dmitrii Medvedev, Ruben Solozabal Ochoa de Retana, and Martin Takáč. On the study of curriculum learning for inferring dispatching policies on the job shop scheduling. In *Proceedings of the Thirty-Second International Joint Conference on Artificial Intelligence (IJCAI)*, pp. 5350–5358, 2023. doi: 10.24963/ijcai.2023/594.

Damjan Kalajdzievski. A rank stabilization scaling factor for fine-tuning with lora, 2023. URL https://arxiv.org/abs/2312.03732.

Eugeniusz Nowicki and Czeslaw Smutnicki. An advanced tabu search algorithm for the job shop problem. *Journal of Scheduling*, 8(2):145–159, 2005. doi: 10.1007/s10951-005-6364-5.

John Schulman, Filip Wolski, Prafulla Dhariwal, Alec Radford, and Oleg Klimov. Proximal policy optimization algorithms. arXiv preprint arXiv:1707.06347, 2017.

Eric Taillard. Benchmarks for basic scheduling problems. *European Journal of Operational Research*, 64(2):278–285, 1993.

Karthik Valmeekam, Alberto Olmo, Sarath Sreedharan, and Subbarao Kambhampati. Large language models still can't plan: A benchmark for llms on planning and reasoning about change. In *NeurIPS 2022 Foundation Models for Decision Making Workshop*, 2022. URL https://openreview.net/forum?id=wUU-7XTL5XO.

Jason Wei, Xuezhi Wang, Dale Schuurmans, Maarten Bosma, Brian Ichter, Fei Xia, Ed H. Chi, Quoc V. Le, and Denny Zhou. Chain-of-thought prompting elicits reasoning in large language models. *Google Research, Brain Team*, 2022.

Chengrun Yang, Xuezhi Wang, Yifeng Lu, Hanxiao Liu, Quoc V Le, Denny Zhou, and Xinyun Chen. Large language models as optimizers. *arXiv preprint arXiv:2309.03409*, 2023.

Mohamed Habib Zahmani, Baghdad Atmani, Abdelghani Bekrar, and Nassima Aissani. Multiple priority dispatching rules for the job shop scheduling problem. In *3rd International Conference on Control, Engineering Information Technology (CEIT'2015)*, Tlemcen, Algeria, 2015. doi: 10.1109/CEIT.2015.7232991.

Cong Zhang, Wen Song, Zhiguang Cao, Jie Zhang, Puay Siew Tan, and Chi Xu. Learning to dispatch for job shop scheduling via deep reinforcement learning. In *34th Conference on Neural Information Processing Systems (NeurIPS)*, 2020.

Jianan Zhao, Le Zhuo, Yikang Shen, Meng Qu, Kai Liu, Michael M. Bronstein, Zhaocheng Zhu, and Jian Tang. Graphtext: Graph learning in text space, 2024. URL https://openreview.net/forum?id=dbcWzalk6G.

A Appendix

B Training Details

Model Overview

The model being fine-tuned is LLaMA 3.1, an 8 billion parameter model from MetaAI (2024a), using a 4-bit quantized version to reduce memory usage. Finetuning was conducted using Stabilized Low-Rank Adaptation (RsLoRA) with rank $r = 64$ to introduce learnable parameters specifically in

targeted layers. Kalajdzievski (2023) Compared to LoraHu et al. (2022) RsLoRa improves the stability of training by modifying the rank during adaptationKalajdzievski (2023). The target modules include:

$$\text{target\_modules} = \{\texttt{q\_proj}, \texttt{k\_proj}, \texttt{v\_proj}, \texttt{o\_proj}, \texttt{gate\_proj}, \texttt{up\_proj}, \texttt{down\_proj}\} \quad (1)$$

The LoRA-specific parameters are configured as follows:

- Rank (r): 64
- LoRA Alpha (α): 64
- LoRA Dropout: 0
- Bias: none

This resulted in number of trainable parameters $= 167,772,160$ or 2 % of the entire Llama 8B model's parameters.

Quantization and Memory Efficiency

The model is loaded in 4-bit precision to reduce memory consumption. Gradient checkpointing is enabled using the `unsloth` AI (2024b) method, to fit longer sequences by saving memory. This reduces the VRAM usage by approximately 30%, enabling larger batch sizes.

Training Parameters

The fine-tuning process is controlled by the following parameters:

- **Batch size per device**: 4
- **Gradient accumulation steps**: 4
- **Max sequence length**: 40,000 tokens
- **Number of epochs**: 2
- **Warmup steps**: 5
- **Learning rate**: 2×10^{-4}
- **Optimizer**: AdamW with 8-bit precision
- **Weight decay**: 0.01
- **Learning rate scheduler**: Linear decay
- **FP16 precision**:True

Listing 1: Prompt format used during training.

```
alpaca_prompt = """Below is an instruction that describes a task,
paired with an input that provides further context.
Write a response that appropriately completes the request.

    ### Instruction:
    {}

    ### Input:
    {}

    ### Response:
    {}"""
```

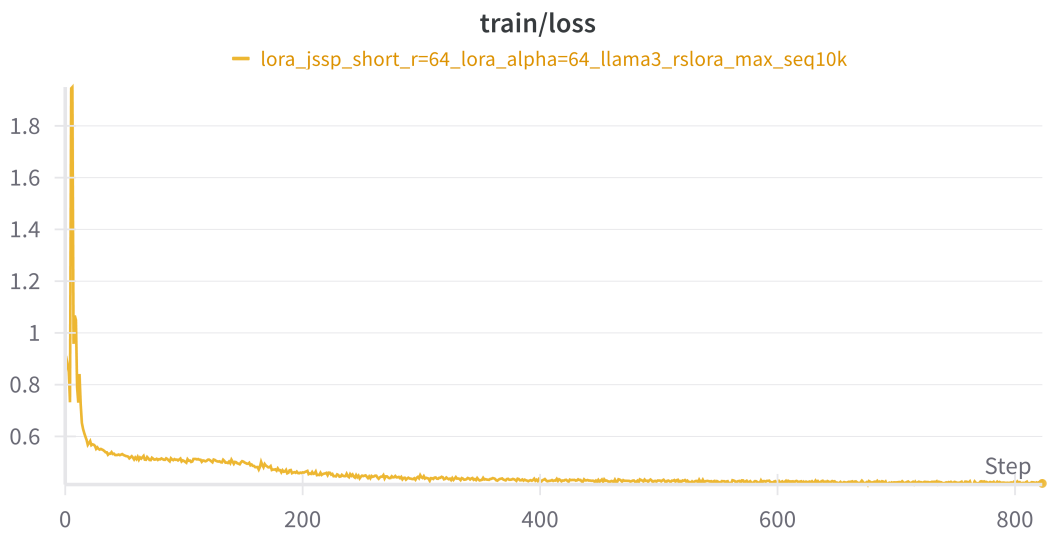


Figure 2: Train loss of Llama 8B 4bt model on Starjob dataset

Figure 3: Eval Loss of Llama 8B 4bt model on Starjob dataset

Data and Dataset Splitting

The dataset used for training is a local version of the proposed Starjob dataset, and it is split into 98% training and 2% evaluation:

$$\text{train} : \text{eval} = 98\% : 2\%$$

The prompts are formatted using a predefined Alpaca-style template, which ensures the model is trained on instruction-following tasks.

Evaluation and Saving Strategy

The best model was loaded at the end of training based on the evaluation loss:

$$\text{Metric for Best Model} = \text{Evaluation Loss}$$

Total number of saved models is limited to 50 to prevent excessive memory usage.

GPU Utilization

The training process takes place on Nvidia A6000 GPU with 48GB of memory. Training took around 70 hours and required 30GB of GPU RAM.

C General Statistics about Dataset

The dataset comprises 130,000 randomly generated JSSP instances with solutions in natural language, provided in `.json` format with the following columns:

- `num_jobs` (int64): 12 unique values.
- `num_machines` (int64): 12 unique values.
- `instruction` (object): 130,000 unique values. Initial problem description detailing jobs and machines.
- `input` (object): 130,000 unique values. Problem description formatted for LLM.
- `output` (object): 130,000 unique values. Solution in LLM format.
- `matrix` (object): 130,000 unique values. OR-Tool makespan and solution in matrix format.

The `output` column serves as the target or label column, providing the solution to the JSSP problem in natural language and the associated makespan.

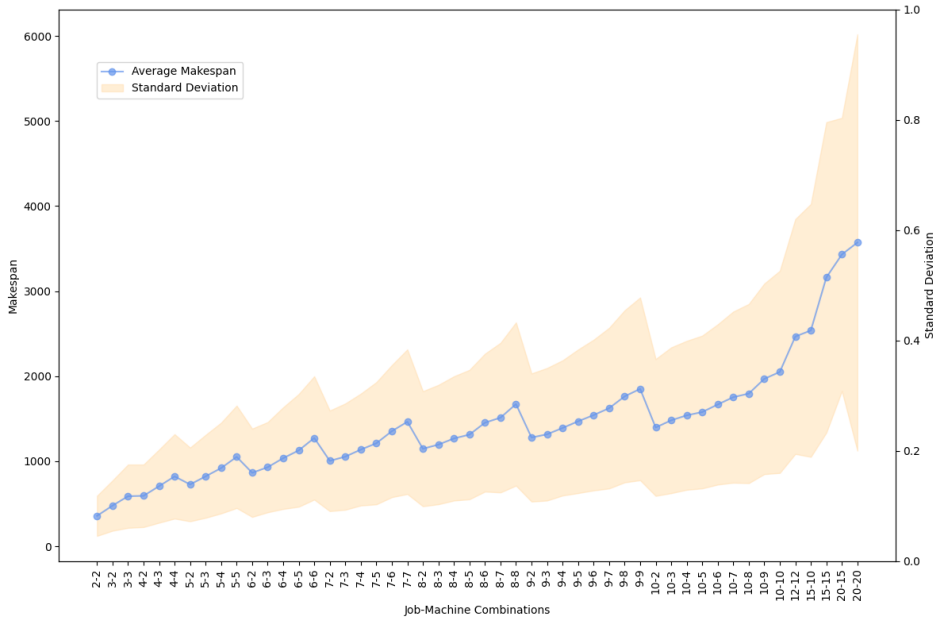


Figure 4: Makespan metrics across different job-machine combinations. The x-axis represents the combinations of jobs and machines (e.g., a 3-2 instance refers to 3 jobs and 2 machines), the right y-axis shows the standard deviation, while the left y-axis shows the makespan values.

Table 4: Comparison of PDRs against L2D against Finetuned Llama on Starjob dataset and the average Gaps on Tai Benchmark Dataset. The lower the value, the closer the schedule is to the optimal solution, thus representing better performance. BFL* indicates best from literature.

J	M	Instance	SPT	MWKR	FDD/WKR	MOPNR	L2D	BFL*	Llama-FT-Ours
15	15	Ta01	1872 (52.1%)	1786 (45.1%)	1841 (49.6%)	1864 (51.4%)	1443 (17.2%)	1231.0*	1453.0 (18.0%)
15	15	Ta02	1709 (37.4%)	1944 (56.3%)	1895 (52.3%)	1680 (35.0%)	1544 (24.1%)	1244.0*	1440.0 (15.8%)
15	15	Ta03	2009 (64.9%)	1947 (59.9%)	1914 (57.1%)	1558 (27.9%)	1440 (18.2%)	1218.0*	1521.0 (24.9%)
15	15	Ta04	1825 (53.3%)	1694 (44.2%)	1653 (40.7%)	1755 (49.4%)	1637 (39.3%)	1175.0*	1387.0 (18.0%)
15	15	Ta05	2044 (67.0%)	1892 (54.6%)	1787 (46.0%)	1605 (31.1%)	1619 (32.3%)	1224.0*	1461.0 (19.4%)
15	15	Ta06	1771 (43.1%)	1976 (59.6%)	1748 (41.2%)	1815 (46.6%)	1601 (29.3%)	1238.0*	1499.0 (21.1%)
15	15	Ta07	2016 (64.3%)	1961 (59.8%)	1660 (35.3%)	1884 (53.5%)	1568 (27.8%)	1227.0*	1473.0 (20.0%)
15	15	Ta08	1654 (35.9%)	1803 (48.2%)	1839 (51.1%)	1839 (51.1%)	1468 (20.6%)	1217.0*	1475.0 (21.2%)
15	15	Ta09	1962 (54.0%)	2215 (73.9%)	1848 (45.1%)	2002 (57.1%)	1627 (27.7%)	1274.0*	1534.0 (20.4%)
15	15	Ta10	2164 (74.4%)	2057 (65.8%)	1937 (56.1%)	1821 (46.7%)	1527 (23.0%)	1241.0*	1465.0 (18.0%)
20	15	Ta11	2212 (63.0%)	2117 (56.0%)	2101 (54.8%)	2030 (49.6%)	1794 (32.2%)	1357.0*	1691.0 (24.6%)
20	15	Ta12	2414 (76.6%)	2213 (61.9%)	2034 (48.8%)	2117 (54.9%)	1805 (32.0%)	1367.0*	1677.0 (22.7%)
20	15	Ta13	2346 (74.7%)	2026 (50.9%)	2141 (59.4%)	1979 (47.4%)	1932 (43.9%)	1343.0*	1749.0 (30.2%)
20	15	Ta14	2190 (56.8%)	2164 (60.9%)	1841 (36.9%)	2036 (51.4%)	1664 (23.7%)	1345.0*	1660.0 (23.4%)
20	15	Ta15	2163 (61.5%)	2180 (62.6%)	2187 (63.3%)	1939 (44.8%)	1730 (29.2%)	1339.0*	1770.0 (32.2%)
20	15	Ta16	2232 (64.1%)	2528 (85.9%)	1926 (41.6%)	1980 (45.6%)	1710 (25.7%)	1360.0*	1731.0 (27.3%)
20	15	Ta17	2185 (49.5%)	2015 (37.8%)	2093 (43.2%)	2211 (51.2%)	1897 (29.8%)	1462.0*	1846.0 (26.3%)
20	15	Ta18	2267 (62.4%)	2275 (63.0%)	2064 (47.9%)	1981 (44.9%)	1794 (28.5%)	1396.0*	1706.0 (22.2%)
20	15	Ta19	2238 (68.0%)	2201 (65.2%)	1958 (47.0%)	1899 (42.6%)	1682 (26.3%)	1332.0*	1685.0 (26.5%)
20	15	Ta20	2370 (75.8%)	2188 (62.3%)	2195 (62.8%)	1986 (47.3%)	1739 (29.0%)	1348.0*	1802.0 (33.7%)
20	20	Ta21	2836 (72.7%)	2622 (59.7%)	2455 (49.5%)	2320 (41.3%)	2252 (37.1%)	1642.0*	2077.0 (26.5%)
20	20	Ta22	2672 (67.0%)	2554 (59.6%)	2177 (36.1%)	2415 (50.9%)	2102 (31.4%)	1600.0*	2443.0 (52.7%)
20	20	Ta23	2397 (53.9%)	2408 (54.7%)	2514 (61.5%)	2194 (40.9%)	2085 (33.9%)	1557.0*	2086.0 (34.0%)
20	20	Ta24	2787 (69.5%)	2553 (55.3%)	2391 (45.4%)	2250 (36.9%)	2200 (33.8%)	1644.0*	2135.0 (29.9%)
20	20	Ta25	2513 (57.6%)	2582 (61.0%)	2267 (42.1%)	2146 (43.4%)	2201 (38.0%)	1595.0*	2304 (44.4%)
20	20	Ta26	2649 (61.2%)	2506 (52.5%)	2484 (60.9%)	2284 (50.9%)	2176 (32.4%)	1643.0*	2195.0 (33.6%)
20	20	Ta27	2707 (61.1%)	2768 (64.8%)	2514 (49.6%)	2298 (36.8%)	2132 (26.9%)	1680.0*	2172.0 (29.3%)
20	20	Ta28	2654 (65.0%)	2370 (47.8%)	2330 (45.0%)	2259 (40.4%)	2146 (33.9%)	1603.0*	2088.0 (30.3%)
20	20	Ta29	2681 (65.0%)	2399 (47.6%)	2322 (37.4%)	2367 (45.7%)	1952 (20.1%)	1625.0*	2209 (35.9%)
20	20	Ta30	2662 (68.1%)	2424 (53.0%)	2348 (48.2%)	2370 (49.6%)	2035 (28.5%)	1584.0*	2038.0 (28.7%)

Table 5: Comparison of PDRs against L2D against Finetuned Llama on Starjob dataset and the average Gaps on DMU Benchmark Dataset. The lower the value, the closer the schedule is to the optimal solution, thus representing better performance. BFL* indicates best from literature.

J	M	Instance	SPT	MWKR	FDD/WKR	MOPNR	L2D	BFL*	Llama-FT-Ours
20	15	Dmu01	4516 (76.2%)	3988 (55.6%)	3535 (37.9%)	3882 (51.5%)	3323 (29.7%)	2563.0*	3064 (19.5%)
20	15	Dmu02	4593 (69.7%)	4555 (68.3%)	3847 (42.2%)	3884 (43.5%)	3630 (34.1%)	2706.0*	3233 (19.5%)
20	15	Dmu03	4438 (62.5%)	4117 (50.8%)	4063 (48.8%)	3979 (45.7%)	3660 (34.0%)	2731.0*	3296 (20.7%)
20	15	Dmu04	4533 (69.8%)	3995 (49.7%)	4160 (55.9%)	4079 (52.8%)	3816 (43.0%)	2669.0*	3299 (23.6%)
20	15	Dmu05	4420 (60.8%)	4977 (81.0%)	4238 (54.2%)	4116 (49.7%)	3897 (41.8%)	2749.0*	3458 (25.8%)
20	15	Dmu41	5283 (62.7%)	5377 (65.5%)	5187 (59.7%)	5070 (56.1%)	4316 (32.9%)	3248.0*	4137 (27.4%)
20	15	Dmu42	5354 (57.9%)	6076 (79.2%)	5583 (64.7%)	4976 (46.8%)	4858 (43.3%)	3390.0*	4169 (23.0%)
20	15	Dmu43	5328 (54.8%)	4938 (43.5%)	5086 (47.8%)	5012 (45.7%)	4887 (42.0%)	3441.0*	4634 (34.7%)
20	15	Dmu44	5745 (64.7%)	5630 (61.4%)	5550 (59.1%)	5213 (49.5%)	5151 (47.7%)	3488.0*	4429 (27.0%)
20	15	Dmu45	5305 (62.1%)	5446 (66.4%)	5414 (65.5%)	4921 (50.4%)	4615 (41.0%)	3272.0*	4423 (35.2%)
20	20	Dmu06	6230 (92.0%)	5556 (71.3%)	5258 (62.1%)	4747 (46.3%)	4358 (34.3%)	3244.0*	4173 (28.6%)
20	20	Dmu07	5619 (84.5%)	4636 (52.2%)	4789 (57.2%)	4367 (43.4%)	3671 (20.5%)	3046.0*	3821 (25.4%)
20	20	Dmu08	5239 (64.3%)	5078 (59.3%)	4817 (51.1%)	4480 (40.5%)	4048 (27.0%)	3188.0*	3982 (24.9%)
20	20	Dmu09	4874 (57.6%)	4519 (46.2%)	4675 (51.2%)	4519 (46.2%)	4482 (45.0%)	3092.0*	4376 (41.5%)
20	20	Dmu10	4808 (61.1%)	4963 (66.3%)	4149 (39.0%)	4133 (38.5%)	4021 (34.8%)	2984.0*	3853 (29.1%)
20	20	Dmu46	6403 (58.7%)	6168 (52.9%)	5778 (43.2%)	6136 (52.1%)	5876 (45.6%)	4035.0*	5447 (35.0%)
20	20	Dmu47	6015 (52.7%)	6130 (55.6%)	6058 (53.8%)	5908 (50.0%)	5771 (46.5%)	3939.0*	4899 (24.4%)
20	20	Dmu48	5345 (42.0%)	5701 (51.5%)	5887 (56.4%)	5384 (43.1%)	5034 (33.8%)	3763.0*	4854 (29.0%)
20	20	Dmu49	6072 (63.7%)	6089 (64.1%)	5807 (56.5%)	5469 (47.4%)	5470 (47.4%)	3710.0*	4674 (26.0%)
20	20	Dmu50	6300 (68.9%)	6050 (62.2%)	5764 (54.6%)	5380 (44.3%)	5314 (42.5%)	3729.0*	4515 (21.1%)

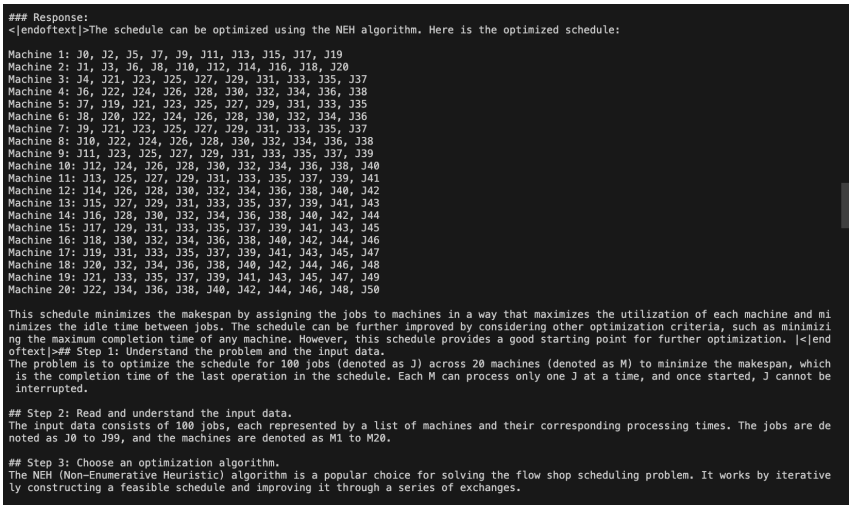


Figure 5: Zero Shot inference on LLama 8B 4bt

D Details of the Baselines

In this section, we show how the baseline PDRs compute the priority index for the operations. We begin by introducing the notations used in these rules, summarized as follows:

$$
\begin{aligned}
&Z_{ij}: \text{ the priority index of operation } O_{ij},\\
&n_i: \text{ the number of operations for job } J_i,\\
&Re_i: \text{ the release time of job } J_i\big(\text{here we assume } Re_i = 0 \text{ for all } J_i,\\
&\qquad \text{i.e. all jobs are available in the beginning, but in general}\\
&\qquad \text{the jobs could have different release times}\big),\\
&p_{ij}: \text{ the processing time of operation } O_{ij}.
\end{aligned}
$$

Based on the above notations, the decision principles for each baseline are given below:

- **Shortest Processing Time (SPT):**

$$\min Z_{ij} \;=\; p_{ij}.$$

- **Most Work Remaining (MWKR):**

$$\max Z_{ij} \;=\; \sum_{k=1}^{n_i} p_{ik}.$$

- **Minimum ratio of Flow Due Date to Most Work Remaining (FDD/MWKR):**

$$\min Z_{ij} \;=\; \frac{Re_i + \sum_{k=1}^{j} p_{ik}}{\sum_{k=1}^{n_i} p_{ik}}.$$

- **Most Operations Remaining (MOPNR):**

$$\max Z_{ij} \;=\; n_i - j + 1.$$

D.1 L2D: MDP Formulation and GNN-Based Policy

Markov Decision Process. Zhang et al. (2020) models a JSSP instance as an MDP, where each step t selects one eligible operation to schedule. The partial schedule at time t is represented by a disjunctive graph $G(t) = (\mathcal{O}, \mathcal{C} \cup \mathcal{D}_u(t), \mathcal{D}(t))$, whose arcs encode machine-ordering constraints. The state s_t specifies (i) which operations are already scheduled and (ii) estimated completion times for each operation. An action a_t picks the next operation to schedule, leading to an updated graph $G(t+1)$ and state s_{t+1}. The reward $R(a_t, s_t) = H(s_t) - H(s_{t+1})$ is the change in a lower bound of the makespan $H(\cdot)$; maximizing the sum of such rewards (with discount $\gamma = 1$) is equivalent to minimizing the final makespan. A policy $\pi(a_t \mid s_t)$ outputs a probability distribution over eligible actions.

Graph Neural Network (GNN). L2D uses a Graph Isomorphism Network (GIN) to learn graph-structured representations. Given a graph $\mathcal{G} = (V, E)$, GIN updates each node embedding $h_v^{(k)}$ iteratively:

$$h_v^{(k)} = \mathrm{MLP}_{\theta_k}\Big(\big(1 + \epsilon^{(k)}\big)h_v^{(k-1)} + \sum_{u \,\in\, \mathcal{N}(v)} h_u^{(k-1)}\Big). \tag{2}$$

After K iterations, a global embedding $h_{\mathcal{G}}$ is obtained by pooling node embeddings, e.g. average-pooling. For action selection, each operation embedding $h_{a_t}^{(K)}$ is concatenated with $h_{\mathcal{G}}$ and passed through an MLP to produce a score; a softmax over these scores yields the policy distribution π_θ. During training, a PPO-based Schulman et al. (2017) actor-critic approach is used, where the critic v_ϕ shares the GIN backbone but includes an additional MLP to estimate cumulative rewards.