Ecological Modeling of Sex Ratio Dynamics and Its Impacts on Ecosystem Stability in Sevengill Eels

***Abstract*—Sex ratio dynamics play a crucial role in the population ecology of sevengill eels, influencing their reproductive success, ecological interactions, and long-term ecosystem stability. This study develops a series of ecological models to explore how variations in sex ratio impact population dynamics, resource competition, and trophic interactions within the marine ecosystem. By integrating the Lotka-Volterra equations and an improved M. Gordon method, we establish a mathematical framework to quantify the effects of sex ratio adjustments on species competition and ecosystem equilibrium. Sensitivity and robustness analyses are conducted to assess the stability of the proposed models under varying ecological conditions. Our results indicate that sex ratio shifts significantly influence population growth rates and predator-prey relationships, yet the ecosystem exhibits self-regulatory mechanisms that restore equilibrium over time. Additionally, we find that resource availability serves as a key factor driving sex ratio adaptations, enhancing species survival in competitive environments. The findings provide valuable insights into adaptive population strategies and contribute to a better understanding of ecological resilience in marine ecosystem.**

***Keywords*—Lotka-Volterra equation, Improved M. Gordon method, Sensitivity analysis**

I. INTRODUCTION

A. Background and Motivation

Sex ratio dynamics play a fundamental role in shaping population structures and ecological interactions across various species. In marine ecosystems, the adaptive adjustment of sex ratios can influence reproductive success, competition for resources, and trophic relationships, thereby affecting ecosystem stability. Sevengill eels (Notorynchus cepedianus), as key predators in their ecological niche, exhibit sex ratio variations that are influenced by environmental conditions, food availability, and intra-species competition. Understanding the mechanisms behind these changes and their broader ecological implications is critical for predicting population trends and maintaining ecosystem balance.

Mathematical modeling provides a powerful approach to analyzing complex ecological systems by integrating population dynamics, species interactions, and environmental factors into quantitative frameworks. In this study, we employ ecological modeling techniques to investigate how sex ratio variations impact sevengill eel populations and their surrounding ecosystems. Specifically, we use differential equation models, including the Lotka-Volterra predator-prey framework and an improved M. Gordon method, to simulate the effects of sex ratio fluctuations on ecosystem dynamics.

B. Research Objectives

This study aims to address the following key problems:

- What ecological and biological factors influence the adaptive sex ratio variations in sevengill eels?
- How do sex ratio shifts affect population growth, reproductive success, and species competition?
- What role does sex ratio adaptation play in maintaining ecological equilibrium, and how does it influence predator-prey dynamics?
- How sensitive are the proposed models to variations in key parameters, and what are the implications for real-world ecosystem stability?

C. Literature Review

Previous studies have explored the evolutionary and ecological significance of sex ratio adjustments in various species. Mateus et al. [1] examined the genetic basis of sex differentiation in lampreys, while York et al. [2] highlighted the relationship between sex ratio imbalances and reproductive efficiency in marine predators. The Lotka-Volterra model has been widely used to describe predator-prey interactions, yet its applications to sex ratio-dependent dynamics remain relatively unexplored. Moreover, recent advances in ecological modeling, such as the improved M. Gordon method, provide novel approaches to assessing ecosystem stability under varying sex ratio conditions.

D. Contributions and Overview of the Study

This study integrates classical ecological models with new perspectives on sex ratio dynamics to offer a comprehensive understanding of sevengill eel population interactions. The key contributions of this research include:

- Developing a dynamic model incorporating sex ratio-dependent population changes.
- Applying sensitivity and robustness analyses to validate model stability.
- Providing ecological insights into adaptive reproductive strategies and ecosystem resilience.

II. ASSUMPTIONS AND JUSTIFICATIONS

Considering that practical problems always contain many complex factors, first of all, we need to make reasonable assumptions to simplify the model, and each hypothesis is closely followed by its corresponding explanation:

- **Assumption 1: The data provided is assumed to be true and valid.**

  Justification: This assumption is crucial for the integrity of the model's input. By presuming the data to be accurate and representative of real-world scenarios, we can rely on the model's outputs to reflect realistic predictions. This foundation is essential for any form of data analysis or modeling, as the conclusions drawn directly depend on the quality and veracity of the input data.

- **Assumption 1: The data provided is assumed to be true and valid.**

  Justification: This assumption is crucial for the integrity of the model's input. By presuming the data to be accurate and representative of real-world scenarios, we can rely on the model's outputs to reflect realistic predictions. This foundation is essential for any form of data analysis or modeling, as the conclusions drawn directly depend on the quality and veracity of the input data.

- **Assumption 2: Our data sources are reliable and accurate.**

  Justification: Since our data comes from international websites, we assume that the data is reliable. On this basis, objective and accurate results can be obtained by applying the data to the establishment of our model.

- **Assumption 3: It is assumed that the studied lampreys population is within a relatively confined space (e.g., a certain geographic area).**

  Justification: The effects of immigration and emigration are not taken into account, or that the phenomenon of migration can be quantitatively described.

Additional assumptions are made to simplify analysis for individual sections. These assumptions will be discussed at the appropriate locations.

III. RELATED WORK

A. Our Work

Our work can be showed by the Figure 1 below.

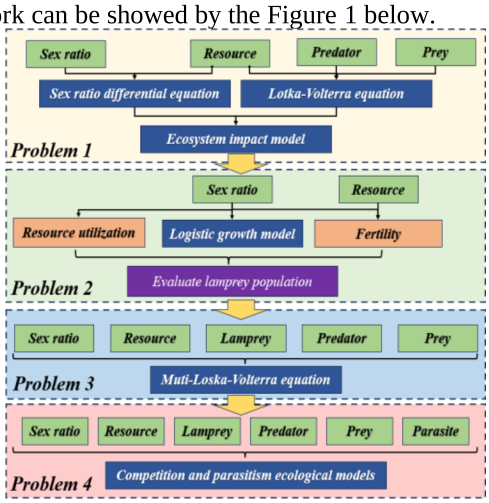

Figure 1. Our work

B. Notations

Some important mathematical notations used in this paper are listed in Table 1 below.

Table 1: Notations used in this paper

Symbol	Description
P_i	The number of species i
λ	Natural growth multiplier
k	Influence factor caused by adaptive sex ratio variation
t	Time
d_{ij}	Influence factor of species i on species j
w_{ij}	Competition factor of species i on species j
g_i	Natural growth rate of species i
d_M	Natural mortality rate of male individuals
d_F	Natural mortality rate of female individuals

Note: There are some variables that are not listed here and will be discussed in detail in each section.

IV. DYNAMIC MODELING OF LAMPREYS SEX RATIO

A. Sex ratio dynamic modeling

Sex ratio plays a crucial role in the dynamics of lampreys populations. Theoretically, the ideal female-to-male ratio should be balanced to ensure that reproductive opportunities are maximized and the population grows steadily. When there is a sex imbalance in a lampreys population, e.g., too many males or too few females, reproductive success may be severely compromised, resulting in the remaining individuals not being able to reproduce effectively and inhibiting population growth. In addition, an imbalanced sex ratio may exacerbate competitive pressures for resources, such as food, territory and mating partner selection, and potentially affect the physiological health of lampreys themselves.

From an ecological perspective, changes in the sex structure of lampreys, as part of the marine ecosystem, can act indirectly on other species through food chain relationships. For example, a reduction in lampreys populations as a result of sex ratio issues may reduce food pressure on predators and may also alter the extent to which they act as parasites on host fish.

Consider the dynamics of the sex ratio R_t over time t in relation to resource availability A_t.

$$-\frac{dR_t}{dt} = f(A_t, R_t) \tag{1}$$

where f is a function that describes the change in the sex ratio and can be set based on actual data or assumptions, e.g..

$$f(A_t, R_t) = \gamma(\mathrm{A_t} - \mathrm{A_{threshold?}}) \tag{2}$$

$A_{\text{threshold?}}$ is the resource availability threshold that affects gender transition, and γ is a moderating coefficient that indicates the strength of the effect of resource availability on the rate of change in the sex ratio.

The effect of resource availability on the rate of change in the sex ratio can be showed by the Figure 2 below.

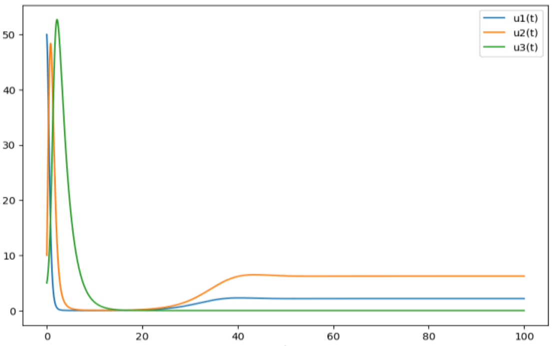

Figure 2. The effect of resource availability on the rate of change in the sex ratio

According to the model, the graph shows the numerical changes of a hypothetical population over time under different conditions of competition coefficient (bM) and natural mortality (dM). When the values of bM and dM are low, the population growth reaches a peak and then declines slowly, reflecting the fact that the population can be sustained for a period of time under the influence of certain resource competition and mortality. However, as bM and dM values increased, the population declined rapidly after reaching a peak, showing that higher competitive pressures and mortality rates accelerate population decline, which may indicate that populations are more susceptible to negative influences and have difficulty in maintaining long-term stability in resource-limited environments. This model further shows that the peak and subsequent rate of decline in population size is strongly correlated with the magnitude of the coefficient of competition and natural mortality.

B. *Ecological model of resource competition*

Key factors in the growth and development of seven gill lampreys need to be considered, such as how food availability directly affects larval growth rates and eventual sex determination. This process involves examining changes in sex ratios within sea lamprey populations, including how sex ratios respond to changes in resources, and how these changes are optimized through natural selection and adaptive behavior in the population. Models will need to quantify the relationship between the amount of food resources and sex ratios, and how such ratios can be regulated to improve the survival and reproductive success of populations. In addition, the effects of changes in sex ratio on other components of the ecosystem will need to be evaluated, considering how changes in sex ratio affect the population's social structure, reproductive strategies, and interactions with other species.

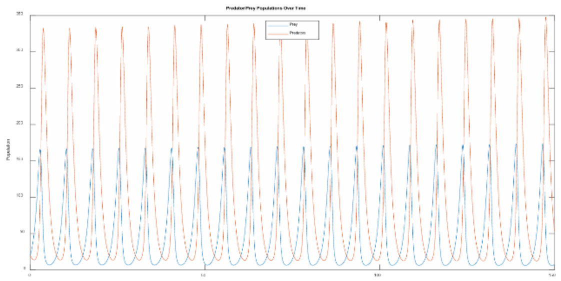
Figure 3. Predator population change

Mathematical modeling allows for the prediction of specific changes in sex ratios under different resource levels and further explores the effects of these changes on ecosystem-wide stability and biodiversity. The population dynamics of males, females and predators in an ecosystem are modeled. The model contains three main populations: males (M), females (F), and predators (P), and the growth rate of each population is affected by natural birth rate, natural mortality rate, competition coefficient, and other factors such as mating success and predation rate. Where r is the birth rate, d is the mortality rate, b is the competition coefficient, α is the mating success rate, and a is the predation rate. By varying the values of the competition coefficient bM and mortality rate dM for males, the model explores how these parameters affect the dynamics of the whole system. The results of the simulations are presented by plots, with time as the horizontal axis and population size as the vertical axis, and different parameter combinations are represented by different colors, providing an intuitive view for understanding population dynamics.

Rate of change of male individuals:

$$\frac{dM}{dt} = r_M \cdot M - d_M \cdot M + b_M \cdot a \cdot M \tag{3}$$

Rate of change in females:

$$\frac{dF}{dt} = r_F \cdot F - d_F \cdot F + b_F \cdot \alpha \cdot F \tag{4}$$

Rate of Predator change:

$$\frac{dP}{dt} = r_P \cdot P - a \cdot P \cdot (M + F) \tag{5}$$

Of which.

M,F,P represents the number of male individuals, female individuals and predators, respectively.

r_M,r_F,r_P are the birth rates of male, female and predator individuals, respectively.

d_M, d_F is the natural mortality rate of male and female individuals.

b_M, b_F is the coefficient of increase in the number of male and female individuals due to competition.

α is the mating success rate.

a is the predation efficiency of predators.

We consider the different species on this area as a whole, through the competition and evolution of the populations, in order to comply with the laws of ecosystem logic. Thus, we can obtain a model of the competitive relationship between lampreys and other species:

$$\frac{dN_D}{dt} = r_D \cdot N_D \cdot \left(1 - \frac{N_D}{K_D} - \beta_1 \frac{N_O}{K_O}\right) \tag{6}$$

$$\frac{dN_O}{dt} = r_O \cdot N_O \cdot \left(1 - \frac{N_O}{K_O} - \beta_2 \frac{N_D}{K_D}\right) \tag{7}$$

Where, r_D>0, r_O>0, β_1>0, β_2>0, N_D is the population size of lampreys and N_O is the population size of other. r_D is the coefficient of population growth for lampreys, and r_O is the coefficient of population growth for other . K_D is the maximum carrying capacity of the area for lampreys, K_O is the maximum carrying capacity of the area for other . $(1 - \frac{N_D}{K_D} - \beta_1 \frac{N_O}{K_O})$ represents the effect of resources consumed by lampreys and resources consumed by other on the growth of lampreys population. β_1 represents the fact that for lampreys, a unit of lampreys consumes β_1 times as much as other . β_2 has a similar meaning.

The model in equation (6) and equation (7) has the following four equilibria, and the results of the stability analysis of these four equilibria are presented in the table below:

Table 2: The results of the stability analysis of these four equilibria

Equilibrium point	Steady condition
(0,0)	Not steady
$(N_D,0)$	$\beta_1 < 1, \beta_2 > 1$
$(0, N_D)$	$\beta_1 > 1, \beta_2 < 1$
$\left(\frac{N_D(1-\beta_1)}{1-\beta_1 * \beta_2}, \frac{N_O(1-\beta_2)}{1-\beta_1 * \beta_2}\right)$	$\beta_1 < 1, \beta_2 < 1$

C. *Model Establishment*

We are now modeling the effects of lampreys on other populations of the same area as it reproduces, as well as on people.As an important link in the ecological chain, changes in the population structure of sevengill eels can cause chain reactions. For example, some species of sevengill eels use fish as hosts, and fluctuations in their populations can affect the abundance and health of host fish populations, which further affects the distribution and abundance of organisms at other levels in the food web. In addition, the recovery or decline of sevengill populations may also reflect indicators of water quality, benthic biodiversity and the overall health of river ecosystems. We propose to model how lampreys coexists and competes with other species under the same area, as well as the impact on humans.

By assigning values to the parameters and drawing diagrams to show the competitive coexistence of lampreys and other on the area under different circumstances.

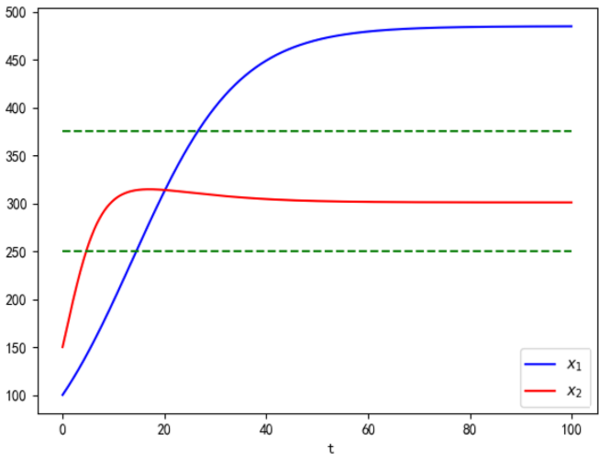

Figure 4. Change curves for different species

Due to having predators, we assume that human harvesting behaviour and the environment allow for balanced coexistence of lampreys and other species.

D. *Ecosystem impact model*

The dynamics of other populations in the introduced ecosystem, as well as changes in resources A_t, can be modeled using the Lotka-Volterra equations:

$$\frac{dN_{prey}}{dt} = r_{prey}N_{prey}\left(1 - \frac{N_{prey}}{K}\right) - \alpha_{\mathrm{prey}}N_{prey}N_{pred} \quad (8)$$

$$\frac{dN_{pred}}{dt} = -r_{pred}N_{pred} + \beta_{\mathrm{pred}}N_{prey}N_{pred} \quad (9)$$

$$\frac{dA_t}{dt} = r_A A_t\left(1 - \frac{A_t}{K_A}\right) - \sigma(N_{lamprey}, R_t, A_t) \quad (10)$$

Where.

N_{prey} *and* N_{pred} denote the size of prey and predator populations, respectively;

r_{prey} , r_{pred} denotes the respective natural growth rate, and

α_{prey} , β_{pred} denotes the coefficients of predation and reproduction rates, and

σ denotes the rate of consumption of the resource by sea lampreys, which may be related to the sex ratio and the amount of the resource.

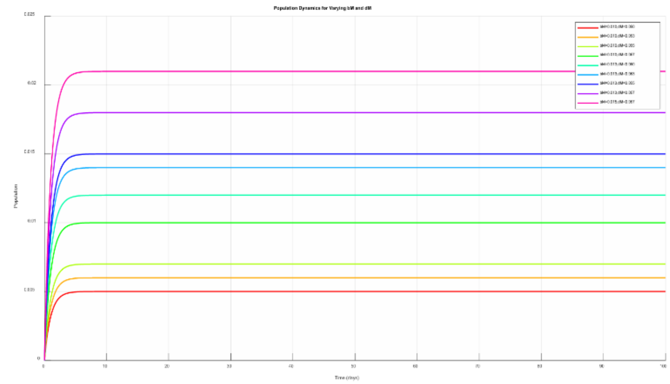

Figure 5. Effect of different rates of change on results

The graphs show the trend of population size over a 100-day period under different competitive coefficients (bM) and natural mortality rates (dM). As bM and dM values increase, the population declines more rapidly after the peak, suggesting that higher competitive pressures and mortality rates negatively affect the long-term survival of the population. Higher competitive coefficients lead to more difficult resource acquisition, while higher mortality rates directly reduce population size, and together they accelerate population decline. These patterns help us understand how different ecological pressures in an ecosystem affect population stability and sustainability. When the competition coefficient and natural mortality rate were low, the population size increased and then decreased slowly; while when the values of these two parameters increased, the population size peaked rapidly and then declined sharply, indicating that higher competition and mortality rates have a greater negative impact on the population, which may lead to difficulties in maintaining long-term population stability. This shows the relationship between population dynamics and competition and mortality in ecosystems.

Consider the effects of resource availability on the sex ratio of sevengill eels and construct an ecological model that incorporates growth rates, sex determination, and resource competition. This model should be able to simulate how the growth rate of sevengill eel larvae affects their sex determination process under different resource conditions, thus changing the sex ratio of the population. How this change in sex ratio affects the competitiveness of the sevengill eel in its ecological niche and the possible effects on predator and prey population dynamics are key points of the analysis. By modeling changes in sex ratios at different resource levels, we can predict the long-term effects of changes in sex ratios on ecosystem diversity, stability, and resilience. Resource availability leads to changes in sex ratios by affecting the growth rate of sevengill eel larvae, which in turn affects their sex-determining mechanisms. Such changes not only affect ecological dynamics within sea lamprey populations, such as reproductive success and population growth rates, but may also have knock-on effects on other species, such as altering food sources for predators or competitive relationships.

V. THE ASSESSMENT MODEL OF LAMPREYS

A. *Model of Population Growth*

Assume that under ideal conditions of infinite resources and no environmental resistance, the population increases at a constant natural rate of growth. The model will include the effect of sex ratio on population growth, such as the sex ratio effect (α) mentioned in the improved Lotka-Volterra model, which affects reproductive success and changes in population size. The base population growth model is as follows

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K\,(R)}\right) \tag{11}$$

where N is the population size, r is the basal growth rate, and K(R) is a function of sex ratio R, which represents the effect of sex ratio on environmental carrying capacity.

B. *Model of reproductive Success*

The model assumes that reproductive success is proportional to the product of resource size and sex ratio.

$$S(R, A) = \alpha R(1 - R)A \tag{12}$$

S denotes reproductive success, A denotes the amount of resources, α is a coefficient.

C. *Model of Resource Efficiency*

$$E(R) = \beta\frac{R}{1 - R} \tag{13}$$

E represents the efficiency of resource utilization, β is the efficiency factor.

From the perspective of population ecology, changes in sex ratio have both advantages and disadvantages for populations:

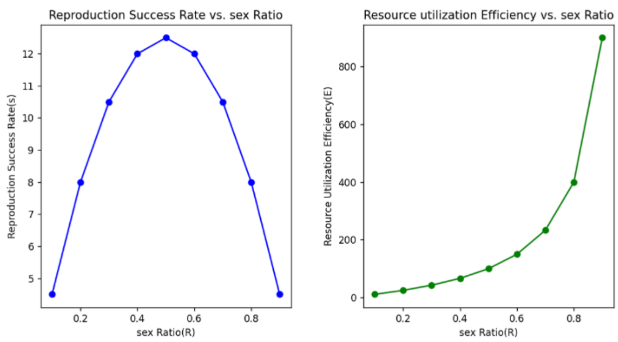

Figure 6. Changes in sex ratio have both advantages and disadvantages

- **Advantages:** Increasing the number of males in a situation where there are more females may help to increase the reproductive capacity of the population, as males can mate with more than one female, promoting genetic diversity and population size growth.
- **Disadvantages:** If there are too many males, this may lead to increased competition for resources, reduced reproductive opportunities, and overutilization of females, all of which may negatively affect the long-term stability of the population. Overall, an appropriate balance of sex ratios is essential for maintaining stable and reproducing populations.

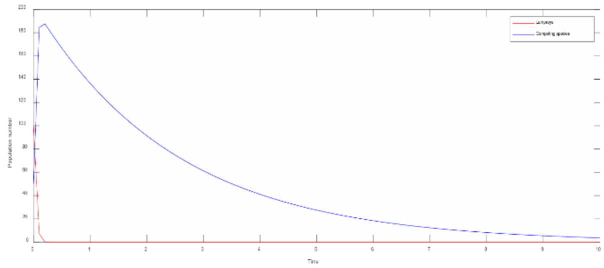
Figure 7. Consequences of the introduction of competing species

These graphs may reflect, to some extent, the effects of factors such as resource availability, mating behavior, and population management strategies on sex ratios.

When there are too many males, this may lead to intense mating competition and female fatigue, which can affect reproductive success. When there are too many females, although the mating opportunities of males increase, if the number of males is not sufficient to meet the mating needs of all females, it may result in females not being inseminated, affecting the number of the next generation. In ecosystems, a proper balance of sex ratios is an important factor in maintaining population health and ecological balance.

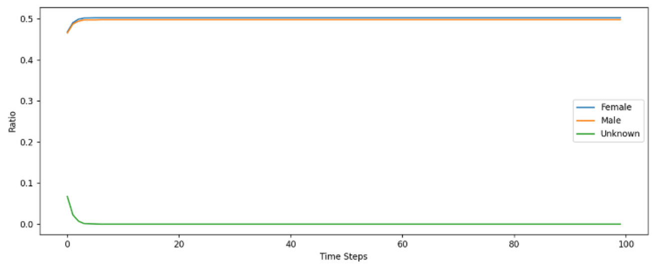

Figure 8. Consequences of the introduction of competing species

When analyzing these graphs, the specific effects that changes in sex ratio may have on populations can be further discussed:

- In the case of a rapid increase in the number of males followed by stabilization, it may mean that there are enough males to ensure that reproductive activity takes place, but beyond a certain threshold, adding more males may not have a significant effect on reproductive success, and may even give rise to internal competition, leading to an unequal distribution of resources and destabilization of the social structure.
- The sharp decline in the number of females suggests that females may become a limiting resource when males are dominant, thus posing a bottleneck to population growth and reproduction. Declining female numbers may lead to reduced reproductive opportunities, which in turn affects the long-term sustainability of the population.
- Sex ratio fluctuations may reflect complex interactions within populations, such as mating strategies, reproductive success, competition and viability. A

high degree of fluctuation may indicate that the population is highly sensitive to environmental changes and internal dynamics, which may lead to large fluctuations in population size under unstable environmental conditions.

In summary, changes in sex ratios affect populations in many ways, both in terms of their impact on the reproductive system and in relation to the social structure and long-term survival of populations.

VI. Sensitivity and Robustness Analysis

A. *Sensitivity Analysis*

We introduced a number of factors to estimate possible changes in different parameters. Changing the magnitude of this parameter implies a change in the parameter that affects population growth of lampreys and the impact on the ecosystem. We wanted to analyze the sensitivity of this parameter to see if our model is sensitive to this parameter.

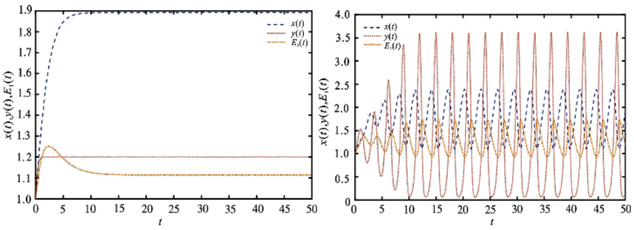

Figure 9. Different parameters as a function of the growth of lampreys populations

The results show that different parameter variations did not lead to changes in the overall trend. Accordingly, this is reasonable and explainable when combined with the real situation. The trends of the curves obtained from the sensitivity tests are consistent with the real situation.

B. *Robustness Analysis*

In addition, we verified the robustness of the model. When we added 5% random perturbation of different parameters, the practical implication is that we wanted to investigate whether our model is as accurate as possible in judging the effect of the sex ratio of lampreys on other species located in the area When there is some deviation in the data, we can find that the perturbation of the different parameters has a certain effect on the results, but it is within an acceptable range. Even if there is a slight error in the distance calculation, accurate judgment can be made to a large extent with the error within an acceptable threshold.

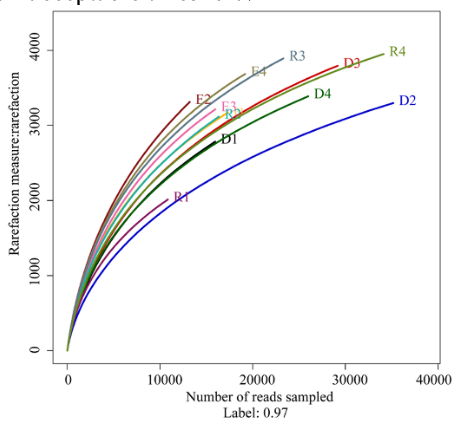

Figure 10. Robustness analysis

VII. Evaluation of Strengths and Weaknesses

A. *Strengths*

Our model has the following strengths:

- **Ecosystem dynamics modeling:** Embedding the sevengill eel agent model into the ecosystem dynamics model to interact with other species. Introduce other ecological factors such as other predators, competitors, etc. to construct a more comprehensive food web model.

- **Realistic, comprehensive and adaptable:** The experimental data in this paper take into account adaptation to different environmental conditions, maintaining relatively high reproductive success and appropriate sex ratios.

- **Complex system response:** Overall, the response of the system to stochastic perturbations shows complexity, where population dynamics are influenced not only by internal biological factors (e.g., sex ratio, reproductive rate) but also by external environmental factors. In this paper, most of these factors are taken into account.

B. *Weaknesses and Further Improvements*

Our model has the following limitations and related improvements:

- Some approximate analysis methods are applied to model other places, which may lead to the situation that not to be the most optimal.

- Due to time constraints, this paper does not consider data from different regions, which can be further considered in the future.

VIII. Conclusion

This study provides a comprehensive investigation into the ecological impact of sex ratio dynamics in sevengill eels through mathematical modeling. Our findings indicate that sex ratio variations significantly influence population growth and predator-prey interactions, yet ecosystems possess self-regulatory mechanisms that stabilize over time. Sensitivity and robustness analyses confirm the reliability of our models under different ecological conditions. Future work should explore the integration of AI-driven ecological modeling techniques to enhance predictive accuracy and assess broader ecosystem interactions.

References

[1] Mateus C S, Rodríguez-Muñoz R, Quintella B R, et al. Lampreys of the Iberian Peninsula: distribution, population status and conservation[J]. Endangered Species Research, 2012, 16(2): 183-198.

[2] Moser M L, Butzerin J A M, Dey D B. Capture and collection of lampreys: the state of the science[J]. Reviews in Fish Biology and Fisheries, 2007, 17: 45-56.

[3] York J R, Thresher R E, McCauley D W. Applying functional genomics to the study of lamprey development and sea lamprey population control[J]. Journal of Great Lakes Research, 2021, 47: S639-S649.

[4] Maitland P S. Review of the ecology of lampreys in northern Europe[J]. Canadian Journal of Fisheries and Aquatic Sciences, 1980, 37(11): 1944-1952

[5] Anisiu, M. C. (2014). Lotka, Volterra and their model. Didáctica mathematica, 32(01).

[6] Gordon, M. S., Mullin, J. M., Pruitt, S. R., Roskop, L. B., Slipchenko, L. V., & Boatz, J. A. (2009). Accurate methods for large molecular systems. *The Journal of Physical Chemistry B*, *113*(29), 9646-9663.

[7] Christopher Frey, H., & Patil, S. R. (2002). Identification and review of sensitivity analysis methods. *Risk analysis*, *22*(3), 553-578.

[8] Fath, B. D., Jørgensen, S. E., Patten, B. C., & Straškraba, M. (2004). Ecosystem growth and development. *Biosystems*, *77*(1-3), 213-228.

[9] National Research Council, Division on Earth, Life Studies, Water Science, Technology Board, Committee on Assessing, ... & Related Terrestrial Ecosystems. (2005). *Valuing ecosystem services: toward better environmental decision-making*. National Academies Press.

[10] Brännström, Å., & Sumpter, D. J. (2005). The role of competition and clustering in population dynamics. *Proceedings of the Royal Society B: Biological Sciences*, *272*(1576), 2065-2072.

[11] Waltman, P. (1983). *Competition models in population biology*. Society for industrial and applied mathematics.

[12] Palmblad, I. G. (1968). Competition in experimental populations of weeds with emphasis on the regulation of population size. *Ecology*, *49*(1), 26-34.