Fault Estimation and Fault-Tolerant Control for Discrete-Time Nonlinear Systems With Perturbation: a Mixed Design Scheme

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***Abstract*—This paper examines the observer-based fault-tolerant control issue for discrete-time nonlinear systems experiencing perturbations and fault signals. The nonlinear term with perturbation is first incorporated into the local nonlinear component, allowing the nonlinear system with perturbation to be represented as an interval type-1 (IT1) T-S fuzzy system. Next, using unknown input observer technology, an IT1 T-S fuzzy fault estimation (FE) observer scheme is developed to obtain real-time FE data and decouple the local nonlinear component from the estimation error system, thereby reducing design complexity and computational load simultaneously. Subsequently, based on the real-time FE data, an FE-based interval type-2 (IT2) T-S fuzzy fault-tolerant control scheme is proposed to compensate for the fault signal's impact and stabilize the system. Unlike traditional approaches, a mixed design strategy combining the IT1 T-S fuzzy fault estimation observer method with the IT2 T-S fuzzy fault-tolerant controller method is introduced in this paper. This strategy not only lessens computational load but also produces less conservative outcomes. Finally, the validity of the mixed design approach is verified through an example.**

***Index Terms*—Fault-tolerant control; fault estimation; perturbation; discrete-time nonlinear systems; IT2 T-S fuzzy system**

I. INTRODUCTION

In practice, sensors, actuators, and other components are prone to faults during system operation. These faults can degrade system performance or even lead to system instability. To enhance the system's reliability and safety, it is essential to implement measures that compensate for the adverse effects of faults on the system. In this context, fault-tolerant control (FTC) emerges as an effective method to mitigate the impact of faults, attracting significant attention and leading to numerous studies on the subject [1]-[8]. Currently, existing FTC methods are generally classified into two categories: passive FTC approaches [9]-[11] and active FTC approaches [12]-[17]. Passive FTC approaches typically apply the same controller strategy in both normal and fault conditions. While passive FTC design schemes are straightforward to implement, the fault estimation (FE) information obtained is limited, making these approaches relatively conservative and potentially leading to unsatisfactory control performance. In contrast, active FTC approaches can acquire more real-time FE information, enabling the synthesis of controllers that better compensate for the effects of faults. Active FTC approaches offer strong robustness and can effectively ensure satisfactory system performance.

On the other hand, nonlinear characteristics are commonly observed in practical systems [18]-[20]. However, traditional linear control methods are inadequate for addressing nonlinear problems, necessitating research into the nonlinear issues present in systems. To tackle this challenge, the T-S fuzzy model method has been introduced [21]. In this approach, the original nonlinear system is decomposed into a set of linear subsystems using IF-THEN rules. Within the T-S fuzzy model framework, many advanced linear control methods can be applied as references, making the T-S fuzzy model method a powerful tool for solving and analyzing nonlinear system design problems. Over the past few decades, significant attention has been directed towards the T-S fuzzy model method, leading to numerous important outcomes in areas such as controller synthesis [22]-[28], filter design [29]-[31], stability analysis [32]-[35], and others. It should be noted that the above results were achieved using the IT1 fuzzy model method. However, when perturbations are present in the nonlinear system, the membership functions derived from modeling the nonlinear system may contain uncertain information, rendering the IT1 T-S modeling method less effective for handling nonlinear systems with perturbations. To address this issue, the IT2 T-S fuzzy modeling method was introduced in [36]. In the IT2 T-S fuzzy model method, the nonlinear system with perturbations is better represented by employing lower and upper membership functions. In recent years, research based on the IT2 T-S fuzzy model has made progress in stability analysis [37], filtering design [38]-[40], and controller synthesis [41]-[43] for nonlinear systems with perturbations. For instance, based on the IT2 T-S fuzzy model, Li et al. [38] proposed a filter design method to ensure the stochastic stability of nonlinear networked systems. In [40], the IT2 T-S fuzzy filter design problem was addressed by proposing an event-based reduced-order fuzzy filter to simplify the implementation of nonlinear systems and conserve network resources. Furthermore, by constructing a delay product-type

Lyapunov-Krasovskii function, the conservatism of controller synthesis conditions for IT2 fuzzy systems with multi-class uncertainties was reduced [41].

Although the IT2 T-S fuzzy model can effectively describe nonlinear systems with perturbations, designing controllers or observers based on this model is challenging because the membership functions of the controller or observer cannot be identical to those of the model. In contrast, with the IT1 T-S fuzzy model, the membership functions of the observer or controller can be selected to match those of the IT1 T-S fuzzy model, thereby overcoming the disadvantages of the IT2 T-S fuzzy model. This naturally leads to the idea of using the IT1 fuzzy model method to achieve better design performance for nonlinear systems with perturbations. Inspired by [44], this paper adopts the local nonlinear modeling method to model the nonlinear system, incorporating the perturbation into the local nonlinear term $\varphi\big(x(k)\big)$ to obtain an IT1 T-S fuzzy model where the premise variables of the model are no longer dependent on the perturbation term. Subsequently, an IT1 T-S fuzzy FE observer is proposed, allowing the premise variables of the observer to be selected identically to those of the model, thereby reducing the complexity of observer design. Additionally, unknown input observer technology is employed to directly isolate the term $\varphi\big(x(k)\big)$ from the estimation error system, eliminating the need for additional conditions to constrain the term $\varphi\big(x(k)\big)$. However, the situation is entirely different in controller design, where unknown input observer technology cannot be applied, and the nonlinear term with perturbation may not satisfy the sector-boundedness condition. In this scenario, the nonlinear system with perturbation is modeled as an IT2 T-S fuzzy model for controller design. Based on this model, an IT2 T-S fuzzy fault-tolerant controller is proposed to compensate for the fault's influence on the nonlinear system and stabilize the system. The above discussion highlights the motivation for developing the mixed design scheme of the IT1 T-S fuzzy FE observer and the IT2 T-S fuzzy fault-tolerant controller presented in this paper.

Based on the aforementioned discussion, this paper investigates the Fault Estimation (FE) and Fault-Tolerant Control (FTC) problems for discrete-time nonlinear systems with perturbations and proposes a mixed design scheme that combines the IT1 T-S fuzzy FE observer method with the IT2 T-S fuzzy fault-tolerant controller method. The main contributions and novelties of this paper are summarized as follows:

First, inspired by the local nonlinear modeling technique, a mixed design scheme of the IT1 T-S fuzzy FE observer and the IT2 T-S fuzzy fault-tolerant controller is proposed. This approach simplifies the design of the observer and reduces the computational burden of the observer-based FTC scheme compared to the traditional IT2 T-S fuzzy design method.

Second, by decoupling the term $\varphi\big(x(k)\big)$ from the estimation error system using unknown input observer technology, the term $\varphi\big(x(k)\big)$ no longer influences the dynamics of the estimation error system. This method avoids the need for the Lipschitz condition used in [44].

In this paper, $P > 0$ $(P < 0)$ denotes a positive (negative) definite matrix; for a matrix A, A^{-1} and A^T denote its inverse and transpose, respectively; *0* and *I* represent the zero matrix and the identity matrix with appropriate dimensions, respectively; $T^{\dagger}$ denotes a generalized inverse of *T*; $l2[0,\infty)$ represents the space of square-integrable vector functions over $[0,\infty)$; and $|*|2$ denotes the usual $l_2[0,\infty)$ norm of a vector.

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