# BAYESIAN COMPRESSED DEEP LEARNING FOR STATE ESTIMATION OF UNOBSERVABLE POWER SYSTEMS

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## Abstract

In recent years, state-of-the-art Deep Learning (DL)-based modeling has been applied to the problem of state estimation of unobservable electrical distribution systems, with promising results. Unfortunately, the definition and training of these flexible models have been largely heuristic, which may result in oversized neural networks, with computationally inefficient layers. In this work, we apply the method of Bayesian Compression for eliminating spurious redundancies of DL-based State Estimation models. Experimental results in four test networks, including two IEEE Test Case Power Networks, corroborate the benefits of the proposed compression approach for obtaining reduced versions of the models without compromising their performance.

# **1** INTRODUCTION

State Estimation in Power Distribution Systems is a crucial procedure for their continuous monitoring, along with its consequent reliability and safety of their operation. Its relevance has continuously grown, in a world with increasingly more distributed and real-time operated electrical networks.

A common pitfall in power networks is their unobservability, which is to say that the available set of measurements correspond to uncountably many possible states. This unavailability may result from insufficient number of sensors available across the network, but also from faulty sensing devices, which may output null or unreliable measurement values.

Recently proposed approaches (Mestav et al. (2018); Mestav & Tong (2019)) based on neural networks with multiple hidden layers, denominated "*Deep Learning*" (DL) (Goodfellow et al. (2016)), have shown promising results on the problem of State Estimation of unobservable electrical networks, taking advantage of the expressiveness and *Universal Approximation* (Csáji (2001)) capabilities of such models for approximating joint distributions of partial measurements' and network states' values.

Unfortunately, much of DL modeling is based on manual tuning, rules of thumb, or just plain trial and error. This lack of rigour in Power System state modeling step is likely to result in oversized neural networks, with weights which exhibit redundancy with respect to the remaining of the model parameters.

For dealing with this accidental model oversizing, several approaches of Model compression were proposed, which act on the reducing the complexity of the model without significantly compromising its performance. Among them, the so-called *Bayesian Compression* (BC) (Louizos et al. (2017)) uses a variational approach for simultaneous model pruning and bit precision reduction, which we will adapt here for the case of state estimation of unobservable Power Systems.

In the following, we describe the steps of our Bayesian Compressed Deep Learning-based State Estimation in Unobservable Power Systems (BCDL-SEUPS), as well as the main improvements with respect to the current works involving DL modeling of Power Systems, mainly introduced in (Mestav et al. (2018); Mestav & Tong (2019)). The main differences are:

1. The Distribution Learning of typical injection/consumption values from historical data, here pursued with standardized implementation of Gaussian Mixture Models (GMM), is

done in a systematic way, through the Bayesian Information Criterion (BIC) for Model Regularization;

2. The DL modeling step for state estimation is followed by a posterior one, which compresses the fitted model through the simultaneous weight pruning and bit precision reduction procedures of the Bayesian Compression method.

The efficacy of proposed improvements is validated through reproducible experiments with four example networks, including two IEEE Test Case networks (14s and 57).

### 2 PROPOSED APPROACH

Let be a Power Distribution System S with N nodes. We denote the State  $\mathbf{Y} \in \mathbb{R}_N^+$  of the System S as a N-sized positive-valued vector  $\mathbf{Y} = \{Y_1, Y_2, ..., Y_N\}$ . The generalization for node states with three-phase voltages and angle components is left for future work.

This state of the system is affected by the architecture of the network, as well as the power injection or consumption acting along each node of the network. Here, we summarize the injection and consumption of power in the form of a N-sized real-valued Input vector  $\mathbf{X} = \{X_1, X_2, ..., X_N\} \in \mathbb{R}_N$ . By definition, a negative entry  $X_i$  corresponds to a resulting power consumption at the i-th node, while a positive value corresponds to power injection.

Associated with the assumed unobservable system, we have a M-sized Partial Measurement vector  $\mathbf{Z} \in \mathbb{R}^+_M$ , with

$$M = 2\lfloor \kappa N \rceil, \text{ s.t. } \kappa \in (0, 1 - \frac{1}{2N})$$
(1)

where the operator  $\lfloor \cdot \rceil$  corresponds to rounding the operand to the nearest integer. In the present work, the M entries of Z correspond to a M-sized randomly sampled subset of indexes from 1 to N.

#### 2.1 DEEP LEARNING FOR STATE ESTIMATION IN POWER SYSTEMS

Inspired by recent works (Mestav et al. (2018); Mestav & Tong (2019)), we propose to model the response of the distribution system through a learning-based strategy, which consists of a model of the conditional probability of the network state given its partial measurement input vector: p(Y|Z)

The DL approach consists of fitting an approximate model  $\tilde{p}_{\theta}(\mathbf{Y}|\mathbf{Z}) \approx p(Y|Z)$  in the form of a flexible enough neural network, to a dataset  $\mathcal{D}$  consisting of D input-output example pairs  $\mathcal{D} = \{\mathbf{Z}_k, \mathbf{Y}_k\}_{k=1}^{D}$  through the minimization of a corresponding loss function, s.a. Mean Absolute Error (MAE) or Mean Squared Error (MSE). The vector  $\theta$  denotes the weights of the neural network.

These examples are generated by applying different injection values of  $X_k$  to the network and running the power flow equations of the networks, for obtaining the corresponding values of  $Z_k$  and  $Y_k$ . For bringing the network model close to real-world operation values, we fit a probabilistic model p(X) of injection values with historical data, such as those of publicly available datasets of household energy consumption across time. More details are given in the Experimental section.

Each sampled value  $X_k$  is then applied to the model of the network, and the corresponding inputoutput pair  $\{Z_k, Y_k\}$  is obtained through running the corresponding network power flow equations. The procedure is repeated until the D samples are obtained.

### 2.2 BAYESIAN COMPRESSION FOR DEEP LEARNING

For circumventing DL models' inefficiencies and redundancies, several compression techniques have been successfully proposed, which aim to optimize the computational cost of the model operation while preserving its accuracy and overall performance indicators through two main strategies: (1) Pruning layers, for removing redundant weights; And (2) reducing the floating bit precision of the weights to the minimum required to maintain the accuracy of the model's computation.

The method denoted *Bayesian Compression* (Louizos et al. (2017)) achieves both strategies by using a variational inference approach to the probabilistic distribution of weights. It assumes that the weights of the neural network can be represented by a probabilistic distribution  $\theta \sim \mathcal{N}(\theta; 0, \eta^2)$ 



Figure 1: Summary of the steps of our Bayesian Compressed DL-based State Estimation method. The steps in red correspond to the ones which were added or modified in our current procedure, when comparing with other DL-based methods.

with mean zero, where the parameter  $\eta$ , associated to the variance, which implicitly controls the sparsity of the weights, is also represented by a parametric probability distribution  $\eta \sim p(\eta)$ , which can be defined, for example, as the hyperparameter free log-uniform prior or the half-Cauchy prior.

It is then possible to define an approximate joint distribution  $q_{\phi}(\theta, \eta)$  so as to maximize a lower bound on the loglikelihood of the observed data  $\mathcal{D}$  regularized by this variational approximation

$$\mathcal{L}(\phi) = \mathbb{E}_{q_{\phi}(\theta,\eta)} \left[\log p(\mathcal{D}|\theta)\right] - \mathbb{E}_{q_{\phi}(\eta)} \left[KL(p(\theta,\eta)||q_{\phi}(\theta,\eta))\right]$$
  
=  $\mathbb{E}_{q_{\phi}(\theta,\eta)} \left[\log p(\mathcal{D}|\theta)\right] - \mathbb{E}_{q_{\phi}(\eta)} \left[KL(p(\theta|\eta)||q_{\phi}(\theta|\eta))\right] - KL(p(\eta)||q_{\phi}(\eta))$  (2)  
where  $KL(\cdot||\cdot)$  refers to the Kullback-Leibler Divergence:  $KL(p(\cdot)||q(\cdot)) = \mathbb{E}_{p(\cdot)} \left[\log\left(\frac{p(\cdot)}{q(\cdot)}\right)\right]$ 

By assuming parametric forms for  $p(\eta)$ , the second and third terms of the right side of Equation 2 can be computed in closed-form. From the final resulting variance of the approximated distribution, it is possible to infer the unit round off required to represent the weights. The steps of our full Bayesian Compressed DL-based State Estimation method are described in Figure 1.

#### **3** EXPERIMENTS

We evaluated the performance of the proposed compressed estimation pipeline on four network test cases. The first two, with 3 and 6 buses, are implemented using the PyPSA library, including the power flow simulations, and the last two are IEEE standard network test cases (14s and 57). We used the standardized versions of both IEEE networks available, as long as the power flow equation simulations, through the Python *pandapower* library. Source code will be made available.

Our goal was to evaluate the performance of the compressed estimation pipeline across different values of  $\kappa$  and also different network architectures. For that purpose, we used four-layered Multilayer Perceptrons (MLPs) and varied the number of neurons in the two hidden layers.

#### 3.1 DISTRIBUTION LEARNING OF NETWORK INJECTION VALUES

For distribution learning of typical injection values, the household consumption data from Ausgrid Solar home electricity data (AG1 (2014)) was used. The Gaussian Mixture Model (GMM) was chosen as the probabilistic model, due to its modeling flexibility. The *scikit-learn* implementation of GMM was fitted in the data from 01/Jul/2011 to 30/Jun/2012 and tested in the data from 01/Jul/2012 to 30/Jun/2013.

For choosing the appropriate number of gaussian components, we used the Bayesian Information Criterion (BIC) (Schwarz (1978)):  $BIC = k \ln(n) - 2 \ln(\hat{L})$ , where k is the number of parameters of the GMM, n is the number of data points, and  $\hat{L}$  is the maximized value of the likelihood of the GMM. It was observed that raising the number of components above 5 has little effect in the overall BIC performance, and so we set number of gaussians for the GMM as 5.

### 3.2 PyPSA and IEEE Network Test Cases

For performance evaluation, we chose four different combinations of Layer Multiplying Factors :{(5,5),(10,5),(15,10),(20,10)}, in which  $(a, b) \in \mathbb{Z}^2_+$  means that the first hidden layer is going to have a \* M weights, while the second hidden layer has b \* M weights.



Figure 2: DL Modeling and Compression performance results for the four Network Test Cases, with the  $\kappa$  (blue) and Layer Multiplying Factors (green) as varying parameters.

We obtained a total of 100 samples and used them to fit the discriminative DL model across 50 epochs. For the opf-storage-hvdc, Case 14s and Case 57 networks, the  $\kappa$  was varied between  $\{0.2, 0.4, 0.6, 0.8\}$ , while the smaller 3-bus network had  $\kappa$  varied among  $\{0.4, 0.6, 0.8, 1.0\}$ , since the vector with only 20 % of its 3 node measurements would be an empty one.

We used 80 % of the available measurements for training and the remaining 20 % for validation. We evaluated both the performance of the DL-based state estimation and the effectiveness of the Bayesian Compression method on the layers. We computed the final MSE on the validation set, and the compression factor, which is the ratio by which the model could be reduced, in terms of number of parameters, without having its performance affected.

It can be seen, in Figure 2, that the compression ratio stays above 2 for a wide range of network architectures and observable nodes' ratio. For the 6-bus (opf-storage-hvdc) network, increasing the complexity of the MLP model, and also the observability parameter  $\kappa$ , decreases the resulting MSE in the test set. For the smaller 3-bus network, increasing the complexity of the MLP decreases its test set performance, which indicates possible overfitting.

It was possible to observe that the Bayesian Compression method performed well across several network scales, but its performance is intrinsically dependent on the effectiveness of Power Flow optimization routines, which showed degraded performance for the larger 14s and 57 networks, not converging after the limit number of 100 iterations. Nevertheless, even with the power flow output variables which were not fully converged, the compression ratio stayed greater or equal to 2.

# 4 CONCLUSION

The Deep Learning (DL)-based approach to state estimation in unobservable Electrical Distribution Networks has recently been introduced, with promising results. However, the architecture choice and model training of deep neural networks is recognizably cumbersome and inefficient, possibly resulting in oversized models, with computationally inefficient layers. In the present work, we apply a technique know as "Bayesian Compression" to the DL modeling of state estimation in power networks, and show its efficacy in producing parsimonious but effective representations of the network underlying dynamics. The method is validated through experimental results in four test networks, including two IEEE standardized Test Case Networks.

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