Value Diffusion Reinforcement Learning

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Abstract

Model-free reinforcement learning (RL) combined with diffusion models has achieved significant progress in addressing complex continuous control tasks. However, a persistent challenge in RL remains the accurate estimation of Q-values, which critically governs the efficacy of policy optimization. Although recent advances employ parametric distributions to model value distributions for enhanced estimation accuracy, current methodologies predominantly rely on unimodal Gaussian assumptions or quantile representations. These constraints introduce distributional bias between the learned and true value distributions, particularly in some tasks with a nonstationary policy, ultimately degrading performance. To address these limitations, we propose value diffusion reinforcement learning (VDRL), a novel model-free online RL method that utilizes the generative capacity of diffusion models to represent multimodal value distributions. The core innovation of VDRL lies in the use of the variational loss of diffusion-based value distribution, which is theoretically proven to be a tight lower bound for the optimization objective under the KL-divergence measurement. Furthermore, we introduce double value diffusion learning with sample selection to enhance training stability and further improve value estimation accuracy. Extensive experiments conducted on the MuJoCo benchmark demonstrate that VDRL significantly outperforms some SOTA model-free online RL baselines, showcasing its effectiveness and robustness.

1 Introduction

Recent advancements in model-free reinforcement learning (RL) integrated with diffusion models [1, 2, 3, 4, 5, 6, 7] have demonstrated remarkable success in solving practical decision-making tasks, such as robotic locomotion control [8], robotic navigation [9], and robot arm manipulation [10, 11, 12]. By harnessing the expressive power of diffusion models [13, 14, 15], these works focus on enhancing the exploration efficiency of policy by effectively capturing the expressiveness and multimodality of policy beyond traditional action distribution assumption (e.g., unimodal Gaussian) [16]. However, a core challenge in RL remains the accurate estimation of state-action values (i.e., Q-values), which is fundamental to both policy evaluation and improvement [17]. Despite their enhanced exploration capabilities, RL methods combined with diffusion policies often suffer from value estimation errors, particularly overestimation [18], leading to suboptimal solutions [19].

To mitigate value overestimation, some techniques like double Q-network [20] and approximate upper-bound [21] have been incorporated into prominent RL algorithms, such as twin delayed deep deterministic policy gradient (TD3) [22] and soft actor-critic (SAC) [23], achieving significant performance improvements. Nevertheless, these methods may introduce a considerable underestimation bias and increase the computation cost for each iteration [24]. Therefore, inaccuracies in value estimation remain a limiting factor for further advancements in policy learning [25].

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On the other hand, distributional RL [26], rooted in the idea of modeling the value distribution rather than estimating only the expected return (i.e., Q-value), has emerged as a novel and promising avenue for improving the accuracy of value estimation. Building on this concept, some early works [27, 28, 29, 30, 31] leverage quantile functions to represent the value distribution. However, these approaches are limited to discrete or low-dimensional action spaces. To address this challenge, the recent representative works such as DSAC [24] and DSACT [32] learn a continuous Gaussian value distribution and provide solid theoretical insights into how value distribution learning reduces overestimation. Despite the reduction in value overestimation, the parametric value distributions of these methods are constrained by the assumption (i.e., unimodal Gaussian), which may introduce bias between the approximated and true value distributions in tasks with non-stationary policies, where the true value distribution often exhibits multimodal characteristics. Consequently, this bias leads to imprecise value estimation, yielding a degraded performance. The above limitation motivates a key question: *Can we develop a universal value distribution representation of RL that simultaneously achieves multimodal expressiveness and flexibility for complex continuous control tasks?*

Inspired by the considerable success of diffusion models applied in RL [4, 5, 6, 7], we propose Value Diffusion Reinforcement Learning (VDRL), a novel model-free online RL method. To the best of our knowledge, this is the first work to integrate diffusion models into distributional RL, enabling precise value estimation. The core idea of VDRL is conceptually elegant yet highly effective: we harness diffusion models to generate more expressive and multimodal distributions for representing value distributions. Our key theoretical insight reveals that the optimization objective of value distribution learning for distributional RL, under the KL-divergence measure, can be reformulated as the variational bound objective of the diffusion model. To mitigate variance introduced by iterative diffusion sampling, we introduce double value diffusion learning with adaptive sample selection, ensuring stable training while preserving distributional fidelity. In summary, the key contributions of this work are as follows:

i) To improve the accuracy of value estimation, we propose the value diffusion reinforcement learning (VDRL) method. VDRL leverages the capability of the diffusion model to fit more expressive and multimodal value distributions. More importantly, we theoretically prove that the variational loss of diffusion-based value distribution is a tight bound for the optimization objective of value distribution learning under the KL-divergence measurement.

ii) To address the challenge of unstable learning introduced by iterative diffusion sampling, we propose the double value diffusion learning with sample selection for value distributions. This mechanism can further reduce the value overestimation bias and might introduce a minor value underestimation, which enhances learning stability and promotes better performance.

iii) We perform extensive experiments on eight popular MuJoCo [33] benchmark tasks, comparing the performance of VDRL to eleven model-free online RL baselines, including traditional RL, distributional RL, and diffusion-based policy RL methods. The experimental results demonstrate that VDRL achieves stable and consistently superior performance across most evaluated tasks, highlighting its robustness and effectiveness.

2 Preliminaries

2.1 Problem Formulation

The standard RL task could be regarded as a Markov decision process (MDP) [17]. Formally, the task can be formulated as a tuple $\mathcal{M} = \langle \mathcal{S}, \mathcal{A}, \mathcal{P}, R, \gamma \rangle$, where each element is defined as follows:

- $\mathcal{S}$: a finite set of states. Given a current state $s \in \mathcal{S}$, we often denote the next state as $s' \in \mathcal{S}$.
- $\mathcal{A}$: a set of actions. Note that the action may be discrete or continuous.
- $\mathcal{P}(s'|s, a)$: the state transition function, which usually denotes the probability from the current state s to the next state s' when the agent selects the action a.
- $R(s, a)$: the reward function. After the agent performs an action a under the environment state s, the agent will receive an immediate global reward $r = R(s, a)$. Notably, in this work, the reward is explicitly treated as a random variable.
- $\gamma \in (0, 1]$: the discount factor used in the computation of cumulative return.

Further, the behavior of the agent is defined by a stochastic policy $\pi(a|s)$, which maps a given state to a probability distribution over actions. The state-action value function $Q^\pi(s,a)$ associated with a given policy π denotes the expectation of cumulative state-action return when the agent taking the action $a \in \mathcal{A}$ under the state $s \in \mathcal{S}$, i.e., $Q^\pi(s,a) := \mathbb{E}_\pi\left[\sum_{k=0}^{\infty}\gamma^k R(s_k,a_k)|s_0=s,a_0=a\right]$.

2.2 Distributional Reinforcement Learning

Distributional Reinforcement Learning [26, 34, 29] learns the full probability distribution of the cumulative state-action return $\mathcal{Z}(\cdot|s,a)$ instead of the expected return, i.e., a single scalar Q-value $Q(s,a)$, which behaves better than traditional RL methods in terms of learning stability and performance. In specific, the cumulative state-action return $Z^\pi(s,a)$ associated with a given policy π is the sum of discounted rewards along the trajectory of interactions with the environment, i.e., $Z^\pi(s,a) = \sum_{k=0}^{\infty}\gamma^k R(s_k,a_k)$. Note that the cumulative state-action return is a random variable, which describes the intrinsic randomness of the agent's interactions with its environment, i.e., the randomness of state transition $P(s'|s,a)$, reward $R(s,a)$, and next state-action return $Z(s',a')$ [26]. The distribution of the cumulative state-action return $\mathcal{Z}^\pi(\cdot|s,a)$ [2] is a mapping from state-action pairs to distribution over the random return, i.e., $Z^\pi(s,a) \sim \mathcal{Z}^\pi(\cdot|s,a)$. Significant to distributional RL is the use of the distributional Bellman operator to describe the value distribution, defined by

$$\mathcal{T}^\pi Z(s,a) \overset{D}{:=} R(s,a) + \gamma P^\pi Z(s',a'), \tag{1}$$

where the equation $X \overset{D}{:=} Y$ represents that the random variable X is distributed according to the same law with Y, $s' \sim P(\cdot|s,a)$, $a' \sim \pi(\cdot|s')$, and $\mathcal{T}^\pi Z(s,a)$ is a new random variable, whose distribution is defined as $\mathcal{T}^\pi \mathcal{Z}(\cdot|s,a)$.

Distributional Soft Actor-Critic. In most previous distributional RL methods, the value distribution is commonly modeled by a discrete probability density function [26, 30, 29] or a quantile function [27, 28, 35] due to its highly expressive advantage [36]. However, these functions restrict direct access to the mean value, necessitating its approximation via a weighted sum of finite quantiles. To overcome this issue, the recent work [24] proposed the distributional soft actor-critic (DSAC) method by directly learning a continuous parameterized probability density function (Gaussian) for the random return $Z(s,a)$, i.e., $\mathcal{Z}_\theta(\cdot|s,a) = \mathcal{N}(Q_\theta(s,a), \sigma^2_\theta(s,a))$. The parameter θ can be updated by optimizing the Kullback-Leibler (KL) divergence between the target value distribution $\mathcal{T}^\pi\mathcal{Z}(\cdot|s,a)$ and value distribution $\mathcal{Z}^\pi(\cdot|s,a)$ via gradient descent, i.e.,

$$J^{\text{DSAC}}_{\mathcal{Z}}(\theta) = \mathbb{E}_{(s,a)\sim\mathcal{B}}\left[D_{\text{KL}}(\mathcal{T}^{\pi_{\bar{\phi}}}\mathcal{Z}_{\bar{\theta}}(\cdot|s,a) \parallel \mathcal{Z}_\theta(\cdot|s,a))\right], \tag{2}$$

where $\mathcal{B}$ is a replay buffer of previously sampled experience, $\bar{\theta}$ and $\bar{\phi}$ represent the parameters of the target value distribution and policy functions.

2.3 Diffusion Model

Diffusion models [13, 14] are highly powerful tools that learn to generate data by gradually denoising a normally distributed variable, which have achieved superior expressiveness in representing complex probability distributions, particularly in image and video generation [37, 38, 39, 40, 41]. A representative algorithm is the well-known denoising diffusion probabilistic models (DDPM) [15], which synthesizes data by learning to reverse a predefined noising process, i.e., forward process, through iterative denoising steps modeled by a neural network, i.e., reverse process. Given the real data x_0 samples from the date distribution $x_0 \sim q(x_0)$, DDPM uses a parameterized latent variable model, i.e., $p_\theta(x_0) = \int p_\theta(x_{0:T})dx_{1:T}$, to model how the isotropic Gaussian noise $x_T \sim \mathcal{N}(\mathbf{0},\mathbf{I})$ is denoised into real data x_0, where x_t denotes the step of diffusion process and T represents the total number of diffusion steps. The forward process is a Markov chain that gradually adds Gaussian noise to the data according to a variance schedule $\beta_1, \cdots, \beta_T$, expressed by

$$q(x_{1:T}|x_0) = \prod_{t=1}^{T} q(x_t|x_{t-1}), \quad q(x_t|x_{t-1}) = \mathcal{N}(x_t; \sqrt{1-\beta_t}x_{t-1}, \beta_t\mathbf{I}). \tag{3}$$

The reverse process reconstructs data distribution starting at $p(x_T) = \mathcal{N}(x_T;\mathbf{0},\mathbf{I})$, represented as

$$p_\theta(x_{0:T}) = p(x_T)\prod_{t=1}^{T} p_\theta(x_{t-1}|x_t), \quad p_\theta(x_{t-1}|x_t) = \mathcal{N}(x_{t-1}; \mu_\theta(x_t,t), \Sigma_\theta(x_t,t)), \tag{4}$$

[2] For notational simplicity, this distribution is called as value distribution in this work.

where $\mu_\theta(x_t, t)$ denotes the mean function approximator, and $\Sigma_\theta(x_t, t)$ is set $\sigma_t^2\mathbf{I}$ to untrained time dependent constants. Inspired by VAE [42], the variational lower bound (VLB) is utilized to optimize the negative log likelihood as the training objective of DDPM:

$$J_{\text{VLB}}(\theta) = \mathbb{E}_{x_0\sim p_0, x_{1:T}\sim q}\left[\log \frac{p_\theta(x_{0:T})}{q(x_{1:T}|x_0)}\right]. \tag{5}$$

To this end, the diffusion model is trained by minimizing the simplified loss function:

$$\mathcal{L}_{\text{DDPM}}(\theta) = \mathbb{E}_{t\sim[1,T],x_0,\epsilon}\left[\|\epsilon - \epsilon_\theta(\sqrt{\bar{\alpha}_t}x_0 + \sqrt{1-\bar{\alpha}_t}\epsilon, t)\|^2\right], \tag{6}$$

where $\bar{\alpha}_t = \prod_{s=1}^t \alpha_s$, $\alpha_t = 1 - \beta_t$ and ϵ is sampled from $\mathcal{N}(\mathbf{0}, \mathbf{I})$.

3 Method

In this section, we propose a novel model-free online RL method called value diffusion reinforcement learning (VDRL) to improve the accuracy of value estimation. The key innovation of VDRL is the use of the variational loss of diffusion-based value distribution, which can be theoretically proven as a tight lower bound for the optimization objective of value distribution learning under the KL-divergence measurement. Furthermore, we introduce double value diffusion learning with sample selection to stabilize training and further enhance estimation accuracy.

3.1 Variational Objective for Value Diffusion Learning

According to the distributional Bellman operator in Eq. (1), the optimization objective of value distribution learning in distributional RL is to minimize the distance between the target value distribution, i.e.,$\mathcal{Z}_{\text{tar}}(\cdot|s,a)$, and the current value distribution, i.e.,$\mathcal{Z}_{\text{cur}}(\cdot|s,a)$, formally,

$$\min_{\mathcal{Z}_{\text{cur}}}\left\{\mathop{\mathbb{E}}_{(s,a)\sim\mathcal{B}}\left[d(\mathcal{Z}_{\text{tar}}(\cdot|s,a), \mathcal{Z}_{\text{cur}}(\cdot|s,a))\right]\right\}, \tag{7}$$

where $\mathcal{Z}_{\text{tar}}(\cdot|s,a) = \mathcal{T}^\pi\mathcal{Z}_{\text{old}}(\cdot|s,a)$ and d is the distance metric for measuring two probability distributions, such as Kullback-Leibler (KL) divergence, Wasserstein distance.

To optimize the objective in Eq. (7), in practice, we need to choose a tractable approximating distribution for modeling the value distribution. For instance, the value distribution is modeled as a unimodal Gaussian distribution in DSAC [24] and DSACT [32]. However, as highlighted in [43], the unimodal Gaussian approximation fails to capture the inherent expressiveness and multimodality of value distributions, which may introduce bias between the approximated and true distributions in tasks with non-stationary policies, leading to imprecise value estimation and then yielding degraded performance. More importantly, we find that when the value distribution is modeled by a diffusion model, the optimization objective under the KL-divergence measurement can be reformulated and simplified into a variational bound objective, which is formalized as follows:

Theorem 1. ***(Lower Bound of Learning Objective for Value Distribution)*** *If the value distribution is modeled by a diffusion model, the variational bound objective*

$$\mathbb{E}_{\substack{(s,a,r,s')\sim\mathcal{B},a'\sim\pi_{\bar{\phi}},\\ Z(s',a')\sim\mathcal{Z}_{\bar{\theta}}(\cdot|s',a')}}\left[\mathbb{E}_{\mathcal{T}^\pi Z_{1:T}\sim q(\mathcal{T}^\pi Z_{1:T}|s,a,\mathcal{T}^\pi Z_0)}\left[\log\frac{P_\theta(\mathcal{T}^\pi Z_{0:T}|s,a)}{q(\mathcal{T}^\pi Z_{1:T}|s,a,\mathcal{T}^\pi Z_0)}\right]\right], \tag{8}$$

is the tight lower bound of the optimization objective of value distribution learning for distributional RL under the KL-divergence measurement

$$\mathbb{E}_{(s,a)\sim\mathcal{B}}\left[D_{KL}(\mathcal{T}^\pi\mathcal{Z}_{\bar{\theta}}(\cdot|s,a) \parallel \mathcal{Z}_\theta(\cdot|s,a))\right], \tag{9}$$

where $\mathcal{Z}_\theta(\cdot|s,a)$ denotes the value distribution to be optimized and $\mathcal{Z}_{\bar{\theta}}(\cdot|s,a)$ is the parameterized target value distribution, and the equality holds when the policy converges.

Proof. The proof is deferred to Appendix A. □

Remark 1. *Although the variational lower bound objective in our method (VDRL), as shown in Eq. (8), appears similar to QVPO [6], they differ fundamentally in both derivation and formulation:*

- ***Derivation****: Our variational lower bound is derived from the optimization objective of the value distribution, whereas QVPO derives its lower bound from the policy objective in RL.*
- ***Formulation****: The variational lower bound objective of QVPO incorporates Q-value under the specific condition, i.e.,* $Q(s, a) \geq 0$*, while our method maintains a straightforward form without introducing additional constraints.*

According to Theorem 1, we can derive the variational loss function of value diffusion learning as Eq. (10), which can be applied to value distribution optimization for distributional RL.

$$\mathcal{L}(\theta) = \mathbb{E}_{\substack{(s,a,r,s')\sim\mathcal{B}, a'\sim\pi_{\bar{\phi}}, \\ Z(s',a')\sim\mathcal{Z}_{\bar{\theta}}(\cdot|s',a'), \boldsymbol{\epsilon}, t}} \left[\left\| \boldsymbol{\epsilon} - \boldsymbol{\epsilon}_\theta \left(\sqrt{\bar{\alpha}_t} \mathcal{T}^\pi Z + \sqrt{1-\bar{\alpha}_t}\boldsymbol{\epsilon}, s, a, t \right) \right\|^2 \right], \tag{10}$$

Where $\mathcal{T}^\pi Z(s,a) = R(s,a) + \gamma Z(s',a')|_{Z(s',a')\sim\mathcal{Z}_{\bar{\theta}}(\cdot|s',a')}$.

3.2 Double Value Diffusion Learning with Sample Selection

While the value diffusion learning captures the inherent expressiveness and multimodality of the true value distribution, it also introduces a large variance, which may lead to unstable learning processes in some tasks. To address this problem, we propose double value diffusion learning with sample selection for value distribution $\mathcal{Z}(\cdot|s,a)$. Specifically, we parameterize two value distribution functions $\mathcal{Z}_{\theta_i}(\cdot|s,a)$, then choose sample with lower random return $Z(s',a')$ to calculate the target random return $\mathcal{T}^\pi Z(s,a)$, formally,

$$\mathcal{T}^\pi Z(s,a) = R(s,a) + \gamma \min_{i=1,2} Z(s',a')|_{Z(s',a')\sim\mathcal{Z}_{\bar{\theta}_i}(\cdot|s',a')}. \tag{11}$$

Furthermore, as shown in Table 2, this mechanism for value distribution can further reduce the value overestimation, and might induce a relatively minor value underestimation. In fact, as discussed in TD3 [22], the slight value underestimation is preferable to value overestimation, as unlike overestimation, underestimated value do not propagate explicitly through policy update during training.

3.3 Value Diffusion Reinforcement Learning

According to the above introduction, we present a practical model-free online off-policy RL algorithm called value diffusion reinforcement learning (VDRL), which includes two value distribution functions $\mathcal{Z}(\cdot|s,a)$ and a policy function $\pi(a|s)$. To that end, we use function approximators for both the value distribution and the policy, and alternate between optimizing both networks with stochastic gradient descent. Specifically, we consider two parameterized value distribution functions $\mathcal{Z}_{\theta_1}(\cdot|s,a), \mathcal{Z}_{\theta_2}(\cdot|s,a)$ and a parameterized policy function $\pi_\phi(a|s)$. The parameters of these networks are θ_1, θ_2, and ϕ, respectively. Furthermore, we employ the idea of a target network, where parameters of the target network can be an exponentially moving average of the main network weights, which has been shown to stabilize training in DQN [44]. According to the theorem 1 in section 3.1, the value distribution functions can be trained to minimize the variational objective,

$$J_{\mathcal{Z}}(\theta_i) = \mathbb{E}_{\substack{(s,a,r,s')\sim\mathcal{B}, a'\sim\pi_{\bar{\phi}}, \\ Z(s',a')\sim\mathcal{Z}_{\bar{\theta}_i}(\cdot|s',a')}} \left[\mathbb{E}_{\mathcal{T}^\pi Z_{1:T}\sim q(\mathcal{T}^\pi Z_{1:T}|s,a,\mathcal{T}^\pi Z_0)} \left[\log \frac{P_{\theta_i}(\mathcal{T}^\pi Z_{0:T}|s,a)}{q(\mathcal{T}^\pi Z_{1:T}|s,a,\mathcal{T}^\pi Z_0)} \right] \right], \tag{12}$$

Then, the policy function parameters can be learned by directly maximizing the expected cumulative state-action return $Z(s,a)$, formally,

$$J_\pi(\phi) = \mathbb{E}_{s\sim\mathcal{B}, a\sim\pi_\phi} \left[\min_{i=1,2} \mathbb{E}_{Z(s',a')\sim\mathcal{Z}_{\bar{\theta}_i}(s',a')} \left[Z(s,a) \right] \right]. \tag{13}$$

In implementation, we employ Monte Carlo estimation to approximate the expectation of each value distributions, i.e., $\mathbb{E}_{Z(s',a')\sim\mathcal{Z}_{\bar{\theta}_i}(s',a')}[Z(s,a)]$. For the sake of comprehensiveness, we provide a complete training process of the proposed VDRL method, as shown in Algorithm 1.

4 Related Work

In this section, we will look over existing works that are the most relevant to our proposed method. We divide our surveys into two main parts. Firstly, we provide an overview of distributional reinforcement

Algorithm 1 Value Diffusion Reinforcement Learning

Input: Diffusion-based value distribution networks $\{\mathcal{Z}_{\theta_i}(Z|s,a)\}_{i=1}^2$, target diffusion-based value distribution networks $\{\mathcal{Z}_{\overline{\theta}_i}(Z|s,a)\}_{i=1}^2$, policy network $\pi_\phi(a|s)$, target policy network $\pi_{\overline{\phi}}(a|s)$, the learning rate of value distribution and policy lr_θ, lr_ϕ, the momentum coefficient τ

Initializing: all networks parameters $\overline{\theta}_1 \leftarrow \theta_1$, $\overline{\theta}_2 \leftarrow \theta_2$, $\overline{\phi} \leftarrow \phi$;
for each iteration **do**
  Initialize the environment;
  for each sampling step **do**
    Select action $a \sim \pi_\phi(s,a)$;
    Execute action a to obtain the reward r and the next state s';
    Store $(s,a,r,s',done)$ in replay buffer $\mathcal{B}$;
  end for
  for each optimization step **do**
    Sample a random minibatch of N samples from replay buffer:$(s,a,r,s',done)$;
    Update the parameters of diffusion-based value distribution networks $\{\theta_i\}_{i=1}^2$: $\theta_i \leftarrow \theta_i - \beta_\theta \nabla_{\theta_i} J_{\mathcal{Z}}(\theta_i)$;
    Update the parameters of policy network ϕ: $\phi \leftarrow \phi + \beta_\phi \nabla_\phi J_\pi(\phi)$;
    Update the parameters of target diffusion-based value distribution networks $\overline{\theta}_i = \tau\theta_i + (1-\tau)\overline{\theta}_i$ for $i = \{1,2\}$;
    Update the parameters of target policy network $\overline{\phi} = \tau\phi + (1-\tau)\phi$;
  end for
end for

learning. Secondly, we discuss the diffusion-based RL methods that use diffusion models as polices for model-free online RL setups.

Distributional Reinforcement Learning. Recent interest in distributional methods [45, 46, 43] for RL has grown with the introduction of deep RL approaches for learning the value distribution. This paradigm was first formalized in the seminal work [26], which introduced the Categorical DQN (C51) algorithm that approximates the value distribution using a fixed categorical distribution over discrete support points and applies a distributional Bellman operator. Building on this foundation, subsequent works [27, 28, 29, 30, 31] leveraged quantile functions to represent the value distribution, enabling significant advancements. However, these approaches are inherently limited to discrete and low-dimensional action spaces. In order to deal with continuous control settings, methods such as [34, 35, 47, 48] have been proposed in recent years. Nevertheless, these approaches approximate the mean value through a weighted average of finite quantiles, which may reduce the accuracy of the mean value estimation, leading to suboptimal performance in some tasks. The distributional soft actor-critic (DSAC) [24] addressed the above limitation by learning a continuous Gaussian value distribution with the Q-value directly parameterized by the critic network, improving value estimation accuracy. Despite its effectiveness, DSAC suffers from training instability and sensitivity. To alleviate these issues, the term DSAC with three refinements (DSACT) [32] introduces three key refinements to refine Q-value estimation. However, both DSAC and DSACT constrain the value distribution and stochastic policy space to unimodal Gaussian distributions, which hinders exploration efficiency and learning performance, particularly in tasks with multiple goals or complex dynamics, where the true distribution often exhibits multimodality. In contrast, *the proposed VDRL method employs diffusion models to capture more expressive and multimodal representations of the value distribution, which further enhances value estimation and learning performance in tasks requiring sophisticated modeling of uncertainty and dynamics.*

Diffusion Models for Model-free Online Reinforcement Learning. Recently, numerous works [1, 49, 50, 51, 52, 53, 54, 2, 3] have employed diffusion models as the policy class in model-free offline RL to tackle decision-making tasks, due to their superior expressive richness in representing complex distributions. In contrast, diffusion policies have been less widely explored in model-free online RL, owing to the noisier and policy-varying nature of value estimations [16]. To date, only several studies [4, 5, 6, 7] have investigated the application of diffusion models for online RL. The pioneering work, DIPO [4], provided a solid convergence guarantee for diffusion policies and facilitated policy improvement by updating the action via the action gradient of the Q-value. However, fully learning of action gradient not only presents a significant challenge but also introduces additional computational costs. Unlike DIPO, Q-score matching (QSM) [5] proposed a novel policy update mechanism by aligning the score of the learned diffusion model, i.e., $\nabla_a \log(\pi(a|s))$ with the action gradient of the Q-value. Unfortunately, it neglects inaccuracies in the value function gradient and biases introduced during the alignment process when updating the policy. To mitigate these limitations, Q-weighted variational policy optimization (QVPO) [6] introduced the Q-weighted variational lower bound (VLO) loss, and then designed a tailored entropy regularization term for diffusion polices. Concurrently, Wang *et al*. [7] proposed the diffusion actor-critic with entropy regulator (DACER) method, and employed Gaussian mixture models for entropy estimation. Despite these great advances, fundamental challenges persist: approximation inaccuracies and computational inefficiencies continue to constrain the effectiveness of diffusion models in model-free online RL. Different from all existing model-free online RL methods with diffusion policy, *from the value perspective, the proposed VDRL method utilizes diffusion models to represent value distributions, which enhances the accuracy of value estimation, yielding significantly improved performance.*

5 Experiments

In this section, we empirically validate the effectiveness of the proposed algorithm on different decision-making tasks and with various hyperparameters. With these experimental results, we aim to answer the following questions: 1) How does VDRL compare to traditionally canonical and existing diffusion-based model-free online RL methods? 2) What contributes to the performance improvement of VDRL on the continuous locomotion tasks? Note that all experiments are conducted on a 2.90GHz Intel Core i7-10700 CPU, 64G RAM, and NVIDIA GeForce RTX 3090 GPU.

5.1 Experimental Setup

Evaluation Environment. In this work, we use eight different level tasks of the popular MuJoCo [33] benchmark [3] to evaluate the performance of all methods, including Ant-v3, HalfCheetah-v3, Hopper-v3, Humanoid-v3, Inverted2Pendulum-v2, Pusher-v2, Swimmer-v3, and Walker2d-v3. The more details of these tasks and the MuJoCo benchmark are deferred to Appendix B.

Compared Baselines. We evaluate the performance of VDRL against eleven model-free online RL algorithms, categorized into three groups. The first group comprises **traditional model-free RL** baselines, including two on-policy methods (PPO [55] and SPO [56]) and three off-policy algorithms (DDPG [57], TD3 [22], and SAC [23]). The second group focuses on **distributional RL** approaches, represented by DSAC [24] and DSACT [32]. Finally, the third group consists of **advanced diffusion-based policy RL** methods, including DIPO [4], QSM [5], QVPO [6], and DACER [7]. The more details of implementation and hyperparameters for all baselines are deferred to Appendix D.

Methodology for Reported Metrics. For the proposed method and all baselines, all experiments are conducted over 800,000 training interaction steps with four runs and different random seeds (0, 50, 100, 200), with each performing one evaluation rollout every 10,000 interaction steps. In particular, each evaluation result is the average of ten episodes. The mean return, standard deviation, and average highest return are logged as the performance of methods.

5.2 Details of Implementation

The value distribution is modeled using the reverse process of a conditional diffusion model, parameterized similarly to DDPM [15], as detailed in Section 2.3. The effects of reverse diffusion steps and noise schedules are analyzed in Section 5.4. For the policy, a Gaussian distribution with a mean and diagonal covariance is adopted. The policy network architecture consists of two hidden layers, each with 256 units and GeLU activation functions. Parameter updates are performed using the Adam optimizer. A detailed description of all training hyperparameters is deferred to Appendix C.

5.3 Comparative Evaluation

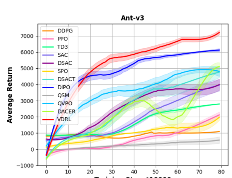
The best performance of the proposed method (VDRL) and all baselines across eight MuJoCo benchmark tasks is presented in Table 1. VDRL achieves results comparable to baseline methods

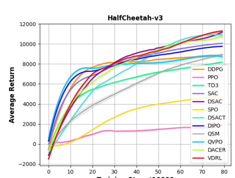
[3] The version of MuJoCo utilized in all experiments of this work is mujoco210, which is the same version used as DSAC [24], QVPO [6], DACER [7], and DSACT [32].

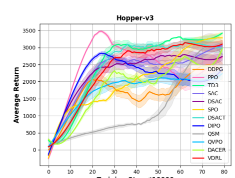
Table 1: Performance on eight tasks of OpenAI Gym MuJoCo benchmark. The results show the best mean returns and standard deviations over 800,000 training interaction steps and four random seeds.

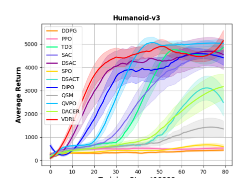
	METHOD	ANT-V3	HALFCHEETAH-V3	HOPPER-V3	HUMANOID-V3
Traditional Model-Free RL	PPO [55]	2129.33 ± 98.68	1757.57 ± 25.86	3291.76 ± 18.28	612.15 ± 117.34
	SPO [56]	2019.02 ± 46.91	4725.37 ± 102.64	3480.20 ± 92.66	749.70 ± 131.01
	DDPG [57]	1124.35 ± 51.49	8993.40 ± 140.39	3326.17 ± 12.22	597.52 ± 104.79
	TD3 [22]	2836.68 ± 41.87	8306.89 ± 82.31	3497.59 ± 4.39	5000.91 ± 13.38
	SAC [23]	4270.41 ± 130.00	10214.06 ± 98.44	3380.51 ± 8.14	5314.84 ± 43.05
Distributional RL	DSAC [24]	4264.95 ± 215.50	11610.33 ± 135.05	3436.95 ± 13.14	5009.60 ± 271.94
	DSACT [32]	5313.04 ± 122.46	11670.82 ± 131.81	3654.39 ± 7.06	5087.93 ± 165.00
Diffusion-based Policy RL	DIPO [4]	6224.93 ± 82.27	10052.71 ± 49.81	3454.60 ± 37.90	5266.87 ± 5.93
	QSM [5]	839.53 ± 510.14	9204.16 ± 85.61	3513.23 ± 22.65	1950.42 ± 1270.96
	QVPO [6]	5222.62 ± 78.02	9067.63 ± 72.47	3369.51 ± 439.56	5252.72 ± 13.75
	DACER [7]	5506.40 ± 99.67	11350.96 ± 119.48	3488.54 ± 2.79	4447.02 ± 750.42
	VDRL (Ours)	$\mathbf{7236.67 \pm 89.44}$	$\mathbf{11815.20 \pm 142.04}$	$\mathbf{3679.41 \pm 7.47}$	$\mathbf{5497.15 \pm 48.58}$
	METHOD	PUSHER-V2	SWIMMER-V3	WALKER2D-V3	INVERTED2PENDULUM-V2
Traditional Model-Free RL	PPO [55]	-22.92 ± 3.47	137.59 ± 1.35	3168.82 ± 104.73	9359.67 ± 0.21
	SPO [56]	-22.29 ± 4.99	136.06 ± 1.41	3849.82 ± 245.47	9359.53 ± 0.35
	DDPG [57]	-35.53 ± 4.94	42.64 ± 1.70	3478.72 ± 1185.37	9359.74 ± 0.22
	TD3 [22]	-23.06 ± 2.33	75.78 ± 0.86	3834.99 ± 64.33	92.13 ± 17.69
	SAC [23]	$\mathbf{-20.28 \pm 1.62}$	77.49 ± 3.07	4486.88 ± 42.63	9359.83 ± 0.15
Distributional RL	DSAC [24]	-31.43 ± 3.93	140.76 ± 9.20	4819.56 ± 171.21	9359.83 ± 0.08
	DSACT [32]	-25.65 ± 2.63	134.46 ± 1.96	5134.79 ± 22.68	9359.75 ± 0.11
Diffusion-based Policy RL	DIPO [4]	-39.60 ± 5.50	55.10 ± 2.01	4508.09 ± 10.41	9359.85 ± 0.16
	QSM [5]	-46.69 ± 3.41	105.04 ± 2.77	3897.01 ± 721.01	2475.43 ± 89.95
	QVPO [6]	-38.52 ± 5.74	124.14 ± 1.44	4198.86 ± 46.44	9354.38 ± 0.86
	DACER [7]	-24.91 ± 3.23	$\mathbf{151.55 \pm 1.04}$	4053.14 ± 62.19	9359.79 ± 0.14
	VDRL (Ours)	$\mathbf{-26.92 \pm 3.95}$	$\mathbf{144.40 \pm 2.27}$	$\mathbf{5223.77 \pm 10.30}$	$\mathbf{9359.89 \pm 0.16}$

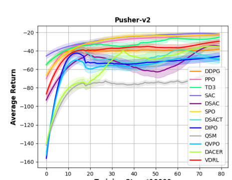
on simpler tasks such as Pusher-v2, Swimmer-v3, and InvertedPendulum-v2, while significantly outperforming them on more complex locomotion tasks, including Ant-v3, Hopper-v3, Humanoid-v3, and Walker2d-v3. The Figure 1 illustrates the learning curves of all algorithms, further demonstrating that VDRL delivers stable and consistently high performance across all evaluated tasks, highlighting its superior robustness. In contrast, baseline methods struggle with poor performance on one or more tasks, emphasizing the limitations of existing approaches.

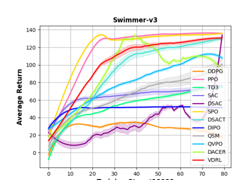

(a) Ant-v3

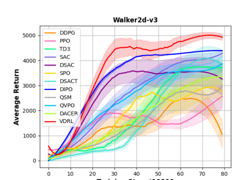

(b) HalfCheetah-v3


(c) Hopper-v3


(d) Humanoid-v3


(e) Pusher-v2


(f) Swimmer-v3


(g) Walker2d-v3

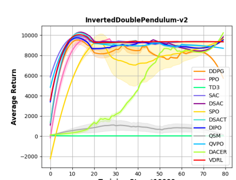

(h) Inverted2Pendulum-v2

Figure 1: Learning curves of the proposed method and all baselines on 8 continuous tasks of MuJoCo benchmark over 10 evaluation episodes every 10k iterations, where the solid lines correspond to the mean and the shaded regions to standard deviations over 4 different random seeds.

Table 2 shows the average value estimation bias, i.e., the difference between the estimated Q-value and the true Q-value, for VDRL and some baselines, including traditional model-free online RL methods and distributional RL approaches. The true Q-value is approximated by the average actual discounted cumulative return across states over ten episodes, evaluated every 10,000 iterations. Notably, the value estimation bias for the InvertedPendulum-v2 task is excluded, as effective policies are learned before the value function converges. The experimental results yield several key insights. First, compared to conventional RL algorithms, both VDRL and distributional RL baselines demonstrate reduced value estimation bias in most tasks, underscoring the effectiveness of value distribution learning in mitigating estimation inaccuracies. Second, VDRL consistently achieves lower bias than DASC and DSACT, highlighting the advantages of leveraging diffusion models to capture multimodal and complex value distributions for improved estimation accuracy. Last, even when compared to on-policy RL methods such as PPO and SPO, VDRL exhibits a significant advantage in value estimation accuracy across most tasks, further validating its effectiveness.

Table 2: Average value estimation bias over four runs for the proposed VDRL method, conventional model-free RL, and distributional RL baselines on eight tasks of MuJoCo benchmark. In particular, the bias is calculated as the difference between the estimated Q-value Q_{est} and the true Q-value Q_{true}, i.e., $Q_{\text{est}} - Q_{\text{true}}$, where the true Q-value is accessed based on the discounted accumulation of rewards. Furthermore, the name of each task are simplified because of the space limitation.

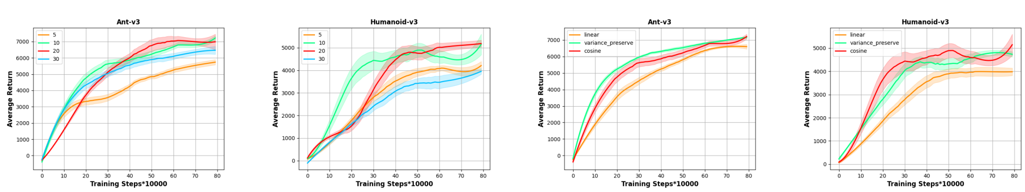
	METHOD	ANT3	HAL3	HOP3	HUM3	PUS2	SWI3	WAL3
Traditional on-policy RL	PPO [55]	9.45	320.36	271.29	16.74	-18.05	0.94	2.51
	SPO [56]	11.52	182.19	275.74	15.26	-21.35	0.88	3.60
Conventional off-policy RL	DDPG [57]	94.63	37.27	476.89	49.08	7.93	12.30	126.04
	TD3 [22]	-373.20	-372.05	-929.33	-225.06	-19.76	-3.07	-58.56
	SAC [23]	-27.94	-6.28	251.92	-92.48	-13.51	-1.06	-3.14
Distributional RL	DSAC [24]	37.94	83.47	2264.98	65.72	-5.15	1.84	69.93
	DSACT [32]	-25.17	36.76	-181.54	-59.24	$\mathbf{-4.73}$	-1.13	-9.80
	VDRL (Ours)	$\mathbf{-7.36}$	$\mathbf{-4.13}$	$\mathbf{-113.70}$	$\mathbf{-12.69}$	$\mathbf{-6.85}$	$\mathbf{0.42}$	$\mathbf{-0.95}$

5.4 Sensitivity Analysis

We perform sensitivity analysis on different diffusion steps and diffusion noise schedule, using the Ant-v3 and Humanoid-v3 tasks as examples.

Diffusion Steps. We evaluate the performance of VDRL under varying diffusion steps ($T = 5, 10, 20, 30$), as shown in Figure 2(a)-2(b). The results indicate that the performance does not improve monotonically with an increasing number of diffusion steps. Adopting larger diffusion steps degrades the performance of VDRL, potentially due to gradient vanishing or exploding issues. Additionally, larger diffusion steps will lead to increased training and evaluation costs. To balance the performance and computational efficiency, we choose 10 diffusion steps for all tasks.

Diffusion Noise Schedule. Figure 2(c)-2(d) illustrates the performance of VDRL with distinct diffusion noise schedules. It can be observed that the cosine and variance preserve schedule obtain comparable results, and both outperform the linear schedule. Thus, we select the cosine schedule for all experiments.



(a) Diffusion Steps (b) Diffusion Steps (c) Noise Schedule (d) Noise Schedule

Figure 2: Ablation learning curves for different diffusion steps and diffusion noise schedule on Ant-v3 and Humanoid-v3 tasks, where the solid lines correspond to the mean and the shaded regions to standard deviations over 4 different random seeds.

6 Conclusion and Discussion

In this work, we presented a novel model-free online RL method called value diffusion reinforcement learning (VDRL) to overcome the representational limitations of learned value distributions. By harnessing the inverse denoising process of diffusion models, VDRL captures expressive, multimodal value distributions, thereby substantially improving value estimation. The theoretical cornerstone of VDRL lies in its diffusion-based variational loss, which we rigorously establish as a tight variational lower bound for optimizing value distributions under the KL-divergence measurement. Furthermore, to improve the training stability and further enhance value estimation, we introduce double value diffusion learning with sample selection. Comprehensive experiments on eight continuous control tasks from the MuJoCo benchmark demonstrate the effectiveness and robustness of VDRL.

Limitations. While value diffusion learning presents a promising advance, its application to online RL faces several unresolved challenges. For instance, the reliance on iterative value diffusion sampling incurs significantly increased computational costs than traditional model-free RL and distributional RL methods, especially in high-dimensional action spaces. In addition, this work is evaluated primarily on MuJoCo benchmarks, leaving its scalability to real-world robotics applications with partial observability and sensor noise as an open question.

Future Work. In our future research, we will pursue two key avenues to advance this paradigm. First, we will develop lightweight diffusion models, potentially through knowledge distillation, to reduce the training and inference latency. Second, we will explore hybrid methods that integrate value diffusion with model-based planning, aiming to tackle the challenges of complex, long-horizon tasks and thereby unlock further performance gains in online RL.

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A Proof

Theorem 1. ***(Lower Bound of Learning Objective for Value Distribution)*** *If the value distribution is modeled by a diffusion model, the variational bound objective*

$$\mathbb{E}_{\substack{(s,a,r,s')\sim\mathcal{B},a'\sim\pi_{\bar{\phi}},\\ Z(s',a')\sim\mathcal{Z}_{\bar{\theta}}(\cdot|s',a')}}\left[\mathbb{E}_{\mathcal{T}^\pi Z_{1:T}\sim q(\mathcal{T}^\pi Z_{1:T}|s,a,\mathcal{T}^\pi Z_0)}\left[\log\frac{P_\theta(\mathcal{T}^\pi Z_{0:T}|s,a)}{q(\mathcal{T}^\pi Z_{1:T}|s,a,\mathcal{T}^\pi Z_0)}\right]\right], \tag{14}$$

is the tight lower bound of the optimization objective of value distribution learning for distributional RL under the KL-divergence measurement

$$\mathbb{E}_{(s,a)\sim\mathcal{B}}\left[D_{KL}(\mathcal{T}^\pi\mathcal{Z}_{\bar{\theta}}(\cdot|s,a)\parallel\mathcal{Z}_\theta(\cdot|s,a))\right], \tag{15}$$

where $\mathcal{Z}_\theta(\cdot|s,a)$ denotes the value distribution to be optimized and $\mathcal{Z}_{\bar{\theta}}(\cdot|s,a)$ is the parameterized target value distribution, and the equality holds when the policy converges.

Proof. The optimization objective of value distribution learning under the KL-divergence measurement is

$$\min_\theta\left\{\mathop{\mathbb{E}}_{(s,a)\sim\mathcal{B}}\left[D_{\mathrm{KL}}(\mathcal{T}^\pi\mathcal{Z}_{\bar{\theta}}(\cdot|s,a)\parallel\mathcal{Z}_\theta(\cdot|s,a))\right]\right\}, \tag{16}$$

where $\mathcal{B}$ is the replay buffer of historical samples, $\mathcal{Z}_\theta(\cdot|s,a))$ denotes the value distribution to be optimized, and $\mathcal{Z}_{\bar{\theta}}(\cdot|s,a))$ is the parameterized target value distribution. According to the definition of KL-divergence, we can show that

$$\begin{aligned}
J_{\mathcal{Z}}(\theta) &= \mathop{\mathbb{E}}_{(s,a)\sim\mathcal{B}}\left[D_{\mathrm{KL}}(\mathcal{T}^\pi\mathcal{Z}_{\bar{\theta}}(\cdot|s,a)\parallel\mathcal{Z}_\theta(\cdot|s,a))\right]\\
&= \mathop{\mathbb{E}}_{(s,a)\sim\mathcal{B}}\left[\int_{\mathcal{T}^\pi Z(s,a)} P_{\bar{\theta}}(\mathcal{T}^\pi Z(s,a))\log\frac{P_{\bar{\theta}}(\mathcal{T}^\pi Z(s,a))}{P_\theta(\mathcal{T}^\pi Z(s,a))}\right]\\
&= -\mathop{\mathbb{E}}_{(s,a)\sim\mathcal{B}}\left[\int_{\mathcal{T}^\pi Z(s,a)} P_{\bar{\theta}}(\mathcal{T}^\pi Z(s,a))\log P_\theta(\mathcal{T}^\pi Z(s,a))\right]+c\\
&= -\mathop{\mathbb{E}}_{(s,a)\sim\mathcal{B}}\left[\mathbb{E}_{\mathcal{T}^\pi Z(s,a)\sim\mathcal{T}^\pi\mathcal{Z}_{\bar{\theta}}(\cdot|s,a)}\left[\log P_\theta(\mathcal{T}^\pi Z(s,a))\right]\right]+c\\
&= -\mathop{\mathbb{E}}_{\substack{(s,a,r,s')\sim\mathcal{B},a'\sim\pi_{\bar{\phi}},\\ Z(s',a')\sim\mathcal{Z}_{\bar{\theta}}(\cdot|s',a')}}\left[\log P_\theta(\mathcal{T}^\pi Z(s,a))\right],
\end{aligned} \tag{17}$$

where the constant c denotes the term independent of θ, $\pi_{\bar{\phi}}$ denotes the parameterized target policy, and $\mathcal{T}^\pi Z(s,a)=R(s,a)+\gamma Z(s',a')$. For simplicity of exposition, we let Y denotes $\mathcal{T}^\pi Z(s,a)$. Thus, the optimization objective of value distribution learning can be formalized as follows:

$$\max_\theta\left\{\mathop{\mathbb{E}}_{\substack{(s,a,r,s')\sim\mathcal{B},a'\sim\pi_{\bar{\phi}},\\ Z(s',a')\sim\mathcal{Z}_{\bar{\theta}}(\cdot|s',a')}}\left[\log P_\theta(Y)\right]\right\}. \tag{18}$$

If $\mathcal{Z}_\theta(\cdot|s,a)$ is a diffusion-based value distribution, we have

$$\begin{aligned}
\log P_\theta(Y) =&\mathbb{E}_{Y_{1:T}\sim q(Y_{1:T}|s,a,Y_0)}\left[\log\frac{P_\theta(Y_{1:T}|s,a,Y_0)}{P_\theta(Y_{1:T}|s,a,Y_0)}P_\theta(Y_0)\right]\\
=&\mathbb{E}_{Y_{1:T}\sim q(Y_{1:T}|s,a,Y_0)}\left[\log\frac{P_\theta(Y_{0:T}|s,a,Y_0)P_\theta(Y_0)}{P_\theta(Y_{1:T}|s,a,Y_0)}\right]\\
=&\mathbb{E}_{Y_{1:T}\sim q(Y_{1:T}|s,a,Y_0)}\left[\log\frac{P_\theta(Y_{0:T}|s,a)}{q(Y_{1:T}|s,a,Y_0)}\frac{q(Y_{1:T}|s,a,Y_0)}{P_\theta(Y_{1:T}|s,a,Y_0)}\right]\\
=&\mathbb{E}_{Y_{1:T}\sim q(Y_{1:T}|s,a,Y_0)}\left[\log\frac{P_\theta(Y_{0:T}|s,a)}{q(Y_{1:T}|s,a,Y_0)}\right]+D_{\mathrm{KL}}(q(Y_{1:T}|s,a,Y_0)\parallel P_\theta(Y_{1:T}|s,a,Y_0)).
\end{aligned} \tag{19}$$

According to the inequality of Jensen, the term $D_{\text{KL}}(q(Y_{1:T}|s,a,Y_0) \parallel P_\theta(Y_{1:T}|s,a,Y_0))$ in Eq. (19) can be shown that

$$\begin{aligned}
D_{\text{KL}}(q(Y_{1:T}|s,a,Y_0) \parallel P_\theta(Y_{1:T}|s,a,Y_0)) =& \mathbb{E}_{Y_{1:T}\sim q(Y_{1:T}|s,a,Y_0)}\left[\log\frac{q(Y_{1:T}|s,a,Y_0)}{P_\theta(Y_{1:T}|s,a,Y_0)}\right] \\
=& \mathbb{E}_{Y_{1:T}\sim q(Y_{1:T}|s,a,Y_0)}\left[-\log\frac{P_\theta(Y_{1:T}|s,a,Y_0)}{q(Y_{1:T}|s,a,Y_0)}\right] \\
\geq& -\log\left(\mathbb{E}_{Y_{1:T}\sim q(Y_{1:T}|s,a,Y_0)}\left[\frac{P_\theta(Y_{1:T}|s,a,Y_0)}{q(Y_{1:T}|s,a,Y_0)}\right]\right).
\end{aligned} \tag{20}$$

When the learning policy converges, the term $\frac{P_\theta(Y_{1:T}|s,a,Y_0)}{q(Y_{1:T}|s,a,Y_0)}$ is equal to 1, and we can show that

$$\begin{aligned}
J_{\mathcal{Z}}(\theta) =& \mathop{\mathbb{E}}_{(s,a)\sim\mathcal{B}}\left[D_{\text{KL}}(\mathcal{T}^\pi \mathcal{Z}_{\bar{\theta}}(\cdot|s,a) \parallel \mathcal{Z}_\theta(\cdot|s,a))\right] \\
=& -\mathop{\mathbb{E}}_{\substack{(s,a,r,s')\sim\mathcal{B},a'\sim\pi_{\bar{\phi}},\\ Z(s',a')\sim\mathcal{Z}_{\bar{\theta}}(\cdot|s',a')}}\left[\log P_\theta(Y)\right] \\
\geq& -\mathop{\mathbb{E}}_{\substack{(s,a,r,s')\sim\mathcal{B},a'\sim\pi_{\bar{\phi}},\\ Z(s',a')\sim\mathcal{Z}_{\bar{\theta}}(\cdot|s',a')}}\left[\mathbb{E}_{Y_{1:T}\sim q(Y_{1:T}|s,a,Y_0)}\left[\log\frac{P_\theta(Y_{0:T}|s,a)}{q(Y_{1:T}|s,a,Y_0)}\right]\right]
\end{aligned} \tag{21}$$

Thus, the variational bound objective

$$\mathbb{E}_{\substack{(s,a,r,s')\sim\mathcal{B},a'\sim\pi_{\bar{\phi}},\\ Z(s',a')\sim\mathcal{Z}_{\bar{\theta}}(\cdot|s',a')}}\left[\mathbb{E}_{\mathcal{T}^\pi Z_{1:T}\sim q(\mathcal{T}^\pi Z_{1:T}|s,a,\mathcal{T}^\pi Z_0)}\left[\log\frac{P_\theta(\mathcal{T}^\pi Z_{0:T}|s,a)}{q(\mathcal{T}^\pi Z_{1:T}|s,a,\mathcal{T}^\pi Z_0)}\right]\right], \tag{22}$$

is the tight lower bound of the learning objective of value distribution for distributional RL under the KL-divergence measurement

$$\mathbb{E}_{(s,a)\sim\mathcal{B}}\left[D_{\text{KL}}(\mathcal{T}^\pi \mathcal{Z}_{\bar{\theta}}(\cdot|s,a) \parallel \mathcal{Z}_\theta(\cdot|s,a))\right]. \tag{23}$$

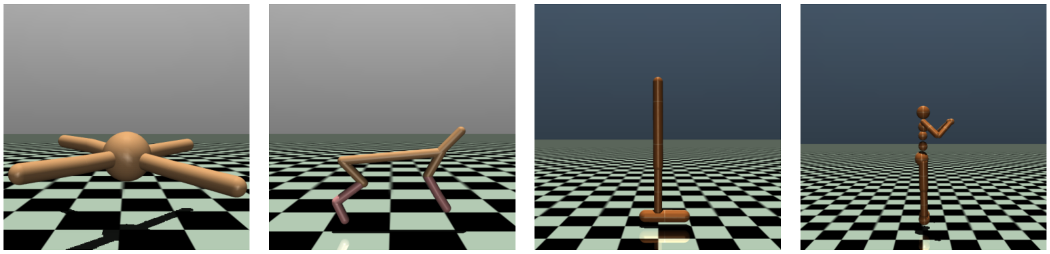
This completes the proof of Theorem 1. □

B Environmental Details

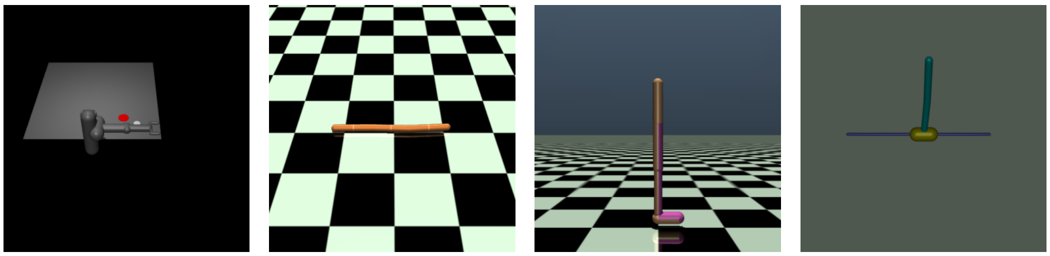
MuJoCo (Multi-Joint dynamics with Contact) [33] is a high-performance physics engine widely used in robotics and biomechanics simulations, renowned for its efficiency and accuracy in modeling complex dynamics. It provides a versatile platform for developing and benchmarking reinforcement learning (RL) algorithms, enabling the simulation of articulated bodies with intricate interactions, including friction, contact, and soft constraints. These features make it particularly suitable for tasks involving multi-joint movement and physical interactions.

In this work, we use eight MuJoCo benchmark tasks, categorized into three classes based on their dynamics and objectives: **locomotion**, **control**, and **navigation**. Specifically, the locomotion tasks focus on the ability of agents to move efficiently across a terrain, including Ant-v3, HalfCheetah-v3, Hopper-v3, Humanoid-v3, and Walker2d-v3. The control tasks, such as Inverted2Pendulum-v2 and Pusher-v2, require agents to manipulate objects or maintain certain positions. The navigation task, exemplified by Swimmer-v3, evaluates the agent's ability to move through a fluid medium. Figure 3 shows the visualization of all tasks used in this study. Furthermore, these tasks are detailed as follows:

- **Ant-v3**: $(s \times a) \in \mathbb{R}^{111} \times \mathbb{R}^8$, a four-legged robot learns to navigate and cover distance by coordinating its legs, with key challenges in balancing and adapting to varying terrains.
- **HalfCheetah-v3**: $(s \times a) \in \mathbb{R}^{17} \times \mathbb{R}^6$, a bipedal robot emulates the running motion of a cheetah, with the objective to maximize forward speed while maintaining stability.
- **Hopper-v3**: $(s \times a) \in \mathbb{R}^{11} \times \mathbb{R}^3$, a single-legged robot learns to hop forward efficiently, focusing on balance and joint control to maximize distance.
- **Humanoid-v3**: $(s \times a) \in \mathbb{R}^{376} \times \mathbb{R}^{17}$, a humanoid robot must learn to walk and run, tackling the complex challenge of maintaining balance during motion.



(a) Ant-v3 (b) HalfCheetah-v3 (c) Hopper-v3 (d) Humanoid-v3



(e) Pusher-v2 (f) Swimmer-v3 (g) Walker2d-v3 (h) Inverted2Pendulum-v2

Figure 3: The screenshots of MuJoCo tasks utilized in this work.

- **Walker2d-v3**: $(s \times a) \in \mathbb{R}^{17} \times \mathbb{R}^{6}$, a two-legged robot learns to walk forward, emphasizing coordination and adaptability in dynamic environments.
- **Inverted2Pendulum-v2**: $(s \times a) \in \mathbb{R}^{6} \times \mathbb{R}^{1}$, a classic control task where the agent balances an inverted pendulum on a pivot, requiring precise application of forces to maintain stability.
- **Pusher-v2**: $(s \times a) \in \mathbb{R}^{23} \times \mathbb{R}^{7}$, the agent controls a robot arm to push a block to a target location, testing precision in force application and obstacle navigation.
- **Swimmer-v3**: $(s \times a) \in \mathbb{R}^{8} \times \mathbb{R}^{2}$, a two-dimensional swimmer propels itself forward by coordinating body segments, emphasizing control over body dynamics in a fluid-like environment.

C Hyperparameters

The setting of all hyperparameters for the training is presented in Table 3.

Table 3: Hyperparameters for training.

name	Value	Description
Optim	Adam	the optimizer of method
n_iteration	8×10^5	Maximum iteration steps until the end of training
buffer_size	1×10^6	capacity of replay buffer
batch_size	256	number of samples from each update
evaluate_cycle	10000	how often to evaluate the model
No. of hidden layers	2	the number of hidden layers
No. of hidden nodes	256	the number of hidden nodes
Activation_π	GeLU	the activation of policy
Activation_$\mathcal{Z}_i$	Mish	the activation of value distributions
lr_ϕ	3×10^{-4}	learning rate for policy
lr_{θ_i}	3×10^{-4}	learning rate for value distributions
τ	0.005	the target momentum coefficient
γ	0.99	discount factor
T	10	diffusion step

D Details of the baseline methods

In this section, we discuss details of the baseline methods with which we compare our method, including the implementation and hyperparameters of each algorithm.

DDPG [57] We use the high-quality implementations of reinforcement learning algorithms, i.e., OpenAI Baselines [58]

`https://github.com/openai/baselines`

with configuration `baselines/ddpg/ddpg.py` to evaluate the DDPG baseline on the MuJoCo benchmark.

PPO [55]. We use the code from the authors' repository

`https://github.com/ikostrikov/pytorch-a2c-ppo-acktr-gail`

for their evaluation in continuous control tasks, i.e., the MuJoCo benchmark. We employ their configuration `/a2c_ppo_acktr/arguments.py`.

TD3 [22]. We use the official code from

`https://github.com/sfujim/TD3`

with hyper-parameters from `main.py` for the MuJoCo experiments.

SAC [23]. We utilize the code from

`https://github.com/toshikwa/soft-actor-critic.pytorch`

with their configuration `code/main.py` for the MuJoCo continuous control tasks in OpenAI gym, which is the PyTorch implementation of Soft Actor-Critic.

DSAC [24]. We use the code from

`https://github.com/Jingliang-Duan/DSAC-v1`

with their configuration `example_train/main.py` for the evaluation of MuJoCo continuous control tasks, which is the PyTorch implementation of Distributional Soft Actor-Critic.

DSACT [23]. We utilize the code from

`https://github.com/Jingliang-Duan/DSAC-v2`

with their configuration `example_train/dsacv2_mlp_mujoco_offserial.py` for the MuJoCo benchmark, which is the PyTorch implementation of Distributional Soft Actor-Critic with Three Refinements.

SPO [56]. We use the official code from

`https://github.com/MyRepositories-hub/Simple-Policy-Optimization`

with hyper-parameters from `mujoco/main.py`.

DIPO [4]. We utilize the code from the authors' repository

`https://github.com/BellmanTimeHut/DIPO`

with hyper-parameters from `main.py`.

QSM [5]. We provide a PyTorch implementation of the QSM baseline based on the authors' repository, which is built on top of a re-implementation of the JAXRL framework

`https://github.com/Alescontrela/score_matching_rl`

with hyper-parameters from `examples/states/configs/score_matching_config.py`.

QVPO [6]. We use the official code from `https://github.com/wadx2019/qvpo`

with hyper-parameters from `main.py`.

DACER [7]. For a fair comparison, we provide an implementation of the DACER method with PyTorch, which is based on the official code (JAX) from

`https://github.com/happy-yan/DACER-Diffusion-with-Online-RL`

with hyper-parameters from `scripts/train_mujoco.py`.

E Training and Inference Time

The training and inference time of VDRL and all baselines on three tasks of MuJoCo, i.e., Ant-v3, Humanoid-v3, and Walker2d-v3, are shown in the Table 4-5. In particular, Notably, the official code of the QSM and DACER methods is based on the JAX framework, but the implementation of other baselines and our method is based on PyTorch. In practice, the algorithm realized with JAX is 6-10 times faster than that realized with PyTorch [6]. To ensure a fair comparison, we utilize the implementation of the QSM and DACER methods with PyTorch, which is based on the official code (JAX). Furthermore, the number of diffusion steps for all diffusion-based online RL baselines (DIPO, QSM, QVPO, and DACER) is set to 20.

Table 4: The training time (h) comparison on three tasks of MuJoCo Benchmarks.

	VDRL	**DIPO**	**QSM**	**QVPO**	**DACER**	*DSAC*	*DSACT*	PPO	SAC
Ant-v3	7.9	11.1	10.2	9.1	9.8	5.3	5.8	0.4	3.8
Humanoid-v3	8.2	11.5	10.7	9.4	10.1	5.5	6.1	0.5	3.9
Walker2d-v3	7.8	11.0	9.9	8.9	9.6	5.2	5.7	0.4	3.8

Table 5: The inference time (ms) comparison on three tasks of MuJoCo Benchmarks.

	VDRL	**DIPO**	**QSM**	**QVPO**	**DACER**	*DSAC*	*DSACT*	PPO	SAC
Ant-v3	3.6	5.8	6.4	6.2	5.4	1.9	2.1	0.2	0.4
Humanoid-v3	3.7	6.0	6.5	6.3	5.5	2.0	2.2	0.2	0.5
Walker2d-v3	3.6	5.7	6.3	6.1	5.3	1.9	2.1	0.2	0.4

From the results of the above tables, we can observe:

i) The training and inference time of VDRL is faster than the diffusion-based online RL methods (i.e., DIPO, QSM, QVPO, and DACER), because the lower diffusion steps of VDRL (T=10) compared with diffusion-based RL baselines (T=20) reduce the computational complexity.

ii) All diffusion-based RL methods are slower than distributional (i.e., DSAC and DSACT) or traditional (i.e., SAC and PPO) ones, due to the iterative denoising process with some steps. Although VDRL is slower than the traditional or distributional RL methods in inference, the inference times (3.7ms) are still tolerable in a real-time application. As discussed in QVPO [6], most existing real robots only require a 50-Hz control policy, i.e., output action per 20 ms. Moreover, the training and inference time of VDRL can be further reduced by using the JAX framework if it is necessary, like the official code of QSM and DACER methods. Hence, the inference time is not a bottleneck to applying our method to real-time applications.

Overall, the performance improvements of VDRL are relatively worth it with appropriate longer training and inference time.

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