ROBUST META-LEARNING WITH SAMPLING NOISE AND LABEL NOISE VIA EIGEN-REPTILE

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ABSTRACT

Recent years have seen a surge of interest in meta-learning techniques for tackling the few-shot learning (FSL) problem. However, the meta-learner is prone to overfitting since there are only a few available samples with sampling noise on a clean dataset. More importantly, when handling the data sampled with noisy labels, meta-learner could be extremely sensitive to label noise on a corrupted dataset. To address these two challenges, we present Eigen-Reptile (ER) that updates the meta-parameters with the main direction of historical task-specific parameters to alleviate sampling and label noise. Specifically, the main direction is computed in a fast way for the required large-scale matrix. Furthermore, to obtain a more accurate main direction for Eigen-Reptile in the presence of label noise, we further propose Introspective Self-paced Learning (ISPL). We have theoretically and experimentally demonstrated the soundness and effectiveness of the proposed Eigen-Reptile and ISPL. Particularly, our experiments on different tasks show that the proposed method is able to outperform or achieve highly competitive performance compared with other gradient-based methods with or without noisy labels.

1 INTRODUCTION

Meta-learning, also known as learning to learn, is the key for few-shot learning (FSL) (Vinyals et al., 2016; Chi et al., 2021). One line of the meta-learning methods are gradient-based, which usually optimize meta-parameters as initialization that can fast adapt to new tasks with few samples. However, fewer samples often lead to a higher risk of overfitting (Zintgraf et al., 2019), as the result of the ubiquitous sampling noise and label noise in practice. Particularly, a popular first-order method, Reptile (Nichol et al., 2018), updates the meta-parameters towards the inner loop direction, which is from the current initialization to the last task-specific parameters.

For sampling noise on a clean dataset, as shown by the bold line of *Reptile* in Figure 1, with the gradient update at the last step, the update direction of meta-parameters has a significant disturbance. It is because sampling noise leads the meta-parameters to overfit on the sampled samples at gradient steps. Many prior works have proposed different solutions for the meta-overfitting problem (Zintgraf et al., 2019), such as using dropout (Bertinetto et al., 2018; Lee et al., 2020), and modifying the loss function (Jamal & Qi, 2019) etc.. This paper casts the meta-overfitting problem as a gradient noise problem that from sampling noise while performing gradient update (Wu et al., 2019). Neelakantan et al. (2015) and other works have proved that introducing additional gradient noise can improve the generalization of neural networks with a large number of samples. However, for FSL, there are only a few samples of each task, and the model will not only learns the contents that need to be identified but also overfits the noise (Zhang et al., 2016).

Meta-learner is also inevitably affected by label noise because of the required large number of tasks. More specifically, high-quality manual labeling data is often time-consuming and expensive. And low-cost approaches to collect low-quality annotated data, such as from search engines, will introduce noisy labels. Conceptually, the initialization learned by existing meta-learning algorithms can severely degrade in the presence of noisy labels. As shown in *Reptile with noisy labels* of Figure 1, noisy labels cause a significant random disturbance in the update direction of Reptile. Furthermore, conventional algorithms about learning with noisy labels require much more data for each class (Yao

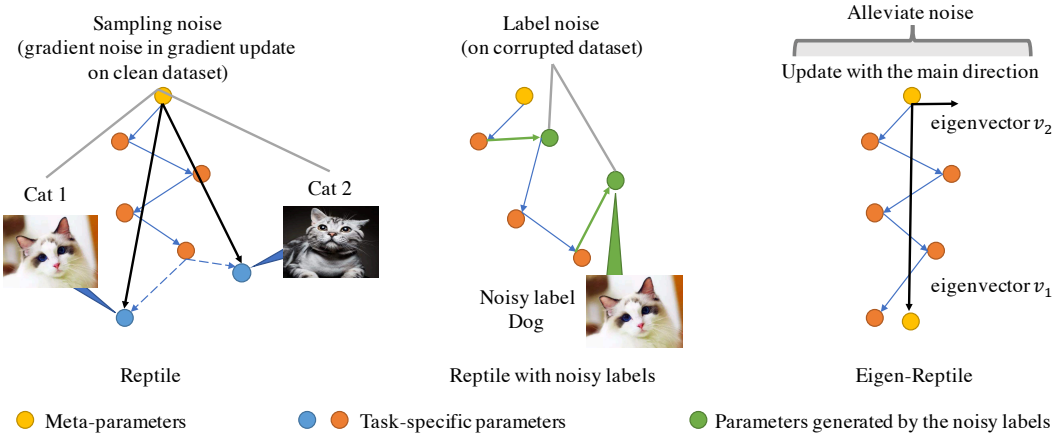


Figure 1: Inner loop steps of Reptile, Eigen-Reptile. Reptile updates meta-parameters towards the last task-specific parameters, which is biased. Eigen-Reptile considers all samples more fair with the main direction of historical task-specific parameters. Note that the main direction is the eigenvector corresponding to the largest eigenvalue.

et al., 2018; Patrini et al., 2017). Therefore, there are few methods that can directly be applied to address noisy FSL problems.

To effectively address these issues, in this paper, we propose a novel method, namely Eigen-Reptile (ER), based on the original Reptile. In particular, as shown in Figure 1, Eigen-Reptile updates the meta-parameters with the main direction of task-specific parameters that can effectively alleviate sampling and label noise. However, it is unrealistic to compute parameters' main direction due to the large scale of neural network parameters. Therefore, we introduce the process of fast computing the main direction (Turk & Pentland, 1991) into FSL, which computes the eigenvectors of the inner loop step scale matrix instead of the parameter scale matrix. Intuitively, noisy labels will degrade the main direction, which in turn degrades the Eigen-Reptile, with the increase of noise ratio. To get a more accurate main direction for Eigen-Reptile in the presence of noisy labels, we propose Introspective Self-paced Learning (ISPL) that constructs multiple prior models with randomly sampling to discard high loss samples from the dataset. We have theoretically and experimentally demonstrated the soundness and effectiveness of the proposed methods.

Experimental results show that Eigen-Reptile significantly outperforms the baseline model Reptile by 22.93% and 5.85% on corrupted Mini-ImageNet of 5-way 1-shot and clean Mini-ImageNet of 5-way 5-shot, respectively. Moreover, the proposed algorithms outperform or are highly competitive with recent gradient-based methods on few-shot classification tasks.

The main contributions of this paper can be summarized as follows:

- We propose Eigen-Reptile that can alleviate noise (both sampling and label noise) effectively. Besides, we propose ISPL, which improves the computed main direction for Eigen-Reptile in the presence of noisy labels.
- We theoretically verify the effectiveness of the proposed Eigen-Reptile for sampling and label noise. Furthermore, we theoretically show that ISPL improves Eigen-Reptile by improving the accuracy of the observed eigenvector learned with corrupted samples.
- The proposed methods outperform or achieve highly competitive performance compared with the recent gradient-based methods on few-shot tasks.

2 Related Work

There are three main types of meta-learning approaches: metric-based meta-learning approaches (Ravi & Larochelle, 2016; Andrychowicz et al., 2016; Santoro et al., 2016), model-based meta-learning approaches (Vinyals et al., 2016; Koch et al., 2015; Mordatch, 2018; Snell et al., 2017;

Oreshkin et al., 2018) and gradient-based meta-learning approaches (Finn et al., 2017; Jamal & Qi, 2019; Zintgraf et al., 2018; Li et al., 2017; Rajeswaran et al., 2019). In this paper, we focus on gradient-based meta-learning approaches which can be viewed as the bi-level loop. The goal of the outer loop is to update the meta-parameters on a variety of tasks, while task-specific parameters are learned through only a small amount of data in the inner loop. In addition, some algorithms achieve state-of-the-art results by additionally training a all-way classification task on meta-training set (Yang et al., 2020; Hu et al., 2020) like transfer learning (Zuo et al., 2018; Liu, 2020), we do not discuss these algorithms in this paper.

Meta-Learning with overfitting. Due to too few samples, meta-learner inevitably tends to overfit in FSL (Mishra et al., 2017). Zintgraf et al. (2018) introduces additional context parameters to the model's parameters, which can prevent meta-overfitting; Bertinetto et al. (2018) find that regularization such as dropout can alleviate meta-overfitting and Yin et al. (2019) propose meta-regularization on weights; Jamal & Qi (2019) propose a novel paradigm of Task-Agnostic Meta-Learning (TAML), which uses entropy or other approaches to minimize the inequality of initial losses beyond the classification tasks to improve the generalizability of meta-learner; Similar to TAML, Rajendran et al. (2020) introduces an information-theoretic framework of meta-augmentation to make meta-learner generalize to new tasks; Ni et al. (2020) improves the performance of meta-learners by data augmentation. All these methods stay at the model level. However, we solve the meta-overfitting problem from the gradient aspect. We propose Eigen-Reptile, which updates the meta-parameters by the main direction of task-specific parameters to alleviate meta-learner overfit on noise.

Learning with noisy labels. Learning with noisy labels has been a long-standing problem (Frénay & Verleysen, 2013; Han et al., 2018b;a). There are many approaches to solve it, such as studying the denoise loss function (Hendrycks et al., 2018; Patrini et al., 2017; Arazo et al., 2019), relabeling (Lin et al., 2014), and so on. Nevertheless, most of these methods require much data for each class. Gao et al. (2019) propose hybrid attention based prototypical networks for noisy few-shot relation classification but without good transferability. For model-agnostic noisy FSL, a gradient-based meta-learner is trained to optimize an initialization on various tasks with noisy labels. As there are few samples of each class, the traditional algorithms for noisy labels cannot be applied. When the existing gradient-based meta-learning algorithms, such as Reptile, update meta-parameters, they focus on the samples that generate the last gradient step. And these samples may be corrupted, which makes the parameters learned by meta-learner susceptible to noisy labels. To better solve the problem of noisy FSL, we further proposed ISPL based on the idea of Self-paced Learning (SPL) (Kumar et al., 2010; Khan et al., 2011; Basu & Christensen, 2013; Tang et al., 2012) to learn more accurate main direction for Eigen-Reptile. ISPL constructs prior models to decide which sample should be discarded when train task-specific models. In contrast, the model with SPL learns the samples gradually from easy to complex, and the model itself decides the order, which can improve the robustness like adversarial training (Neelakantan et al., 2015; Zhang et al., 2020; Gao et al., 2020; Du et al., 2021). However, SPL is not applicable in meta-learning because of the number of tasks.

3 PRELIMINARIES

Gradient-based meta-learning aims to learn meta-parameters $\boldsymbol{\phi}$ as initialization that can adapt to new tasks after a few iterations. The dataset D is usually divided into the meta-training set $D_{meta-train}$ and meta-testing set $D_{meta-test}$, which are used to optimize meta-parameters and evaluate its generalization, respectively. For meta-training, we have tasks $\{\mathcal{T}_i\}_{i=1}^{B}$ drawn from task distribution $p(\mathcal{T})$, each task has its own train set D_{train} and test set D_{test}, and the tasks in $D_{meta-test}$ are defined in the same way. Note that there are only a small number of samples for each task in FSL. Specifically, the N-way K-shot classification task refers to K examples for each of the N classes. Generally, the number of shots in meta-training should match the one at meta test-time to obtain the best performance (Cao et al., 2019). In this paper, we follow Lee et al. (2019); Cao et al. (2019) to increase the training shots appropriately to get the main direction of individual tasks during meta-training. To minimize the test loss $L(D_{test}, \widetilde{\boldsymbol{\phi}})$ of an individual task, meta-parameters $\boldsymbol{\phi}$ need to be updated n times to get suitable task-specific parameters $\widetilde{\boldsymbol{\phi}}$. That is minimizing

$$L(D_{test}, \widetilde{\boldsymbol{\phi}}) = -\frac{1}{N} \sum \mathbb{E} \left[\frac{1}{K} \sum_{(x,y) \in \mathcal{D}_{test}} \log q \left(\hat{y} = y \mid x, \boldsymbol{\phi}, \widetilde{\boldsymbol{\phi}} \right) \right] \tag{1}$$

where $\widetilde{\boldsymbol{\phi}} = U^n(D_{train}, \boldsymbol{\phi})$, U^n represents n inner loop steps through gradient descent or Adam (Kingma & Ba, 2014) on batches from D_{train}, $q(\cdot)$ is the predictive distribution.

When considering updating the meta-parameters in the outer loop, different algorithms have different rules. In the case of Reptile, after n inner loop steps, the meta-parameters can be updated as:

$$\boldsymbol{\phi} \longleftarrow \boldsymbol{\phi} + \beta(\widetilde{\boldsymbol{\phi}} - \boldsymbol{\phi}) \tag{2}$$

where β is a scalar stepsize hyperparameter that controls the update rate of meta-parameters.

4 EIGEN-REPTILE FOR CLEAN AND CORRUPTED DATA

The proposed Eigen-Reptile alleviates the meta-learner overfitting on sampling noise (on a clean dataset) and label noise (on a corrupted dataset). Furthermore, ISPL improves the performance of Eigen-Reptile by computing a more accurate main direction when there are noisy labels in the dataset.

4.1 THE EIGEN-REPTILE ALGORITHM

To alleviate overfitting on sampling and label noise and improve the generalizability of meta-learner, we propose Eigen-Reptile, which updates d-dimensional meta-parameters with the main direction of historical task-specific parameters. We train the task-specific model with n inner loop steps that start from the meta-parameters $\boldsymbol{\phi}$. Let i-th column $\boldsymbol{W}_{:,i} \in R^{d\times 1}$ of historical task-specific parameter matrix $\boldsymbol{W} \in R^{d\times n}$ be the parameters after i-th gradient update, e.g., $\boldsymbol{W}_{:,i} = U^i(D_{train}, \boldsymbol{\phi})$. And treat $\boldsymbol{W}_{:,i}$ as a d-dimensional parameter point $\boldsymbol{w}_i$ in the parameter space. $\boldsymbol{e} \in R^{d\times 1}$ is a unit vector that represents the main direction of n parameter points in $\boldsymbol{W}$. Intuitively, projecting all parameter points onto $\boldsymbol{e}$ should retain the most information.

We represent the parameter points by a straight line of the form $\boldsymbol{w} = \overline{\boldsymbol{w}} + l\boldsymbol{e}$, which shows that the straight line passes through the mean point $\overline{\boldsymbol{w}}$ and the signed distance of a point $\boldsymbol{w}$ from $\overline{\boldsymbol{w}}$ is l. Then we get the loss function $J(l_1, l_2, \cdots, l_n, e) = \sum_{i=1}^{n} \parallel \overline{\boldsymbol{w}} + l_i\boldsymbol{e} - \boldsymbol{w_i} \parallel^2$. And determine the signed distance l of each point by partially differentiating J with respect to l_i, we get $l_i = \boldsymbol{e}^\top(\boldsymbol{w_i} - \overline{\boldsymbol{w}})$. Plugging in this expression for l_i in J, we get

$$J(\boldsymbol{e}) = -\sum_{i=1}^{n} \boldsymbol{e}^\top(\boldsymbol{w_i} - \overline{\boldsymbol{w}})(\boldsymbol{w_i} - \overline{\boldsymbol{w}})^\top\boldsymbol{e} + \sum_{i=1}^{n} \parallel \boldsymbol{w_i} - \overline{\boldsymbol{w}} \parallel^2 = -\boldsymbol{e}^\top\boldsymbol{S}\boldsymbol{e} + \sum_{i=1}^{n} \parallel \boldsymbol{w_i} - \overline{\boldsymbol{w}} \parallel^2 \tag{3}$$

where $\boldsymbol{S} = \sum_{i=1}^{n} (\boldsymbol{w_i} - \overline{\boldsymbol{w}})(\boldsymbol{w_i} - \overline{\boldsymbol{w}})^\top$ is a scatter matrix. According to Eq.3, minimizing J is equivalent to maximizing $\boldsymbol{e}^\top\boldsymbol{S}\boldsymbol{e}$. Note that $\boldsymbol{e}$ needs to be roughly consistent with the gradient update direction $\overline{\boldsymbol{V}}$ in the process of learning task-specific parameters. Use Lagrange multiplier method as

$$\max \boldsymbol{e}^\top\boldsymbol{S}\boldsymbol{e} \qquad \text{s.t.} \begin{cases} \overline{\boldsymbol{V}}\boldsymbol{e} > 0 \\ \boldsymbol{e}^\top\boldsymbol{e} = 1 \end{cases}, \text{where} \quad \overline{\boldsymbol{V}} = \frac{1}{\lfloor n/2 \rfloor} \sum_{i=1}^{\lfloor n/2 \rfloor} \boldsymbol{w}_{n-i+1} - \boldsymbol{w}_i \tag{4}$$

We get the objective function

$$g(\mu, \boldsymbol{e}, \lambda, \eta) = \boldsymbol{e}^\top\boldsymbol{S}\boldsymbol{e} - \lambda(\boldsymbol{e}^\top\boldsymbol{e} - 1) + \mu(-\overline{\boldsymbol{V}}\boldsymbol{e} + \eta^2), \text{where} \quad \lambda \neq 0, \mu \geq 0 \tag{5}$$

then partially differentiating g in Eq.5 with respect to μ, e, λ, η,

$$\begin{cases} -\overline{\boldsymbol{V}}\boldsymbol{e} + \eta^2 = 0 \\ 2\boldsymbol{S}\boldsymbol{e} - 2\lambda\boldsymbol{e} - \mu\overline{\boldsymbol{V}} = 0 \\ \boldsymbol{e}^\top\boldsymbol{e} - 1 = 0 \\ 2\mu\eta = 0 \end{cases} \tag{6}$$

According to Eq.6, if $\eta = 0$, then $\overline{\boldsymbol{V}}$ and $\boldsymbol{e}$ are orthogonal, which obviously does not meet our expectations. So we get $\eta \neq 0$, and $\mu = 0$, then $\boldsymbol{S}\boldsymbol{e} = \lambda\boldsymbol{e}$. We can see $\boldsymbol{e}$ is the eigenvector of $\boldsymbol{S}$ corresponding to the largest eigenvalue λ, which is the required main direction. It should be noted that even if $\overline{\boldsymbol{V}}$ is not directly related to $\boldsymbol{e}$, in Eigen-Reptile, the linear constraint $\overline{\boldsymbol{V}}\boldsymbol{e} > 0$ in Eq.4 must be retained as it determines the update direction of the outer-loop. Otherwise, the algorithm will not converge.

A concerned question about $\boldsymbol{S}\boldsymbol{e} = \lambda \boldsymbol{e}$ is that the scatter matrix $\boldsymbol{S} \in R^{d \times d}$ grows quadratically with the number of parameters d. As the large number of of parameters typically used in neural networks, computing eigenvalues and eigenvectors of $\boldsymbol{S}$ could come at a prohibitive cost (the worst-case complexity is $\mathcal{O}\left(\mathrm{d}^3\right)$). To avoid calculating the eigenvectors of $\boldsymbol{S}$ directly, we focus on $\boldsymbol{W}^\top \boldsymbol{W}$ (centralize $\boldsymbol{W}$ by subtracting the mean $\overline{\boldsymbol{w}}$, and the scatter matrix $\boldsymbol{S} = \boldsymbol{W}\boldsymbol{W}^\top$). As $\boldsymbol{W}^\top \boldsymbol{W}\widehat{\boldsymbol{e}} = \widehat{\lambda}\widehat{\boldsymbol{e}}$, multiply both sides of the equation with $\boldsymbol{W}$,

$$\boldsymbol{W}\boldsymbol{W}^\top \underbrace{\boldsymbol{W}\widehat{\boldsymbol{e}}}_{\boldsymbol{e}} = \underbrace{\widehat{\lambda}}_{\lambda} \underbrace{\boldsymbol{W}\widehat{\boldsymbol{e}}}_{\boldsymbol{e}} \tag{7}$$

It can be found from Eq.7 that $\boldsymbol{W}^\top \boldsymbol{W} \in R^{n \times n}$ and $\boldsymbol{W}\boldsymbol{W}^\top \in R^{d \times d}$ have the same eigenvalue, $\lambda = \widehat{\lambda}$. Furthermore, we get the eigenvector of $\boldsymbol{W}\boldsymbol{W}^\top$ as $\boldsymbol{e} = \boldsymbol{W}\widehat{\boldsymbol{e}}$. The main advantage of Eq.7 is that the intermediate matrix $\boldsymbol{W}^\top \boldsymbol{W}$ now grows quadratically with the inner loop steps. As we are interested in FSL, n is very small. It will be much easier to compute the eigenvector $\widehat{\boldsymbol{e}}$ of $\boldsymbol{W}^\top \boldsymbol{W}$. Then we get the eigenvector $\boldsymbol{e}$ of $\boldsymbol{W}\boldsymbol{W}^\top$ based on $\widehat{\boldsymbol{e}}$. Moreover, we project parameter update vectors $\boldsymbol{w}_{i+1} - \boldsymbol{w}_i, i = 1, 2, \cdots, n-1$ on $\boldsymbol{e}$ to get the corresponding update stepsize ν, so meta-parameters $\boldsymbol{\phi}$ can be updated as

$$\boldsymbol{\phi} \longleftarrow \boldsymbol{\phi} + \beta \nu \zeta \boldsymbol{e}, \quad \text{where} \quad \zeta = \frac{\lambda}{\sum_{m=1}^{n} \lambda_m} \tag{8}$$

where β is a scalar stepsize hyperparameter that controls the update rate of meta-parameters, ζ is the proportion of the largest eigenvalue to the sum of all eigenvalues. The larger the value of ζ, the more accurate the meta-parameter update direction.

The Eigen-Reptile algorithm is summarized in Algorithm 1 of ***Appendix*** A. To illustrate the validity of Eigen-Reptile for sampling noise (gradient noise in gradient updating), we present Theorem 1:

Theorem 1 *Assume that the gradient noise variable x follows Gaussian distribution (Hu et al., 2017; Jastrzbski et al., 2017; Mandt et al., 2016), $x \sim \mathrm{N}\left(0, \sigma^2\right)$. Moreover, x and neural network parameter variable are assumed to be uncorrelated. The observed covariance matrix $\boldsymbol{C}$ equals noiseless covariance matrix $\boldsymbol{C}_t$ plus gradient noise covariance matrix $\boldsymbol{C}_x$. Then, we get*

$$\boldsymbol{C} = \frac{1}{n-1}\boldsymbol{S} = \boldsymbol{C}_t + \boldsymbol{C}_x = \boldsymbol{P}_t(\Lambda_t + \boldsymbol{\Lambda}_x)\boldsymbol{P}_t^\top = \boldsymbol{P}_t(\boldsymbol{\Lambda}_t + \sigma^2 \boldsymbol{I})\boldsymbol{P}_t^\top = \boldsymbol{P}_t \Lambda \boldsymbol{P}_t^\top = \boldsymbol{P}\Lambda\boldsymbol{P}^\top \tag{9}$$

*where $\boldsymbol{P}_t$ and $\boldsymbol{P}$ are the orthonormal eigenvector matrices of $\boldsymbol{C}_t$ and $\boldsymbol{C}$ respectively, $\boldsymbol{\Lambda}_t$ and $\boldsymbol{\Lambda}$ are the corresponding diagonal eigenvalue matrices, and $\boldsymbol{I}$ is an identity matrix. It can be seen from Eq.9 that $\boldsymbol{C}$ and $\boldsymbol{C}_t$ has the same eigenvectors. We defer the proof to the **Appendix** B.*

Theorem 1 shows that eigenvectors are not affected by gradient noise. Therefore, Eigen-Reptile can find a more generalizable starting point for new tasks without overfitting sampling noise (on the clean dataset). As for label noise (on the corrupted dataset), the analysis is shown in ***Appendix*** D. We also show the complexity analysis in ***Appendix*** C, which illustrates that Reptile and Eigen-Reptile are the same in spatial complexity and time complexity.

4.2 The Introspective Self-paced Learning for More Accurate Main Direction

As shown in ***Appendix*** D, Eigen-Reptile addresses the noisy FSL problem by separating noisy information. However, with the increase of noise ratio, the eigenvector will gradually become invalid. To get a more accurate eigenvector, we propose Introspective Self-paced Learning (ISPL).

Self-paced learning (SPL) learns the samples from low losses to high losses, which is proven beneficial in achieving a better generalization result (Khan et al., 2011; Basu & Christensen, 2013; Tang et al., 2012). Besides, some previous work (Zhu et al., 2019) solve the problem of traditional noisy labels by SPL. Nevertheless, in a meta-learning setting, a meta-learner is trained on various tasks; the initial model may have

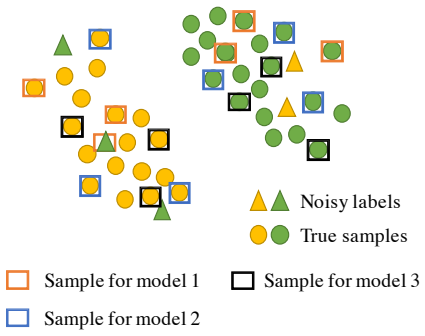


Figure 2: Randomly sample examples to build different prior models.

lower losses for trained classes and higher losses for unseen classes or noisy samples. For this reason, we cannot train the task-specific model in the same way as SPL to solve the noisy FSL problem. To this end, we propose an improved SPL algorithm to help Eigen-Reptile achieve better performance for the problem of the noisy labels. As shown in Figure 2, even though the two categories of the yellow and green show an excellent distribution that can be well separated, some samples are marked wrong. To address this noisy label problem, we build three prior models. Specifically, we randomly sample three times, and model 1 is trained with a corrupted label. Due to different samples learned by prior models, building multiple models to vote on the data will obtain more accurate losses. Moreover, samples with losses above a certain threshold will be discarded. Furthermore, we imitate SPL to add the hidden variable $v = 0$ or 1 that is decided by Q prior models before the loss of each sample to control whether the sample should be abandoned. And we get the task-specific loss as

$$L_{ISPL}\left(\boldsymbol{\phi}, \boldsymbol{v}\right) = \sum_{i=1}^{h} v_i L\left(x_i, y_i, \boldsymbol{\phi}\right), \text{where} \quad v_i = \arg\min_{\mathbf{v}} \frac{v_i}{Q} \sum_{j=1}^{Q} L_j\left(x_i, y_i, \boldsymbol{\phi}_j\right) - \gamma v_i \tag{10}$$

where h is the number of samples x from dataset D_{train}, y is label, γ is the sample selection parameter, which gradually decreases, parameter of model j is $\boldsymbol{\phi}_j = U^n(D_j, \boldsymbol{\phi}), D_j \in D_{train}$. Note that we update the meta-parameters with the model trained on h samples from D_{train}. The ISPL is summarized in Algorithm 2 of ***Appendix*** A. Intuitively, it is difficult to say whether discarding high-loss samples containing correct and wrong samples will improve the accuracy of eigenvector, so we prove the effectiveness of ISPL by Theorem 2.

Theorem 2 *Let $\boldsymbol{W}_o$ be the parameter matrix only generated by the corrupted samples. Compute the eigenvalues and eigenvectors of the observed expected parameter matrix*

$$\frac{1}{\lambda}\mathbb{E}(\boldsymbol{C}_{tr})\boldsymbol{e} = \boldsymbol{P}_o(\boldsymbol{I} - \frac{\boldsymbol{\Lambda}_o}{\lambda})\boldsymbol{P}_o^\top \boldsymbol{e} \approx \boldsymbol{P}_o(\boldsymbol{I} - \frac{\lambda_o}{\lambda}\boldsymbol{I})\boldsymbol{P}_o^\top \boldsymbol{e} > \boldsymbol{P}_o(\boldsymbol{I} - \frac{\lambda_o - \xi}{\lambda - \xi}\boldsymbol{I})\boldsymbol{P}_o^\top \boldsymbol{e} \tag{11}$$

*where $\boldsymbol{C}_{tr}$ is the covariance matrix generated by clean samples, λ is the observed largest eigenvalue, λ_o is the largest eigenvalue in the corrupted diagonal eigenvalue matrix $\boldsymbol{\Lambda}_o$, $\boldsymbol{P}_o$ is the orthonormal eigenvector matrix of corrupted covariance matrix. According to Eq.11, if λ_o/λ is smaller, the observed eigenvector e is more accurate. Assume that the discarded high loss samples have the same contributions ξ to λ and λ_o, representing the observed and corrupted main directional variance, respectively. Note that these two kinds of data have the same effect on the gradient updating of the model, so this assumption is relatively reasonable. Furthermore, it is easy to find that $(\lambda_o - \xi)/(\lambda - \xi)$ is smaller than λ_o/λ. We defer the proof to the **Appendix** E.*

Theorem 2 shows that discard high loss samples can help improve the accuracy of the observed eigenvector learned with corrupted labels. Therefore, ISPL can improve the performance of Eigen-Reptile but not other meta-learning algorithms.

5 Experimental Results and Discussion

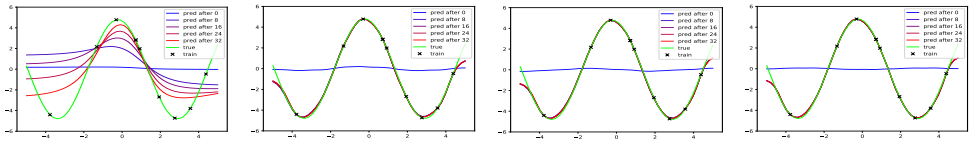
In our experiments, we aim to (1) evaluate the effectiveness of Eigen-Reptile to alleviate overfitting on sampling and label noise, (2) test the robustness of Eigen-Reptile to some hyperparameters, (3) evaluate the improvement of ISPL to Eigen-Reptile in the presence of noisy labels. The code and data for the proposed model are provided for research purposes [1] and all experiments run on a 2080 Ti.

5.1 Meta-learning with Clean Toy Data on Regression

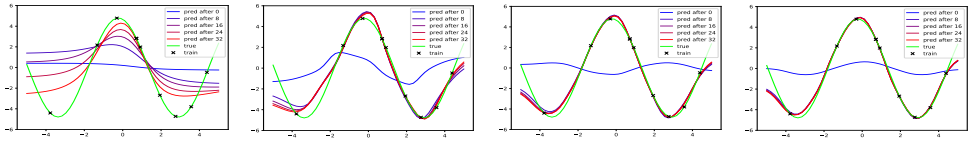
In this experiment, we evaluate Eigen-Reptile by the 1D sine wave K-shot regression problem (Nichol et al., 2018). Each task is defined by a sine curve $y(x) = Asin(x + b)$, where the amplitude $A \sim U([0.1, 5.0])$ and phase $b \sim U([0, 2\pi])$. The amplitude A and phase b are varied between tasks. The goal of each task is to fit a sine curve with the data points sampled from the corresponding $y(x)$. We calculate loss in ℓ_2 using 50 equally-spaced points from the whole interval $[-5.0, 5.0]$ for each task. The loss is

$$\int_{-5.0}^{5.0} \parallel y(x) - \widehat{y}(x) \parallel^2 dx \tag{12}$$

[1] Code is included in the supplemental material and will be released upon the paper acceptance.



(a) ER iteration 1 (b) ER iteration 10000 (c) ER iteration 20000 (d) ER iteration 30000



(e) R iteration 1 (f) R iteration 10000 (g) R iteration 20000 (h) R iteration 30000

Figure 3: Eigen-Reptile (ER) and Reptile (R) training process on the regression toy test. (a), (b), (c), (d) and (e), (f), (g), (h) show that after the gradient update 0, 8, 16, 24, 32 times based on initialization learned by Eigen-Reptile and Reptile respectively.

where $\widehat{y}(x)$ is the predicted function that start from the initialization learned by meta-learner.

The K-shot regression task fits a selected sine curve through K points, here $K = 10$. For the regressor, we use a small neural network, which is the same as (Nichol et al., 2018), except that the activation functions are Tanh. Specifically, the small network includes an input layer of size 1, followed by two hidden layers of size 64, and then an output layer of size 1.

In this part, we mainly compare Reptile and Eigen-Reptile. Both meta-learners use the same regressor and are trained for 30000 iterations with inner loop steps 5, batch size 10, and a fixed inner loop learning rate of 0.02.

We report the results of Reptile and Eigen-Reptile in Figure 3. It can be seen that the curve fitted by Eigen-Reptile is closer to the true green curve, which shows that Eigen-Reptile performs better. According to Jamal & Qi (2019), the initial model with a larger entropy before adapting to new tasks would better alleviate meta-overfitting. As shown in Figure 3, from 1 to 30000 iterations, Eigen-Reptile is more generalizable than Reptile as the initial blue line of Eigen-Reptile is closer to a straight line, which shows that the initialization learned by Eigen-Reptile is less affected by gradient noise. Furthermore, Figure 4 shows that Eigen-Reptile converges faster and gets a lower loss than Reptile.

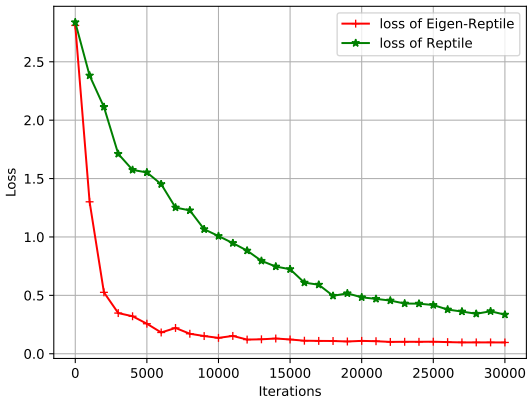


Figure 4: Loss of the 10-shot regression.

5.2 Meta-learning in Clean Realistic Problem

We verify the effectiveness of Eigen-Reptile on two realistic few-shot classification dataset Mini-Imagenet (Vinyals et al., 2016) and CIFAR-FS (Bertinetto et al., 2018), and **the results of CIFAR-FS are shown in Appendix** F.

The Mini-Imagenet dataset contains 100 classes, each with 600 images. We follow Ravi & Larochelle (2016) to divide the dataset into three disjoint subsets: meta-training set, meta-validation set, and meta-testing set with 64 classes, 16 classes, and 20 classes, respectively. We follow the few-shot learning protocols from prior work (Vinyals et al., 2016), except that the number of the meta-training shot is 15 as Lee et al. (2019); Cao et al. (2019), which is still much smaller than the number of samples required by traditional tasks. Moreover, we run our algorithm on the dataset for the different

Table 1: Accuracy of FSL on Mini-Imagenet N-way K-shot. The $\pm$ shows 95% confidence interval over tasks.

Algorithm	5-way 1-shot	5-way 5-shot
Matching Nets (Vinyals et al., 2016)	$43.56 \pm 0.84\%$	$55.31 \pm 0.73\%$
MAML (Finn et al., 2017)	$48.70 \pm 1.84\%$	$63.11 \pm 0.92\%$
FOML (Finn et al., 2017)	$48.07 \pm 1.75\%$	$63.15 \pm 0.91\%$
Meta-SGD (Li et al., 2017)	$50.47 \pm 1.87\%$.	$64.03 \pm 0.94\%$
GNN (Gidaris & Komodakis, 2018)	50.30%	66.40%
CAML (Zintgraf et al., 2018)	$51.82 \pm 0.65\%$	$65.85 \pm 0.55\%$
TAML (Jamal & Qi, 2019)	$51.77 \pm 1.86\%$	$65.60 \pm 0.93\%$
Meta-dropout (Lee et al., 2019)	$51.93 \pm 0.67\%$	$67.42 \pm 0.52\%$
Warp-MAML (Flennerhag et al., 2020)	$52.30 \pm 0.80\%$	$68.4 \pm 0.60\%$
MC (128) (Park & Oliva, 2020)	$\mathbf{54.08 \pm 0.93\%}$	$67.99 \pm 0.73\%$
ARML (Yao et al., 2020)	$50.42 \pm 1.73\%$	–
Reptile (32) (Nichol et al., 2018)	$49.97 \pm 0.32\%$	$65.99 \pm 0.58\%$
Eign-Reptile (32)	$51.80 \pm 0.90\%$	$68.10 \pm 0.50\%$
Eign-Reptile (64)	$\mathbf{53.25 \pm 0.45\%}$	$\mathbf{69.85 \pm 0.85\%}$

number of test shots and compare our results to other meta-learning algorithms. What needs to be reminded is that approaches that use deeper, residual networks or pretrained all-way classifier on meta-training set can achieve higher accuracies (Gidaris & Komodakis, 2018; Yang et al., 2020; Hu et al., 2020). So for a fair comparison, we only compare algorithms that use convolutional networks without a pretrained model as Reptile does. Specifically, our model follows (Nichol et al., 2018), which has 4 modules with a 3×3 convolutions and 64 filters, 2×2 max-pooling etc.. The images are downsampled to 84×84, and the loss function is the cross-entropy error. We use Adam with $\beta_1 = 0$ in the inner loop. Our model is trained for 100000 iterations with a fixed inner loop learning rate of 0.0005, 7 inner-loop steps and batch size 10.

The results of Eigen-Reptile and other meta-learning approaches are summarized in Table 1. The proposed Eigen-Reptile (64 filters) outperforms and achieves highly competitive performance compared with other algorithms for 5-shot and 1-shot classification problems, respectively. More specifically, for 1-shot, the results of MC are similar to that of Eigen-Reptile. However, as a second-order optimization algorithm, the computational cost of MC is much higher than that of Eigen-Reptile (as shown in ***Appendix*** C, Eigen-Reptile is a first-order algorithm like Reptile). Furthermore, the result of Eigen-Reptile is much better than Reptile for each task. Compared with Reptile, Eigen-Reptile uses the main direction to update the meta-parameters to alleviate the meta-overfitting caused by gradient noise. More importantly, Eigen-Reptile outperforms the state-of-the-art meta-overfitting preventing method Meta-dropout (Lee et al., 2019), which is based on regularization. This result shows the effectiveness of addressing the meta-overfitting problem from the perspective of alleviating gradient noise.

We follow Lee et al. (2019); Cao et al. (2019) to vary the number of inner-loops and the number of corresponding training shots to show the robustness of Eigen-Reptile on the 5-way 5-shot problem. Besides, other hyperparameters are not changed.

As shown in Figure 5, after the number of inner-loops i reaches 7, the test accuracy tends to be stable, which shows that changing the number of inner-loops within a specific range has little effect on Eigen-Reptile. That is, Eigen-Reptile is robust to this hyperparameter. As for train shot, to make the trained task-specific parameters as unbiased as possible, we specify train shot roughly satisfies $\lceil \frac{i \times batch\_size}{N} \rceil + 1$, where

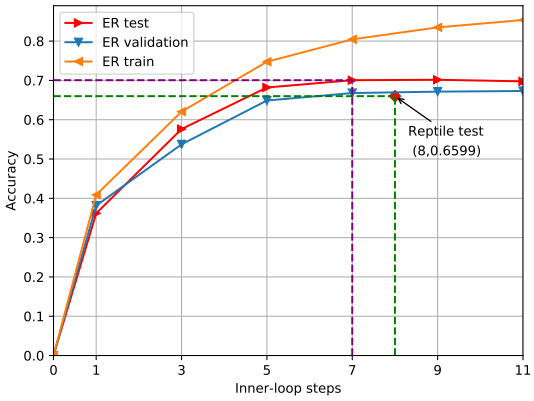


Figure 5: The number of inner-loop and accuracy of 5-way 5-shot task on Mini-Imagenet.

Table 2: Average test accuracy of 5-way 1-shot on the Mini-Imagenet with label noise. S denotes symmetric label noise, AS denotes asymmetric label noise

Algorithm	$p = 0.0$	$p = 0.2$		$p = 0.5$	
		S	AS	S	AS
Reptile (Nichol et al., 2018)	47.64%	43.49%	45.51%	23.33%	42.03%
Reptile+ISPL	47.23%	43.70%	45.42%	21.83%	41.09%
Eigen-Reptile	$\mathbf{47.87}\%$	45.01%	46.50%	27.23%	42.29%
Eigen-Reptile+ISPL	47.26%	$\mathbf{45.49}\%$	$\mathbf{46.83}\%$	$\mathbf{28.68}\%$	$\mathbf{43.71}\%$

N is the number of classes. So when $i = 7$, the number of train shots is 15. It is important to note that in our experiments, Reptile with the hyperparameters set by its authors, the number of inner-loops is 8, the number of train shots is 15, and the corresponding accuracy is 65.99%, which is much lower than the result of Eigen-Reptile.

5.3 Meta-learning with Label Noise

We conduct the 5-way 1-shot experiment with noisy labels generated by corrupting the original labels of Mini-Imagenet. There are symmetric label noise and asymmetric label noise in this experiment. For symmetric label noise, correct labels are flipped to other labels with equal probability, i.e., in the case of symmetric noise of ratio p, a sample retains the correct label with probability $1 - p$, and it becomes some other label with probability $p/(N - 1)$. On the other hand, for asymmetric label noise, we randomly flip the labels of one class to the labels of another fixed class with probability p.

Note that we only introduce noise in the train set during meta-training, where the meta-training shot is 30. Moreover, all meta-learners with 32 filters are trained for 10000 iterations to alleviate overfitting on corrupted samples, and the learning rate is 0.001 in the inner loop. The sample selection parameter $\gamma = 10$ that decreases by 0.6 every 1000 iterations.

As shown in Table 2, for symmetric label noise, with the increase of the ratio p, the performance of Reptile decreases rapidly. When $p = 0.5$, the initialization learned by Reptile can hardly meet the requirements of quickly adapting to new tasks with few samples. On the other hand, Eigen-Reptile is less affected by symmetric label noise, especially when the noise ratio is high, i.e., $p = 0.5$. As for asymmetric label noise, meta-learners are trained on tasks with the same noise transition matrix, which allows meta-learners to learn more useful information, so the results are higher than that with symmetric noise. Like the results of symmetric label noise, Eigen-Reptile outperforms Reptile in all tasks.

As shown in Table 2, Eigen-Reptile+ISPL achieves better results than that of Eigen-Reptile when $p \neq 0$. Specifically, ISPL plays a more significant role when p is higher. However, when $p = 0$, ISPL harms Eigen-Reptile, as ISPL only discards correct samples. These results corresponding to the conclusion in ***Appendix*** D, with the increase of noise ratio, the eigenvector will gradually become invalid, which demonstrates the effectiveness of ISPL and verify the idea that ISPL collaborates with Eigen-Reptile by providing improved main direction. In addition, ISPL does not significantly improve or even degrades the results of Reptile. This is because too many high-loss samples are removed, causing Reptile to fail to converge quickly with the same number of iterations. These experimental results show that Eigen-Reptile and ISPL can effectively separate noisy information, thereby alleviating the meta-overfitting on corrupted samples.

6 Conclusion

This paper proposes a gradient-based meta-learning algorithm Eigen-Reptile. It updates the meta-parameters through the main direction, proven by theory and experiments to effectively alleviate the overfitting on sampling and label noise. Furthermore, to get closer to real-world situations, we introduce noisy labels into the meta-training dataset. The proposed ISPL constructs prior models to select samples for Eigen-Reptile to get a more accurate main direction.

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A PSEUDO-CODE

Algorithm 1 Eigen-Reptile

Input: Distribution over tasks $P(\mathcal{T})$, outer step size β.

Initialize meta-parameters $\boldsymbol{\phi}$
while not converged **do**
  $\boldsymbol{W} = [\,], \nu = 0$
  Sample batch of tasks $\{\mathcal{T}_i\}_{i=1}^{B} \sim P(\mathcal{T})$
  for each task $\mathcal{T}_i$ **do**
    $\boldsymbol{\phi}_i = \boldsymbol{\phi}$
    Sample train set D_{train} of $\mathcal{T}_i$
    for $j = 1, 2, 3, ..., n$ **do**
      $\boldsymbol{\phi}_i^j = U^j(D_{train}, \boldsymbol{\phi}_i)$
      $\boldsymbol{W}$ appends $\boldsymbol{W}_{:,j} = flatten(\boldsymbol{\phi}_i^j), \quad \boldsymbol{W}_{:,j} \in R^{d\times 1}$
    end for
    Mean centering, $\boldsymbol{W} = \boldsymbol{W} - \overline{\boldsymbol{w}}, \quad \overline{\boldsymbol{w}} \in R^{d\times 1}$
    Compute eigenvalue matrix $\widehat{\boldsymbol{\Lambda}}$ and eigenvector matrix $\widehat{\boldsymbol{P}}$ of scatter matrix $\boldsymbol{W}^\top \boldsymbol{W}$
    Eigenvalues $\lambda_1 > \lambda_2 > \cdots > \lambda_n$ in $\widehat{\boldsymbol{\Lambda}}$
    Compute eigenvector matrix of $\boldsymbol{W}\boldsymbol{W}^\top$, $\boldsymbol{P} = \boldsymbol{W}\widehat{\boldsymbol{P}}$
    Let the eigenvector corresponding to λ_1 be a unit vector, $\| \boldsymbol{e}_i^1 \|_2^2 = 1$
    for $j = 1, 2, 3, ..., n-1$ **do**
      $\nu = \nu + (\boldsymbol{W}_{:,j+1} - \boldsymbol{W}_{:,j})\boldsymbol{e}_i^1$
    end for
    $\boldsymbol{e}_i^1 = \frac{\lambda_1}{\sum_{m=1}^{n}\lambda_m} \times \boldsymbol{e}_i^1$
    Calculate the approximate direction of task-specific gradient update $\overline{\boldsymbol{V}}$:
    $\overline{\boldsymbol{V}} = \frac{1}{\lfloor n/2 \rfloor} \sum_{i=1}^{\lfloor n/2 \rfloor} \boldsymbol{W}_{:,n-i+1} - \boldsymbol{W}_{:,i}$
    if $\boldsymbol{e}_i^1 \cdot \overline{\boldsymbol{V}} < 0$ **then**
      $\boldsymbol{e}_i^1 = -\boldsymbol{e}_i^1$
    end if
  end for
  Average the main directions to get $\tilde{\boldsymbol{e}} = (1/B)\sum_{i=1}^{B} \boldsymbol{e}_i^1$
  Update meta-parameters $\boldsymbol{\phi} \longleftarrow \boldsymbol{\phi} + \beta \times \nu/B \times \tilde{\boldsymbol{e}}$
end while

Algorithm 2 Introspective Self-paced Learning

Input: Dataset D_{train}, initialization $\boldsymbol{\phi}$, batch size b, selection parameter γ, attenuation coefficient μ, the number of prior models Q.

Initialize network parameters $\boldsymbol{\phi}^* = \boldsymbol{\phi}$ for a sampled task
for $j = 1, 2, 3, \cdots, Q$ **do**
  Sample examples D_j from D_{train} for training $model_j$, $\boldsymbol{\phi}_j = U^m(D_j, \boldsymbol{\phi}^*)$
end for
Train task-specific parameters:
for $i = 1, 2, 3, \cdots, n$ **do**
  Compute hidden variable vector $\boldsymbol{v}$:
  $v = \arg\min_{\mathbf{v}} v_q \sum_{q=1}^{b} L_q - \gamma \sum_{q=1}^{b} v_q$,
  where $L_q = \frac{1}{Q}\sum_{j=1}^{Q} L_j\left(x_q, y_q, \boldsymbol{\phi}_j\right)$
  Update task-specific parameters ϕ^*:
  $\boldsymbol{\phi}^* = \arg\min_{\boldsymbol{\phi}^*} L_{ISPL}\left(\boldsymbol{\phi}^*, v\right)$
  $\gamma = \gamma - \mu$
end for

B PROOF OF THEOREM 1

Gradient update always with gradient noise inserted at every iteration, which caused Reptile, MAML, etc. cannot find accurate directions to update meta-parameters. In this section, we will prove that Eigen-Reptile can alleviate gradient noise.

Theorem 1 *Assume that the gradient noise variable x follows Gaussian distribution (Hu et al., 2017; Jastrzbski et al., 2017; Mandt et al., 2016), $x \sim \mathrm{N}\left(0, \sigma^2\right)$. Furthermore, x and neural network parameter variable are assumed to be uncorrelated. The observed covariance matrix $\boldsymbol{C}$ equals noiseless covariance matrix $\boldsymbol{C}_t$ plus gradient noise covariance matrix $\boldsymbol{C}_x$. Then, we get*

$$\boldsymbol{C} = \frac{1}{n-1}\boldsymbol{S} = \boldsymbol{C}_t + \boldsymbol{C}_x = \boldsymbol{P}_t(\Lambda_t + \boldsymbol{\Lambda}_x)\boldsymbol{P}_t^\top = \boldsymbol{P}_t(\boldsymbol{\Lambda}_t + \sigma^2\boldsymbol{I})\boldsymbol{P}_t^\top = \boldsymbol{P}_t\Lambda\boldsymbol{P}_t^\top = \boldsymbol{P}\Lambda\boldsymbol{P}^\top \quad (13)$$

where $\boldsymbol{P}_t$ and $\boldsymbol{P}$ are the orthonormal eigenvector matrices of $\boldsymbol{C}_t$ and $\boldsymbol{C}$ respectively, $\boldsymbol{\Lambda}_t$ and $\boldsymbol{\Lambda}$ are the corresponding diagonal eigenvalue matrices, and $\boldsymbol{I}$ is an identity matrix. It can be seen from Eq.13 that $\boldsymbol{C}$ and $\boldsymbol{C}_t$ has the same eigenvectors.

Proof B.1 *In the following proof, we assume that the probability density function of gradient noise variable x follows Gaussian distribution, $x \sim \mathrm{N}\left(0, \sigma^2\right)$. Treat the parameters in the neural network as variables, and the parameters obtained by each gradient update as samples. Furthermore, gradient noise and neural network parameters are assumed to be uncorrelated.*

For observed parameter matrix $\boldsymbol{W} \in R^{d\times n}$, there are n samples, let $\boldsymbol{W}_{i,:} \in R^{1\times n}$ be the observed values of the i-th variable $\boldsymbol{W}_i$, and $\boldsymbol{W} = [\boldsymbol{W}_{1,:}^\top, \cdots, \boldsymbol{W}_{i,:}^\top, \cdots, \boldsymbol{W}_{d,:}^\top]^\top$. Similarly, we denote the noiseless parameter matrix by $\boldsymbol{W}^t = [(\boldsymbol{W}_{1,:}^t)^\top, \cdots, (\boldsymbol{W}_{i,:}^t)^\top, \cdots, (\boldsymbol{W}_{d,:}^t)^\top]^\top$, and

$$\boldsymbol{W} = \boldsymbol{W}^t + \boldsymbol{X} \quad (14)$$

Where $\boldsymbol{X} = [\boldsymbol{X}_{1,:}^\top, \cdots, \boldsymbol{X}_{i,:}^\top, \cdots, \boldsymbol{X}_{d,:}^\top]^\top$ is the dataset of noise variables. Then, centralize each variable by

$$\overline{\boldsymbol{W}}_k = \boldsymbol{W}_k - \frac{1}{n}\sum_{i=1}^{n}\boldsymbol{W}_{k,:}(i) \quad (15)$$

So we get $\overline{\boldsymbol{W}} = [\overline{\boldsymbol{W}}_1^\top, \cdots, \overline{\boldsymbol{W}}_d^\top]^\top$. Suppose $\boldsymbol{W}^t$ is also centralized by the same way and get $\overline{\boldsymbol{W}^t} = [\overline{\boldsymbol{W}^t}_1^\top, \cdots, \overline{\boldsymbol{W}^t}_d^\top]^\top$. Then, we have:

$$\overline{\boldsymbol{W}} = \overline{\boldsymbol{W}^t} + \boldsymbol{X} \quad (16)$$

Computing the covariance matrix of $\overline{\boldsymbol{W}}$:

$$\begin{aligned}\boldsymbol{C} &= \frac{1}{n}\overline{\boldsymbol{W}}\overline{\boldsymbol{W}}^\top \\ &= \frac{1}{n}(\overline{\boldsymbol{W}^t} + \boldsymbol{X})(\overline{\boldsymbol{W}^t}^\top + \boldsymbol{X}^\top) \\ &= \frac{1}{n}(\overline{\boldsymbol{W}^t}\overline{\boldsymbol{W}^t}^\top + \overline{\boldsymbol{W}^t}\boldsymbol{X}^\top + \boldsymbol{X}\overline{\boldsymbol{W}^t}^\top + \boldsymbol{X}\boldsymbol{X}^\top)\end{aligned} \quad (17)$$

Since $\overline{\boldsymbol{W}^t}$ and $\boldsymbol{X}$ are uncorrelated, $\overline{\boldsymbol{W}^t}\boldsymbol{X}^\top$ and $\boldsymbol{X}\overline{\boldsymbol{W}^t}^\top$ are approximately zero matrices. Thus:

$$\boldsymbol{C} \approx \frac{1}{n}(\overline{\boldsymbol{W}^t}\overline{\boldsymbol{W}^t}^\top + \boldsymbol{X}\boldsymbol{X}^\top) = \boldsymbol{C}_t + \boldsymbol{C}_x \quad (18)$$

The component $\boldsymbol{C}_x(i,j)$ is the correlation between $\boldsymbol{X}_i$ and $\boldsymbol{X}_j$ which corresponds to the i-th and j-th rows of $\boldsymbol{X}$. As the two noise variables are not related to each other, if $i \neq j$, then $\boldsymbol{C}_x(i,j) = 0$. So $\boldsymbol{C}_x \in R^{d\times d}$ is a diagonal matrix with diagonal elements σ^2. Decompose $\boldsymbol{C}_t$ as:

$$\boldsymbol{C}_t = \boldsymbol{P}_t\boldsymbol{\Lambda}_t\boldsymbol{P}_t^\top \quad (19)$$

where $\boldsymbol{P}_t$ is the noiseless orthonormal eigenvector matrix and $\boldsymbol{\Lambda}_t$ is the noiseless diagonal eigenvalue matrix, then

$$\boldsymbol{C}_x = \boldsymbol{\Lambda}_x\boldsymbol{P}_t\boldsymbol{P}_t^\top = \boldsymbol{P}_t\boldsymbol{\Lambda}_x\boldsymbol{P}_t^\top = \boldsymbol{P}_t\boldsymbol{C}_x\boldsymbol{P}_t^\top \quad (20)$$

where $\mathbf{\Lambda}_x = \sigma^2 \boldsymbol{I}$, and $\boldsymbol{I}$ is the identity matrix. Thus,

$$\begin{aligned} \boldsymbol{C} &= \boldsymbol{C}_t + \boldsymbol{C}_x \\ &= \boldsymbol{P}_t \mathbf{\Lambda}_t \boldsymbol{P}_t^\top + \boldsymbol{P}_t \mathbf{\Lambda}_x \boldsymbol{P}_t^\top \\ &= \boldsymbol{P}_t (\mathbf{\Lambda}_t + \mathbf{\Lambda}_x) \boldsymbol{P}_t^\top \\ &= \boldsymbol{P}_t \mathbf{\Lambda} \boldsymbol{P}_t^\top \end{aligned} \tag{21}$$

where $\mathbf{\Lambda} = \mathbf{\Lambda}_t + \mathbf{\Lambda}_x$. It can be seen from Eq.21 that $\boldsymbol{C}$ and $\boldsymbol{C}_t$ has the same eigenvector matrix. In other words, ***eigenvector is not affected by gradient noise.***

C Algorithm Complexity Analysis of Eigen-Reptile

As for the time complexity of Eigen-Reptile, the cost of single gradient descent in the inner-loop is $\mathcal{O}(d)$, where d is the number of network parameters. The cost of the scatter/covariance matrix computations is $\mathcal{O}(n^2 d)$, where n is the number of inner-loop. Moreover, the worst-case complexity of computing eigenvalue decomposition is $\mathcal{O}(n^3)$. Finally, the computational complexity of restoring eigenvector is $\mathcal{O}(nd)$. We set the maximal number of outer-loop to T. Hence the time complexity of Eigen-Reptile is

$$\mathcal{O}(T(n^3 + n^2 d + nd + nd)) = \mathcal{O}(Td) \tag{22}$$

In FSL, n is small (in this paper $n = 7$), so the overall time complexity is $\boldsymbol{\mathcal{O}(Td)}$.

As for Reptile, the time complexity is also $\mathcal{O}(Td)$, which means that the time complexity of both Reptile and Eigen-Reptile is much lower than the second-order optimization algorithms.

As for spatial complexity, Eigen-Reptile needs to store a $d \times n$ matrix and a $n \times n$ matrix. The overall spatial complexity is $\boldsymbol{\mathcal{O}(d)}$, while the spatial complexity of Reptile is $\mathcal{O}(d)$, too.

It can be seen that, compared to Reptile, Eigen-Reptile is the same in spatial complexity and time complexity. Still, the accuracy of Eigen-Reptile is much higher than that of Reptile.

D Effectiveness of Eigen-Reptile for Noisy FSL

Eigen-Reptile addresses the noisy FSL problem by separating noisy information. More specifically, from the perspective of signal to noise ratio (SNR) :

$$SNR = \frac{\delta^2_{clean}}{\delta^2_{noise}} \tag{23}$$

where δ^2_{clean} is the variance of weight points introduced by clean data, and δ^2_{noise} is the variance of weight points introduced by corrupted data.
Normally, SNR$\gg$1, which shows that the effective information is far more than noisy information. Specifically, in a task-specific training, weight matrix $\mathbf{W}$ is composed of all weight points, and we decompose it as:

$$\mathbf{W} = \mathbf{U}\mathbf{S}\mathbf{V}^T = \sum_{k=1}^{r} \sigma_k \mathbf{u_k} \mathbf{v_k}^T \tag{24}$$

where r is the rank of W, u_k is the eigenvector of the covariance matrix WW^T, v_k is the eigenvector of the covariance matrix $W^T W$.
The singular values are ordered in decreasing magnitude as $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_r$ which are the positive square roots of the eigenvalues of WW^T. In Eigen-Reptile, we only remain the eigenvector corresponding to the largest eigenvalue, e.g., σ_1^2, which can be viewed as the δ^2_{clean} in SNR. In contrast, the noisy information δ^2_{noise} is removed by omitting the low singular values. However, with the increase of noise ratio, Eigen-Reptile is more and more likely to suffer from the problem that the influence of corrupted samples gradually dominates the main update direction.

E PROOF OF THEOREM 2

In this section, we will prove that discarding high loss samples will result in a more accurate main direction in noisy FSL.

Theorem 2 *Let $\boldsymbol{W}_o$ be the parameter matrix only generated by the corrupted samples. Compute the eigenvalues and eigenvectors of the observed expected parameter matrix*

$$\frac{1}{\lambda}\mathbb{E}(\boldsymbol{C}_{tr})\boldsymbol{e} = \boldsymbol{P}_o(\boldsymbol{I} - \frac{\boldsymbol{\Lambda}_o}{\lambda})\boldsymbol{P}_o^\top \boldsymbol{e} \approx \boldsymbol{P}_o(\boldsymbol{I} - \frac{\lambda_o}{\lambda}\boldsymbol{I})\boldsymbol{P}_o^\top \boldsymbol{e} > \boldsymbol{P}_o(\boldsymbol{I} - \frac{\lambda_o - \xi}{\lambda - \xi}\boldsymbol{I})\boldsymbol{P}_o^\top \boldsymbol{e} \tag{25}$$

where $\boldsymbol{C}_{tr}$ is the covariance matrix generated by clean samples, λ is the observed largest eigenvalue, λ_o is the largest eigenvalue in the corrupted diagonal eigenvalue matrix $\boldsymbol{\Lambda}_o$, $\boldsymbol{P}_o$ is the orthonormal eigenvector matrix of corrupted covariance matrix. According to Eq.25, if λ_o/λ is smaller, the observed eigenvector e is more accurate. Assume that the discarded high loss samples have the same contributions ξ to λ and λ_o, representing the observed and corrupted main directional variance, respectively. Note that these two kinds of data have the same effect on the gradient updating of the model, so this assumption is relatively reasonable. Furthermore, it is easy to find that $(\lambda_o - \xi)/(\lambda - \xi)$ is smaller than λ_o/λ.

Proof E.1 *Here, we use $\boldsymbol{w}$ to represent the parameter point obtained after a gradient update. For convenience, let $\boldsymbol{w}$ be generated by a single sample, $\boldsymbol{w} \in R^{d\times 1}$. Then the parameter matrix can be obtained,*

$$\boldsymbol{W} = \begin{bmatrix} \boldsymbol{w}_1^{tr} & \boldsymbol{w}_2^{tr} & \cdots & \boldsymbol{w}_1^{o} & \cdots & \boldsymbol{w}_m^{o} & \cdots & \boldsymbol{w}_n^{tr} \end{bmatrix} \tag{26}$$

where $\boldsymbol{w}^o$ represents the parameters generated by the corrupted sample, and $\boldsymbol{w}^{tr}$ represents the parameters generated by the true sample. Furthermore, there are n parameter points generated by n samples. Moreover, there are m corrupted parameter points generated by m corrupted samples. Mean centering $\boldsymbol{W}$, and show the observed covariance matrix $\boldsymbol{C}$ as

$$\begin{aligned} \boldsymbol{C} &= \frac{1}{n}\boldsymbol{W}\boldsymbol{W}^\top \\ &= \frac{1}{n}\begin{bmatrix} \boldsymbol{w}_1^{tr} & \boldsymbol{w}_2^{tr} & \cdots & \boldsymbol{w}_n^{tr} \end{bmatrix} \begin{bmatrix} (\boldsymbol{w}_1^{tr})^\top \\ (\boldsymbol{w}_2^{tr})^\top \\ \vdots \\ (\boldsymbol{w}_n^{tr})^\top \end{bmatrix} \\ &= \frac{1}{n}(\boldsymbol{w}_1^{tr}(\boldsymbol{w}_1^{tr})^\top + \cdots + \boldsymbol{w}_1^{o}(\boldsymbol{w}_1^{o})^\top + \cdots + \boldsymbol{w}_m^{o}(\boldsymbol{w}_m^{o})^\top \\ &\quad + \cdots + \boldsymbol{w}_n^{tr}(\boldsymbol{w}_n^{tr})^\top) \end{aligned} \tag{27}$$

It can be seen from the decomposition of $\boldsymbol{C}$ that the required eigenvector is related to the parameters obtained from the true samples and the parameters obtained from the noisy samples. For a single parameter point

$$\boldsymbol{w}\boldsymbol{w}^\top = \begin{bmatrix} a_1 \\ \vdots \\ a_i \\ \vdots \\ a_d \end{bmatrix} \begin{bmatrix} a_1 & \cdots & a_i & \cdots & a_d \end{bmatrix} = \begin{bmatrix} a_1^2 & a_1a_2 & \cdots & a_1a_d \\ a_2a_1 & a_2^2 & \cdots & a_2a_d \\ \vdots & \vdots & \ddots & \vdots \\ a_da_1 & a_da_2 & \cdots & a_d^2 \end{bmatrix} \tag{28}$$

As we discard all high loss samples that make the model parameters change significantly, and the randomly generated noisy labels may cause the gradient to move in any direction, we assume that the variance of corrupted parameter point variables is δ. Compute the expectations of all variables in the corrupted parameter point

$$\mathbb{E}(\boldsymbol{w}\boldsymbol{w}^\top) = \begin{bmatrix} \delta_1^2 + \mathbb{E}(a_1)^2 & \mathbb{E}(a_1a_2) & \cdots & \mathbb{E}(a_1a_d) \\ \mathbb{E}(a_2a_1) & \delta_2^2 + \mathbb{E}(a_2)^2 & \cdots & \mathbb{E}(a_2a_d) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbb{E}(a_da_1) & \mathbb{E}(a_da_2) & \cdots & \delta_d^2 + \mathbb{E}(a_d)^2 \end{bmatrix} = \Omega \tag{29}$$

Let the sum of all corrupted $\frac{1}{n}\mathbb{E}(\boldsymbol{w}\boldsymbol{w}^\top)$ *be* $\boldsymbol{\Omega}_o$*, then*

$$\boldsymbol{\Omega}_o = \frac{1}{n}\begin{bmatrix} m\delta^2 + \sum_{j=1}^m \mathbb{E}(a_{j1})^2 & \cdots & \sum_{j=1}^m \mathbb{E}(a_{j1}a_{jd}) \\ \sum_{j=1}^m \mathbb{E}(a_{j2}a_{j1}) & \cdots & \sum_{j=1}^m \mathbb{E}(a_{j2}a_{jd}) \\ \vdots & \ddots & \vdots \\ \sum_{j=1}^m \mathbb{E}(a_{jd}a_{j1}) & \cdots & m\delta^2 + \sum_{j=1}^m \mathbb{E}(a_{jd})^2 \end{bmatrix} \tag{30}$$

And let the sum of all true $\frac{1}{n}\boldsymbol{w}\boldsymbol{w}^\top$ *be* $\boldsymbol{C}_{tr}$*. So the expectation of* $\boldsymbol{C}$ *can be written as,*

$$\mathbb{E}(\boldsymbol{C}) = \mathbb{E}(\boldsymbol{C}_{tr}) + \boldsymbol{\Omega}_o \tag{31}$$

Treat eigenvector and eigenvalue as definite values, we get

$$(\boldsymbol{\Omega}_o + \mathbb{E}(\boldsymbol{C}_{tr}))\boldsymbol{e} = \lambda\boldsymbol{e} \tag{32}$$

where $\mathbf{e}$ *is the observed eigenvector,* λ *is the corresponding eigenvalue. Divide both sides of the equation by* λ*.*

$$\begin{aligned} \frac{1}{\lambda}\mathbb{E}(\boldsymbol{C}_{tr})\boldsymbol{e} &= (\boldsymbol{I} - \frac{1}{\lambda}\boldsymbol{\Omega}_o)\boldsymbol{e} \\ &= \boldsymbol{P}_o(\boldsymbol{I} - \frac{1}{\lambda}\boldsymbol{\Lambda}_o)\boldsymbol{P}_o^\top\boldsymbol{e} \\ &\approx \boldsymbol{P}_o(\boldsymbol{I} - \frac{\lambda_o}{\lambda}\boldsymbol{I})\boldsymbol{P}_o^\top\boldsymbol{e} \end{aligned} \tag{33}$$

where λ_o *is the largest eigenvalue in the corrupted diagonal eigenvalue matrix* $\boldsymbol{\Lambda}_o$*,* $\boldsymbol{P}_o$ *is the orthonormal eigenvector matrix of* $\boldsymbol{\Omega}_o$*. According to Eq.33, if* λ_o/λ *is smaller,* $\mathbf{e}$ *is more accurate. Discard some samples with the largest losses, which may contain true samples and noisy samples. Assume that the discarded high loss samples have the same contributions* ξ *to* λ *and* λ_o*, as these two kinds of data have the same effect on the gradient updating of the model. Compare the ratio of eigenvalues before and after discarding, get*

$$\underbrace{\frac{\lambda_o}{\lambda}}_{before} - \underbrace{\frac{\lambda_o - \xi}{\lambda - \xi}}_{after} = \frac{\xi(\lambda - \lambda_o)}{\lambda(\lambda - \xi)} > 0 \tag{34}$$

Obviously, $\lambda > \lambda_o$*, and if we don't discard all samples, then* $\lambda > \xi$*. So Eq.34*> 0*, which means discarding high loss samples could reduce* λ_o/λ*.* ***Therefore, discarding high loss samples can improve the accuracy of eigenvector in the presence of noisy labels.***

For further analysis, we assume that any two variables are independently and identically distributed, the expectation of variable a*,* $\mathbb{E}(a) = \epsilon$*. Thus,*

$$\frac{1}{\lambda}\boldsymbol{\Omega}_o = \frac{p}{\lambda}\begin{bmatrix} \delta^2 + \epsilon^2 & \cdots & \epsilon^2 \\ \epsilon^2 & \cdots & \epsilon^2 \\ \vdots & \ddots & \vdots \\ \epsilon^2 & \cdots & \delta^2 + \epsilon^2 \end{bmatrix} \tag{35}$$

where p *is the proportion of noisy labels,* $np = m$*. As can be seen from Eq.35, if* $p\epsilon^2/\lambda \approx 0$*, then* $\boldsymbol{\Omega}_o/\lambda$ *is a diagonal matrix. According to proof. B.1, the observed eigenvector* $\mathbf{e}$ *is unaffected by noisy labels with the corresponding eigenvalue* $\frac{p(\delta^2+\epsilon^2)}{\lambda}$*.*

F EXPERIMENTS ON CIFAR-FS

In this part, we will test Eigen-Reptile on another realistic dataset. Bertinetto et al propose CIFAR-FS (CIFAR100 few-shots), which is randomly sampled from CIFAR-100, containing images of size 32×32. The settings of Eigen-Reptile (64 filters) in this experiment are the same as in the mini-imagenet experiment. Moreover, we do not compare algorithms with additional tricks, such as higher way. It can be seen from Table 3 that on CIFAR-FS, the performance of Eigen-Reptile is still far better than Reptile without any parameter adjustment.

Table 3: Few Shot Classification on CIFAR-FS N-way K-shot accuracy. The $\pm$ shows 95% confidence interval over tasks.

Algorithm	5-way 1-shot	5-way 5-shot
MAML(Finn et al., 2017)	$58.90 \pm 1.90\%$	$71.50 \pm 1.00\%$
PROTO NET (Snell et al., 2017)	$55.50 \pm 0.70\%$	$72.00 \pm 0.60\%$
GNN (Satorras & Estrach, 2018)	61.90%	75.30%
Embedded Class Models (Ravichandran et al., 2019)	$55.14 \pm 0.48\%$	$71.66 \pm 0.39\%$
Reptile (Nichol et al., 2018)	$58.30 \pm 1.20\%$	$75.45 \pm 0.55\%$
Eign-Reptile	$\mathbf{61.90 \pm 1.40\%}$	$\mathbf{78.30 \pm 0.50\%}$