TOWARDS SCIENTIFIC DISCOVERY WITH DICTIONARY LEARNING: EXTRACTING BIOLOGICAL CONCEPTS FROM MICROSCOPY FOUNDATION MODELS

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ABSTRACT

Dictionary learning (DL) has emerged as a powerful interpretability tool for large language models. By extracting known concepts (e.g., Golden-Gate Bridge) from human-interpretable data (e.g., text), sparse DL can elucidate a model's inner workings. In this work, we ask if DL can also be used to discover *unknown* concepts from less human-interpretable scientific data (e.g., cell images), ultimately enabling modern approaches to scientific discovery. As a first step, we use DL algorithms to study microscopy foundation models trained on multi-cell image data, where little prior knowledge exists regarding which high-level concepts should arise. We show that sparse dictionaries indeed extract biologically-meaningful concepts such as cell type and genetic perturbation type. We also propose a new DL algorithm, Iterative Codebook Feature Learning (ICFL) and combine it with a pre-processing step which uses PCA whitening from a control dataset. In our experiments, we demonstrate that both ICFL and PCA improve the selectivity or "monosemanticity" of extracted features compared to TopK sparse autoencoders.

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1 INTRODUCTION

029 Large scale machine learning systems are extremely effective at generating realistic text and images. However, these models remain black boxes: it is difficult to understand how they produce such 031 detailed reconstructions, and to what extent they encode semantic information about the target 032 domain in their internal representations. One approach to better understanding these models is to 033 investigate how models encode and use high-level, human-interpretable concepts. A challenge to this 034 endeavor is the "superposition hypothesis" (Bricken et al. 2023), which states that neural networks encode many more concepts than they have neurons, and as a result, one cannot understand the model 035 by inspecting individual neuron. One hypothesis for how neurons encode multiple concepts at once 036 is that they are low-dimensional projections of some high-dimensional, sparse feature space. Quite 037 surprisingly, there is now a large body of empirical evidence that supports this hypothesis in language models (Mikolov et al., 2013; Elhage et al., 2022; Park et al., 2023), games (Nanda et al., 2023) and multimodal vision models (Rao et al., 2024), by showing that high-level features are typically 040 predictable via *linear* probing. Further, recent work has shown that model representations can be 041 decomposed into human-interpretable concepts using a dictionary learning model, estimated via 042 sparse autoencoders (Templeton, 2024; Rajamanoharan et al., 2024b;a; Gao et al., 2024). 043

However, all of these successes have relied on some form of text supervision, either directly through 044 next-token prediction or indirectly via contrastive objectives like CLIP (Radford et al., 2021), which align text and image representations. Further, these successes appear in domains which are naturally 046 human-interpretable (i.e. text, games and natural images), and as a result, one may worry that 047 high-level features can be extracted only in settings that we already understand. This raises a natural 048 question: can we extract similarly meaningful high-level concepts from completely unsupervised models in domains where we lack strong prior knowledge? For example, in computational biology, masked autoencoders (MAE) trained on cellular microscopy images have been shown to be very 051 effective at learning representations that recover known biological relationships (Kraus et al., 2024). However, it is not known whether analogous high-level concepts can be extracted from these large 052 MAEs. These settings are precisely where extracting high-level concepts could be most valuable: given that models can detect subtle differences in images (even those that are very challenging for



Figure 1: Cell images ranked according to the correlation strength with three selected features learned by our dictionary learning algorithm. Each feature captures distinct cellular morphologies: Feature A activates for cells with an elongated, spindle-like shape (left) and anti-correlates for sparser or aggregated cells (right); Feature B activates for cells that are densely packed with closely arranged nuclei (left) and deactivates when cell density drops (right); and Feature C activates for small-shaped, compact, brights cells without cell-cell contacts almost entirely made up from just nuclei (left), in contrast to multi-nucleated cells which occupy larger areas (right).

human experts to interpret), we might hope that we can use these techniques to better understandsubtle differences.

We study the extraction of high-level concepts from large-scale MAEs trained on microscopy images of cells that have been perturbed in genetic and small molecule perturbations screens (Fay et al., 2023). Understanding the morphological changes induced by genetic and small molecule perturbations is an inherently difficult and fundamental problem that plays a crucial role in drug discovery (Celik et al., 2022). Recent progress in this field using machine learning has been made by building similarity maps of genetic perturbations via cosine-similarities of post-processed representations from MAEs (Kraus et al., 2024; Celik et al., 2022; Lazar et al., 2024). However, a limitation of these deep learning-based methods is that we only gain limited insights about the morphological changes arising from the perturbations: we can tell whether two perturbations are similar (or dissimilar) via cosine similarity, but we cannot tell why (or the ways in which) they are different. That is, we collapse the multidimensional similarities and dissimilarities down to a single score.

In this paper, we train dictionary learners on top of intermediate representations of large-scale MAEs (Kraus et al., 2024) and find features correlated with single concepts such as individual cell types or genetic perturbations in an unsupervised manner. Moreover, via linear probing, we show that the reconstructed representations from the sparse features preserve significant amounts of biologically-meaningful information. Through this research, we make several key contributions:

- We show that dictionary learning can be used to extract biologically-meaningful concepts from microscopy foundation models (see Figure 1), opening the path to scientific discovery using tools from mechanistic interpretability.
- We propose a new dictionary learning algorithm—Iterative Codebook Feature Learning (ICFL) which naturally avoids "dead" features (Section 4).
- We further show how PCA whitening on a control dataset can act as a form of weak supervision for dictionary learning (Section 5), resulting in more meaningful features.
- We demonstrate empirically that both ICFL and PCA improve the selectivity or "monosemanticity" of extracted features compared to TopK sparse autoencoders (Section 6).

108 Algorithm 1 Iterative Codebook Feature Learning 109 1: Input: Parameters W_{dec} , b_{pre} ; model representation x; # sparse features K and iterations J110 2: Initialize $x^{(1)} := x - b_{pre}$ 111 3: for t = 1 to J do 112 Select top K columns of W_{dec} which maximize $\langle W_{\text{dec},m}, x^{(t)} \rangle$ 4: 113 Solve $z^{(t)} = \arg \min_{z} ||x^{(t)} - W_{dec}z||_2^2$ with z non-zero only for selected columns 5: 114 Update $x^{(t+1)} := x^{(t)} - W_{dec} z^{(t)}$ 6: 115 7: end for 116 8: **Output:** Sparse features $z := \sum_{t=1}^{J} z^{(t)}$ 117

2 **RELATED WORK**

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121 The disentanglement and causal representation literature (CRL) share the goal of learning high-level, 122 interpretable concepts (Bengio et al., 2013; Kulkarni et al., 2015; Higgins et al., 2017; Chen et al., 123 2016; Eastwood & Williams, 2018; Schölkopf et al., 2021). Two key differences with the dictionary learning approach are: (i) disentanglement/CRL methods consider low-dimensional representa-124 125 tions to capture the factors of variation in data, whereas overcomplete dictionary learning seeks a higher-dimensional representation to capture a large set of sparsely-firing concepts; and (ii) disentan-126 glement/CRL methods aim to be inherently interpretable, whereas this paper considers a post-hoc 127 approach to interpret pre-trained models. Related work on post-hoc explainability also learns "concept 128 vectors" in neural network internal states (Kim et al., 2018; Ghorbani et al., 2019); a key difference 129 is that these methods use class-labeled data, whereas this paper uses an unsupervised approach to 130 discover concepts. Additionally, feature-visualization works aim to interpret internal states/neurons 131 by finding the data points (or gradient-optimized inputs) that lead to maximal activation (Mordvintsev 132 et al., 2015; Olah et al., 2017; Borowski et al., 2021). 133

3 BACKGROUND

136 The superposition hypothesis. Let $x_i \in \mathbb{R}^d$ denote a representation for token *i*; as an example, 137 x_i may be the embedding of token i after a transformer layer. Bricken et al. (2023) hypothesize 138 that (i) such token representations $x_i \in \mathbb{R}^d$ are linear combinations of concepts; (ii) the number of 139 available concepts M significantly exceed the dimension of the representation d; and (iii) each token 140 representation is the sum of a sparse set of concepts. These desiderata are satisfied by the following 141 model that is widely studied in compressed sensing and dictionary learning: 142

$$x_i \approx W z_i$$
 where $\|z_i\|_0 \ll d$ (1)

where $W \in \mathbb{R}^{d \times M}$ is a latent concept matrix and $z_i \in \mathbb{R}^M$ is a sparse latent concept-selector (resp. 144 feature) vector. 145

146 **Feature learning using Topk SAEs.** Given a set of token representations $\{x_i\}_{i=1}^N$, learning both 147 W and $\{z_i\}_{i=1}^N$ is a dictionary learning or sparse coding problem Olshausen & Field (1997), with 148 a long history of works proposing efficient algorithms with provable guarantees (Aharon et al., 149 2006; Arora et al., 2014; 2015). In the context of mechanistic interpretability, the dominant choice 150 for learning these parameters are two-layer sparse autoencoders. In this paper, we compare to the 151 state-of-the-art method called TopK SAE, originally proposed by Makhzani & Frey (2013) and 152 recently studied by Gao et al. (2024). Following their notation, the model is:

$$x_i = W_{\text{dec}} z_i + b_{\text{pre}}, \text{ with } z_i = \text{TopK}(W_{\text{enc}} x_i - b_{\text{pre}})$$

154 where $TopK(\cdot)$ is an operator that sets all but the K largest elements to zero. The parameters 155 $\{W_{dec}, W_{enc}, b_{pre}\}$ are learned by minimizing the reconstruction loss: 156

$$L(W,b) := \sum_{i} \|x_i - \hat{x}_i\|_2^2, \quad \text{where } \hat{x}_i = W_{\text{dec}} \text{TopK}(W_{\text{enc}} x_i - b_{\text{pre}}) + b_{\text{pre}}$$
(2)

A problem with the above optimization is that some concept vectors $W_{\text{dec},m}$ are barely used; that is, 159 features $z_{im} = 0$ for almost all $i \in [N]$. This is called the "dead feature" phenomenon. To reduce the 160 amount of dead features, Gao et al. (2024) introduce an additional reconstruction error term using 161 only these concept vectors to encourage their usage in the model (see Table 1).

162 **ITERATIVE CODEBOOK FEATURE LEARNING (ICFL)** 4

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Sparse autoencoders such as TopK SAEs face two major limitations: (i) they require regularization to 165 avoid "dead features" after training (Gao et al., 2024; Bricken et al., 2023) and (ii) some concepts may 166 be overrepresented in the samples $\{x_i\}_{i=1}^N$, biasing the estimation. To overcome these limitations, we propose Iterative Codebook Feature Learning (ICFL). ICFL retains the decoder of TopK SAEs, 168 however, instead of using an encoder to learn the features z, ICFL updates z using a variant of the orthogonal matching pursuit algorithm of Mallat & Zhang (1993) as described in Algorithm 1. Specifically, given the current decoder/feature matrix W_{dec} , we first select the top-k columns most 170 aligned with $x^{(1)} = x$. Then, we learn the features $z^{(1)}$ that best reconstruct $x \approx W_{\text{dec}} z^{(1)}$, using only these columns (i.e. $z^{(1)}$ is K-sparse). Next, to obtain $z^{(2)}$, we repeat this step, but replace 172 x with the residual $x^{(2)} = x - W_{dec} z^{(1)}$. Repeating this process, the final output z is taken to be 173

 $z = \sum_{t=1}^{J} z^{(t)}$. Consequently, z is at most Jk-sparse. 174

175 The key idea of ICFL is that early iterations subtract dominant concepts from x, allowing the 176 algorithm in later iterations to select a broader set of concepts that are not as correlated with the main 177 concepts in x. After updating z as detailed in Algorithm 1, the decoder parameters $\{W_{dec}, b_{pre}\}$ are 178 updated to minimize the reconstruction loss from equation 2 with $\hat{x} = W_{dec}z + b_{pre}$. As z is fixed in 179 this gradient step, the algorithm does not propagate gradients through z. Consequently, the algorithm 180 results in very few "dead" features. As a result, we do not require any additional regularization to address this "dead feature" issue that often hinders SAEs, as shown in Table 1. 181

182 In practice, we leverage random resets to ensure that the columns of 183 $W_{\rm dec}$ are not too correlated. To prevent the collapse of the feature directions (columns of W_{dec}), after every 100 stochastic gradient 185 descent steps, we take every pair of columns of W_{dec} that have cosine-similarity above 0.9 and randomly initialize one of the pairs with a vector selected uniformly at random from the hypersphere. 187 Before running Algorithm 1, we always center the representations 188 by the average representation with unperturbed samples from the 189 control distribution. By doing so, we center the representations 190 such that the origin represents the unperturbed state. Finally, we 191 normalize the representations before applying the dictionary learner. 192

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Table 1: The number of "dead features" (out of 8192) that have been activated less than a fraction of 10^{-5} many times during the last 1000 training steps, for both TopK and ICFL with and without PCA whitening (see Section 5).

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EXPERIMENTAL SETUP 5

Data source and foundation model We evaluated our dictionary learning approach on two large-196 scale masked autoencoders trained on cellular microscopy Cell Painting image data using 256x256x6 197 pixel crops as input and a patch size of 8, following the same procedures as those described in Kraus et al. (2024). These models were trained on data from multiple cell types that were perturbed with 199 both CRISPR gene-knockouts and small molecule perturbations. Both models used the architecture 200 hyperparameters from Kraus et al. (2024), with the smaller of the two using the ViT-L/8 configuration, 201 while the larger model used the ViT-G/8 configuration. We refer to these models as MAE-L and MAE-202 G, respectively. We obtain a single token per input crop by aggregating all patch tokens (excluding 203 the class token). For both the residual stream and the attention output (after the out-projection), the 204 dimension d of the tokens (representations) are 1024 and 1664 for MAE-L and MAE-G, respectively. All the visualizations used Cell Painting microscopy images from the public RxRx1 (Sypetkowski 205 et al., 2023) and RxRx3 (Fay et al., 2023) datasets. 206

207 We extract the tokens from layer 16 (MAE-L) and layer 33 (MAE-G), respectively. The motivation for 208 using intermediate instead of final layers is that these tokens are more-likely to capture abstract high 209 level concepts that are *internally used* by the model to solve the SSL task (Alkin et al., 2024). We 210 selected this layer by finding the layer which maximized linear probing performance on the functional 211 group tasked (described below) from the original embeddings.

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213 **Preserving linear probing signals** To investigate whether the features found by sparse dictionary learning retain important information from the original representation, we define five different 214 classification tasks, summarized in Table 2. For each classification task, we use a separate (potentially 215 overlapping) dataset and split it into train and test data to distinguish labels across:

Task	Cell Type	Experiment Batch	siRNA Perturbation	CRISPR Perturbation	Functional Gene Group
# Classes	23	272	1 138	5	39
# Samples	110,971	80,000	81,224	79,555	57,863
Bal. Test Acc.	97.2%	87.8%	51.6%	94.6%	32.1%

Table 2: The five classification tasks and the test bal. acc. for linear probes trained on well-level aggregated representations from the residual stream from an intermediate layer from *MAE-G*.

- (1) 23 different cell types which are almost perfectly distinguishable via linear classification.
- (2) 272 different experiment batches. Even in controlled conditions, subtle changes in experimental conditions can induce strong *batch effects*, *i.e.* changes in experimental outcomes due to experiment-specific variations unrelated to the perturbation that is being tested.
- (3) 1138 siRNA perturbations from the RxRx1 dataset (Sypetkowski et al., 2023), where the single-gene expression (i.e. gene mRNA level) is partially (or completely) silenced using short interfering (si-)RNA. siRNA targets the gene mRNA for destruction via the RNA interference pathway (Tuschl, 2001). As the extent of siRNA knock-downs is hard to quantify and prone to significant but consistent off-target effects, we also evaluated:
- (4) 5 single-gene CRISPR perturbation knockouts which induce strong and consistent morphological profiles across cell types, known as "perturbation signal benchmarks" (Celik et al., 2024). Unlike the siRNA approach, CRISPR cuts the gene DNA directly, which induces mutation in the sequence and represses the gene function. To evaluate whether our method retrieves signal which corresponds to similar phenotypes, we also assessed:
- (5) 39 functional gene groups composed of CRISPR single-gene knockouts categorized by phenotypic relationships between the genes, including major protein complexes, metabolic and signaling pathways. Each gene group targets similar or related cellular process, which results in inducing morphologically similar changes in the cells (Celik et al., 2022).
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To remove the impact of spurious correlations between perturbations and batch effects on the test accuracy, we always use mutually exclusive experiments for test and train data, except for (ii) where the task is to predict the experiment. Except for (i), all classification tasks use HUVEC cells and always use well-level aggregated representations: that is, we take the mean over tokens from all 36 non-edge crops from an image of a given well of cells. Because some of the classes are heavily imbalanced (particularly for Task (1)), we always report the *balanced test accuracy* and train our linear probes using logistic regression on a class-balanced cross-entropy loss.

PCA whitening using a control dataset As dictionary learners seek to minimize the Euclidean 254 distance between the model representations x and their reconstructions $\hat{x} = Wz$, the learned features z are naturally biased towards capturing the dominant directions in the data (i.e., those that explain 256 the most variance). Unfortunately, these directions often do not align with meaningful concepts. To 257 address this, we use a dataset of control samples as a form of weak supervision, downweighting 258 dominant directions in this control dataset as we know they do not correspond to the biological pertur-259 bations of interest. In particular, we learn a PCA-and-centerscale transform on this control dataset and 260 apply it to the entire dataset before normalization. For our multi-cell data, unperturbed HUVEC-cell 261 images act as our control dataset. Note that similar PCA whitening on a control dataset has been used 262 to improve the quality of the learned multi-cell image representations (Kraus et al., 2024).

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Training the DL models By default, we always choose a sparsity of K = 100 for TopK SAEs and J = 20, k = 5 (resulting in a max sparsity of 100) for ICFL as described in Section 4, and use a total of 8192 features. Unless otherwise specified, we always apply the PCA whitening described in Section 5 and use representations from the residual stream. We train the sparse autoencoders using 40M tokens (one token per crop) with a batch size of 8192 for 300k iterations. Our learning rate is 5×10^{-5} for all experiments. Similar to Gao et al. (2024), we observed that changing the learning rate has a limited impact on the outcome. We present an ablation for the learning rate in Appendix C.



Figure 2: *Top row:* a) Test bal. acc. of linear probes trained on the original representation (solid line) and reconstructions from ICFL and TopK SAEs in combination with PCA whitening and with out. b) Test bal. acc. as a function of the sparsity (dashed line is the original representation) for classification Task 5. c) Cosine similarity of reconstruction and original representations as a function of sparsity for tokens from a hold-out validation dataset. *Bottom row:* The highest selectivity scores among all features for each label. We separately order the labels for each line starting with the maximum score. We plot the avg (solid) and max (dashed) selectivity scores.

6 EXPERIMENTAL RESULTS

In this section we present our experimental results. If not further specified, we always use features extracted from ICFL in combination with PCA whitening.

6.1 DICTIONARY FEATURES ARE CORRELATED WITH BIOLOGICAL CONCEPTS

301 **Preserving linear probing signals** By comparing linear probes on the representations and recon-302 structions from ICFL sparse features, we can measure how much "biologically-relevant" information 303 is lost when extracting sparse features. Figure 2a shows that almost the entire signal is preserved 304 for simple concepts such as cell types (1), batch effects (2) and perturbations with strong morpho-305 logical changes (4). For the difficult tasks of distinguishing between many genetic perturbations 306 (3,5), a substantial amount of the linear signal is preserved. Both TopK SAEs and ICFL features yield a similar linear probing accuracy, while we can see a clear drop if no PCA whitening is used 307 during pre-processing. We further present in Figure 2b an ablation for the sparsity of the extracted 308 feature vector. While increasing the number of non-zeros improves the accuracy, the effect is limited 309 compared to PCA whitening. 310

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Reconstruction loss To evaluate the quality of unsupervised DL, the cosine similarity (or ℓ_2 -error) has been often used as a benchmark (Rajamanoharan et al., 2024a; Gao et al., 2024). Figure 2c shows that the reconstruction quality of ICFL is much higher than TopK SAE for the same sparsity constraints when using PCA whitening. We provide further ablations in Appendix C.

316 **Selectivity of features for biological concepts** As a third experiment, we investigate how strongly 317 correlated the features are with labels from the classification tasks in Table 2. For each dataset 318 associated with a classification task, we extract from every image a feature vector using the center 319 crop as input to the MAE. For each feature, we then compute two selectivity scores: the avg selectivity 320 score, which is the % of times that the feature is active given that label *i* occurs minus the % of times 321 the feature is active given any other label. As a stronger notion of correlation, we also use the **max** selectivity score, that subtracts the maximum % for any other label. The selectivity score has been 322 originally proposed in the context of neuroscience (Hubel & Wiesel, 1968) and has also been used by 323 Madan et al. (2022) to measure the "monosemanticity" of neurons.



(e) Comparison with CP features (g) Correlation with CP features (f) Average score vs. sparsity

Figure 3: Top row: Cosine-similarity histograms for selected pairs of representations from perturbations from Task 3 and features directions (as shown in the caption), given that its associated perturbation is applied (blue) and that any other perturbation is applied (orange). Bottom row: Comparison of the average selectivity score 345 of features from CP and ICFL. a) Maximum average selectivity scores for each label, displayed in descending 346 order. b) The scores from (a) averaged across labels at different thresholds for CP and sparsity levels for ICFL, as a function of the average number of non-zero values. c) Correlation of maximum average selectivity scores for each label between CP and ICFL. 348

349 We plot in Figure 2d-2f the selectivity scores for both ICFL features and TopK SAEs. We see that 350 ICFL features consistently achieve higher selectivity scores than TopK SAE features. Moreover, 351 especially for cell types, we observe a high max selectivity across almost all cell types, while for 352 more complex features we still observe a moderate selectivity score of more than 0.1 across all 353 labels. We present in Table 3 the number of features exhibiting an average selectivity greater than 354 a given threshold for at least one label, across all five classification tasks. This is done using three 355 different thresholds for ICFL with PCA whitening. We observe that dominant concepts, such as cell 356 types, batch effects, and siRNA perturbations that induce strong morphological changes, lead to a 357 substantial portion of features displaying high selectivity. However, also for labels from the functional 358 gene groups (Task 5), we identify more than 100 features with selectivity scores of at least 0.1.

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Separation along feature directions The selectivity score analysis showed that activation patterns 361 of the sparse features can be strongly correlated with genetic perturbations. To further strengthen 362 this argument, we illustrate in Figure 3 the cosine similarities between representations from different genetic perturbations and selected feature directions, that is the *i*-th column of W_{dec} for Feature *i*. 364 While we could also directly look at the feature values z_i , due to the sparsity, most of the values are 0.

We plot Figure 3 the cosine similarities between selected feature directions and the crop-level 366 aggregated tokens. The histogram in blue represent tokens from specific siRNA perturbations, while 367 the histogram in orange represent all other tokens from Task 3. The plot shows that feature directions 368 effectively separate the two groups, showing that certain features capture important biological 369 information, which shed light on the morphological changes caused by genetic perturbations. 370

Threshold	Cell Type	Experiment Batch	siRNA Perturbation	CRISPR Perturbation	Functional Gene Group
0.5	73	11	0	0	0
0.2	455	77	141	2	37
0.1	928	243	681	13	166

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Table 3: Feature count (max 8192) with avg selectivity above thresholds for at least one label

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3796.2COMPARISON WITH FEATURES FROM CellProfiler

380 As a second set of experiments, we compare the average selectivity scores of features from ICFL 381 with those from a set of 964 handcrafted features generated by CellProfiler (CP) (Carpenter et al., 2006). These features are designed by domain experts and are widely used for microscopy image 382 analysis. This task compares the monosemanticity of unsupervised features extracted from foundation 383 models to that of human expert-designed features. We obtain sparse features by thresholding the 384 average CP features obtained from all cells from a multi-cell image taken form a subset of the public 385 RxRx1-dataset (Sypetkowski et al., 2023). We threshold at the α and $1 - \alpha$ quantiles with α such 386 that the average number of non-zeros is ≈ 100 . A feature was classified as "activated" when its value, 387 under perturbation conditions, exceeded these quantiles. The selectivities corresponding to both CP 388 and our SAEs, measured using the same datasets.

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Comparison of selectivity scores In Figure 3e, we plot the highest average selectivity score for 391 each genetic perturbation (a subset of Task 3 in sorted order for both CP features and ICFL features). 392 The results show that the features extracted by ICFL almost match the selectivity scores of the 393 handcrafted, human-designed features. Additionally, in Figure 3f, we show the average score across 394 all labels as a function of various thresholding levels for the CP features. On the x-axis, we plot 395 the average number of non-zero elements. We again observe that our features perform comparably 396 to CP features. Interestingly, CP features peak at high levels of non-zeros (≈ 300), leaving future work to assess whether this peak selectivity can be matched using deep learning-based approaches 397 while using significantly fewer non-zero elements. We further illustrate the correlation between the 398 best average selectivity scores from the CP and ICFL features for each label (Figure 3g). The plot 399 demonstrates a strong correlation (Pearson coefficient of 0.71), suggesting that ICFL is capable of 400 identifying features that capture patterns similar to those detected by CP. 401

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7 QUALITATIVE ANALYSIS OF SELECTED FEATURES

In this section, we illustrate striking, non-trivial patterns captured by selected features and provide an example for how domain experts can interpret, study and validate features found by DL. To study the "semanticity" of features in ViTs, we propose interpreting them at the pixel level by examining which patches exhibit the highest cosine similarities with the feature directions. More precisely, for the multi-cell image crops strongly correlated with selected feature directions, we compute heatmaps of the cosine-similarities of the individual tokens from 8×8 patches and feature directions (Figure 4 and Figure 5, top rows).

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7.1 TOKEN-LEVEL FEATURES IN FUNCTIONAL GENE GROUP

414 We begin by examining a single feature for our interpretabiliy analysis, and its corresponding feature 415 direction, that we chose because it demonstrated a clear biological relationship. The feature is strongly 416 correlated with gene knockouts from the *adherens junctions* pathway, a label from the functional 417 gene perturbation group from Task 5 (§ 5). The adherens junctions connect cell membranes to 418 cytoskeletal elements and form cell-cell adhesions; they can be thought of as "glue proteins" that 419 stick cells together. Our microscopy images visualization in Figure 4 strongly correlated with the 420 feature direction, reflecting this disrupted cellular morphology. The images comprise of small, bright 421 and isolated cells which appear unable to establish proper connections with the neighboring cells 422 (Figure 4, middle row). Note that despite the similar appearance, the images do not originate from the perturbation of a single gene, but rather from a group of genes related in a functional family (Task 5). 423 The regions *most* correlated with the concept direction (Figure 4, top row, most white) belong to areas 424 surrounding perturbed cells. These tokens appear to form a ring-like pattern around the compact cells 425 (top row), which suggests that the concept corresponds to the expected but missing actin (rendered in 426 red) around the cell nucleus (rendered in blue), which is indicative of the perturbation phenotype. 427

By contrast, the tokens that are *not* aligned with the feature direction associate with cells where
the actin meshwork extensively protrudes away from the cell center (yellow bounding boxes). The
CRISPR gene editing process (§ 5) is imperfect and as a result in any well, a small proportion of cells
remains unperturbed. We found that tokens *least* correlated with the concept direction (Figure 4, top
row, most black) belong to what appear to be unperturbed cells (yellow bounding boxes). Examining



Figure 4: Visualization of composite images (middle row) and their actin-staining channel (bottom row) which strongly correlate with a selected feature from a single functional gene group — *adherens junctions*. Plotted above are the token-level heatmaps of the inner products of the individual tokens with the selected feature direction (top row) for 5 out of 8 strongest correlated images per feature direction. Highlighted are the cells which most likely remain unperturbed (yellow bounding boxes), which are the only instances attempting to establish cell-cell connections (cyan arrows) as they produce the gene to form functional *adherens junctions*.

the corresponding channel-specific image for actin (Figure 4, bottom row) clearly shows that these cells differ from the rest of the well in that they do not contribute to the overall morphology of the image as they manage to form an extensive actin meshwork, and are the only instances which attempt to make connections with neighboring cells (cyan arrows).

7.2 CHANNEL-SPECIFIC PROPERTIES OF SINGLE-GENE PERTURBATIONS

Finally, we examine the extent to which we can recover channel-specific signal associated with the gene perturbations. For this exercise, we queried 3 specific gene perturbations: (i) OPA-1, which contributes to the maintenance of correct shape of mitochondria, (ii) ALG-3, which aids in the modification of proteins and lipids in the endoplasmic reticulum (ER) after synthesis, and (iii) TSC-2, which contributes to the control of the cell size (Figure 5).

OPA-1 The mitochondrial channel shows that most correlated tokens are overlaid with distant regions where enlarged mitochondria are present (pink arrows). Quantitatively, this nuanced relation-ship does not show a strong correlation in the mitochondrial channel (0.41) due to the aberrant image background, but qualitative examination of the mitochondrial channel highlights this delicate detail which is not obvious from the composite images (Figure 5, 1st column, middle row).

ALG-3 The most aligned tokens appear specific to regions of endoplasmic reticulum (ER) and
RNA with which ALG-3 co-localizes, where it aids with attachment of a sugar-like groups to proteins.
In this dense image, we report that the correlation of endoplasmic reticulum (0.63) and RNA-specific
channels (0.63) are much higher than for channels staining other cellular compartments, *e.g.* plasma
membrane (0.24) or actin (0.16). This suggests that our token heatmap is prevalently focused on
ER-specific information (Figure 5, 2nd column), which is consistent with what we would expect from
our understanding of the protein function.

TSC-2 We examine the plasma membrane- and Golgi apparatus-specific channel to relate perturbed
 cell size control to the token alignment. We confirm that this channel correlates most strongly with
 the queried concept direction, but this time in a negative direction. As the plasma membrane — and,
 hence, cytoplasmic area — are the most extensive from the cell center, the mostly aligned tokens
 appear to focus specifically on regions which are not covered by the cell membrane, or the membrane



Figure 5: Visualization of representative images from selected single-gene perturbations as in Figure 4 out of 25 strongest correlated images per feature direction. Plotted are the channel-specific staining images of subcellular compartments: mitochondria (orange), endoplasmic reticulum (green) and membrane with Golgi (yellow). Displayed are per-image correlation coefficients between token heatmaps and channel-specific images.

pixel intensity fades away (Figure 5, 3rd column, bottom row). This relationship shows highly 513 negative correlation (-0.71), making it a stronger signal than actin (-0.43) or mitochondria (-0.49), 514 and is likely monitoring the lack of channel-specific signal, similarly to the finding from Section 7.1. 515

516 **Inverse focus** Finally, we show that the tokens are not always co-localized with regions occupied 517 by cells. Here, we selected two genes which appear to follow an "inverse" trend, namely affecting PLK-1, which enables cell cycle progression through mitosis, and TMED-2, which helps to regulate 518 intracellular protein transport. While both of these gene perturbations render the cells in a charac-519 teristic affected state (small, clumped cells struggling to divide vs. large, spread out and actively 520 dividing cells), it appears that their most aligned tokens correspond to areas *not* covered by cells, 521 which we confirm with highly negative correlations across all channels (Figure 5, last 2 columns). 522 This suggests that the salient feature for these perturbations is the *lack* of cell density in a well. 523

- 524 CONCLUSION 8
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526 In this paper we have explored the extent to which dictionary learning can be used to extract 527 biologically-meaningful concepts from microscopy foundation models. The results are encouraging: 528 with the right approach, we were able to extract sparse features that are associated with distinct and 529 biologically-interpretable morphological traits. That said, these sparse features are clearly incomplete: 530 we see significant drops in their linear-probing performance on tasks that involve more subtle changes in morphology. It is not clear to what extent this is a limitation of our current dictionary learning 531 techniques, the scale of our models, or whether these more subtle changes are simply not represented 532 linearly in embedding space. Nonetheless, it is clear that the choice of dictionary learning algorithm 533 matters to extract meaningful features. 534

535 We also proposed a new dictionary learning algorithm, Iterative Codebook Feature Learning (ICFL), and the use of PCA whitening on a control dataset as a form of weak supervision for the feature 536 extraction. In our experiments, we found that both ICFL and PCA significantly improve the selectivity 537 or "monosemanticity" of extracted features, compared to TopK sparse autoencoders. We hope that 538 future work further explores the use of dictionary learning for scientific discovery, as well as the use of ICFL for other modalities like text.

540 REFERENCES

548

553

561

- 542 Michal Aharon, Michael Elad, and Alfred Bruckstein. K-svd: An algorithm for designing over 543 complete dictionaries for sparse representation. *IEEE Transactions on signal processing*, 54(11):
 544 4311–4322, 2006.
- Benedikt Alkin, Lukas Miklautz, Sepp Hochreiter, and Johannes Brandstetter. Mim-refiner:
 A contrastive learning boost from intermediate pre-trained representations. *arXiv preprint arXiv:2402.10093*, 2024.
- Sanjeev Arora, Rong Ge, and Ankur Moitra. New algorithms for learning incoherent and overcomplete dictionaries. In *Conference on Learning Theory*, pp. 779–806. PMLR, 2014.
- Sanjeev Arora, Rong Ge, Tengyu Ma, and Ankur Moitra. Simple, efficient, and neural algorithms for sparse coding. In *Conference on learning theory*, pp. 113–149. PMLR, 2015.
- Yoshua Bengio, Aaron Courville, and Pascal Vincent. Representation learning: A review and new perspectives. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 35(8):1798–1828, 2013.
- Judy Borowski, Roland Simon Zimmermann, Judith Schepers, Robert Geirhos, Thomas S. A. Wallis,
 Matthias Bethge, and Wieland Brendel. Exemplary natural images explain {cnn} activations better
 than state-of-the-art feature visualization. In *International Conference on Learning Representations*,
 2021.
- Trenton Bricken, Adly Templeton, Joshua Batson, Brian Chen, Adam Jermyn, Tom Conerly, Nick
 Turner, Cem Anil, Carson Denison, Amanda Askell, et al. Towards monosemanticity: Decomposing language models with dictionary learning. *Transformer Circuits Thread*, 2, 2023.
- Anne E Carpenter, Thouis R Jones, Michael R Lamprecht, Colin Clarke, In Han Kang, Ola Friman,
 David A Guertin, Joo Han Chang, Robert A Lindquist, Jason Moffat, et al. Cellprofiler: image
 analysis software for identifying and quantifying cell phenotypes. *Genome biology*, 7:1–11, 2006.
- Safiye Celik, Jan-Christian Huetter, Sandra Melo, Nathan Lazar, Rahul Mohan, Conor Tillinghast,
 Tommaso Biancalani, Marta Fay, Berton Earnshaw, and Imran S Haque. Biological cartography:
 Building and benchmarking representations of life. In *NeurIPS 2022 Workshop on Learning Meaningful Representations of Life*, 2022.
- Safiye Celik, Jan-Christian Hütter, Sandra Melo Carlos, Nathan H Lazar, Rahul Mohan, Conor
 Tillinghast, Tommaso Biancalani, Marta M Fay, Berton A Earnshaw, and Imran S Haque. Building,
 benchmarking, and exploring perturbative maps of transcriptional and morphological data. *bioRxiv*,
 2024. doi: 10.1101/2022.12.09.519400.
- 577 Xi Chen, Yan Duan, Rein Houthooft, John Schulman, Ilya Sutskever, and Pieter Abbeel. InfoGAN:
 578 Interpretable representation learning by information maximizing generative adversarial nets. In
 579 Advances in Neural Information Processing Systems, volume 29, pp. 2180–2188, 2016.
- Timothée Darcet, Maxime Oquab, Julien Mairal, and Piotr Bojanowski. Vision transformers need
 registers. *arXiv preprint arXiv:2309.16588*, 2023.
- 583
 584
 585
 Cian Eastwood and Christopher K I Williams. A framework for the quantitative evaluation of disentangled representations. In *International Conference on Learning Representations*, 2018.
- Nelson Elhage, Tristan Hume, Catherine Olsson, Nicholas Schiefer, Tom Henighan, Shauna Kravec,
 Zac Hatfield-Dodds, Robert Lasenby, Dawn Drain, Carol Chen, et al. Toy models of superposition.
 arXiv preprint arXiv:2209.10652, 2022.
- Marta M Fay, Oren Kraus, Mason Victors, Lakshmanan Arumugam, Kamal Vuggumudi, John Urbanik, Kyle Hansen, Safiye Celik, Nico Cernek, Ganesh Jagannathan, et al. Rxrx3: Phenomics map of biology. *bioRxiv*, pp. 2023–02, 2023.
- ⁵⁹³ Javier Ferrando, Gabriele Sarti, Arianna Bisazza, and Marta R Costa-jussà. A primer on the inner workings of transformer-based language models. *arXiv preprint arXiv:2405.00208*, 2024.

626

- 594 Leo Gao, Tom Dupré la Tour, Henk Tillman, Gabriel Goh, Rajan Troll, Alec Radford, Ilya 595 Sutskever, Jan Leike, and Jeffrey Wu. Scaling and evaluating sparse autoencoders. arXiv preprint 596 arXiv:2406.04093, 2024. 597
- Amirata Ghorbani, James Wexler, James Y Zou, and Been Kim. Towards automatic concept-based 598 explanations. In Neural Information Processing Systems, 2019.
- 600 Irina Higgins, Loic Matthey, Arka Pal, Christopher Burgess, Xavier Glorot, Matthew Botvinick, 601 Shakir Mohamed, and Alexander Lerchner. β -VAE: Learning basic visual concepts with a 602 constrained variational framework. In International Conference on Learning Representations, 603 2017. 604
- Po-Yao Huang, Vasu Sharma, Hu Xu, Chaitanya Ryali, Haoqi Fan, Yanghao Li, Shang-Wen Li, Gargi 605 Ghosh, Jitendra Malik, and Christoph Feichtenhofer. MAViL: Masked Audio-Video Learners. 606 arXiv, 2022. doi: 10.48550/arxiv.2212.08071. 607
- 608 David H Hubel and Torsten N Wiesel. Receptive fields and functional architecture of monkey striate 609 cortex. The Journal of physiology, 195(1):215–243, 1968. 610
- 611 Been Kim, Martin Wattenberg, Justin Gilmer, Carrie Cai, James Wexler, and Fernanda Viegas. Interpretability beyond feature attribution: Quantitative testing with concept activation vectors 612 (tcav). In International Conference on Machine Learning, 2018. 613
- 614 Oren Kraus, Kian Kenyon-Dean, Saber Saberian, Maryam Fallah, Peter McLean, Jess Leung, Vasudev 615 Sharma, Ayla Khan, Jia Balakrishnan, Safiye Celik, et al. Masked autoencoders for microscopy are 616 scalable learners of cellular biology. In Proceedings of the IEEE/CVF Conference on Computer 617 Vision and Pattern Recognition, pp. 11757–11768, 2024.
- Tejas D Kulkarni, William F Whitney, Pushmeet Kohli, and Josh Tenenbaum. Deep convolutional 619 inverse graphics network. In Advances in Neural Information Processing Systems, volume 28, pp. 620 2539-2547, 2015. 621
- 622 Nathan H Lazar, Safiye Celik, Lu Chen, Marta M Fay, Jonathan C Irish, James Jensen, Conor A 623 Tillinghast, John Urbanik, William P Bone, Christopher C Gibson, et al. High-resolution genome-624 wide mapping of chromosome-arm-scale truncations induced by crispr-cas9 editing. Nature 625 Genetics, pp. 1–12, 2024.
- Spandan Madan, Timothy Henry, Jamell Dozier, Helen Ho, Nishchal Bhandari, Tomotake Sasaki, 627 Frédo Durand, Hanspeter Pfister, and Xavier Boix. When and how convolutional neural networks 628 generalize to out-of-distribution category-viewpoint combinations. Nature Machine Intelligence, 629 4(2):146-153, 2022. 630
- 631 Alireza Makhzani and Brendan Frey. K-sparse autoencoders. arXiv preprint arXiv:1312.5663, 2013. 632
- Stéphane G Mallat and Zhifeng Zhang. Matching pursuits with time-frequency dictionaries. IEEE 633 Transactions on signal processing, 41(12):3397–3415, 1993. 634
- 635 Tomáš Mikolov, Wen-tau Yih, and Geoffrey Zweig. Linguistic regularities in continuous space word 636 representations. In Proceedings of the 2013 Conference of the North American chapter of the association for computational linguistics: Human language technologies, pp. 746–751, 2013. 638
- 639 Alexander Mordvintsev, Christopher Olah, and Mike Tyka. Inceptionism: Going deeper into neural networks. Google research blog, 20(14):5, 2015. 640
- 641 Neel Nanda, Andrew Lee, and Martin Wattenberg. Emergent linear representations in world models 642 of self-supervised sequence models. arXiv preprint arXiv:2309.00941, 2023. 643
- 644 Chris Olah, Alexander Mordvintsev, and Ludwig Schubert. Feature visualization. Distill, 2(11):e7, 645 2017. 646
- Bruno A Olshausen and David J Field. Sparse coding with an overcomplete basis set: A strategy 647 employed by v1? Vision research, 37(23):3311-3325, 1997.

- Kiho Park, Yo Joong Choe, and Victor Veitch. The linear representation hypothesis and the geometry of large language models. *arXiv preprint arXiv:2311.03658*, 2023.
- Alec Radford, Jong Wook Kim, Chris Hallacy, Aditya Ramesh, Gabriel Goh, Sandhini Agarwal,
 Girish Sastry, Amanda Askell, Pamela Mishkin, Jack Clark, et al. Learning transferable visual
 models from natural language supervision. In *International conference on machine learning*, pp.
 8748–8763. PMLR, 2021.
- Senthooran Rajamanoharan, Arthur Conmy, Lewis Smith, Tom Lieberum, Vikrant Varma, János
 Kramár, Rohin Shah, and Neel Nanda. Improving dictionary learning with gated sparse autoen coders. *arXiv preprint arXiv:2404.16014*, 2024a.
- Senthooran Rajamanoharan, Tom Lieberum, Nicolas Sonnerat, Arthur Conmy, Vikrant Varma, János Kramár, and Neel Nanda. Jumping ahead: Improving reconstruction fidelity with jumprelu sparse autoencoders. *arXiv preprint arXiv:2407.14435*, 2024b.
- Sukrut Rao, Sweta Mahajan, Moritz Böhle, and Bernt Schiele. Discover-then-name: Task-agnostic
 concept bottlenecks via automated concept discovery. *arXiv preprint arXiv:2407.14499*, 2024.
 - Bernhard Schölkopf, Francesco Locatello, Stefan Bauer, Nan Rosemary Ke, Nal Kalchbrenner, Anirudh Goyal, and Yoshua Bengio. Toward causal representation learning. *Proceedings of the IEEE*, 109(5):612–634, 2021.
- Maciej Sypetkowski, Morteza Rezanejad, Saber Saberian, Oren Kraus, John Urbanik, James Taylor, Ben Mabey, Mason Victors, Jason Yosinski, Alborz Rezazadeh Sereshkeh, Imran Haque, and Berton Earnshaw. Rxrx1 a dataset for evaluating experimental batch correction methods. In *Proceedings of the IEEE CVF Conference on Computer Vision and Pattern Recognition*, pp. 4285–4294, 2023.
- Alex Tamkin, Mohammad Taufeeque, and Noah D Goodman. Codebook features: Sparse and discrete
 interpretability for neural networks. *arXiv preprint arXiv:2310.17230*, 2023.
- Adly Templeton. Scaling monosemanticity: Extracting interpretable features from claude 3 sonnet.
 Anthropic, 2024.
- ⁶⁷⁸ Thomas Tuschl. Rna interference and small interfering rnas. *Chembiochem*, 2(4):239–245, 2001.
- Kiaohua Zhai, Alexander Kolesnikov, Neil Houlsby, and Lucas Beyer. Scaling vision transformers.
 In *Proceedings of the IEEE/CVF conference on computer vision and pattern recognition*, pp. 12104–12113, 2022.
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Figure 6: a) Reconstructions when decoding from intermediate layers. b) The relative linear probing accuracy when using the component from the null space, row space and a random 512-dimensional subspace as component compared to the full component. Both Figures use the MAE-G model.

A DISCUSSION: LINEAR CONCEPT DIRECTIONS IN VIT MAES

We have shown that DL is a powerful approach for finding linear concept directions (features) that 721 are strongly correlated with biological concepts such as cell-types and genetic perturbations. From an 722 interpretability perspective, a question that remains, however, is whether these correlations solely 723 appear due to first order effects of complex non-linear structures used by the model to store abstract 724 information, or whether linear directions are actually inherently meaningful to the model? While 725 linear causal interventions offer strong evidence that the latter may indeed at least be partially true for 726 large language models (see e.g., (Ferrando et al., 2024) for an overview), there exists relatively little 727 evidence for ViT MAEs besides the high linear probing accuracies on e.g., natural and microscopy 728 image classification tasks Huang et al. (2022); Alkin et al. (2024).

In this section, we provide an argument further supporting the hypothesis that MAEs may rely on
 linear concept directions when processing data by analyzing at which point in the model are the
 concepts are the most linearly separable.

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733 **Separation into row- and nullspace.** We note that standard MAE architectures (Huang et al., 2022) 734 use two different embedding dimensions for the encoder block and the decoder block. Both blocks 735 are connected via an *encoder-decoder projection matrix* $W : \mathbb{R}^{d_e \times d_d}$ with, in our case, $d_e = 1664$ 736 (ViT-G model from (Zhai et al., 2022)) and $d_d = 512$. This projection matrix gives raise to a 737 separation of the tokens into the row-space and null space of W, $x = x_{row} + x_{null}$ where only 738 the information stored in x_{row} is passed to the decoder. ViTs and more generally transformer models have shown to align the basis across layers, allowing for decoding of tokens from intermediate layers 739 (Alkin et al., 2024). We visualize this behavior in Figure 6a where we show the reconstructions when 740 using the tokens from intermediate layers. Thus, we observe that the row-space component x_{row}^l of 741 tokens from early and intermediate layers $x^{(l)}$ already store a reconstruction of the masked image 742 that is refined over the layers. Thus the question appears what is the role of the null space component 743 x_{null} which won't be passed to the decoder and thus serves as a "register" (in analogy to Darcet et al. 744 (2023))?745

746 **Component-wise linear probing** We analyze in Figure 6b the different components, showing the 747 relative linear probing accuracy of the probing accuracies using the *null* and *row space* components, 748 compared to the entire token (dashed line at 1) across different layers. As observed, the null 749 space component consistently yields the same probing accuracy as the entire token, while the row 750 space component yields significantly lower accuracy. For comparison, we also show the relative 751 probing accuracy when using a random d_d -dimensional subspace (the same dimension as the row 752 space), which consistently yields higher accuracy than that obtained from the row space. These 753 findings suggest that biological concepts (i.e., genetic perturbations) are most linearly separable in the component used only for internal processing during the forward pass and not passed to the 754 decoder, and therefore aligns with the hypothesis that the model represents abstract concepts as linear 755 directions accessed by the layers while processing the data Bricken et al. (2023).



Figure 7: The cosine similarity between the original tokens and the reconstructed tokens for ICFL and TopK-SAE, a) with PCA whitening and b) without, as a function of the sparsity (first and third), i.e. # of non-zeros, and \log_{10} learning rate (second and fourth).

768 B INTERPRETABLE FEATURES 769

In this section, we present additional visualizations of crops strongly correlated with selected feature directions. In the spirit of recent works for LLMs (Bricken et al., 2023), we only present a qualitative analysis that aims to highlight non-trivial, complex, and interpretable patterns captured by these features.

For completeness, Figure 8 shows the same crops as Figure 1 but this time all 6 most correlated and anti-correlated crops. We further present in Figures 9 to 13 additional examples similar to Figure 4 for images strongly correlated with different features. In addition to the heat-map and the entire crop, we also plot the patches that are most strongly correlated with the feature. We make two important observations: a) we can see clear interpretable patterns for which patches are most strongly correlated with the cells, posing a promising area for future research on interpreting and validating concept directions found in large foundation models for microscopy image data; b) we see that the most correlated patches are robust to light artifacts, which can be seen best in the last column in Figure 9.

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C ABLATIONS

In this section we present ablations on type of token, model size, sparsity and learning rate. If not further specified, we always use features extracted from ICFL using PCA whitening.

788 **Attention block** It is common in the literature to use representations from the MLP output or 789 the attention output (Bricken et al., 2023; Tamkin et al., 2023; Rajamanoharan et al., 2024a). We 790 compare in Table 4 the test balanced accuracy when taking representations from the residual stream and attention output. We observe that both result in similaraccuracies. We make the same observation 791 in Figure 14a and 14b showing an ablation for the linear probes trained on the reconstruction using 792 the same setting as described in Section 6. Moreover, we compare in Figure 15 the selectivity scores 793 as in Figure 2, confirming further that the residual stream and the attention output show a similar 794 behavior. The only exception is TopK for cell types, where the attention outputs result in significantly 795 better selectivity scores, however, still substantially below the ones obtained by ICFL. 796

Residual stream	97.2%	87.8%	51.6%	94.6%	32.1%
Attention output	96.8%	85.8%	52.5%	94.6%	32.1%

Table 4: The test bal. acc. for representations taken from the residual stream (Test. Bal. Acc. row from Table 4) and the attention output.

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Model size We further investigate the model size, as shown in Figures 14a and 14b, where we compare the linear probes for the MAE-G (referred to as Ph2 with 1.9B parameters) with the much smaller model MAE-L (referred to as Ph1 with 330M parameters). We observe that for simple tasks like classifying cell types, both models yield similar performances. However, we observe consistent improvements on complex classification tasks (3,5), both for the probes trained on the original representations, as well as the reconstructions from ICFL and TopK. This demonstrates that dictionary learning benefits from scaling the model size.

We further plot in Figure 16 the selectivity scores. For ICFL, we consistently observe improvements when increasing the model size, while for TopK SAE, we see a significant drop. Interestingly, this drop does not occur for the probing accuracy on the reconstructions in Figures 14a and 14b. This suggests that, although capturing meaningful signals in the reconstructions, TopK SAE faces more difficulties in finding "interpretable" features with high selectivity scores from richer representations post-processed using PCA whitening.

Sparsity As a third ablation, we plot in Figure 7 the cosine similarity of the original tokens and the reconstructed token from the DL for both TopK-SAE and ICFL. We observe that the reconstruction quality of ICFL is much higher than TopK SAE for the same sparsity constraints. This trend persists across all levels of sparsity. The unsupervised reconstruction quality measured by the cosine similarity (or the related ℓ_2 -error) has been often used as a benchmark for SAEs (Rajamanoharan et al., 2024a; Gao et al., 2024).

Learning rate As a last ablation, we also plot in Figure 7 the cosine-similarity for different learning rates. Since PCA whitening leads to more dense tokens, we expect that a decrease in the cosinesimilarities, which is also the case when comparing the solid lines (w/o PCA whitening) with the dashed lines (w PCA whitening). Except for TopK-SAE with PCA whitening the reconstruction quality slightly increases with the learning rate (likely due to too few training for small learning rates) and flattens after a learning rate of 5×10^{-5} , which we choose for all experiments in this paper. Moreover, we observe that TopK-SAE experiences high instabilities when combined with PCA whitening, which is not the case for ICFL.

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Figure 9: This feature appears to be focusing on the endoplasmic reticuli and nucleoli channel (cyan area) surrounding the nucleus. These are expanded relative to the usual morphology of HUVEC cells.



Figure 10: This feature appears to be firing for cells that are unusually large with spread out actin. Note that the feature focuses on the actin channel (red) surrounding the cell.



Figure 11: This feature appears to be active for long spindly cells, with the features are most aligned for the long "stretched out" section of the cells.



Figure 12: This feature is active for tightly clumped cells. The heatmaps are less clearly interpretable for these images, but appear to be active when neighboring nuclei are touching.







Figure 15: The selectivity scores as in Figure 2 for ICFL (first row) and TopK (second row) when using representations from the residual stream (green) and the attention block (yellow).



Figure 16: The selectivity scores as in Figure 2 for ICFL (first row) and TopK (second row) when using representations from the residual stream from Ph2 (green) and Ph1 (yellow) using PCA whitening.