ADAPTIVE SELF-IMPROVEMENT LLM AGENTIC SYSTEM FOR ML LIBRARY DEVELOPMENT

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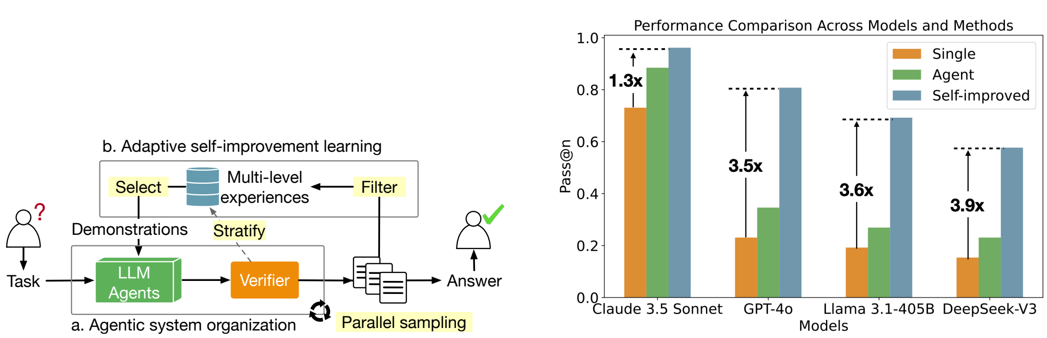
ABSTRACT

ML libraries, often written in architecture-specific programming languages (ASPLs) that target domain-specific architectures, are key to efficient ML systems. However, writing these high-performance ML libraries is challenging because it requires expert knowledge of ML algorithms and the ASPL. Large language models (LLMs), on the other hand, have shown general coding capabilities. However, challenges remain when using LLMs for generating ML libraries using ASPLs because 1) this task is complicated even for experienced human programmers and 2) there are limited code examples because of the esoteric and evolving nature of ASPLs. Therefore, LLMs need complex reasoning with limited data in order to complete this task. To address these challenges, we introduce an adaptive self-improvement agentic system. In order to evaluate the effectiveness of our system, we construct a benchmark of a typical ML library and generate ASPL code with both open and closed-source LLMs on this benchmark. Our results show improvements of up to 3.9× over a baseline single LLM.

1 INTRODUCTION



(a) System diagram.

(b) Performance comparison

Figure 1: **Left:** we propose an adaptive self-improvement LLM agentic system. Similar to human experiential learning Kolb (2014), LLM agents start from their base knowledge and accumulate experiences through parallel sampling. Our adaptive self-improvement learning algorithm filters high-quality answers, stratifies the earned experiences by difficulty, and adaptively selects demonstrations to enhance LLM agents. **Right:** the portion of completed tasks (Pass@n) across models using single LLM, agentic system, and adaptive self-improvement agentic system, highlighting performance improvement.

With the ending of Dennard Scaling and Moore's Law, computer architectures are specializing in domain applications to achieve greater performance and efficiency and will continue to do so Hennessy & Patterson (2019). New domain-specific architectures (DSA) typically come with new architecture-specific programming languages (ASPL), such as CUDA for NVIDIA GPUs NVIDIA (2025), HIP for AMD GPUs AMD (2025), and Pallas for Google TPUs Google (2025). Even existing ASPLs change as generations of DSAs evolve because new DSAs introduce specialized functions to these existing ASPLs Choquette (2023). Efficiently utilizing these new functions requires

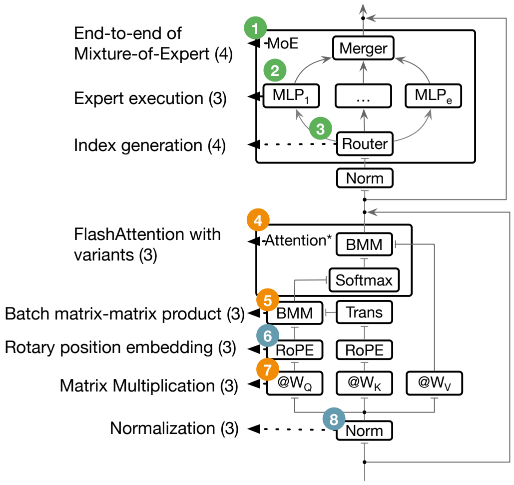


Figure 2: We benchmarked 8 groups of ML operators for a common LLM layer Jiang et al. (2024). They can be categorized into dynamic, static matrix, and static vector operators where parentheses indicate the number of tasks associated with each group of ML operators.

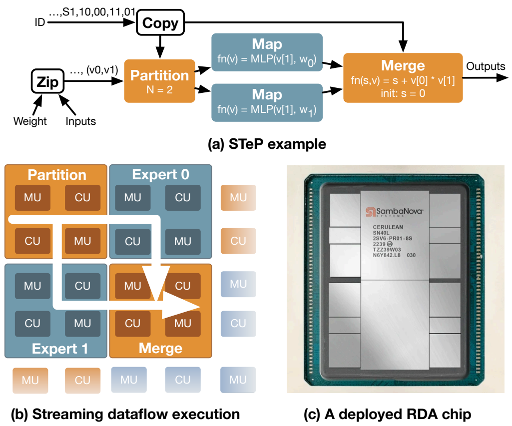


Figure 3: (a) diagrams a STeP program for a simplified MoE module. More details can be found in appendix A.2. (b) illustrates the streaming dataflow execution of (a) on an RDA where "MU" stands for memory unit, and "CU" stands for compute unit. (c) is the SN40L, a deployed RDA chip.

fundamentally different programming styles and thus new ASPLs Thakkar et al. (2023); Hagedorn et al. (2023).

Each DSA needs a corresponding ML library, a collection of ML operators written in its ASPL, before programmers can effectively use the DSA to accelerate ML applications. ML library development is challenging because it requires expertise in both ML algorithms and the target ASPL. Essentially, library development is a generation process that composes low-level ASPL primitives into high-level ML operators Dong et al. (2024); Ye et al. (2025).

ML library development using ASPLs requires complex reasoning while minimizing data requirements. ML libraries are developed simultaneously as the chip is manufactured to meet the production timeline Villa et al. (2021). This library–chip co-design process creates such a tight timeline that ASPLs have only a limited number of code examples. Moreover, this task is complicated even for experienced human programmers. For example, the publication of FlashAttention-3 Shah et al. (2024) lagged behind the release of the H100 by two years. Directly adapting FlashAttention-2 Dao (2023) from A100 to H100 GPU witnessed a 47% performance drop Spector et al. (2024).

The challenges in ML library development call for more automatic solutions. Furthermore, these automatic solutions need to self-improve to perform complex reasoning starting from simple and limited data. Large language models (LLMs) have demonstrated emerging capabilities in code generation Kaplan et al. (2020); Wei et al. (2022a). Moreover, empirical evidence implies that LLMs already have the base knowledge of ML algorithms Ouyang et al. (2024). Therefore, we explore the use of LLM agents to develop ML libraries with emerging ASPLs.

Current self-improvement methods for LLM agents fall short because of limited exploration or low data efficiency. LLM agents can enhance their performance by synthesizing semantically similar data Yu et al. (2023); Shinn et al. (2024); Zhao et al. (2024). Although these methods are effective for local exploration Chen et al. (2024), they are insufficient for tasks that require substantial cognitive effort Huang et al. (2023). Self-improvement learning can significantly improve reasoning ability through reinforcement learning Cobbe et al. (2021); Bai et al. (2022); Singh et al. (2023). This approach, however, currently requires hundreds of effective trajectories sampled from LLM agents for each problem Wang et al. (2024), making it unsuitable for complex scenarios with limited data.

To address these limitations, we design an adaptive self-improvement learning algorithm integrated with an agentic system organization. This approach not only produces a self-improving agentic system to assist humans but also generates high-quality ML operators that can be leveraged by other systems. We show our system in fig. 1a. Our techniques create a self-improvement cycle: LLM agents evolve through earned experiences and these evolved agents can earn more experiences. This self-improvement cycle is fully automated, involving no human experts beyond the ASPL designers themselves, who initially tell the models how to use the ASPL primitives.

Inspired by curriculum learning Bengio et al. (2009), our algorithm prioritizes hard-earned experiences gained from challenging task completion. When these hard-earned experiences are used up, the algorithm adaptively expand demonstrations by mixing experiences earned from less challenging tasks. Section 6.1 shows that hard-earned experiences improve LLM agents more efficiently than mixed ones. Mixed experiences, while sometimes diluting demonstrations, can help agents overcome learning obstacles and complete more tasks. As a byproduct, the algorithm adaptively increases test-time compute on challenging tasks until they are finished or the data is used up.

To emulate the library–chip co-design process, we choose Streaming Tensor Programs (STeP) as the target ASPL for library generation. STeP is an emerging ASPL designed for next-generation reconfigurable dataflow architectures Prabhakar et al. (2017), a family of DSAs for AI Prabhakar & Jairath (2021); Chen et al. (2023); Prabhakar et al. (2024). The only public document of STeP is a non-archival three-page workshop publication Sohn et al. (2024a), which defines its semantics without any code examples or execution environments. Therefore, STeP programs do not exist in the training corpus of any LLM.

Putting all these together, our system solves up to 96% of the tasks in our benchmark and achieves up to $3.9\times$ improvements over a baseline single LLM, as shown in fig. 1b. The contributions of this paper are: (1) an adaptive self-improvement learning algorithm that enables LLM agents to continuously construct ML libraries through adaptive experience-driven evolution; (2) an end-to-end agentic system that uses adaptive self-improvement to develop an ML library for STeP, an ASPL for a next-generation AI accelerator; (3) a complete evaluation of the adaptive self-improvement learning algorithm and the integrated agentic system on a realistic benchmark constructed from common ML operators.

2 Background

In this section, we provide background on how DSAs are programmed through their ASPLs, describe how these ASPLs are used to create end-user ML libraries, and identify the key challenges of this library generation process. We also establish STeP as the target ASPL to explore LLM techniques for ML library development. Key concepts related to the background are listed in table 5.

2.1 Architecture-specific programming languages

ASPLs describe the low-level programming interface of a DSA using primitives and specialized functions. Primitives model the basic execution pattern similar to general-purpose programming language constructs, and specialized functions control specialized accelerator units that are optimized for domain applications on the DSA. Unlike domain-specific languages (DSLs), which are a top-down distillation of the domain algorithms, ASPLs refer to a bottom-up abstraction of the underlying chip architecture.

2.2 ML library development using ASPLs

ML libraries developed in ASPLs face portability challenges because ASPLs rapidly evolve to align with DSA updates to meet the demand of growing ML workloads. For example, the matrix multiplication units on NVIDIA GPUs and their corresponding MMA instructions have been updated every generation since introduction NVIDIA (2017). Consequently, every library function that uses MMA instructions must be rewritten in a new ASPL per generation. Moreover, ML libraries need to be shipped with the chip simultaneously. In this case, library development costs are not negligible.

To solve these challenges, we propose to enhance users' learning capabilities for a given ASPL. This approach offers an alternative to current automation techniques that focus on simplifying the learning curve for new ASPLs. Mainstream techniques compromise by optimizing ML operators whose performance is most significantly affected by the ASPL update Tillet et al. (2019); Thakkar et al. (2023). Only focusing on certain ML operators does not fully incorporate some ASPL updates, such as memory optimizations, which could potentially accelerate any ML operator. Meanwhile, new ML operators are being proposed Gu & Dao (2023); Sun et al. (2024). Given these factors, we need better automation to improve the productivity of ML library development using APSLs.

2.3 STeP FOR NEXT-GENERATION RDA

We chose STeP as our target ASPL due to its potential for better efficiency and its status as a research prototype ASPL. STeP's efficiency potential stems from its role as the ASPL for next-generation RDAs, which have emerged as a promising alternative to GPUs. The SN40L, a deployed RDA implementation shown in Figure 3(c), demonstrates record-breaking inference speeds for the Llama 3.1 405B model SambaNova (2024). Although STeP does not yet have a path to a fabricated chip, developing ML libraries in STeP still presents similar challenges as other ASPLs. Writing STeP programs requires complex reasoning about streaming dataflow execution, and our work began without any existing executable STeP programs to reference.

Like other ASPLs, STeP's operational semantics are expressed by primitives and specialized functions. Specifically, STeP primitives describe different stream token manipulation strategies, and STeP specialized functions express different configurations of RDA units. STeP adds streaming to the conventional dataflow execution model of RDAs as shown in fig. 3(b). Streaming dataflow unifies both data values and control signals as stream tokens. The streaming dataflow execution model in STeP is inspired by parallel patterns and array programming Hsu et al. (2023); Rucker et al. (2024). A stream can be consumed by at most one primitive because of the queueing nature of dataflow Zhang et al. (2021), which is called an affine type constraint in programming language theory Wikipedia (2024). The affine type constraint is a global property of the program since it counts the usage of a variable in the whole program.

STeP primitives are categorized as either arithmetic or shape manipulation. Arithmetic primitives apply computations and control flow to stream tokens. Shape manipulation primitives reshape the data within the stream by changing the control tokens. Figure 3(a) and Figure 16 are example STeP programs that contain 5 arithmetic primitives and only shape manipulation primitives, respectively.

3 ADAPTIVE SELF-IMPROVEMENT LEARNING

Our adaptive self-improvement learning evolves LLM agentic systems with data generated by themselves. This algorithm parallel samples the agentic system for correct answers with the success rate, filters high-quality correct answers, stratifies the earned experiences, and adaptively updates demonstrations until all the tasks are solved or the demonstrations are used up. Algorithm 1 shows the complete algorithm for adaptive self-improvement learning. As a byproduct, the algorithm adaptively assigns more test-time compute to harder tasks because it excludes a task from the task set after completion. Figure 4 shows a running example, where only the unfinished tasks are fed to the agentic system. This algorithm is independent of the specific organization of the agentic system.

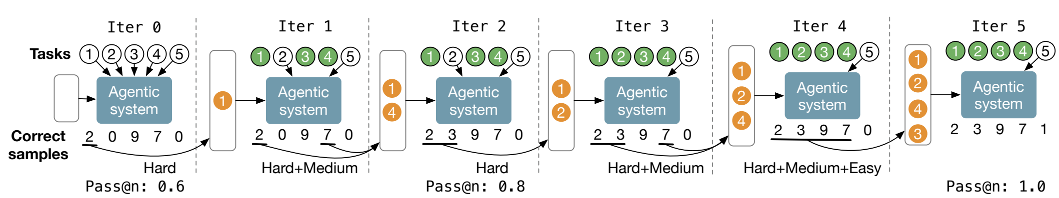


Figure 4: Running example of algorithm 1 with $m = 3$ and $|\mathcal{X}| = 5$. Orange circles are demonstrations d_j for the current iteration. Green circles are finished tasks and white ones are not. $m = 3$ means β stratifies demonstrations into 3 levels: hard, medium, and easy. Iter 1 and 3 consider hard-only examples, and the other iterations consider mixed examples.

3.1 FILTERING HIGH-QUALITY ANSWERS

The filter function, σ, collects earned experience $\mathcal{D}$ by filtering one answer for each newly solved task in $\mathcal{S}_t$ and records the success rate of this answer. σ first groups the correct answers $\mathcal{B}_t$ by the isomorphic abstract syntax tree Knuth (1968). Then, it randomly selects one answer from each isomorphic group. After that, it selects the one with the minimal length of pure code (excluding comments and empty lines) from these canonical representatives as the final answer for the task in $\mathcal{S}_t$. It also stores the success rate from $\mathcal{C}_t$ for β. The minimal length selection follows the Minimum Description Length principle for higher information density Rissanen (1978). On the other hand, shorter text might lose chain-of-thought comments Wei et al. (2022b). To balance these two contradicting intuitions, we introduce randomness to the selection of representative answers for each isomorphic group.

3.2 STRATIFICATION AND SELECTION

The selection function β stratifies the earned experiences $\mathcal{D}$ by binning them into m levels of difficulty and demonstrations d_j are selected incrementally from the stratified experiences $\mathcal{E}$. We define the difficulty as the opposite of the success rate following Lightman et al. (2023). β sorts $\mathcal{D}$ in ascending order of success rate and then bins the tasks as evenly as possible to get the boundaries of each bin. Then tasks are rebinned using these boundary values. This selection strategy can cause repetitive steps as exemplified by the dash line circles (Iter 4&5 in fig. 6(a)) when the newly finished tasks are easier than the demonstrations. Our algorithm keeps these repetitive steps instead of avoiding them because the tasks with low success rates have a better chance of getting one correct answer with the number of samples doubled. For example, Iter 4 performs better than Iter 3 in fig. 6(b) with the same demonstrations. This method can also cause later iterations to have fewer tokens but with higher quality than the previous iteration (Iter 2&3 in fig. 6(a)) when the boundary value crosses two bins and the newly finished tasks are easier.

3.3 DISCUSSION

To use this technique in a continuous learning setting, users can add new tasks to the initial $\mathcal{X}$ and apply algorithm 1. This approach, however, does not guarantee success and might exceed context window constraints. If the task set $\mathcal{X}$ is finite, as in our case, the algorithm will terminate.

4 AGENTIC SYSTEM ORGANIZATION

In this section, we introduce a specific agentic system organization tailored for ML library development using STeP as shown in fig. 5. It includes LLM agents, a code generator, and verifiers with a structural intermediate representation as the interface between users and system components.

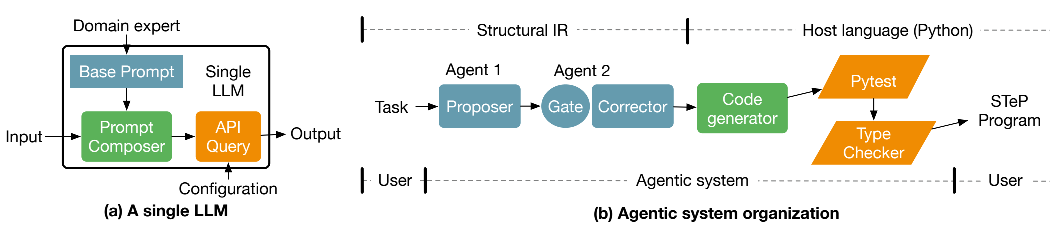


Figure 5: Details of our agentic system organization. (a) shows a single LLM. (b) shows the system components and their representations. The user is either a human or a self-improvement learning process. The filled colors align with the text colors in section 4.

4.1 LLM AGENTS

As shown in fig. 5(a), each single LLM is designed for a purpose assigned by the domain expert through the base prompt. The base prompt contains the task description, base knowledge, and demonstrations. The prompt composer chains the base prompt and task-specific input, which is then fed into a configured LLM serving API.

We design two agents, *a proposer and a guardian*. As explained in section 2.3, the affine type constraint in STeP is a global property that requires thinking back and forth beyond step-by-step reasoning, which is challenging for the causal generation of LLMs. Therefore, we design a guardian agent to check and correct the affine type error globally. We exclude the demonstration tasks in the base prompt of the agents from the benchmark to avoid the model directly copying the answer. The proposer agent generates a candidate STeP implementation whose base prompt comes from ASPL designers. The base prompt is composed of STeP references and usage patterns. Figure 13 shows the reference for the `Accum` primitive, and fig. 16 exemplifies one of the usage patterns for shape manipulation. The guardian agent decides whether the output of the proposer violates the affine type constraint and corrects the implementation when necessary. The guardian agent consists of a fused gate and corrector. Figure 19 shows the base prompt for the guardian agent, which provides input and output examples of variable reuse where the variable is reused zero, one, or two times.

4.2 Code generator

After the LLM agents, a code generator takes in the generated implementation and outputs a self-contained pytestable Python script. With this code generator, LLM agents only need to output implementations without other helper code. We embed the STeP specifications written in natural language to Python (as exemplified in fig. 20 to fig. 21). Beneath this Python frontend is a functional simulator that calculates the result of STeP programs. Since the essence of each ASPL is the programming abstraction (semantics) instead of its syntax Liskov (2011), we choose to prompt LLMs with their familiar Python syntax. In this way, LLMs can focus more on reasoning about the STeP programming abstraction without being distracted by alien syntax.

4.3 Verifier

Fast verification is vital because it bottlenecks adaptive self-improvement learning. ASPLs further increase this complexity by requiring a simulator for the library–chip co-design process. Simulating STeP as a general dataflow system in Python would be slow because of its high dynamism Zhang et al. (2024a). Since we only focus on functional correctness in this work, we meticulously limit the level of dynamism to the degree that ML operators require. Consequently, our simulator emulates stream execution using tensor computation with necessary control flows.

Our system organization contains two verifiers. As a result, the reward in algorithm 1 is $r(x, y) = 1$ for task y if answer x passes these two verifiers and $r(x, y) = 0$ otherwise. One verifier checks for functional correctness. Users program PyTorch to express their ML operators, which elicits LLMs' base knowledge of ML algorithms. The verifier compares the execution results of our simulator with the result tensors of the corresponding PyTorch program on a single set of shapes with random input values. The fidelity of our unit test method builds on practice Jia et al. (2019) and theory Gulwani & Necula (2003). The other verifier checks the affine type constraint by performing static analysis on the abstraction syntax tree of the STeP program with the Python ast module Ronacher (2008).

4.4 Structual intermediate representation

Good interfaces can improve the performance of agentic coding systems Yang et al. (2024); Wei et al. (2024). We borrow the intermediate representation (IR) technique from compiler literature Lattner et al. (2021) and use a structural IR to unify the interfaces of our agentic system. Specifically, the structural IR is used as the interface between users and the agentic system and as the interface between LLM agents and the code generator within the system. Our structural IR encodes necessary information using a data serialization language. It externalizes and condenses programs instead of simply formatting the prompt without changing the content. Comparing the structural IR in fig. 17 with the equivalent bare Python in fig. 18 for the same task, users only need to state two things: the ML operator to implement and the specialized functions as in fig. 14 without any redundant glue string. Moreover, structural IR saves tokens by reducing redundant prompts, allowing for more demonstrations.

5 BENCHMARK

We construct a set of tasks to measure the adaptive self-improvement agentic system proposed in section 3 and section 4. This benchmark should cover a diverse set of popular ML operators and specialized functions. In total, we collect 26 tasks covering 8 groups of ML operators in common LLM model architectures, as shown in fig. 2.

We choose pass@k Chen et al. (2021) as the metric for task completion. Pass@k is calculated as eq. (2) given T tasks, n samples of the agentic system, and c_i correct responses for each task i. It is useful for us to analyze the metric at two extremes: pass@1 and pass@n. Pass@1 is the expectation of the success rate across tasks. Pass@n is the expectation of the portion of tasks that can be solved given all samples.

We construct the benchmark from first principles and do not favor any kind of task. Firstly, the number of tasks in each group is nearly the same as shown in fig. 2. Secondly, the benchmark has an even distribution of difficulties. Table 2 shows that both tasks requiring shape and arithmetic primitives and tasks with and without reused variables distribute fairly evenly. We provide a reference implementation for each task. These oracle implementations ensure that each task has at least one correct answer. Each task of one type has either different specialized functions for the same operator or different operators with different specialized functions. More details on the benchmark are in appendix A.3.

6 EXPERIMENTS

Detailed experimental settings are in appendix A.4. We also benchmark the tasks with OpenAI-o1 in table 4 but do not include it in the following experiments to control for test-time compute. All prompts are formatted in YAML because structural prompts generally benefit He et al. (2024).

6.1 ANALYSIS OF ADAPTIVE SELF-IMPROVEMENT LEARNING

The hard-only examples improve the performance more effectively than examples mixed with easier ones. As shown in fig. 6, the Pareto optimal is composed of hard examples (denoted by "H") for all three models. Moreover, hard examples bring the most significant improvement along the learning curve. In some cases, fewer hard examples may perform better than more examples mixed with easier examples. For example, Iter 3 has better performance than Iter 2 for gpt-4o.

Mixed examples are required to generate better hard-only examples. In DeepSeek-V3, although HM_1 performs the same as H_1 and HME_1 performs worse than H_2 while taking more tokens, H_2 would not be discovered without HM_1 and HME_1. Although the new hard-only examples do not necessarily improve the performance, they can save input tokens, as exemplified by HM_4&H_5 of gpt-4o and HM_5&H_6 of llama.

Adaptive granularity affects token efficiency and peak performance. If the adaptive granularity m is too small, then easy examples might dilute the difficulty of training data. If m is too large, then exploration steps might be too conservative and thus waste input tokens. Therefore, there is a sweet spot that balances the training data difficulty and cost of input tokens. As shown in fig. 7, $m = 3$ is that spot. $m = 3$ saves $1.07\times$ tokens over $m = 4$ while maintaining performance. Additionally, $m = 3$ improves the performance by $1.5\times$ at a similar input token cost when compared to $m = 1$.

6.2 ABLATION STUDY OF AGENTIC SYSTEM ORGANIZATION

Agentic system can discover non-trivial STeP programs. Surprisingly, the agentic system composes attention operators with over 50% Pass@1 as shown in fig. 12. That means the agentic system can discover online softmax Milakov & Gimelshein (2018) and memory-free streaming attention Sohn et al. (2024b) with specialized functions provided in fig. 15, which is considered challenging for ordinary programmers.

Our structual IR improves performance by increasing the sample diversity. Figure 8 shows the efficacy of our structural IR and agentic method. We calculate the semantic diversity of sampled answers. A group of answers is considered to have the same semantics if their abstract syntax

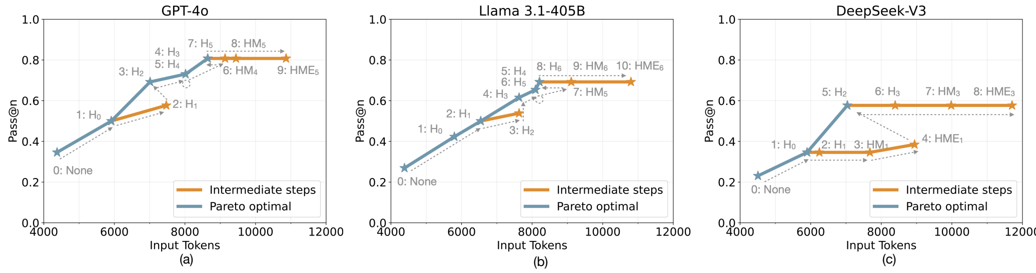


Figure 6: Adaptive self-improvement learning improves the agentic system with data generated by itself. The input tokens are averaged across tasks. A task takes increasing input tokens until success or data is used up. "9: HME_5" means 9-th iteration, 5-th cycle of adaptive sampling, and demonstrations contain hard (H), medium (M), and easy (E)-earned experiences. "None" means no examples for the first iteration. For Iter i, if Pass@n_{i-1} > Pass@n_{i-2}, then a new cycle of adaptive sampling starts from "H". Otherwise, the current cycle continues in the order of H→HM→HME. The dash lines are connected in the iteration order. Claude 3.5 Sonnet result is in fig. 10.

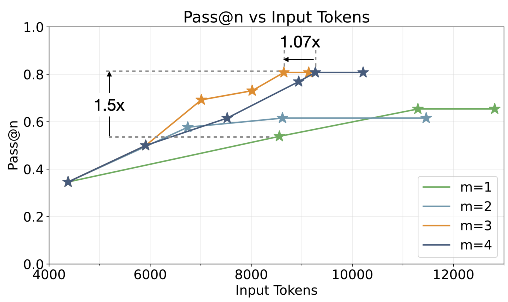


Figure 7: Hyperparameter tuning of adaptive granularity m on GPT-4o. This supports using m=3 for fig. 6.

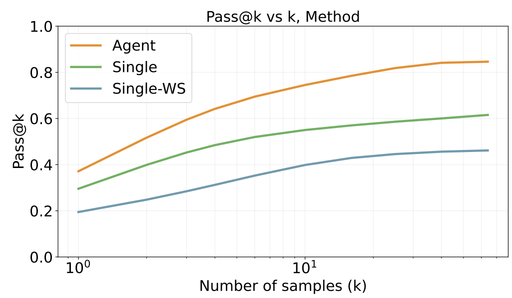


Figure 8: Pass@k against the number of samples for a single LLM without structural IR, baseline single LLM, and agentic system..

trees are isomorphic. We calculate success diversity as the number of semantically different correct answers divided by the total number of correct answers, averaged across all tasks. Failure diversity is calculated in a similar way but for wrong answers. Overall, diversity combines both metrics. As shown in table 1, Pass@n has a positive correlation with failure, overall diversity, and the complexity of the methods. However, structural IR can hurt success diversity.

Method	Success diversity	Failure diversity	Overall diversity	Pass@n
Single-WS	0.32	0.47	0.41	0.46
Single	0.27	0.64	0.50	0.62
Agent	**0.34**	**0.68**	**0.52**	**0.85**

Table 1: Analysis of the correlation between semantic diversity of answers and the performance. "-WS" means "without structural IR". Higher values indicate higher semantic diversity.

The guardian agent can correct affine type errors but might also corrupt correct answers output by the proposer agent. As shown in Table 2, the guardian agent effectively corrects the proposer agent, solving 5 extra reuse tasks (Pass@n of Avg-Reuse from 6/12 to 11/12). Notably, all the Arith-Reuse tasks can be solved by the agentic system (Pass@n of Arith-Reuse from 5/8 to 8/8). Surprisingly, the agentic system also helps with non-reuse tasks (Pass@n of Shape-Once from 3/7 to 4/7). However, the agentic system reduces the Pass@1 of Arith-Once from 0.685 to 0.663, implying that the guardian agent might corrupt the proposer's output. The agentic system can compensate for such corruption by finishing more tasks, resulting in an unchanged Pass@1 of Avg-Once.

Mode	Metric	Method	Once	Reuse	Avg
Arith	Pass@1	Single	**0.685**	0.232	0.444
		Agent	0.663	**0.455**	**0.552**
	Pass@n	Single	**7**/7	5/8	12/15
		Agent	**7**/7	**8**/8	**15**/15
Shape	Pass@1	Single	0.145	0.004	0.094
		Agent	**0.167**	**0.051**	**0.125**
	Pass@n	Single	3/7	1/4	4/11
		Agent	**4**/7	**3**/4	**7**/11
Avg	Pass@1	Single	**0.415**	0.156	0.296
		Agent	**0.415**	**0.320**	**0.371**
	Pass@n	Single	10/14	6/12	16/26
		Agent	**11**/14	**11**/12	**22**/26

Table 2: Analysis of improvement brought by the agentic method. "Arith" and "Shape" mean the oracle implementation only involves arithmetic primitives and involves shape manipulation primitives as introduced in section 2.3, respectively. "Once" and "Reuse" mean all the streams are used once and more than once, respectively.

7 Related Work

7.1 Self-improvement learning for LLMs

Self-improvement learning for LLMs typically involves two stages: scoring generated samples (trajectories) and incorporating those samples to enhance the model. Scoring can be achieved through human labeling Cobbe et al. (2021); Lightman et al. (2023) or through automated methods such as verifiers and heuristics Wang et al. (2024); Singh et al. (2023). Our method stands out in this context by utilizing AST analysis, offering a more interpretable approach to scoring. When it comes to incorporating samples, models may rely on retrieval Zhao et al. (2024); Park et al. (2023), reflection Shinn et al. (2024); Liu et al. (2023), or reward feedback Opsahl-Ong et al. (2024); Fernando et al. (2023). Our method introduces a new mechanism in this stage by adaptively extending and prioritizing high-scoring samples.

Our method shares similar reinforcement principles with self-improvement learning at the post-training stage Bai et al. (2022); Gulcehre et al. (2023); Tian et al. (2024) but is rewarded at a task-level granularity instead of token-level. Specifically, each action is a program instead of token prediction and the state is defined by earned experiences rather than generated sequences.

7.2 Agentic system organization for specialized tasks

Task-specific organization has proven effective in enhancing the performance of agentic systems across diverse coding tasks Zhang et al. (2024b); Fang et al. (2024); Guan et al. (2024). We adopt an agentic system organization specifically for ML library development using an ASPL. Such domain knowledge can be further augmented with automatic agentic system design tools Khattab et al. (2023); Hu et al. (2024). Furthermore, well-designed interfaces between agents, tools, and other agents have been shown to improve performance Schick et al. (2023); Yang et al. (2024); Wu et al. (2023), and a structural IR enables these interfaces to be highly task-aligned in our system.

8 Conclusion

ML library development using ASPLs is a critical component of the ML ecosystem, but it remains poorly automated. To address this limitation, we co-design the learning process and agentic system around a central objective: enabling complex reasoning with limited data. Our methods simultaneously implement non-trivial ML operators and produce a self-improving agent. Consequently, our system provides not only support to experts in developing ML libraries but also valuable resources for other systems.

Impact Statement

Our work on adaptive self-improvement learning LLM agentic systems for automating ML library development using ASPLs has several potential societal implications. The primary impact is the potential to significantly enhance the productivity of ML library developers, which could accelerate the development of more efficient ML systems. This advancement could democratize access to ML development tools and reduce the technical barriers to entry in the field. Additionally, our technique offers an approach for deploying LLM agents in scenarios requiring complex reasoning with limited data availability. While these developments primarily aim to advance the field of Machine Learning, we acknowledge that increased automation in software development could impact the nature of programming work and skills required in the field. We believe these potential implications warrant ongoing discussion and careful consideration as the technology develops.

Acknowledgement

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A APPENDIX

A.1 ADAPTIVE SELF-IMPROVEMENT LEARNING ALGORITHM

Algorithm 1 Adaptive self-improvement learning

Input: $\mathcal{X}$: task set, m: adaptive granularity
Require: θ: LLM agentic system, r: reward from verifier, σ: filter function, β: selection function

$\mathcal{D} \leftarrow \emptyset$
$t \leftarrow 0$ ▷ iteration
repeat
  $\mathcal{E} \leftarrow \beta(\mathcal{D}, m)$ ▷ stratification
  for $e_j \in \mathcal{E}$ **do**
    $d_j \leftarrow [e_0, e_1, ...e_j]$ ▷ selection
    // Parallel sampling
    $\mathcal{C}_t \leftarrow \{\mathbb{E}_{y \sim p_\theta(y|x_i, d_j)}[r(x_i, y)] \mid x_i \in \mathcal{X}\}$
    $\mathcal{B}_t \leftarrow \{(x_i, y) \mid r(x_i, y) = 1, x_i \in \mathcal{X}\}$
    $\mathcal{S}_t \leftarrow \{x_i \mid c_i > 0, c_i \in \mathcal{C}_t\}$
    if $\mathcal{B}_t \neq \emptyset$ **then**
      $\mathcal{D} \leftarrow \mathcal{D} \cup \sigma(\mathcal{B}_t, \mathcal{C}_t, \mathcal{S}_t)$ ▷ filtering
      $\mathcal{X} \leftarrow \mathcal{X} \setminus \mathcal{S}_t$
      $t \leftarrow t + 1$
      break
    end if
    $t \leftarrow t + 1$
  end for
until $\mathcal{X} = \emptyset \vee (d_j = \mathcal{D} \wedge \mathcal{B}_{t-1} = \emptyset)$
Output: Solutions: $\mathcal{D}$

Discussion. To use this technique in a continuous learning setting, users can add new tasks to the initial $\mathcal{X}$ and apply algorithm 1. This approach, however, does not guarantee success and might exceed context window constraints. If the task set $\mathcal{X}$ is finite, as in our case, the algorithm will terminate.

A.2 STEP INTRODUCTION

As shown in fig. 9, Copy duplicates a stream for the affine type constraint. Zip combines two streams of values into a stream of tuples. Map applies the function (**fn**) on each input value. Partition routes tokens of the data stream to experts assigned by the index stream (S_1^0). Merge accumulates tokens from experts. The accumulation of Merge is parameterized by **fn** and **init** where **init** initializes the state and **fn** updates the state with the input value. Partition and Merge are used in pairs, sharing the same index stream. $\langle\rangle$ represents the Tuple type of value tokens. Buffer, Multihot, and Scalar are also types of value tokens, parametrized by the generic data type like float and half, which is omitted for simplicity in the algorithm. Buffer and Multihot types are further parametrized by the shape in the paratheses. Shape manipulation primitives include `Promote`, `Repeat` and `RepeatRef`. Figure 3(a) also shows two streams. `S1` is a control token signaling the end of rank-1. `01` is a value token of multihot vector type served in index streams. `(v0,v1)` is a value token of type Tuple(Scalar, Reference) because the weight and input streams are composed of scalar and reference values, respectively.

$$\sum_{i=0}^{1} G_i \cdot \text{gelu}(W_i X) \text{ with } N_i = \mathbf{I}[G_i > 0] \tag{1}$$

Algorithm 2 STeP for a simplified MoE module

Require: X: [m,n] of Buffer(k), N: [m,n] of Multihot(e), G: [m,n] of Buffer(e)
Output: [m,n] of Buffer(k)
param W_0: [k,d], W_1: [k,d]
func weightedsum ▷ external function
 type: Buffer(k) $\rightarrow$ $\langle$Buffer(k), Scalar$\rangle$ $\rightarrow$ Buffer(k)
 fn (s,v) = s + $v_0 * v_1$
 init: s = 0
func expert_0
 type: $\langle$Buffer(k), Buffer(e)$\rangle$ $\rightarrow$ $\langle$Buffer(k), Scalar$\rangle$
 fn (v) = gelu($W_0 v_0$), v_1[0])
func expert_1
 type: $\langle$Buffer(k), Buffer(e)$\rangle$ $\rightarrow$ $\langle$Buffer(k), Scalar$\rangle$
 fn (v) = gelu($W_1 v_0$), v_1[1])
1: S_0 = Zip(X, G)
2: S_1^0, S_1^1 = Copy(N)
3: S_2 = Partition(2, S_0, S_1^0)
4: S_3 = [Map(expert_0, S_2[0]), Map(expert_1, S_2[1])]
5: S_4 = Merge(weightedsum, S_3, S_1^1)

Figure 9: Algorithm 2 is a STeP program example for eq. (1). The type signature follows the Haskell style where $\rightarrow$ connects a sequence of argument types with one return type. Three **func**s are external functions provided by the hardware.

A.3 Benchmark details

The pass@k is defined as:

$$\text{pass@k} := \frac{1}{T}\sum_{i=1}^{T}\left[1 - \frac{\binom{n-c_i}{k}}{\binom{n}{k}}\right] \tag{2}$$

As shown in fig. 2, Group 4 tasks have the same operator: $softmax(S) \cdot V$ where S equals QK^T. They differ in external functions as shown in fig. 15. Group 4 can use RDA's on-chip fusion to compose scale-dot-product attention with Group 5. Group 7 also differs in external functions. Group 5 contains three dataflow orders: inner-product("$mnk, mdk \rightarrow mnd$"), row-wise("$mnk, mkd \rightarrow mnd$"), and outer-product("$mkn, mkd \rightarrow mnd$"). Group 6 contains GptJ and NeoX styles which differ in pairing even and odd or the first and second half positions vLLM (2023). Group 8 contains LayerNorm and RMSNorm.

The last three in table 3: index, expert, and etoe all come from MoE. MoE contains token Shazeer et al. (2017) eq. (3) and expert choice routing Zhou et al. (2022) eq. (4).

$$\begin{aligned} &S = softmax(X \cdot W_g), S \in \mathbb{R}^{n\times e} \\ &G, I = TopK(S) \qquad \text{Along expert dimension} \end{aligned} \tag{3}$$

$$\begin{aligned} &S = softmax(X \cdot W_g), S \in \mathbb{R}^{n\times e} \\ &G, I = TopK(S^T) \qquad \text{Along token dimension} \end{aligned} \tag{4}$$

The expert choice routing can have another MLP auxiliary predictor for causal inference when it is a binary choice Raposo et al. (2024) eq. (5).

$$\begin{aligned} &S = \sigma((gelu(X \cdot W_{g_0})) \cdot W_{g_1}), S \in \mathbb{R}^{n\times 1} \quad \text{expert=2} \\ &G, I = S > 0.5 \end{aligned} \tag{5}$$

Group	Description	Count
attn	Softmax(S)@V part	3
gemm	Matrix multiplication with expanded reduction dimension	3
bmm	Batch matrix-matrix product	3
norm	RMSNorm and LayerNorm (without bias and gain)	3
rope	GptJ and NeoX style of RoPE	3
index	Index generation of MoE router	4
expert	Expert execution of MoE	3
etoe	End-to-end of MoE module	4

Table 3: Description of each type of tasks. Matmul is short for matrix multiplication.

A.4 Experiment settings

In section 6.1 we sample 64 times for each temperature of 0.4, 0.7, and 1.0, recording the best result. In section 6.2, we sample 64 times at temperature 0.7 on Claude 3.5 Sonnet to control variables. Four models are: claude-3-5-sonnet-20241022 of Anthropic API (Claude 3.5 Sonnet), gpt-4o-2024-11-20 of OpenAI API (GPT-4o), deepseek-chat of DeepSeek API (DeepSeek-V3), and Meta-Llama-3-1-405B-Instruct-Turbo of TogetherAI API (Llama 3.1-405B). Maximum output tokens are set as 1024 and the seed for GPT-4o is 42. Section 6.1 uses the agentic system organization described in section 4.1 for the best possible base learning capability. Section 6.2 studies the base learning capability of agentic systems without the self-improvement process.

A.5 Extra results

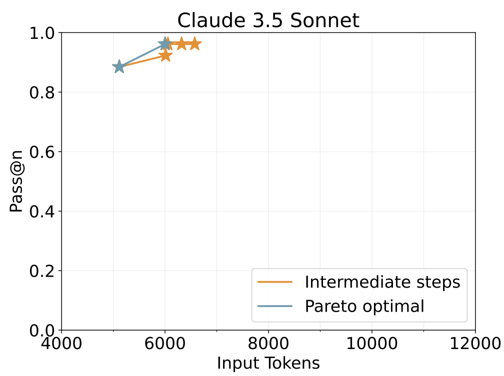

Figure 10: Self-improvement learning curve with m=3. Claude 3.5 Sonnet consumes much fewer tokens than other models because the input tokens are counted by the least necessary number of tokens averaged across tasks. A task can take increasing input tokens but still fail. Sonnet only has 3 unsolved tasks left. Therefore, although it has the most example tokens, the average number of input tokens across tasks is still less than others.

As shown in table 4, OpenAI-o1 achieves similar performance at the cost of more tokens than a single Claude-3-5-Sonnet. This observation aligns with previous findings that scaling pretraining is preferable over inference for challenging tasks Snell et al. (2024). Meanwhile, a single Claude-3-5-Sonnet proposer can finish more tasks than the OpenAI-o1 using fewer tokens. Table 3 contains all abbreviations.

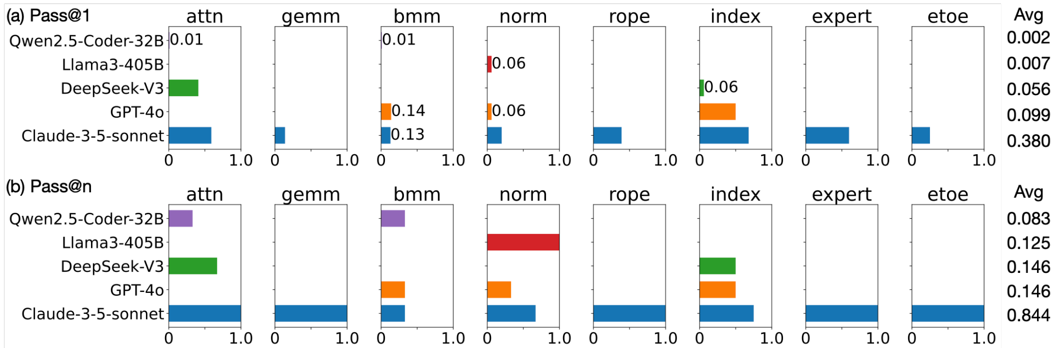

Figure 11: Result of five models at temperature 0.7.

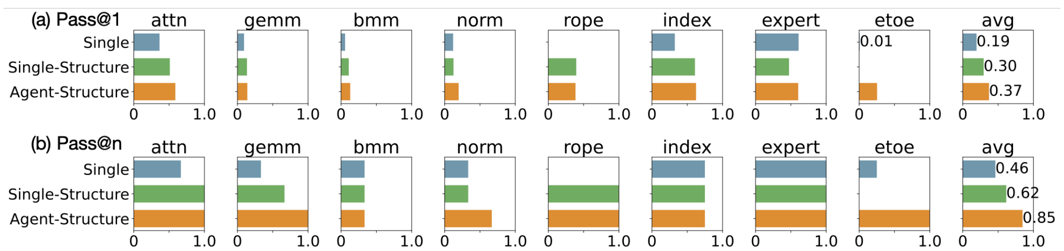

Figure 12: Result of three methods on Claude 3.5 Sonnet at temperature 0.7.

A.6 Additional explanation

Model	Metric (↑)	attn	gemm	bmm	norm	rope	index	expert	etoe	avg
Qwen2.5-Coder-32B	Pass@1	0.010	0	0.052	0	0	0.004	0	0	0.008
	Pass@n	0.33	0	**0.33**	0	0	0.25	0	0	0.115
Llama3-405B	Pass@1	0.010	0	0	0.057	0.089	0.016	0	0	0.020
	Pass@n	0.33	0	0	**1.00**	0.67	0.25	0	0	0.269
DeepSeek-V3	Pass@1	0.438	0	0	0	0	0.113	0	0	0.068
	Pass@n	**1.00**	0	0	0	0	**0.75**	0	0	0.231
GPT-4o	Pass@1	0.021	0.016	**0.214**	0.094	0	0.258	0.005	0	0.080
	Pass@n	0.33	0.67	**0.33**	0.67	0	0.5	0.33	0	0.346
Claude-3-5-sonnet	Pass@1	**0.620**	**0.229**	0.146	**0.208**	**0.526**	**0.676**	**0.688**	**0.324**	**0.433**
	Pass@n	**1.00**	**1.00**	**0.33**	**1.00**	**1.00**	**0.75**	**1.00**	**1.00**	**0.885**
OpenAI-o1 (n=8)	Pass@1	0.208	0.042	0	0	0.083	0.343	0.583	0	0.159
	Pass@n	0.67	0.33	0	0	0.67	0.5	**1.00**	0	0.385

Table 4: The performance of the self-improvement agentic system across models.

Abbv.	Description
DSA	Domain Specific Architectures
DSL	Domain Specific Language
ASPL	Architecture Specific Programming Language
RDA	Reconfigurable Dataflow Architecture, a DSA for AI
STeP	Streaming Tensor Program, an ASPL for next-generation RDA

Table 5: Explanation of the abbreviations.

A.7 CODE AND PROMPT EXAMPLES

```
- name: Accum
  desc: |
  Accum is a primitive operation that applies a function to a stream in a recursive manner.
  The function is applied to the first element of the stream and the initial state to produce
       the first output element.
  The function is then applied to the second element of the stream and the output of the
       previous application to produce the second output element, and so on.
  The state is initialized at rank `b` of the input stream. The output stream's shape is the
       input stream's shape excluding the first `b` dimensions.

  examples:
  - inputs:
    - name: E0
      dtype: fp32
      dims: [M, N]
      data_gen: torch.rand
    fns:
    - name: Sum
      apply: |
        return [state[0] + input[0]]
      init: [0]
      input_dtype: fp32
      output_dtype: fp32
      func_name: fn_sum
    outputs:
    - name: S0
      dtype: fp32
      dims: [N]
      data_transform:
        - |
          torch.sum(input_data['E0'], 1, keepdim=False)
    impl: |
      E1 = step.Accum(fn=fn_sum, b=1).apply(E0)
      return E1
```

Figure 13: The reference of `Accum` that contains the definition and an example. Each example in the examples field is composed of task description and implementation.

```yaml
inputs:
  - name: E0
    dtype: fp32
    dims: [M, N]
    data_gen: torch.rand
  - name: E1
    dtype: Buffer(fp32, [D])
    dims: [M, N]
    data_gen: torch.rand

fns:
  - name: MaxSum
    apply: |
      m_t, l_t, o_t = state # scalar, scalar, [D]
      s_t, v_t = input # scalar, [D]
      m_next = torch.max(m_t, s_t) # scalar
      l_prim_t = torch.exp(m_t - m_next) * l_t
      p_t = torch.exp(s_t - m_next)
      l_next = p_t + l_prim_t
      o_next = l_prim_t * o_t / l_next + p_t * v_t / l_next
      return [m_next, l_next, o_next]
    init: [-inf, 0, 0]
    input_dtype: [fp32, "Buffer(fp32, [D])"]
    output_dtype: [fp32, fp32, "Buffer(fp32, [D])"]
    func_name: fn_maxsum

  - name: GetThird
    apply: |
      return [input[2]]
    input_dtype: [fp32, fp32, "Buffer(fp32, [D])"]
    output_dtype: Buffer(fp32, [D])
    func_name: fn_getthird

outputs:
  - name: S0
    dtype: fp32
    dims: [D, N]
    data_transform:
      - |
        torch.bmm(torch.softmax(input_data['E0'], 1).unsqueeze(1),
             input_data['E1']).squeeze(1)

impl: |
```

Figure 14: Task description of attn task 0 in our structural IR, where the LLM needs to complete `impl`.

```
# Task 0
# MaxSum
def apply(state, input):
    m_t, l_t, o_t = state
    s_t, v_t = input
    m_{t+1} = max(m_t, s_t)
    l'_t = l_t * e^(m_t - m_{t+1})
    p_t = e^(s_t - m_{t+1})
    l_{t+1} = p_t + l'_t
    o_{t+1} = 1/l_{t+1} (l'_t * o_t + p_t * v_t)

    return (m_{t+1}, l_{t+1}, o_{t+1})

def init():
    return (-∞, 0, 0)

# GetThrid
def apply(input):
    return input[2]
```

```
# Task 1
# ExpMaxDiff
def apply(state, input):
    m_t, e_t, d_t = state
    s_t, = input
    m_{t+1} = max(m_t, s_t)
    Δm = m_t - m_{t+1}
    e_{t+1} = e^(s_t - m_{t+1})
    d_{t+1} = e^Δm
    return (m_{t+1}, e_{t+1}, d_{t+1})

def init():
    return (-∞, 0, 0)

# DivSum
def apply(state, input):
    v_t, e_t, d_t = input
    l_t, o_t = state
    l'_t = d_t * l_t
    l_{t+1} = e_t + l'_t
    o_{t+1} = 1/l_{t+1} (l'_t * o_t + e_t * v_t)
    return (l_{t+1}, o_{t+1})

def init():
    return (0, 0)

# GetSecondThrid
def apply(input):
    return input[1],input[2]

# GetSecond
def apply(input):
    return input[1]
```

```
# Task 2
# ExpMaxDiff
def apply(state, input):
    m_t, e_t, d_t = state
    s_t, = input
    m_{t+1} = max(m_t, s_t)
    Δm = m_t - m_{t+1}
    e_{t+1} = e^(s_t - m_{t+1})
    d_{t+1} = e^Δm
    return (m_{t+1}, e_{t+1}, d_{t+1})

def init():
    return (-∞, 0, 0)

# GetSecondThrid
def apply(input):
    return input[1],input[2]

# WeightedSumSingle
def apply(state, input):
    e_t, d_t = input
    r_t = state
    return (r_t * d_t + e_t)

def init():
    return 0

# WeightedSumDouble
def apply(state, input):
    v_t, e_t, d_t = input
    return (state * d_t + e_t * v_t)

def init():
    return 0

# Div
def apply(input):
    r_t, l_t = input
    return l_t / r_t
```

Figure 15: Inner functions for 3 tasks of attn. Task 0 encapsulates the whole innermost loop body of FlashAttention Dao et al. (2022) in the `MaxSum` function. Task 1 splits the `MaxSum` into `ExpMaxDiff` and `DivSum`. Task 2 postpones the division of summation as in FlashAttention2 Dao (2023). The bold symbols are streams with type 1D Buffer, and the symbols are streams with type Scalar.

```
name: Stashing dimension
desc: |
  When the pritmives require a non-one dimension to be inserted as a non-innermost dimension,
      a Bufferize&Streamify pair can wrap the primitives to adjust the dimension.
  This pattern is useful for Repeat and RepeatRef primitives.
examples:
  - inputs:
    - name: E0
      dtype: fp32
      dims: [M, N, K]
      data_gen: torch.rand
    outputs:
    - name: S0
      dtype: fp32
      dims: [M, N, D, K]
      data_transform:
        - |
          input_data['E0'].unsqueeze(1).repeat(1, D_value, 1, 1)
    impl: |
      E1 = step.Bufferize(a=2).apply(E0) # E1: {dtype: Buffer(fp32, [M, N]), dims: [K]}
      E2 = step.Repeat(n=D).apply(E1) # E2: {dtype: Buffer(fp32, [M, N]), dims: [D, K]}
      E3 = step.Streamify().apply(E2) # E3: {dtype: fp32, dims: [M, N, D, K]}
      return E3
```

Figure 16: An example of usage pattern that contains 3 shape manipulation primitives: Bufferize, Repeat, and Streamify.

```
inputs:
  - name: E0
    dtype: fp32
    dims: [M, N]
    data_gen: torch.rand
  - name: E1
    dtype: Buffer(fp32, [D])
    dims: [M, N]
    data_gen: torch.rand

fns:
  - name: MaxSum
    apply: |
      m_t, l_t, o_t = state # scalar, scalar, [D]
      s_t, v_t = input # scalar, [D]
      m_next = torch.max(m_t, s_t) # scalar
      l_prim_t = torch.exp(m_t - m_next) * l_t
      p_t = torch.exp(s_t - m_next)
      l_next = p_t + l_prim_t
      o_next = l_prim_t * o_t / l_next + p_t * v_t / l_next
      return [m_next, l_next, o_next]
    init: [-inf, 0, 0]
    input_dtype: [fp32, "Buffer(fp32, [D])"]
    output_dtype: [fp32, fp32, "Buffer(fp32, [D])"]
    func_name: fn_maxsum

  - name: GetThird
    apply: |
      return [input[2]]
    input_dtype: [fp32, fp32, "Buffer(fp32, [D])"]
    output_dtype: Buffer(fp32, [D])
    func_name: fn_getthird

outputs:
  - name: S0
    dtype: fp32
    dims: [D, N]
    data_transform:
      - |
        torch.bmm(torch.softmax(input_data['E0'], 1).unsqueeze(1),
             input_data['E1']).squeeze(1)

impl: |
  E3 = step.Zip().apply((E0, E1))
  E4 = step.Accum(fn=fn_maxsum, b=1).apply(E3)
  E5 = step.Map(fn=fn_getthird).apply(E4)
  E2 = step.Streamify().apply(E5)
  return E2
```

Figure 17: The complete implementation of attn task 0 written in structural IR.

```
import step
from sympy import Symbol
import torch

M = Symbol("M")
N = Symbol("N")
K = Symbol("K")
D = Symbol("D")
M_value = 5
N_value = 7
K_value = 9
D_value = 11
ctx = {M: M_value, N: N_value, K: K_value, D: D_value}
input_dtype = {'E0': step.Scalar("float"), 'E1': step.Buffer(step.Scalar("float"), [D])}
input_data = {'E0': torch.rand(N_value, M_value), 'E1': torch.rand(N_value, M_value, D_value)}

class MaxSum(step.Fn):
    def __init__(self, input, output):
        super().__init__("MaxSum", input, output)
    def getInit(self):
        return [torch.tensor(float('-inf')), torch.tensor(0), torch.zeros(D_value)]
    def apply(self, state, input):
        m_t, l_t, o_t = state # scalar, scalar, [D]
        s_t, v_t = input # scalar, [D]
        m_next = torch.max(m_t, s_t) # scalar
        l_prim_t = torch.exp(m_t - m_next) * l_t
        p_t = torch.exp(s_t - m_next)
        l_next = p_t + l_prim_t
        o_next = l_prim_t * o_t / l_next + p_t * v_t / l_next
        return [m_next, l_next, o_next]
fn_maxsum = MaxSum(step.STuple((step.Scalar("float"), step.Buffer(step.Scalar("float"),
     [D]))), step.STuple((step.Scalar("float"), step.Scalar("float"),
     step.Buffer(step.Scalar("float"), [D]))))

class GetThird(step.Fn):
    def __init__(self, input, output):
        super().__init__("GetThird", input, output)
    def apply(self, input):
        return [input[2]]
fn_getthird = GetThird(step.STuple((step.Scalar("float"), step.Scalar("float"),
     step.Buffer(step.Scalar("float"), [D]))), step.Buffer(step.Scalar("float"), [D]))

def prepare():
    E0 = step.Stream("E0", step.Scalar("float"), 1, [M, N])
    E0.ctx = ctx
    E0.data = [input_data['E0']]
    E1 = step.Stream("E1", step.Buffer(step.Scalar("float"), [D]), 1, [M, N])
    E1.ctx = ctx
    E1.data = [input_data['E1']]
    return E0, E1

def check_shape(S0):
    assert S0.shape == [D, N]
    assert S0.dtype == step.Scalar("float")

def check_data(S0):
    S0_data_0 = torch.bmm(torch.softmax(input_data['E0'], 1).unsqueeze(1),
        input_data['E1']).squeeze(1)
    torch.testing.assert_close(S0.data[0], S0_data_0)

def test():
    E0, E1 = prepare()
    S0 = body(E0, E1)
    check_shape(S0)
    check_data(S0)

def body(E0, E1):
    E3 = step.Zip().apply((E0, E1))
    E4 = step.Accum(fn=fn_maxsum, b=1).apply(E3)
    E5 = step.Map(fn=fn_getthird).apply(E4)
    E2 = step.Streamify().apply(E5)
    return E2
```

Figure 18: The unit test of the implementation of attn task 0 in Python produced by the code generator from structural IR shown in fig. 17.

```
desc: |
  Streaming Tensor Programs (STeP) provides a higher-level abstraction for dataflow systems.
  The streams can be only consumed once. Your task is to use Copy primitives to create a new
       stream that is a copy of the input stream when necessary.

examples:
  - input_impl: |
      E2 = step.Partition(N=E_value).apply((E0, E1))
      E3 = [step.Map(fn=fn).apply(s) for fn, s in zip(matmul_fns, E2)]
      E4 = step.Merge(fn=fn_sum).apply((E3, E1))
      return E4

    output_impl: |
      E1_0, E1_1 = step.Copy().apply(E1)
      E2 = step.Partition(N=E_value).apply((E0, E1_0))
      E3 = [step.Map(fn=fn).apply(s) for fn, s in zip(matmul_fns, E2)]
      E4 = step.Merge(fn=fn_sum).apply((E3, E1_1))
      return E4

    explanation: |
      Stream E1 is consumed twice in the input implementation. To ensure that the stream is
           consumed only once, we create a copy of the stream E1 and use the copy in the
           second step.

  - input_impl: |
      E1 = step.Map(fn=fn_predict).apply(E0)
      E2 = step.Map(fn=fn_router).apply(E1)
      E3 = step.Map(fn=fn_affinity).apply(E0)
      E4 = step.Zip().apply((E0, E3))
      return E4

    output_impl: |
      E0_0, E0_1 = step.Copy().apply(E0)
      E0_2, E0_3 = step.Copy().apply(E0_0)
      E1 = step.Map(fn=fn_predict).apply(E0_1)
      E2 = step.Map(fn=fn_router).apply(E1)
      E3 = step.Map(fn=fn_affinity).apply(E0_2)
      E4 = step.Zip().apply((E0_3, E3))
      return E4

    explanation: |
      Stream E0 is consumed 3 times in the input implementation. To ensure that all streams
           are consumed only once, we create a copy of the stream E0 and use the copy in the
           subsequent steps.

  - input_impl: |
      E1 = step.Bufferize(a=1).apply(E0)
      E2 = step.Map(fn=fn_gate).apply(E1)
      E3 = step.Map(fn=fn_top2).apply(E2)
      return E3, E2

    output_impl: |
      E1 = step.Bufferize(a=1).apply(E0)
      E2 = step.Map(fn=fn_gate).apply(E1)
      E3 = step.Map(fn=fn_top2).apply(E2)
      return E3, E2

    explanation: |
      All streams are consumed only once in the input implementation. No need to create a
           copy of any stream.
```

Figure 19: Base prompt for the guardian agent.

```
### <a name="Accum"></a>Accum
Accumulate the lower `b` dimensions in `Stream<A,a>` into a single value of type `B`.
     **Accum** will continue to dequeue and accumulate to a value of type `B` by calling the
     given accumulation function (`Fn(A,B)->B`) until it sees a `.Sb` in the input stream.
     Then, it will emit the accumulated value of type `B` into the output stream and
     initialize the accumulator with the given initialize function.
```
Accum<A,B,a,b>: Fn(A,B) -> B, Fn() -> B, Stream<A,a> -> Stream<B,a-b>
                (accumulate) (initialize)
Precondition: 0 < b <= a
```
We can think of `b` as the minimum stop token level **Accum** has to see before emitting the
     accumulated values. More details on how to set `b` according to the type of reduction we
     do can be found in the below examples.

<details>

<summary>
Examples
</summary>

**Example1: Rowmax**  <br/>

```
Goal: [B,N,E] -> [B,N] (Reduce over the inner-most dim)
Accum<A=f32, B=f32, a=3, b=1>:
  Fn(f32,f32)->f32, Fn()->f32, Stream<f32,3>->Stream<f32,2>

Precondition: 0 < b <= a
                (=1) (=3)

```
We will call the given function (max) on every dequeue and emit the accumulated value when we
     see a `.Sx(x>=b)`. b is 1 in this example because we have to see the whole vector to
     obtain the reduced value.
<br/>
```

Figure 20: The specification of `Accum` primitive in the STeP document.

```
class Accum(OpBase):
    def __init__(self, **kwargs):
        super().__init__("Accum", **kwargs)

    def apply(self, input: base.Stream, name=""):
        b = self.config["b"]
        fn: base.Fn = self.config["fn"]
        assert isinstance(fn, base.Fn), f"Accum should take one of provided fns as input, but
             get {type(fn)}"
        assert fn.input == input.dtype, f"Accum should take {fn.input} as input, but get
             {input.dtype}"
        assert b > 0 and b <= input.rank, f"Accum should take a positive integer b less than
             or equal to the rank of the input, but get b: {b} and input rank: {input.rank}"

        result = base.Stream(
            self.getName(name), fn.output, input.rank - b, input.shape[b:]
        )
        if input.data is not None:
            result.ctx = input.ctx
            # TODO: Construct a general application function here
            output_indices = get_full_indices(base.subsOuterShape(result.shape, result.ctx))
            input_indices = get_full_indices(base.subsOuterShape(input.shape[:b], input.ctx))
            if isinstance(result.dtype, base.Element) or isinstance(result.dtype,
                 base.Buffer):
                output_shapes = [base.subsFullShape(result.dtype, result.shape,
                     result.ctx)[::-1]]
            elif isinstance(result.dtype, base.STuple):
                output_shapes = [base.subsFullShape(r, result.shape, result.ctx)[::-1] for r
                     in result.dtype]
            else:
                raise ValueError("Invalid dtype")
            result.data = [torch.zeros(shape) for shape in output_shapes]
            for idx in output_indices:
                state = fn.getInit()
                for i in input_indices:
                    full_idx = idx + i
                    partial_data = [d[full_idx + (...,)] for d in input.data]
                    state = fn.apply(state, partial_data)
                for n, s in enumerate(state):
                    result.data[n][idx] = s
        return result
```

Figure 21: The definition of `Accum` primitive in the Python frontend.