

# RAPIDLY ADAPTING POLICIES TO THE REAL WORLD VIA SIMULATION-GUIDED FINE-TUNING

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## ABSTRACT

Robot learning requires a considerable amount of high-quality data to realize the promise of generalization. This amount is challenging to collect entirely in the real world. Simulation can help greatly, wherein techniques such as reinforcement learning can provide broad coverage over states and actions. However, high-fidelity physics simulators are fundamentally misspecified approximations to reality, making direct zero-shot transfer challenging, especially in tasks where precise and force-sensitive manipulation is necessary. This makes real-world fine-tuning of policies pretrained in simulation an attractive approach to robot learning in principle, but for many practical applications exploring the real-world dynamics with standard RL fine-tuning techniques is too inefficient. This paper introduces Simulation-Guided Fine-Tuning, a general framework that leverages the structure of a value function learned in simulation to guide and accelerate real-world exploration. We demonstrate our approach across several real-world manipulation tasks for which learning successful policies using purely real-world data is very hard. Last but not least, we provide theoretical justification for this new paradigm.

## 1 INTRODUCTION

Robot learning offers a pathway to building robust, general-purpose robotic agents that can rapidly adapt their behavior to new environments and tasks. This shifts the burden from designing accurate environment models and task-specific controllers by hand to the problem of collecting large behavioral datasets with sufficient coverage. This raises a fundamental question: *how do we cheaply obtain and leverage such datasets at scale?* Real-world data collection via teleoperation (Walke et al., 2023; team, 2024) can generate high-quality trajectories but is tedious and scales linearly with human effort. Even with community-driven teleoperation (Mandlekar et al., 2018; Collaboration, 2024), current robotics datasets are still orders of magnitude smaller than those powering vision and language applications.

Massively parallelized physics simulation (Todorov et al., 2012; Makoviychuk et al., 2021) is a potential solution for cheaply generating vast quantities of synthetic robotics data. Moreover, applying techniques such as reinforcement learning (RL) in simulation can cheaply generate datasets with extensive *coverage*, when coupled with techniques such as automatic scene generation (Chen et al., 2024; Deitke et al., 2022) and extensive randomization of initial conditions and dynamics parameters (Peng et al., 2018; Andrychowicz et al., 2020).

Unfortunately, simulation-generated data is not a silver bullet, as it provides cheap but ultimately *off-domain data*. Namely, simulators are fundamentally *misspecified* approximations to reality. This is highlighted in tasks like hammering in a nail, where the modeling of high-impact, deformable contact remains an open problem (Acosta et al., 2022; Levy et al., 2024). In these regimes, *no choice of*

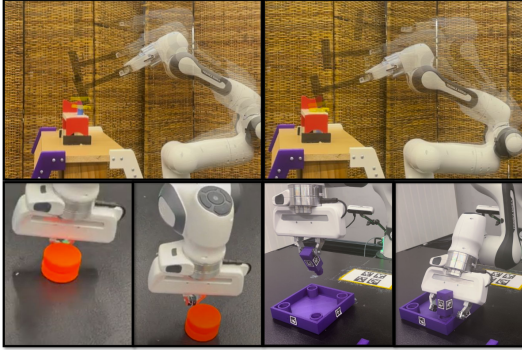


Figure 1: Three dynamic, contact-rich manipulation tasks – hammering (**top**), pushing (**bottom left**), and inserting (**bottom right**) – solved in the real world using our method *SGFT*.

parameters for the physics simulator accurately capture the real-world dynamics. This gap persists despite efforts towards improving existing physics simulators with system identification (Memmel et al., 2024; Huang et al., 2023) identification and learning generative world models directly from large, but ultimately finite, real-world data sets Yang et al. (2023); Bruce et al.. Thus, despite impressive performance for many tasks, methods that transfer policies from simulation to reality zero-shot (Kumar et al., 2021; Lee et al., 2020; Peng et al., 2018; Andrychowicz et al., 2020) still run into failure modes when they encounter situations outside simulated training distribution (Smith et al., 2022b).

The question becomes: *can inaccurate simulation models be useful in the face of fundamental misspecifications?* A natural technique is to fine-tune policies pre-trained in a simulator using real-world experience (Smith et al., 2022b; Zhang et al., 2023). This approach can overcome misspecification by training directly on data from the target domain. However, existing approaches employ the unstructured exploration strategies used by standard, general reinforcement learning algorithms Haarnoja et al. (2018), which were designed not for fine-tuning but for *tabula rasa* learning. As a result, current RL fine-tuning frameworks remain to sample inefficient for real-world deployment.

We argue the following: despite inherently getting the finer details wrong, generative physics simulators capture the rough structure of real-world dynamics well enough to distill targeted exploration strategies which can substantially accelerate real-world learning. Leveraging this insight, we propose *Simulation-Guided Fine-Tuning (SGFT)*, a general framework for rapid adaptation of behaviors learned in simulation to the nuances of real-world dynamics. The key ingredient of *SGFT* is to use a value function  $V_{sim}$  learned in the approximate simulator to guide exploration in the real-world. We demonstrate this general framework can be built on top of generic reinforcement learning algorithms, substantially accelerating real-world learning.

In more detail, unlike most fine-tuning strategies which aim to solve an intractable infinite-horizon objective, *SGFT* solves a short-horizon  $H$ -step policy optimization problem with a reshaped objective. Specifically, we use  $V_{sim}$  to provide guidance to base policy optimization strategies by leveraging the Potential-based Reward-Shaping Formalism Ng et al. (1999). This horizon-shortening approach effectively uses  $V_{sim}$  to boot-strap long-horizon returns, leading to a tractable search problem Westenbroek et al. (2022); Cheng et al. (2021). This paradigm is particularly powerful when built on top of a base model-based reinforcement learning (MBRL) algorithm Hafner et al. (2019); Janner et al. (2019); Hansen et al. (2024). The central challenge for these methods (when solving standard long-horizon objectives) is that small errors in the model rapidly compound when unrolling predictions over multiple steps Janner et al. (2019). The short-horizon *SGFT* effectively leverages experience gained in simulation to circumvent this challenge, unlocking the potential of generative environment modeling.

We outline our contributions as follows:

1. We introduce the *SGFT* framework, and illustrate through extensive experiments how it is a light-weight modification to existing sim-to-real RL finetuning frameworks Smith et al. (2022b); Zhang et al. (2019); indeed, because these approaches generally employ algorithms which already learn a value function  $V_{sim}$  in simulation, implementing the reshaped *SGFT* objective is only a few lines of code.
2. We implement two versions of *SGFT* on top of different base MBRL algorithms, and demonstrate that *SGFT* can learn highly effective policies in regimes where existing zero-shot transfer techniques fail. Moreover, *SGFT* provides substantial sample complexity gains over existing fine-tuning method. In particular, *SGFT* provides a powerful framework for learning policy with near 100% success rates, which we observe to be a significant challenge for existing fine-tuning approaches.
3. We demonstrate theoretically how *a)* *SGFT* can learn highly performant policies, despite the bias introduced by the short-horizon objective and *b)* *SGFT* renders standard MBRL approaches robust to large errors in generative dynamics models, which are bound to arise in the fine-tuning setting where only a small amount of on-task data is available. Together, these insights underscore how *SGFT* effectively leverages plentiful off-domain data from a simulator alongside small real-world data sets to substantially accelerate real-world fine-tuning.

## 2 RELATED WORK

**Simulation-to-Reality Transfer.** In this work, we assume that perception in simulation and reality is approximately matched and focus primarily on the dynamics gap. To bridge this dynamics gap, two classes of methods have been popular: 1) adapting simulation parameters to real-world data and 2) learning adaptive or robust policies to account for changing real-world dynamics. While going back from the real world to simulation can help target the simulation parameters more accurately (Chebotar et al., 2019; Ramos et al., 2019; Memmel et al., 2024), it cannot overcome inherent model misspecification, as we show in our experimental evaluation. Learning adaptive policies to account for changing real-world dynamics (Qi et al., 2022; Kumar et al., 2021; Yu et al., 2017) can help to some extent but is unable to guide exploration and adapt beyond the training dynamics range. Another appealing approach is to search for distributions of domain randomization parameters that will lead to maximally robust transfer in a principled fashion. For example, Tiboni et al. (2023a) uses entropy maximization to find simulator parameters which will lead to maximally robust transfer while accurately describing a small number of real world trajectories. This is taken further by Tiboni et al. (2023b), which automatically evolves training distributions purely in simulation to generate a curricula of environments which leads to policies which are both performant and robust.

Perhaps most related is Smith et al. (2022b); Zhang et al. (2023), where policies learned in simulation are fine-tuned with off-policy RL. However, besides initialization, simulation is not used to guide exploration throughout fine-tuning. Prior work has additionally considered mixing simulated and real data during policy optimization – either through co-training (Torne et al., 2024), simply initializing the replay-buffer with simulation data (Smith et al., 2022a; Ball et al., 2023), or adaptively sampling the simulated dataset and up-weighting transitions that approximately match the true dynamics (Eysenbach et al., 2020; Liu et al., 2022; Xu et al., 2023; Niu et al., 2022). In contrast, our approach focuses on distilling useful *exploration strategies from simulation*.

**Fine-tuning in Reinforcement Learning.** Our work is related to algorithms for RL-based fine-tuning with online data collection, primarily starting from offline RL-aided or imitation learning-aided initializations. These algorithms typically aim to provide an initialization that can continue improving with standard off-policy RL (Rajeswaran et al., 2018; Nair et al., 2020; Kostrikov et al., 2021; Hu et al., 2023; Nakamoto et al., 2024). They initialize policies and Q-functions from offline data and continue training them with standard RL methods, but do not use the pre-training data beyond initialization and populating the replay buffer. We utilize the pretraining in simulation not just for initialization but also to provide guidance throughout the policy improvement process.

**Reward Design in Reinforcement Learning.** A significant component of our methodology is learning dense shaped reward in simulation to guide real-world fine-tuning. This is closely tied to the problem of reward design and reward inference in RL (Gupta et al., 2022). This is a challenging problem if attempted tabula rasa but prior techniques have tried to infer rewards from expert demos (Ziebart et al., 2008; Ho & Ermon, 2016), success examples (Fu et al., 2018; Li et al., 2021a), LLMs (Ma et al., 2023; Yu et al., 2023), and heuristics (Margolis et al., 2024; Berner et al., 2019). We rely on simulation to provide reward supervision using the PBRs formalism (Ng et al., 1999), and shorten the horizon of the learning task to improve the sample complexity of real-world learning (Cheng et al., 2021; Westenbroek et al., 2022). Another line of work complementary to our comes from Eysenbach et al. (2020); Liu et al. (2022); Xu et al. (2023); Niu et al. (2022), which relabel rewards from off-task (simulated) data, effectively up-weighting transitions that approximately match the dynamics observed in the target domain. These works focus on the *retrieval* of useful samples from prior datasets with shifted dynamics. In contrast, our approach uses prior data to guide the discovery of novel sequences of states and actions in the target domain. In principle, these techniques could be used in conjunction; we leave this to future work.

**Model-Based Reinforcement Learning.** A significant body of work on model-based RL learns dynamics models to perform data augmentation (Sutton, 1991; Wang & Ba, 2019; Janner et al., 2019; Yu et al., 2020; Kidambi et al., 2020) for downstream policy optimization algorithms, plan online using the model (Ebert et al., 2018; Zhang et al., 2019), or to use the model as a control variate to reduce variance of policy gradient methods (Chebotar et al., 2017; Cheng et al., 2019; 2020; Che et al., 2018). The central challenge for each of these model-based methods is that small inaccuracies in predictive models can quickly compound over time, leading to large *model-bias*. An effective critic can be used to shorten search horizons (Hansen et al., 2024; Bhardwaj et al., 2020; Hafner et al., 2019) yielding easier decision-making problems, but learning such a critic from scratch can still require large amounts of on-task data. We demonstrate that, for many real-world continuous

control problems, critics learned entirely in simulation can be robustly transferred to the real-world and substantially accelerate model-based learning.

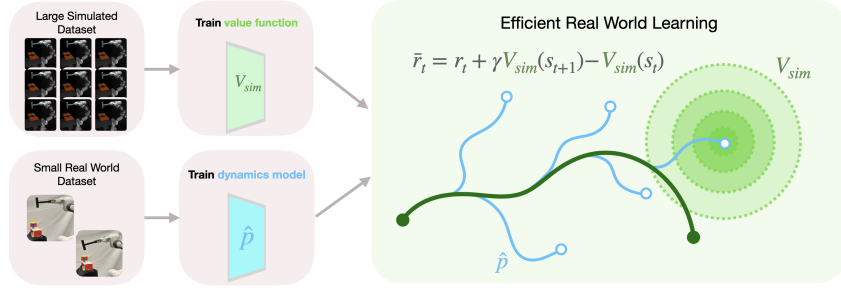


Figure 2: Depiction of *SGFT*: Rather than simply fine-tuning from a policy initialization, *SGFT* uses short horizon rollouts from the learned model along with potential-based reward shaping using the learned value function to inform efficient sim-to-real fine-tuning.

### 3 PRELIMINARIES

Let  $\mathcal{S}$  and  $\mathcal{A}$  be state and action spaces. Our goal is to control a real-world system defined by an unknown Markovian dynamics  $s' \sim p_{real}(\cdot|s, a)$ , where  $s, s' \in \mathcal{S}$  are states and  $a \in \mathcal{A}$  is an action. The usual formalism for solving tasks with RL is to define a Markov Decision Process of the form  $\mathcal{M}_r = (\mathcal{S}, \mathcal{A}, p_{real}, \rho_{real}^0, r, \gamma)$  with initial real-world state distribution  $\rho_{real}^0$ , reward function  $r$ , and discount factor  $\gamma \in [0, 1]$ . Given a policy  $\pi$ , we let  $d_{real}^\pi(s)$  denote the distribution over trajectories  $\tau = (s_0, a_0, s_1, a_1, \dots)$  generated by applying  $\pi$  starting at initial state  $s_0$ . Defining the value function under  $\pi$  as  $V_{real}^\pi(s) = \mathbb{E}_{s_t \sim d_{real}^\pi(s)}[\sum_t \gamma^t r_t(s_t)]$ , our objective is to find  $\pi_{real}^* \leftarrow \sup_\pi \mathbb{E}_{s \sim \rho_{real}^0}[V_{real}^\pi(s)]$ . We define the optimal value function as  $V_{real}^*(s) := \sup_\pi V_{real}^\pi(s)$ .

Unfortunately, as discussed above, obtaining a good approximation to  $\pi_{real}^*$  using only real-world data is often impractical. Thus, many approaches leverage an approximate simulation environment  $s' \sim p_{sim}(s, a)$  and solve an approximate MDP of the form  $\mathcal{M}_{sim} := (\mathcal{S}, \mathcal{A}, p_{sim}, \rho_{sim}^0, r, \gamma)$  to train a policy  $\pi_{sim}$  meant to approximate  $\pi_{real}^*$ . We let  $V_{sim}$  denote  $\pi_{sim}$ 's value function with respect to  $\mathcal{M}_{sim}$ . We do not require  $\pi_{sim}$  to be optimal in simulation, but merely an approximately optimal one that can reliably solve the given task under  $p_{sim}$ . Here,  $\rho_{sim}^0$  is the distribution over initial conditions in the simulator.

### 4 SIMULATION-GUIDED FINE-TUNING

Simulation-Guided Fine-Tuning (*SGFT*) takes the perspective that  $\pi_{sim}$  can be viewed as an approximate expert for controlling  $\mathcal{M}_{real}$ . That is,  $\pi_{sim}$  can capture the rough structure of behaviors that solve the task in reality, even if it fails to transfer to  $\mathcal{M}_{real}$  zero-shot. Specifically, we contend that these behaviors are encoded into  $V_{sim}$  in a way that is robust to dynamics shifts. Indeed,  $V_{sim}$  defines an ordering over states that are desirable to reach in the future, *but says nothing about the actions needed to reach those states*. *SGFT* uses small amounts of real-world data to learn actions that will increase  $V_{sim}$  over short horizons, and thus leverages  $V_{sim}$  to guide real-world exploration towards behaviors that can solve the entire (longer-horizon) task.

#### 4.1 REWARD SHAPING AND HORIZON SHORTENING

We implement our horizon-shortening approach using the Potential-Based Reward Shaping (PRBS) formalism (Ng et al., 1999), which replaces the reward  $r(s)$  with  $\bar{r}(s, s') = r(s) + \gamma\Phi(s') - \Phi(s)$  for some function  $\Phi$  and is usually applied to infinite horizon MDPs  $\mathcal{M}$  to form a new MDP  $\bar{\mathcal{M}}$ . Intuitively, the addition of the potential term encourages policy search algorithms to follow  $\Phi$  by increasing its value during each transition. A well-designed  $\Phi$  can guide exploration and decrease the variance of policy search by effectively acting as a *baseline*, but the reshaped reward does not change the optimal policy. Indeed, by telescoping out the rewards one may observe that  $\sum_{t=0}^{\infty} \bar{r}_t = \sum_{t=0}^{\infty} r_t - \Phi(s_0)$ . Because  $\Phi(s_0)$  does not depend on the choice of the policy, the reshaped reward does not affect the ordering over policies. However, even with an informative reshaped reward, solving the infinite horizon reshaped MDP is still costly. This motivates us to shorten the horizon of

the returns that we optimize in the real-world, and thus we will investigate optimizing  $H$ -step returns of the form  $\sum_{t=0}^{H-1} \bar{r}_t = \sum_{t=0}^{H-1} r_t + \gamma^H \Phi(s_H) - \Phi(s_0)$ . Here, the  $\gamma^H \Phi(s_H)$  term can be interpreted as a fixed approximation to the true long-horizon returns  $\gamma^H V_{real}^*(s_H)$ , while  $-\Phi(s_0)$  again acts as a baseline to reduce variance. While this introduces bias into the returns, optimizing the  $H$ -step return from a given initial condition presents a significantly more tractable problem.

#### 4.2 $H$ -STEP SIMULATION-GUIDED EXPERT POLICIES

We propose to set  $\Phi(s) = V_{sim}(s)$  (i.e. the value of the policy  $\pi_{sim}$  with respect to  $\mathcal{M}_{sim}$ ) and optimize the reshaped reward  $\bar{r}(s, s') = r(s) + \gamma V_{sim}(s') - V_{sim}(s)$  over  $H$ -steps from every initial condition  $s \in \mathcal{S}$ , as a mean to adapt  $\pi_{sim}$  from  $\mathcal{M}_{sim}$  to  $\mathcal{M}_{real}$ . We will use ‘tilde’ notation  $\tilde{\pi}_H = \{\pi_{H,0}, \pi_{H,1}, \dots, \pi_{H,H-1}\}$  to denote non-stationary policies of horizon  $H$ , where  $\pi_{H,t}$  is the policy applied at time  $t$ . Consider the following  $H$ -step returns under the real-world dynamics:

$$V_H^{\tilde{\pi}_H}(s) = \mathbb{E} \left[ \gamma^H V_{sim}(s_H) + \sum_{t=0}^{H-1} \gamma^t r(s_t) - V_{sim}(s_0) \middle| s_0 = s, a_t \sim \pi_{H,t}(\cdot | s_t) \right]. \quad (1)$$

$$V_H^*(s) := \sup_{\tilde{\pi}_H} V_H^{\tilde{\pi}_H}(s) \quad Q_H^*(s, a) = \mathbb{E}_{s' \sim p_{real}(s, a)} [\gamma V_{H-1}^*(s') + \bar{r}(s, s')] \quad (2)$$

Note that the  $-V_{sim}(s_0)$  term in Equation (3) is not affected by the choice of  $\tilde{\pi}_H$ , and does not affect the ordering of policies. Thus,  $V_H^{\tilde{\pi}_H}$  is equivalent to the planning objective used by model predictive control (MPC) methods (Jadbabaie et al., 2001; Hansen et al., 2024; Sun et al., 2018; Bhardwaj et al., 2020) with  $H$ -step look-ahead and a terminal reward of  $V_{sim}$  (when the ground truth dynamics is known). Thus, we define the  $H$ -step simulation-guided MPC expert via  $\pi_H^*(\cdot | s) = \pi_{H,0}^*(\cdot | s)$ , where  $\pi_0^*$  is taken from the first step of the solution to  $\tilde{\pi}_H^* \leftarrow \max_{\tilde{\pi}_H} V_H^{\tilde{\pi}_H}(s)$ . In other words, this policy simply applies the first action generated by the optimal  $H$ -step policy at each state. Abusing notation, note that this policy can also be calculated as  $\pi_H^*(\cdot | s) \leftarrow \max_{\pi} Q_H^*(s, \pi(s))$ .

Intuitively,  $\pi_H^*$  will greedily follow  $V_{sim}$  at every state when  $H = 1$ , and as we take  $H \rightarrow \infty$  the behavior of  $\pi_H$  will recover the behavior of  $\pi_{real}^*$ . Thus,  $\pi_H^*$  can be viewed as a policy that has adapted the behavior of  $\pi_{sim}$  to follow  $V_{sim}$  along the real-world dynamics, and for smaller values of  $H$  we should expect  $\pi_H^*$  to retain more of the behavior of  $\pi_{sim}$ . As we discuss further in Section 5, these idealized MPC policies have a number of desirable properties, when compared to directly transferring  $\pi_{sim}$  to  $\mathcal{M}_{real}$ . Of course, because these expert policies depend on the real-world dynamics, we do not have direct access to their actions. Next, We introduce a general framework that implicitly attempts to mimic these experts through interactions with the real-world dynamics.

#### 4.3 THE GENERAL SGFT FRAMEWORK

Even though we do not have direct access to  $\pi_H^*$ , we can implicitly learn its actions by optimizing policies to maximize the  $H$ -step return (Equation (3)) starting from every initial condition  $s \in \mathcal{S}$ . Specifically, note that the expert actions are given by  $\pi_H^* \leftarrow \max_{\pi} Q_H^*(s, \pi)$ , and that many standard (model-based and model-free) RL methods can be leveraged to solve this short-horizon policy optimization problem. This motivates our conceptual *Simulation-Guided Fine-Tuning (SGFT)* framework, which is defined via the pseudo-code in Algorithm 1. *SGFT* fine-tunes  $\pi_{sim}$  to succeed under the real-world dynamics by iteratively 1) unrolling the current policy to correct transitions from  $p_{real}$  and 2) using the current dataset  $\mathcal{D}$  of transitions to approximately optimize  $\pi \leftarrow \max_{\pi} Q_H^*(s, \pi(s_j))$  at each state  $s_j$  the agent has visited. In Sun et al. (2018) a model-

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##### Algorithm 1 Simulation-Guided Fine-tuning (SGFT)

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**Require:** Pretrained policy  $\pi_{sim}$  and value function  $V_{sim}$

- 1:  $\pi \leftarrow \pi_{sim}, \mathcal{D} \leftarrow \emptyset$
  - 2: **for** each iteration  $k$  **do**
  - 3:   **for** time step  $t = 1, \dots, T$  **do**
  - 4:      $a_t \sim \pi(\cdot | s_t)$
  - 5:     Observe the state  $s_{t+1}$  and the reward  $r_t$ .
  - 6:      $\bar{r}_t \leftarrow r_t + \gamma V_{sim}(s_{t+1}) - V_{sim}(s_t)$
  - 7:      $\mathcal{D} \leftarrow \mathcal{D} \cup \{(s_t, a_t, \bar{r}_t, s_{t+1})\}$
  - 8:   **end for**
  - 9:   Approx. optimize  $\pi \leftarrow \max_{\pi} Q_H^*(s, \pi(s_j))$  using transitions in  $\mathcal{D} \forall s_j$  in  $\mathcal{D}$ ’s transitions.
  - 10: **end for**
-

free method for approximately achieving the optimization in step 2) is proposed<sup>1</sup>, and we discuss how this step can be performed with model-based methods below. By approximately optimizing  $\pi \leftarrow \max_{\pi} Q_H^*(s, \pi(s_j))$ , *SGFT* is implicitly attempting to learn and approximate the actions taken  $\pi_H^*$  at every state the agent has visited (which is reminiscent of the learning loop used by DAgger (Ross et al., 2011), with the obvious caveat that in our case the expert is not directly available).

**What are we taking away from simulation?** We are explicitly interested in applying *SGFT* with small values of  $H$ , as this will yield real-world learning problems that are significantly more tractable than the original infinite-horizon problem (Laidlaw et al., 2023; Li et al., 2021b). In this regime, *SGFT* provides an ideal separation between what is learned using the simulated dynamics  $p_{sim}$  and what is learned from interacting with real dynamics  $p_{real}$ . For small values of  $H$ , optimizing (3) guides policy optimization algorithms towards policies that increase the value of  $V_{sim}$  over  $H$ -step windows. In the extreme case where  $H = 1$ , *SGFT* greedily attempts to increase the value of  $V_{sim}$  each step. Conceptually, in this special case we have reduced the policy search problem to a contextual bandits problem, which is much easier to solve than the original infinite horizon objective. Intuitively, this is because the critic is frozen and no bootstrapping in the real-world is required. More generally, as  $H$  becomes larger the dependence on  $V_{sim}$  is lessened while the dependence on the real returns increase. In simulation, we can easily generate enough data to explore many paths through the state space and discover which motions lead to higher returns (e.g., reach towards an object to be manipulated). This information is distilled into  $V_{sim}$  during the learning process, which provides information about *where* the agent should go  $H$  steps into the future. On the other hand, by optimizing Equation (3) over short horizons, we only need to learn short sequences of actions to move the system to states where  $V_{sim}$  is higher. That is, the short-horizon problem makes it easy to learn *how to get to these desirable states*, even if the required sequences of intervening states and actions differ substantially under  $p_{sim}$  and  $p_{real}$ . Thus, *SGFT* approximately decomposes into a) learning *where to go* in the simulator and b) learning the finer details of *how to get there* with small amounts of real-world data.

#### 4.4 LEVERAGING SHORT MODEL ROLL-OUTS

Model-based reinforcement learning (MBRL) holds the promise of learning a generative model  $\hat{p}$  to rapidly identify effective policies with significantly less data than model-free methods Janner et al. (2019). In particular, generating rollouts with the model enables an agent to reason about trajectories not contained in the dataset the agent has observed. However, as discussed in Section 2, the central challenge for MBRL is that small errors in  $\hat{p}$  can quickly compound over multiple steps, degrading the quality of the predictions. As a consequence, learning a model accurate enough to solve long-horizon problems can often take as much data as solving the task with modern model-free methods (Chen et al., 2021; Hiraoka et al., 2021). By bootstrapping  $V_{sim}$  in simulation, where data is plentiful, the *SGFT* framework enables agents to act effectively over long horizons using only short, local predictions about the real-world dynamics. As our experiments demonstrate, this substantially improves performance compared to model-free approaches.

In what follows, we denote the following returns under the model for  $\tilde{\pi}_H = \{\pi_{H,t}\}_{t=0}^{H-1}$ :

$$\hat{V}_H^{\tilde{\pi}_H}(s) = \mathbb{E} \left[ \gamma^H V_{sim}(s_H) + \sum_{t=0}^{H-1} \gamma^t r(s_t) - V_{sim}(s_0) \middle| s_0 = s, a_t \sim \pi_{H,t}(\cdot | s_t), s_{t+1} \sim \hat{p}(s_t, a_t) \right]. \quad (3)$$

$$\hat{V}_H^*(s) := \sup_{\tilde{\pi}_H} \hat{V}_H^{\tilde{\pi}_H}(s) \quad \hat{Q}_H^*(s, a) = \mathbb{E}_{s' \sim \hat{p}(s, a)} [\gamma \hat{V}_{H-1}^*(s') + \bar{r}(s, s')] \quad (4)$$

The corresponding MPC that uses  $\hat{p}$  is given by:

$$\hat{\pi}_H^*(\cdot | s) \leftarrow \arg \max_{\pi} \hat{Q}_H^*(s, \pi). \quad (5)$$

Next, we discuss two broad approaches that use  $\hat{p}$  to approximately learn  $\hat{\pi}_H^*$  in each iteration.

<sup>1</sup>In particular, in Sun et al. (2018) (where the authors consider general potentials  $\Phi$  in the  $H$ -step rewards), instead of optimizing the non-stationary objective Equation (3) directly, the  $H$ -step return  $\hat{V}_H^{\pi}(s) = \mathbb{E}_{d^{\pi}(s)} [\gamma^H \Phi(s_H) + \sum_{t=0}^{H-1} \gamma^t r(s_t) - \Phi(s_0)]$  for the current (stationary) policy is used as a stand-in. This approach uses Monte-Carlo returns to estimate the advantage  $\bar{A}_H^{\pi}(s, a) = \bar{Q}(s, a) - V(s, a)$  for  $\pi$  and update  $\pi \leftarrow \max_{\pi} \bar{A}_H^{\pi}(s, \pi)$ . Repeating this optimization fits policies which optimize  $\pi \leftarrow \max_{\pi} Q_H^*(s, \bar{\pi}(s))$ .

### Improved Sample Efficiency with Data Augmentation (Algorithm 2).

The generative model  $\hat{p}$  can be used for *data augmentation* by generating a dataset of synthetic rollouts  $\hat{\mathcal{D}}$  to supplement the real-world dataset  $\mathcal{D}$  (Janner et al., 2019; Sutton, 1990; Gu et al., 2016). The combined dataset can then be fed to any policy optimization strategy, such as generic model-free algorithms. We are specifically interested in state-of-the-art Dyna-style algorithms (Janner et al., 2019), which, in our context, branch  $H$ -step rollouts from states the agent has visited previously. As Algorithm 2 shows, after each data-collection phase, this approach updates the generative model  $\hat{p}$  and then repeatedly *a)* generates a dataset  $\hat{\mathcal{D}}$  of synthetic  $H$ -step rollouts under the current policy  $\pi$  starting from states in  $\mathcal{D}$ , and *b)* approximately solves  $\pi \leftarrow \max_{\pi} Q_H^*(s, \pi(s))$  at the observed real-world states using the augmented dataset  $\hat{\mathcal{D}} \cup \mathcal{D}$  and a base model-free method. For example, the method from Sun et al. (2018) can be used for this purpose. In Section 6, we describe how to implement this approach with 1-step hallucinated trajectories and SAC (Haarnoja et al., 2018) as a base model-free algorithm.

**Online Planning (Algorithm 3).** The most straightforward way to approximate the behavior of  $\pi_H^*$  is simply to apply the MPC controller  $\hat{\pi}_H^*$  generated using the current best guess for the dynamics  $\hat{p}$ , as in Equation (5). Algorithm 3 provides general pseudocode for this approach, which iteratively 1) rolls out  $\hat{\pi}_H^*$  (which is calculated using online optimization and  $\hat{p}$  (Williams et al., 2017)) then 2) updates the model on the current dataset of transitions  $\mathcal{D}$ . This high-level approach encompasses a wide array of methods proposed in the literature, e.g., Ebert et al. (2018) and Zhang et al. (2019). In Section 6, we implement this approach using the TDMPC-2 (Hansen et al., 2024).

## 5 THEORETICAL ANALYSIS

In this section we analyze the effectiveness of the  $H$ -step simulation-guided expert  $\pi_H^*$ . Specifically, we seek suboptimality bounds for this agent and a characterization of situations when the behavior of this idealized policy will be robust to the errors made by MBRL techniques, which leverage an approximate model  $\hat{p}$  for  $p_{real}$  that has been learned from real-world data. We are particularly interested in understanding how *SGFT* with short prediction horizons  $H$  can mitigate errors in  $\hat{p}$  with a small number of samples. To set the stage for this analysis, we first recall several relevant theoretical results from prior RL work.

**Proposition 1.** Suppose that  $\max_{s \in \mathcal{S}} |V_{sim}(s) - V_{real}^*(s)| \leq \epsilon$  and that  $\|\hat{p}(s, a) - p_{real}(s, a)\| \leq \alpha$ . Then for  $H$  sufficiently small and for each  $s \in \mathcal{S}$  we have:

$$V_{real}^*(s) - V_{real}^{\hat{\pi}_H^*}(s) \leq O\left(\frac{\gamma}{1-\gamma}\alpha H + \frac{\gamma^H}{1-\gamma^H}\epsilon\right), \quad (6)$$

where  $\hat{\pi}_H^*$  is the MPC policy under the approximate mode as in Equation (5)

This result is a direct translation of (Bhardwaj et al., 2020, Theorem 3.1) to our setting, where the big- $O$  notation suppresses problem-dependent constants and lower-order terms for small values of  $H$  which are not important for our discussion. To understand the bound, first set  $\alpha = 0$  so that there is no modeling error (and we recover the behavior of  $\pi_H^*$ ). In this case, we are incentivized to make  $H$  larger. Intuitively, this follows from the fact that  $V_{sim}(s_H)$  can be viewed as an approximation to  $V_{real}^*(s_H)$  in the  $H$ -step look ahead objective Equation (3). Because this approximation is scaled by

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### Algorithm 2 Dyna-SGFT

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**Require:** Pretrained policy  $\pi_{sim}$  and value function  $V_{sim}$

- 1:  $\pi \leftarrow \pi_{sim}$
- 2: **for** each iteration  $k$  **do**
- 3:   Generate rollout  $\{(s_t, a_t, r_t, s_{t+1})\}_{t=0}^T$  under  $\pi$ .
- 4:    $\bar{r}_t \leftarrow r_t + \gamma V_{sim}(s_{t+1}) - V_{sim}(s_t)$
- 5:    $\mathcal{D} \leftarrow \mathcal{D} \cup (s_t, a_t, \bar{r}_t, s_{t+1})$
- 6:   Fit generative model  $\hat{p}$  with  $\mathcal{D}$ .
- 7:   **for**  $G$  policy updates **do**
- 8:     Generate synthetic branched rollouts  $\hat{\mathcal{D}}$  under  $\pi$ .
- 9:     Approx. optimize  $\pi \leftarrow \max_{\pi} Q_H^*(s, \pi(s_j))$   
 $\quad \forall s_j \in \mathcal{D}$  using augmented dataset  $\hat{\mathcal{D}} \cup \mathcal{D}$
- 10:   **end for**
- 11: **end for**

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### Algorithm 3 MPC-SGFT

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**Require:** Pretrained value  $V_{sim}$  and initialized model  $\hat{p}$ .

- 1: **for** each iteration  $k$  **do**
- 2:   Generate rollout  $\{(s_t, a_t, r_t, s_{t+1})\}_{t=0}^T$  under  $\hat{\pi}_H^*$ .
- 3:    $\bar{r}_t \leftarrow r_t + \gamma V_{sim}(s_{t+1}) - V_{sim}(s_t)$
- 4:    $\mathcal{D} \leftarrow \mathcal{D} \cup \{(s_t, a_t, \bar{r}_t, s_{t+1})\}$ .
- 5:   Fit generative model  $\hat{p}$  with  $\mathcal{D}$ .
- 6: **end for**

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$\gamma^H$ , the effect of errors in  $V_{sim}$  is diminished for larger values of  $H$ . Note, however, that this result scales poorly for small values of  $H$ . This is especially true for long-horizon problems where  $\gamma \approx 1$ . On the other hand, the term involving  $\alpha$  captures how errors in the model accumulate for different horizons  $H$ , leading to mistakes in decision making. This term incentivizes us to make  $H$  as small as possible. In particular, we are interested in understanding how we can mitigate large values of  $\alpha$ , which will arise in low-data regimes.

Since results like Proposition 1 simply assume uniform worst-cases bounds on the difference between the magnitudes of  $V_{sim}$  and  $V_{real}^*$ , they do not capture the fact that  $V_{sim}$  may still preserve an ordering over states that is useful for guiding real-world decision making, i.e., when the *geometry* of  $V_{sim}$  enables *SGFT* to guide policy search algorithms towards effective policies under the real dynamics. We use the following definition from Cheng et al. (2021), which is similar to properties from other works (Westenbroek et al., 2022; Grune & Rantzer, 2008):

**Definition 1.** We say that  $V_{sim}$  is improvable with respect to  $\mathcal{M}_{real}$  if for each  $s \in \mathcal{S}$  we have:

$$\max_a \mathbb{E}_{s' \sim p_{real}(s,a)} [\gamma V_{sim}(s')] - V_{sim}(s) \geq -r(s). \quad (7)$$

Namely,  $V_{sim}$  is improvable with respect to  $\mathcal{M}_{real}$  if there exists a policy that can increase  $V_{sim}$  enough over time for each state (with respect to the reward function). A quick intuition we make precise later is the following: as long as  $V_{sim}$  reaches a maximum at desirable states in the real world (e.g., at desired positions for an object being manipulated), then if  $V_{sim}$  is improvable with respect to  $\mathcal{M}_{real}$  we can greedily follow  $V_{sim}$  over a short horizon to reach these desirable states. As defined in Section 3,  $V_{sim}$  is learned under the simulation dynamics and policy  $\pi_{sim}$  such that  $V_{sim}(s) = \mathbb{E}[\gamma V_{sim}(s') + r(s) | s' \sim p_{sim}(s,a), a \sim \pi_{sim}(\cdot|s)]$ , and thus is constructed to be improvable with respect to  $\mathcal{M}_{sim}$ . We provide a pedagogical example in Section F highlighting why this is a plausible property to assume for real-world transfer.

We now present our main theoretical result:

**Theorem 1.** Let the Assumptions of Proposition 1 hold. Further, suppose that  $V_{sim}$  is improvable with respect to  $\mathcal{M}_{real}$ . Then for  $H$  sufficiently small and each  $s \in \mathcal{S}$  we have:

$$V_{real}^*(s) - V_{\hat{\pi}_H^*}^*(s) \leq O\left(\frac{\gamma}{1-\gamma}\alpha H + \gamma^H \epsilon\right), \quad (8)$$

where  $\hat{\pi}_H^*$  is the MPC policy under the approximate model, as in Equation (5).

The proof can be found in Appendix A. At a high-level, the proof uses arguments similar to Grune & Rantzer (2008) to bound the suboptimality of the expert policy  $\pi_H^*$  and then combines this bound with the perturbation bounds from Bhardwaj et al. (2020) to bound how errors in the dynamics lead to additional suboptimality. Note that the dependence on  $H$  and  $\alpha$  is identical to the bound from Proposition 1 above. However, the scaling for the term involving  $\epsilon$  is improved substantially for small values of  $H$ , especially for long-horizon problems where  $\gamma \approx 1$ . Thus, we can more readily use small values of  $H$  to combat large model bias when  $V_{sim}$  is improvable with respect to  $\mathcal{M}_{real}$ . This provides insight into how *SGFT* can rapidly learn effective policy in the real world by using short model rollouts with a coarse model to approximate the behaviors of  $\hat{\pi}_H^*$ .

## 6 EXPERIMENTS

We aim to answer the following questions: **(1)** Can *SGFT* facilitate online fine-tuning of policies for dynamic, real-world robotic manipulation tasks using only small amounts of real-world data? **(2)** Does *SGFT* improve the sample efficiency of online fine-tuning compared benchmarks? Can *SGFT* learn policies that outperform direct transfer techniques which leverage extensive domain randomization and/or system identification?

### 6.1 METHODS EVALUATED

Our experiments compare several approaches from three families:

**SGFT Instantiations.** We implement concrete instantiations of the general Dyna-*SGFT* and MPC-*SGFT* frameworks sketched in Algorithms 2 and 3. **SGFT-SAC** fits a model to real world transitions to perform data augmentation and uses SAC as a base model-free policy optimization algorithm. Specifically, after collecting each real-world trajectory for  $G$  gradient steps we 1) generate a dataset  $\hat{\mathcal{D}}$  of synthetic rollouts branched from the real dataset  $\mathcal{D}$ , and then 2) sample from



the joint replay buffer  $\hat{\mathcal{D}} \cup \mathcal{D}$  to perform one policy updates with SAC using the reshaped reward. We use  $H = 1$  in all our experiments. Crucially, we set the ‘done’ flag to true at the end of each rollout – this ensures SAC does not bootstrap its own critic from the real-world data and only uses  $V_{sim}$  to bootstrap long-horizon returns. **SGFT-TDMP-2** uses TDMP-2 Hansen et al. (2024) as a backbone. The base method learns a critic, a policy, and an approximate dynamics model through interactions with the environment. When acting in the world and MPC controller solves online planning problems using the approximate model, the critic as a terminal reward, and uses the policy prior to seed an MPPI planner Williams et al. (2017). To integrate this method with SGFT, when transferring to the real world we simply freeze the critic learned in simulation and use the reshaped objective in Equation (3) as the online planning objective. For our experiments, we use  $H = 4$  and the default hyperparameters reported in Hansen et al. (2024).

**Baseline Fine-tuning Methods.** The **SAC** baseline fine-tunes the pre-trained policy to solve the original MDP  $\mathcal{M}_{real}$  using SAC Haarnoja et al. (2018) – it does not use shaping or horizon shortening. **PBR** fine-tunes the policy under a reshaped infinite-horizon MDP using the reshaped reward  $\bar{r}$  and SAC (Haarnoja et al., 2018) as the policy optimizer. Namely, this approach does not leverage horizon shortening. **TDMP-2** fine-tunes the entire TDMP-2 architecture Hansen et al. (2024) in the real world, but does not leverage reward shaping. *This serves as a state-of-the-art baseline for MBRL.* **IQL** fine-tunes the pre-trained policy to solve the original MDP  $\mathcal{M}_{real}$  using IQL (Kostrikov et al., 2021). It does not make use of reward shaping or horizon shortening. *This serves as a state-of-the-art baseline for fine-tuning methods.*

**Baseline Sim-to-Real Methods.** Our **Domain Randomization** baseline refers to policies trained with extensive domain randomization in simulation and transferred directly to the real world. These policies rely only on the previous observation. **Recurrent Policy + Domain Randomization** uses policies conditioned on histories of observations, similar to methods such as Kumar et al. (2021). The history enables the agent to infer information about the dynamics parameters of the environment it is operating in. **ASID** (Mammel et al., 2024) is a system identification method that performs targeted exploration in the real world to identify the dynamics parameters of the simulator that best match the real-world scene. Once the parameters are identified, a policy is trained under the parameters in simulation and then deployed zero-shot to the real world.

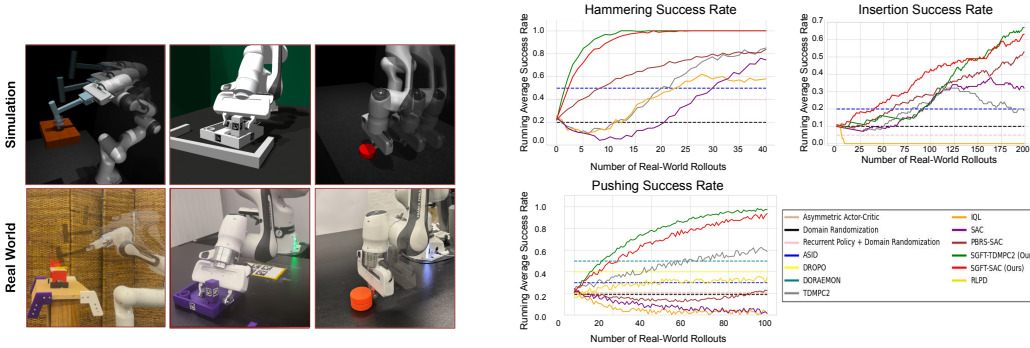


Figure 3: **Sim-to-Real Setup** Simulation setup for pretraining (top) and execution of real-world fine-tuning (bottom) of real-world hammering (left), insertion (middle), and pushing (right).

Figure 4: **Real-world success rates during the course of online fine-tuning.** We plot task success rates over number of fine-tuning rollouts for the tasks described in Sec. 6. We see that **SGFT** yields significant improvements in success and efficiency.

## 6.2 SIM-TO-REAL EVALUATIONS

We test each methods on three real-world manipulation tasks illustrated in Figure 3, demonstrating that both the **SGFT-SAC** and **SGFT-TDMP-2** instantiations of **SGFT** excel at learning policies with minimal real-world data.

**Hammering** is a highly dynamic task involving force and contact dynamics that are impractical to precisely model in simulation. In our setting, the robot is tasked with hammering a nail in a board. The nail has high, variable dry friction along its shaft. In order to hammer the nail into the board,

the robot must hit the nail with high force repeatedly. The dynamics are inherently misspecified between simulation and reality here due to the infeasibility of accurately modeling the properties of the nail and its contact interaction with the hammer and board.

**Insertion** (Heo et al., 2023) involves the robot grasping a table leg and accurately inserting it into a table hole. The contact dynamics between the leg and the table differ between simulation and real-world conditions. In the simulation, the robot successfully completes the task by wiggling the leg into the hole, but in the real world this precise motion becomes challenging due to inherent noise in the real-world observations as well as contact discrepancies between the leg and the table hole.

**Pushing** requires pushing a puck of unknown mass and friction forward to the edge of the table without it falling off the edge. Here, the underlying feedback controller of the real world robot inherently behaves differently from simulation. Additionally, retrieving and processing sensor information from cameras incurs variable amounts of latency. As a result, the controller executes each commanded action for variable amounts of time. These factors all contribute to the sim-to-real dynamics shift, requiring real-world fine-tuning to reconcile.

Each of these tasks is evaluated on a physical setup using a Franka FR3 robot operating with either Cartesian position control or joint position control at 5Hz. We compute object positions by color-thresholding pointclouds or by Aruco marker tracking, although this approach could easily be upgraded. Further details of all tasks, reward functions, robot setups, environments and implementation details can be found in the Appendix.

### 6.3 ANALYSIS

The results of the real-world evaluation during fine-tuning on these three tasks are presented in Figure 4. For all three tasks, zero-shot performance seen at the start of the plot is quite poor due to the dynamics sim-to-real gap. Moreover, the poor performance of system identification methods such as ASID highlights the fact that these gaps are due to more than parameter misidentification, but rather stem from fundamental misspecification.

The second class of comparison methods includes offline pretraining with online fine-tuning techniques like IQL Kostrikov et al. (2021) and SAC Haarnoja et al. (2018). Both flavors of *SGFT*, the model-free *SGFT-SAC* and model-based *SGFT-TDMPC-2*, substantially outperform these prior techniques in terms of efficiency and asymptotic performance. This suggests that simulation can offer more guidance during real-world policy search than just an initialization for subsequent fine-tuning. Our full system consistently leads to significant improvement from fine-tuning, achieving 100% success for hammering and pushing within an hour of fine-tuning and 70% success for inserting within two hours of fine-tuning. The fact that *SGFT* outperforms both TD-MPC2 Hansen et al. (2024) and PBR-SAC, suggests that efficient fine-tuning requires a *combination* of both short model rollouts and value-driven reward shaping.

Last but not least, note that *SGFT* offers improvements on top of both SAC and TDMPC2, showing the generality of the proposed paradigm. Additional evaluations and visualizations are in the Appendix, namely a set of sim-to-sim transfer results following standard benchmarks (Appendix E), and visualizations of transferred value functions (Appendix D).

## 7 LIMITATIONS AND FUTURE WORK

In this work, we present *SGFT*, a technique for efficient sim-to-real fine-tuning using off-policy RL. The key idea in *SGFT* is to leverage learned value functions and models from simulation to provide guidance for exploration even when simulation does not perfectly match reality through a combination of short-horizon model hallucinations and potential-based reward shaping. There are several limitations of *SGFT* that open avenues for improvement. Firstly, scaling *SGFT* to work from raw perceptual inputs such as camera images rather than low-dimensional states would make this paradigm more broadly applicable. Secondly, it is important to scale *SGFT* to higher-dimensional action spaces and longer-horizon tasks. Thirdly, our choice of off-policy RL method can display a degree of instability and a more efficient and stable base algorithm should be considered.

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## A PROOFS

We first present several Lemma's used in the proof of 1.

**Lemma 1.** (Bhardwaj et al., 2020, Lemma A.1.) Suppose that  $\|\hat{p}(s, a) - p_{real}(s, a)\|_1 \leq \alpha$ . Further suppose  $\Delta r = \max_{sim} r(s) - \min_{sim} r(s)$  and  $\Delta V = \max_{sim} V_{sim}(s) - \min_{sim} V_{sim}(s)$  are finite. Then, for each policy  $\tilde{\pi}$  we may bound the  $H$ -step returns under the model and true dynamics by:

$$\|\hat{V}_H^{\tilde{\pi}}(s) - V_H^{\tilde{\pi}}(s)\|_\infty \leq \gamma \left( \frac{1 - \gamma^{H-1}}{1 - \gamma} \frac{\Delta r}{2} + \gamma^H \frac{\Delta V}{2} \right) \cdot \alpha H. \quad (9)$$

*Proof.* This result follows imediatly from the proof of (Bhardwaj et al., 2020, Lemma A.1.), with changes to notation and noting that we assume access to the true reward.  $\square$

**Lemma 2.** Suppose that  $\|\hat{p}(s, a) - p_{real}(s, a)\|_1 \leq \alpha$ . Further suppose  $\Delta r = \max_{sim} r(s) - \min_{sim} r(s)$  and  $\Delta V = \max_{sim} V_{sim}(s) - \min_{sim} V_{sim}(s)$  are finite. Then for each state  $s \in \mathcal{S}$  we have:

$$V_H^{\hat{\pi}_H^*}(s) - V_H^*(s) \leq \left( \frac{1 - \gamma^{H-1}}{1 - \gamma} \Delta r + \gamma^H \Delta V \right) \quad (10)$$

where  $\hat{\pi}_H^* \leftarrow \max_{\tilde{\pi}_H} \hat{V}_H^{\tilde{\pi}_H}(s)$ .

*Proof.* Let  $\tilde{\pi}_H^* \leftarrow \max_{\tilde{\pi}_H} V_H^{\tilde{\pi}_H}(s)$  be the optimal policy under the true dynamics. By Lemma 1 we have both that

$$V_H^*(s) \leq \hat{V}_H^{\tilde{\pi}_H^*}(s) + \gamma \left( \frac{1 - \gamma^{H-1}}{1 - \gamma} \frac{\Delta r}{2} + \gamma^H \frac{\Delta V}{2} \right) \cdot \alpha H. \quad (11)$$

$$\hat{V}_H^{\hat{\pi}_H^*}(s) \leq V_H^{\hat{\pi}_H^*}(s) + \gamma \left( \frac{1 - \gamma^{H-1}}{1 - \gamma} \frac{\Delta r}{2} + \gamma^H \frac{\Delta V}{2} \right) \cdot \alpha H. \quad (12)$$

Combining these two bounds with the fact that  $\hat{V}_H^{\hat{\pi}_H^*}(s) \leq V_H^{\hat{\pi}_H^*}(s)$  yields the desired result.  $\square$

**Lemma 3.** Suppose that  $\sup_a \mathbb{E}_{s \sim p_{real}(s, a)} [\gamma V_{sim}(s')] - V_{sim}(s) > -r(s)$ . Then we have  $V_H^*(s) \geq V_{H-1}^*(s)$  for each  $s \in \mathcal{S}$ . Then for each  $s \in \mathcal{S}$  we have:

$$V_H^*(s) \geq V_{H-1}^*(s) \quad (13)$$

*Proof.* Fix an initial condition  $s_0 \in \mathcal{S}$ . Let  $\pi$  be arbitrary, and fix the shorthand  $\pi^* = \{\pi_0^*, \dots, \pi_{H-1}^*\}$  for the time-varying policy  $\pi^* \leftarrow \max_{\tilde{\pi}} V_H^{\tilde{\pi}}(s_0)$ . Then, concatenate these policies to define:  $\tilde{\pi} = \{\pi_1^*, \dots, \pi_{H-2}^*, \pi\}$ , which is simply the result of applying the optimal policy for the  $(H-1)$ -step look ahead objective Equation (3) starting from  $s_0$ , followed by applying  $\pi$  for a single step. Letting the following distributions over trajectories be generated by  $\pi^*$ , by the definition of  $V_H^*$ :

$$\begin{aligned} & V_H^*(s_0) \\ & \geq \mathbb{E} \left[ \gamma^H V_{sim}(s_H) + \sum_{t=1}^{H-1} \gamma^t r(s_t) - V_{sim}(s_0) \right] \\ & = \mathbb{E} [\gamma^H V_{sim}(s_H) - \gamma^{H-1} V_{sim}(s_{H-1}) + \gamma^H r(s_{H-1})] \\ & \quad + \mathbb{E} \left[ \gamma^{H-1} V_{sim}(s_{H-1}) + \sum_{t=1}^{H-2} \gamma^t r(s_t) - V_{sim}(s_0) \right] \\ & = \mathbb{E} [\gamma^H V_{sim}(s_H) - \gamma^{H-1} V_{sim}(s_{H-1}) + \gamma^H r(s_{H-1}) + V_{H-1}^*(s_0)] \end{aligned}$$

Now, since our choice of  $\pi$  used to define  $\tilde{\pi}$  was arbitrary, we choose  $\pi$  to be deterministic and such that  $\mathbb{E}_{s' \sim p_{real}(s, a)} [\gamma V_{sim}(s')] - V_{sim}(s) > -r(s)$  at each state  $s \in \mathcal{S}$ , as guaranteed by the assumption made for the result. This choice of policy grants that:

$$\mathbb{E} [\gamma^H V_{sim}(s_H) - \gamma^{H-1} V_{sim}(s_{H-1}) + \gamma^H r(s_{H-1})] \geq 0. \quad (14)$$

The desired result follows immediately by combining the two preceding bounds, and noting that our choice of initial condition was arbitrary, meaning the preceding analysis holds for all initial conditions.  $\square$



**Lemma 4.** Suppose that  $V_{sim}$  is improvable and further suppose that  $\max_{s \in \mathcal{S}} |V_{sim}(s) - V_{real}^*(s)| < \epsilon$ . Then any policy  $\pi$  which satisfies  $A_H^*(s, \pi) = Q_H^*(s, \pi) - V_H^*(s) \geq -\delta$  will satisfy:

$$V_{real}^*(s) - V_{real}^\pi(s) \leq \gamma^H \epsilon + \frac{\delta}{1 - \gamma}. \quad (15)$$

*Proof.* Our goal is first to bound how  $Q_H^*(s, \pi)$  changes on expectation when applying the given policy for a single step. We have that:

$$Q_H^*(s, \pi) + \delta \geq V_H^*(s) \quad (16)$$

$$V_H^*(s) \geq V_{H-1}^*(s) \quad (17)$$

$$Q_H^*(s, \pi) = \mathbb{E}[\gamma V_{H-1}^*(s') + \bar{r}(s, s')] \quad (18)$$

where the first inequality follows from the Assumption of the theorem, the second inequality follows from Lemma 3 and is simply the definition of  $Q_H^*$ . Letting  $s' \sim p_{real}(s, a)$  with  $a \sim \pi(\cdot|s)$ , we can take expectations can combine the previous relation to obtain:

$$\gamma \mathbb{E}[Q_H^*(s', \pi) + \bar{r}(s, s')] + \gamma \delta \geq \gamma \mathbb{E}[V_H^*(s') + \bar{r}(s, s')] + \geq \mathbb{E}[\gamma V_{H-1}^*(s') + \bar{r}(s, s')] = Q_H^*(s, \pi). \quad (19)$$

That is:

$$\gamma \mathbb{E}[Q_H^*(s', \pi)] + \bar{r}(s) + \gamma \delta \geq Q_H^*(s, \pi). \quad (20)$$

Alternatively:

$$\bar{r}(s) \geq Q_H^*(s, \pi) - \gamma \mathbb{E}[Q_H^*(s', \pi)] - \gamma \delta. \quad (21)$$

Next, we use this bound to provide a lower bound for  $V_{real}^\pi(s)$ . Because the previous analysis holds at all states when we apply  $\pi$ , the following holds over the distribution of trajectories generated by applying  $\pi$  starting from the initial condition  $s_0$ :

$$\begin{aligned} \mathbb{E}_{\rho_{real}^\pi(s)} \left[ \sum_{t=0}^{\infty} \gamma^t \bar{r}(s_t) \right] &= V_{real}^\pi(s) - V_s(s_0) \\ &\geq \mathbb{E}_{\rho_{real}^\pi(s)} \left[ \sum_{t=0}^{\infty} \gamma^t \left( Q_H^*(s_t, \pi) - \gamma Q_H^*(s_{t+1}, \pi) \right) \right] - \gamma \delta \sum_{t=0}^{\infty} \gamma^t \\ &= Q_H^*(s_0, \pi) - \frac{\gamma \delta}{1 - \gamma}, \end{aligned}$$

where we have repeatedly telescoped out sums to cancel out terms.

Thus, we have the lower-bound:

$$V_{real}^\pi(s) \geq Q_H^*(s, \pi) + V_{sim}(s_0) - \frac{\gamma \delta}{1 - \gamma} \quad (22)$$

Next, we may bound:

$$\begin{aligned} V_H^*(s_0) + V_{sim}(s_0) &\geq \mathbb{E}_{\rho_{real}^{\pi^*}(s_0)} \left[ \gamma^H V_{sim}(s_H) + \sum_{t=0}^{H-1} \gamma^t r(s_t) \right] \\ &= \mathbb{E}_{\rho_{real}^{\pi^*}(s_0)} \left[ \gamma^H V_{sim}(s_H) - \gamma^H V_{real}^*(s_H) + \gamma^H V_{real}^*(s_H) + \sum_{t=0}^{H-1} \gamma^t r(s_t) \right] \\ &= \mathbb{E}_{\rho_{real}^{\pi^*}(s_0)} [\gamma^H V_{sim}(s_H) - \gamma^H V_{real}^*(s_H)] + V_{real}^*(s_0). \end{aligned} \quad (23)$$

Invoking the assumption that  $\max_s |V_{sim}(s) - V_{real}^*(s)| < \epsilon$ , we can combined this with the preceding bound to yield:

$$V_H^*(s_0) + V_{sim}(s_0) \geq V_{real}^*(s) - \gamma^H \epsilon. \quad (24)$$

Finally, once more invoking the fact that  $Q_H^*(s, \pi) + \delta \geq V_H^*(s)$  for each  $s \in S$  and combining this with Equation (22) and Equation (23), we obtain that:

$$\begin{aligned} V_{real}^\pi(s) &\geq Q_H^*(s, \pi) + V_{sim}(s_0) - \frac{\gamma\delta}{1-\gamma} \\ &\geq V_H^*(s_0) + V_{sim}(s_0) - \frac{\gamma\delta}{1-\gamma} - \delta \\ &\geq V_{real}^*(s) - \frac{\gamma\delta}{1-\gamma} - \delta - \gamma^H \epsilon \\ &= V_{real}^*(s) - \frac{\delta}{1-\gamma} - \gamma^H \epsilon \end{aligned}$$

from which the state result follows immediately.  $\square$

### Proof of Theorem 1:

*Proof.* The result follows directly from a combination of Lemma 4 and Lemma 2 by suppressing problem-dependent constants and lower order terms in the discount factor  $\gamma$ .  $\square$

## B ENVIRONMENT DETAILS

**Sim2Real Environment.** We use a 7-DoF Franka FR3 robot with a 1-DoF parallel-jaw gripper. Two calibrated Intel Realsense D455 cameras are mounted across from the robot to capture position of the object by color-thresholding pointcloud readings or retrieving pose estimation from aruco tags. Commands are sent to the controller at 5Hz. We restrict the end-effector workspace of the robot in a rectangle for safety so the robot arm doesn’t collide dangerously with the table and objects outside the workspace. We conduct extensive domain randomization and randomize the initial gripper pose during simulation training. The reward is computed from measured proprioception of the robot and estimated pose of the object. Details for each task are listed below.

**Hammering.** For hammering, the action is 3-dimensional and sets delta joint targets for 3 joints of the robot using joint position control. The observation space is 12-dimensional and includes end-effector cartesian xyz, joint angles of the 3 movable joints, joint velocities of the 3 movable joints, the z position of the nail, and the xz position of the goal. Each trajectory is 50 timesteps. In simulation, we randomize over the position, damping, height, radius, mass, and thickness of the nail. Details are listed in Tab. 1.

The reward function is parameterized as  $r(t) = -10 \cdot r_{\text{nail-goal}}(t)$  where  $r_{\text{nail-goal}} = (\mathbf{r}_{\text{nail}})_z - (\mathbf{r}_{\text{goal}})_z$  represents the distance in the z dimension of the nail head to the goal, which we set to be the height of the board the nail is on.

**Puck Pushing.** For puck pushing, the action is 2-dimensional and sets delta cartesian xy position targets using end-effector position control. The observation space is 4-dimensional and includes end-effector cartesian xy and the xy position of the puck object. Each trajectory is 40 timesteps. In simulation, we randomize over the position of the puck. Details are listed in Tab. 3.

Let  $\mathbf{r}_{\text{ee}}$  be the cartesian position of the end effector and  $\mathbf{r}_{\text{obj}}$  be the cartesian position of the puck object. The reward function is parameterized as  $r(t) = -r_{\text{ee-goal}}(t) - r_{\text{obj-goal}}(t) + r_{\text{threshold}}(t) - r_{\text{table}}(t)$  where  $r_{\text{ee-goal}}(t) = \|\mathbf{r}_{\text{ee}}(t) - \mathbf{r}_{\text{obj}}(t) + [3.5\text{cm}, 0.0\text{cm}, 0.0\text{cm}]\|$  represents the distance of the end effector to the back of the puck,  $r_{\text{obj-goal}}(t) = \|(\mathbf{r}_{\text{obj}}(t))_x - 55\text{cm}\|$  represents the distance of the puck to the goal (which is the edge of the table along the x dimension),  $r_{\text{threshold}}(t) = \mathbb{I}[r_{\text{obj-goal}}(t) \geq 2.5\text{cm}]$  represents a goal reaching binary signal, and  $r_{\text{table}}(t) = \mathbb{I}[(r_{\text{obj}}(t))_z \leq 0.0]$  represents a binary signal for when the object falls off the table.

**Inserting.** For inserting, the action is 3-dimensional and sets delta cartesian xyz position targets using end effector position control. The observation space is 9-dimensional and includes end-effector cartesian xyz, the xyz of the leg, and the xyz of the table hole. Each trajectory is 40 timesteps. In simulation, we randomize the initial gripper position, position of the table, and friction of both the table and the leg.

Let  $\mathbf{r}_{\text{pos1}}(t)$  and  $\mathbf{r}_{\text{pos2}}(t)$  represent the Cartesian positions of the leg and table hole. Let:

$$x_{\text{distance}}(t) = \text{clip}(|\mathbf{r}_{\text{pos1},x}(t) - \mathbf{r}_{\text{pos2},x}(t)|, 0.0, 0.1)$$

$$y_{\text{distance}}(t) = \text{clip}(|\mathbf{r}_{\text{pos1},z}(t) - \mathbf{r}_{\text{pos2},z}(t)|, 0.0, 0.1)$$

$$z_{\text{distance}}(t) = \text{clip}(|\mathbf{r}_{\text{pos1},y}(t) - \mathbf{r}_{\text{pos2},y}(t)|, 0.0, 0.1)$$

Let the success condition be defined as:

$$r_{\text{success}}(t) = \mathbb{I}[x_{\text{distance}}(t) < 0.01 \text{ and } y_{\text{distance}}(t) < 0.01 \text{ and } z_{\text{distance}}(t) < 0.01]$$

The reward function is now:

$$r(t) = r_{\text{success}}(t) - 100 * (x_{\text{distance}}(t)^2 + y_{\text{distance}}(t)^2 + z_{\text{distance}}(t)^2)$$

**Sim2Sim Environment.** We additionally attempt to model a sim2real dynamics gap in simulation by taking the hammering environment and create a proxy for the real environment by fixing the domain randomization parameters, fixing the initial gripper pose, and rescaling the action magnitudes before rolling out in the environment.

## C IMPLEMENTATION DETAILS

**Algorithm Details.** We use SAC as our base off-policy RL algorithm for training in simulation and fine-tuning in the real world. For our method, we additionally add in two networks: a dynamics model that predicts next state given current state and action, and a state-conditioned value network which regresses towards the Q-value estimates for actions taken by the current policy. These networks are training jointly with the actor and critic during SAC training in simulation.

**Network Architectures.** The Q-network, value network, and dynamics model are all parameterized by a two-layer MLP of size 512. The dynamics model is implemented as a delta dynamics model where model predictions are added to the input state to generate next states. The policy network produces the mean  $\mu_a$  and a state-dependent log standard deviation  $\log \sigma_a$  which is jointly learned from the action distribution. The policy network is parameterized by a two-layer MLP of size 512, with a mean head and log standard deviation head on top parameterized by a FC layer.

**Pretraining in Simulation.** For hammering and puck pushing, we collect 25,000,000 transitions of random actions and pre-compute the mean and standard deviation of each observation across this dataset. We train SAC in simulation on the desired task by sampling 50-50 from the random action dataset and the replay buffer. We normalize our observations by the pre-computed mean and standard deviation before passing them into the networks. We additionally add Gaussian noise centered at 0 with standard deviation 0.004 to our observations with 30% probability during training. For inserting, we train SAC in simulation with no normalization. We train SAC with autotuned temperature set initially to 1 and a UTD of 1. We use Adam optimizer with a learning rate of  $3 \times 10^{-4}$ , batch size of 256, and discount factor  $\gamma = .99$ .

**Fine-tuning in Real World.** We pre-collect 20 real-world trajectories with the policy learned in simulation to fill the empty replay buffer. We then reset the critic with random weights and continue training SAC with a fixed temperature of  $\alpha = 0.01$  and with a UTD of  $2d$  with the pretrained actor and dynamics model. We freeze the value network learned from simulation and use it to relabel PBRS rewards during fine-tuning. During fine-tuning, for each state sampled from the replay buffer, we additionally hallucinate 5 branches off and add it to the training batch. As a result, our batch size effectively becomes 1536. The policy, Q-network, and dynamics model are all trained jointly on the real data during SAC fine-tuning. We don't train on any simulation data during real-world fine-tuning because we empirically found it didn't help fine-tuning performance in our settings.

Table 1: Domain randomization of hammering task in simulation

Name	Range
Nail x position (m)	[0.3, 0.4]
Nail z position (m)	[0.55, 0.65]
Nail damping	[250.0, 2500.0]
Nail half height (m)	[0.02, 0.06]
Nail radius (m)	[0.005, 0.015]
Nail head radius (m)	[0.03, 0.04]
Nail head thickness (m)	[0.001, 0.01]
Hammer mass (kg)	[0.015, 0.15]

Table 2: Domain randomization of puck pushing task in simulation

Name	Range
Puck x position (m)	[0.0, 0.3]
Puck y position (m)	[-0.25, 0.25]

Table 3: Domain randomization of inserting task in simulation

Name	Range
Parts x/y position (m)	[-0.05, 0.05]
Parts rotation (degrees)	[0, 15]
Parts friction	[-0.01, 0.01]

## D QUALITATIVE RESULTS

We analyze the characteristics of hallucinated states and value functions in Fig. 5. We visualize a trajectory of executing puck pushing in simulation using the learned policy in this plot. The red dots indicate states along a real rollout in simulation. The blue dots indicate hallucinated states branching off real states along a real rollout in simulation. The blue dots indicate hallucinated states generated by the learned dynamics model. The green heatmap indicates the value function estimates at different states. A corresponding image of the state is shown for two states. The trajectory shown in the figure shows the learned policy moving closer to the puck before pushing it. The value function heatmap shows higher values when the end effector is closer to the puck and lower values when further. Hallucinated states branching off each state show generated states for fine-tuning the learned policy.

Note that it is hard to directly visualize states and values due to the high-dimensionality of the state space. To get around this for puck pushing, we only show a part of the trajectory where the puck does not move. This allows us to visualize states and values along changes in only end effector xy.

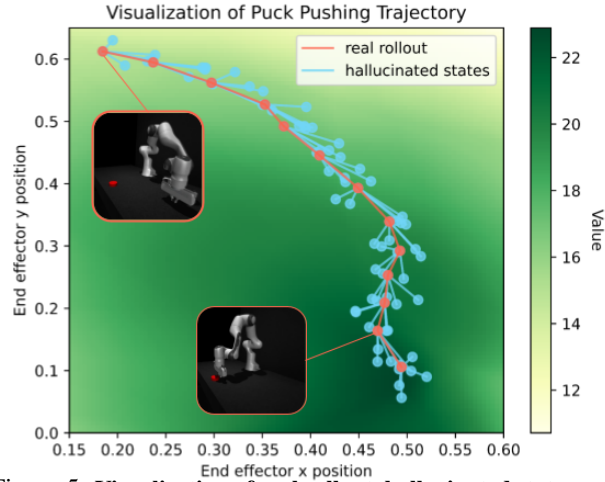


Figure 5: **Visualization of real rollout, hallucinated states, and value function.** The red dots indicate states along a real rollout in simulation. The blue dots indicate hallucinated states branching off real states generated by the learned dynamics model. The green heatmap indicates the value function estimates at different states. A corresponding image of the state is shown for two states. Since it is hard to directly visualize states and values due to the high-dimensionality of the state space, we only show a part of the trajectory where the puck does not move. This allows us to visualize states and values along changes in only end effector xy.

## E SIM-TO-SIM EXPERIMENTS

Here we additionally test each of the proposed methods on the sim-to-sim set-up from Du et al. (2021), which is meant to mock sim-to-real gaps but for familiar RL benchmark tasks. The results are depicted in Figure 6 for the Walker Walk and Cheetah run environments. For both tasks, we use the precise settings from Du et al. (2021). Note that the general trend of these results matches our real world experiments – *SGFT* substantially accelerates learning and overcoming the dynamics gap between the ‘simulation’ and ‘real environments’.

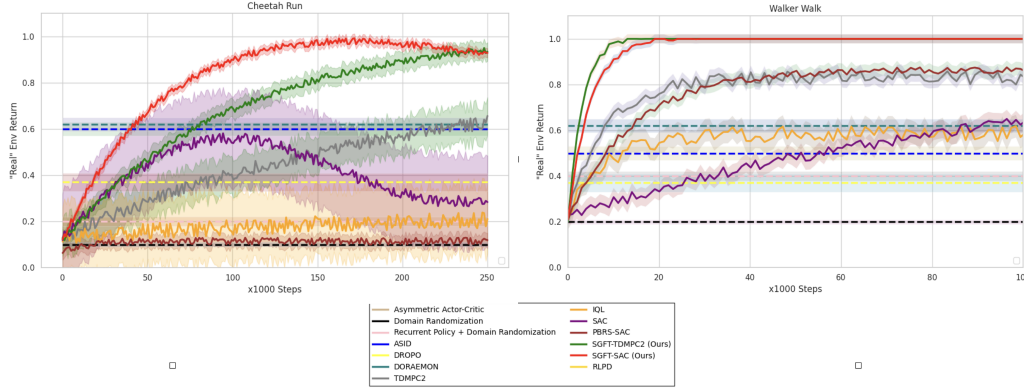


Figure 6: **Normalized Rewards for Sim-to-Sim Transfer.** We plot the normalized rewards for two sim-to-sim transfer tasks, where the rewards are normalized by the maximum reward achieved by any method.

## F PEDIGOGICAL EXAMPLE:

We use the following pedagogical example to begin building an intuition for why we might expect  $V_{sim}$  to also be improvable with respect to  $\mathcal{M}_{real}$ .

**Pedagogical Example.** Consider the following case where the real and simulated dynamics are both deterministic, namely,  $s' = f_{real}(s, a)$  and  $s' = f_{sim}(s, a)$  for some real and simulated transition maps  $f_{real}$  and  $f_{sim}$ . Further, assume for simplicity that  $\pi_{sim}$  is deterministic. Specifically, consider the case where  $s = (s_1, s_2) \in \mathcal{S} \subset \mathbb{R}^2$ ,  $a \in \mathcal{A} = \mathbb{R}$ , and the dynamics are given by:

$$f_{sim}(s, a) = \begin{bmatrix} s'_1 \\ s'_2 \end{bmatrix} = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \Delta t \begin{bmatrix} x_2 \\ \frac{g}{l} \sin(x_1) + a \end{bmatrix}$$

$$f_{real}(s, a) = \begin{bmatrix} s'_1 \\ s'_2 \end{bmatrix} = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \Delta t \begin{bmatrix} x_2 \\ \frac{g}{l} \sin(x_1) + a + e(s_1, s_2) \end{bmatrix}$$

These are the equations of motion for a simple pendulum (Wang et al., 2022) under an Euler discretization with time step  $\Delta t$ , where  $s_1$  is the angle of the arm,  $s_2$  is the angular velocity,  $a$  is the torque applied by the motor,  $g$  is the gravitational constant, and  $l$  is the length of the arm. The real-world dynamics contains unmodeled terms  $e(s_1, s_2)$ , which might correspond to complex frictional or damping effects. Consider the policy for the real world given by  $\pi_{real}(s) = \pi_{sim}(s) - e(s_1, s_2)$  and observe that  $f_{sim}(s, \pi_{sim}(s)) = f_{real}(s, \pi_{real}(s))$ . This implies that  $\gamma V_{sim}(f_{real}(s, \pi_{real}(s))) - V_{sim}(s) = \gamma V_{sim}(f_{sim}(s, \pi_{sim}(s))) - V_{sim}(s) = -r(s)$ , and thus  $V_{sim}$  is improvable with respect to  $\mathcal{M}_{real}$ , because it is improvable with respect to  $\mathcal{M}_{sim}$  by definition. Note that  $\pi_{sim}$  and  $\pi_{real}$  can differ substantially for a large gap  $e(s_1, s_2)$ .

**Main Insight.** To make this property precise, we observe the following fact: if we assume that for any state  $s$ , there is some  $a$  such that  $p_{real}(\cdot|s, a) = p_{sim}(\cdot|s, \pi_{sim}(s))$ , then  $V_{sim}$  is improvable with respect to  $\mathcal{M}_{real}$ . More generally, for many continuous control tasks, it is reasonable to expect that  $p_{sim}$  approximately captures the geometry of what motions are possible under  $p_{real}$ , even if the actions required to realize those motions in the two MDPs differ substantially, and thus it is

1134 reasonable to assume  $V_{sim}$  is improvable. This intuition is highlighted by our real-world learning  
1135 examples in cases where we use *SGFT* with a prediction horizon of  $H = 1$ . In these cases the  
1136 learned policy is able to greedily follow  $V_{sim}$  at each state and reach the goal, even in the face of  
1137 large dynamics gaps.  
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