Adaptive Fuzzy Finite-Time Prescribed Performance Tracking Control for Strict-Feedback Nonlinear Systems with Unknown Control Directions and Input Dead-Zone

Zongsheng Huang
School of Automation Engineering
University of Electronic Science and Technology of China
Chengdu 611731, China
zs__Huang@163.com

***Abstract*—The adaptive fuzzy finite-time prescribed performance tracking control problem is stressed for a class of uncertain strict-feedback nonlinear systems with unknown control directions and input dead-zone in this paper. The fuzzy logic systems are introduced to approximate the unknown nonlinearities. Firstly, by adopting a characteristic function, the input dead-zone is converted as a linear model with bounded perturbation. Then, an adaptive fuzzy controller based on command filter is built to settle the "computation complexity" issue. Meanwhile, the Nussbaum function is implemented in controller design to counter the hardship of unknown control directions. Besides, the tracking error can be restricted in the prescribed boundary in finite time with the designed performance function. The presented control scheme can not only release computation burden, but also guarantee the finite-time convergence property of tracking error and the boundedness of all signals in the closed-loop system. Finally, the simulation examples illustrate the validity of the designed control scheme.**

***Index Terms*—Fuzzy control, finite-time performance function, command filter, unknown control directions, input dead-zone, strict-feedback nonlinear systems.**

I. Introduction

As a matter of fact, strict-feedback nonlinear form can be adopted to characterize many classical physical systems, such as the hypersonic flight vehicle [1], spacecrafts [2], robot manipulators [3] and so on. Thus, strict-feedback nonlinear systems have received extensive attention and lots of remarkable control approaches have been discussed. In particular, adaptive backstepping method has been regarded as an effective approach in the control field of strict-feedback nonlinear systems with uncertainties. It is noteworthy that there is a widespread class of uncertainties named unstructured uncertainties, which cannot be modeled or repeated and bring a great difficulty for controller design. To handle this challenge, adaptive control approaches with approximators were explored to handle unstructured uncertainties using neural network (NN) or fuzzy logic systems (FLSs) approximators [4]. However, these control schemes mentioned above suffered from a major limitation of "computation complexity" problem.

The "computation complexity" issue is occurred by the repeated derivation of the virtual controllers in every step within the conventional backstepping method. To cope with this weakness, the dynamic surface control (DSC) technique was firstly constructed in [5], in which the filtering variable was generated via a first-order filter and the "computation complexity" can be eliminated. The DSC technique was expanded to adaptive control area for strict-feedback nonlinear systems with parametric uncertainties in [6]. Later, the adaptive control on the basis of NN for the investigated systems with arbitrary uncertainty was proposed in [7]. Thereafter, in [8], combined with "minimal learning parameter" technique, the adaptive NN DSC control method proposed in [7] was improved to settle the " explosion of learning parameters" issue caused by introducing NN as approximator for the investigated systems. Recently, the DSC-based adaptive NN control scheme in [7] was further extended for pure-feedback stochastic nonlinear systems [9], nonstrict-feedback stochastic nonlinear systems [10] and so on. However, the DSC method leads to the filtering errors, which may affect the control quality [11]. As an alternative, a modified backstepping technique named command filtering control (CFC) was firstly proposed in [12], in which the command filters were incorporated in design procedure, and error compensation signal was provided to handle the filtering errors. Further, the CFC method was expanded to adaptive control filed in [13] and numerous valuable achievements have been accomplished. For instance, a command filter-based fuzzy controller for multi-input multi-output (MIMO)-switched nonstrict-feedback nonlinear systems was designed in [14]. In [15], the finite-time adaptive controller on the basis of CFC was built for fractional-order nonlinear systems.

Although adaptive intelligent control algorithms mentioned above for the investigated systems with uncertainties have achieved much success, there are still many nonsmooth nonlinear factors should come into notice in lots of industrial processes, such as input dead-zone nonlinearity. Input dead-zone exists universally in practical engineering, which causes poor performance and instability of the considered system and draws significant attentions in control community for long time. To handle input dead-zone, Tao and Kokotovic firstly developed the adaptive dead-zone inverse function [16]. Then, the adaptive dead-zone inverse function was applied for discrete-time linear systems in [17]. This method was further continued in [18], [19] and so on. Different from the dead-zone inverse approach that aimed at minimizing the effect of dead-zone, in [20], a linear function with bounded perturbation was proposed to model the input dead-zone, which intuitively demonstrated the property of input dead-zone by mathematical model. Subsequently, the method designed in [20] was employed for input dead-zone of MIMO nonlinear systems in [21].

It is recognized that a conventional assumption of most control schemes previously discussed is that the sign of control coefficient is determined, that is, the control direction is known. However, there is no prior knowledge of control directions in practice, which leads it more complex for controller design. Hence, the Nussbaum gain function was firstly put forward in [22] for the matter of unknown control directions for the first-order linear systems. Later, in [23] and [24], the Nussbaum function was implemented in backstepping design procedure to settle the adaptive tracking control problem for the considered systems with unknown control directions. The Nussbaum function was expanded for MIMO nonlinear systems in [25]. Currently, many results concerned with unknown control directions were presented. In [26], an adaptive fuzzy controller was obtained for pure-feedback nonlinear systems with unknown control directions. An innovative Nussbaum function was built for fractional-order interconnected systems to solve the hardship that brought by the multiple unknown control directions in [27]. Lv *et al.* designed a logic-based online estimation of the control directions to settle the matter of multiple unknown control directions [28].

Some of the aforementioned control schemes can allow that the error converges to a small residual set with unknown size. In other words, control accuracy cannot be specified a priori [29]. Nevertheless, in engineering fields, the designed control scheme often requires to meet the prescribed transient and steady-state performance. As an alternative, the prescribed performance (PP) technique was developed in [30] for the first time, which was an effective method to change the steady state and transient state tracking error performance into the performance constraints. With an error transformation, an expected tracking performance can be obtained accordingly. Subsequently, Bechlioulis and Rovithakis extended PP to MIMO strict-feedback nonlinear systems in [31]. Combined with PP, many researches have been carried out, including fuzzy control [32], neural control [33], sliding mode control [34] and so on. From the aspect of engineering, finite-time convergence is one of the most significant indexes of control performance. Thus, an improved PP technique named finite-time PP technique was recently reported in [35] for the sake of ensuring tracking error converges to a boundary within a given convergence time.

Inspired by these works, in this article, the command filter-based adaptive fuzzy finite-time prescribed performance tracking control scheme for the considered systems with unknown control directions and input dead-zone is designed. The main contributions of this paper are summarized as follows:

(1) Distinct from [9], [10] that did not take the filtering errors into account, which may affect the control quality. The CFC-based adaptive fuzzy control scheme is developed related to less computation complexity, and the presented controller is simple to realize in practice applications in this article.

(2) In this paper, the issue of unknown control directions and the phenomenon of input dead-zone are considered simultaneously from the perspective of engineering. On the basis of Nussbaum functions, the limitation that the control directions are known in advance is relaxed. Furthermore, the input dead-zone is converted as a linear model with bounded perturbation. The designed control scheme is more practical.

(3) Compared with [25] that did not employ PP technique, in this paper, by implementing a finite-time performance function in the controller design procedure, the steady-state and transient-state tracking performance can be satisfied. In addition, the tracking error can converge to a predefined boundary in finite-time, which represents the finite-time convergence index can be ensured. Thus, the control performance has been improved.

Notations: Throughout this article, for any value or function $\mathfrak{F}$, $\hat{\mathfrak{F}}$ denotes the estimation value. $\varepsilon_{fi}(\cdot), i = 1, \cdots, n$ is the fuzzy approximation error.

II. Problem Formulation and Preliminaries

A. System Model

Give the model as

$$\begin{cases} \dot{x}_i = \psi_i g_i(\bar{x}_i) x_{i+1} + f_i(\bar{x}_i), i = 1, 2, \cdots, n-1 \\ \dot{x}_n = \psi_n g_n(\bar{x}_n) u + f_n(\bar{x}_n) \\ y = x_1 \end{cases} \tag{1}$$

where $\bar{x}_i = [x_1, x_2, \dots, x_i]^T \in \mathbb{R}^i$, $i = 1, \dots, n$ denotes the state vector of the system, $y \in \mathbb{R}$ is the output of (1). $f_i(\cdot) \in \mathbb{R}$, $i = 1, \dots, n$ denotes the unknown smooth nonlinear function. $\psi = 1$ or $\psi = -1$ denotes the unknown control direction; $g_i(\cdot)(g_i(\cdot) \neq 0)$, $i = 1, \dots, n$ is known smooth nonlinear function and $\psi_i g_i(\bar{x}_i)$ is the control coefficient. According to [20], $u \in \mathbb{R}$ is both the output of dead-zone and the control input of system and can be displayed as

$$u = D(v) = \begin{cases} M_r(v - a_r), & v \geq a_r \\ 0, & a_l < v < a_r \\ M_l(v - a_l), & v \leq a_l \end{cases} \tag{2}$$

where $v \in \mathbb{R}$ is the input of dead-zone. M_r and M_l are the slopes, a_r and a_l are the breakpoints. M_r, M_l, a_r, a_l are unknown parameters with known signs, $M_r > 0, M_l > 0, a_r > 0, a_l < 0$. Also, M_r, M_l, a_r, a_l are bounded, $M_r \in [\underline{M}_r, \overline{M}_r]$, $M_l \in [\underline{M}_l, \overline{M}_l]$, $a_r \in [\underline{a}_r, \overline{a}_r]$ and $a_l \in [\underline{a}_l, \overline{a}_l]$ and $\underline{M}_r, \overline{M}_r, \underline{M}_l, \overline{M}_l, \underline{a}_r, \overline{a}_r, \underline{a}_l, \overline{a}_l$ are known parameters.

Then, (2) is simplified below

$$u = D(v) = Mv + h(v)$$

where

$$M = \begin{cases} M_r & v > 0 \\ M_l, & v \le 0 \end{cases}$$

$$h(v) = \begin{cases} -M_r a_r, & v \ge a_r \\ -Mv, & a_l < v < a_r \\ -M_l a_l, & v \le a_l \end{cases}$$

where $h(v) \le \bar{h}$, and $\bar{h} = \max\{\overline{M}_r \bar{a}_r, -\overline{M}_l \underline{a}_l\}$, where $\underline{a}_l < 0$. Define $M_0 = \min\{M_r, M_l\}, M_1 = \max\{M_r, M_l\}$.

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