Learning Compact Regular Decision Processes using Priors and Cascades

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Abstract

In this work we study offline Reinforcement Learning (RL), and extend the previous work on learning Regular Decision Processes (RDPs), which are a class of non-Markovian environments, where the unknown dependency of future observations and rewards from the past interactions can be captured by some hidden finite-state automaton. We utilise the language metric introduced by Deb et al. [2025], and introduce a novel algorithm to learn a significantly more compact RDP with cycles, which are crucial for scaling to larger, more complex environments. Key to our results is a novel notion of *priors* for automaton learning, that allows us to exploit prior domain-related knowledge, used to factor out of the state space any feature that is known a priori. We validate our approach experimentally and provide a Probably Approximately Correct (PAC) analysis of our algorithm, showing it enjoys a sample complexity polynomial in the problem parameters.

1 Introduction

Reinforcement Learning (RL) is a family of powerful algorithms for learning behaviour from repeated interactions with a stochastic dynamical system. A key assumption that underlies the majority of existing RL algorithms is the Markov property, which implies that the current observation of the environment, together with the agent's current action, is sufficient to predict the future evolution of the system [Puterman, 1994, Sutton et al., 1998]. Although it offers a great mathematical appeal that proves instrumental in many RL developments, there exist many interesting application scenarios –e.g., in robotics– where the Markov property does not hold. In such *non-Markov* cases, RL becomes considerably more difficult. A classical approach to deal with such cases has been to consider a hidden state [Whitehead and Lin, 1995], to account for missing piece of information from the agent's current observation. This is most notably studied in the context of Partially Observable Markov Decision Processes, or POMDPs [Kaelbling et al., 1998]. Although the POMDP framework offers very expressive representations and is of great relevance in practice, it suffers from intractability in both planning and learning, and consequently the corresponding learning algorithms become impractical in large problems, unless some restrictive assumptions are imposed. An alternative recently proposed framework is Regular Decision Process, or RDP [Brafman and De Giacomo, 2019, 2024], wherein the past interaction history is compactly represented by a finite state automaton. In essence, an RDP is a special POMDP, whose hidden dynamics evolve according to some (unobservable) finite-state automaton featuring a controlled form of stochasticity, ensuring key favourable properties. Although RDPs by construction are less generic compared to POMDPs, they are computationally and statistically tractable. This has led to a growing interest in developing RDP learning algorithms from trajectories [Abadi and Brafman, 2020, Ronca and De Giacomo, 2021, Ronca et al., 2022, Cipollone et al., 2023, Deb et al., 2025].

We investigate offline RL in episodic RDPs, where the goal is to find a near-optimal policy from a dataset pre-collected using a behaviour policy. This problem was first studied by Cipollone et al.

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[2023], where a first algorithm with provable PAC-type performance guarantee in terms of sample complexity was proposed. Despite its appeal, this bound may imply a sample complexity growing exponentially in episode length in some problem instance. This was remedied in [Deb et al., 2025] through statistical tests defined via a novel metric called the *language metric*, specifically designed for traces, borrowing ideas from the theory of formal languages. However, the models presented in these papers consider an unstructured (i.e., atomic) hidden state modelling, which are incompetent to leverage some prior information one has about the structure of the hidden states.

In this paper, we extend previous work for RDP learning along two dimensions. The first contribution is to introduce *priors* for automaton learning. A prior is an automaton that incorporates prior knowledge about a problem. Given one or more priors, an RDP can be expressed as the *cascade* composition of the prior automata and a domain-specific automaton. This amounts to factoring out the features provided by the priors, and hence learning compact domain-specific automata. Notable priors include the *timestep prior* to factor out timesteps from the state space while still considering them, the *Markov prior* to specify that the previous observation may be relevant and avoid learning to remember it (notably this ensures that the domain-specific automaton will be a trivial single-state automaton if the RDP is in fact an MDP), and *spatial priors* that provide a description of the physical space of the domain and relieve the domain-specific automaton from having to learn it.

The second contribution is to allow cycles in the learned domain-specific automaton. In previous work the RDP states are organized in layers, one for each timestep. Introducing cycles can make the learned automaton significantly more compact, especially for episodic problems with long horizons. We identify conditions under which RDPs with cycles can be correctly learned, and demonstrate in experiments that the learned RDPs are often much smaller than in previous work. To learn RDPs with cycles our algorithm has to compare suffix distributions with different lengths, which is possible by exploiting the *language metric* [Deb et al., 2025]. In addition to experiments, we perform a theoretical analysis of the sample complexity of our algorithm, showing it enjoys a sample complexity polynomial in the problem parameters.

1.1 Related work

Offline RL under the Markov property is by now well-established, and there exists a large and growing literature covering a broad range of MDP settings. In many settings, algorithms with optimal sample complexity bounds exist. To mention some notable studies, we refer to [Chen and Jiang, 2019, Jin et al., 2021, Li et al., 2024b, Rashidinejad et al., 2021, Uehara and Sun, 2022].

Research on decision making under non-Markov assumption dates back to, at least, three decades ago; some early attempts include [Schmidhuber, 1990, Whitehead and Lin, 1995, Bacchus et al., 1996, Bakker, 2001]. A classical and effective approach to tackle non-Markov problems was through considering hidden states [Whitehead and Lin, 1995], which related such problems to partially-observable problems. We discuss below the most relevant lines of research that can handle non-Markov problems, while excusing ourselves to give a through overview of all related developments.

POMDPs, PSRs, and State Representation There exist at least two major lines of research to handle hidden information states in the context of partial observability: POMDPs and state representations. Since RDPs are special POMDPs –with underlying dynamics evolving according to some finite state automaton–, RL algorithms for POMDPs also apply to RDPs. Unfortunately, tractable learning for general POMDPs remain to be an open problem, and to the best of our knowledge has only been achieved in subclasses such as ergodicity [Azizzadenesheli et al., 2016], undercomplete POMDPs [Guo et al., 2022, Jin et al., 2020], few-step decodability [Efroni et al., 2022, Krishnamurthy et al., 2016], or weakly-revealing [Liu et al., 2022]. In this context, Hahn et al. [2024] introduces a generalization of RDPs with ω-regular lookahead called Omega-Regular Decision Processes (ODPs) and provide classical complexity results. In the case of state representation, the most notable notion is Predictive State Representation (PSR) [Bowling et al., 2006, James and Singh, 2004, Kulesza et al., 2015, Singh et al., 2003], which provide general descriptions of dynamical systems; they capture POMDPs and therefore RDPs. However, existing work on PSRs [Zhan et al., 2023] rely on PSR-specific parameters and are therefore not directly applicable to RDPs.

Reward Machines and RDPs Some early work on non-Markov decision making restrict attention to non-Markov rewards, while assuming Markov dynamics. This is, for instance, considered in

[Bacchus et al., 1996], where the reward function is specified in a temporal logic of the past. Revisiting this setting has led to some fast growing lines of research that notably include *reward machines* [Toro Icarte et al., 2018] and temporal logics of the future on finite traces [Brafman et al., 2018, Giacomo et al., 2020]. A reward machine is a finite automaton (or transducer) used to specify a non-Markovian reward function. Reward machines have been introduced in [Toro Icarte et al., 2018] along with an RL algorithm that assumes the reward machine to be known. There is a fast growing line of research on reward machines in a variety of settings; see, e.g., [Gaon and Brafman, 2020, Xu et al., 2020, Dohmen et al., 2022, Furelos-Blanco et al., 2023, Varricchione et al., 2024, Parać et al., 2024, Li et al., 2024a, Bourel et al., 2023]. Reward machines have been generalised so as to predict observations as well [Toro Icarte et al., 2019, Hasanbeig et al., 2021], which makes them equivalent to RDPs—as mentioned above. Although some of these algorithms tackle the case of unobservable reward machines, they do not report performance guarantees on the proposed methods. Following their introduction by Brafman and De Giacomo [2019], RDPs were studied in the RL setting; in the online RL setting, some attempts include [Ronca and De Giacomo, 2021, Ronca et al., 2022, Abadi and Brafman, 2020]. They are recently studied in the offline RL setting –the same setting considered here– following the work by Cipollone et al. [2023].

2 Preliminaries

Notation We use $\Delta(\mathcal{X})$ to denote the set of probability distributions over a set $\mathcal{X}$. A conditional probability distribution is a function $p : \mathcal{X} \to \Delta(\mathcal{Y})$ whose elements equal $p(y \mid x)$. We use $\mathbb{I}(E)$ to denote the indicator function of an event E. Given integers m and n such that $0 \leq m \leq n$, let $[\![m, n]\!] := \{m, \dots, n\}$ and $[\![n]\!] := [\![1, n]\!]$. The notation $\widetilde{\mathcal{O}}(\cdot)$ hides poly-logarithmic terms.

2.1 Language metrics

The notion of language metric has been introduced by Deb et al. [2025], and here we present a close variant. Let Γ be an alphabet, i.e. a finite set of symbols. Given a natural number $\ell \in \mathbb{N}$, let Γ^ℓ be the set of strings of symbols in Γ of length ℓ, and let $\Gamma^+ = \cup_{\ell=1}^{\infty} \Gamma^\ell$ be the set of non-empty strings of any length. The empty string is denoted ε. A *language* $X \subseteq \Gamma^+$ is a subset of non-empty strings. Let $\mathcal{X}$ be a set of languages. The *language metric* in $\mathcal{X}$ is a function $L_{\mathcal{X}} : \Delta(\Gamma^+) \times \Delta(\Gamma^+) \to \mathbb{R}$, on pairs of probability distributions $p, p' \in \Delta(\Gamma^+)$, defined as $L_{\mathcal{X}}(p, p') := \max_{X \in \mathcal{X}} |p(X) - p'(X)|$, where the probability of a language is $p(X) := \sum_{x \in X} p(x)$.

To learn cyclic automata in episodic RDPs we necessarily have to compare probability distributions over strings of different lengths. To do so we exploit the fact that the language metric $L_{\mathcal{X}}$ is a *pseudo-metric*: two different distributions $p \neq p'$ may satisfy $L_{\mathcal{X}}(p, p') = 0$. We are therefore interested in languages that are invariant to the string length. One such example is the family of languages that contain some pattern, e.g. any string that contains a given symbol $\gamma \in \Gamma$. Even if p and p' assign non-zero probability to strings of different lengths, we may still have $L_{\mathcal{X}}(p, p') = 0$.

2.2 Episodic decision processes and regular decision processes

An *episodic decision process* is a tuple $\mathbf{P} = \langle \mathcal{O}, \mathcal{A}, \mathcal{R}, \bar{T}, \bar{R}, H, \nu \rangle$, where $\mathcal{O}$ is a finite set of observations, $\mathcal{A}$ is a finite set of actions, $\mathcal{R} \subset [0, 1]$ is a finite set of rewards, $H > 0$ is an integer horizon, and $\nu \in \Delta(\mathcal{O})$ is an initial distribution on observations. We frequently consider the concatenation $\mathcal{AO}$ of the sets $\mathcal{A}$ and $\mathcal{O}$. Let $\mathcal{H}_t = (\mathcal{AO})^{t+1}$ be the set of histories of length $t+1$, and let $h_{m:n} \in \mathcal{H}_{n-m}$ denote a history from time m to time n, both included. Each action-observation pair $ao \in \mathcal{AO}$ in a history has an associated reward label $r \in \mathcal{R}$, which we write $ao/r \in \mathcal{AO}/\mathcal{R}$ with the understanding that the slash corresponds to string concatenation. A *trajectory* $e_{0:T}$ is the full history generated until (and including) time T.

We assume that a trajectory $e_{0:T}$ can be partitioned into *episodes* $e_{\ell:\ell+H} \in \mathcal{H}_H$ of length $H + 1$. In each episode $e_{0:H}$, a_0 is a dummy action and o_0 is sampled from the distribution ν. The transition function $\bar{T} : \mathcal{H} \times \mathcal{A} \to \Delta(\mathcal{O})$ and the reward function $\bar{R} : \mathcal{H} \times \mathcal{A} \to \Delta(\mathcal{R})$ depend on the current history in $\mathcal{H} = \cup_{t=0}^{H} \mathcal{H}_t$. Given $\mathbf{P}$, a *generic policy* is a function $\pi : (\mathcal{AO})^* \to \Delta(\mathcal{A})$ that maps trajectories to distributions over actions. The *value function* $V^\pi : [\![0, H]\!] \times \mathcal{H} \to \mathbb{R}$ of a policy π is a mapping that assigns real values to histories. For $h \in \mathcal{H}$, it is defined as $V^\pi(H, h) := 0$ and

$$V^\pi(t, h) := \mathbb{E}\left[\sum_{i=t+1}^{H} r_i \,\middle|\, h, \pi \right], \quad \forall t \in [\![0, H]\!], \forall h \in \mathcal{H}_t.$$

For brevity, we write $V_t^\pi(h) := V^\pi(t,h)$. The *optimal value function* V^* is defined as $V_t^*(h) := \sup_\pi V_t^\pi(h), \forall t \in [\![0,H]\!], \forall h \in \mathcal{H}_t$, where sup is taken over all policies $\pi : (\mathcal{AO})^* \to \Delta(\mathcal{A})$. Any policy achieving V^* is called an *optimal policy*, which we denote by π^*; namely $V^{\pi^*} = V^*$. In what follows, we consider simpler policies of the form $\pi : \mathcal{H} \to \Delta(\mathcal{A})$ mapping finite histories to distributions over actions. Let $\Pi_\mathcal{H}$ denote the set of such policies. It can be shown that $\Pi_\mathcal{H}$ always contains an optimal policy, i.e. $V_t^*(h) := \max_{\pi \in \Pi_\mathcal{H}} V_t^\pi(h), \forall t \in [H], \forall h \in \mathcal{H}_t$. A policy $\widehat{\pi}$ is ε-optimal iff $\mathbb{E}_{h_0}[V_0^*(h_0) - V_0^{\widehat{\pi}}(h_0)] \leq \varepsilon$, where $h_0 = a_\perp o_0$ for some $o_0 \sim \nu$.

Each history $h \in \mathcal{H}_t$ and policy π induces a probability distribution over suffixes $p_h^\pi \in \Delta(\Gamma^{H-t})$, where $\Gamma = \mathcal{AO}/\mathcal{R}$ is the alphabet of action-observation-reward triplets. Concretely, the probability of a suffix $e_{t+1:H} = a_{t+1}o_{t+1}/r_{t+1} \cdots a_H o_H/r_H$ is given by

$$p_h^\pi(e_{t+1:H}) = \textstyle\prod_{i=t+1}^H \pi(a_i|h_{i-1})\, \bar{T}(o_i|h_{i-1}, a_i)\, \bar{R}(r_i|h_{i-1}, a_i),$$

where $h_{i-1} = h a_{t+1} o_{t+1} \cdots a_{i-1} o_{i-1}$ for each $i \in [\![t+1, H]\!]$. Two histories h and h' are *equivalent* w.r.t. a class of policies Π if $p_h^\pi = p_{h'}^\pi$ for every policy $\pi \in \Pi$; we write equivalence as $h \sim_\Pi h'$.

Observation 1. Specific policies may induce the same distribution for histories that are not equivalent. Namely, for a class of policies Π, and two histories h and h', a policy $\pi_1 \in \Pi$ may induce different distributions $p_h^{\pi_1} \neq p_{h'}^{\pi_1}$, while a second policy $\pi_2 \in \Pi$ may induce identical distributions $p_h^{\pi_2} = p_{h'}^{\pi_2}$ (as shown in Example 5, Appendix B).

Episodic RDPs We adopt the episodic variant of RDPs by Deb et al. [2025], a minor modification of the one by Cipollone et al. [2023]. An *episodic regular decision process* is an episodic decision process $\mathbf{R} = \langle \mathcal{O}, \mathcal{A}, \mathcal{R}, \bar{T}, \bar{R}, H, \nu \rangle$ described by a *probabilistic-deterministic finite automaton*, or simply *automaton* for us, of the specific form $\mathbf{A} = \langle \mathcal{U}, \Sigma, \Omega, \tau, \theta, u_0 \rangle$ with $\mathcal{U}$ a finite set of states, $\Sigma = \mathcal{AO}$ a finite input alphabet composed of actions and observations, Ω a finite output alphabet, $\tau : \mathcal{U} \times \Sigma \to \mathcal{U}$ a transition function, $\theta : \mathcal{U} \to \Omega$ an output function, and $u_0 \in \mathcal{U}$ an initial state. Let τ^{-1} denote the inverse of τ, i.e., $\tau^{-1}(u) \subseteq \mathcal{U} \times \mathcal{AO}$ is the subset of state-input pairs that map to $u \in \mathcal{U}$. An RDP $\mathbf{R}$ implicitly represents a function $\bar{\tau} : \mathcal{H} \to \mathcal{U}$ from histories in $\mathcal{H}$ to states in $\mathcal{U}$, recursively defined as $\bar{\tau}(h_0) := \tau(q_0, a_0 o_0)$ and $\bar{\tau}(h_t) := \tau(\bar{\tau}(h_{t-1}), a_t o_t)$. We use A, O, R, U to denote the cardinality of $\mathcal{A}, \mathcal{O}, \mathcal{R}, \mathcal{U}$, respectively, and assume $H \geq 2$, $A \geq 2$ and $O \geq 2$.

The output function $\theta : \mathcal{U} \to \Omega$ maps the current state to an output in Ω. The output space $\Omega = \Omega_\mathsf{o} \times \Omega_\mathsf{r}$ consists of a finite set of functions that specify the conditional probabilities of observations and rewards, of the form $\Omega_\mathsf{o} \subseteq \mathcal{A} \to \Delta(\mathcal{O})$ and $\Omega_\mathsf{r} \subseteq \mathcal{A} \to \Delta(\mathcal{R})$. For convenience, we often split the output function into two functions $\theta_\mathsf{o} : \mathcal{U} \times \mathcal{A} \to \Delta(\mathcal{O})$ and $\theta_\mathsf{r} : \mathcal{U} \times \mathcal{A} \to \Delta(\mathcal{R})$ specifying the conditional probabilities separately. The transition function and reward function of $\mathbf{R}$ are defined as $\bar{T}(o \mid h, a) = \theta_\mathsf{o}(o \mid \bar{\tau}(h), a)$ and $\bar{R}(r \mid h, a) = \theta_\mathsf{r}(r \mid \bar{\tau}(h), a)$ for each history $h \in \mathcal{H}$ and action-observation-reward triplet $ao/r \in \mathcal{AO}/\mathcal{R}$. An RDP is *minimal* if its automaton is minimal, i.e., without redundant states, and hence *unique*, cf. [Hartmanis and Stearns, 1966].

The class $\Pi_\mathbf{R}$ of policies acting according to the states of an RDP $\mathbf{R}$ is of particular importance. They are called *regular policies*, and they are defined as the policies $\pi : \mathcal{H} \to \Delta(\mathcal{A})$ satisfying the equality $\pi(h_1) = \pi(h_2)$ for all pairs of equivalent histories h_1, h_2 mapping to same state $u = \bar{\tau}(h) = \bar{\tau}(h')$. Hence, we can compactly define a regular policy as a function of the state, i.e., $\pi : \mathcal{U} \to \Delta(\mathcal{A})$. Regular policies exhibit *key properties*: (*P1*) under a regular policy, suffixes have the same probability of being generated for histories that map to the same state in $\mathcal{U}$; (*P2*) there exists at least one optimal policy that is regular; (*P3*) in the special case where an RDP is Markovian in both observations and rewards, it is sufficient for the states in $\mathcal{U}$ to track the observation in $\mathcal{O}$.

In the case of RDPs, under regular policies, the notion of history equivalence admits an alternative and insightful form. Two histories h and h' are equivalent if and only if they map to the same state, i.e., $h \sim_{\Pi_\mathbf{R}} h' \Leftrightarrow \bar{\tau}(h) = \bar{\tau}(h') = u$ for $u \in \mathcal{U}$. Thus, in this setting, we can also write p_u^π in place of the identical distributions p_h^π and $p_{h'}^\pi$. This shows that the meaning of a history is captured by the state $u = \bar{\tau}(h)$ the history maps to.

Distinguishing RDP states We will use language metrics $L_\mathcal{X}$ to learn RDPs from data. Thus we are interested in language sets $\mathcal{X}$ that correctly distinguish an RDP $\mathbf{R}$. For a given regular policy π, a language set *distinguishes* an RDP $\mathbf{R}$ if the following two conditions hold:

- For each pair of histories h, h' such that $\bar{\tau}(h) = \bar{\tau}(h')$, $L_\mathcal{X}(p_h^\pi, p_{h'}^\pi) = 0$.
- For each pair of histories h, h' such that $\bar{\tau}(h) \neq \bar{\tau}(h')$, $L_\mathcal{X}(p_h^\pi, p_{h'}^\pi) \geq \mu_\mathcal{X} > 0$.

Note that two histories h, h' such that $\bar{\tau}(h) = \bar{\tau}(h')$ may have different lengths. In this case we have $p^\pi_h \neq p^\pi_{h'}$, but $L_{\mathcal{X}}(p^\pi_h, p^\pi_{h'}) = 0$ may still hold due to $L_{\mathcal{X}}$ being a pseudo-metric. The quantity $\mu_{\mathcal{X}} := \inf_{h,h':\bar{\tau}(h)\neq\bar{\tau}(h')} L_{\mathcal{X}}(p^\pi_h, p^\pi_{h'})$ is called the *distinguishability* of an RDP $\mathbf{R}$ under a language set $\mathcal{X}$ and a regular policy π. A language set will not distinguish an RDP under policies causing collapse of distributions as pointed out in Observation 1. Hence, we say that a policy π is *admissible* if it ensures that $p^\pi_h \neq p^\pi_{h'}$ for every two histories h, h' such that $\bar{\tau}(h) \neq \bar{\tau}(h')$. We highlight this property to isolate the cases where the impossibility of distinguishing states depends on the policy and not on the considered language set $\mathcal{X}$.

Automata cascades Representing automata with a state space $\mathcal{U}$ of atomic elements does not allow for specifying the complex meaning of a state and the single functionalities implemented by the transition function τ in order to perform state updates. Cascades offer a richer way to to represent automata and overcome such limitations. A *cascade* is an automaton $\mathbf{C} = \langle \Sigma, \mathcal{U}, \tau, u_0, \Omega, \theta \rangle$ given by the composition $\mathbf{A}_1 \ltimes \cdots \ltimes \mathbf{A}_d$ where every $\mathbf{A}_i = \langle \Sigma_i, \mathcal{U}_i, \tau_i, u_0^i \rangle$ is a partial automaton that only specifies the components relevant to describe transitions (called a *semiautomaton*, following the terminology of automata theory). Every $\mathbf{A}_i$ is called a *component* of the cascade, and its input alphabet is $\Sigma_i := \mathcal{U}_1 \cdots \mathcal{U}_{i-1}\Sigma$, allowing it to read the states of the preceding components in addition to inputs from Σ. Then, the states of $\mathbf{C}$ are given by the states of the single components, with $\mathcal{U} := \mathcal{U}_1 \times \cdots \times \mathcal{U}_d$ and $u_0 := \langle u_0^1, \ldots, u_0^d \rangle$, and the transition function is

$$\tau(u_1, \ldots, u_d, \sigma) := \langle \tau_1(u_1, \sigma), \tau_2(u_2, u_1\sigma), \ldots, \tau_d(u_d, u_1 \cdots u_{d-1}\sigma) \rangle,$$

where the transition function τ_i of the i-th cascade component is applied to the component's state u_i and to the extended input $u_1 \cdots u_{i-1}\sigma$ containing the states of the preceding cascade components in addition to the input σ. We note that a component does not need to depend on all preceding components necessarily—in some cases, its transition function may ignore the state of some of the preceding components. This can be specified through the *cross-product* notation. For example, we can write $(\mathbf{A}_1 \times \mathbf{A}_2) \ltimes \mathbf{A}_3$ to say that $\mathbf{A}_2$ ignores the state of $\mathbf{A}_1$, and then $\mathbf{A}_3$ reads the state of both $\mathbf{A}_1$ and $\mathbf{A}_2$—note that parentheses are important to make it clear that we are not stating that $\mathbf{A}_3$ is independent from $\mathbf{A}_1$. We remark that cascades offer an advanced representation formalism–compared to conventional representations that are oblivious of the structure of states and transition function—as they allow for specifying how an automaton is realised by the composition of several components, each implementing a specific functionality, building on information already computed by the preceding components. We remark that cascades are an advanced representation formalism because they allow for specifying how an automaton is realised by the composition of several components, each implementing a specific functionality, building on information already computed by the preceding components. This observation applies directly to the transition function τ, and indirectly also to the output function θ. In fact, the output function $\theta : \mathcal{U}_1 \times \cdots \times \mathcal{U}_d \to \Omega$ of a cascade is over a factored state space, which allows for richer descriptions that make it explicit how the function depends on the single state components.

2.3 Offline RL in episodic RDPs

Consider a batch dataset $\mathcal{D}$ comprising episodes sampled using an *admissible regular* behavior policy π^{b}. Specifically, the k-th episode (or episode trace) in $\mathcal{D}$ is of the form $e^k_{0:H} = a^k_0 o^k_0 / r^k_0 \cdots a^k_H o^k_H / r^k_H$ where, for each $t \in [\![H]\!]$,

$$o^k_0 \sim \nu, \quad u^k_0 = u_0, \quad a^k_t \sim \pi^{\mathsf{b}}(u^k_t), \quad o^k_t \sim \theta_{\mathsf{o}}(u^k_t, a^k_t), \quad r^k_t \sim \theta_{\mathsf{r}}(u^k_t, a^k_t), \quad u^k_{t+1} = \tau(u^k_t, a^k_t o^k_t).$$

The learner seeks an ε-optimal policy $\widehat{\pi}$ for a given accuracy $\varepsilon \in (0, H]$, using the smallest dataset $\mathcal{D}$ possible, without further exploration. More precisely, we aim at finding $\widehat{\pi}$ satisfying $V^*_0(h) - V^{\widehat{\pi}}_0(h) \leq \varepsilon$ for each $h \in \mathcal{H}$ with probability at least $1 - \delta$, using the smallest dataset $\mathcal{D}$ possible. We stress that in so doing π^{b} and underlying RDP states u^k_t are unknown to the learner. It suffices to restrict attention to regular ε-optimal policies (cf. Proposition 5 in Deb et al. [2025]). However, some assumptions must be imposed on π^{b} to provably guarantee that an ε-optimal regular policy can be learned from $\mathcal{D}$.

Given a regular policy $\pi : \mathcal{U} \to \Delta(\mathcal{A})$, let $d^\pi_t \in \Delta(\mathcal{U} \times \mathcal{AO})$ be the induced *occupancy*, i.e., a probability distribution over candidate states $u, ao \in \mathcal{U} \times \mathcal{AO}$, recursively defined as

$$d^\pi_0(u_0, a_0 o_0) = \nu(o_0),$$

$$d^\pi_t(u_t, a_t o_t) = \textstyle\sum_{u, ao \,\in\, \tau^{-1}(u_t)} d^\pi_{t-1}(u, ao)\, \pi(a_t \mid u_t)\, \theta_{\mathsf{o}}(o_t \mid u_t, a_t), \quad \forall t \in [\![H]\!].$$

Of particular interest is the occupancy $d_t^* := d_t^{\pi^*}$ associated with an optimal policy π^*, which is unique if we assume that π^* is unique. Likewise, let $d_t^{\mathsf{b}} := d_t^{\pi^{\mathsf{b}}}$ be the occupancy associated with π^{b}. Since a state $u \in \mathcal{U}$ may appear at different time steps, we often abuse notation and write $d^{\mathsf{b}}(u, ao)$ or $d^*(u, ao)$ to denote the occupancy of u, ao for the *first* timestep at which u may appear.

As in offline RL in MDPs, it is necessary to control the mismatch in occupancy between the behavior policy π^{b} and the optimal policy π^*. Concretely, the single-policy RDP concentrability coefficient associated with RDP $\mathbf{R}$ and behavior policy π^{b} is defined as

$$C^*_{\mathbf{R}} = \max_{u, ao \in \mathcal{U} \times \mathcal{AO}} \frac{d^*(u, ao)}{d^{\mathsf{b}}(u, ao)} .$$

It is generally impossible to learn an RDP correctly from samples collected under a behaviour policy that does not have a finite concentrability coefficient, since this describes a situation where important states are not explored. Thus, we assume concentrability to be bounded away from infinity, $C^*_{\mathbf{R}} < \infty$, which further implies that for every $u, ao \in \mathcal{U} \times \mathcal{AO}$, $d^{\mathsf{b}}(u, ao) > 0$ whenever $d^*(u, ao) > 0$.

3 Novel Techniques and Concepts

Equipped with the notions and definitions introduced in Section 2, we introduce two key notions that prove instrumental in the design of our proposed algorithm (Section 2.3). The first one deals with incorporating and leveraging some prior knowledge in RDPs, while the second characterises particularly-favourable cases for learning RDPs with priors, also extending the stationarity assumption in terms of timestep priors. We believe these notions could be of independent interest beyond RDPs.

3.1 Priors for RDPs

We introduce the novel notion of *priors* for RDPs, that allow for shaping the state space of an RDP with fundamental structures known a priori. This enables learning algorithms to focus on domain-specific aspects, relieving them from the burden of having to learn fundamental structures that are known to be present in a domain. A *prior* is an automaton without output components (a semiautomaton), $\mathbf{A}_{\mathsf{p}} = \langle \Sigma_{\mathsf{p}}, \mathcal{U}_{\mathsf{p}}, \tau_{\mathsf{p}}, u_0^{\mathsf{p}} \rangle$ with input alphabet $\Sigma_{\mathsf{p}} = \mathcal{AO}$, or alternatively $\Sigma_{\mathsf{p}} = \mathcal{U}'_{\mathsf{p}}\mathcal{AO}$ in the case it is part of a cascade where it depends on additional priors that precede it in the cascade and provide it with states from $\mathcal{U}'_{\mathsf{p}}$. Priors are included in the representation of an RDP by expressing its automaton $\mathbf{A}$ as a cascade $\mathbf{A} = \mathbf{A}_{\mathsf{p}} \ltimes \mathbf{A}_{\mathrm{r}}$ where $\mathbf{A}_{\mathrm{r}}$ is a second 'remainder' semiautomaton. In general, we can include several priors as $\mathbf{A} = \mathbf{A}^1_{\mathsf{p}} \ltimes \cdots \ltimes \mathbf{A}^m_{\mathsf{p}} \ltimes \mathbf{A}_{\mathrm{r}}$. We can specify independence between some of the cascade components as, e.g., $\mathbf{A} = (\mathbf{A}^1_{\mathsf{p}} \times \mathbf{A}^2_{\mathsf{p}}) \ltimes \mathbf{A}_{\mathrm{r}}$. Effectively, cascades allow for decomposing $\mathbf{A}$ into several components, each factoring out a specific feature implicit in the states of $\mathbf{A}$. The cascade decomposition focuses on states and transitions, but also provides a structured state space that allows for richer descriptions of the output function of $\mathbf{A}$. In fact, output functions will be over a factored state space $\mathcal{U}^1_{\mathsf{p}} \times \cdots \times \mathcal{U}^m_{\mathsf{p}} \times \mathcal{U}_{\mathrm{r}}$ (abbreviated as $\mathcal{U}^{1:m}_{\mathsf{p}} \times \mathcal{U}_{\mathrm{r}}$), and they can be seen as functions of the overall state as in (a), or as functions of $\mathcal{U}_{\mathrm{r}}$ mapping to functions over the prior state space $\mathcal{U}^{1:m}_{\mathsf{p}}$ as in (b),

$$\text{(a)} \quad \theta : \big(\mathcal{U}^{1:m}_{\mathsf{p}} \times \mathcal{U}_{\mathrm{r}}\big) \to \big(\mathcal{A} \to \Delta(\mathcal{OR})\big), \qquad \text{(b)} \quad \theta : \mathcal{U}_{\mathrm{r}} \to \big(\mathcal{U}^{1:m}_{\mathsf{p}} \to \big(\mathcal{A} \to \Delta(\mathcal{OR})\big)\big).$$

Note that, although the output function of $\mathbf{A}$ has an extended domain, the automaton $\mathbf{A}$ still represents the functions $\bar{T}$ and $\bar{R}$ of the RDP over histories as usual. Specifically, the cascade decomposition only changes the way we express the (hidden) states of an RDP, that are now seen as consisting of several components focusing on specific aspects. It is also important to note that, although the factored state space may contain extra states compared to the standard state space consisting of atomic elements, this redundancy does not prevent the cascaded automaton from representing the RDP correctly, since redundant states can be 'collapsed' by the output function—formally, there may not be a bijection (isomorphism), but there is always an injection (homomorphism) that maps factored states to the corresponding atomic states.

Next we describe three of the most fundamental priors, and showcase their usage in RDPs.

Markov priors Markov priors allow for specifying that the previous observation may be a relevant feature in determining distributions over episode suffixes. Markov priors are simple semiautomata that store the previous observation. Specifically, the *Markov prior* for observations $\mathcal{O}$ is $\mathbf{M}_{\mathcal{O}} = \langle \mathcal{AO}, \mathcal{O} \cup \{\star\}, \tau_{\mathrm{o}}, \star \rangle$ where the initial state '$\star$' is an arbitrary element not in $\mathcal{O}$, and the transition function is the function $\tau_{\mathrm{o}}(o, ao') := o'$, that simply returns o' disregarding o and a. Including a Markov prior in the RDP automaton as $\mathbf{A} = \mathbf{M}_{\mathcal{O}} \ltimes \mathbf{A}_{\mathrm{r}}$ allows for factoring out the functionality of storing the previous observations, hence avoiding that this aspect is factored into the state space of $\mathbf{A}_{\mathrm{r}}$, which is left more compact and cleaner.

Timestep priors Timestep priors allow for specifying that the current timestep in an episode may be a relevant feature in determining distributions over episode suffixes. Timestep priors are simple semiautomata that count the number of timesteps elapsed. Specifically, the *timestep prior* for horizon H is $\mathbf{T}_H = \langle \mathcal{AO}, [\![0, H]\!], \tau_{\mathrm{t}}, 0 \rangle$ where the transition function is defined as $\tau_{\mathrm{t}}(t, ao) := t + 1$. Including a timestep prior in the RDP automaton as $\mathbf{A} = \mathbf{T}_H \ltimes \mathbf{A}_{\mathrm{r}}$ allows for factoring out the functionality of keeping track of the current timestep, hence avoiding that this aspect is factored into the state space of $\mathbf{A}_{\mathrm{r}}$, which is left more compact and cleaner.

Spatial priors Spatial priors allow for describing the physical space (its geometry) of a domain, and specify that the current position in such space may be a relevant feature in determining distributions over episode suffixes. Automata allow for describing all finite spaces. A notable instance is the $m \times n$ *grid prior* for an RDP including motion actions $\mathcal{A}_{\mathrm{m}} = \{\rightarrow, \leftarrow, \uparrow, \downarrow\} \subseteq \mathcal{A}$, defined as $\mathbf{G}_{m \times n} = \langle \mathcal{AO}, [\![m]\!] \times [\![n]\!], \tau_{m \times n}, \langle x_0, y_0 \rangle \rangle$ with transition function $\tau_{m \times n}(x, y, ao)$ returning updated coordinates when a is one of the motion actions.

Example (RDPs with Markov and timestep priors) To convey a clearer idea of the effect of priors, we show explicitly what the automaton of an RDP looks like when Markov and timestep priors are included at the same time. In particular, the two priors do not depend on each other, and hence they are composed as $\mathbf{T}_H \times \mathbf{M}_{\mathcal{O}}$. Then, the automaton of the RDP is expressed as $(\mathbf{T}_H \times \mathbf{M}_{\mathcal{O}}) \ltimes \mathbf{A}_{\mathrm{r}}$. The resulting state space is $\mathcal{U} = [\![0, H]\!] \times \mathcal{O} \times \mathcal{U}_{\mathrm{r}}$, and the transition function is

$$\tau(t, o, u_{\mathrm{r}}, ao') \;=\; \langle \tau_{\mathrm{t}}(t, ao'), \tau_{\mathrm{o}}(o, ao'), \tau_{\mathrm{r}}(u_{\mathrm{r}}, toao') \rangle \;=\; \langle t + 1, o', u'_{\mathrm{r}} \rangle,$$

where $u'_{\mathrm{r}} = \tau_{\mathrm{r}}(u_{\mathrm{r}}, toao')$ is the result of applying the transition function τ_{r} of $\mathbf{A}_{\mathrm{r}}$ to the previous state u_{r} and the extended input $toao'$, which includes the current timestep t and the previous observation o, in addition to the current action a and observation o'.

3.2 Partial independence from priors (and semi-stationarity)

In some special cases, the domain-specific automaton can be learned without considering priors explicitly at learning time. Let us consider an RDP expressed as a cascade $\mathbf{A}_{\mathsf{p}} \ltimes \mathbf{A}_{\mathrm{r}}$ where $\mathbf{A}_{\mathsf{p}}$ is a prior and $\mathbf{A}_{\mathrm{r}}$ is a domain-specific automaton. This yields a state space $\mathcal{U} = \mathcal{U}_{\mathsf{p}} \times \mathcal{U}_{\mathrm{r}}$, and hence an output function of the form $\theta : \mathcal{U}_{\mathsf{p}} \times \mathcal{U}_{\mathrm{r}} \to (\mathcal{A} \to \Delta(\mathcal{OR}))$. Intuitively, this cascade representation amounts to a factoring out the cascade features from $\mathbf{A}$. Then, the special case when priors can be considered separately is captured by the following notion.

Definition 1. An RDP represented by the cascade $\mathbf{A}_{\mathsf{p}} \ltimes \mathbf{A}_{\mathrm{r}}$ is *partially independent from priors* when the following conditions hold: (I) the two cascade components are independent, $\mathbf{A} = \mathbf{A}_{\mathsf{p}} \times \mathbf{A}_{\mathrm{r}}$, (II) the observation function θ_{o} of $\mathbf{A}$ can be expressed as the product of two independent functions as $\theta_{\mathsf{o}}(o \mid u_{\mathrm{p}}, u_{\mathrm{r}}, a) = \theta^{\mathrm{p}}_{\mathsf{o}}(o \mid u_{\mathrm{p}}, a) \cdot \theta^{\mathrm{r}}_{\mathsf{o}}(o \mid u_{\mathrm{r}}, a)$, and (III) the reward function θ_{r} can be expressed as the product of two independent functions as $\theta_{\mathsf{r}}(r \mid u_{\mathrm{p}}, u_{\mathrm{r}}, a) = \theta^{\mathrm{p}}_{\mathsf{r}}(r \mid u_{\mathrm{p}}, a) \cdot \theta^{\mathrm{r}}_{\mathsf{r}}(r \mid u_{\mathrm{r}}, a)$. When Conditions (I) and (III) hold, we say the RDP is *partially independent from priors w.r.t. rewards.* When an RDP is partial independent from a timestep prior $\mathbf{T}_H$, we say the RDP is *semi-stationary.*

The definition applies to the case of multiple priors, as they can all be seen as part of $\mathbf{A}_{\mathsf{p}}$.

Example 1. The T-maze of length N and horizon H [Deb et al., 2025], when represented as $\mathbf{T}_H \ltimes \mathbf{A}_{\mathrm{r}}$ is partially independent from the timestep prior $\mathbf{T}_H$, or semi-stationary. Furthermore, when represented as $(\mathbf{T}_H \times \mathbf{G}_{3 \times (N+1)}) \ltimes \mathbf{A}_{\mathrm{r}}$, with $\mathbf{G}_{3 \times (N+1)}$ the grid prior, the RDP is partially independent from both priors w.r.t. rewards only. Details in Appendix B.3.

Partial independence is important because it allows for learning the domain-specific automaton $\mathbf{A}_{\mathrm{r}}$ while ignoring the prior $\mathbf{A}_{\mathsf{p}}$ at learning time, since states $u_{\mathrm{r}} = \bar{\tau}_{\mathrm{r}}(h)$ and the corresponding transition function τ_{r} can be learned by checking similarity of the distributions they induce on episode suffixes, which are independent of any feature provided by the priors. If independence is only w.r.t. rewards, only the reward function can be captured correctly by a cascade where independence from priors is included, which can still be useful to learn optimal policies. We explore partial independence in our experiments.

4 Algorithm and PAC analysis

In this section we present ADACT–L, our algorithm for learning RDPs with priors and cycles. The algorithm assumes that we are provided with a prior automaton $\mathbf{A}_\mathsf{p} = \langle \Sigma, \mathcal{U}_\mathsf{p}, \tau_\mathsf{p}, u_\mathsf{p}^0 \rangle$, and the aim is to learn a problem-specific automaton $\mathbf{A}_\mathsf{r} = \langle \Sigma, \mathcal{U}_\mathsf{r}, \tau_\mathsf{r}, u_\mathsf{r}^0 \rangle$ such that the RDP is expressed as a cascade $\mathbf{A}_\mathsf{p} \ltimes \mathbf{A}_\mathsf{r}$. For this purpose, the transition function $\tau_\mathsf{r} : \mathcal{U}_\mathsf{r} \times \mathcal{U}_\mathsf{p}\, \Sigma \to \mathcal{U}_\mathsf{r}$ incorporates the states of the prior automaton as part of its input. We remark that the prior automaton $\mathbf{A}_\mathsf{p}$ could itself be a cascade of automata, and that the algorithm can learn an RDP without prior knowledge by defining a prior automaton $\mathbf{A}_\mathsf{p}$ with a single state.

```
Function ADACT–L($\mathcal{U}_\mathsf{r}$, $\mathcal{D}$, $\mathbf{A}_\mathsf{p}$, $\delta$)
Input: Automaton states $\mathcal{U}_\mathsf{r} = \{u_\mathsf{r}^0, u_\mathsf{r}^1, u_\mathsf{r}^2, \ldots\}$, dataset $\mathcal{D}$ of traces in $\Gamma^{H+1}$,
       prior automaton $\mathbf{A}_\mathsf{p} = \langle \Sigma, \mathcal{U}_\mathsf{p}, \tau_\mathsf{p}, u_\mathsf{p}^0 \rangle$, failure probability $0 < \delta < 1$
Output: Transition function $\tau_\mathsf{r} : \mathcal{U}_\mathsf{r} \times \mathcal{U}_\mathsf{p}\, \mathcal{A}\, \mathcal{O} \to \mathcal{U}_\mathsf{r}$
foreach $u_\mathsf{p} \in \mathcal{U}_\mathsf{p}$ do $i(u_\mathsf{p}) \leftarrow 0$
$Q \leftarrow \{u_\mathsf{p}^0 u_\mathsf{r}^0\}$                         // queue data structure containing $u_\mathsf{p}^0 u_\mathsf{r}^0$
$i(u_\mathsf{p}^0) \leftarrow 1$, $t(u_\mathsf{p}^0 u_\mathsf{r}^0) \leftarrow 0$, $\mathcal{Z}(u_\mathsf{p}^0 u_\mathsf{r}^0) \leftarrow \mathcal{D}$
while $Q$ is not empty do
    dequeue $u_\mathsf{p} u_\mathsf{r}$ from $Q$                                   // next joint state
    for $ao \in \mathcal{AO}$ do
        $u'_\mathsf{p} \leftarrow \tau_\mathsf{p}(u_\mathsf{p}, ao)$                                   // next prior state
        $\mathcal{Z}(ao) \leftarrow \{e_{t+1:H} \mid ao/re_{t+1:H} \in \mathcal{Z}(u_\mathsf{p} u_\mathsf{r})\}$          // compute suffixes
        $j \leftarrow i(u'_\mathsf{p})$
        for $k = 0, \ldots, i(u'_\mathsf{p}) - 1$ do
            if not TESTDISTINCT($\mathcal{Z}(u'_\mathsf{p} u_\mathsf{r}^k$, $\mathcal{Z}(ao)$, $\delta$) then $j \leftarrow k$
        end
        $\tau(u_\mathsf{r}, u_\mathsf{p} ao) \leftarrow u_\mathsf{r}^j$                           // define transition function
        if $j = i(u'_\mathsf{p})$ then
            enqueue $u'_\mathsf{p} u_\mathsf{r}^j$ in $Q$
            $i(u'_\mathsf{p}) \leftarrow j + 1$, $t(u'_\mathsf{p} u_\mathsf{r}^j) \leftarrow t(u_\mathsf{p} u_\mathsf{r}) + 1$, $\mathcal{Z}(u'_\mathsf{p} u_\mathsf{r}^j) \leftarrow \mathcal{Z}(ao)$
        else if $t(u'_\mathsf{p} u_\mathsf{r}^j) = t(u_\mathsf{p} u_\mathsf{r}) + 1$ then $\mathcal{Z}(u'_\mathsf{p} u_\mathsf{r}^j) \leftarrow \mathcal{Z}(u'_\mathsf{p} u_\mathsf{r}^j) \cup \mathcal{Z}(ao)$
    end
end
return $\tau$
Function TESTDISTINCT($\mathcal{Z}_1$, $\mathcal{Z}_2$, $\delta$)
    return $L_\mathcal{X}(\mathcal{Z}_1, \mathcal{Z}_2) \geq \sqrt{\log(2|\mathcal{X}|/\delta) / \min(|\mathcal{Z}_1|, |\mathcal{Z}_2|)}$      // statistical test
```

The algorithm is based on the fact that the transition function τ_r is invariant to the specific order of pairs $u_\mathsf{p} u_\mathsf{r}$ as long as the initial state u_p^0 of $\mathbf{A}_\mathsf{p}$ is always paired with the initial state u_r^0 of $\mathbf{A}_\mathsf{r}$. For example, let u_r^1 and u_r^2 be two states of $\mathbf{A}_\mathsf{r}$ and let u_p be a state of $\mathbf{A}_\mathsf{p}$. We can construct an equivalent RDP by swapping the definitions of $\tau_\mathsf{r}(u_\mathsf{r}^1, u_\mathsf{p}\, \cdot)$ and $\tau(u_\mathsf{r}^2, u_\mathsf{p}\, \cdot)$ and changing the definition of $\tau(\cdot, u'_\mathsf{p} ao)$ from u_r^1 to u_r^2 or vice versa whenever $\tau_\mathsf{p}(u'_\mathsf{p}, ao) = u_\mathsf{p}$.

As a consequence of the above fact, when we discover a new transition for a given prior state u_p, the identity of the associated state in $\mathcal{U}_\mathsf{r}$ can be arbitrary. In the algorithm, we simply assign the transition to the next available state in $\mathcal{U}_\mathsf{r}$ for u_p. To do so, we assume that we have access to a sequence of states $u_\mathsf{r}^0, u_\mathsf{r}^1, u_\mathsf{r}^2, \ldots$ and for each prior state $u_\mathsf{p} \in \mathcal{U}_\mathsf{p}$ we remember the index $i(u_\mathsf{p}) \geq 0$ of the next available state in $\mathcal{U}_\mathsf{r}$. We also remember the first timestep $t(u_\mathsf{p} u_\mathsf{r})$ of each state pair in order to add all suffixes of the same length to the associated multiset.

In Appendix A we prove the following sample complexity bound for ADACT–L.

Theorem 1. ADACT–L$(\mathcal{D}, \delta)$ *returns a minimal automaton* $\mathbf{A}_\mathsf{r}$ *with probability at least* $1 - 2AOUU_p\, \delta$ *when using a language set* $\mathcal{X}$ *that distinguishes* $\mathbf{A}_\mathsf{p} \ltimes \mathbf{A}_\mathsf{r}$ *under the behavior policy* π^b

with associated distinguishability $\mu_{\mathcal{X}}$ and the size of the dataset $\mathcal{D}$ is at least

$$|\mathcal{D}| \geq \widetilde{\mathcal{O}}\left(\frac{C^*_{\mathbf{R}} \log(1/\delta) \log|\mathcal{X}|}{d^*_m \cdot \mu^2_{\mathcal{X}}}\right),$$

where $d^*_m = \min_{u,ao} d^*(u, ao)$ is the minimum occupancy of the optimal policy π^*.

5 Experimental Evaluation

We conduct numerical experiments to further demonstrate the performance and properties of AdaCT–L. We present our results for five familiar domains in the literature of POMDPs and RDPs: Corridor [Ronca and De Giacomo, 2021], T-maze(c) [Bakker, 2001], Cookie [Toro Icarte et al., 2019], Cheese [McCallum, 1992] and Mini-hall [Littman et al., 1995], and summarize our results in Table 1. We compare against FlexFringe [Baumgartner and Verwer, 2023], a state-of-the-art algorithm for learning probabilistic-deterministic finite automata, which includes RDPs as a special case, and ADACT-H [Deb et al., 2025]. FlexFringe can learn RDPs with cycles, but includes several heuristics that do not preserve high-probability sample complexity guarantees. ADACT-H learns RDPs without cycles. The proposed algorithm AdaCT–L can learn cycles in addition to providing sample complexity guarantees. In all experiments we use a Markov prior and a language set $\mathcal{X}$ consisting of one language per action-observation-reward triplet, containing all strings of any length that includes the triplet. This language set assumes partial independence of the learned RDP, sometimes only w.r.t. rewards (cf. Section 3.2), but may only learn an approximate RDP in some domains.

From our results in Table 1, we can see that AdaCT–L learns much smaller automata, while also achieving the highest average reward. In T-maze(c), FlexFringe fails to find the optimal policy, since the heuristics defined for FlexFringe are not optimized to preserve reward. In the domains Cheese and Minihall, all the algorithms fail to learn the optimal policy owing to the complexity of the POMDP environments; however, AdaCT–L outperforms the other approaches by getting a higher average reward as well as learning significantly smaller automata.

		FlexFringe			ADACT-H			AdaCT–L		
Name	H	U	r	time	U	r	time	U	r	time
Corridor	5	11	**1.0**	0.03	11	**1.0**	**0.01**	**3**	**1.0**	**0.01**
T-maze(c)	5	29	0.0	0.11	18	**1.0**	0.26	**5**	**1.0**	**0.15**
Cookie	9	220	**1.0**	0.36	91	**1.0**	**0.08**	**11**	**1.0**	**0.08**
Cheese	6	669	$0.69 \pm .04$	19.28	1178	$0.87 \pm .03$	12.11	**85**	$\mathbf{0.89 \pm .04}$	**7.27**
Mini-hall	15	897	$0.33 \pm .04$	25.79	6098	$0.86 \pm .03$	29.90	**65**	$\mathbf{0.87 \pm .04}$	**25.18**

Table 1: For each domain, H, U are the horizon and the number of states in the learned automaton respectively, r is the average normalised reward (over 100 episodes) of the derived policy, and 'time' is the running time in seconds of automaton learning. Best results emphasised in bold.

6 Conclusions

In this work, we introduce a novel algorithm AdaCT–L utilizing the language metric introduced by Deb et al. [2025], which allows us to learn a significantly smaller RDP with cycles, and also identify conditions under which RDPs with cycles can be correctly learned which makes it possible to scale to larger and more complex domains. Further to exploit domain-related knowledge, we also introduce the notion of *priors* for automaton learning, that can be used to factor out of the state space any feature that is known a priori. We further validate our approach experimentally over five familiar domains in the POMDP and RDP literature, and compare the performance of our algorithm to FlexFringe, a state-of-the-art algorithm for learning PDFA. Finally, as future work, we plan to explore the approximate version of our algorithm and also to extend our work to the online setting.

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A Technical Lemmas

The technical lemmas are reformulated from [Deb et al., 2025] for our setting. Following the proof-structure, we first provide the high probability upper bound on the language metric $L_{\mathcal{X}}$ adapted to our setting.

Lemma 2. *Let $\mathcal{X}$ be a language set. Given a candidate state $u, ao \in \mathcal{U} \times \mathcal{AO}$ and a multiset $\mathcal{Z}(uao)$ of suffixes in Γ^+, with probability at least $1 - \delta$ the language metric $L_{\mathcal{X}}$ satisfies*

$$L_{\mathcal{X}}(\widehat{p}_{uao}, p_{uao}) \leq \sqrt{\frac{\log(2|\mathcal{X}|/\delta)}{2|\mathcal{Z}(uao)|}},$$

where $p_{uao} \in \Delta(\Gamma^+)$ is the true distribution on suffixes induced by the candidate uao and the behavior policy π^{b}, and $\widehat{p}_{uao} \in \Delta(\Gamma^+)$ is the empirical estimate on suffixes induced by $\mathcal{Z}(uao)$.

Proof. Let $p_{uao}(X) = \sum_{x \in X} p_{uao}(x)$ be the true probability of each language language $X \in \mathcal{X}$, and let $\widehat{p}_{uao}(X) = \sum_{x \in \mathcal{Z}(uao)} \mathbb{I}(x \in X)/|\mathcal{Z}(uao)|$ be the empirical estimate of $p_{uao}(X)$. Following Hoeffding's inequality we get

$$\mathbb{P}\left(|\widehat{p}_{uao}(X) - p_{uao}(X)| > \sqrt{\frac{\log(2/\delta_{\mathsf{s}})}{2|\mathcal{Z}(uao)|}}\right) \leq \delta_{\mathsf{s}}.$$

Choosing $\delta_{\mathsf{s}} = \delta/|\mathcal{X}|$ and taking a union bound implies that $L_{\mathcal{X}}$ satisfies

$$L_{\mathcal{X}}(\widehat{p}_{uao}, p_{uao}) = \max_{X \in \mathcal{X}} |\widehat{p}_{uao}(X) - p_{uao}(X)| \leq \sqrt{\frac{\log(2|\mathcal{X}|/\delta)}{2|\mathcal{Z}(uao)|}}$$

with probability $1 - |\mathcal{X}|\delta_{\mathsf{s}} = 1 - \delta$, which completes the proof. □

Next, we define an associated event $\mathcal{E}_{\mathcal{X}}$ to correctly bound the language metric $L_{\mathcal{X}}$ for all candidate states:

$$\mathcal{E}_{\mathcal{X}} = \left\{\forall u, ao \in \mathcal{U} \times \mathcal{AO} : L_{\mathcal{X}}(\widehat{p}_{uao}, p_{uao}) \leq \sqrt{\frac{\log(2|\mathcal{X}|/\delta)}{2|\mathcal{Z}(uao)|}}\right\}.$$

We next prove a high-probability sample complexity bound for accurately estimating the occupancy $d^{\mathsf{b}}(u, ao)$ of each candidate state. Let $\widehat{d}(uao)$ be the empirical occupancy of uao. Given a number of episodes N, an empirical Bernstein inequality yields

$$\mathbb{P}\left(\left|\widehat{d}(uao) - d^{\mathsf{b}}(u, ao)\right| > \sqrt{\frac{2\widehat{d}(uao)\log(4/\delta)}{N}} + \frac{14\log(4/\delta)}{3N}\right) \leq \delta. \tag{1}$$

We can next define G_δ as the function for the bound in the empirical Bernstein inequality where δ is the given failure probability, given by

$$G_\delta(\widehat{d}, N) = \sqrt{\frac{2\widehat{d}\log(4/\delta)}{N}} + \frac{14\log(4/\delta)}{3N}$$

where G_δ is monotonically increasing in $\widehat{d}$ and monotonically decreasing in N. We can further define an associated event $\mathcal{E}_B$ to correctly bound $|\widehat{d}(uao) - d^{\mathsf{b}}(u, ao)|$ for all hao:

$$\mathcal{E}_B = \left\{\forall u, ao \in \mathcal{U} \times \mathcal{AO} : \left|\widehat{d}(uao) - d^{\mathsf{b}}(u, ao)\right| \leq G_\delta(\widehat{d}(uao), N)\right\}.$$

The following lemma shows that we can control the number of episodes N to obtain an upper bound on the function G_δ.

Lemma 3. *For fixed probabilities δ and $\widehat{d}$, if $N \geq 16\log(4/\delta)/\widehat{d}$ it holds that $3G_\delta(\widehat{d}, N) < 2\widehat{d}$.*

Proof. We first show that the inequality holds for $N = 16\log(4/\delta)/\widehat{d}$. In this case we have

$$3G_\delta(\widehat{d}, N) = 3\sqrt{\frac{2\widehat{d}^2\log(4/\delta)}{16\log(4/\delta)}} + \frac{14\widehat{d}\log(4/\delta)}{16\log(4/\delta)} = \left(\frac{3}{\sqrt{8}} + \frac{14}{16}\right)\widehat{d} < 2\widehat{d}.$$

The case $N > 16\log(4/\delta)/\widehat{d}$ follows from the fact that G_δ is monotonically decreasing in N. □

Since $\widehat{d}(uao) = |\mathcal{Z}(uao)|/N$ implies $N = |\mathcal{Z}(uao)|/\widehat{d}(uao)$, we obtain the following corollary.

Corollary 4. *Under event $\mathcal{E}_B$, if $|\mathcal{Z}(uao)| \geq 16\log(4/\delta)$, it holds that $|\widehat{d}(uao) - d^{\mathsf{b}}(u, ao)| \leq 2\widehat{d}(uao)/3$.*

We show that under event $\mathcal{E}_B$, we can choose the sample complexity N to ensure that we obtain at least a certain number of elements in $\mathcal{Z}(uao)$.

Lemma 5. *Given a candidate state $u, ao \in \mathcal{U} \times \mathcal{AO}$, under event $\mathcal{E}_B$, it holds that $|\mathcal{Z}(uao)| \geq b\log(4/\delta)$ if the sample complexity N satisfies*

$$N \geq \frac{\log(4/\delta)}{d^{\mathsf{b}}(u, ao)}\left(2b + 31/6\right).$$

Proof. Letting $M = |\mathcal{Z}(uao)|$, due to event $\mathcal{E}_B$ and the given bound on N it holds that

$$\begin{aligned} d^{\mathsf{b}}(u, ao) - \frac{M}{N} &\leq G_\delta(M/N, N) \\ \Leftrightarrow \quad 0 &\leq M + NG_\delta(M/N, N) - Nd^{\mathsf{b}}(u, ao) \\ &\leq M + \sqrt{2M\log(4/\delta)} + 14\log(4/\delta)/3 - \log(4/\delta)\left(2b + 31/6\right) \\ &= M + \sqrt{2\log(4/\delta)}\sqrt{M} - \log(4/\delta)\left(2b + 1/2\right). \end{aligned}$$

Solving the quadratic inequality for positive $\sqrt{M}$ yields

$$\begin{aligned} \sqrt{M} &\geq -\sqrt{\frac{\log(4/\delta)}{2}} + \sqrt{\frac{\log(4/\delta)}{2} + \log(4/\delta)\left(2b + 1/2\right)} \\ &= -\sqrt{\frac{\log(4/\delta)}{2}} + \sqrt{\log(4/\delta) + 2b\log(4/\delta)} \\ &\geq -\sqrt{\frac{\log(4/\delta)}{2}} + \frac{\sqrt{\log(4/\delta)} + \sqrt{2b\log(4/\delta)}}{\sqrt{2}} = \sqrt{b\log(4/\delta)}, \end{aligned}$$

where we have used the inequality $\sqrt{x+y} \geq (\sqrt{x} + \sqrt{y})/\sqrt{2}$. Hence the bound on N in the lemma implies that $M = \sqrt{M}^2 \geq b\log(4/\delta)$. □

A.1 Proof of Theorem 1

We first prove two lemmas very similar to Lemmas 16 and 17 of [Deb et al., 2025].

Lemma 6. *Let $\mathbf{R}$ be an RDP and let $\mathcal{X}$ be a language set that distinguishes $\mathbf{R}$ under the behavior policy π^{b}. Given a candidate state $u, ao \in \mathcal{U} \times \mathcal{AO}$ and a reference state $u' \in \mathcal{U}$, let $\mathcal{Z}_1$ and $\mathcal{Z}_2$ be two multisets sampled from the true distributions p_{uao} and $p_{u'}$ on suffixes in Γ^+, respectively. Under event $\mathcal{E}_{\mathcal{X}}$, if $\tau(u, ao) = u'$ then* TESTDISTINCT$(\mathcal{Z}_1, \mathcal{Z}_2, \delta)$ *returns false.*

Proof. Since $\tau(u, ao) = u'$, any pair of histories h_1 and h_2 associated with u, ao and u' satisfy $\bar{\tau}(h_1) = \bar{\tau}(h_2) = u'$. Since $\mathcal{X}$ distinguishes $\mathbf{R}$, this implies that $L_{\mathcal{X}}(p_{uao}, p_{u'}) = 0$ holds. Letting $\widehat{p}_{uao}$ and $\widehat{p}_{u'}$ be the empirical distributions on suffixes induced by the multisets $\mathcal{Z}_1$ and $\mathcal{Z}_2$, we can now use the event $\mathcal{E}_{\mathcal{X}}$, Lemma 2 and the triangle inequality to obtain

$$\begin{aligned} L_{\mathcal{X}}(\widehat{p}_{uao}, \widehat{p}_{u'}) &\leq L_{\mathcal{X}}(\widehat{p}_{uao}, p_{uao}) + L_{\mathcal{X}}(p_{uao}, p_{u'}) + L_{\mathcal{X}}(p_{u'}, \widehat{p}_{u'}) \\ &\leq \sqrt{\frac{\log(2|\mathcal{X}|/\delta)}{2|\mathcal{Z}_1|}} + 0 + \sqrt{\frac{\log(2|\mathcal{X}|/\delta)}{2|\mathcal{Z}_2|}} \leq \sqrt{\frac{2\log(2|\mathcal{X}|/\delta)}{\min(|\mathcal{Z}_1|, |\mathcal{Z}_2|)}}. \end{aligned}$$

This is precisely the condition for which TESTDISTINCT returns false. □

Lemma 7. *Let $\mathbf{R}$ be an RDP and let $\mathcal{X}$ be a language set that distinguishes $\mathbf{R}$ under the behavior policy π^{b}. Given a candidate state $u, ao \in \mathcal{U} \times \mathcal{AO}$ and a reference state $u' \in \mathcal{U}$, let $\mathcal{Z}_1$ and $\mathcal{Z}_2$ be two multisets sampled from the true distributions p_{uao} and $p_{u'}$ on suffixes in Γ^+, respectively. Under event $\mathcal{E}_{\mathcal{X}}$, if $\tau(u, ao) \neq u'$ then* TESTDISTINCT$(\mathcal{Z}_1, \mathcal{Z}_2, \delta)$ *answers true if $\mathcal{Z}_1$ and $\mathcal{Z}_2$ satisfy $\min(|\mathcal{Z}_1|, |\mathcal{Z}_2|) \geq 8 \log(2|\mathcal{X}|/\delta)/\mu_{\mathcal{X}}^2$.*

Proof. Since $\tau(u, ao) \neq u'$, any pair of histories h_1 and h_2 associated with u, ao and u' satisfy $\bar{\tau}(h_1) \neq \bar{\tau}(h_2)$. Since $\mathcal{X}$ distinguishes $\mathbf{R}$, this implies that $L_{\mathcal{X}}(p_{uao}, p_{u'}) \geq \mu_{\mathcal{X}}$ holds. Letting $\widehat{p}_{uao}$ and $\widehat{p}_{u'}$ be the empirical distributions on suffixes induced by the multisets $\mathcal{Z}_1$ and $\mathcal{Z}_2$, we can now use the event $\mathcal{E}_{\mathcal{X}}$, Lemma 2 and the triangle inequality to obtain

$$\begin{aligned}
L_{\mathcal{X}}(\widehat{p}_{uao}, \widehat{p}_{u'}) &\geq L_{\mathcal{X}}(p_{uao}, p_{u'}) - L_{\mathcal{X}}(\widehat{p}_{uao}, p_{uao}) - L_{\mathcal{X}}(p_{u'}, \widehat{p}_{u'}) \\
&\geq \mu_{\mathcal{X}} - \sqrt{\frac{\log(2|\mathcal{X}|/\delta)}{2|\mathcal{Z}_1|}} - \sqrt{\frac{\log(2|\mathcal{X}|/\delta)}{2|\mathcal{Z}_2|}} \\
&\geq \mu_{\mathcal{X}} - \sqrt{\frac{2\log(2|\mathcal{X}|/\delta)}{\min(|\mathcal{Z}_1|, |\mathcal{Z}_2|)}} \geq \mu_{\mathcal{X}} - \sqrt{\frac{\mu_{\mathcal{X}}^2}{4}} = \frac{\mu_{\mathcal{X}}}{2} \geq \sqrt{\frac{2\log(2|\mathcal{X}|/\delta)}{\min(|\mathcal{Z}_1|, |\mathcal{Z}_2|)}},
\end{aligned}$$

where we have used the given condition on $\min(|\mathcal{Z}_1|, |\mathcal{Z}_2|)$ twice on the last line. This is precisely the condition for which TESTDISTINCT returns true. □

The following lemma shows that the algorithm ADACT–L returns a minimal RDP if the multisets $\mathcal{Z}$ associated with candidate states satisfy $|\mathcal{Z}| \geq 16 \log(4/\delta) \log |\mathcal{X}|/\mu_{\mathcal{X}}^2 \equiv M_{\mathcal{X}}$.

Lemma 8. *Under event $\mathcal{E}_{\mathcal{X}}$,* ADACT–L *outputs a minimal automaton $\mathbf{A}_{\mathsf{r}}$ if the language set $\mathcal{X}$ distinguishes $\mathbf{A}_{\mathsf{p}} \ltimes \mathbf{A}_{\mathsf{r}}$ under the behavior policy π^{b} and the multiset $\mathcal{Z}(uao)$ associated with each candidate state $u, ao \in \mathcal{U} \times \mathcal{AO}$ satisfies $|\mathcal{Z}(uao)| \geq M_{\mathcal{X}}$.*

Proof. We prove the lemma using induction on RDP states $u = u_{\mathsf{p}} u_{\mathsf{r}} \in \mathcal{U}_{\mathsf{p}} \mathcal{U}_{\mathsf{r}}$. Since the algorithm uses a queue data structure, such state pairs are visited in breadth-first order. The base case is given by the initial state pair $u_{\mathsf{p}}^0 u_{\mathsf{r}}^0$ and the associated multiset $\mathcal{Z}(u_{\mathsf{p}}^0 u_{\mathsf{r}}^0) = \mathcal{D}$. This state pair is covered by the single initial state u_{r}^0 that has to be part of any minimal automaton $\mathbf{A}_{\mathsf{r}}$.

The inductive case is given by a state pair $u_{\mathsf{p}} u_{\mathsf{r}}$ visited by the algorithm, and the associated multiset $\mathcal{Z}(u_{\mathsf{p}} u_{\mathsf{r}})$ induced by all shortest histories mapping to $u_{\mathsf{p}} u_{\mathsf{r}}$. By hypothesis of induction, all state pairs visited by the algorithm prior to (and including) $u_{\mathsf{p}} u_{\mathsf{r}}$ are induced by the known prior $\mathbf{A}_{\mathsf{p}}$ and a minimal automaton $\mathbf{A}_{\mathsf{r}}$. Consider an action-observation $ao \in \mathcal{AO}$ and let $\mathcal{Z}(ao)$ be the multiset of suffixes in $\mathcal{Z}(u_{\mathsf{p}} u_{\mathsf{r}})$ consistent with ao. Let $u_{\mathsf{p}}' = \tau_{\mathsf{p}}(u_{\mathsf{p}}, ao)$ be the resulting next state of the prior automaton, and let $u_{\mathsf{r}}' = \tau_{\mathsf{r}}(u_{\mathsf{r}}, u_{\mathsf{p}} ao)$ be the next state of a minimal automaton $\mathbf{A}_{\mathsf{r}}$. If $u_{\mathsf{p}}' u_{\mathsf{r}}'$ is visited before $u_{\mathsf{p}} u_{\mathsf{r}}$, then Lemma 6 implies that TESTDISTINCT$(\mathcal{Z}(ao), \mathcal{Z}(u_{\mathsf{p}}' u_{\mathsf{r}}'), \delta)$ returns false. In this case the algorithm correctly defines $\tau_{\mathsf{r}}(u_{\mathsf{r}}, u_{\mathsf{p}} ao) = u_{\mathsf{r}}'$, and does not enqueue a new state pair. On the other hand, if $u_{\mathsf{p}}' u_{\mathsf{r}}'$ is not visited before $u_{\mathsf{p}} u_{\mathsf{r}}$, then if the multisets associated with all candidate states have cardinality at least $M_{\mathcal{X}}$, Lemma 7 implies that TESTDISTINCT$(\mathcal{Z}(ao), \mathcal{Z}(\hat{u}_{\mathsf{p}} \hat{u}_{\mathsf{r}}), \delta)$ returns true for all state pairs $\hat{u}_{\mathsf{p}} \hat{u}_{\mathsf{r}}$ visited before $u_{\mathsf{p}} u_{\mathsf{r}}$. In this case the algorithm defines $\tau_{\mathsf{r}}(u_{\mathsf{r}}, u_{\mathsf{p}} ao) = u_{\mathsf{r}}'$ for the next available state $u_{\mathsf{r}}' \in \mathcal{U}_{\mathsf{r}}$ associated with u_{p}', and enqueues a new state pair $u_{\mathsf{p}}' u_{\mathsf{r}}'$. This proves that the output of the algorithm is the transition function τ_{r} of a minimal automaton $\mathbf{A}_{\mathsf{r}}$. □

To complete the proof of the theorem we need to select a minimum number of episodes to ensure that $|\mathcal{Z}(uao)| \geq M_{\mathcal{X}}$ for each u, ao. Choosing $b = 16 \log |\mathcal{X}|/\mu_{\mathcal{X}}^2$ in Lemma 5, we get the following bound:

$$N \geq \max_{u,ao} \left\{ \frac{\log(4/\delta)}{d^{\mathsf{b}}(u, ao)} \left(\frac{32 \log |\mathcal{X}|}{\mu_{\mathcal{X}}^2} + 31/6 \right) \right\}.$$

Since $\mathcal{X}$ distinguishes $\mathbf{A}_{\mathsf{p}} \ltimes \mathbf{A}_{\mathsf{r}}$ and event $\mathcal{E}_{\mathcal{X}}$ holds, Lemma 8 now directly applies. It is sufficient to choose $\delta_0 = \delta/2UU_{\mathsf{p}}AO$ to ensure that events $\mathcal{E}_{\mathcal{X}}$ and $\mathcal{E}_B$ hold for all candidate states. Using the lower bound $d^{\mathsf{b}}(u, ao) \geq d^*(u, ao)/C_{\mathbf{R}}^* \geq d_{\mathsf{m}}^*/C_{\mathbf{R}}^*$ yields

$$N \geq \frac{C_{\mathbf{R}}^* \log(8UU_{\mathsf{p}}AO/\delta_0)}{d_{\mathsf{m}}^*} \left(\frac{32 \log |\mathcal{X}|}{\mu_{\mathcal{X}}^2} + 31/6 \right) = \widetilde{\mathcal{O}} \left(\frac{C_{\mathbf{R}}^* \log(1/\delta) \log |\mathcal{X}|}{d_{\mathsf{m}}^* \cdot \mu_{\mathcal{X}}^2} \right).$$

which concludes the proof. We remark that Deb et al. [2025] present an improved analysis for an approximate version of their algorithm, but we leave a similar analysis for future work.

B Examples

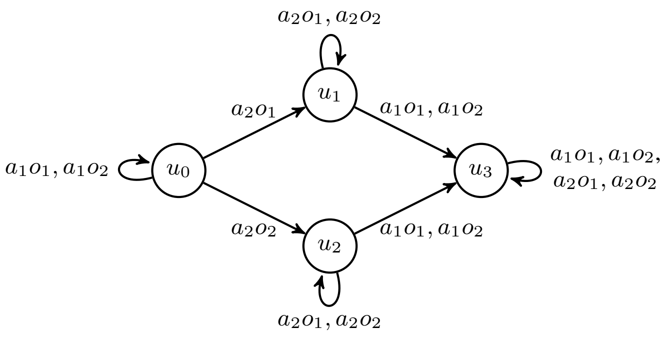
We provide several examples that help to understand important aspects of RDPs, as well as of our novel notions.

B.1 Example RDPs with a focus on distinguishability

Example 2. Consider an RDP defined by $\mathbf{R} = \langle \mathcal{O}, \mathcal{A}, \mathcal{R}, \bar{T}, \bar{R}, H, \nu \rangle$ and $\mathbf{A} = \langle \mathcal{U}, \Sigma, \Omega, \tau, \theta, u_0 \rangle$ with components given by

$$\mathcal{O} = \{o_1, o_2\}, \quad \mathcal{A} = \{a_1, a_2\}, \quad \mathcal{R} = \{0, 1\}, \quad \mathcal{U} = \{u_0, u_1, u_2, u_3\}.$$

The (semi-)automaton $\mathbf{A}$ is illustrated in the following figure:



The output function θ is defined as follows:

- $\theta_{\mathsf{o}}(o \mid u, a) = 0.5$ for each $o \in \mathcal{O}$, $u \in \{u_0, u_3\}$ and $a \in \mathcal{A}$.
- $\theta_{\mathsf{o}}(o \mid u, a_2) = 0.5$ for each $o \in \mathcal{O}$ and $u \in \{u_1, u_2\}$.
- $\theta_{\mathsf{o}}(o_1 \mid u_1, a_1) = \theta_{\mathsf{o}}(o_2 \mid u_2, a_1) = 0.75$.
- $\theta_{\mathsf{o}}(o_2 \mid u_1, a_1) = \theta_{\mathsf{o}}(o_1 \mid u_2, a_1) = 0.25$.
- $\theta_{\mathsf{r}}(0 \mid u, a) = 1$ for each $u \in \{u_0, u_3\}$ and $a \in \mathcal{A}$.
- $\theta_{\mathsf{r}}(1 \mid u, a_1) = 1$ for each $u \in \{u_1, u_2\}$.
- $\theta_{\mathsf{r}}(0 \mid u, a_2) = 1$ for each $u \in \{u_1, u_2\}$.

Let π be the regular policy defined as $\pi(a|u) = 0.5$ for each $a \in \mathcal{A}$ and each $u \in \mathcal{U}$. Let X be the language defined by the regular expression `.*(.`o_1`1).*`. Hence a string in $\Gamma^+ = (\mathcal{A}\mathcal{O}/\mathcal{R})^+$ belongs to X if and only if the observation-reward pair $o_1 1$ appears in the string. Let $\mathcal{X} = \{X\}$ be the language set containing only X.

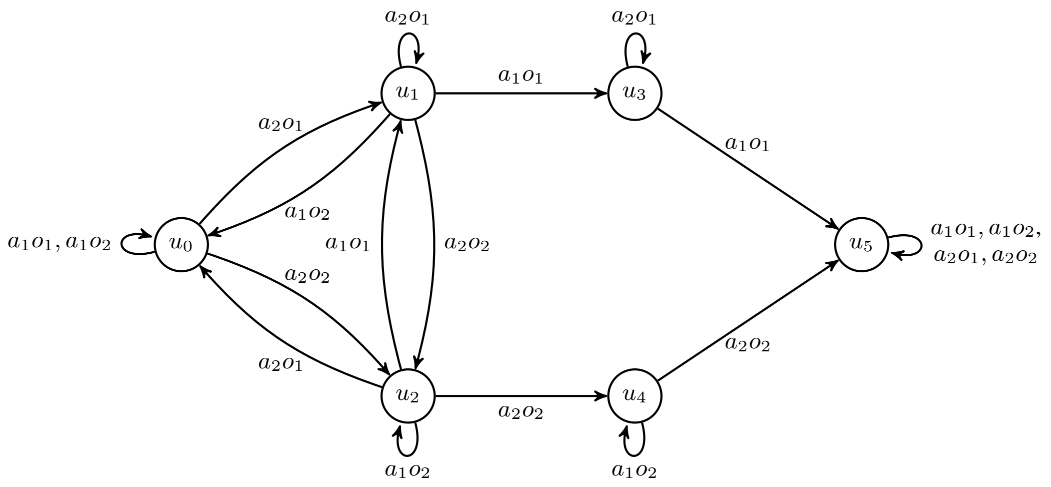
We claim that $\mathcal{X}$ distinguishes the RDP $\mathbf{R}$ under the regular policy π. For any history h mapping to state u_3, the probability of the language X is $p_h^\pi(X) = 0$ since the reward 1 can never appear. For any history h mapping to state u_1, eventually the policy π will select action a_1 and the probability of $o_1 1$ is $\theta_{\mathsf{o}}(o_1 \mid u_1, a_1)\theta_{\mathsf{r}}(1 \mid u_1, a_1) = 0.75 \cdot 1 = 0.75$, implying $p_h^\pi(X) = 0.75$. For any history h mapping to state u_2, eventually the policy π will select action a_1 and the probability of $o_1 1$ is $\theta_{\mathsf{o}}(o_1 \mid u_2, a_1)\theta_{\mathsf{r}}(1 \mid u_2, a_1) = 0.25 \cdot 1 = 0.25$, implying $p_h^\pi(X) = 0.25$. For any history h mapping to state u_0, eventually the policy π will select action a_2. This always causes a reward of 0 and transitions to u_1 or u_2 with equal probability. Hence the probability of $o_1 1$ is $0.5 \cdot 0.75 + 0.5 \cdot 0.25 = 0.5$, implying $p_h^\pi(X) = 0.5$.

As a consequence, given two histories $h, h' \in \mathcal{H}$, if $h \sim h'$ the language metric is given by $L_{\mathcal{X}}(p_h^\pi, p_{h'}^\pi) = |p_h^\pi(X) - p_{h'}^\pi(X)| = 0$, while if $h \not\sim h'$ we have $L_{\mathcal{X}}(p_h^\pi, p_{h'}^\pi) = |p_h^\pi(X) - p_{h'}^\pi(X)| \geq 0.25$. Hence $\mathcal{X}$ distinguishes $\mathbf{R}$ for π and has distinguishability $\mu_{\mathcal{X}} = 0.25$. ∎

Example 3. Another example RDP is the following one.

$$\mathcal{O} = \{o_1, o_2\}, \quad \mathcal{A} = \{a_1, a_2\}, \quad \mathcal{R} = \{0, 1\}, \quad \mathcal{U} = \{u_0, u_1, u_2, u_3, u_4, u_5\}.$$

The (semi-)automaton $\mathbf{A}$ is illustrated in the following figure:



The output function θ is defined as follows:

- $\theta_o(o \mid u, a) = 0.5$ for each $o \in \mathcal{O}$, $u \in \{u_0, u_5\}$ and $a \in \mathcal{A}$.
- $\theta_o(o_1 \mid u_1, a) = \theta_o(o_2 \mid u_2, a) = 0.75$ for each $a \in \mathcal{A}$.
- $\theta_o(o_2 \mid u_1, a) = \theta_o(o_1 \mid u_2, a) = 0.25$ for each $a \in \mathcal{A}$.
- $\theta_o(o_1 \mid u_3, a) = \theta_o(o_2 \mid u_4, a) = 1$ for each $a \in \mathcal{A}$.
- $\theta_r(0 \mid u, a) = 1$ for each $u \in \{u_0, u_1, u_2, u_5\}$ and $a \in \mathcal{A}$.
- $\theta_r(0 \mid u_3, a_2) = \theta_r(0 \mid u_4, a_1) = 1$.
- $\theta_r(1 \mid u_3, a_1) = \theta_r(1 \mid u_4, a_2) = 1$.

Consider the regular policy π defined as $\pi(a|u) = 0.5$ for each $u \in \mathcal{U}$ and $a \in \mathcal{A}$. Some facts about the RDP:

- From state u_5 we can never observe reward 1.
- From state u_3 we eventually observe $o_1 1$.
- From state u_4 we eventually observe $o_2 1$.
- From state u_1 we eventually reach u_3 with probability 0.75 and u_4 with probability 0.25.
- From state u_2 we eventually reach u_3 with probability 0.25 and u_4 with probability 0.75.
- From state u_0 we eventually reach u_3 with probability 0.5 and u_4 with probability 0.5.

To prove the last three facts, let p_0, p_1, p_2 be the probability of reaching u_3 from u_0, u_1, u_2 respectively. These probabilities satisfy the following system of linear equations:

$$\begin{aligned} p_0 &= 0.5p_1 + 0.5p_2, \\ p_1 &= 0.2p_0 + 0.2p_2 + 0.6, \\ p_2 &= 0.2p_0 + 0.2p_1. \end{aligned}$$

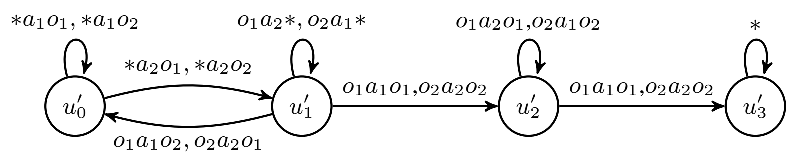
The solution is given by $p_0 = 0.5$, $p_1 = 0.75$, $p_2 = 0.25$.

Consider the language set $\mathcal{X} = \{X_1, X_2\}$, where X_1 is the language defined by the regular expression `.*(.`o_1`1).*` and X_2 is the language defined by the regular expression `.*(.`o_2`1).*`. For each state $u \in \mathcal{U}$, the probabilities of the two languages for histories h that map to u, i.e. $\bar{\tau}(h) = u$, are given by

$$\begin{array}{lll}
u_0\colon & p_h^\pi(X_1) = 0.5, & p_h^\pi(X_2) = 0.5, \\
u_1\colon & p_h^\pi(X_1) = 0.75, & p_h^\pi(X_2) = 0.25, \\
u_2\colon & p_h^\pi(X_1) = 0.25, & p_h^\pi(X_2) = 0.75, \\
u_3\colon & p_h^\pi(X_1) = 1, & p_h^\pi(X_2) = 0, \\
u_4\colon & p_h^\pi(X_1) = 0, & p_h^\pi(X_2) = 1, \\
u_5\colon & p_h^\pi(X_1) = 0, & p_h^\pi(X_2) = 0.
\end{array}$$

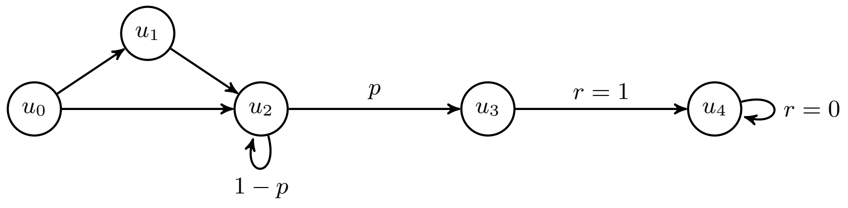
It is easy to verify that for the given language set $\mathcal{X}$ and two histories $h, h' \in \mathcal{H}$, $L_{\mathcal{X}}(p_h^\pi, p_{h'}^\pi) = 0$ if $h \sim h'$ and $L_{\mathcal{X}}(p_h^\pi, p_{h'}^\pi) \geq 0.25$ if $h \not\sim h'$. Hence $\mathcal{X}$ distinguishes $\mathbf{R}$ and the distinguishability is $\mu_{\mathcal{X}} = 0.25$.

We can represent the RDP more compactly using a cascade $\mathbf{A}_o \ltimes \mathbf{A}_r$, where $\mathbf{A}_o$ is a Markov prior and $\mathbf{A}_r$ is the following automaton:



Concretely, the state $o_1u'_1$ in the cascade corresponds to the state u_1 in the original RDP, while $o_2u'_1$ corresponds to u_2. Likewise, $o_1u'_2$ in the cascade corresponds to the state u_3 in the original RDP, while $o_2u'_2$ corresponds to u_4. Both $o_1u'_0$ and $o_2u'_0$ map to u_0, and both $o_1u'_3$ and $o_2u'_3$ map to u_5. Note that the automaton $\mathbf{A}_r$ is more compact than the original RDP. ∎

Example 4. A third example to illustrate the difficulty of suffixes with different lengths. Here I have omitted actions and observations and focus only on probability distributions over suffixes (under the given behavior policy). For simplicity, assume that all transitions are deterministic except for $u_2 \to u_3$, which has probability p (else the agent remains in u_2).



We can reach u_2 in two different ways: directly from u_0 (history h), or via u_1 (history h'). Let us assume that the only language in $\mathcal{X}$ checks if reward 1 is present in a suffix. The current algorithm will estimate $L_{\mathcal{X}}(p_h^\pi, p_{h'}^\pi)$ using two multisets of suffixes: one whose suffixes have length $H-1$, and one whose suffixes have length $H-2$.

The probability of *not* reaching u_3 in k steps is $(1-p)^k$, since the agent will attempt to reach u_3 every timestep and fails with probability $1-p$. Hence the probability of observing reward 1 in suffixes of length $H-1$ is $1-(1-p)^{H-2}$, and the probability of observing reward 1 in suffixes of length $H-2$ is $1-(1-p)^{H-3}$. To observe reward 1 in k steps we have to reach u_3 in $k-1$ steps to have time for the last transition from u_3 to u_4. For example, if $p = 0.1$ and $H = 10$ we have

$$\begin{aligned}
1 - (1-p)^{H-2} &= 1 - 0.9^8 = 0.57, \\
1 - (1-p)^{H-3} &= 1 - 0.9^7 = 0.52.
\end{aligned}$$

∎

B.2 Examples for Section 2 (Preliminaries)

Example 5. Specific policies may induce the same distribution for histories that are not equivalent, as noted in Observation 1. This phenomenon can be observed in the following example, which focuses on the probability of observations, omitting rewards since they follow the same argument,

$$\mathcal{A} = \{a_1, a_2\}, \quad \mathcal{O} = \{o_1, o_2\}, \quad \mathcal{U} = \{u_0, u_1, u_2\},$$

$$\tau(u_0, ao_1) = u_1 \;\; \forall a \in \mathcal{A}, \quad \tau(u_0, ao_2) = u_2 \;\; \forall a \in \mathcal{A}, \quad \tau(u_i, ao) = u_i \;\; \forall ao \in \mathcal{AO}, \forall i \in \{1, 2\},$$

$$\begin{aligned}
&\theta_\mathsf{o}(o_1 \mid u_1, a_1) = 0.1, \;\; \theta_\mathsf{o}(o_1 \mid u_1, a_2) = 0.9, \;\; \theta_\mathsf{o}(o_2 \mid u_1, a_1) = 0.9, \;\; \theta_\mathsf{o}(o_2 \mid u_1, a_2) = 0.1,\\
&\theta_\mathsf{o}(o_1 \mid u_2, a_1) = 0.9, \;\; \theta_\mathsf{o}(o_1 \mid u_2, a_2) = 0.1, \;\; \theta_\mathsf{o}(o_2 \mid u_2, a_1) = 0.1, \;\; \theta_\mathsf{o}(o_2 \mid u_2, a_2) = 0.9.
\end{aligned}$$

In this example, a regular policy causing the collapse of distributions over observations determined by the two different states u_1, u_2 is the following one, defined as a function of RDP states,

$$\pi(a_1 \mid u_1) = 0.9, \quad \pi(a_2 \mid u_1) = 0.1, \quad \pi(a_1 \mid u_2) = 0.1, \quad \pi(a_2 \mid u_2) = 0.9.$$

For instance, we have that the probability of o_1 coincides in the two states u_1 and u_2,

$$\begin{aligned}
\mathbb{P}(o_1 \mid u_1, \pi) = \theta_\mathsf{o}(o_1 \mid u_1, a_1) \cdot \pi(a_1 \mid u_1) + \theta_\mathsf{o}(o_1 \mid u_1, a_2) \cdot \pi(a_2 \mid u_1) = 0.18,\\
\mathbb{P}(o_1 \mid u_1, \pi) = \theta_\mathsf{o}(o_1 \mid u_1, a_1) \cdot \pi(a_1 \mid u_1) + \theta_\mathsf{o}(o_1 \mid u_1, a_2) \cdot \pi(a_2 \mid u_1) = 0.18.
\end{aligned}$$

Similarly for o_2, we have $\mathbb{P}(o_2 \mid u_1, \pi) = \mathbb{P}(o_2 \mid u_2, \pi) = 0.82$. In general $p^\pi_{h_1} = p^\pi_{h_2}$ for histories h_1, h_2 mapping to u_1, u_2 respectively, even though $h_1 \not\sim h_2$ since $u_1 \neq u_2$. ∎

B.3 Extended version of Example 1 (Partial independence from priors)

The T-maze with corridor length N and horizon H has observations, actions, and rewards,

$$\begin{aligned}
&\mathcal{O} = \{\mathit{InCorridor}, \mathit{InJunction}, \mathit{GoalNorth}, \mathit{GoalSouth}\},\\
&\mathcal{A} = \{\mathit{North}, \mathit{South}, \mathit{East}, \mathit{West}\},\\
&\mathcal{R} = \{0, 1\}\\
&\mathcal{U} = \big(\{\mathit{corridor}\} \times [\![0, N]\!] \cup \{\mathit{junction}\} \times [\![-1, +1]\!]\big) \times \{\mathit{GoalNorth}, \mathit{GoalSouth}\},
\end{aligned}$$

and it is represented by the cascade cascade $\mathbf{T}_H \times \mathbf{A}$ where $\mathbf{T}_H$ is the timestep prior and the semiautomaton $\mathbf{A} = \langle \mathcal{U}, \mathcal{AO}, \tau, u_0 \rangle$ is defined as follows.

States,

$$\mathcal{U} = \{u_0\} \cup \Big(\big(\{\mathit{corridor}\} \times [\![0, N]\!] \cup \{\mathit{junction}\} \times [\![-1, +1]\!]\big) \times \{\mathit{GoalNorth}, \mathit{GoalSouth}\}\Big).$$

The transition function is defined as follows, where all variables range over their entire respective domains,

$$\tau(u_0, a\,\mathit{goal}) = \begin{cases} \langle \mathit{corridor}, 1, \mathit{goal} \rangle & \text{if } a = \mathit{West} \\ \langle \mathit{corridor}, 0, \mathit{goal} \rangle & \text{otherwise} \end{cases}$$

$$\tau(\mathit{corridor}, x, \mathit{goal}, ao) = \begin{cases} \langle \mathit{corridor}, x, \mathit{goal} \rangle & \text{if } a = \mathit{North} \text{ or } a = \mathit{South} \\ \langle \mathit{corridor}, \max(0, x-1), \mathit{goal} \rangle & \text{if } a = \mathit{East} \\ \langle \mathit{corridor}, x+1, \mathit{goal} \rangle & \text{if } a = \mathit{West} \text{ and } x < N \\ \langle \mathit{junction}, 0, \mathit{goal} \rangle & \text{if } a = \mathit{West} \text{ and } x = N \end{cases}$$

$$\tau(\mathit{junction}, y, \mathit{goal}, ao) = \begin{cases} \langle \mathit{junction}, \min(1, y+1), \mathit{goal} \rangle & \text{if } a = \mathit{North} \\ \langle \mathit{junction}, \max(-1, y-1), \mathit{goal} \rangle & \text{if } a = \mathit{South} \\ \langle \mathit{junction}, y, \mathit{goal} \rangle & \text{if } a = \mathit{West} \\ \langle \mathit{junction}, y, \mathit{goal} \rangle & \text{if } a = \mathit{East} \text{ and } y \neq 0 \\ \langle \mathit{corridor}, N, \mathit{goal} \rangle & \text{if } a = \mathit{East} \text{ and } y = 0 \end{cases}$$

Let $\perp$ represents the observation symbol marking the end of an episode. When the symbol is produced, the generated episode trace is to be considered complete.

The (deterministic) observation output function over the cascade state space $\theta_{\mathsf{o}} : [\![0, H]\!] \times \mathcal{U} \times \mathcal{A} \to \mathcal{O}$ is defined as follows, where t ranges over $[\![0, H-1]\!]$.

$$\theta_{\mathsf{o}}(t, \mathit{corridor}, x, \mathit{goal}, a) = \begin{cases} \mathit{corridor} & \text{if } a = \mathit{North} \text{ or } a = \mathit{South} \\ \mathit{corridor} & \text{if } a = \mathit{East} \\ \mathit{corridor} & \text{if } a = \mathit{West} \text{ and } x < N \\ \mathit{junction} & \text{if } a = \mathit{West} \text{ and } x = N \end{cases}$$

$$\theta_{\mathsf{o}}(t, \mathit{junction}, y, \mathit{goal}, a) = \begin{cases} \mathit{junction} & \text{if } a = \mathit{North} \\ \mathit{junction} & \text{if } a = \mathit{South} \\ \mathit{junction} & \text{if } a = \mathit{West} \\ \mathit{junction} & \text{if } a = \mathit{East} \text{ and } y \neq 0 \\ \mathit{corridor} & \text{if } a = \mathit{East} \text{ and } y = 0 \end{cases}$$

$$\theta_{\mathsf{o}}(H, u_1, u_2, u_3, a, o) = \bot$$

The (deterministic) reward output function over the cascade state space $\theta_{\mathsf{r}} : [\![0, H]\!] \times \mathcal{U} \times \mathcal{A} \to \mathcal{R}$ is defined as follows, where all variables range over their entire respective domains (including t),

$$\theta_{\mathsf{r}}(t, \mathit{corridor}, x, \mathit{goal}, a) = 0$$

$$\theta_{\mathsf{r}}(t, \mathit{junction}, y, \mathit{goal}, a) = \begin{cases} 1 & \text{if } y = 0 \text{ and } a = \mathit{North} \text{ and } \mathit{goal} = \mathit{GoalNorth} \\ 1 & \text{if } y = 0 \text{ and } a = \mathit{South} \text{ and } \mathit{goal} = \mathit{GoalSouth} \\ 0 & \text{otherwise} \end{cases}$$

Showing partial independence from the timestep prior (semi-stationarity) The automaton above already satisfies the cascade condition (I) since it is given by $\mathbf{T}_H \times \mathbf{A}$. We show its output functions satisfy conditions (II) and (III). The observation output function (seen as returning distributions) can be factored into the following two functions,

$$\theta_{\mathsf{o}}^{\mathrm{r}}(o \mid \mathit{corridor}, x, \mathit{goal}, a) = \begin{cases} 1 & \text{if } o = \theta_{\mathsf{o}}(\mathit{corridor}, x, \mathit{goal}, a) \\ 0 & \text{otherwise} \end{cases}$$

$$\theta_{\mathsf{o}}^{\mathrm{t}}(o \mid t, a) = \begin{cases} 1 & \text{if } o \neq \bot \text{ and} \\ 1 & \text{if } o = \bot \text{ and } t = H \\ 0 & \text{otherwise} \end{cases}$$

Remark 1. The above function $\theta_{\mathsf{o}}^{\mathrm{r}}$ does not specify episode termination, and hence, at learning time, the distributions it induces must be assessed by a language metric that ignores string length—as we do when relevant in our experiments.

The reward output function (seen as returning distributions) can be factored into the following functions, where all variables range over their entire respective domains,

$$\theta_{\mathsf{r}}^{\mathrm{r}}(r \mid \mathit{corridor}, x, \mathit{goal}, a) = 0$$

$$\theta_{\mathsf{r}}^{\mathrm{r}}(r \mid \mathit{junction}, y, \mathit{goal}, a) = \begin{cases} 1 & \text{if } r = \theta_{\mathsf{r}}(\mathit{junction}, y, \mathit{goal}, a) \\ 0 & \text{otherwise} \end{cases}$$

and

$$\theta_{\mathsf{r}}^{\mathrm{t}}(r \mid t, a) = 1$$

The above shows that the T-maze is partially independent from the timestep prior, i.e., it is semi-stationary.

Preserving rewards only In the T-maze automaton, we can also factor out a spatial prior as $(\mathbf{T}_H \times \mathbf{S}) \ltimes \mathbf{A}$ where $\mathbf{S}$ describes the space of the maze, using states

$$\mathcal{U}_\mathrm{s} = \{\mathit{corridor}\} \times [\![0, N]\!] \cup \{\mathit{junction}\} \times [\![-1, +1]\!].$$

Note that we could also use the spatial prior $\mathbf{G}_{3\times(N+1)}$ (introduced earlier) as a correct over-approximation. However, introducing the independence $(\mathbf{T}_H \times \mathbf{S}) \times \mathbf{A}$ allows only for representing an approximation of the original automaton. Specifically, we can still represent the reward function exactly, clear from the fact that the function θ_r above is independent of its first three arguments. However, we can no longer represent precisely distributions on observations, since the function θ_o depends on its second and third argument. The advantage is that the domain-specific automaton $\mathbf{A}$ is very compact. It only needs to remember the goal position communicated at the beginning of an episode, and it can do so by using the two states $\{\mathit{GoalNorth}, \mathit{GoalSouth}\}$.