

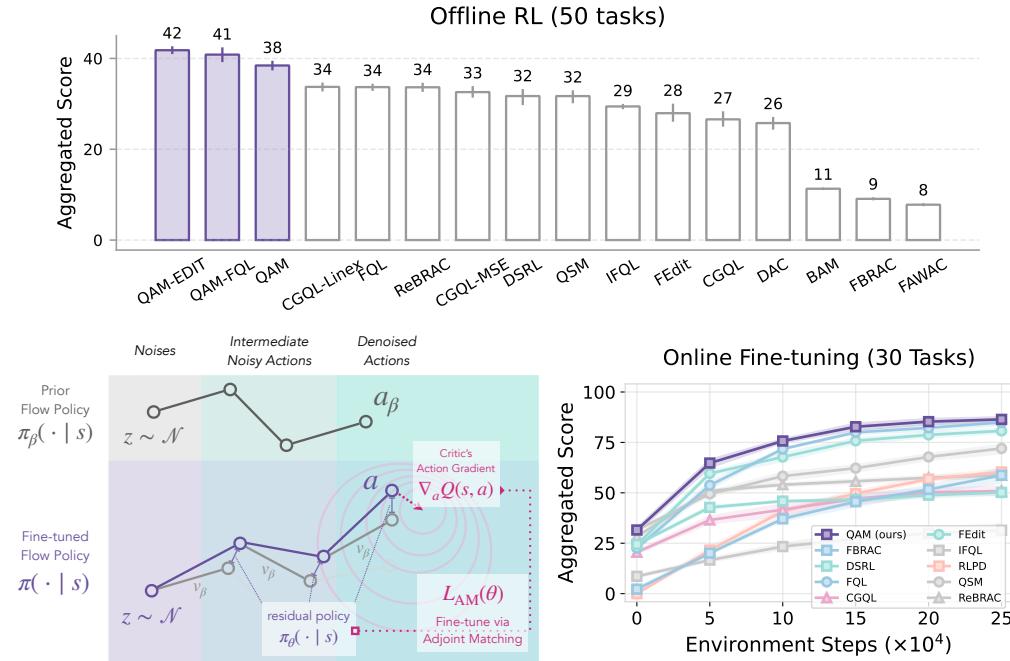
Q-LEARNING WITH ADJOINT MATCHING

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ABSTRACT

010 We propose Q-learning with Adjoint Matching (QAM), a novel TD-based reinforcement
011 learning (RL) algorithm that tackles a long-standing challenge in continuous-
012 action RL: efficient optimization of an expressive diffusion or flow-matching policy
013 with respect to a parameterized value function (*i.e.*, the critic $Q_\phi(s, a)$). Effective
014 optimization requires exploiting the first-order information of the critic (*i.e.*, the
015 action gradient, $\nabla_a Q_\phi(s, a)$), but it is challenging to do so for flow or diffusion
016 policies because direct gradient-based optimization via backpropagation through
017 their multi-step denoising process is numerically unstable. Existing methods work
018 around this either by only using the value and discarding the gradient information,
019 or by relying on approximations that sacrifice policy expressivity or bias the learned
020 policy. QAM sidesteps both of these challenges by leveraging adjoint matching, a
021 recently proposed technique in generative modeling, which transforms the critic’s
022 action gradient to form a step-wise objective function that is free from unstable
023 backpropagation, while providing an unbiased, expressive policy at the optimum.
024 Combined with temporal-difference (TD) backup for critic learning, QAM con-
025 sistently outperforms prior approaches across challenging, sparse reward tasks in
026 both offline and offline-to-online RL settings.
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1 INTRODUCTION



051 **Figure 1: QAM: Q-learning with Adjoint Matching.** *Bottom-left:* QAM uses adjoint matching ob-
052 jective (Domingo-Enrich et al., 2025) that leverages the critic’s action gradient directly to fine-tune a residual
053 flow policy such that the combined policy converges to the optimal prior-constrained policy: $\pi(\cdot | s) \propto \pi_\beta(\cdot | s) e^{Q(s, \cdot)}$. *Top/Bottom-right:* Aggregated score for offline RL and online fine-tuning (8 seeds).

054 A long-standing tension in continuous-action reinforcement learning (RL) especially in the
 055 offline/offline-to-online setting is between policy expressivity and optimization tractability with re-
 056 spect to a critic (*i.e.*, $Q(s, a)$). Simple policies, such as single-step Gaussian policies, are easy to
 057 train, since they can directly leverage the critic’s action gradient (*i.e.*, $\nabla_a Q(s, a)$) via the repara-
 058 meterization trick (Haarnoja et al., 2018). This optimization tractability, however, often comes at the
 059 cost of expressivity. Some of the most expressive policy classes today, such as flow policies, generate
 060 actions through a multi-step denoising process. While this allows flow policies to represent complex,
 061 multi-modal action distributions, leveraging the action gradient requires backpropagation through the
 062 entire denoising process, which often leads to instability (Park et al., 2025b). Prior work has there-
 063 fore resorted to either (1) discarding the critic’s action gradient entirely and only using its value (Ren
 064 et al., 2024; Zhang et al., 2025; McAllister et al., 2025), or (2) distilling expressive, multi-step flow
 065 policies into one-step noise-conditioned approximations (Park et al., 2025b). The former sacrifices
 066 learning efficiency and often under-performs methods that use the critic’s action gradient (Park et al.,
 067 2024b; 2025b), while the latter compromises expressivity. This raises a question: can we somehow
 068 keep the full expressivity of flow policies while incorporating the critic’s action gradient directly into
 069 the denoising process without backpropagation instability?
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071 One might be tempted to directly apply the critic’s action gradient to intermediate noisy actions
 072 within the denoising process, as in diffusion classifier guidance (with the critic function being the
 073 classifier) (Dhariwal & Nichol, 2021). Intuitively, this blends two generative process together: one
 074 that generates a behavior action distribution, and another that hill-climbs the critic to maximize
 075 action value. While this approach bypasses the backpropagation instability and retains full policy
 076 expressivity, it relies on the assumption that the critic’s gradient at a noisy action is a good proxy for
 077 its gradient at the corresponding denoised action. In practice, this assumption often breaks down:
 078 when the offline dataset has limited action coverage, the critic is well-trained only on a narrow
 079 distribution of noiseless actions, rendering its gradients unreliable for intermediate noisy actions that
 080 are out of distribution. As what we will show in our experiments, methods that use the gradients at
 081 intermediate noisy actions underperform (CGQL in Section 5, Figure 2).
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083 We propose **Q-learning with Adjoint Matching (QAM)**, a novel RL algorithm that leverages adjoint
 084 matching (Domingo-Enrich et al., 2025), a recently developed technique in generative modeling, to
 085 effectively use the critic’s action gradient for training flow policies to maximize returns subject to
 086 a prior constraint (*e.g.*, behavior or entropy constraint) (Figure 1). In general, such a constrained
 087 optimization problem on a flow model can be formulated as a stochastic optimal control (SOC)
 088 objective, which can be solved by using the continuous adjoint method (Pontryagin et al., 1962).
 089 However, this standard formulation has the same loss landscape as directly backpropagating through
 090 the SOC objective, causing instability. Instead, we leverage a modified objective from Domingo-
 091 Enrich et al. (2025) that admits the same optimal solution, but does not suffer from the instability
 092 challenge. At a high level, the critic’s gradient at noiseless actions is directly transformed by a flow
 093 model constructed from the prior, independent from the possibly ill-conditioned flow model that
 094 is being optimized, to construct unbiased gradient estimates for optimizing the state-conditioned
 095 velocity field at intermediate denoising steps. This allows the flow policy’s velocity field to align
 096 directly with the optimal state-conditioned velocity field implied by the critic and the prior, without
 097 direct and potentially unstable backpropagation, while preserving the full expressivity of multi-step
 098 flow models. By combining this policy extraction procedure with a standard temporal-difference
 099 (TD) backup for critic learning, QAM enables the flow policy to efficiently converge to the optimal
 100 policy subject to the prior constraint. In contrast, approximation methods that rely on the critic’s
 101 gradients at noisy intermediate actions lack such convergence guarantees.
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103 Our main contribution is a novel TD-based RL algorithm that leverages adjoint matching to perform
 104 policy extraction effectively on a critic function. Unlike prior Q-learning methods with flow-
 105 matching that rely on approximations or throwing away the action gradient of the critic altogether,
 106 our algorithm directly uses the gradient to form an objective that at convergence recovers the
 107 optimal behavior-regularized policy. We conduct a comprehensive empirical study comparing policy
 108 extraction methods for flow/diffusion policies, including recent approaches and new baselines, and
 109 show that QAM consistently achieves strong performance across both offline RL and offline-to-online
 110 RL benchmarks.
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2 RELATED WORK111
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RL with diffusion and flow policies. Diffusion and flow policies have been explored in both policy
gradient methods (Ren et al., 2024) and actor-critic methods (Fang et al., 2025; Kang et al., 2023;
Chen et al., 2024c;a; Lu et al., 2023b; Ding et al., 2024b; Wang et al., 2023; He et al., 2023a; Ding &
Jin, 2024; Ada et al., 2024; Zhang et al., 2024; Hansen-Estruch et al., 2023). The key challenge of
leveraging diffusion/flow policies in TD-based RL methods is to optimize these policies against the
critic function (*i.e.*, $Q(s, a)$). Prior work can be largely put into three categories based on how the
value function is used:117
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(1) *Post-processing* approaches refine the action distribution from a base diffusion/flow policy with
rejection sampling based on the critic value (Hansen-Estruch et al., 2023; Mark et al., 2024; Li
et al., 2025; Dong et al., 2025), or using additional gradient steps to hill climb the critic (Mark et al.,
2024) (*i.e.*, $a_t \leftarrow a_t + \nabla_a Q(s, a)$). These approaches often reliably improve the quality of extracted
policy but at the expense of additional computation during evaluation or even training (*i.e.*, rejection
sampling for value backup target (Li et al., 2025; Dong et al., 2025)). Alternatively, one may train
a residual policy that modifies a base behavior policy in either the noise space (Singh et al., 2020;
Wagenmaker et al., 2025) or in the action space directly (Yuan et al., 2024; Dong et al., 2025).125
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(2) *Backprop-based* approaches perform direct backpropagation through both the critic and the
policy (Wang et al., 2023; He et al., 2023b; Ding & Jin, 2023; Zhang et al., 2024; Park et al., 2025b;
Espinosa-Dice et al., 2025; Chen et al., 2025). While this is the most-straightforward implementation-
wise, it requires backpropagation through the diffusion/flow policy’s denoising process which has
been observed to be unstable Park et al. (2025b), or instead learns a distilled policy (Ding & Jin,
2023; Chen et al., 2024b; Park et al., 2025b; Espinosa-Dice et al., 2025; Chen et al., 2025), in the
expense of policy expressivity.132
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(3) *Intermediate fine-tuning* approaches, which our method also belongs to, mitigate the need of the
stability/expressivity trade-off in *backprop-based* approaches by leveraging the critic to construct an
objective that provides direct step-wise supervision to the intermediate denoising process (Psenka
et al., 2023; Fang et al., 2025; Ding et al., 2024a; Li et al., 2024b; Frans et al., 2025; Zhang et al.,
2025; Ma et al., 2025; Koirala & Fleming, 2025). While these approaches remove the need for
backpropagation through the denoising process completely, the challenge lies in carefully crafting
the step-wise objective that does not introduce additional biases and learning instability. Compared
to prior methods that either rely on approximations (Lu et al., 2023a; Fang et al., 2025) that do not
provide theoretical guarantees (see more discussions in Appendix A) or directly throwing away the
critic’s action gradient (and use its value instead) (Ding et al., 2024a; Zhang et al., 2025; Ma et al.,
2025; Koirala & Fleming, 2025), we leverage adjoint matching (Domingo-Enrich et al., 2025) which
allows us to use the critic’s action gradient directly to construct an direct step-wise objective for our
flow policy that recovers the optimal prior regularized policy at the optimum of the objective.145
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Offline-to-online reinforcement learning methods focus on leveraging offline RL to first pretrain
on an offline dataset, and then use the pretrained policy and value function(s) as initialization to
accelerate online RL (Xie et al., 2021; Song et al., 2023; Lee et al., 2022; Agarwal et al., 2022;
Zhang et al., 2023; Zheng et al., 2023; Ball et al., 2023; Nakamoto et al., 2024; Li et al., 2024a;
Wilcoxon et al., 2024; Zhou et al., 2025). While it is possible to skip the offline pre-training phase
altogether and use online RL methods directly by treating the offline dataset as additional off-policy
data that is pre-loaded into the replay buffer (Lee et al., 2022; Song et al., 2023; Ball et al., 2023),
these methods often under-perform the methods that leverage explicit offline pre-training, especially
on more challenging tasks (Nakamoto et al., 2024; Park et al., 2025b). Our method also operates in
this regime where we first perform offline RL pre-training and then perform online fine-tuning from
the offline pre-trained initialization. In addition, we follow a common design in prior work where
the same offline RL objective is used for both offline pre-training and online fine-tuning (Kostrikov
et al., 2021; Fujimoto & Gu, 2021; Tarasov et al., 2023; Park et al., 2025b). While we focus on
evaluating our method in the offline-to-online RL setting, the idea of using adjoint matching to train
an expressive flow policy can be applied to other settings such as online RL.159
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Diffusion and flow-matching with guidance. Diffusion and flow-matching models have been used
for generating data with different modalities ranging from images (Rombach et al., 2022), videos (Ho
et al., 2022), and text (Lou et al., 2023). In most applications, the generative models trained on large-
scale unlabeled data do not provide high-quality samples when conditioned on some context (*e.g.*,

language description), and a common practice is to augment the sampling process with classifier guidance/classifier-free guidance (Dhariwal & Nichol, 2021; Ho & Salimans, 2022), with the goal of aligning the sampling distribution better with the posterior distribution conditioned on the context. However, most of the guidance methods suffer from a bias problem that is tricky to tackle, stemming from the fact that simply adding or interpolating two diffusion/flow-matching sampling processes (*i.e.*, $v^1(\cdot, t) + v^2(\cdot, t)$ for flow or $\log p_t^1 + \log p_t^2$ for diffusion) does not lead to the correct composite distribution (*i.e.*, $\propto \pi_1 \pi_2$) in general (Du et al., 2023; Bradley & Nakkiran, 2024). One solution is to use Langevin dynamics sampling approaches (Song & Ermon, 2019) where only the score function for the noise-free distribution is required, but they have been known to under-perform diffusion/flow models due to the challenge of accurately estimating the score functions in low-density regions (Song & Ermon, 2020). Since then, a line of work has proposed solutions to generate the correct composite distribution. Du et al. (2023), Phillips et al. (2024), Thornton et al. (2025), Singhal et al. (2025) and Skreta et al. (2025) propose to use Sequential Monte Carlo (SMC) that uses resampling procedures to leverage additional test-time compute to correct such bias. Rather than correcting the distribution at test-time, Domingo-Enrich et al. (2025) and Havens et al. (2025) take a different perspective by formulating it as a stochastic optimal control (SOC) objective that can be efficiently optimized as a fine-tuning process while providing guarantee that the model converges to the correct distribution at the optimum. The flow/diffusion policy optimization problem in actor-critic RL methods shares a similarity to the aforementioned classifier/classifier-free guidance problem in generative modeling literature where the critic function serves as the guidance to the generative policy model. This allows our method builds directly on top of the algorithm developed by Domingo-Enrich et al. (2025) while enjoying the guarantee that our policy converges to the optimal prior regularized solution (*i.e.*, $\pi \propto \pi_\beta \exp(Q(s, a))$).

3 PRELIMINARIES

Reinforcement learning and problem setup. We consider a Markov Decision Process (MDP), $\mathcal{M} = (\mathcal{S}, \mathcal{A}, P, \gamma, R, \mu)$, where \mathcal{S} is the state space, $\mathcal{A} = \mathbb{R}^A$ ($A \in \mathbb{Z}^+$) is the action space, $P : \mathcal{S} \times \mathcal{A} \rightarrow \Delta_{\mathcal{S}}$ is the transition function, $\gamma \in [0, 1]$ is the discount factor, $R : \mathcal{S} \times \mathbb{R}^A \rightarrow \mathbb{R}$ is the reward function, and $\mu \in \Delta_{\mathcal{A}}$ is the initial state distribution. We have access to a dataset D consisting of a set of transitions $\{(s_i, a_i, s'_i, r_i)\}_{i=1}^{|D|}$, where $s' \sim P(\cdot | s, a)$ and $r = R(s, a)$. Our first goal (*offline RL*) is to learn a policy $\pi_\theta : \mathcal{S} \rightarrow \mathcal{A}$ from D that maximizes its expected discounted return,

$$U_\pi = \mathbb{E}_{s_0 \sim \mu, s_{t+1} \sim P(\cdot | s_t, a_t), a_t \sim \pi(\cdot | s_t)} \left[\sum_{t=0}^{\infty} \gamma^t R(s_t, a_t) \right]. \quad (1)$$

The second goal (*offline-to-online RL*) is to fine-tune the offline pre-trained policy π_θ by continuously interacting with the MDP through trajectory episodes with a task/environment dependent maximum episode length of H (*i.e.*, the maximum number of time steps before the agent is reset to μ). The central challenge of offline-to-online RL is to maximally leverage the behavior prior π_β in D to learn as sample-efficiently as possible online.

Flow-matching generative model. A flow model uses a time-variant velocity field $v : \mathbb{R}^d \times [0, 1] \rightarrow \mathbb{R}^d$ to estimate the marginal distribution of a denoising process from noise, $X_0 = \mathcal{N}(0, I_d)$, to data, $X_1 = D$, at each intermediate time $t \in [0, 1]$:

$$X_t = (1 - t)X_0 + tX_1. \quad (2)$$

In particular, the flow model approximates the intermediate X_t via an ordinary differential equation (ODE) starting from the noise: $X^0 = \mathcal{N}$:

$$d\hat{X}_t = \mathbf{f}(\hat{X}_t, t)dt. \quad (3)$$

Flow models are typically trained with a *flow matching* objective (Liu et al., 2022):

$$L_{\text{FM}}(\theta) = \mathbb{E}_{t \sim \mathcal{U}[0,1], x_0 \sim \mathcal{N}, x_1 \sim D} [\|\mathbf{f}_\theta((1 - t)x_0 + tx_1, t) - x_1 + x_0\|_2^2], \quad (4)$$

where any optimal velocity field, $v_{\theta*}$, results in \hat{X}_t where its marginal distribution $p_f(x_t)$ exactly recovers the marginal distribution of the original denoising process X_t , $p_D(x_t)$, for each $t \in$

[0, 1] (Lipman et al., 2024). Furthermore, one may use the Fokker-Planck equations to construct a family of stochastic differential equations (SDE) that admits the same marginals as well:

$$d\hat{X}_t = \left(\mathbf{f}(\hat{X}_t, t) + \frac{\sigma_t^2 t}{2(1-t)} \left(\mathbf{f}(\hat{X}_t, t) + X_t/t \right) \right) dt + \sigma_t dB_t \quad (5)$$

with B_t being a Brownian motion and $\sigma_t > 0$ being any noise schedule.

Adjoint matching is a technique developed by Domingo-Enrich et al. (2025) with the goal of modifying a base flow generative model f_β such that it generates the following *tilt* distribution:

$$p^*(x_1) \propto p_\beta(x_1) e^{\mathbf{Q}(x_1)} \quad (6)$$

where $\mathbf{Q} : \mathbb{R}^d \rightarrow \mathbb{R}$ is any value function that up-weights or down-weights the probability of each example in the domain \mathbb{R}^d . Domingo-Enrich et al. (2025) uses a marginal-preserving SDE with a ‘memoryless’ noise schedule (i.e., X_0 and X_1 are independent), $\sigma_t = \sqrt{2(1-t)/t}$:

$$dX^t = (2\mathbf{f}(X_t, t) - X_t/t) dt + \sqrt{2(1-t)/t} dB_t, \quad (7)$$

because solving the following stochastic optimal control equation (with X^t sampling from the joint distribution defined by the SDE in Equation (7)),

$$L(\theta) = \mathbb{E}_{\mathbf{X}=\{X_t\}_t} \left[\int_0^1 \left(\frac{1}{2} \|\mathbf{f}_\theta(X_t, t) - \mathbf{f}_\beta(X_t, t)\|_2^2 \right) - \mathbf{Q}(X_1) \right] \quad (8)$$

gives the correct marginal *tilt* distribution for X^1 :

$$p(X_1) \propto p_\beta(X_1) e^{\mathbf{Q}(X_1)}. \quad (9)$$

Let the adjoint state be the gradient of the tilt function applied at the denoised X_1 :

$$g(\mathbf{X}, t) = \nabla_{X_t} \left[\int_t^1 \frac{1}{2} \|\mathbf{f}_\theta(X_{t'}, t') - \mathbf{f}_\beta(X_{t'}, t')\|_2^2 dt' - \mathbf{Q}(X_1) \right], \quad (10)$$

which satisfies the following ODE:

$$dg(\mathbf{X}, t) = -g(\mathbf{X}, t)^\top \nabla_{X_t} [2\mathbf{f}_\theta(X_t, t) - X_t/t] + \nabla_{X_t} \|\mathbf{f}_\theta(X_t, t) - \mathbf{f}_\beta(X_t, t)\|_2^2 / (2\sigma_t^2) dt \quad (11)$$

with the boundary condition $g(\mathbf{X}, 1) = -\nabla_{X_1} \mathbf{Q}(X_1)$. We can compute the adjoint states by stepping through the reverse ODE (which can be efficiently computed with the Jacobian-vector product (JVP) in most modern deep learning frameworks). Then, it can be shown that it equivalently optimizes the ‘basic’ adjoint matching objective below:

$$L_{\text{BAM}}(\theta) = \mathbb{E}_{\mathbf{X}} \left[\int_0^1 \|2(\mathbf{f}_\theta(X_t, t) - \mathbf{f}_\beta(X_t, t)) / \sigma_t + \sigma_t g(\mathbf{X}, t)\|_2^2 dt \right]. \quad (12)$$

The optimal \mathbf{f}_θ coincides with the optimal solution in the original SOC equation (Equation (8)), which gives the correct marginal distribution of X^1 as a result. However, the objective is equivalent to the objective used in the continuous adjoint method (Pontryagin et al., 1962) with its gradient equivalent to that of backpropagation through the denoising process.

Instead, Domingo-Enrich et al. (2025) derive the ‘lean’ adjoint state where all the terms in the adjoint state that are zero at the optimum are removed from the state. The ‘lean’ adjoint state satisfies the following ODE:

$$d\tilde{g}(\mathbf{X}, t) = -\tilde{g}(\mathbf{X}, t)^\top \nabla_{X_t} [2\mathbf{f}_\beta(X_t, t) - X_t/t] dt, \quad (13)$$

with the same boundary condition $\tilde{g}(\mathbf{X}, 1) = -\nabla_{X_1} \mathbf{Q}(X_1)$.

Note that computing the ‘lean’ adjoint state only requires the base flow model $\mathbf{f}_\beta(\mathbf{X}_t, t)$ and no longer needs to use $\mathbf{f}_\theta(\mathbf{X}_t, t)$ as needed in either the basic adjoint matching objective (Equation (12)) or naive backpropagation through the denoising process. The resulting adjoint matching objective is

$$L_{\text{AM}}(\theta) = \mathbb{E}_{\mathbf{X}} \left[\int_0^1 \|2(\mathbf{f}_\theta(X_t, t) - \mathbf{f}_\beta(X_t, t)) / \sigma_t + \sigma_t \tilde{g}(\mathbf{X}, t)\|_2^2 dt \right], \quad (14)$$

where again \mathbf{X} is sampled from the marginal preserving SDE in Equation (7). Because the terms omitted in the ‘lean’ adjoint state are zero at the optimum, and thus do not change the optimal solution for \mathbf{f}_θ . Thus, the optimal solution for the adjoint matching gives the correct tilt distribution.

270 4 Q-LEARNING WITH ADJOINT MATCHING (QAM)

272 In this section, we describe in details how our method leverages adjoint matching to directly align the
 273 flow policy to prior regularized optimal policy without suffering from backpropagation instability.

274 To start with, we first define the optimal policy that we want to learn as the solution of the best policy
 275 the under the standard KL behavior constraint:

$$276 \arg \max_{\pi} \mathbb{E}_{a \sim \pi(\cdot|s)}[Q(s, a)] \quad \text{s.t.} \quad D_{\text{KL}}(\pi_{\beta} \parallel \pi) \leq \epsilon(s). \quad (15)$$

278 or equivalently, for an appropriate $\tau(s)$,

$$280 \pi^*(\cdot | s) \propto \pi_{\beta}(\cdot | s) e^{\tau(s) Q_{\phi}(s, a)} \quad (16)$$

281 where $\tau : \mathcal{S} \rightarrow \mathbb{R}^+$ is the inverse temperature coefficient that controls the strength of the behavior
 282 constraint at each state.

284 We approximate the behavior policy using a flow-matching behavior policy, $\textcolor{blue}{f}_{\beta} : \mathcal{S} \times \mathbb{R}^A \times [0, 1] \rightarrow$
 285 \mathbb{R}^A that is optimized with the standard flow-matching objective:

$$286 L_{\text{FM}}(\beta) = \mathbb{E}_{(s, a) \sim D, u \sim [0, 1], z \sim \mathcal{N}} \left[\|\textcolor{blue}{f}_{\beta}(s, (1-u)z + uz, t) - a + z\|_2^2 \right] \quad (17)$$

288 We then parameterize our approximation of the optimal policy as a sum of the behavior flow model
 289 $\textcolor{blue}{f}_{\beta}$ and a residual flow model $\textcolor{blue}{f}_{\theta} : \mathcal{S} \times \mathbb{R}^A \times [0, 1] \rightarrow \mathbb{R}^A$ and solve the following SOC equation:

$$291 L(\theta) = \mathbb{E}_{s \sim D, a^u} \left[\int_0^1 \frac{1}{2} \|\textcolor{blue}{f}_{\theta}(s, a^u, t)\|_2^2 - \tau(s) Q_{\phi}(s, a^1) du \right], \quad (18)$$

294 where a^u is defined by the following ‘memoryless’ SDE (e.g., a^0 is independent from a^1):

$$295 296 da^u = (2\textcolor{blue}{f}_{\theta}(s, a^u, u) + 2\textcolor{blue}{f}_{\beta}(s, a^u, u) - a^u/u) du + \sqrt{2(1-u)/u} dB_u. \quad (19)$$

297 Similar to the derivation by Domingo-Enrich et al. (2025), the memoryless property allows us to
 298 directly conclude that the SOC equation has the optimum at

$$300 \pi_{\theta}(\cdot | s) \propto \pi_{\beta}(\cdot | s) e^{\tau(s) Q_{\phi}(s, a)} \quad (20)$$

301 where $\pi_{\theta}(\cdot | s)$ and $\pi_{\beta}(\cdot | s)$ are the corresponding action distributions defined by $\textcolor{blue}{f}_{\theta} + \textcolor{blue}{f}_{\beta}$ and $\textcolor{blue}{f}_{\beta}$.

303 However, directly solving the SOC equation involves backpropagation through time that introduces
 304 additional stability. To circumvent this issue, we use the adjoint matching objective proposed by
 305 Domingo-Enrich et al. (2025) (Equation (14)) to construct a similar objective for policy optimization
 306 in our case:

$$307 308 L_{\text{AM}}(\theta) = \mathbb{E}_{s \sim D, \{a^u\}_u} \left[\int_0^1 \|2\textcolor{blue}{f}_{\theta}(s, a^u, u)/\sigma_u + \sigma_u \tilde{g}^u\|_2^2 du \right] \quad (21)$$

309 where \tilde{g}^u is the ‘lean’ adjoint state defined by a reverse ODE constructed from a^u defined by the
 310 forward SDE:

$$312 d\tilde{g}^u = -\tilde{g}^{u\top} \nabla_{a^u} [2\textcolor{blue}{f}_{\beta}(s, a^u, u) - a^u/u] du. \quad (22)$$

314 Unlike the original SOC objective (Equation (18)) from which calculating the gradient requires back-
 315 propagating through an SDE, which suffers from stability challenges, the adjoint matching objective
 316 is constructed without backpropagation. Instead, it uses the behavior velocity field $\textcolor{blue}{f}_{\beta}$ to calculate
 317 the ‘lean’ adjoint states $\{\tilde{g}^u\}_u$ through a series of JVPs for every SDE trajectory $\{a^u\}_u$, which are
 318 then used to form a squared loss in the adjoint matching objective. Mathematically, backpropagation
 319 can also be interpreted as calculating the adjoint states through a series of JVPs, with the key distinc-
 320 tion that the JVPs are computed under the flow model that is being optimized (i.e., $\textcolor{blue}{f}_{\theta} + \textcolor{blue}{f}_{\beta}$). This is
 321 an important distinction because for direct backpropagation, any ill-conditioned action gradient in
 322 $\textcolor{blue}{f}_{\theta}$ (i.e., $\nabla_a \textcolor{blue}{f}_{\theta}(s, a, t)$) would compound over the entire denoising process, contributing to the ‘ill-
 323 condition-ness’ of the overall gradient to the parameter θ , which can in turn destabilize the whole
 324 optimization process. In contrast, in adjoint matching, action gradient of $\textcolor{blue}{f}_{\theta}$ has no contribution to the
 325 overall gradient to θ , which allows the optimization to be much more stable.

Finally, we combine the policy optimization with the standard critic learning objective in TD-based RL algorithms:

$$L(\phi) = \mathbb{E}_{s,a,s',r \sim D} [(Q(s,a) - r - \gamma Q_{\bar{\phi}}(s',a'))], \quad a' \leftarrow \text{ODE}(\mathbf{f}(s',\cdot,\cdot), a'^0 \sim \mathcal{N}) \quad (23)$$

where $\mathbf{f}(s,\cdot,\cdot) = \mathbf{f}_\beta(s,\cdot,\cdot) + \mathbf{f}_\theta(s,\cdot,\cdot)$ is the summation of the behavior velocity field and the residual velocity field and $\bar{\phi}$ is the exponential moving average of ϕ with a time-constant of $\lambda = 0.005$ (i.e., $\bar{\phi}_{i+1} \leftarrow (1 - \lambda)\bar{\phi}_i + \lambda\phi_i$ for each training step i).

Practical considerations. In practice, following Domingo-Enrich et al. (2025), we solve both the SDE and the reverse ODE with discrete approximation and a fixed step size of $h = 1/T$, where T is the number of discretization steps. In particular, with $a^0 \sim \mathcal{N}$ and $z^u \sim \mathcal{N}, \forall u \in \{0, h, \dots, (T-1)h\}$, the forward SDE process is approximated by

$$a^{u+h} \leftarrow a^u + h \cdot (2\mathbf{f}_\theta(s, a^u, u) + 2\mathbf{f}_\beta(s, a^u, u) - a^u/u) + \sqrt{2h(1-u)/u}z^u. \quad (24)$$

We set the boundary condition as $\tilde{g}^1 = -\tau \nabla_{a^1} Q_\phi(s, a^1)$, where we use a state *independent* inverse temperature coefficient τ to modulate the influence of the prior π_β and **we additionally clip the magnitude of the parameter gradient element-wise by 1 for numerical stability**. The backward adjoint state calculation process is then approximated by

$$\tilde{g}^{u-h} \leftarrow \tilde{g}^u + h \cdot \text{JVP}(\nabla_{a^u}(2\mathbf{f}_\beta(s, a^u, u) - a^u/u), \tilde{g}^u), \quad (25)$$

with $\text{JVP}(\nabla_y b(y), x) = x^\top \nabla_y b(y)$ being the Jacobian-vector product and it can be practically implemented by carrying the ‘gradient’ x with backpropagation through \mathbf{f} . For the critic, we use an ensemble of $K = 10$ critic functions ϕ^1, \dots, ϕ^K and use the pessimistic target value backup with a coefficient of $\rho = 0.5$ (Ghasemipour et al., 2022). The loss function for each $\phi^j, j \in \{1, 2, \dots, K\}$ is

$$L(\phi^j) = (Q_{\phi^j}(s, a) - r - \gamma [\bar{Q}_{\text{mean}}(s', a') - \rho \bar{Q}_{\text{std}}(s', a')])^2, \quad (26)$$

where $\bar{Q}_{\text{mean}}(s', a') := \frac{1}{K} \sum_k Q_{\bar{\phi}^k}(s', a')$, $\bar{Q}_{\text{std}}(s', a') = \sqrt{\sum_k (Q_{\bar{\phi}^k}(s', a') - \bar{Q}_{\text{mean}}(s', a'))^2}$, and a' is the action sampled from the combined flow model: $\mathbf{f}(s', \cdot, \cdot) = \mathbf{f}_\beta(s, \cdot, \cdot) + \mathbf{f}_\theta(s, \cdot, \cdot)$. For all our experiments, we do not use a separate training process for \mathbf{f}_β and instead training it at the same to as \mathbf{f}_θ and Q_ϕ , following Park et al. (2025b); Li et al. (2025), using the standard flow-matching objective described in Equation (17). For all our loss functions, the transition tuple (s, a, s', r) is drawn from D uniformly. During offline training, D is the offline data. During online fine-tuning, D is combination of the offline and online replay buffer data without any re-weighting.

Theoretical guarantees. As our algorithm builds off from Domingo-Enrich et al. (2025), we can directly extend their theoretical results to our setting as follows (proof in Appendix E):

Proposition 1 (Extension of Proposition 7 in Domingo-Enrich et al. (2025) to Policy Optimization.) Take $L_{\text{AM}}(\theta)$ in Equation (14), there is a unique f_θ such that

$$\frac{\partial}{\partial f_\theta} L_{\text{AM}} = 0, \quad (27)$$

and for all $s \in \text{supp}(D)$,

$$\pi_\theta(\cdot | s) \propto \pi_\beta(\cdot | s) e^{\tau Q_\phi(s, a)}. \quad (28)$$

The importance of this result is that as long as the loss function L_{AM} is optimized to convergence (i.e., with $\partial L / \partial f_\theta = 0$), the learned policy coincides with the optimal behavior-constrained policy.

Combining with existing flow/diffusion RL methods. In practice, we also find it to be beneficial to combine QAM with existing methods such as FQL (Park et al., 2025b) and edit policies (Dong et al., 2025) to further boost the performance. For the QAM-FQL variant, we learn the 1-step noise-conditioned policy from Park et al. (2025b), $\mu_\omega(s, z)$, and optimize the following objective (with α being a tunable BC coefficient),

$$L_{\text{QAM-FQL}}(\omega) = \mathbb{E}_{z \sim \mathcal{N}} [-Q_\phi(s, \mu_\omega(s, z)) + \alpha \|\mu_\omega(s, z) - \text{ODE}(f(s, \cdot, \cdot), z)\|_2^2]. \quad (29)$$

378 **Algorithm 1** Learning procedure in QAM.

379
 380 **Input:** (s, a, s', r) : off-policy transition tuple, \mathbf{f}_β : behavior velocity field, \mathbf{f}_θ residual velocity
 381 field, Q_ϕ : critic function.
 382 $\mathbf{f}(s, \cdot) \leftarrow \mathbf{f}_\theta(s, \cdot, \cdot) + \mathbf{f}_\beta(s, \cdot, \cdot)$ \triangleright *Memoryless SDE (Equation (24))*
 383 $\mathbf{a} = \{a^0, a^h, \dots, a^1\} \leftarrow \text{SDE}_{\text{am}}(\mathbf{f}(s, \cdot, \cdot))$ \triangleright *Computing the critic's action gradient*
 384 $\tilde{g}^1 \leftarrow -\tau \nabla_{a^1} Q_\phi(s, a^1)$
 385 $\tilde{g}^0, \tilde{g}^h, \dots, \tilde{g}^{1-h} \leftarrow \text{LeanAdj}_{\text{am}}(\mathbf{f}_\beta(s, \cdot, \cdot), \tilde{g}^1, \mathbf{a})$ \triangleright *Lean adjoint states (Equation (25))*
 386 Optimize θ w.r.t $L(\theta) = \sum_u \|2\mathbf{f}_\theta(s, a^u, u)/\sigma_u + \sigma_u \tilde{g}^u\|_2^2$ \triangleright *Adjoint matching (Equation (21))*
 387 $a' \leftarrow \text{ODE}(\mathbf{f}(s, \cdot, \cdot), z \sim \mathcal{N}(0, I_A))$
 388 Optimize ϕ w.r.t $L(\phi) = (Q_\phi(s, a) - r - \gamma Q_{\bar{\phi}}(s', a'))^2$
 389 **Output:** $\mathbf{f}_\theta, Q_\phi$

390
 391 We then use this 1-step policy both to interact with the environment and to compute value targets (*i.e.*,
 392 $\bar{Q}(s', a' = \mu(s, z \sim \mathcal{N}))$). Intuitively, the 1-step policy is optimized to remain close to the QAM-fine-
 393 tuned flow policy while also maximizing the action value under the current critic. For the QAM-EDIT
 394 variant, we optimize a Gaussian edit policy from Dong et al. (2025), $\pi_\omega(\cdot | s, \tilde{a})$, to modify the output
 395 from the QAM-fine-tuned flow policy (*i.e.*, \tilde{a}) for further action refinements with the objective below:
 396

397
$$L_{\text{QAM-EDIT}}(\omega) = \mathbb{E}_{\Delta a \sim \pi_\omega(\cdot | s, \tilde{a}), z \sim \mathcal{N}} [-Q_\phi(s, \Delta a + \tilde{a})], \quad \text{where } \tilde{a} := \text{ODE}(f(s, \cdot, \cdot), z) \quad (30)$$

398 where Δa is restricted to be within a L_∞ ball (with a tunable scaling parameter of σ_a). See more
 399 details on the implementation on the edit policy and the FQL 1-step policy in Appendix C.
 400

401 **5 EXPERIMENTS**

402 We conduct experiments to evaluate the effectiveness of our method on a range of long-horizon,
 403 sparse-reward domains and compare it against a set of representative baselines.

404 **Domains and datasets.** We consider 10 domains from OGbench (Park et al.,
 405 2024a): `scene`, `puzzle-3x3` (p33), `puzzle-4x4` (p44), `cube-double` (c2), `cube-triple`
 406 (c3), `cube-quadruple` (c4), `humanoidmaze-medium` (hm), `humanoidmaze-large` (hl),
 407 `antmaze-large` (al), and `antmaze-giant` (ag). For `antmaze-*` and `humanoidmaze-*`, we use
 408 the default navigate datasets. For `scene`, `puzzle-*`, and `cube-*`, we use the default play datasets
 409 except for c4 and p44 where we use the larger 100M-size dataset from Park et al. (2025a), and use
 410 the sparse reward definition for {p33, p44, scene}, following Li et al. (2025). All of these do-
 411 mains require the RL agent to solve long-horizon tasks from diverse offline behavior data that can
 412 only be accurately captured by expressive policies like flow/diffusion policies. Furthermore, the
 413 harder domains (Figure 4) are difficult to solve from offline data alone, making these benchmarks
 414 great for evaluating the online fine-tuning effectiveness of our approach. In addition, for all {cube-*,
 415 `scene-*`, p33-*, p44-*} domains, we follow Li et al. (2025) to learn action chunking policies
 416 with an action chunking size of $h = 5$. Action chunking policies output high-dimensional actions that
 417 exhibits a much more complex behavior distribution, where the policy extraction becomes critical.
 418 Since our approach primarily focuses on the policy extraction aspect, these domains make an ideal
 419 testbed us to compare our method to prior work. See Appendix B for more details on these domains.
 420

421 **Comparisons.** To provide a comprehensive empirical evaluation of our method, we carefully select
 422 8 representative, strong baselines that can be roughly categorized into the following 5 categories—
 423 **(1) Gaussian: ReBRAC** (Tarasov et al., 2023), **(2) Backprop:** FBRAC (Park et al., 2025b) (back-
 424 prop through the flow policy's denoising step directly), FQL (backprop through a 1-step distilled
 425 policy) (Park et al., 2025b); **(3) Adv. Weighted:** FAWAC (Park et al., 2025b) (advantage weighted
 426 actor critic, AWAC (Nair et al., 2020), with flow policy); **(4) Guidance with the critic's action**
 427 **gradient:** DAC (Fang et al., 2025), QSM (Psenka et al., 2023), and CGQL/CGQL-MSE/CGQL-Linex
 428 (three variants of classifier guidance-based methods inspired by Dhariwal & Nichol (2021)); **(5) Post-**
 429 **processing-based:** DSRL (Wagenmaker et al., 2025), FEdit (flow + Gaussian edit policy from Dong
 430 et al. (2025)), and IFQL (flow counterpart of IDQL (Hansen-Estruch et al., 2023)). For offline-to-
 431 online evaluations we additionally compare with RLPD (Ball et al., 2023). Finally, we compare with
 432 **BAM**, a direct ablation from our method QAM where we use the ‘basic’ adjoint matching objective in
 433 Equation (12) instead of the adjoint matching objective in Equation (14), and keep the rest of the

			a1	ag	hm	hl	scene	p33	p44	c2	c3	c4	all
			5 tasks	50 tasks									
432 433 434 435	GAUSSIAN	ReBRAC	94	38	42	8	61	72	0	8	1	2	34
		FBRAC	1	0	28	0	43	0	18	0	0	0	9
	BACKPROP	BAM	70	4	20	1	13	2	0	2	1	0	11
		FQL	66	0	59	3	67	100	5	30	2	1	34
436 437 438 439 440 441 442 443 444 445	ADV. WEIGHTED	FAWAC	17	0	19	0	37	3	0	1	0	0	8
		CGQL	69	0	51	4	26	43	30	37	5	0	27
	GUIDANCE	CGQL-MSE	67	8	39	0	66	100	0	40	5	0	33
		CGQL-Linex	58	0	48	3	86	96	0	41	5	0	34
	446 447	DAC	79	10	68	0	59	16	0	20	4	3	26
		QSM	72	4	68	6	71	34	0	42	3	16	32
	POST-PROCESSING	DSRL	37	1	40	0	100	80	0	59	0	0	32
		FEdit	37	0	3	2	68	99	29	30	1	0	28
		IFQL	23	0	70	10	82	100	0	9	0	0	29
448 449 450 451	ADJOINT MATCHING	QAM	63	11	60	2	98	97	0	57	5	3	38
		QAM-FQL	73	8	59	5	98	100	8	49	1	9	41
		QAM-EDIT	67	2	61	4	98	100	29	57	4	2	42

Table 1: **Offline RL performance at 1M training steps (50 tasks, 8 seeds).** Our method (QAM) and two of its variants, QAM-FQL and QAM-EDIT outperform all prior baselines. We use abbreviations of the domains as follows: a1=antmaze-large, ag=antmaze-giant, hm=humanoidmaze-medium, hl=humanoidmaze-large, p33=puzzle-3x3, p44=puzzle-4x4, c2=cube-double, c3=cube-triple, c4=cube-quadruple.

implementation exactly the same. We categorize it as a “backprop” method because its gradient is equivalent to that of backpropagating through the memoryless SDE as we discuss above in Section 3.

Among them, RLPD does not employ any behavior constraint, so we directly train them from scratch online with 50/50 offline/online sampling (*i.e.*, half of the training batch comes from offline and half of the training batch comes from the online replay buffer). To make the comparison fair, we use $K = 10$ critic networks, pessimistic value backup with $\rho = 0.5$ (except on humanoidmaze-large where we find $\rho = 0$ to work better), no best-of-N sampling (*i.e.*, $N = 1$) for both our method and all our baselines except for IFQL where best-of-N is used for policy extraction. We refer the reader to Appendix C for detailed description and implementation detail for each of the baselines. We also include the domain-specific hyperparameters for each baseline in Appendix D.

6 RESULTS

In this section, we present our experimental results to answer the following three questions:

(Q1) How effective is our method for offline RL?

Table 1 reports the offline RL performance across 10 different domains (50 tasks in total). QAM outperforms all prior methods with an aggregated score of 38. Furthermore, combining QAM with FQL and FEdit can push the performance even further. QAM-FQL achieves an aggregated score of 40 and QAM-EDIT achieves an even higher aggregated score of 42.

(Q2) How effective is our method for offline-to-online fine-tuning?

Next, we take the best performing variant of QAM, QAM-EDIT, and evaluate its ability to online fine-tune from its offline RL initialization. Figure 2 shows the sample efficiency curve (with x-axis being the number of environment steps). QAM outperforms all prior methods on cube-triple and is the most robust method across the board. For example, compared to ours, QSM fine-tunes better on a1, ag but struggles on all other tasks except on c2 where it performs similar to our method. FQL fine-tunes slightly better on ag and much slower on both p44 and c3.

(Q3) How sensitive is our method to hyperparameters? Finally, we conduct sensitivity analyses for various components of our method including pessimistic backup (ρ), gradient clipping, number of

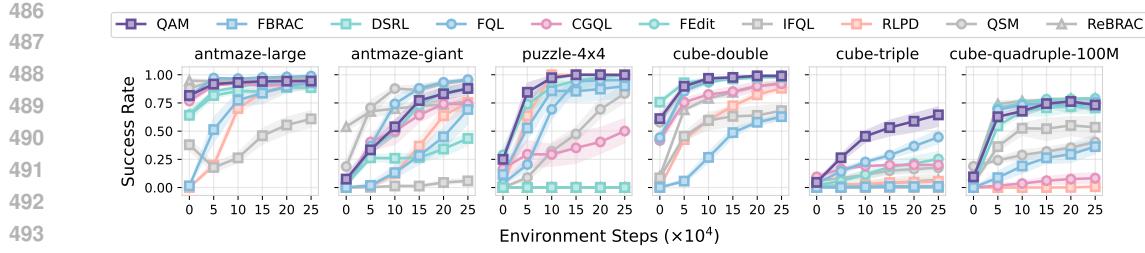


Figure 2: **QAM online fine-tunes more effectively than prior methods (30 tasks, 8 seeds).** For online fine-tuning experiments, we use the QAM-FEdit variant for QAM and we use CGQL-Linex variant for CGQL due to their good performance in our offline experiments.

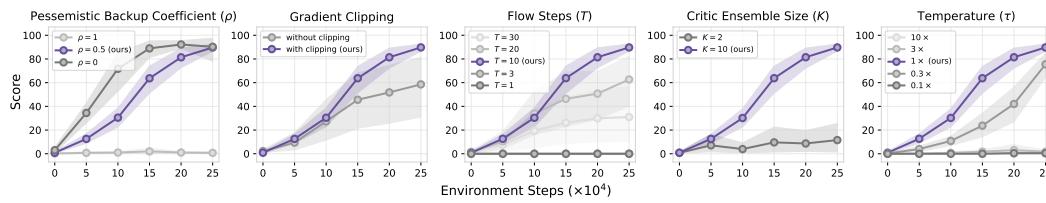


Figure 3: **Sensitivity analysis on cube-triple-task2 (8 seeds).** *Pessimistic Backup Coefficient (ρ):* this parameter controls how the standard deviation multiplier in the TD backup target $\bar{Q}_{\text{mean}} - \rho \bar{Q}_{\text{std}}$ (Equation (26)); *Gradient Clipping:* whether to use gradient clipping in our optimizer; *Flow Steps (T):* this parameter indicates the number of numerical integration steps that we use for the flow model. *Critic Ensemble Size (K):* number of critic network in the ensemble; *Temperature (τ):* the parameter that modulates the influence of the prior. We rerun our method with $0.1 \times, 0.3 \times, 3 \times$, and $10 \times$ the temperature value we obtain from our tuning runs.

flow steps (T), critic ensemble size (K) and the temperature coefficient (τ). As we show in Figure 3, all of these components contribute to QAM’s effectiveness. Among them, the pessimistic backup coefficient, and the temperature parameter (τ) have the biggest impact on QAM’s performance and need to be tuned. For the other components, we find enabling gradient clipping and having a large critic ensemble size ($K = 10$) always helps. We also find setting the number of flow steps to $T = 10$ works the best empirically.

7 DISCUSSION

We present Q-learning with Adjoint Matching (QAM), a novel TD-based RL method that effectively leverages the critic’s action gradient to extract an optimal prior-constrained policy while circumventing common limitations of prior approaches (*e.g.*, approximations that do not guarantee to converge to the desired optimal solution, learning instability, or reduced expressivity from distillation). Our empirical results suggest that QAM is an effective policy extraction method in both the offline RL setting and the offline-to-online RL setting, performing on par or better than prior methods. There are still practical challenges associated with QAM. While QAM’s effectiveness can be largely attributed to how well it is able to leverage the critic’s action gradient, this can be a double-edge sword—for cases where the critic function is ill-conditioned, it could lead to optimization stability issue. Gradient clipping (as done in our method) can alleviate this issue, but a more principled method that combines both value and gradient information could further improve robustness and performance. Another possible extension is to apply QAM in real-world robotic settings with action chunking policies. Our initial success (especially in the manipulation domains where we also leverage action chunking policies) may suggest that our method might work more effectively in complex real-world scenarios.

REPRODUCIBILITY STATEMENT

We include our source code as part of the supplementary materials (including installation instructions and example scripts for running our method and all our baselines). We describe our evaluation

540 domains in Appendix B, hyperparameters in Appendix D, and implementation details for each of our
 541 baselines in Appendix C.
 542

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810 A ADDITIONAL DISCUSSIONS FOR RELATED WORK 811

812 **CEP (Lu et al., 2023a) and CFGRL (Frans et al., 2025).** Both of them build off from the idea of
813 classifier/classifier-free guidance, which combines the denoising step of a base diffusion/flow policy
814 to the denoising step of for a tilt distribution.

$$816 \text{ CEP (diffusion): } \log \pi^t(s, a^t, t) \leftarrow \alpha \log \pi_\beta^t(s, a^t, t) + (1 - \alpha) Q^t(s, a^t, t) \quad (31)$$

$$817 \text{ CFGRL (flow): } v(s, a^t, t) \leftarrow \alpha v_\beta(s, a^t, t) + (1 - \alpha) v_{o=1}(s, a^t, t) \quad (32)$$

819 where $Q^t(s, a^t, t) \leftarrow \log \mathbb{E}_{a^t|a} [e^{Q(s, a)}]$ is the score of the Boltzmann distribution (i.e., $\propto e^{Q(s, a)}$) at
820 denoising time t and v_o is the velocity field of the policy that is conditioned on a optimality variable,
821 a binary indicator of whether the policy is ‘optimal’ ($o = 1$ means it is). CEP aims at approximating
822 $\pi \propto \pi_\beta^\alpha e^{(1-\alpha)Q(s, a)}$ whereas CFGRL aims at approximating $\pi \propto \pi_\beta^\alpha \pi_{o=1}^{(1-\alpha)}$.

823 However, as discussed in many prior work (Du et al., 2023; Bradley & Nakkiran, 2024), even when
824 both the denoising steps are exact ($\log \pi^t$ for diffusion and $v(\cdot, \cdot, t)$ for flow), the denoising process
825 that uses a summation of them do not lead to the correct distribution:

$$826 \nabla_{a^t} \log \pi^t(a^t | s) \neq \nabla_{a^t} \log \pi_\beta^t(a^t | s) + \tau \nabla_{a^t} Q^t(s, a^t). \quad (33)$$

827 **DAC (Fang et al., 2025).** Diffusion actor critic uses the diffusion formulation where the goal is to
828 find a policy that satisfies $\pi(\cdot | s) \propto \pi_\beta(\cdot | s) e^{Q(s, \cdot)}$. However, their training objective is derived
829 based on the assumption that

$$832 \nabla_{a^u} \log p^u(a^u | s) \approx \nabla_{a^u} \log p_\beta^u(a^u | s) + \tau \nabla_{a^u} Q^u(s, a^u), \quad (34)$$

834 and additionally

$$835 \nabla_{a^u} Q^u(s, a^u) \approx \nabla_{a^u} Q(s, a^u). \quad (35)$$

837 While these assumptions provide a convenient approximation of the objective function, it does not
838 provide guarantees on where policy converges to at the optimum.

840 B DOMAIN AND EXPERIMENT DETAILS 841

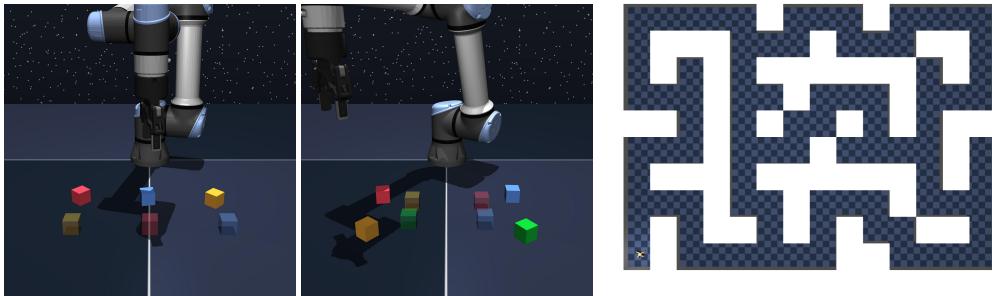
842 We consider 10 domains in our experiments. The dataset size, episode length, and the action
843 dimension for each domain is available in Table 2. For each method and each task, we run 8 seeds. All
844 plots and tables report the means with 95% confidence intervals computed via bootstrapping. All our
845 experiments are run on NVIDIA RTX-A5000 GPU and our code is written in JAX (Bradbury et al.,
846 2018). We use NVIDIA-A5000 GPU to run all our experiments. Each complete offline-to-online
847 experiment run takes around 3 hours. To reproduce all our results in Table 1 and Figure 2, we estimate
848 that it would take around $\underbrace{3}_{\text{hours per single run}} \times \underbrace{17}_{\text{\# of methods}} \times \underbrace{50}_{\text{\# of tasks}} \times \underbrace{8}_{\text{\# of seeds}} = 20\,400$ GPU hours.

$$849 \quad \underbrace{3}_{\text{hours per single run}} \times \underbrace{17}_{\text{\# of methods}} \times \underbrace{50}_{\text{\# of tasks}} \times \underbrace{8}_{\text{\# of seeds}} = 20\,400 \text{ GPU hours}$$

851 Tasks	852 Dataset Size	853 Episode Length	854 Action Dimension (A)
853 cube-double-*	854 1M	855 500	856 5
854 cube-triple-*	855 3M	856 1000	857 5
855 cube-quadruple-100M-*	856 100M	857 1000	858 5
856 antmaze-large-*	857 1M	858 1000	859 8
857 antmaze-giant-*	858 1M	859 1000	860 8
858 humanoidmaze-medium-*	859 2M	860 2000	861 21
859 humanoidmaze-large-*	860 2M	861 2000	862 21
860 scene-sparse-*	861 1M	862 750	863 5
861 puzzle-3x3-sparse-*	862 1M	863 500	864 5
862 puzzle-4x4-100M-sparse-*	863 100M	864 500	865 5

866 Table 2: Domain metadata.

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875 Figure 4: **OGBench domains**. In this paper, we primarily focus on evaluating our method on some of the
876 hardest domains on OGBench (Park et al., 2024a). `cube-{triple, quadruple}` (left) requires a robot arm to
877 manipulate up to 3/4 cubes from an initial arrangement to a goal arrangement. `antmaze-giant` (right) requires
878 an ant robot agent to navigate from one location to another location. All of these domains are long-horizon by
879 design and are difficult to solve from offline data alone. The offline dataset in these domains contain diverse multi-
880 modal behaviors that can only be accurately captured by expressive generative models like flow/diffusion policies.
881

C BASELINES

883 In this section, we describe in details how each of our baselines are implemented. In all the loss
884 functions below, unless specified otherwise, s, a, r, s' are assumed to be sampled from D and as
885 part of the expectation even when it is not explicitly written under the expectation (i.e., $\mathbb{E}_{z \sim \mathcal{N}}[\cdot] :=$
886 $\mathbb{E}_{(s, a, r, s') \sim D, z \sim \mathcal{N}}[\cdot]$).
887

C.1 BASELINE IMPLEMENTATION DETAILS

1. Backprop-based.

888 **FBRAC** is a baseline considered in FQL (Park et al., 2025b) as a flow counterpart of diffusion Q-
889 learning (DQL) (Wang et al., 2023), where the multi-step flow policy is directly optimized against
890 the Q-function with backpropagation through time (BPTT). In addition to maximizing the Q-value,
891 FBRAC also has a behavior cloning term where the flow policy is trained with the standard flow-
892 matching objective on the dataset action with a BC coefficient α .
893

894 We implement this baseline by training a flow-matching policy with the following loss:
895

$$L_{\text{FBRAC}}(\theta) = \alpha L_{\text{FM}}(\theta) + L_{\text{BPTT}}(\theta), \quad (36)$$

896 where L_{FM} is the standard flow-matching loss that clones the data behavior (i.e., Equation (17)) and
897

$$L_{\text{BPTT}}(\theta) = -\mathbb{E}_{a^0=z \sim \mathcal{N}(0, I_A)} \left[Q_\phi \left(s, \text{Clip} \left[z + h \sum_{k=0}^{T-1} v_\theta(s, a^{k-1}, kh) \right]_{-1}^1 \right) \right], \quad (37)$$

898 where $\text{Clip}[\cdot]_a^b$ is an element-wise clipping function that makes sure the actions generated from the
899 flow model v_θ are within the valid range $[-1, 1]$ and $\{a^i\}_i$ is the discrete approximation of the ODE
900 trajectory using Euler's method with a step size of $h = 1/T$:
901

$$a^{i+1} := a^i + h v_\theta(s, a^{i-1}, ih), \forall i \in \{0, 1, \dots, T\}. \quad (38)$$

902 We use

$$\text{ODE}(v_\theta(s, \cdot, \cdot), z) := \text{Clip} \left[z + h \sum_{i=0}^{T-1} v_\theta(s, a^{i-1}, ih) \right]_{-1}^1 \quad (39)$$

903 as the abbreviation for the rest of this section, and additionally use π_{v_θ} to denote the distribution of
904 actions generated by v_θ .
905

906 The critic loss is the standard TD backup:
907

$$L(\phi) = \mathbb{E}_{z \sim \mathcal{N}} [(Q_\phi(s, a) - r - Q_{\bar{\phi}}(s', \text{ODE}(v_\theta(s', \cdot, \cdot), z))^2)] \quad (40)$$

In practice, we also use $K = 10$ critic functions and pessimistic target value backup as described in Equation (26) and the policy we use to interact with the environment is π_{v_θ} .

FQL (Park et al., 2025b) distill a multi-step flow policy into a one-step distillation to avoid BPTT.

This baseline is implemented by training a behavior cloning flow-matching policy (i.e., $v_\theta : \mathcal{S} \times \mathbb{R}^A \times [0, 1] \rightarrow \mathbb{R}^A$), and a 1-step distilled noise-conditioned policy (i.e., $\Omega_\omega : \mathcal{S} \times \mathbb{R}^A \rightarrow \mathbb{R}^A$):

$$L_{\text{FQL}}(\theta, \omega) = L_{\text{FM}}(\theta) + L_{\text{onestep}}(\omega) \quad (41)$$

where $L_{\text{FM}}(\theta)$ is the standard flow-matching loss (i.e., Equation (17)) and

$$L_{\text{onestep}}(\omega) = \mathbb{E}_{z \sim \mathcal{Z}} [\alpha \|\Omega_\omega(s, z) - \text{ODE}(v_\theta(s', \cdot, \cdot), z)\|_2^2 - Q(s, \Omega_\omega(s, z))] \quad (42)$$

where α is the BC coefficient that controls how close the 1-step distilled policy should be relative to the BC policy π_{v_θ} . Finally, the critic loss is the standard TD backup:

$$L(\phi) = \mathbb{E}_{z \sim \mathcal{N}} [(Q_\phi(s, a) - r - Q_{\bar{\phi}}(s', \Omega_\omega(s', z))^2] \quad (43)$$

In practice, we also use $K = 10$ critic functions and pessimistic target value backup as described in Equation (26) and the policy we use to interact with the environment is π_{Ω_ω} where the action is sampled by first drawing a Gaussian noise $z \sim \mathcal{N}$ and then obtain the action by running through the one-step distilled model: $a = \Omega_\omega(s, z)$.

2. Directly using the action gradient of the critic (i.e., $\nabla_a Q(s, a)$) with approximations.

QSM (Psenka et al., 2024) uses the action gradient of the critic to train a diffusion model such that it can reconstruct the policy that follows the Boltzmann distribution of the Q-function:

$$\pi(\cdot | s) \propto \tau Q_\phi(s, \cdot). \quad (44)$$

To approximate the score of the intermediate actions, $\nabla_{a^i} \log \pi^i(a^i | s)$, QSM directly uses the critic evaluated at a^i :

$$\nabla_{a^i} \log \pi^i(a^i | s) \approx \tau \nabla_{a^i} Q_\phi(s, a^i) \quad (45)$$

where $a^i = \sqrt{\alpha^i} a + \sqrt{1 - \alpha^i} \varepsilon$ with $\varepsilon \sim \mathcal{N}$ and $\{\alpha^0, \dots, \alpha^{T-1}\}$ being any diffusion schedule with the number of diffusion steps of T . For all diffusion methods in this paper, we follow Hansen-Estruch et al. (2023) to use the variance preserving diffusion schedule:

$$\beta^i = 1 - \exp \left(-\frac{b_{\min}}{T} - \frac{(b_{\max} - b_{\min})(2i + 1)}{2T^2} \right), \quad (46)$$

$$\alpha^i = \prod_{j=0}^i (1 - \beta^j), \quad (47)$$

for all $i \in \{0, 1, \dots, T - 1\}$ with $b_{\min} = 0.1$, $b_{\max} = 10$ and T being the number of diffusion steps.

To train the diffusion model, QSM uses the following loss function:

$$L_{\text{qsm}}(\theta) = \mathbb{E}_{\varepsilon \sim \mathcal{N}, i \sim \mathcal{U}\{0, \dots, T-1\}} [\|\varepsilon - \tau \nabla_a Q_\phi(s, a^i) - f_\theta(s, a^i, i)\|_2^2], \quad (48)$$

where again $a^i = \sqrt{\alpha^i} a + \sqrt{1 - \alpha^i} \varepsilon$. In practice, we find that using $\nabla_a Q_{\bar{\phi}}(s, a^i)$ (the action gradient of the critic network) helps learning stability, so we use that instead.

To adopt QSM into the offline-to-online RL setting, we additionally augment the loss function with the standard diffusion loss (Ho et al., 2020) as the behavior regularization:

$$L_{\text{ddpm}}(\theta) = \mathbb{E}_{\varepsilon \sim \mathcal{N}, i \sim \mathcal{U}\{0, \dots, T-1\}} [\|\varepsilon - f_\theta(s, a^i, i)\|_2^2]. \quad (49)$$

The overall actor loss is thus

$$L(\theta) = L_{\text{qsm}}(\theta) + \eta L_{\text{ddpm}}(\theta). \quad (50)$$

QSM then uses the standard diffusion denoising procedure (with the clipping to make sure generated actions are within $[-1, 1]^{|A|}$):

$$a^{i-1} \leftarrow \text{Clip} \left[\frac{1}{\sqrt{1 - \beta^i}} \left(a^i - \frac{\beta^i}{\sqrt{1 - \alpha^i}} f_\theta(s, a^i, i) \right) + \sqrt{\beta^i} \varepsilon^i \right]_{-1}^1, \quad \varepsilon^i \sim \mathcal{N}, \quad (51)$$

972 where $a^T \sim \mathcal{N}$. We denote $\text{Diff}(f_\theta(s, \cdot, \cdot))$ as the resulting a^0 after going through the diffusion
 973 procedure above and $\pi_{f_\theta} : s \mapsto \text{Diff}(f_\theta(s, \cdot, \cdot))$ as the policy that generates actions using f_θ .
 974

975 The critic loss can now be defined as follows:

$$976 \quad L(\phi) = \mathbb{E}_{z \sim \mathcal{N}} [(Q_\phi(s, a) - r - Q_{\bar{\phi}}(s', a' \sim \pi_{f_\theta}(\cdot | s')))^2] \quad (52)$$

977 In practice, we also use $K = 10$ critic functions and pessimistic target value backup as described in
 978 Equation (26) and the policy we use to interact with the environment is π_{f_θ} .

979 **DAC** (Fang et al., 2025) is another method that uses a similar approximation with a behavior prior
 980 (*i.e.*, $\nabla_{a^i} \log \pi^i(a^i | s) \approx \nabla_{a^i} \log \pi_\beta^i(a^i | s) + \tau \nabla_{a^i} Q_\phi(s, a^i)$). To train the diffusion model, DAC
 981 matches f_θ to a linear combination of ε (behavior cloning) and $\nabla_{a^i} Q_\phi(s, a^i)$ (Q-maximization).
 982

$$983 \quad L_{\text{dac}}(\theta) = \mathbb{E}_{\varepsilon \sim \mathcal{N}, i \sim \mathcal{U}\{0, \dots, T-1\}} [\|\varepsilon - \tau \nabla_{a^i} Q_\phi(s, a^i) - f_\theta(s, a^i, i)\|_2^2] \\ 984 \quad = \mathbb{E}_{\varepsilon \sim \mathcal{N}, i \sim \mathcal{U}\{0, \dots, T-1\}} [\|\varepsilon - f_\theta(s, a^i, i)\|_2^2 + \eta^{-1}(f_\theta(s, a^i, i) \cdot \nabla_{a^i} Q_\phi(s, a^i))] + C, \quad (53)$$

985 with $\eta = 2/\tau$. In practice, it is implemented as a combination of

$$986 \quad L(\theta) = \mathbb{E}_{\varepsilon \sim \mathcal{N}, i \sim \mathcal{U}\{0, \dots, T-1\}} [\|(f_\theta(s, a^i, i) \cdot \nabla_{a^i} Q_\phi(s, a^i))\| + \eta L_{\text{ddpm}}(\theta) \quad (54)$$

987 DAC uses a slightly different way to clip the actions:

$$988 \quad \hat{a}^{i-1} \leftarrow \text{Clip} \left[\frac{1}{\sqrt{1-\beta^i}} \left(a^i - \frac{\beta^i}{\sqrt{1-\alpha^i}} f_\theta(s, a^i, i) \right) \right]_{-1}^1 \quad (55)$$

$$989 \quad a^{i-1} \leftarrow \frac{\beta^i \sqrt{\alpha^{i-1}}}{1-\alpha^i} \hat{a}^{i-1} + \sqrt{1-\beta^i}(1-\alpha^{i-1})a^i + \sqrt{\beta^i} \varepsilon^i, \quad \varepsilon^i \sim \mathcal{N} \quad (56)$$

990 where $a^T \sim \mathcal{N}$ and \hat{a}^{i-1} can be interpreted as an approximation of the denoised action (*e.g.*, in the
 991 DDIM sampler (Song et al., 2020)). This approximated denoised action is first clipped to be within
 992 $[-1, 1]^{|\mathcal{A}|}$ and then used to reconstruct a^{i-1} using the closed-form Gaussian conditional probability
 993 distribution of $p(a^{i-1} | a^0 = \hat{a}^{i-1}, a^T = a)$ (*e.g.*, Eq. (7) in Ho et al. (2020)). In practice, DAC
 994 also uses $\sqrt{\beta^i}$ as standard deviation for this conditional distribution instead of the correct one $\tilde{\beta}^i =$
 995 $\frac{\sqrt{1-\alpha^{i-1}}}{\sqrt{1-\alpha^i}} \beta^i$ as an approximation. This is why in the expression above the multiplier before ε^i is $\sqrt{\beta^i}$.
 996

997 Similar to QSM, the critic loss can now be defined as follows:

$$998 \quad L(\phi) = \mathbb{E}_{z \sim \mathcal{N}} [(Q_\phi(s, a) - r - Q_{\bar{\phi}}(s', a' \sim \pi_{f_\theta}(\cdot | s')))^2], \quad (57)$$

999 where π_{f_θ} is the policy that generates actions using the procedure above in Equation (55) and
 1000 Equation (56). In practice, we also use $K = 10$ critic functions and pessimistic target value backup
 1001 as described in Equation (26) and the policy we use to interact with the environment is π_{f_θ} .
 1002

1003 **CGQL** is a novel baseline built on top of the idea of classifier guidance (Dhariwal & Nichol, 2021).
 1004 In particular, we combine the velocity field of a behavior cloning flow policy and the gradient field of
 1005 the Q-function to form a new velocity field that approximates the velocity field that generates the
 1006 optimal behavior-constrained action distribution.
 1007

1008 More specifically, we implement this baseline by interpreting $Q_\phi(s, \cdot)$ as the score of the optimal
 1009 entropy-regularized distribution $\log \pi^*(\cdot | s)$ (where $\pi^*(\cdot | s) \propto e^{\tau Q_\phi(s, \cdot)}$). The corresponding
 1010 velocity field that generates this distribution of actions can be obtained through a simple conversion
 1011 (*e.g.*, following Equation 4.79 from Lipman et al. (2024)):

$$1012 \quad v_\phi(s, a, u) := \frac{(1-u)\tau \nabla_a Q_\phi^u(s, a) + a}{u}, \quad (58)$$

1013 where

$$1014 \quad Q_\phi^u(s, a) := \frac{1}{\tau} \log \mathbb{E}_{z \sim \mathcal{N}(0, I_A)} [e^{\tau Q_\phi(s, (1-u)z + ua)}], \quad (59)$$

1015 is the score of the distribution over the noisy intermediate actions. In the classifier guidance literature,
 1016 the score of the noisy examples is approximated by the score of the noise-free examples at the noisy
 1017 examples (Dhariwal & Nichol, 2021). In our setting this translates to
 1018

$$1019 \quad \hat{v}_\phi(s, a^u, u) := \frac{(1-u)\tau \nabla_a Q_\phi(s, a^u) + a^u}{u}. \quad (60)$$

1026 Empirically, both versions (v_ϕ and \hat{v}_ϕ) perform similarly and we opt for a simpler design \hat{v} as it does
 1027 not require learning or approximating $Q_\phi^u(s, a)$ for all $u \in [0, 1]$. Finally, we add the velocity field
 1028 defined by Q_ϕ directly to the behavior cloning velocity field to form our policy:
 1029

$$v = v_\beta + \vartheta \hat{v}_\phi, \quad (61)$$

1030 where v_β is trained with the standard flow-matching loss (i.e., $L(\beta) = L_{\text{FM}}(\beta)$) and ϑ is coefficient
 1031 that modulates influence of the guidance that we find to be helpful (e.g., $\vartheta < 1$ often works better
 1032 than $\vartheta = 1$). The critic loss uses π_v to backup the target Q -value:
 1033

$$L(\phi) = \mathbb{E}_{z \sim \mathcal{N}} [(Q_\phi(s, a) - r - Q_{\bar{\phi}}(s', \text{ODE}(v(s', \cdot, \cdot), z))^2]. \quad (62)$$

1034 In practice, we also use $K = 10$ critic functions and pessimistic target value backup as described
 1035 in Equation (26) and the policy we use to interact with the environment is π_v (generated from the
 1036 summation of v_β and \hat{v}_ϕ).
 1037

1038 **CGQL-MSE/Linex.** Alternatively, we can approximate Q_ϕ^u in Equation (59) more closely with a
 1039 training objective. In particular, we explore the following two regression objectives:
 1040

$$\text{MSE: } L_{\text{MSE}}(\zeta) = \mathbb{E}_{u \sim \mathcal{U}[0,1], z \sim \mathcal{N}} \left[\left(\hat{Q}_\zeta^u(s, a^u) - Q_\phi(s, a) \right)^2 \right] \quad (63)$$

$$\text{Linex: } L_{\text{Linex}}(\zeta) = \mathbb{E}_{u \sim \mathcal{U}[0,1], z \sim \mathcal{N}} \left[\exp(\tau(Q_\phi(s, a) - \hat{Q}_\zeta^u(s, a^u))) + \tau \hat{Q}_\zeta^u(s, a^u) \right] \quad (64)$$

1041 where $a^u := (1 - u)z + ua$. The optimal solution of \hat{Q}_ζ^u in Equation (63) is not exactly the same as
 1042 the desired Q_ϕ^u in Equation (59) but constitutes a lower-bound due to Jensen's inequality:
 1043

$$Q_{\text{mse}}^{u*} = \mathbb{E}_{z, u} [Q_\phi(s, a^u)] \leq \frac{1}{\tau} \log \mathbb{E}_{z \sim \mathcal{N}(0, I_A)} \left[e^{\tau Q_\phi(s, a^u)} \right]. \quad (65)$$

1044 The second objective resembles the classic Linex objective (Parsian & Kirmani, 2002) where the
 1045 optimal solution of \hat{Q}_ζ^u in Equation (64) is the same as the desired Q_ϕ^u in Equation (59):
 1046

$$Q_{\text{linex}}^{u*} = \frac{1}{\tau} \log \mathbb{E}_{z \sim \mathcal{N}(0, I_A)} \left[e^{\tau Q_\phi(s, (1-u)z + ua)} \right]. \quad (66)$$

1047 A discussion on this can be found in Myers et al. (2025). For completeness, we show this below.
 1048 Without loss of generality, we just need to show that $\exp(y) = \mathbb{E}[\exp(x)] = \int p(x) \exp(x) dx$ is the
 1049 unique minimum for the following loss function:
 1050

$$L(y) = \int p(x) [\exp(x - y) + y] dx \quad (67)$$

1051 Taking the first and second derivatives with respect to y gives
 1052

$$\frac{dL}{dy} = -\exp(-y) \mathbb{E}[\exp(x)] + 1 \quad (68)$$

$$\frac{d^2L}{dy^2} = \exp(-y) \mathbb{E}[\exp(x)] \quad (69)$$

1053 Setting Equation (68) gives $y = \log \mathbb{E}[\exp(x)]$ at which the second derivative is 1 (i.e., $\frac{d^2L}{dy^2} = 1$).
 1054 Furthermore, to prevent the exponential blow up of $\exp(Q_\phi(s, a) - \hat{Q}_\zeta^u(s, a^u))$, we follow Myers
 1055 et al. (2025) to use a Huber-style loss that locally behaves like a Linex loss but with a linear penalty
 1056 when the exponential term is too large. In particular, we use the following loss:
 1057

$$L_{\text{Linex}^+}(\zeta) = \begin{cases} \mathbb{E} \left[\exp(\Delta) + \tau \hat{Q}_\zeta^u(s, a^u) \right], & \Delta > 5 \\ \mathbb{E} [\Delta], & \Delta \leq 5 \end{cases} \quad (70)$$

1058 where $\Delta = \tau(Q_\phi(s, a) - \hat{Q}_\zeta^u(s, a^u))$.
 1059

1060 In practice, however, both the MSE and Linex objectives ($L_{\text{Linex}^+}, L_{\text{MSE}^+}$) can still be unstable and
 1061 exhibit high variances. Instead of learning \hat{Q}_ζ^u and Q_ϕ separately, we find it is often better to directly
 1062 learn both of them in a single network $Q_\phi(s, a, u) : \mathcal{S} \times \mathcal{A} \times [0, 1] \rightarrow \mathbb{R}$ as follows:
 1063

$$L(\phi) = \mathbb{E}_{z \sim \mathcal{N}} [(Q_\phi(s, a, 1) - r - Q_{\bar{\phi}}(s', \text{ODE}(v(s', \cdot, \cdot), z))^2] + \\ \varrho \mathbb{E}_{u \sim \mathcal{U}[0,1], z \sim \mathcal{N}} [(Q_\phi(s, (1-u)z + ua, u) - Q_{\bar{\phi}}(s, a, 1))^2], \quad (71)$$

1080 where ϱ is the coefficient that balances the TD backup for $Q_\phi(s, a, 1)$ and the noisy target regression
 1081 for $Q_\phi(s, a, u < 1)$. With $Q_\phi(s, \cdot, u)$, we can define our velocity field (which is also used as the TD
 1082 backup above) as

$$1084 v := v_\beta + \vartheta \hat{v}_\phi, \quad \text{where } \hat{v}_\phi(s, a^u, u) := \frac{(1-u)\tau \nabla_a Q_\phi(s, a^u, u) + a^u}{u}, \quad (72)$$

1085 and ϑ again modulates the guidance strength.

1086 **3. Directly using the critic value (i.e., $Q(s, a)$).**

1087 **FAWAC** is a baseline considered in FQL (Park et al., 2025b) where it uses AWR to train the flow
 1088 policy similar to QIPO (Zhang et al., 2025).

1089 We implement it by training a flow-matching policy with the weighted flow-matching loss:

$$1090 L_{\text{FAWAC}}(\theta) = \tilde{w}(s, a) L_{\text{FM}}(\theta) \quad (73)$$

$$1091 = \tilde{w}(s, a) \mathbb{E}_{u \sim \mathcal{U}[0,1], z \sim \mathcal{N}} [\|v_\theta(s, (1-u)z + ua, u) - z + a\|_2^2] \quad (74)$$

1092 where $\tilde{w}(s, a) = \min(e^{\tau(Q_\phi(s, a) - V_\xi(s))}, 100.0)$. The inverse temperature parameter τ controls how
 1093 sharp the prior regularized optimal policy distribution is.

1094 The critic function $Q_\phi(s, a)$ is trained with the standard TD backup and the value function $V_\xi(s)$
 1095 regresses to the same target:

$$1096 L(\phi) = \mathbb{E}_{z \sim \mathcal{N}} [(Q_\phi(s, a) - r - Q_{\bar{\phi}}(s', \text{ODE}(v(s', \cdot, \cdot), z))^2] \quad (75)$$

$$1097 L(\xi) = \mathbb{E}_{z \sim \mathcal{N}} [(V_\xi(s) - r - Q_{\bar{\phi}}(s', \text{ODE}(v(s', \cdot, \cdot), z))^2] \quad (76)$$

1098 The second line can also be alternatively implemented by regressing to the critic function $Q(s, a)$
 1099 directly. We implement in this particular way because we can re-use the Q -target computed.

1100 In practice, we also use $K = 10$ critic functions and pessimistic target value backup as described in
 1101 Equation (26) and the policy we use to interact with the environment is π_{v_θ} .

1102 **4. Post-processing-based.**

1103 **FEdit** is a baseline that uses the policy edit from a recent offline-to-online RL method conceptually
 1104 similar to EXPO (Dong et al., 2025). We implement a Gaussian edit policy on top of a BC flow
 1105 policy rather than a diffusion policy used in EXPO. EXPO also uses the standard sample-and-rank
 1106 trick where it samples multiple actions and rank them based on the value. To keep computational
 1107 cost down and comparisons fair to other methods, we only use a single edited action for both value
 1108 backup and evaluation.

1109 We implement this baseline by training a flow-matching policy (i.e., $v_\theta : \mathcal{S} \times \mathbb{R}^A \times [0, 1] \rightarrow \mathbb{R}^A$),
 1110 and a 1-step Gaussian edit policy (i.e., $\pi_\omega : \mathcal{S} \times \mathcal{A} \rightarrow \Delta_{\mathcal{A}}$) implemented with an entropy regularized
 1111 SAC policy (Haarnoja et al., 2018). The loss function can be described as follows:

$$1112 L_{\text{FEdit}}(\theta, \omega) = L_{\text{FM}}(\theta) + L_{\text{Gaussian}}(\omega), \quad \text{s.t. } \mathbb{E}_{s \sim D} [\mathbb{H}(\pi_\omega(\cdot | s))] \geq H_{\text{target}} \quad (77)$$

1113 where L_{FM} is the standard flow-matching loss that clones the data behavior (i.e., Equation (17)),
 1114 H_{target} is the target entropy that the Gaussian policy is constrained to be above of, and

$$1115 L_{\text{Gaussian}}(\omega) = \mathbb{E}_{\Delta a \sim \pi_\omega(\cdot | s, \tilde{a}), z \sim \mathcal{N}} \left[-Q_\phi(s, \text{Clip}[\sigma_a \cdot \Delta a + \tilde{a}]_{-1}^1) \right] \quad (78)$$

1116 where $\tilde{a} = \text{ODE}(v_\theta(s, \cdot, \cdot), z)$ and $\text{Clip}[\cdot]_a^b$ is an element-wise clipping function that makes sure the
 1117 actions are within the valid range $[-1, 1]$.

1118 Intuitively, the Gaussian SAC policy edits the behavior flow policy by modifying its output action
 1119 where σ_a is the hyperparameter that controls how much the original behavior actions can be edited.

1120 The critic loss is the standard TD backup:

$$1121 L(\phi) = \mathbb{E}_{z \sim \mathcal{N}, \Delta a' \sim \pi_\omega(\cdot | s', \tilde{a}')} \left[(Q_\phi(s, a) - r - Q_{\bar{\phi}}(s', \text{Clip}[\tilde{a}' + \sigma_a \cdot \Delta a']_{-1}^1))^2 \right] \quad (79)$$

1122 where again $\tilde{a}' = \text{ODE}(v_\theta(s', \cdot, \cdot), z)$. In practice, we also use $K = 10$ critic functions and
 1123 pessimistic target value backup as described in Equation (26) and the policy we use to interact with

the environment is a combination of the BC policy π_{v_θ} and the Gaussian edit policy. We first sample $z \sim \mathcal{N}$ and then run it through the BC flow policy to obtain an initial action $\tilde{a} \leftarrow \text{ODE}(v_\theta(s, \cdot, \cdot), z)$ and then both the initial action and the state is fed into the edit policy to generate the final action $a \leftarrow \tilde{a} + \sigma_a \cdot \Delta a$ where $\Delta a \sim \pi_\omega(\cdot | s, \tilde{a})$.

DSRL (Wagenmaker et al., 2025) is a recently proposed method that performs RL directly in the noise-space of a pre-trained expressive BC policy (flow or diffusion). We use the flow-matching version of DSRL as our method is also based on flow-matching policies. The original DSRL implementation does not fine-tune the BC policy during online learning while all our baselines do fine-tune the BC policy online. To make the comparison fair, we modify the DSRL implementation such that it also fine-tunes the BC policy. One additional implementation trick that allows this modification to work well is the use of target policy network for the noise-space policy. In general, we find that fine-tuning the BC policy yields better online performance, so we adopt this new design of DSRL in our experiments.

More specifically, we train a flow-matching policy (*i.e.*, $v_\theta : \mathcal{S} \times \mathbb{R}^A \times [0, 1] \rightarrow \mathbb{R}^A$), and a 1-step Gaussian edit policy (*i.e.*, $\pi_\omega : \mathcal{S} \times \mathcal{A} \rightarrow \Delta_{\mathcal{A}}$) implemented with an entropy regularized SAC policy (Haarnoja et al., 2018). The loss function can be described as follows:

$$L_{\text{DSRL}}(\theta, \omega) = L_{\text{FM}}(\theta) + L_{\text{LatentGaussian}}(\omega), \quad \text{s.t.} \quad \mathbb{E}_{s \sim D} [\mathbb{H}(\pi_\omega(\cdot | s))] \geq H_{\text{target}} \quad (80)$$

where L_{FM} is the standard flow-matching loss that clones the data behavior (*i.e.*, Equation (17)), H_{target} is the target entropy that the Gaussian policy is constrained to be above of, and

$$L_{\text{LatentGaussian}}(\omega) = \mathbb{E}_{z \sim \pi_\omega(\cdot | s)} [-Q_\psi^z(s, z)] \quad (81)$$

where $Q_\psi^z(s, z)$ is a distilled critic function in the noise space that is regressed to the original critic function, $Q_\phi(s, a)$:

$$L(\psi) = \mathbb{E}_{z \sim \mathcal{N}} [(Q_\psi^z(s, z) - Q_\phi(s, \text{ODE}(v_{\bar{\theta}}(s, \cdot, \cdot), z))^2]. \quad (82)$$

Intuitively, DSRL directly learns a policy in the noise space by hill-climbing a distilled critic that also operates in the noise space. Finally, the critic loss for the original critic function in the action space is

$$L(\phi) = \mathbb{E}_{z \sim \pi_\omega(\cdot | s')} [(Q_\phi(s, a) - r - Q_{\bar{\phi}}(s', \text{ODE}(v_{\bar{\theta}}(s', \cdot, \cdot), z))^2] \quad (83)$$

We use $K = 10$ critic functions and pessimistic target value backup as described in Equation (26). The policy we use to interact with the environment is a combination of the BC policy π_{v_θ} and the Gaussian noise-space policy. We first sample $z \sim \pi_\omega(\cdot | s)$ and then run it through the BC flow policy to obtain the final action $a \leftarrow \text{ODE}(v_\theta(s, \cdot, \cdot), z)$. One important implementation detail for stability in the offline-to-online setting is to use the target network for the BC flow policy $v_{\bar{\theta}}$ (instead of v_θ). Without it DSRL can become unstable sometimes when the BC flow policy changes too fast online.

IFQL is a baseline considered in FQL (Park et al., 2025b) as a flow counterpart of implicit diffusion Q-learning (IDQL) (Hansen-Estruch et al., 2023), where IQL (Kostrikov et al., 2021) is used for value learning and the policy extraction is done by sampling multiple actions from a behavior cloning diffusion policy and select the one that maximizes the Q -function value.

More specifically, we train a critic function $Q_\phi(s, a)$ and a value function $V_\xi(s)$ with implicit value backup:

$$L(\phi) = (Q_\phi(s, a) - r - V_\xi(s'))^2 \quad (84)$$

$$L(\xi) = f_{\text{exp}}^\kappa(Q_{\bar{\phi}}(s, a) - V_\xi(s)) \quad (85)$$

where $f_{\text{exp}}^\kappa(u) = |\kappa - \mathbb{I}_{u < 0}|u^2$ is the expectile regression loss function.

On top of that, we also $K = 10$ critic functions and pessimistic target value backup as described in Equation (26) for training the value function $V_\xi(s)$. To extract a policy from $Q_\phi(s, a)$, IFQL uses rejection sampling with a base behavior cloning flow policy that is trained with the standard flow-matching objective. In particular, the output action a^* for s is selected as the following:

$$a^* \leftarrow \arg \max_{a_1, \dots, a_N} Q(s, a_i), \quad a_1, \dots, a_N \sim \pi_{v_\beta}(\cdot | s). \quad (86)$$

5. Gaussian.

RLPD (Ball et al., 2023) is a strong offline-to-online RL method that trains a SAC agent from scratch online with a 50/50 sampling scheme (*i.e.*, 50% of training examples in a batch comes from the offline dataset wheras the other 50% of training examples comes from the online replay buffer).

1188
 1189 **ReBRAC** (Tarasov et al., 2024) is a strong offline RL method that trains a TD3 (Fujimoto et al., 2018)
 1190 agent with behavior cloning loss. In practice, we find two hyperparameters to impact performance
 1191 the most. The first hyperparameter is α which controls the strength of the behavior cloning loss. The
 1192 second hyperparameter is σ which controls the magnitude of the Gaussian noise added in the TD3
 1193 policy. We keep the action noise clip to be 0.5 and an actor delay of 2, following the original paper.
 1194

1194 C.2 CHARACTERIZATION BY BEHAVIOR CONSTRAINT SHAPE

1195
 1196 **Reversed KL.** The reversed KL constraint is the most commonly used constraint in offline RL and
 1197 offline-to-online RL. It amounts to the following constraint on the learned policy π :

$$1198 D_{\text{KL}}(\pi(\cdot | s) \| \pi_{\beta}(\cdot | s)) = \int \pi_{\theta}(a | s) (\log \pi_{\beta}(a | s) - \log \pi(a | s)) da \quad (87)$$

1200 When combined with the Q -maximization objective,

$$1202 L(\pi) = -\mathbb{E}_{a \sim \pi(\cdot | s)} [Q(s, a)] + \beta(s) D_{\text{KL}}(\pi \| \pi_{\beta}), \quad (88)$$

1204 the optimal policy admits the following closed-form solution:

$$1205 \pi^*(\cdot | s) \propto \pi_{\beta}(\cdot | s) \exp(\tau(s) Q(s, \cdot)) \quad (89)$$

1207 where $\tau(s)$ depends on $\beta(s)$ and the magnitude of Q .

1208 In practice, we often use a state-independent τ for simplicity:

$$1209 \pi^*(\cdot | s) \propto \pi_{\beta}(\cdot | s) \exp(\tau Q(s, \cdot)). \quad (90)$$

1211 Our method, QAM, and many of our baselines, FAWAC, CGQL*, DAC, QSM approximate π^* in different
 1212 ways. FAWAC uses advantage-weighted behavior cloning (flow-matching loss since we are using flow
 1213 policies) like the following:

$$1215 L_{\text{FAWAC}}(\pi) = \mathbb{E}_{a \sim \mathcal{D}} [\exp(Q(s, a) - V(s)) L_{\text{FM}}^{\pi}(s, a)] \quad (91)$$

1217 such that the policy converges to the desired optimal policy at the optimum:

$$1218 \pi^*(\cdot | s) \propto \pi_{\beta}(\cdot | s) \exp(\tau(Q(s, \cdot) - V(s))) \propto \pi_{\beta}(\cdot | s) \exp(\tau Q(s, \cdot)). \quad (92)$$

1220 The value function does not change the optimum but is important to reduce the learning variance.

1221 CGQL*, DAC, QSM are based on the observation that

$$1223 \nabla_a \log \pi^*(a | s) = \nabla_a \log \pi_{\beta}(a | s) + \tau \nabla_a Q(s, a) \quad (93)$$

1224 make the approximation that

$$1225 \nabla_{a^u} \log \pi^{u*}(a^u | s) \approx \nabla_{a^u} \log \pi_{\beta}^u(a^u | s) + \tau \nabla_{a^u} Q(s, a^u) \quad (94)$$

1227 where a^u is the intermediate noisy action at some denoising time u and π^u, π^{u*} are the distributions
 1228 of the noisy actions from the behavior and optimal respectively from the corresponding time u .

1229 In contrast to these prior methods, QAM does not rely on approximations while leveraging $\nabla_a Q(s, a)$
 1230 to align the flow-matching policy directly to π^* .

1232 In general, reversed KL behavior constraint respects the support of the behavior distribution π_{β} the
 1233 best because at the optimum, π^* cannot assign non-zero probability to any action a that is outside the
 1234 support of π_{β} (i.e., $\pi_{\beta}(a | s) = 0$). The geometry-agnostic nature of the KL divergence, in principle,
 1235 allows these methods to deal with any behavior distribution and value function landscapes. This is in
 1236 contrast to geometry-aware methods which we will discuss below.

1237 **Wasserstein distance.** Another rising class of behavior constraint is the Wasserstein distance
 1238 constraint. It amounts to the following constraint:

$$1239 W_p(\pi, \pi_{\beta}) = \min_{P \in \mathcal{C}(\pi, \pi_{\beta})} \mathbb{E}_{(a^{\pi}, a^{\beta}) \sim P} [d(a^{\pi}, a^{\beta})]^p \quad (95)$$

1241 where $d : \mathcal{A} \times \mathcal{A}$ is some distance metric that is commonly picked to be L_2 or L_{∞} .

1242 With $d(a, a') = \|a - a'\|_2$ and $p = 2$, FQL (Park et al., 2025b) optimizes the following objective
 1243

$$L(\pi) = -\mathbb{E}_{a \sim \pi(\cdot|s)} [Q(s, a)] + \alpha W_2(\pi_\beta, \pi)^2 \quad (96)$$

1244 where α is the behavior regularization strength.
 1245

1246 Unlike reversed KL behavior constraint, the Wasserstein behavior constraint does not necessarily
 1247 respect the support constraint and can assign high probability mass to an action as long as it is close
 1248 to some behavior actions in terms of d .
 1249

1250 Loosely speaking, FEdit also imposes the behavior constraint in a similar spirit but with a non-metric
 1251 d . In particular, it can be interpreted as using
 1252

$$d(a^\pi, a^\beta) = \mathbb{I} [\|a^\pi - a^\beta\|_\infty > \sigma_e] \quad (97)$$

1253 with $\alpha \rightarrow \infty$. This is equivalent to parameterizing $\pi(\cdot|s)$ as $\tilde{a} \leftarrow \pi_{\text{edit}}(s, a \sim \pi_\beta(\cdot|s))$ where a
 1254 residual edit policy $\pi_{\text{edit}} : \mathcal{S} \times \mathcal{A} \rightarrow \mathcal{A}$ is used to modify the original behavior policy up to σ_e (in
 1255 terms of the infinity norm of the action difference):
 1256

$$L(\pi) = -\mathbb{E}_{a \sim \pi_\beta(\cdot|s)} [Q(s, \pi_{\text{edit}}(s, a))], \quad \text{s.t.} \quad \|\pi_{\text{edit}}(s, a) - a\|_\infty \leq \sigma_e \quad (98)$$

1257 Though strictly speaking it is no longer a Wasserstein distance as d is no longer a metric, it is still
 1258 a valid behavior constraint that shares a similar geometry property as the Wasserstein behavior
 1259 constraint—it does not respect the support of π_β and can assign large probability mass that is close to
 1260 the behavior action in terms of d .
 1261

1262 While not respecting the support constraint might seem to be a bad thing, these methods, conceptually,
 1263 may attain a better performance compared to the reversed KL methods, especially when the Q-
 1264 function generalizes well beyond the behavior support.
 1265

1266 **Behavior support constraint.** The behavior constraint shape in our discussion is the support
 1267 constraint. This is similar to the reversed KL constraint but with a more uniform weight over the
 1268 actions in the support:
 1269

$$D_{\text{supp}}(\pi \parallel \pi_\beta) = \int \pi_\theta(a|s) \mathbb{I} [\pi_\beta(a|s) > 0] da. \quad (99)$$

1270 Compared to the reversed KL constraint, which up-weights actions that have higher probability in the
 1271 behavior distribution, the behavior support constraint puts a uniform weight over all actions that are in
 1272 the support. While achieving this exactly is difficult, a line of work (DSRL (Wagenmaker et al., 2025)
 1273 and Singh et al. (2020)) can be seen as an approximation of this by learning a policy in the noise-
 1274 space of a diffusion/flow policy. To approximate the support of the diffusion/flow policy, it uses a L_∞ -
 1275 bounded action space in the noise space $a_z \in [-\sigma_z, \sigma_z]$ and then optimize the following objective:
 1276

$$L(\pi) = -\mathbb{E}_{a_z \sim \pi(\cdot|s)} [Q(s, a = F_\beta(s, a_z))], \quad (100)$$

1277 where $F_\beta : \mathcal{S} \times \mathcal{Z} \rightarrow \mathcal{A}$ is the behavior policy that maps from a noise space to the action space.
 1278 There are many choices of F_β (e.g., Singh et al. (2020) use normalizing flow (Dinh et al., 2016)) and
 1279 in our experiments, we use DSRL, which uses a flow-matching policy. One practical trick that makes
 1280 DSRL work is to use a distilled Q-network that directly estimates the action value in the noise space.
 1281 We refer the reader to the original paper for more details. We describe the implementation details
 1282 regarding the distilled Q-network in Appendix C.1.
 1283

D HYPERPARAMETERS

1284 While most methods share a common set of hyperparameters (Table 3 for a fair comparison, most
 1285 methods need to be tuned for each domain. We include the domain-specific in Table 4 and the tuning
 1286 range of them in Table 5. To pick the hyperparameter for each domain for each method, we first run
 1287 a sweep over all hyperparameters in the range (specified by Table 5), or all combinations of them if
 1288 there are multiple hyperparameters involved. The hyperparameter tuning runs use 4 seeds for each
 1289 method for each hyperparameter configuration for each of the two tasks per domain. For locomotion
 1290 domains, we use task 1 (the default task) and task 4. For manipulation domains, we use task 2 (the
 1291 default task) and task 4. We use task 4 in addition to the default task recommended by OGBench
 1292 because we often find the combination of these two cover the characteristics of each domain better.
 1293

We then use the combined performance of the two tuning tasks per domain per method to pick the hyperparameter configuration. We include them in Table 4. Finally, for all our main results, we run all methods on all five tasks for each domain on 8 new seeds (different from the tuning seeds). We pick the hyperparameter range such that the total number of tuning runs are similar across methods. To achieve this, we use the following strategy: For methods where more than one hyperparameter needs to be tuned, we use a coarser hyperparameter range. For methods where there is only one hyperparameter, we use a more fine-grained sweep. The only exception is our method QAM where we find 4 tuning hyperparameters (*i.e.*, $\{1, 3, 10, 30\}$) are enough to outperform all prior methods.

1304

1305	Parameter	Value
1306	Batch size	256
1307	Discount factor (γ)	0.99 for {puzzle/scene/cube/antmaze-large}-* 0.995 for {humanoidmaze/antmaze-giant}-*
1308	Optimizer	Adam
1309	Learning rate	3×10^{-4}
1310	Target network update rate (λ)	5×10^{-3}
1311	Critic ensemble size (K)	10
1312	Critic target pessimistic coefficient (ρ)	0.5 for {puzzle/scene/cube/antmaze}-* 0 for humanoidmaze-*
1313	UTD Ratio	1
1314	Number of flow steps (T)	10
1315	Number of offline training steps	10^6 ; except RLPD (0)
1316	Number of online environment steps	0.5×10^6
1317	Network width	512
1318	Network depth	4 hidden layers
1319	Optimizer Gradient clipping	1 for QSM, DAC and QAM*. No clipping for others.
1320		

1322

Table 3: **Common hyperparameters.**

1323

1324

1325

Domains	ReBRAC (α, σ)	QSM (τ, η)	DAC	FBRAC α	FQL	DSRL σ_z	FEdition σ_a	IFQL	CGQL τ	CGQL-MSE (ϑ, ϱ, τ)	CGQL-Linex (ϑ, ϱ, τ)	QAM & BAM τ	QAM-Edit (τ, σ_a)	QAM-FQL (τ, α)
scene-sparse-*	(0.03, 0)	(3, 30)	1	100	300	0.4	0.2	0.9	(10, 0.1)	(10, 0.1, 0.1)	(10, 0.1, 0.1)	1	(1, 0)	(1, ∞)
puzzle-3x3-sparse-*	(0, 1, 0)	(10, 30)	1	0.3	300	1	0.2	0.95	(10, 0.1)	(10, 0.1, 0.1)	(10, 0.001, 0.1)	3	(3, 0.1)	(3, ∞)
cube-4x4-100M-sparse-*	(0.01, 0.2)	(10, 1)	1	0.3	1	1	0.8	0.9	(10, 0.1)	(10, 0.001, 1)	(10, 0.001, 1)	30	(3, 0.9)	(1, 3)
cube-double-*	(0, 01, 0)	(1, 10)	3	0.1	300	1	0.2	0.9	(10, 0.01)	(10, 0.001, 0.01)	(10, 0.001, 0.01)	1	(1, 0)	(1, 0)
cube-triple-*	(0.01, 0.2)	(10, 10)	0.3	0.03	30	1.4	0.3	0.95	(10, 0.1)	(10, 0.001, 0.1)	(10, 0.001, 0.1)	10	(10, 0.1)	(10, 30)
cube-quadruple-100M-*	(0.01, 0.2)	(1, 10)	1	1	100	1.4	0.4	0.95	(10, 0.1)	(10, 0.1, 0.01)	(10, 0.1, 0.01)	1	(3, 0.5)	(1, 30)
antmaze-large-*	(0, 01, 0)	(10, 10)	0.3	0.1	3	0.8	0.2	0.9	(10, 0.1)	(10, 0.1, 0.1)	(10, 0.001, 0.1)	10	(1, 0.1)	(3, 30)
antmaze-giant-*	(0, 01, 0)	(10, 10)	0.3	0.1	3	1.2	0.3	0.8	(10, 0.1)	(10, 0.001, 0.1)	(10, 0.001, 0.1)	3	(3, 0.1)	(3, 300)
humanoidmaze-medium-*	(0, 01, 0)	(10, 30)	1	30	30	0.6	0.5	0.7	(10, 0.01)	(10, 0.1, 0.1)	(10, 0.1, 0.1)	3	(1, 0.1)	(3, ∞)
humanoidmaze-large-*	(0, 01, 0.1)	(10, 30)	0.3	10	30	0.8	0.1	0.8	(10, 0.01)	(10, 0.1, 0.1)	(10, 0.1, 0.1)	3	(1, 0.1)	(1, 30)

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Table 4: **Domain-specific hyperparameters.** The best hyperparameter configuration obtained from our tuning runs. We use the same hyperparameter configuration for all tasks in each domain.

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E PROOF OF PROPOSITION 1

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Proposition 1 (Extension of Proposition 7 in Domingo-Enrich et al. (2025) to Policy Optimization.) Take $L_{\text{AM}}(\theta)$ in Equation (14), there is a unique f_θ such that

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$$\frac{\partial}{\partial f_\theta} L_{\text{AM}} = 0, \quad (27)$$

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Proof. Our proof mainly rewrites the assumptions and the statement of Proposition 7 of Domingo-Enrich et al. (2025) in our notations with a simple extra step that extends it to the state-conditioned version (*e.g.*, for policy optimization).

1350	Method	Hyperparameter(s)	Sweep Range
1351	ReBRAC	(α, σ)	$(\{0.01, 0.03, 0.1, 0.3, 1\}, \{0, 0.1, 0.2\})$
1352	FBRAC	α	$\{0.03, 0.1, 0.3, 1.0, 3.0, 10.0, 30.0, 100.0\}$
1353	CGQL	(ϑ, τ)	$(\{0.01, 0.1, 1\}, \{0.1, 1, 10\})$
1354	CGQL-{\MSE, Linex}	$(\vartheta, \tau, \varrho)$	$(\{0.01, 0.1, 1\}, \{0.1, 1, 10\}, \{0.001, 0.1\})$
1355	FQL	α	$\{0.3, 1, 3, 10, 30, 100, 300, 1000\}$
1356	DSRL	σ_z	$\{0.1, 0.2, 0.4, 0.6, 0.8, 1, 1.2, 1.4\}$
1357	FEdit	σ_a	$\{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8\}$
1358	FAWAC	τ	$\{0.1, 0.2, 0.4, 0.8, 1.6, 3.2, 6.4, 12.8\}$
1359	IFQL	κ	$\{0.5, 0.6, 0.7, 0.8, 0.9, 0.95\}$
1360	DAC	η	$\{0.1, 0.3, 1, 3.0, 10, 30, 100, 300\}$
1361	QSM	(τ, η)	$(\{1.0, 3.0, 10.0, 30\}, \{1, 3, 10, 30\})$
1362	QAM	τ	$\{1, 3, 10, 30\}$
1363	QAM-Edit	(τ, σ_a)	$(\{1, 3, 10\}, \{0, 0.1, 0.5, 0.9\})$
1364	QAM-FQL	(τ, α)	$(\{1, 3, 10\}, \{3, 30, 300, \infty\})$

Table 5: **Domain-specific hyperparameter tuning range.** For QAM-FQL, $\alpha = \infty$ is implemented by QAM. Similarly, for QAM-FQL, $\sigma_a = 0$ is implemented by QAM. For methods with more than one parameter, we tune all possible combinations within the sweep range. For example, for CGQL, we sweep over all $3 \times 3 = 9$ configurations with 3 possible values of ϑ and 3 possible values of τ .

We first define (from Equation 27 in Domingo-Enrich et al. (2025)) the residual

$$v_{\text{res}}(s, a^u, u) := \sqrt{\frac{2}{\beta_u(\beta_u \dot{\alpha}_u / \alpha_u - \dot{\beta}_u)}}(v_\theta(s, a^u, u)). \quad (101)$$

While the original result showed for a more general case with a family of α, β (and later on σ), in our work we assume

$$\alpha(u) = u \quad (102)$$

$$\beta(u) = 1 - u \quad (103)$$

$$\sigma(u) = \sqrt{2(1 - u)}/u \quad (104)$$

This allows us to simplify the expression for v_{res} as

$$v_{\text{res}}(s, a^u, u) = \sqrt{\frac{2u}{1-u}}v_\theta(s, a^u, u) \quad (105)$$

Then, we restate the definition of b (from Equation 27 in Domingo-Enrich et al. (2025)):

$$b(s, a^u, u) = 2(v_\beta(s, a^u, u) + v_\theta(s, a^u, u)) - \frac{\dot{\alpha}_u}{\alpha_u}a^u - \sigma(u)v_{\text{res}}(s, a^u, u) \quad (106)$$

$$= 2(v_\beta(s, a^u, u) + v_\theta(s, a^u, u)) - a^u/u - 2v_\theta(s, a^u, u) \quad (107)$$

$$= 2v_\beta(s, a^u, u) - a^u/u \quad (108)$$

We can now rewrite our ‘lean’ adjoint state definition as

$$d\tilde{g}^u = -\tilde{g}^{u^\top} \nabla_{a^u} [b(s, a^u, u)], \quad \tilde{g}^1 = -\tau \nabla_{a^1} Q_\phi(s, a^1) \quad (109)$$

which coincides with the definition in Equation 38 in Domingo-Enrich et al. (2025), with $f = 0$ and $g(\cdot) = -\tau Q_\phi(s, \cdot)$. Now, we can rewrite our adjoint matching objective as

$$L_{\text{AM}}(\theta) := \mathbb{E}_{s \sim D, \{a^u\}_u} \left[\tilde{L}_{\text{AM}}(v_\theta, \{a^u\}_u) \right] \quad (110)$$

1404 where

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$$\tilde{L}_{\text{AM}}(s, v_{\theta}, \{a^u\}_u) := \int_0^1 \|v_{\text{res}}(s, a^u, u) + \sigma(u)\tilde{g}^u\| \, du \quad (111)$$

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$$= \frac{1}{2} \int_0^1 \left\| \sqrt{\frac{2u}{1-u}} v_{\theta}(s, a^u, u) + \sigma(u)\tilde{g}^u \right\|_2^2 \, du \quad (112)$$

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$$= \frac{1}{2} \int_0^1 \|2v_{\theta}(s, a^u, u)/\sigma(u) + \sigma(u)\tilde{g}^u\|_2^2 \, du \quad (113)$$

1413 Comparing $\tilde{L}_{\text{AM}}(s, v_{\theta}, \{a^u\}_u)$ to the definition in Equation 37 in Domingo-Enrich et al. (2025), they
1414 are different by only a factor of 2 *conditioned on a fixed s*. Thus, their critical points are the same.
14151416 By triggering Proposition 7 in Domingo-Enrich et al. (2025), we can conclude that for a fixed s , the
1417 only critical point of the following loss function,

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$$\mathbb{E}_{\{a^u\}_u} \left[\tilde{L}_{\text{AM}}(s, v_{\theta}, \{a^u\}_u) \right], \quad (114)$$

1420 is $v_{\text{res}}^*(s, a^u, u)$, the velocity field that generates the following distribution,

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$$\pi^*(\cdot | s) \propto \pi_{\beta}(\cdot | s) \exp(\tau Q_{\phi}(s, \cdot)). \quad (115)$$

1423 Finally, since the $L_{\text{AM}}(\theta)$ is a linear combination of $\mathbb{E}_{\{a^u\}_u} \left[\tilde{L}_{\text{AM}}(s, v_{\theta}, \{a^u\}_u) \right]$ over different
1424 s , the critic point of $L_{\text{AM}}(v_{\theta})$ is simply the cartesian product of over the critic points for each
1425 $s \in \text{supp}(D)$. Since there is only one critical point for each $\mathbb{E}_{\{a^u\}_u} \left[\tilde{L}_{\text{AM}}(s, v_{\theta}, \{a^u\}_u) \right]$, $L_{\text{AM}}(v_{\theta})$
1426 also has only one critical point and coincides with $v_{\text{res}}^*(s, a^u, u)$. This concludes that the only critical
1427 point of $L_{\text{AM}}(v_{\theta})$ results in velocity fields that satisfy
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$$\pi^*(\cdot | s) \propto \pi_{\beta}(\cdot | s) \exp(\tau Q_{\phi}(s, \cdot)), \quad \forall s \in \text{supp}(D). \quad (116)$$

1431 \square

Method	Training Speed (milliseconds/step)	Parameter Count
ReBRAC	2.97	18 238 534
FBRAC	4.54	17 426 989
CGQL	9.62	18 275 398
CGQL-{\MSE, Linex}	10.40	18 275 398
FQL	3.58	18 264 646
DSRL	5.63	27 397 251
FEdit	3.77	18 277 472
FAWAC	3.44	18 243 630
IFQL	2.66	18 243 630
DAC	4.44	18 509 901
QSM	4.44	18 509 901
QAM	5.83	19 941 496
QAM-Edit	6.61	20 791 979
QAM-FQL	6.36	20 779 153

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1450 Table 6: **Training speed and parameter count for each method on cube-triple.**1451
1452 F ADDITIONAL RESULTS1453 In this section, we analyze the performance of our method, QAM, when subject to data of different
1454 equalities. Figure 5 reports the performance of our method on cube-quadruple-task2 with
1455 different dataset sizes. Figure 6 reports the comparison of our method with representative baselines
1456 (FQL, ReBRAC and FEdit) on cube-double-noisy-task4 and cube-triple-noisy-task4,
1457 two of the hardest tasks in the benchmark. Figure 7 contains additional results that aggregate over all
5 tasks for each domain.

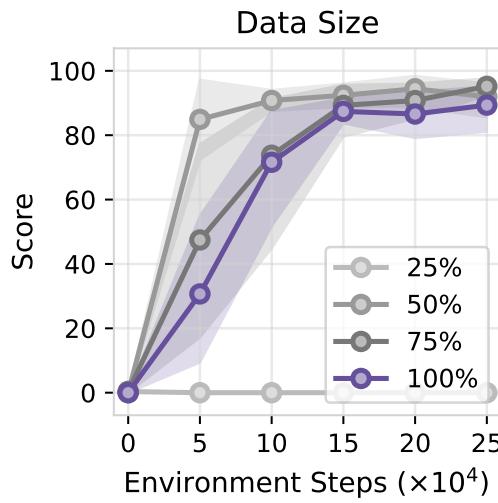


Figure 5: **Dataset size analysis on cube-quadruple-task2 (6 seeds).** 100%, 75%, 50%, 25% corresponds to a dataset size of 100M, 75M, 50M, and 25M. Our method can also work on dataset with smaller size until 25M-size dataset where it completely fails.

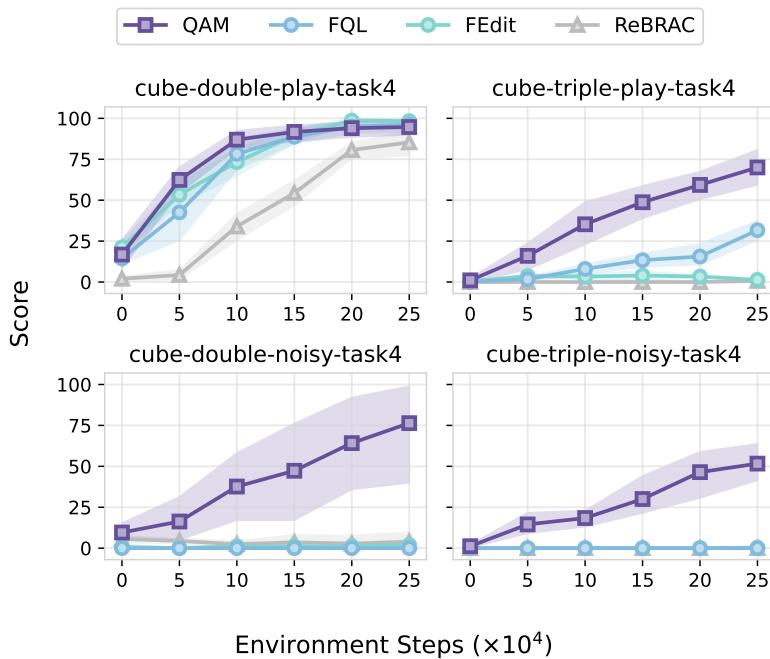


Figure 6: **Experiments on cube-double-task4 and cube-triple-task4 on play and noisy datasets (6 seeds).** *Top:* the original play-style datasets we use in our main experiments; *Bottom:* the noisy-style datasets that were collected by expert policies with large uncorrelated, Gaussian action noises. We use the QAM-EDIT variant as it is the best-performing variant in our offline RL experiments. We use the same hyperparameters for experiments on noisy-style and play-style datasets. Our method exhibits strong robustness against action noises. While all our baselines fail completely on the *noisy* datasets, our method only suffers minor performance drop.

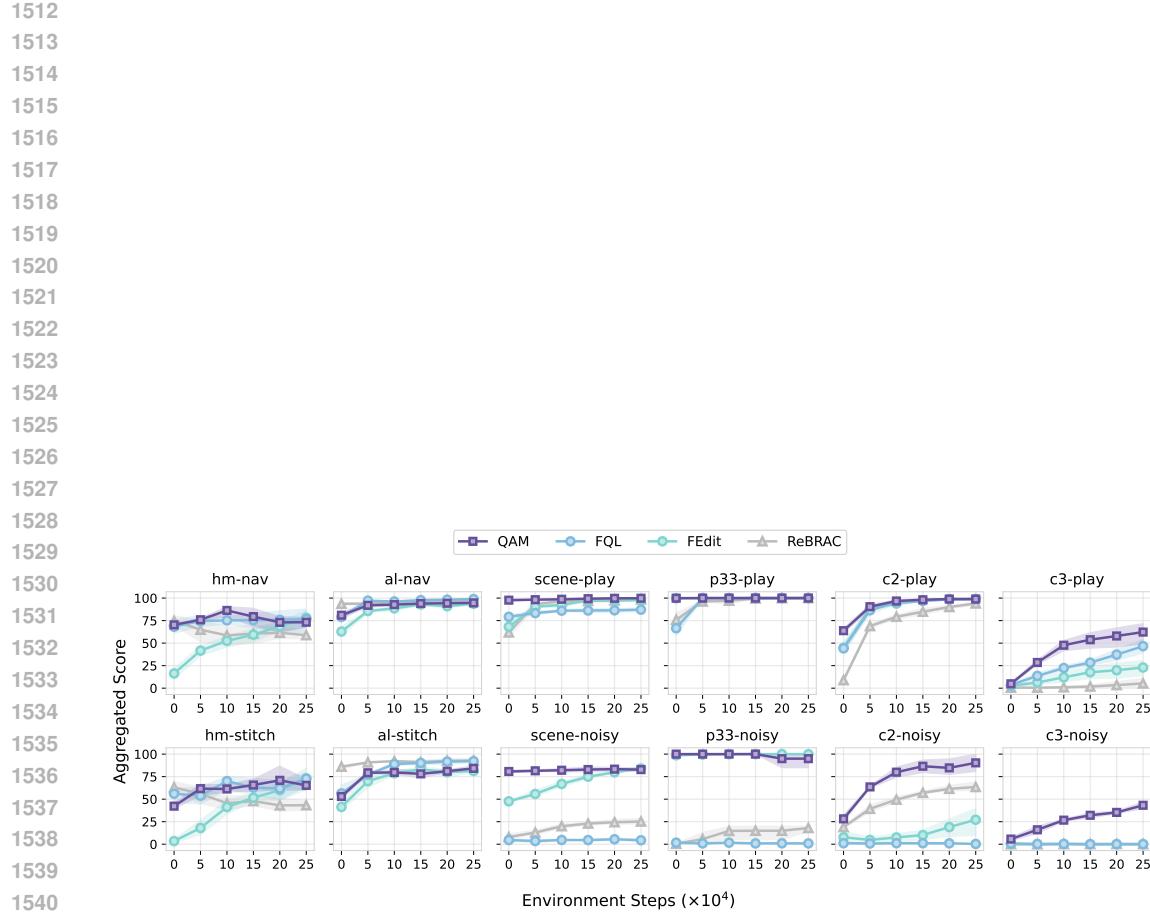


Figure 7: **Data quality analysis (4 seeds).** *Top:* performance on the original {navigate/play}-style datasets we use in our main experiments; *Bottom:* performance on the {stitch/noisy}-style datasets. **stitch**-style datasets were collected the same way as **navigate**-style datasets but with much shorter trajectory segments. **noisy**-style datasets were collected by expert policies with large uncorrelated, Gaussian action noises. We use the QAM-EDIT variant as it is the best-performing variant in our offline RL experiments. We use the same hyperparameters from our main experiments. Each subplot reports the aggregated score over 5 tasks in each domain. For locomotion tasks (e.g., `humanoidmaze-medium` and `antmaze-large`), we observe almost no performance difference when switching from **navigate**-style to **stitch**-style dataset. For manipulation tasks, our method exhibits strong robustness against action noises. In particular, while all our baselines fail completely on the `cube-triple-noisy` environments, our method only suffers minor performance drop.