# ROLL-AE: A SPATIOTEMPORAL INVARIANT AUTOEN CODER FOR NEURONAL ELECTRO-PHYSIOLOGY

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#### ABSTRACT

Micro-electrode array (MEA) assays enable high-throughput recording of the electrophysiological activity in biological tissues, both in vivo and in vitro. While various classical and deep learning models have been developed for MEA signal analysis, the majority focus on in vivo experiments or specific downstream applications in vitro. Consequently, extracting relevant features from in vitro MEA recordings has remained largely dependent on particular curated features known as neural metrics. In this work, we introduce Roll-AE, a novel autoencoder designed to extract spatiotemporally invariant features from in vitro MEA recordings. Roll-AE serves as a foundational model that facilitates a wide range of downstream tasks. We demonstrate that 1) Roll-AE's embeddings outperform those from standard autoencoders across various classification tasks, and 2) Roll-AE's embeddings effectively characterize electrophysiological phenotypic traits in induced Pluripotent Stem Cells (iPSC)-derived neuronal cultures.

- 1 INTRODUCTION
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In vitro micro-electrode array (MEA) assays provide a unique opportunity to obtain rich highthroughput electro-physiological data from induced Pluripotent Stem Cell (iPSC) (Takahashi & Yamanaka, 2006)-derived neuronal cultures (Fukushima et al., 2016; Kayama et al., 2018). MEA enables monitoring and recording of the extra-cellular action potentials in a non-invasive manner and provides valuable insights into the development and organization of neuronal networks (Novellino et al., 2011; Maeda et al., 2016). MEA has been particularly useful in the context of disease modeling, where excitability phenotypes are used to quantify the effects of genetic mutations and potential treatments of Parkinson's disease, Amyotrophic Lateral Sclerosis (ALS), Tuberous Sclerosis Complex (TSC), etc. (Woodard et al., 2014; Wainger et al., 2014; Winden et al., 2019)

Although MEAs on in-vitro cell cultures have increased in popularity over the past decade, existing 036 methodologies to analyse this type of data remain limited. Most existing methods are based on a 037 collection of hand-crafted features called neural metrics (Mossink et al., 2021; Wainger et al., 2014; 038 Kim et al., 2020). These methods (Mack et al., 2014; Bryson et al., 2022; Kapucu et al., 2022; Passaro et al., 2021) apply principal component analysis, factor analysis, or similar dimension reduction techniques to distill and analyze electro-physiological recordings. Neural metrics generally include 040 descriptive statistics on different activity patterns such as sporadic neuronal firing, synchronous firing 041 across electrodes, rapid consecutive firings (bursts), and bursts across multiple electrodes (network 042 bursts). Fig. 7a lists the neural metrics curated by the commonly used Axion Biosystems (Biosystems, 043 2024) algorithm. Analysis of these neural metrics comes with several limitations. First, data 044 aggregation in a predefined manner results in a loss of resolution. A typical MEA recording involves 045 action-potential data on 6-96 wells per plate and 8-64 electrodes per well at a millisecond sampling 046 rate. This high-resolution data is then compressed to 30-40 predefined metrics, many of which are 047 redundant because they are functionally linked to other metrics. This compression often leads to 048 significant loss of information which adversely impacts the quality of both phenotypes and disease models. Second, these neural metrics depend on multiple manually picked hyperparameters (e.g., burst threshold). This may lead to high variance with respect to different experimental conditions 051 (e.g., culture media, seeding cell density, etc.). Third, when an event either rarely occurs or does not occur in a recording (e.g., absence of bursts during low electro-physiological activity), a substantial 052 portion of the neural metrics becomes unavailable. Handling such missingness in the data analysis stage is not trivial. Traditional imputation methods assume that the metric exists in reality but was

not observed. In this case, the metric is unavailable or undefined rather than not unobserved. This
 further creates a potential for neuronal metrics to be volatile and attain outlier values.

Over the past decade, deep learning models have revolutionized the fields of computer vision, natural 057 language processing, video and speech recognition (LeCun et al., 2015; Chai et al., 2021; Chai & Li, 2019). Recent applications of deep learning in neuroscience have shown promising advancement especially in processing electroencephalogram (EEG) data or in vivo recordings (van Leeuwen et al., 060 2019; Schirrmeister et al., 2017; Buccino et al., 2018). Several recent statistical and machine-learning 061 methods have been developed to understand the neuronal biology from live electro-physiological 062 recordings of mouse-brains (Wu et al., 2017; Williams et al., 2020; Valente et al., 2022; Keeley et al., 063 2020). On passive MEA recordings of in vitro tissue cultures however, such applications are still 064 limited. Recent works (Matsuda et al., 2022; Zhao et al., 2019), mainly deriving inspiration from computer vision, use of end-to-end convolutional neural networks (CNN) (Krizhevsky et al., 2012) on 065 rasterized MEA signals for classifying gene knockouts and drug responses. Beyond models developed 066 for specific tasks, foundational models such as autoencoders, or in general, feature extraction through 067 self-supervised learning models, remains unexplored. 068

069 The MEA data modality is different from the image data modality and requires different calibration techniques. An important requirement for MEA is that the recordings remain invariant to shifts in 071 time or changes in the orientation of the electrodes. When learning features from MEA recordings, it is important for a model to learn features that do not change if the recording is moved in time 072 by a fixed amount, nor should they change if the spatial arrangement of the electrodes is permuted 073 keeping the inter-electrode distances intact. These invariances are crucial as any temporal shift is 074 determined by when the recording started and they have no biological relevance. Similarly, different 075 spatial arrangements of the electrodes only reflect the arbitrary order in which they were arranged in 076 the data matrix. Classical point process models (Snyder & Miller, 2012; Deutsch & Pfeifer, 1981) 077 for analyzing spatiotemporal time-series data, in theory, can address this invariance issue. However, these models often require specific parametric assumptions, which can be overly restrictive for feature 079 learning. Additionally, there is no widely adopted point process model for the analysis of in vitro MEA data; instead, in the broader field of neuronal electrophysiology, ad-hoc models (Amarasingham 081 et al., 2006; Bogaard et al., 2009) are typically devised based on specific research hypotheses.

082 The spatiotemporal invariance for MEA recordings is analogous to orientational (rotation, translation, 083 scale, mirror-flip) invariance of images for computer vision, where data augmentation is a standard 084 technique of choice for model calibration (Perez & Wang, 2017). However, data augmentation 085 does not guarantee that the encoded embeddings generated from an original image and a differently oriented image will be the same. Recent works (Burgess et al., 2024; Lohit & Trivedi, 2020) 087 have proposed autoencoder architectures that guarantee such orientational invariance for 2D and 880 spherical (3D) images by using orientation-equivariant convolution layers and a spatial pooling layer. Moreover, theoretical developments in the deep set literature (Zaheer et al., 2017) and group-089 equivariant methods (Cohen & Welling, 2016) provided necessary building blocks in understanding 090 these invariance principles. Our work is deeply inspired by these recent developments. 091

092 In this paper, we show that augmentation techniques, even though slightly improve the model performances, still fail to learn important features for detecting subtle and complex firing patterns. For this reason, we propose Roll-AE, an novel autoencoder architecture that explicitly and completely 094 calibrates for spatiotemporal invariance while extracting relevant features from MEA recordings. 095 Roll-AE constructs invariant sets from given recordings and learns a set-to-set mapping with a low-096 dimensional bottleneck. Roll-AE is intended to be a foundational model for in vitro MEA recordings and its learned features can be used for multiple downstream tasks. We first demonstrate on a 098 synthetic dataset that the Roll-AE embeddings have superior performance in identifying unique and complex firing patterns compared to standard autoencoders with augmentation. Then, we demonstrate 100 multiple downstream applications of Roll-AE embeddings on an real iPSC-derived neuronal culture to 101 illustrate that the proposed architecture captures meaningful multi-dimensional biological phenotypes 102 useful for disease modeling and treatment discovery.

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### 108 2 ROLL-AE

## 110 2.1 NOTATIONS

112 Let  $x \in \Omega \subseteq \mathbb{R}^D$  be a *D*-length time-series (or *spike-train*) from a feature space  $\Omega$  and let  $x_i$  be 113 the *i*-th element of x. Let  $\mathbb{N}_D = \{0, 1, \dots, D-1\}$ , and  $\pi_i : \Omega \to \Omega$  for  $i \in \mathbb{N}_D$ , be a cyclic 114 permutation function where  $\pi_i(x) = (x_{D-i+1}, x_{D-i+2}, \dots, x_D, x_1, x_2, \dots, x_{D-i})$ . Intuitively, 115  $\pi_i(x)$  cyclically shifts x's elements by i positions, and we can define such cyclic permutations as 116 *shifts*. Let  $\Pi(x) = \{\pi_i(x) : i \in \mathbb{N}_D\}$  be the set of all D shifts of x and  $\Pi(\Omega) = \{\Pi(x) : x \in \Omega\}$ 117 be the set of all  $\Pi(x)$  where  $x \in \Omega$ .

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#### 2.2 STANDARD AUTOENCODER ARCHITECTURE

The standard autoencoder on the input feature space  $\Omega$  is a map  $f_{\theta,\phi} : \Omega \to \Omega$  with the encoder  $g_{\theta} : \Omega \to \mathbb{R}^k$ , decoder  $h_{\phi} : \mathbb{R}^k \to \Omega$ , and loss function  $\ell : \Omega \times \Omega \to \mathbb{R}$ . Here,  $\theta$  and  $\phi$  are parameters of the encoder and decoder, respectively. Let the output of  $g_{\theta}(x)$  to be the encoded embedding of  $x \in \Omega$  and  $\mathbb{E} = \{g_{\theta}(x) : x \in \Omega\}$  to be the embedding space.

The goal of an autoencoder is to reconstruct a given input. Specifically, the network is trained to minimize a reconstruction loss  $\ell(x, \hat{x})$  where  $\hat{x} = f_{\theta,\phi}(x) = h_{\phi}(g_{\theta}(x))$  is the forward propagation reconstruction of a given input  $x \in \Omega$ . When  $\Omega = \mathbb{R}^D$ , a commonly adopted loss function is the mean-squared loss, defined as  $\ell(x, \hat{x}) = \frac{1}{D} \sum_{i=1}^{D} (x_i - \hat{x}_i)^2$ .

A limitation of the standard autoencoder is that it suffers from the lack of shift-invariance. This means that two spike-trains x and  $\pi_i(x)$ , which are just shifted versions of each other, are interpreted as different spike-trains and encoded into different embeddings in the embedding space, i.e.,  $g_\theta(x) \neq$  $g_\theta(\pi_i(x))$ . In the electro-physiology context, a shift is purely determined by when the recording started and has no biological relevance. Hence, any difference between the embeddings of two shifted spike-trains should only represent noise or potentially confounding information.

135 Shift-invariance in time-series data is analogous to the rotational invariance of autoencoders applied 136 in image processing, where we want the network to learn the same encoded embeddings from 137 different rotations of the same image. A standard way to handle such invariances is to apply 138 augmentations (Perez & Wang, 2017; Caron et al., 2021). In the context of MEA, this means, at 139 each training iteration, a randomly selected shifted spike-train  $\pi_i(x)$  is used as the input instead of x, and the reconstruction loss is calculated with respect to the original input x. Notice that even if 140 augmentation encourages the network to reconstruct the original spike-train x, it does not guarantee 141 that  $g_{\theta}(x) = g_{\theta}(\pi_i(x))$  for all *i*, which means the invariance problem is only partially tackled by 142 augmentation. Drawing inspiration from recently proposed methods (Burgess et al., 2024; Lohit & 143 Trivedi, 2020) that achieves orientation-invariance for 2D and spherical (3D) images, in the following 144 section we introduce a novel architecture that enforces shift-invariance directly in the encoding 145 process, ensuring that  $q_{\theta}(x) = q_{\theta}(\pi_i(x))$ . 146

147 148 2.3 ROLL-AE

149 A shift-invariant loss  $\rho: \Omega \to \Omega$  is defined such that the distance  $\rho(\pi_i(x), \pi_i(x))$  between two 150 shifts  $\pi_i(x)$  and  $\pi_i(x)$  of the same spike-train is zero for any  $i, j \in \mathbb{N}_D$ , or in other words, the 151 shifted spike-trains are treated as the same spike-train. This can be achieved by defining the distance 152 metric to be  $\rho(x, x') = L(\Pi(x), \Pi(x'))$  where L is a set-based loss such as Chamfer loss (Zhang 153 et al., 2019), Linear assignment loss (Zhang et al., 2020), etc. Therefore, shift-invariance can be 154 achieved in an autoencoder architecture by modifying the objective, specifically, by reconstructing entire sets  $\Pi(x)$  instead of single spike-trains and back-propagating on the set-based loss function 155 L. Since L is invariant to the ordering of the elements in a set, it guarantees  $\rho(\pi_i(x), \pi_i(x)) =$ 156  $L(\Pi(x),\Pi(x)) = 0$ . Formally, Roll-AE can be defined as a network  $f_{\theta,\phi}: \Pi(\Omega) \to \Pi(\Omega)$  with 157 the encoder  $g_{\theta} : \Pi(\Omega) \to \mathbb{R}^k$ , decoder  $h_{\phi} : \mathbb{R}^k \to \Pi(\Omega)$ , and loss L (see Eq. 1). 158

**Encoder** Since Roll-AE is a set-to-set mapping, the encoder is constructed based on ideas from the Deep Set literature (Zaheer et al., 2017; Soelch et al., 2019; Zhang et al., 2019). Explicitly, for any  $\Pi(x) \in \Pi(\Omega)$ , the encoder is defined as  $g_{\theta}(\Pi(x)) = a(\{\widetilde{g_{\theta}}(x') : x' \in \Pi(x)\})$ , where



Figure 1: Roll-AE architecture. The input spike-train x is converted into its cyclic permutation set  $\Pi(x)$  which is passed to the encoder  $\widetilde{q}_{\theta}$ . The encoder outputs are grouped by the aggregation function a into a single embedding. The decoder  $h_{\phi}$  reconstructs the putative spike-train  $\hat{x}$  which is converted into its cyclic permutation set  $\Pi(\hat{x})$ . The reconstruction loss is computed between  $\Pi(x)$  and  $\Pi(\hat{x})$ .

 $\widetilde{g}_{\theta}(:) \Omega \to \mathbb{R}^k$  is a Multi-layer Perceptron (MLP) and a is an aggregation function that aggregates the set of k-dimensional outputs from  $\tilde{q}_{\theta}$  into a single k-dimensional embedding. The purpose of 182 the aggregation function is to make the learned embeddings invariant of the shifts. Notice that 183  $a\left(\{\widetilde{g_{\theta}}(x'):x'\in\Pi(x)\}\right)=a\left(\{\widetilde{g_{\theta}}(x'):x'\in\Pi(\pi_{i}(x))\}\right)$ , and hence  $g_{\theta}(\Pi(x))=g_{\theta}(\Pi(\pi_{i}(x)))$ . Typically, the average function is used as the aggregator, although other methods can be defined (Soelch et al., 2019). In the case of Roll-AE, the average function is utilized as the aggregation 185 function. 186

**Decoder** The decoder is defined as  $h_{\phi}(e) = \Pi\left(\widetilde{h_{\phi}}(e)\right)$  for  $e \in \mathbb{R}^k$ , where  $\widetilde{h_{\phi}}: \mathbb{R}^k \to \Omega$  is 188 189 an MLP. A crucial challenge in constructing Deep Set autoencoders is finding a suitable mapping 190 from the output of  $h_{\phi}$  to the space of sets. Roll-AE does not face this challenge as this mapping is 191 deterministic and known. Therefore, the forward propagation of the overall Roll-AE architecture is 192 defined as the mapping  $f_{\theta,\phi}(\Pi(x)) = \Pi\left(\widetilde{h_{\phi}}\left(a\left(\{\widetilde{g_{\theta}}(x'): x' \in \Pi(x)\}\right)\right)\right)$ . Fig. 1 shows Roll-AE 193 architecture. We note that the trainable parameters in Roll-AE are all contained in the encoder MLP 194  $\widetilde{q_{\theta}}$  and the decoder MLP  $h_{\phi}$ , which are functions that simply map  $\Omega$  onto  $\mathbb{R}^k$  and back. As a result, 195 the number of trainable parameters in Roll-AE remains the same as a standard autoencoder. 196

**Reconstruction Loss** Roll-AE uses the Linear Assignment loss as the reconstruction loss between  $X = \Pi(x)$  and  $X' = f_{\theta,\phi}(\Pi(x))$ . Specifically, for any arbitrary ordering of the elements X = $\{x^{(1)}, x^{(2)}, \dots, x^{(D)}\}$  and  $X' = \{x'^{(1)}, x'^{(2)}, \dots, x'^{(D)}\}$ , and denoting  $\Psi$  to be the set of all possible permutations (not just cyclic) of  $(1, 2, \ldots, D)$ , the Linear Assignment loss is defined as,

$$L(X, X') = \min_{\psi \in \Psi} \sum_{i \in \mathbb{N}_D} \ell\left(x^{(i)}, x'^{\psi(i)}\right).$$
(1)

205 Computing the Linear assignment loss, in general, is extremely expensive with complexity  $O(s^3)$ 206 using the Hungarian algorithm, where s is the cardinality of X (in this context, s = D). However, in 207 our problem, since both the sets X and X' are closed under the cyclic permutation operation, the 208 computation can be substantially improved to O(r) complexity where r is the order of the cyclic 209 permutation operation (in this context, r = s = D). Lemma A.1 then simplifies this reconstruction 210 loss as  $L(\Pi(x), \Pi(\hat{x})) = D[\min_{i \in \mathbb{N}_D} \ell(\pi_i(x), \hat{x})]$ , where  $\hat{x} = h_{\phi}(a(\{\widetilde{g}_{\theta}(x') : x' \in \Pi(x)\}))$  is 211 the putative output train. Further, it trivially follows from Lemma A.1 that the above expression also applies to Chamfer loss. 212

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**Stochastic Shift-invariance** Roll-AE further implements stochastic shift-invariance, i.e., instead of 214 passing all possible shifted spike-trains in  $\Pi(x)$  on each forward propagation iteration, it uniformly 215 samples spike-trains from  $\Pi(x)$  with sampling rate  $\tau$  and uses the set of sampled spike-trains as

inputs. Once the model is trained, the final embeddings can then be calculated by running a final forward propagation of the encoder with the entire  $\Pi(x)$  as input. This ensures the invariance in the final embeddings remains intact. In Appendix C, we demonstrate through extensive simulation studies that stochastic shift-invariance can perform as good or sometimes even better than complete shift-invariance ( $\tau = 1$ ) while reducing the memory requirement and computation time.



Figure 2: Electro-physiological activity recording and electrode symmetry analysis in a well. On the left, an image of an well and electrodes from Axon Biosystems (Biosystems, 2024). The eight circular dots near the center are the recording electrodes, and are arranged in this 3-2-3 (row-wise) arrangement; on the right; all possible electrode symmetries and corresponding indices. In a typical recording from (Biosystems, 2024), the electrodes are labeled following the pattern on the top left corner.

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244 **Generic Permutations** Roll-AE is not limited to temporal shifts or permutation of input spike-245 trains. The above definitions and properties hold as long as  $\Pi(x)$  is a closed group with respect to any 246 class of cyclic permutations. This allows Roll-AE to handle other interesting classes of permutation that are relevant for our biological assays. For instance, mirror symmetries can be used to permute 247 electrodes in a well and enforce spatial invariance on our embeddings. When multiple spike-trains 248 are simultaneously recorded by different electrodes placed on the same cell culture, they need to be 249 arranged and inputted into the model in a specific reference order. However, this order is arbitrary, and 250 any permutation should be biologically equivalent to the reference order as long as the inter-electrode 251 distances are preserved. Such permutations are given by the mirror flip operation. Fig. 2a shows an 252 image of an actual eight-electrode well used to record neuronal electrophysilogical activity using an 253 Axion Biosystems (Biosystems, 2024) MEA machine. Fig. 2b shows all possible electrode orderings 254 with the top-left graph matching the reference orientation. Notice that the four permutations defined 255 by mirror symmetry form a closed group under the mirror flip operation which further is a special 256 case of cyclic permutations. Folding in the mirror symmetries into the previously defined Roll-AE 257 architecture results in the cardinalities of the input and output sets to become 4D with D temporal permutations, and four spatial permutations. 258

In Sec. 3.2, we will consider an actual dataset collected using the system depicted in Fig. 2a and train Roll-AE using both temporal shift and electrode mirror-symmetry invariances.

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#### 3 EVALUATIONS

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In this section we evaluate the proposed architecture on two case studies. First, we compare a standard autoencoder, a standard autoencoder with augmentation, and Roll-AE on a synthetic dataset. Second, we train and evaluate Roll-AE on a dataset of electro-physiology recordings of neuronal induced Pluripotent Stem Cells (iPSC) (Takahashi & Yamanaka, 2006) subject to different Small Interfering RNA (siRNA) (Fire et al., 1998) treatments.

#### 270 3.1 SYNTHETIC DATA EVALUATION 271

272 Just like in real recordings, we simulate recordings with spike-trains on eight electrodes. To mimic 273 the electro-physiology and firing patterns of real neuronal cultures, our synthetic dataset has four tunable parameters that determine the level of activity along four different types of source-events, 274 namely sporadic spikes (spike), sporadic single-channel bursts (burst), cyclic bursts (cycle), and 275 network bursts (network). These parameters are formally defined as follows: *spike*: the probability 276 of a sporadic firing event at a particular time instance; *burst*: the probability of a sequence of firing 277 events starting at a particular time instance for a random duration; cycle: presence or absence of a 278 repeating pattern of sequence of firing events with a random phase and duration; network: presence 279 or absence of network bursts where sequences of firing events are recorded simultaneously across 280 multiple electrodes. Each network burst starts with an originating electrode, and the probability of observing firing events in other electrodes depend on their proximity to the originating electrode. 282



Figure 3: Examples of synthetically generated spike-trains from eight electrodes tuning the probabilities of spike, burst, cycle, or network firing events.

295 Fig. 3 depicts some examples of firing patterns where one type of firing event was kept at a high 296 probability (or present) and the others are kept at a low probability (or absent). For instance, Fig. 3a 297 shows a recording obtained with high spike but low burst probabilities with no cycle or network 298 behaviors, resulting in a dense randomly scattered stack of spike-trains. Similarly, Fig. 3d depicts a 299 recording obtained with the presence of network behavior, but with low spike and burst probabilities, 300 and absence of cyclic bursts. This results in sparse but vertically aligned spike-trains, mimicking a 301 low sporadic activity but a well-synchronized network. The explicit details of the data generating model for the synthetic data are presented in Appendix B. For each of the tunable firing parameters, 302 we considered two classes, high probability and low probability classes for the spike and burst 303 parameters, and present and absent classes for cyclic and network parameters. This resulted in a total 304 of 16 combined classes of firing patterns. For each of these 16 classes, we generated 500 synthetic 305 recordings, each recording having 300-length spike-trains across eight channels (electrodes). 306

307 **Model Training** We trained three models: a standard autoencoder, a standard autoencoder with 308 augmentation, and Roll-AE on the synthetic dataset. The hyperparameters (such as training batch-size, 309 embedding dimension etc.) for each model were selected based on a two-step training/validation 310 scheme (see Appendix D for training details). The embeddings generated by the trained models were 311 then compared based on downstream classification tasks.

312 We trained five classifiers (with 70/30% training/validation data split) using the embeddings from each 313 autoencoder: four binary classifiers (e.g., high vs low probability of spikes, high vs low probability 314 of a bursts, etc.) and a multi-class classifier with 16 classes encompassing all combinations of our 315 synthetic dataset parameters (e.g., low spike, low burst, no cycle, no network vs low spike, low 316 burst, no cycle, present network vs low spike, low burst, present cycle, present network, and so 317 on). Specifically, we trained logistic regressors with L2 regularization with the penalty parameter 318 determined using a 4-fold cross-validation. The best trained logistic regressor was then used to make 319 predictions on the validation data.

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321 **Results** Fig. 4 reports the obtained accuracies together with the confusion matrices for the multiclass task. Overall, Roll-AE outperforms the standard autoencoders across all classification tasks 322 (see Fig. 4a). Remarkably, Roll-AE achieved a +30% accuracy on the hardest multi-class task. It 323 is also worth noticing, that augmentation improves the standard autoecoder across all tasks (except



(b) Confusion matrices of standard (left), standard with augmentation (center), and Roll-AE (right) autoencoders for multi-class classification. The 16 class labels were each formatted as {spike probability}-{burst probability}-{cycle present}-{network present}.

Figure 4: Roll-AE consistently outperforms standard autoencoders (both with and without augmentation) on all binary and multiclass classification tasks.

*cycle*), while still being sensibly inferior to Roll-AE. Fig. 4b reports the confusion matrices of the
 three models for the multiclass task. These plots highlight how Roll-AE embeddings lead to a high
 predictive accuracy in our downstream classification tasks while the two standard autoencoders tend
 to misclassify similar classes in a multi-class regime.

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#### 3.2 SIRNA TREATMENT EVALUATION

As a second case study, we applied the three models on a real electrophysiological dataset obtained from induced Pluripotent Stem Cells (iPSC) (Takahashi & Yamanaka, 2006) derived neurons subject to Small Interfering RNA (siRNA) (Fire et al., 1998) treatments. Technically, iPSCs are cells that have been reprogrammed from skin or blood cells to become other types of cells, in our case, neurons. siRNAs are artificially synthesized RNA molecules commonly used in molecular biology for silencing genes of interest. In our case, we apply a double siRNA treatment: the first to silence a gene to trigger the disease, the second to investigate a possible cure to counteract the effect of the first intervention.

392 Our neuronal culture was organized on two 96-well plates. Half of the sample set was subject to a 393 siRNA knockdown (siKD) of the gene of interest, mimicking the effect of the considered disease, while the other half was subject to a non-targeting control siRNA sequence (NTS) designed to target 394 no known genes. The NTS treatment is our negative control, i.e., a condition that does not affect the 395 neuron state. Both siKD and NTS samples have been then treated with 24 different siRNA potential 396 cures. Among these, there is an additional negative control NTS and a positive control CTRL+ 397 known to reduce neuronal excitability. For each condition, we cultured 4 replicates recorded at nine 398 different days up to 24 days in vitro. In total we obtained 2 (siKD or NTS)  $\times$  24 (siRNA treatments)  $\times$ 399 4 (replicates)  $\times$  9 (days in vitro)  $\times$  8 (electrodes) = 13824 raw spike-trains. Each spike-train was 400 recorded with one milli-second sampling rate for 10 minutes. We bucketized the raw spike-trains 401 to 500 milli-second bins to end up with spike-train length of D = 1200. To train the models, we 402 first performed a hyperparameter selection of training batch-size, embedding dimension, learning 403 rate, and shift-sampling rate (see details in Appendix D). Models with the selected hyperparameters were then trained to generate the final embeddings. First, we compare the embeddings on an siKD 404 phenotyping task. 405

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**Results - siKD Phenotyping** We compared the embeddings from the three models based on their performance in the classification of siKD wells from NTS wells on each of the 24 siRNA treatments. Biologically, the effect of siKD is very subtle, and discovering a classifier with good classification accuracy helps uncover subtle phenotypes in our disease model. We applied a leave-one-well-out approach, where at each iteration, we left out all the recordings from a particular well with a particular siRNA treatment, trained a siKD vs NTS classifier on the rest of the wells, and predicted the recordings of the left-out well. Logistic regressors with L2 regularization and 4-fold cross-validation were used as the classifiers.



Figure 5: Accuracy comparison between autoencoders for classification of siKD vs NTS on different siRNA treatments. Roll-AE has the highest accuracy for 15 our of the 24 treatments, in particular, on NTS.

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The average classification accuracies are presented in Fig. 5. For 15 out of the 24 treatments, Roll-A8
 AE embeddings had the highest classification accuracy, and most importantly, Roll-AE had the highest accuracy on NTS. Next, we demonstrate the application of the Roll-AE embeddings on two downstream tasks: treatment clustering and neural metrics credentialing.

Results - Treatment Clustering For this study, we used Roll-AE embeddings to extract biological insights and characterize treatment similarities. To do so, we considered cells cultured for 24 days in vitro, reduced the dimension of our embeddings to 10 principal components explaining at least 95% of the variance, computed the centroids of each treatment cluster, and calculated the pairwise distances between treatment centroids. The obtained results for NTS and siKD treated cells are organized in the two dendrograms shown in Fig. 6a and Fig. 6b, respectively.





(a) Pairwise distances between genes for NTS cells.



Figure 6: Dendrograms of embedding pairwise distances between gene centroids applied to unper-turbed (NTS) and diseased (siKD) cells.

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For cells with NTS, we observe a clear distinction between CTRL+ and NTS treatments, indicating that the model effectively differentiates CTRL+ (known to reduce cell excitability) from unperturbed NTS cells. On the other hand, gene\_21, which clusters far from CTRL+, is known for inducing hyperexcitability and stimulating neuronal activity. Furthermore, gene\_9, gene\_10, and gene\_11, which clustered together, are all components of the same signaling pathway involved in stress and inflammatory responses.

For cells with siKD, we observe again that NTS and CTRL+ treatments form distinct clusters, with gene\_21 clustering on the opposite side of the hyperexcitability spectrum of the dendrogram. Interestingly, gene\_10 and 11 are still clustering together but gene\_9 does not. Additionally, gene\_3 and gene\_4, which were grouped as hyperexcitable in the NTS cells study, now cluster among hypoexcitable treatments. This suggests that these genes interact differently with the induced disease state, leading to different types of neuronal activity depending on the disease state of the cell.

This study is an example of how extracting insights and clustering gene treatments can aid biologists
 in formulating hypotheses and identifying recurring patterns across various gene treatments.

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Results - Neural Metrics Credentialing The goal of this study is to verify whether the obtained
 embeddings retain relevant information on the curated neural metrics provided by the Axon Biosystems instrument that are explicitly computed from raw recordings. The full metric list is reported in
 Fig. 7a.

To asses the ability of our embeddings to retain relevant information on these curated neural metrics, for each metric, we trained a Ridge regressor with an *L*2 penalty of 1.0. This was trained on 80% of the entire dataset and then used to predict each metric on the remaining 20%. The correlation (r-score) between the predicted and observed neuronal metrics from the validation data is shown in Fig. 7a. The metrics are ranked from the highest to the lowest correlation.

Roll-AE embeddings predict most metrics with high correlation. Out of 44 regressed metrics, 25 477 had r-score above 0.75, 9 between 0.25 and 0.75, and 10 between -0.1 and 0.25. Fig. 7b illustrates 478 some scatter plots of actual metric values against the predicted ones. We observed that metrics 479 useful for phenotypic analyses, including firing, spike, or burst counts and rates, are accurately 480 captured and predicted. However, certain metrics, such as those related to the inter-burst interval 481 (IBI coefficient) (Di Credico et al., 2021), demonstrated lower correlations. We hypothesize that this 482 discrepancy may be attributed to the selected binning size during the compression of the raw signal, 483 which can potentially eliminate inter-burst information. 484

485 Overall, this study demonstrates Roll-AE's embedding effectiveness in capturing explicit neural metrics and hence their potential as a tool in phenotypic analyses and downstream tasks.



 (a) r-scores between observed and predicted neural metrics from Roll-AE embeddings. High r-scores indicate Roll-AE's embedding ability to capture explicit neural metrics.

(b) Examples of observed vs predicted neural metrics metrics from Roll-AE embeddings with high (top row) and low (bottom row) r-scores.

Figure 7: Credentialing the Roll-AE embeddings by evaluating its performance in predicting the neural metrics.

#### 4 CONCLUSION

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512 In this paper, we proposed Roll-AE, a novel spatiotemporal invariant autoencoder for feature extrac-513 tion from passive recordings of MEA assays. By leveraging and explicitly imposing the invariances 514 in the architecture, Roll-AE can extract features that are relevant for identifying unique firing pat-515 terns. On synthetic data, we demonstrated that the Roll-AE embeddings far outperform standard autoencoders (with or without augmentation) in discriminating different individual source-events. 516 Roll-AE is particularly accurate in the multi-class classification task and showed a +30% accuracy 517 gain compared to standard autoencoders suggesting Roll-AE's ability to identify features relevant to 518 complex and subtle phenotypes. On the siRNA experiment data, we further considered multi-faceted 519 downstream applications of Roll-AE generated embeddings. We demonstrated the superior perfor-520 mance of Roll-AE embeddings in discriminating siKD from NTS highlighting its use in phenotype 521 discovery. The concordance of the relative clustering of treatments with previous biological evidences 522 supports the validity of these machine learnt features. Finally, we showed that these embeddings 523 retain explicit metrics and can be used to predict manually curated features. 524

The original formulation of Roll-AE came with a few limitations, mainly in regard to computational 525 efficiency. Constructing the entire set of cyclic permutations and evaluating the Linear Assignment 526 loss can be computationally demanding. To tackle this issue, we have proposed the stochastic shift-527 invariance approach. Another possible approach is to apply discrete time Fourier transformation 528 to each spike-train to transform them from time domain to frequency domain and apply invariance 529 to analogous operations to temporal shifts. In our siRNA experiment example, we found that the 530 embeddings were not able to predict some neural metrics well, predominantly those with inter-spike 531 interval-related metrics. This could be a consequence of the adopted 500 milli-seconds bin size. While we have demonstrated its efficacy using in vitro MEA data, the autoencoder design could be 532 easily generalized for other types of data where such spatiotemporal invariance is relevant. Other 533 potential use cases for such architectures would be to identify arrhythmia from ECG data and anomaly 534 detection from sensors. 535

In conclusion, Roll-AE provides a foundational model for extracting features from in vitro MEA
 recordings. The features from Roll-AE enables better identification of unique electro-physiological
 activity patterns from MEA recordings, and can be used for a multitude of downstream applications
 including the identification of complex cellular phenotypes of different treatments such as siRNA
 knock-down, gene knock-outs etc.

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#### A MATHEMATICAL RESULTS

**Lemma A.1.** For two sets  $\Pi(x)$  and  $\Pi(y)$ ;  $x, y \in \Omega$ ,

$$\min_{\psi \in \Psi} \sum_{i \in \mathbb{N}_D} \ell\left(\pi_i(x), \pi_{\psi(i)}(y)\right) = D \times \left[\min_{i \in \mathbb{N}_D} \ell\left(\pi_i(x), y\right)\right].$$

*Proof.* First, we note that, for any  $i \in \mathbb{N}_D, \psi \in \Psi$ ,

$$\ell\left(\pi_i(x), \pi_{\psi(i)}(y)\right) \ge \min_{j,j' \in \mathbb{N}_D} \ell\left(\pi_j(x), \pi_{j'}(y)\right),$$

which implies

$$\min_{\psi \in \Psi} \sum_{i \in \mathbb{N}_D} \ell\left(\pi_i(x), \pi_{\psi(i)}(y)\right) \ge D \times \left[\min_{j, j' \in \mathbb{N}_D} \ell\left(\pi_j(x), \pi_{j'}(y)\right)\right].$$
(2)

Lets denote  $(j^*, j'^*) = \arg \min_{j,j'} \ell(\pi_j(x), \pi_{j'}(y))$ . Further, let the permutation  $\psi * \in \Psi$  to be a cyclic permutation that maps  $j^*$  to  $j'^*$ , explicitly,  $\psi^*(j^*) = j'^*$ . Then,

$$D \times \left[ \min_{j,j' \in \mathbb{N}_D} \ell\left(\pi_j(x), \pi_{j'}(y)\right) \right] = D\ell\left(\pi_{j^*}(x), \pi_{j'^*}(y)\right)$$
$$= D\ell\left(\pi_{j^*}(x), \pi_{\psi^*(j^*)}(y)\right) = D\ell\left(\pi_{i^*}(x), \pi_{\psi^*(i^*)}(y)\right).$$

The above is true for any  $i^* \in \mathbb{N}_D$ . Therefore, for  $\psi = \psi^*$ , the equality holds in 2, i.e.,

$$\min_{\psi \in \Psi} \sum_{i \in \mathbb{N}_D} \ell\left(\pi_i(x), \pi_{\psi(i)}(y)\right) = D\ell\left(\pi_{i^*}(x), \pi_{\psi^*(i^*)}(y)\right) \quad \forall i^* \in \mathbb{N}_D.$$

The final part of the proof holds by selecting  $i^*$  such that  $\psi^*(i^*) = 0$ ,

$$\ell\left(\pi_{i^*}(x), \pi_0(y)\right) = \min_{i \in \mathbb{N}_D} \ell\left(\pi_i(x), y\right)$$

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#### **B** SYNTHETIC DATA GENERATION

738 The synthetic data are simulated as normalized binary tensors. Lets denote the synthetic data 739 corresponding to the tunable parameters  $\beta_s, \beta_b, \beta_c, \beta_n$  as  $X(\beta_s, \beta_b, \beta_c, \beta_n)$  with dimensions  $N \times$ 740  $E \times D$ , where N = 500 is the number of recordings, E = 8 is the number of electrodes, and 741 D = 300 is the recording time duration. The parameter  $\beta_s = \{0.02, 0.1\}$  represents the probability 742 of a sporadic firing event. The parameter  $\beta_b = \{0.005, 0.025\}$  represents the probability of a sporadic 743 sequence of multiple firing event, or a burst on a single electrode. The parameter  $\beta_c = \{0, 1\}$ represents the absence or presence of cyclic burst firing pattern, and the parameter  $\beta_n = \{0, 1\}$ 744 represents the absence or presence of network burst firing pattern. Let the indices n, e, and d represent 745 single instances of recordings, electrodes, and timepoints. Let us also denote the four different firing 746 patterns sporadic single firing, sporadic single-channel burst, cyclic single-channel burst, and network 747 burst as source events. 748

To simulate the synthetic recordings, first, four source-specific binary recordings were simulated corresponding to each of the four different source events, and then those recordings were combined using the binary OR ( $\lor$ ) operation. This means, at any given time-point on a given electrode, a neuronal firing can be observed due to any combination of the source events. The synthetic data simulation algorithm is as follows:

**Algorithm B.1** (Algorithm to simulate the synthetic recordings). *First, initialize the following parameters: Firing frequency within a burst*  $\gamma_b = 0.9$ , *probability of a network burst starting at a given time point*  $\gamma_n = 0.035$ , *and network decay factor*  $\delta = 0.8$ .

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756 1. Sporadic single firings (Spike): Simulate  $Z_{n.e.d}^{(S)} \sim Bernoulli(\beta_s)$  i.i.d. 758 2. Sporadic single-channel bursts (Burst): 759 (a) Initialize  $Z^{(B)} = 0$ . 760 (b) Simulate burst initiation indicators  $S_{n.e.d} \sim Bernoulli(\beta_b)$  i.i.d. 761 (c) For each  $S_{n,e,d} = 1$ , 762 *i.* Select a burst duration  $\Delta = min(D, \Delta^*)$  where  $\Delta^* \sim DiscUnif(\{3, 4, 5\})$ , the 763 764 discrete Uniform distribution. ii. Simulate the source-specific firings  $Z_{n,e,i}^{(B)} \sim Bernoulli(\gamma_b)$  for  $i = d, \ldots, d + \Delta$ . 765 766 3. Cyclic single-channel bursts (Cycle): 767 768 (a) Initialize  $Z^{(C)} = 0$ . If  $\beta_c = 0$ , then skip to step 4. Else, move to the next step. 769 (b) For each recording n and electrode e, 770 i. Select a cycle period  $Q_c \sim DiscUnif(\{15, 16, \ldots, 19\})$  and phase  $P_c \sim$ 771  $DiscUnif(\{0, 1, ..., 14\}).$ 772 ii. Set the burst initiation indicators  $S_{n,e,d} = 1$  if  $(d - P_c)$  is divisible by  $Q_c$ , 0 773 otherwise. 774 *iii.* For each  $S_{n,e,d} = 1$ , 775 A. Select a burst duration  $\Delta = min(D, \Delta^*)$  where  $\Delta^* \sim DiscUnif(\{3, 4, 5\})$ , 776 the discrete Uniform distribution. 777 B. Simulate the source-specific firings  $Z_{n,e,i}^{(C)} \sim Bernoulli(\gamma_b)$  for  $i = d, \ldots, d + d$ 778 Δ. 779 780 4. Network bursts (Network): 781 (a) If  $\beta_n = 0$ , set  $Z^{(N)} = 0$  and skip to step 5. Else, move to the next step. 782 (b) Simulate burst initiation indicators  $S_{n,e,d} \sim Bernoulli(\gamma_n/E)$  i.i.d. 783 784 (c) For each  $S_{n,e,d} = 1$ , 785 i. Denote e to be the starting electrode, and d to be starting time-point for the network 786 burst. ii. Select a burst duration  $\Delta = min(D, \Delta^*)$  where  $\Delta^* \sim DiscUnif(\{3, 4, 5\})$ , the 787 discrete Uniform distribution. 788 iii. Simulate the source-specific firings  $Z_{n,e',i}^{(N)} \sim Bernoulli\left(\gamma_b \delta^{\alpha(e,e')}\right)$  for i =789  $d, \ldots, d + \Delta$  and  $e' \in \{1, \ldots, E\}$ . Here,  $\alpha(e, e')$  represents the physical distance 791 between the electrodes e and e' assuming the distance between electrodes 0 and 1 792 in the configuration described in Fig. 2 to be one unit. 793 5. Combine the recordings: Obtain the combined recording  $Z = Z^{(S)} \vee Z^{(B)} \vee Z^{(C)} \vee Z^{(N)}$ . 794 6. Normalize: The final dataset  $X(\beta_s, \beta_b, \beta_c, \beta_n)$  is obtained by normalizing the dataset Z. 796 797 **EVALUATING STOCHASTIC SHIFT-INVARIANCE** 798 С 799 800 Here, we evaluate the accuracy, computation time, and memory requirements of the Roll-AE model 801 under the stochastic shift-invariance strategy based on the synthetic data (generative model B). We trained the Roll-AE model with different training batch-sizes, embedding dimensions (k), and shift-802 sampling rates  $(\tau)$ . We evaluated the accuracy under each selection of hyperparameters using the 803

Fig. 8 shows the classification accuracy, computation time, and memory requirements for each selection of the hyperparameters. The accuracies were similar across all choices of hyperparameters, and except for the case with batch-size = 64 and embedding dimension k = 128, the shift-sampling rate  $\tau = 0.01$  resulted in the best accuracy for the 16-class classification task on the second-level validation dataset (see D). As expected, the computation time was the longest for  $\tau = 1$  which implies the entire set  $\Pi(x)$  was used in each pass of model training. However, the strongest contributor to

same evaluation scheme outlined in D. All models were trained with learning rate 0.0001.

810 Memory Requirement Accurac Computation Time 6000 811 5000 812 0.1 (BM 4000 3000 813 2000 814 0.2 100 815 16/128 16/512 64/128 Batch Size/Embedding Size 16/128 16/512 64/128 64/512 16/128 16/512 64/128 Batch Size/Embedding Size Batch Size/Embedding Size 816

Figure 8: Accuracy, Computation time (Hour), and Memory requirement (MB) for different training batch-sizes and embedding dimensions (k), with respect to different shift-sampling rates ( $\tau$ ). All models were trained on an NVIDIA<sup>®</sup> V100 GPU.

the computation time was batch size, with batch-size 16 requiring  $\sim 1.5 - 2$  times more computation time than batch-size 64.

The most important benefit of the stochastic shift-invariance is in the memory requirement. Training with  $\tau = 0.01$  required half as much memory for batch-size = 16, and nearly one-seventh as much memory for batch-size 64 compared to  $\tau = 1$ . By requiring substantially lower memory than training with complete shift-invariance (entire  $\Pi(x)$ ), in large datasets, the stochastic shift-invariance strategy can allow for training with larger batch-sizes on limited GPU memory, which in turn can help reduce the computation time without impacting the accuracy.

#### D MODEL TRAINING AND HYPERPARAMETER SELECTION

#### D.1 MODEL TRAINING

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835 For the synthetic data, we trained the three models with different hyperparameters (listed in Table 1) 836 on 70% of the data (training) randomly selected, and then applied the trained model on the remaining 837 30% of the data (validation) to obtain the embeddings. The embeddings of the validation dataset were 838 then further split into a second-level of 70%-30% training/validation data to evaluate the predictive 839 accuracy of those embeddings for the 16-class classification task. For this purpose, a 16-class 840 logistic regression classifier with L2-regularization was trained on the second-level training data, and then the classes were predicted on the second-level validation data. The penalty parameter 841 for the L2-regularization was selected based on four-fold cross-validation. Using the two levels of 842 training/validation data splits, we ensured that both the autoencoder model, and the downstream 843 classifier are generalizable to previously unseen data. For each of the three models, whichever 844 hyperparameters led to the highest predictive accuracy in the second-level validation data, were 845 selected and the models with those selected hyperparameters were then trained on the entire dataset 846 to generate the final embeddings. 847

Similar two-level validation approach was taken for selecting hyperparameters in the siRNA data. We 848 first trained the three models with the same set of hyperparameters (listed in Table 1) and generated 849 the embeddings based on a 70%-30% training/validation data split. Then, with the embeddings of the 850 validation dataset, we evaluated a logistic regression classifier (with L2 regularization) of the siKD vs 851 NTS samples based on a second-level 70% - 30% split of the data. The hyperparameters which led 852 to the highest accuracy in the second-level validation data were selected for each model. Finally, we 853 applied the three models with the selected hyperparameters on the entire dataset to generate the final 854 embeddings. 855

856 D.2 HYPERPARAMETERS

Table 1 lists all choices of the training parameters that were considered to train the three autoencoder
models. For the standard autoencoder with augmentation, two augmentation sampling schemes were
evaluated, namely Uniform and Half-mass. For the Uniform sampling scheme, on each epoch, the
augmented recording was randomly selected from the set of all possible shifted (and mirror-flipped)
recordings uniformly. On the contrary, for the Half-mass sampling scheme, with probability 0.5 the
original recording was used as the augmented spike-train, and the with the rest 0.5 probability, the
other shifted (and mirror-flipped) recordings were uniformly selected. The best training parameter

choice for each autoencoder model was determined based on the performance of the embeddings in
 the multi-class classification task for synthetic data, or in the binary classification of siKD and NTS
 in the siRNA experiment data. These parameter choices are highlighted in Table 1 in bold.

Table 1: Choice of training parameters for the three autoencoder models. The best choice of parameters for each autoencoder model are highlighted in bold font (for the synthetic data) or with an asterisk (for the siRNA experiment data).

Γ	Training parameter	Standard AE	Standard AE + aug.	Roll-AE
Г	Training batch-size	<b>8</b> , 16, 32, 64*	8, <b>16</b> *, 32, 64	8, 16, 32, <b>64</b> *
	Embedding dimension $(k)$	128*, <b>256</b> , 512	128, <b>256</b> *, 512	128, 256, <b>512</b> *
	Learning rate	0.0001, <b>0.001</b> *, 0.01, 0.1	<b>0.0001</b> *, 0.001, 0.01, 0.1	<b>0.0001</b> *, 0.001, 0.01, 0.1
	Augmentation scheme	N/A	Uniform, Half-mass*	N/A
	Shift-sampling rate $(\tau)$	N/A	N/A	<b>0.01</b> *, 0.05, 0.1, 1

All models had two hidden layers in each of their encoder and decoder MLPs. The hidden layers in the encoder MLPs had 4k and 2k neurons sequentially, where k is the embedding dimension. Conversely, the hidden layers in the decoder MLPs had 2k and 4k neurons sequentially. Counting the k parameters for the output layer of the encoder MLP and the ED parameters for the output layer of the decoder MLP, our models had a total of (13k + ED) trainable parameters. Mean-squared error loss was used for the standard autoencoders (with or without augmentation), and Linear assignment loss was used for Roll-AE. Adam optimizer Kingma & Ba (2017) was used for the back-propagation in all models. Each model was trained for 200 epochs.