Internal Chain-of-Thought: Empirical Evidence for Layer-wise Subtask Scheduling in LLMs

Anonymous ACL submission

Abstract

We show that large language models (LLMs) exhibit an internal chain-of-thought: they sequentially decompose and execute composite tasks layer-by-layer. Two claims ground our study: (i) distinct subtasks are learned at different network depths, and (ii) these subtasks are executed sequentially across layers. On a benchmark of 15 two-step composite tasks, we employ layer-from context-masking and propose a novel cross-task patching method, confirming (i). To examine claim (ii), we apply LogitLens to decode hidden states, revealing a consistent layerwise execution pattern. We further replicate our analysis on the real-world TRACE benchmark, observing the same stepwise dynamics. Together, our results enhance LLMs transparency by showing their capacity to internally plan and execute subtasks (or instructions), opening avenues for fine-grained, instruction-level activation steering.

1 Introduction

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Large Language Models (LLMs) excel at solving complex tasks such as instruction following, and multi-step problem solving (Zhang et al., 2024; Zeng et al.; Wang et al., 2024). Much recent progress relies on explicit "chain of thought" (Wei et al., 2022; Zhang et al.), which guides models to decompose multi-step problems into intermediate reasoning stages. This raises a foundational question: Do LLMs also perform such multi-step reasoning internally, without revealing steps in their output? In this work, we answer yes: LLMs exhibit an internal chain-of-thought (ICoT), meaning they internally break down composite tasks and process their components sequentially across network layers. Going beyond interpretability studies on latent factual multi-hop reasoning (Yang et al., 2024b; Biran et al., 2024; Yu et al., 2025; Anthropic., 2025), we investigate task-level reasoning rather than just chains of facts.



Figure 1: Evidence of an internal chain-of-thought. Selectively masking (bottom left) from specific layers can preserve the first subtask (antonym) while ablating the second (uppercase). Decoding hidden states (bottom right) on a clean run shows the intermediate answer ("slow") peaks at middle layers.

To illustrate this concept, consider a composite task that requires two steps: antonym then uppercase. For example, given the input "fast", solving this involves an intermediate step-finding the antonym "slow"-followed by capitalizing it to "Slow" (with S capitalized). If an internal chain-ofthought exists, we would expect the hidden state at some intermediate layer to represent "slow", and later layers to transform it into "Slow". In general, the presence of an ICoT implies that distinct phases of computation occur inside the model, each corresponding to a subtask in the overall problem. Figure 1 illustrates our core findings: selectively masking context from specific layers can preserve the first subtask (antonym) while ablating the second (uppercase). Meanwhile, decoding results of hidden state on clean run show that intermediate answer ("slow") often peaks at middle layers.

For tractability, we study tasks that decompose into two sequential subtasks (denoted $t := s_1 \circ s_2$). The task t maps an input x to an output $y^t = s_2(s_1(x))$. We frame our analysis through Task Vector framework (Hendel et al., 2023; Todd et al.; Li et al.; Saglam et al., 2025), which divides incontext learning (ICL) into two phases: (1) a *learning phase*, where the model abstracts task rules into a hidden representation, task vector, and (2) a *rule application phase*, where the query is processed using this vector. For a model T, given demonstrations S and a query x, the process is modeled as:

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$$T([S,x]) \Rightarrow \underbrace{\theta = \mathcal{A}(S)}_{\text{learning phase application phase}}, \underbrace{y = f(x;\theta)}_{\text{plearning phase application phase}},$$
 (1)

where \mathcal{A} abstracts the task vector $\theta \in \mathbb{R}^d$, and f denotes the application of task-specific rules. Extending to composite tasks yields two claims:

- Claim 1. Subtasks are learned at different network depths, inducing distinct subtask vectors θ^{s1} and θ^{s2} which generalize to their respective subtasks.
- Claim 2. Subtasks are executed sequentially across layers. At depth l_1 , the model applies $f^{l_1}(x; \theta^{s_1})$ to compute the first subtask. At a later depth l_2 , it applies $f^{l_2}(x; \theta^{s_2})$, yielding the final result.

Crucially, we distinguish between two processes: "learning" (Claim 1) and "execution" (Claim 2), corresponding to the learning phase and application phase in Task Vector framework, respectively.

We first introduce a benchmark of 15 two-step composite tasks spanning four categories (Section 2.2). We present two lines of evidence for Claim 1: (1) layer-from-context masking (Sia et al., 2024), which blocks attention to demonstrations after layer l to reveal where each subtask is learned, and (2) cross-task patching, a novel method which inserts residual activations from a composite prompt into zero-shot sub-task queries to detect reusable "subtask vectors". Across four models and 15 two-step tasks, masking reveals a sharp "X-shape" (Figure 2), indicating a sequential learning dynamics: the model first abstracts the rule for s_1 at an earlier layer, and later learns s_2 at a deeper layer. Meanwhile, patching activations in Llama-3.1-8B (see Table 2) yield transferable subtask vectors to a significant degree (66% on average).

Next, to verify Claim 2, We decode every layer with LogitLens (Nostalgebraist., 2020), projecting

hidden states into token space and tracking the mean reciprocal rank of the first-step target $(y^{s_1}$ or $y^{s_2})$ versus the final answer $(y^{s_1 \circ s_2})$. Decoding results show the same "handoff" (see Figure 3 and 4): intermediate answer peaks in mid-layers, then is overtaken a few layers later by the final answer. Finally, we replicate layer-from-context masking on TRACE (Zhang et al., 2024), a complex instruction-following benchmark, demonstrating that the same sequential learning dynamics emerge in real-world settings (see Figure 5). The primary contributions of this study are as follows:

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- We construct a curated benchmark of 15 composite tasks spanning four categories.
- We employ context-masking and propose **cross-task patching**, demonstrating that sub-tasks are learned at different depths, inducing distinct subtask vectors.
- We use LogitLens to decode hidden states, revealing a consistent layerwise execution pattern.
- We replicate our method on the TRACE benchmark, confirming the same finding also emerge in practical settings.

Our findings enhance LLM transparency by revealing their capacity to internally plan and execute subtasks (or instructions). This aligns with, and extends, prior interpretability studies on multihop reasoning (Yang et al., 2024b; Biran et al., 2024; Yu et al., 2025; Anthropic., 2025) and lookahead planning (Men et al., 2024). While those often focus on factual recall or predictive steps, our work investigates task-level reasoning rather than just chains of facts. Furthermore, the discovery of ICoT opens exciting avenues for fine-grained, instruction-level behavior control. For instance, by identifying the layers responsible for processing specific (potentially harmful) instructions within a user's prompt, we could directionally intervene to steer their execution for safer LLM behavior.

2 Experimental Setup

2.1 Prompt Design

We focus on composite tasks that naturally decompose into two sequential subtasks—for example, retrieving domain knowledge and then translating it, or extracting information followed by format transformation. Formally, we represent a composite task as $t := s_1 \circ s_2$, where s_1 and s_2 are sequentially applied subtasks. Given a query x,

Category	Task Description	Example (Input $ ightarrow$ Output)	
Knowledge–Algorithmic	s_1 : antonym	$\texttt{fast} \rightarrow \texttt{Slow}$	
Rilowiedge Augoritanine	s_2 : uppercase		
Extractive-Knowledge	s_1 : select adjective	artistic, captain, bring $ ightarrow$ creative	
Extractive=Knowledge	s_2 : synonym		
Extractive-Algorithmic	s_1 : select last item	spicy, cowardly, hoop $ ightarrow$ h	
Extractive-Algorithmic	s_2 : first letter		
Knowledge–Translation	s_1 : retrieve country	Cenepa River $ ightarrow$ Pérou	
Knowledge-Italislation	s_2 : translate to French		

Table 1: Representative examples from the composite task benchmark across four categories. Each task involves a sequential application of two subtasks, though no intermediate outputs are shown in-context. See Appendix A for a complete task list and definitions.

the final output is $y^t = s_2(s_1(x))$. For analysis purposes, we also compute intermediate outputs corresponding to the isolated application of each subtask: $y^{s_1} = s_1(x)$ and $y^{s_2} = s_2(x)$. As an illustrative example, consider $s_1 =$ "antonym" and $s_2 =$ "uppercase". For input x = "fast", the correct intermediate and final outputs would be $y^{s_1} =$ "slow", $y^{s_2} =$ "Fast", and $y^t =$ "Slow".

For each composite task $t \in \mathcal{T}$ in our task suite \mathcal{T} , we construct a dataset \mathcal{P}_t consisting of in-context prompts $p_i^t \in \mathcal{P}_t$. Each prompt includes N input-output demonstration pairs of the form (x, y^t) , showing the full composite transformation, followed by a query input x_{iq} for which the model is expected to predict the corresponding target y_{iq}^t Notably, no intermediate outputs or reasoning steps are included in the prompt. The in-context learning (ICL) prompt format is:

$$p_i^t = \left[(x_{i1}, y_{i1}^{s_1 \circ s_2}), \dots, (x_{iN}, y_{iN}^{s_1 \circ s_2}), x_{iq} \right].$$
(2)

2.2 Dataset

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We construct a benchmark of 15 composite tasks spanning four categories:

- **Knowledge–Algorithmic:** Tasks that combine factual knowledge retrieval (e.g., country capitals, antonyms) with deterministic transformations (e.g., uppercase conversion).
- Extractive–Knowledge: Tasks that require identifying items from a list (e.g., selecting the last item) followed by a knowledge-based operation (e.g., finding a related concept).
- Extractive-Algorithmic: Tasks that involve list-based selection followed by symbolic transformations (e.g., case conversion, character extraction).

• **Knowledge–Translation:** Tasks that combine knowledge retrieval with language translation (e.g., translating the capital city of a given country into French or Spanish).

Each query in the dataset requires the sequential execution of two subtasks in a fixed order as defined by the task specification. However, we do not assume that LLMs necessarily follow this order during internal processing. To probe the latent execution path, we measure intermediate outputs corresponding to the isolated application of each subtask: $y^{s_1} = s_1(x)$ and $y^{s_2} = s_2(x)$. Table 1 presents illustrative examples for each category. Full details and descriptions for all 15 composite tasks can be found in Appendix A.

3 Background

We consider an autoregressive transformer language model T that takes an input prompt p and outputs a next-token distribution T(p) over a vocabulary \mathcal{V} . Internally, T consists of L transformer layers connected via a residual stream (Elhage et al., 2021). We focus our analysis on the residual stream at the final token position. Embedding matrix $\mathbf{W}_E \in \mathbb{R}^{|\mathcal{V}| \times d}$ first maps the last token to a hidden representation as initial residual stream $\mathbf{h}^0 \in \mathbb{R}^d$. At each layer l, the model adds the outputs of the self-attention and feedforward network (FFN) modules to the residual stream from the previous layer. Formally, the residual stream at layer lis given by:

$$\mathbf{h}^{l} = \mathbf{h}^{l-1} + \mathbf{A}^{l} + \mathbf{F}^{l}, \qquad (3)$$

where $\mathbf{A}^{l} \in \mathbb{R}^{d}$ and $\mathbf{F}^{l} \in \mathbb{R}^{d}$ denote the attention and FFN outputs at the final token position, respectively, at layer l.

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Figure 2: Layer-from context-masking results for six composite tasks across four models. The "X-shape" pattern reveals sequential learning dynamics.

4 **Claim 1: Distinct Subtask** Representations

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We present two lines of evidence for **Claim 1**: (1) using layer-from context-masking, we show that different layers are responsible for learning each subtask; (2) with a novel cross-task patching method, we demonstrate that subtask-specific vectors emerge at the final token position, serving as abstract representations that generalize across tasks.

4.1 Layer-from Context-Masking

In-Context Learning requires model to infer a task from examples and apply it to a new input, as for-238 malized in Equation 1. If LLMs follow a internal chain-of-thought, then each subtask in a composite task should be learned at a distinct point in the network. That is, subtask learning should un-242 fold sequentially across layers-rather than all at 243 once-making intermediate learning states observ-244 able. To investigate this, we employ layer-from context-masking (Sia et al., 2024). This technique 246 disables access to the in-context examples (task demonstrations) from a specific layer onward by masking all attention to context tokens. If mask-249 ing is applied from the input layer (l = 0), the model cannot attend to any demonstrations, and ICL should fail. However, if masking begins only 252

after the model has learned the task, then its performance should remain intact. Crucially, by gradually shifting the start-masking layer from early to late, we can infer the sequential dynamics of the model's learning process.

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Let $\mathbf{A} = \frac{\mathbf{Q}\mathbf{K}^{\top}}{\sqrt{D}}$ denote the raw attention scores in a decoder-only transformer, where Q and K are the query and key matrices, respectively, and D is the dimensionality of the hidden states. For token positions i and j, the element A_{ij} represents how much token *i* attends to token *j*. We apply a context masking to disable attention to in-context examples, as $A_{ij} + m(j, \mathbf{u})$. The mask $m(j, \mathbf{u})$ is defined as:

$$m(j, \mathbf{u}) = \begin{cases} 0 & \text{if } x_j \notin \mathbf{u}, \\ -\infty & \text{if } x_j \in \mathbf{u}, \end{cases}$$
(4)

where **u** is the set of all in-context example tokens. The mask is applied from layer *l* onward, such that for all $l' \ge l$, attention to context is zeroed out after Softmax. For each test prompt, we progressively increase the masking layer l from 1 to L and record the model's prediction accuracy on both intermediate and final outputs. For composite tasks, we aim to identify a two-phase masking pattern. At early layers, masking may lead the model to predict intermediate outputs—e.g., y^{s_1} or y^{s_2} —indicating that the model has learned the first subtask but not

Composite Task	Llama-3.1-8B				
antonym-uppercase					
s_1 (antonym)	0.92 ± 0.02				
s_2 (uppercase)	0.24 ± 0.03				
country_capital-lowercase					
s_1 (country_capital)	0.88 ± 0.03				
s_2 (lowercase)	0.49 ± 0.05				
<pre>choose_last-country_capital</pre>					
s_1 (choose_last)	0.44 ± 0.03				
s_2 (country_capital)	0.98 ± 0.02				
adjective_v_verb-antonym					
s_1 (adj_v_verb)	0.38 ± 0.11				
s_2 (antonym)	0.94 ± 0.03				
choose_last-first_letter					
s_1 (choose_last)	0.29 ± 0.03				
s_2 (first_letter)	1.01 ± 0.01				
antonym-english_spanish					
s_1 (antonym)	0.91 ± 0.02				
s_2 (english_spanish)	0.45 ± 0.03				
Average	0.66				

Table 2: Subtask vector strength for six representative composite tasks in Llama-3.1-8B. Each composite task's average residual activation is patched into each subtask, with results shown for s_1 and s_2 respectively.

yet the second. As masking is delayed to deeper layers, the model's predictions should transition sharply from intermediate answers to the final answer y^t , revealing a layered acquisition of subtasks. By contrast, if the model transitions directly from generating no meaningful output to the correct final answer as masking depth increases, without producing intermediate completions, this would suggest a monolithic in-context learning process. This distinction is central to testing whether subtask learning unfolds sequentially across layers.

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Experiment. We conduct our layer-from contextmasking analysis on four LLMs: Llama-3.1-8B (Grattafiori et al., 2024), Mistral-7B (Jiang et al., 2024), Qwen2.5-7B (Yang et al., 2024a), and Llama-3.2-3B (Meta., 2024), evaluating their behavior across all 15 composite tasks. For each task, we generate 500 test prompts, sampled uniformly at random from the corresponding dataset. Each prompt includes N in-context examples (following prior work (Hendel et al., 2023), we set N = 5). To ensure robustness, all experiments are repeated across five random seeds, and we report averaged results. In the resulting sequential learning dynamics plots (see Figure 2 and Appendix B), we observe a striking "X-shape" pattern across composite tasks. Specifically, as context masking is delayed to deeper layers, the model's output transitions from generating one of the intermediate answers (e.g., the result of s_1) to producing the correct final answer. The intersection point-where performance on the intermediate answer begins to drop while performance on the final answer rises-suggests a boundary between subtask learning phases. This structure provides compelling evidence for sequential learning dynamics: the model first abstracts the rule for s_1 at an earlier layer, and later learns s_2 at a deeper layer.

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4.2 Cross-Task Patching

While context-masking reveals when subtask information is acquired, it does not directly test whether LLMs represent individual subtasks as reusable, abstract vectors. To address this, we introduce crosstask patching, a novel method that investigates whether sequential learning dynamics produce distinct subtask vectors. Prior work suggests that the residual stream at the final token position encodes a latent task representation derived from in-context examples (Hendel et al., 2023; Li et al.; Todd et al.). These representations can be replaced into the hidden states while running model on other prompts to influence model behavior. Here, we extend this idea to composite tasks. Specifically, we examine whether the activations obtained from a composite prompt can be used to improve performance on each subtask individually. We compute the average residual stream activation across composite task prompts, then patch it into zero-shot prompts from the subtask datasets. If performance on the subtask improves, we infer that the composite prompt's activation encodes the corresponding subtask vector.

Formally, we begin by running the model on a set of composite prompts $p_i^t \in \mathcal{P}_t$, each containing N examples of task t, and extract activation vector at the final token position from each layer l. Averaging over all prompts yields a layerwise task representation:

$$\bar{\mathbf{h}}_{l}^{t} = \frac{1}{|\mathcal{P}_{t}|} \sum_{p_{i}^{t} \in \mathcal{P}_{t}} \mathbf{h}^{l}(p_{i}^{t}).$$
(5)

We then patch this vector into a set of zero-shot 347 subtask prompts $\tilde{p}_i \in \tilde{\mathcal{P}}_{s_i}$ (i.e., prompts with no 348

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$$\operatorname{Acc}(\tilde{\mathcal{P}}_{s_j}, l) = \frac{1}{|\tilde{\mathcal{P}}_{s_j}|} \sum_{\tilde{p}_i} \mathbb{I}\left[T(\tilde{p}_i \mid \mathbf{h}^l := \bar{\mathbf{h}}_l^t) = y_i\right].$$
(6)

To quantify how well this patched vector recovers the subtask behavior, we define a normalized *subtask vector strength*. As the patching is used on each layer, we choose the best result to calculate the subtask vector strength:

$$\text{Strength}^{s_j} = \frac{\max_l \operatorname{Acc}(\tilde{\mathcal{P}}_{s_j}, l) - \operatorname{Acc}(\tilde{\mathcal{P}}_{s_j})}{\operatorname{Acc}(\mathcal{P}_{s_j}) - \operatorname{Acc}(\tilde{\mathcal{P}}_{s_j})},$$
(7)

where $\operatorname{Acc}(\mathcal{P}_{s_j})$ is the subtask's performance under standard ICL (with N examples), and $\operatorname{Acc}(\tilde{\mathcal{P}}_{s_j})$ is the zero-shot baseline. A strength of 1 implies full recovery of subtask performance, indicating a fully formed subtask vector; a strength of 0 implies no transfer. We test the subtask vector strength on both two subtasks s_1 and s_2 .

Experiment. To ensure independence between datasets, we first split each subtask dataset into disjoint train and test subsets (see Appendix A for details about subtask dataset). Composite datasets are constructed using only the train set, while zero-shot patching is evaluated on held-out subtask examples. We compute \bar{h}_l^t using 100 composite prompts and test patching strength on 500 zero-shot subtask prompts. We repeat this process across 15 composite tasks, 4 models, and 5 random seeds.

Table 2 and Appendix C report the patching strength across tasks. We find that most composite tasks yield transferable subtask vectors to a significant degree (0.66 on average). Interestingly, some composite tasks exhibit asymmetric transfer—for instance, the composite vector may strongly support s_1 but only weakly support s_2 . This asymmetry may reflect either the task type of s_2 (e.g., extractive tasks), or that s_2 is applied in a more entangled fashion atop the result of s_1 , making its representation more context-dependent.

5 Claim 2: Layer-wise Rule Application

Claim 2 hypothesizes that LLMs apply rules for composite tasks in a staged process: at an earlier layer l_1 , the model applies a function $f^{l_1}(x; \theta^{s_1})$ to perform the first subtask; later, at layer $l_2 > l_1$, it applies a second function $f^{l_2}(x; \theta^{s_2})$, integrating



Figure 3: Heatmaps of attention and MLP block decoding results for the country_capital-lowercase task in Llama-3.1-8B.



Figure 4: Decoding results of the residual stream for the country_capital-lowercase task in Llama-3.1-8B.

this intermediate representation with the second subtask's logic to produce the final answer. Crucially, we should be able to trace this transformation through the model's residual stream, which accumulates the outputs of each attention and MLP block.

We decode the next-token probabilities for each intermediate layer using LogitLens (Nostalgebraist., 2020). This method aims to project hidden states into the vocabulary space. Formally, let \mathbf{h}^l denote the residual stream at the final token position, at layer l. To decode their outputs into probability distributions \mathbf{p} over vocabulary tokens, we use the unembedding matrix $\mathbf{W}_{\mathbf{U}} \in \mathbb{R}^{d \times |\mathcal{V}|}$, along with a normalization that rescales component activations relative to the final-layer logits:

$$\mathbf{p} = \text{Softmax}\left(\mathbf{W}_{\mathbf{U}} \cdot \frac{\mathbf{h}^{l} - \bar{\mathbf{h}}^{l}}{\alpha^{*}}\right), \qquad (8)$$

where $\bar{\mathbf{h}}^l$ are the mean component outputs for nor-

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Figure 5: Layer-wise context-masking analysis on TRACE benchmark. Each curve shows model performance (scored 0–10) on a distinct constraint type, evaluated by DeepSeek-V3.

malization, and α^* is a scaling factor derived from the final layer's residual norm. Besides, we also decode each attention and MLP block's output \mathbf{A}^l and \mathbf{F}^l . We then measure the **Mean Reciprocal Rank (MRR)** for three specific targets: y^t , y^{s_1} and y^{s_2} .

Experiment. We conduct this analysis on 500 prompts for each of the 15 composite tasks described earlier. For each prompt, we extract and decode the attention outputs, MLP outputs, and residual stream at each layer, compute mean reciprocal ranks (MRRs) for the three target outputs described above, and plot the resulting trajectories.

Figure 3 shows the heatmaps of attention and MLP block decoding results for the country_capital-lowercase task in Llama-3.1-8B. Figure 4 displays the decoding results of the residual stream (see Appendix D for full results). We observe a clear layerwise task execution pattern: the model produces intermediate answers in middle layers, which are progressively surpassed by the final answer in later layers. The crossover point—where MRR for y^{s_1} declines while MRR for $y^{s_1 \circ s_2}$ increases—mirrors the two-stage task execution hypothesized in Claim 2.

6 A Practical Case: TRACE Dataset

437To evaluate the applicability of our analysis in
real-world scenarios, we extend our experiments to
TRACE (Zhang et al., 2024), a complex instruction
following benchmark. TRACE is built on a manu-
ally curated taxonomy of complex instructions, in-
corporating 26 constraint dimensions grouped into
five high-level categories. Each prompt in TRACE

consists of two components: a *Task Description*, which defines the core objective (e.g., "Introduce Beijing"), and a set of *Constraints*, which specify additional requirements that the model must satisfy. For example, a representative prompt might be:

Task Description: Can you introduce Beijing to me?
Constraints: 1. The generated answer format needs to be JSON;
2. The generated answer must be objective facts and cannot contain any subjective opinions or guesses;
3. All characters need to be encoded in UTF-8 to ensure support for multilingual responses.

We provide the entire prompt to the model and use a LLM evaluator to assign a score (from 0 to 10) for each constraint based on how well it is satisfied. In this section, we apply layer-from context-masking, but with a twist: we selectively mask only the *Constraints* portion of the prompt from each layer onward, while retaining access to the *Task Description* throughout. Our goal is to determine whether different types of constraints are learned at different depths, thereby exhibiting a multi-step learning trajectory.

Experiment. We select a subset of TRACE (69 prompts) that include all the following constraint types: **Inclusion, Output Format**, and **Tone and Style**. We use Qwen2.5-7B-Instruct (Yang et al., 2024a) as the test model to complete the instructions, applying context-masking to the constraint tokens from each layer onward. To evaluate the quality of constraint satisfaction at each masking depth, we use DeepSeek-V3 (Liu et al., 2024a) as an evaluator model, producing per-constraint scores from 0 to 10.

Figure 5 shows the resulting line chart. We observe notable differences in the learning dynamics across constraint types:

- **Output Format:** The learning curve is characterized by a sharp increase in score between layers 17–20, suggesting that formatting constraints (e.g., JSON structure, character encoding) are learned relatively late in the network.
- **Inclusion** and **Tone and Style:** These constraints show more gradual and smooth improvements across layers, indicating a slower

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or more distributed learning process. These results demonstrate that different con-

straint types are learned at different depths in the model, further supporting our hypothesis of sequentially learning dynamics in composite instructionfollowing tasks.

Related Work 7

Multi-Hop Reasoning in LLMs. Recent studies have examined how large language models (LLMs) perform latent factual multi-hop reasoning (Press et al., 2023; Yang et al., 2024c; Li et al., 2024; Ju et al., 2024). (Yang et al., 2024b) finds that LLMs often reliably recall intermediate entities but inconsistently use them to complete complex prompts. (Biran et al., 2024) shows that LLMs resolve intermediate entities early when answering multi-hop queries, and proposes a back-patching method to improve the performance. (Yu et al., 2025) introduces logit flow to analyze latent multihop reasoning in LLMs and proposes back attention to improve accuracy. (Anthropic., 2025) identifies intermediate entities and reasoning path by Cross-Layer Transcoder. While those often focus on factual recall, our work investigates task-level reasoning rather than just chains of facts.

Task Representations in ICL. The ability of LLMs to perform In-Context Learning (ICL) (Brown et al., 2020) has spurred rich research into its internal mechanism. A prominent line of inquiry focuses on explicit task representations (Hendel et al., 2023; Liu et al., 2024b; Todd et al.; Li et al.; Saglam et al., 2025; Yang et al., 2025). Initial work by (Hendel et al., 2023) derived task vectors from layer activations. Other approaches include In-Context Vectors (ICVs) (Liu et al., 2024b) derived from principal components of activation differences, and Function Vectors (FVs) (Todd et al.) which emphasize the role of specific attention heads. While these foundational studies demonstrate how a singular task can be abstracted into a vector, our work extends this by investigating how composite tasks are handled.

Mechanistic Interpretability. Mechanistic inter-525 pretability (Elhage et al., 2021) aims to reverse 527 engineer the internal mechanisms of LLMs. One type of studies focus on constructing the circuit in the model (Olsson et al., 2022; Wang et al.; Gould et al.; Marks et al., 2024). Another line of work focuses on understanding intermediate representa-531

tions through tools such as the LogitLens (Nostalgebraist., 2020). This technique has been extended to trace hidden states in LLMs (Dar et al., 2023; Halawi et al.; Merullo et al., 2024; Wiegreffe et al., 2024). Another major methodology is causal mediation analysis (Todd et al.; Vig et al., 2020; Meng et al., 2022; Geva et al., 2023; Hendel et al., 2023; Wu et al., 2023; Dumas et al., 2024), which measures the effect of intervening on a hidden state to determine its causal contribution to the model's output. Recent work also investigates the superposition hypothesis (Elhage et al., 2022; Scherlis et al., 2022). To disentangle such representations, sparse autoencoders (SAEs) have been employed to extract interpretable features from high-dimensional activations (Gao et al., 2024; Marks et al., 2024; Anthropic., 2024; Ferrando et al., 2024).

8 Conclusion

We show that large language models (LLMs) exhibit an internal chain-of-thought. Two claims ground our study: (i) distinct subtasks are learned at different network depths, and (ii) these subtasks are executed sequentially across layers. On a benchmark of 15 two-step composite tasks, we employ layer-from context-masking and propose a novel cross-task patching method, confirming (i). To examine claim (ii), we apply LogitLens to decode hidden states, revealing a consistent layerwise execution pattern. We further replicate our analysis on the real-world TRACE benchmark, observing the same stepwise dynamics. Together, our results enhance LLMs transparency by showing their capacity to internally plan and execute subtasks (or instructions), opening avenues for fine-grained, instruction-level activation steering.

Limitations

While our findings offer mechanistic insights into how LLMs internally decompose and execute composite tasks, several limitations must be acknowledged:

Model and Scale Scope. Our experiments are conducted on four mid-sized open-source models (3B-8B parameters). While these models are representative of common deployment settings, it remains an open question whether the observed phenomena generalize to larger frontier models (e.g., GPT-4, Claude). Differences in architecture, training corpus, and alignment objectives may yield

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distinct patterns of subtask representation or execu-tion.

Task Construction Bias. The distinct "X-shape" 582 pattern observed in our context-masking experiments (Figure 2) is facilitated by the deliberate design of our benchmark tasks, which feature clearly distinguishable subtask types (e.g., knowledge re-586 trieval followed by algorithmic transformation). 587 This separation likely leads to more temporally 588 distant "learning points" for each subtask across layers. However, when faced with a greater num-590 ber or more nuanced types of subtasks, particularly those with high conceptual similarity, the context-592 masking technique might be less effective at clearly 593 disentangling their individual learning stages. In-594 deed, as observed in our TRACE analysis (Section 595 6), the learning dynamics for closely related constraints can be more intertwined.

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A Datasets

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We evaluate our two central claims on 15 composite tasks spanning four categories. Each composite task is composed of two sequential subtasks, denoted as $s_1 \circ s_2$, and is designed to probe whether LLMs internally represent and apply these subtasks in a layered fashion. A summary of all composite tasks can be found in Table 3. We also describe the subtasks used in the cross-task patching experiment.

Antonym–Uppercase This composite dataset is constructed by capitalizing answers from an antonym dataset. The underlying antonym pairs are drawn from (Nguyen et al., 2017), which includes both antonyms and synonyms (e.g., "good \rightarrow bad"). We follow the preprocessing procedure described in (Todd et al.), then capitalize the antonym response, producing pairs like "good \rightarrow Bad". Intermediate answers are defined as the antonym in lowercase (e.g., "bad") and the capitalized form of the query (e.g., "Good").

Synonym–Uppercase Constructed in the same
way as Antonym–Uppercase, using synonym pairs
from (Nguyen et al., 2017). We capitalize the synonym to form the composite answer, and treat the
lowercase synonym and capitalized query as intermediate outputs.

Country_Capital-Lowercase This dataset is built from a country-capital mapping dataset (Todd et al.). We lowercase the capital names to form the composite answers. For example, "France \rightarrow paris".

Landmark_Country-Lowercase Pairs landmark names with their respective countries, based
on data from (Hernandez et al.). The country name
is lowercased to form the composite answer.

874 Product_Company-Lowercase This dataset
875 contains commercial products paired with the
876 companies that produce them, also curated
877 from (Hernandez et al.). The company name is
878 lowercased to produce the final output.

879 Choose_Last-Country_Capital We use the
880 country-capital dataset (Todd et al.) to create lists
881 of three countries sampled at random. The final
882 answer is the capital of the last country in the list.
883 Intermediate outputs include the last country name
884 and the capital of the first country.

Choose_Last-Landmark_Country Follows the same format as Choose_Last-Country_Capital, using landmark-country pairs from Hernandez et al.. The model must extract the last landmark and map it to its corresponding country.

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Adjective_v_Verb-Antonym This dataset tests syntactic category identification and semantic reasoning. From the antonym dataset (Nguyen et al., 2017), we select words that are unambiguously adjectives or verbs. Each list contains two verbs and one adjective. The model must identify the adjective and return its antonym.

Adjective_v_Verb–Synonym Constructed identically to Adjective_v_Verb–Antonym, but with synonym retrieval instead.

Choose_Last–First_Letter Constructed from a simple list-based selection dataset (Todd et al.). The model is prompted with a list of three items and must return the first letter of the last item.

Choose_Last-Uppercase Similar to Choose_Last-First_Letter, but instead of returning the first letter, the model is required to return the last item in uppercase form.

Antonym–English_French We translate answers from the antonym dataset to French using the Google Translate API. The composite task consists of performing the antonym transformation and then translating the result. Intermediate answers include the English antonym and the French translation of the query.

Antonym–English_Spanish Same as Antonym–English_French, but translated to Spanish.

Landmark_Country-English_French Based on (Hernandez et al.), we first retrieve the country associated with a landmark, then translate the country name to French.

Landmark_Country-English_Spanish

Constructed in the same way as Landmark_Country–English_French, using Spanish as the target language.

Below, we describe the individual subtasks $(s_1$ and $s_2)$ used in the cross-task patching experiments. Each subtask is a functional unit that appears as part of one or more composite tasks.

Category	Composite Tasks		
Knowledge-Algorithmic	Antonym–Uppercase		
	Synonym–Uppercase		
	Country_Capital-Lowercase		
	Landmark_Country-Lowercase		
	Product_Company-Lowercase		
Extractive-Knowledge	Choose_Last-Country_Capital		
	Choose_Last-Landmark_Country		
	Adjective_v_Verb-Antonym		
	Adjective_v_Verb-Synonym		
Extractive-Algorithmic	Choose_Last-First_Letter		
	Choose_Last-Uppercase		
Knowledge-Translation	Antonym–English_French		
	Antonym–English_Spanish		
	Landmark_Country-English_French		
	Landmark_Country-English_Spanish		

Table 3: Summary of the 15 composite tasks used in our experiments. Each task consists of a pair of subtasks $(s_1 \circ s_2)$, spanning four categories: Knowledge–Algorithmic, Extractive–Knowledge, Extractive–Algorithmic, and Knowledge–Translation.

Antonym The antonym dataset is based on data from (Nguyen et al., 2017), which contains word pairs that are either antonyms or synonyms (e.g., "good \rightarrow bad", "spirited \rightarrow fiery"). We follow the same preprocessing protocol as in (Todd 935 et al.).

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Synonym This dataset is also derived from (Nguyen et al., 2017), containing word pairs with synonym relationships. Preprocessing follows the same steps as the antonym dataset.

Country_Capital This dataset consists of country–capital pairs (e.g., "France \rightarrow Paris"), taken 941 from (Todd et al.). 942

land-943 Landmark_Country Includes mark-country pairs such as "Eiffel Tower \rightarrow France", based on the dataset from (Hernandez et al.).

Product_Company Contains entries mapping 947 commercial products to the companies that produce or sell them (e.g., "iPhone \rightarrow Apple"). Also sourced from (Hernandez et al.). 950

Choose_Last Constructed by sampling three 951 items and asking the model to return the last item. 952 Data sourced from (Todd et al.). 953

Adjective_v_Verb This dataset is designed to test part-of-speech reasoning. Each example contains a list of two verbs and one adjective, and the model must identify the adjective. Source: (Todd et al.).

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Uppercase A simple string transformation task where the model is required to convert the input to uppercase. Examples and format are adapted from (Todd et al.).

Lowercase Analogous to the Uppercase task, but the model is required to convert the input to lowercase. Based on the same dataset used in (Todd et al.).

First_Letter The task involves selecting the first letter of a given word. We construct this by reusing inputs from the Uppercase dataset and extracting only the first character.

Translation (English–French / English–Spanish) We use bilingual word pairs from (Conneau et al., 2017) for English–French and English–Spanish translations. Each example consists of an English word and its corresponding translation. We follow the preprocessing pipeline used in (Todd et al.).

B Results of Layer-from Context-Masking

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We present the complete results of the Layer-from Context-Masking experiments across four models.
Each figure visualizes the layer-wise performance on all 15 composite tasks, showing how masking context information from progressively later layers affects the model's ability to complete subtasks and composite outputs.

- Figure 6 shows results for Llama-3.1-8B.
- Figure 7 shows results for Mistral-7B.
- Figure 8 shows results for Qwen2.5-7B.
- Figure 9 shows results for Llama-3.2-3B.

C Results of Cross-Task Patching

We report the full results of the Cross-Task Patching experiment across all four models. Table 4 summarizes the subtask vector strength for each model, indicating how well activations from composite tasks can transfer to individual subtasks.

D Results of Logit Decoding

We present the complete results of the Logit Decoding analysis for all four models. Each model has two figures: (1) Mean Reciprocal Rank (MRR) scores of component outputs (attention and MLP layers), and (2) Mean Reciprocal Rank (MRR) scores of residual stream.

- Figures 10 and 11 show results for Llama-3.1-8B.
- Figures 12 and 13 show results for Mistral-7B.
- Figures 14 and 15 show results for Qwen2.5-7B.
- Figures 16 and 17 show results for Llama-3.2-3B.



Figure 6: Layer-from context-masking results for all composite tasks in Llama-3.1-8B.



Figure 7: Layer-from context-masking results for all composite tasks in Mistral-7B.



Figure 8: Layer-from context-masking results for all composite tasks in Qwen2.5-7B.



Figure 9: Layer-from context-masking results for all composite tasks in Llama-3.2-3B.

Composite Task	Llama-3.1-8B	Mistral-7B	Qwen2.5-7B	Llama-3.2-3B
antonym-uppercase				
s_1 (antonym)	0.92 ± 0.02	0.26 ± 0.07	0.83 ± 0.02	0.87 ± 0.05
s_2 (uppercase)	0.24 ± 0.03	0.35 ± 0.02	0.03 ± 0.02	0.34 ± 0.04
synonym-uppercase				
s_1 (synonym)	0.90 ± 0.05	0.06 ± 0.03	0.39 ± 0.02	0.82 ± 0.04
s_2 (uppercase)	0.19 ± 0.05	0.40 ± 0.03	0.09 ± 0.03	0.11 ± 0.02
country_capital-lowercase				
s_1 (country_capital)	0.88 ± 0.03	0.20 ± 0.03	0.49 ± 0.05	1.02 ± 0.02
s_2 (lowercase)	0.49 ± 0.05	0.45 ± 0.08	0.34 ± 0.03	0.31 ± 0.03
landmark_country-lowercase				
s_1 (landmark_country)	0.96 ± 0.02	0.92 ± 0.02	0.67 ± 0.02	0.96 ± 0.01
s_2 (lowercase)	0.07 ± 0.03	0.11 ± 0.04	0.17 ± 0.04	0.01 ± 0.00
product_company-lowercase				
s_1 (product_company)	1.01 ± 0.01	0.86 ± 0.03	0.65 ± 0.02	0.99 ± 0.03
s_2 (lowercase)	0.05 ± 0.01	0.46 ± 0.04	0.36 ± 0.05	0.06 ± 0.03
choose_last-country_capital			5.20 ± 0.02	5.00 ± 0.00
s_1 (choose_last)	0.44 ± 0.03	0.32 ± 0.03	0.37 ± 0.05	0.26 ± 0.04
s_1 (country_capital)	0.98 ± 0.02	0.92 ± 0.03 0.99 ± 0.01	0.97 ± 0.03 0.97 ± 0.01	0.99 ± 0.01
choose_last-landmark_country	0.90 ± 0.02	0.00 ± 0.01	0.97 ± 0.01	0.00 ± 0.01
s_1 (choose_last)	0.03 ± 0.02	0.00 ± 0.00	0.05 ± 0.04	0.03 ± 0.01
s_1 (encose_last) s_2 (landmark_country)	0.05 ± 0.02 0.94 ± 0.01	0.00 ± 0.00 0.96 ± 0.01	0.03 ± 0.04 0.91 ± 0.01	0.05 ± 0.01 0.96 ± 0.01
adjective_v_verb-antonym	0.94 ± 0.01	0.90 ± 0.01	0.91 ± 0.01	0.90 ± 0.01
s_1 (adjective_v_verb) antonym	0.38 ± 0.11	0.21 ± 0.05	0.54 ± 0.08	0.16 ± 0.04
s_1 (adjective_v_verb) s_2 (antonym)	0.93 ± 0.01 0.94 ± 0.03	0.21 ± 0.03 0.96 ± 0.04	0.94 ± 0.03 0.95 ± 0.01	0.10 ± 0.04 0.91 ± 0.05
adjective_v_verb-synonym	0.94 ± 0.05	0.90 ± 0.04	0.95 ± 0.01	0.91 ± 0.05
s_1 (adjective_v_verb)	0.29 ± 0.04	0.39 ± 0.05	0.26 ± 0.07	0.17 ± 0.04
s_1 (adjective_v_verb) s_2 (synonym)	0.29 ± 0.04 0.99 ± 0.05	0.39 ± 0.03 0.76 ± 0.05	0.20 ± 0.07 0.83 ± 0.07	0.17 ± 0.04 0.79 ± 0.02
choose_last-first_letter	0.99 ± 0.03	0.70 ± 0.03	0.85 ± 0.07	0.79 ± 0.02
s_1 (choose_last)	0.27 ± 0.03	0.08 ± 0.02	0.25 ± 0.04	0.41 ± 0.02
s_1 (choose_last) s_2 (first_letter)	0.27 ± 0.03 1.01 ± 0.01	0.08 ± 0.02 0.54 ± 0.03	0.25 ± 0.04 0.95 ± 0.05	$\begin{array}{c} 0.41 \pm 0.02 \\ 0.97 \pm 0.03 \end{array}$
	1.01 ± 0.01	0.34 ± 0.03	0.93 ± 0.03	0.97 ± 0.03
choose_last-uppercase	0.44 + 0.02	0.10 + 0.00	0.00 + 0.04	0.56 ± 0.02
s_1 (choose_last)	0.44 ± 0.03	0.12 ± 0.02	0.26 ± 0.04	0.56 ± 0.03
s_2 (uppercase)	0.99 ± 0.00	0.99 ± 0.00	1.00 ± 0.00	0.98 ± 0.01
antonym-english_french	0.02 0.02	0.52 + 0.05	0.00 + 0.00	0.76 ± 0.07
s_1 (antonym)	0.92 ± 0.02	0.53 ± 0.05	0.88 ± 0.02	0.76 ± 0.07
s_2 (english_french)	0.39 ± 0.04	0.58 ± 0.05	0.45 ± 0.01	0.60 ± 0.06
antonym-english_spanish	0.01 + 0.02	0.05 + 0.05	0.06 1.0.02	0.70 - 0.00
s_1 (antonym)	0.91 ± 0.02	0.35 ± 0.05	0.86 ± 0.02	0.70 ± 0.08
s_2 (english_spanish)	0.45 ± 0.03	0.69 ± 0.02	0.57 ± 0.03	0.70 ± 0.01
landmark_country-english_french	0.00	0.01.0.0.		0.0= 1.0.0;
s_1 (landmark_country)	0.93 ± 0.01	0.96 ± 0.01	0.92 ± 0.01	0.87 ± 0.01
s_2 (english_french)	0.02 ± 0.00	0.02 ± 0.01	0.02 ± 0.01	0.00 ± 0.00
landmark_country-english_spanish				
s_1 (landmark_country)	0.93 ± 0.01	0.95 ± 0.01	0.92 ± 0.01	0.90 ± 0.01
s_2 (english_spanish)	0.01 ± 0.00	0.01 ± 0.01	0.01 ± 0.01	0.00 ± 0.00

Table 4: Subtask vector strength for all composite tasks across four models.



Figure 10: Heatmaps of attention and MLP block decoding results for all tasks in Llama-3.1-8B.



Figure 11: Decoding results of residual stream for all tasks in Llama-3.1-8B.



Figure 12: Heatmaps of attention and MLP block decoding results for all tasks in Mistral-7B.



Figure 13: Decoding results of residual stream for all tasks in Mistral-7B.



Figure 14: Heatmaps of attention and MLP block decoding results for all tasks in Qwen2.5-7B.



Figure 15: Decoding results of residual stream for all tasks in Qwen2.5-7B.



Figure 16: Heatmaps of attention and MLP block decoding results for all tasks in Llama-3.2-3B.



Figure 17: Decoding results of residual stream for all tasks in Llama-3.2-3B.