

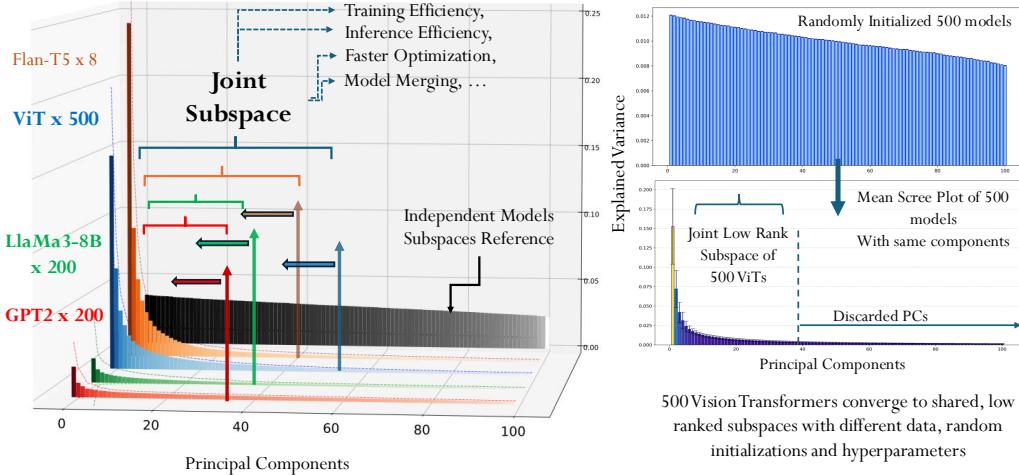
# DO NEURAL NETWORKS LEARN SIMILAR SUBSPACES? AN EMPIRICAL EXPLORATION OF JOINT PARAMETRIC SUBSPACES IN DEEP NEURAL NETWORKS

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## ABSTRACT

013 We show that deep neural networks trained across diverse tasks exhibit remarkably  
 014 similar low-dimensional parametric subspaces. We provide the first large-scale  
 015 empirical evidence that demonstrates that neural networks systematically con-  
 016 verge to shared spectral subspaces regardless of initialization, task, or domain.  
 017 Through mode-wise spectral analysis of over 1100 models - including 500 Mistral-  
 018 7B LoRAs, 500 Vision Transformers, and 50 LLaMA-8B models - we identify  
 019 universal subspaces capturing majority variance in just a few principal directions.  
 020 By applying spectral decomposition techniques to the weight matrices of various  
 021 architectures trained on a wide range of tasks and datasets, we identify sparse, joint  
 022 subspaces that are consistently exploited, within shared architectures across diverse  
 023 tasks and datasets. Our findings offer new insights into the intrinsic organization of  
 024 information within deep networks and raise important questions about the possi-  
 025 bility of discovering these universal subspaces without the need for extensive data  
 026 and computational resources. Furthermore, this inherent structure has significant  
 027 implications for model reusability, multi-task learning, model merging, and the  
 028 development of training and inference-efficient algorithms, potentially reducing  
 029 the carbon footprint of large-scale neural models.



045 **Figure 1: Deep Networks Converge to Shared, Low-Rank (Universal) Subspaces.** Across distinct archi-  
 046 tectures and modalities, neural networks systematically learn to operate within remarkably similar low-dimensional  
 047 parameter subspaces. **Left:** Principal component analysis of 200 GPT2, 500 Vision Transformers, 50 LLaMA-  
 048 8B, and 8 Flan-T5 models reveals consistent sharp spectral decay - strong evidence that a small number of weight  
 049 directions capture dominant variance despite vast differences in training data, objectives, and initialization. The  
 050 black baseline (independent subspaces reference) represents the naive expectation that models would learn  
 051 distinct directions; our empirical findings contradict this. **Right:** Strikingly, 500 randomly initialized ViT  
 052 models converge to a common low-rank subspace, demonstrating this is a fundamental neural network property.  
 053 This emergent structure unlocks powerful applications: parameter-efficient adaptation, efficient model merging,  
 compressed storage, and accelerated training and inference. Further discussion in Section A.

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## 1 INTRODUCTION

058 We show that backpropagated neural networks trained on a variety of datasets - which could be  
 059 disjoint and unrelated - diverse hyper-parameter settings, initializations and regularization methods,  
 060 often learn an architecture-specific, layer-wise similar, low-rank joint subspaces (we refer to this  
 061 as the **Universal Subspace**). We provide the first large-scale empirical analysis - across a diverse  
 062 set of models - that neural networks tend to converge to these joint subspaces, largely independent  
 063 of their initialization or the specific data used for training. Our study encompasses different model  
 064 architectures trained on a variety of datasets, sometimes with different loss functions and tasks.  
 065 Our spectral subspace analysis of the weights of all these models (Figure 1) suggests that although  
 066 individual tasks appear to induce distinct subspaces, individually, they are all part of an unusually  
 067 low-ranked joint subspace. Our work extends the scientific community's understanding of what  
 068 neural networks learn. This universality could explain several puzzling neural properties: why  
 069 overparameterized models with millions more parameters than training samples still generalize;  
 070 how different initializations converge to similar representations; and why techniques like weight  
 071 sharing and parameter-efficient fine-tuning succeed across architectures. If networks indeed learn  
 072 within shared subspaces, this would provide a supporting explanation for implicit regularization,  
 073 transferability, and the effectiveness of sparse training methods, while also opening up avenues for  
 074 applications like efficient merging, new optimization techniques, faster and more efficient learning  
 075 and inference.

076 Several works have hinted at phenomena consistent with our joint (universal) subspace hypothesis.  
 077 For example, Neural Tangent Kernel (NTK) theory demonstrates that, in the infinite-width limit,  
 078 the training dynamics of deep networks are governed by a kernel that is largely invariant to task  
 079 specifics (Jacot et al., 2018). Similarly, research in mechanistic interpretability's own universality  
 080 hypothesis (Olah et al., 2020; Chughtai et al., 2023) has uncovered recurring circuits and patterns  
 081 within some layers of toy or vision networks, lending indirect support to the universality hypothesis.  
 082 Other works, including the lottery ticket hypothesis (Frankle & Carbin, 2019) and studies on mode  
 083 connectivity (Garipov et al., 2018), provide further evidence for the existence of reusable, low-  
 084 dimensional representations in neural networks. Notably, Krizhevsky et al. (2012) observed that  
 085 the first layer of convolutional networks tends to learn Gabor-like filters across various vision tasks.  
 Recent studies by (Guth & Ménard, 2024; Guth et al., 2024) have also shown initial evidence of  
 recurring eigenvectors for some layers of convolutional neural networks trained on natural images.

086 In our analysis, we present compelling empirical evidence for the existence of universal subspaces  
 087 within LoRA adapters across different modalities and tasks. We initially focus on LoRA adapters due  
 088 to their ease of training and the ability to collect a large number of adapters for diverse tasks, models,  
 089 and datasets, which enables robust evaluation of our hypothesis. E.g., we demonstrate the emergence  
 090 of a universal subspace across approximately 500 LoRA adapters for the Mistral-7B (Jiang et al.,  
 091 2023) model. We further extend our investigation to the full weight space, where we observe similar  
 092 universality, extracting sparse, low-rank universal subspaces from about 500 Vision Transformer  
 093 models and 50 LLaMA3-8B models, each trained on different datasets and initializations.

094 Although the underlying causes and broader implications of this universal property remain an open  
 095 area of investigation, even an initial understanding of parameter subspace universality has pro-  
 096 found implications for neural network efficiency and interpretability. Shared subspaces could enable:  
 097 (1) massive model compression by storing only subspace coefficients rather than full weights; (2)  
 098 rapid adaptation to new tasks within learned subspaces; (3) theoretical insights into generalization  
 099 bounds and optimization landscapes; and (4) environmental benefits through reduced computational  
 100 requirements for training and inference.

101 The remainder of this paper is organized as follows. We first define the problem set up formally in  
 102 Section 2 followed by listing of essential properties and conditions with corresponding empirical  
 103 justifications. Section 3.3.1 proposes the method to adapt to new tasks leveraging the shared  
 104 approximate universal subspace. Section 3.1 explains our analysis methodology and Section 3.2  
 105 presents the comprehensive empirical evidence of the Universal subspaces. Section 4 briefly discusses  
 106 the analysis providing useful insights and answers the fundamental questions raised in the introduction.  
 107 We discuss related work in appendix A.1 and discuss limitations and scope for future work in Section  
 5. Our primary contributions include

- 108 • We empirically demonstrate the existence of a lower-dimensional shared universal subspace  
109 in backpropagated neural networks, and also provide relevant theoretical analysis.
- 110 • Illustrate the approach to learning an approximate low-dimensional shared subspace using  
111 the available set of tasks. Propose conditions for convergence of this learned subspace to the  
112 true universal shared subspace.
- 113 • Reuse the learned shared subspace to efficiently adapt to new unseen tasks with significantly  
114 fewer of trainable parameters. Our experiments across a wide variety of large pretrained  
115 models across various architectures and data modalities extensively verify and validate our  
116 hypothesis and theoretical findings.

## 118 2 NOTATIONS, DEFINITIONS AND THEORETICAL ANALYSIS

120 Our theoretical analysis models predictors as elements of a Hilbert space, for example a reproducing  
121 kernel Hilbert space (RKHS), while our experiments are conducted with practical large-scale models  
122 such as transformers and LoRA-based variants. Modeling predictors in a Hilbert space (kernel)  
123 framework is standard when analyzing aspects such as generalization and inductive bias of modern  
124 deep architectures, and has been widely used to approximate or interpret the behavior of large neural  
125 networks in practice (Ortiz-Jimenez et al., 2023; Wei et al., 2019; Chen & Xu, 2021; Belfer et al.,  
126 2024; Bietti et al., 2019). We aim to understand whether the shared structure across tasks can be  
127 consistently recovered from data as number of tasks increase. Specifically, each task has an associated  
128 ground-truth predictor  $f_t^*$ , and we are interested in the covariance (second-moment) operator  $\mathcal{S}$  that  
129 captures the common subspace spanned by these predictors. Since in practice we only observe finite  
130 samples per task and learn approximate predictors  $\hat{f}_t$ , two sources of error arise: (i) variability due  
131 to having finitely many tasks, and (ii) estimation noise within each task. Our goal is to establish  
132 conditions under which the empirical operators built from  $\hat{f}_t$  concentrate around  $\mathcal{S}$ , and to show that  
133 the learned top- $k$  subspace converges to the true one, with convergence rates that separately reflect  
134 the number of tasks and the accuracy of per-task learning.

135 **Setup.** Let  $(\mathcal{H}, \langle \cdot, \cdot \rangle)$  be a separable Hilbert space with norm  $\|\cdot\| = \|\cdot\|_{\mathcal{H}}$ . For  $a, b \in \mathcal{H}$ , the  
136 rank-one operator  $a \otimes b : \mathcal{H} \rightarrow \mathcal{H}$  is  $(a \otimes b)g = \langle b, g \rangle a$ ; in particular  $\|a \otimes b\|_{\text{op}} = \|a\| \|b\|$ . Tasks  
137  $t = \{1, 2, 3, \dots, T\}$  are drawn i.i.d. from distribution  $\mathcal{T}$  and each task dataset  $S_t = \{(x_{t,i}, y_{t,i})\}_{i=1}^{n_t}$   
138 with  $n_t$  samples is drawn independently from  $D_t$ . Let  $f_t^* \in \mathcal{H}$  denote the (unknown) ground-truth  
139 predictor for task  $t$  and  $\hat{f}_t \in \mathcal{H}$  be the learned predictor for the task.

140 **Definition 2.1** (Task second-moment operator). The *population*, *true empirical*, and *learned empirical*  
141 task second-moment operators are respectively,

$$143 \quad \mathcal{S} := \mathbb{E}_{t \sim \mathcal{T}} [f_t^* \otimes f_t^*], \quad \hat{\mathcal{S}} := \frac{1}{T} \sum_{t=1}^T f_t^* \otimes f_t^*, \quad \tilde{\mathcal{S}} := \frac{1}{T} \sum_{t=1}^T \hat{f}_t \otimes \hat{f}_t.$$

146 where  $\mathcal{S}, \hat{\mathcal{S}}, \tilde{\mathcal{S}}$  are self-adjoint and positive semi-definite such that  $\text{tr}(\mathcal{S}) < \infty$ . Its top- $k$  eigenspace  
147  $\mathcal{H}_k^*$  is the population rank- $k$  *shared subspace* of tasks.

148 *Remark 2.2.* We work with the second-moment operator (rather than centered covariance), so the top  
149 eigenspace may include the mean direction of  $\{f_t^*\}_{t \sim \mathcal{T}}$ .

150 Let  $\lambda_1 \geq \lambda_2 \geq \dots$  be the eigenvalues of  $\mathcal{S}$  with orthonormal eigenvectors  $\{\phi_i\}_{i \geq 1}$ . Write  $P_k =$   
151  $\sum_{i=1}^k \phi_i \otimes \phi_i$  for the projector onto the population top- $k$  subspace  $\mathcal{H}_k^* = \text{span}\{\phi_1, \dots, \phi_k\}$ , and  
152 let  $\tilde{P}_k$  be the projector onto the top- $k$  eigenspace of  $\tilde{\mathcal{S}}$  (the learned shared subspace). Define the  
153 eigengap  $\gamma_k := \lambda_k - \lambda_{k+1} > 0$ .

154 **Assumption 2.3** (Realizability, bounded second moment and effective rank). *For a constant  $B > 0$   
155 and for all tasks,  $f_t^* \in \mathcal{H}$  almost surely,  $\|f_t^*\| \leq B$  a.s.,  $\mathbb{E}_{t \sim \mathcal{T}} \|f_t^*\|^2 = \text{tr}(\mathcal{S}) < \infty$ . In addition,  $\mathcal{S}$   
156 has bounded effective rank,  $\frac{\text{tr}(\mathcal{S})}{\|\mathcal{S}\|_{\text{op}}} \leq \kappa$*

157 *Assumption 2.3 ensures that all ground-truth predictors are bounded and have finite second moment,  
158 so the population covariance operator  $\mathcal{S}$  is well-defined. The bounded effective rank condition  
159 further guarantees that the shared structure of the tasks is not arbitrarily infinite-dimensional, making  
160 subspace recovery feasible.*

162 **Assumption 2.4** (Per-task estimation accuracy in  $\mathcal{H}$ ). For any  $\delta_t \in (0, 1)$  with probability at least  
 163  $1 - \delta_t$  over the draw of  $S_t$ ,

$$165 \quad \|\hat{f}_t - f_t^*\| \leq \eta_t, \text{ ...where } \eta_t = \mathcal{R}_{n_t, D_t}(\mathcal{H}) + \sqrt{\frac{\ln(1/\delta_t)}{2n_t}}$$

168 Here  $\mathcal{R}_{n_t, D_t}(\mathcal{H})$  represents Rademacher complexity of the solutions within Hilbert space  $\mathcal{H}$  over  
 169  $n_t$  samples drawn i.i.d. from  $D_t$ . This form is satisfied, for example, by strongly convex regularized  
 170 ERM in an RKHS (e.g., kernel ridge regression or NTK ridge), under bounded kernel norm and  
 171 sub-Gaussian response noise (Bartlett & Mendelson, 2003).

172 Assumption 2.4 requires that each task predictor  $\hat{f}_t$  is learned accurately from its finite dataset. In  
 173 other words,  $\hat{f}_t$  is close to the true  $f_t^*$  in  $\mathcal{H}$ -norm with high probability, at a rate governed by sample  
 174 size and complexity of the hypothesis space.

175 **Theorem 2.5** (Two-level convergence to the shared subspace). Assume 2.3–2.4. Let  $c_1, c_2$  be any  
 176 absolute constants. For any  $\delta \in (0, 1)$ , choose  $\delta_t = \delta/(2T)$  and set  $\delta_T = \delta/2$ . With probability at  
 177 least  $1 - \delta$  (over tasks and all per-task samples),

$$179 \quad \|\tilde{\mathcal{S}} - \mathcal{S}\|_{\text{op}} \leq c_1 B^2 \sqrt{\frac{\log(c_2/\delta)}{T}} + (2B\bar{\eta} + \bar{\eta}^2) \quad (1)$$

182 If moreover  $\gamma_k > 0$ , then

$$183 \quad \|\tilde{P}_k - P_k\|_{\text{op}} \leq \frac{2}{\gamma_k} \left( c_1 B^2 \sqrt{\frac{\log(c_2/\delta)}{T}} + (2B\bar{\eta} + \bar{\eta}^2) \right). \quad (2)$$

186 where  $\bar{\eta} = \frac{1}{T} \sum_{t=1}^T \eta_t$ ,  $\bar{\eta}^2 = \frac{1}{T} \sum_{t=1}^T \eta_t^2$  and  $\eta_t$  is defined same as in assumption 2.4

188 Proof of Theorem 2.5 can be found in appendix Section A.2. The Theorem 2.5 shows that the  
 189 empirical second-moment operator built from the learned predictors converges to the true operator  $\mathcal{S}$ ,  
 190 and the learned top- $k$  subspace  $\hat{P}_k$  converges to the true subspace  $P_k$ . The rates capture two sources of  
 191 error: averaging across tasks (scaling with  $1/\sqrt{T}$ ) and per-task estimation errors (through  $\bar{\eta}$  and  $\bar{\eta}^2$ ).  
 192 A larger eigengap  $\gamma_k$  makes the subspace recovery more stable. In practice, we obtain the eigenvectors  
 193 of  $\tilde{\mathcal{S}}$  using HOSVD (Higher-Order Singular Value Decomposition) of the concatenated weight matrix  
 194  $\mathcal{X}$  highlighted in Section 3. Motivated by our theoretical analysis, we try to approximate  $\hat{\mathcal{S}}$  for a set  
 195 of tasks by extracting principal directions from as many trained models as possible.

### 197 3 ANALYSIS

#### 199 3.1 ANALYSIS METHODOLOGY

201 Since there is no current method that enables us to compare subspaces of models with different  
 202 architectures, we focus on large number of models trained on the same architecture. To this end,  
 203 we perform analysis using Low rank adapters (Hu et al., 2021) (LoRA) as well as classical weights  
 204 of transformer and CNN (Convolutional Neural Network) architectures. For all our experiments,  
 205 unless stated otherwise, we perform Order 1-2 HOSVD only, to ensure that our methodology works  
 206 even in the simplest case. Algorithm 1 provides the algorithm we implement. Refer to Section B for  
 207 discussion regarding secondary subspace and how to choose the number of top components.

#### 209 3.2 RESULTS FROM JOINT SUBSPACES’ ANALYSIS

211 We present empirical results using method shown in Section 3.1, extracting our layer wise universal  
 212 subspace approximations using thousands of publicly available models for most of our experiments.  
 213 This choice allows us to have no training costs whatsoever, for extracting the universal subspace.  
 214 Spectral analysis relies on efficient spectral decomposition libraries, and can even be run on CPUs.  
 215 We run all our analysis and experiments on one Nvidia A5000 GPU. The presented large scale  
 empirical results forms the crux of our work and provide strong evidence for the presence of such

216 **Algorithm 1** Truncated Zero-Centered Higher-Order SVD (HOSVD)  
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218 **Require:** A high-order tensor  $\mathcal{X} \in \mathbb{R}^{I_1 \times \dots \times I_N}$  constructed by stacking  $N$  rank- $r_n$  task matrices  
219 along mode  $n$ , where  $1 \leq r_n \leq I_n$  and  $n \in [1, N]$ .  
220 **Ensure:** Mean tensor  $\mu$ ; factor matrices  $U^{(n)} \in \mathbb{R}^{I_n \times \hat{r}_n}$  (orthonormal columns), where  $\hat{r}_n$  is  
221 chosen as the smallest number of left singular vectors whose cumulative explained variance  
222 is at least  $\tau$ ; and the truncated core tensor  $\mathcal{S} \in \mathbb{R}^{\hat{r}_1 \times \dots \times \hat{r}_N}$ . Reconstruction is given by  $\hat{\mathcal{X}} =$   
223  $\mu + \mathcal{S} \times_1 U^{(1)} \dots \times_N U^{(N)}$ , where  $\times_n$  denotes mode- $n$  tensor–matrix multiplication.  
224 1: **Zero-centering:**  $\mu \leftarrow \text{mean}(\mathcal{X})$  ▷ elementwise mean over all entries  
225 2:  $\mathcal{X}_c \leftarrow \mathcal{X} - \mu$  ▷ broadcast  $\mu$  to the shape of  $\mathcal{X}$   
226 3: **for**  $n = 1$  **to**  $N$  **do**  
227 4:    $X_{(n)} \leftarrow \text{unfold}(\mathcal{X}_c, n)$  ▷ mode- $n$  matricization;  $X_{(n)} \in \mathbb{R}^{I_n \times \prod_{m \neq n} I_m}$   
228 5:   Compute thin SVD:  $X_{(n)} = \tilde{U}^{(n)} \Sigma^{(n)} \tilde{V}^{(n)\top}$   
229 6:    $U^{(n)} \leftarrow \tilde{U}^{(n)}(:, 1:\hat{r}_n)$  ▷ keep first  $\hat{r}_n$  left singular vectors (variance  $\geq \tau$ )  
230 7: **end for**  
231 8: **Truncated core:**  $\mathcal{S} \leftarrow \mathcal{X}_c \times_1 U^{(1)\top} \times_2 U^{(2)\top} \dots \times_N U^{(N)\top}$   
232 9: **return**  $\mu, \{U^{(n)}\}_{n=1}^N, \mathcal{S}$  ▷ Optionally compute  $\hat{\mathcal{X}} = \mu + \mathcal{S} \times_1 U^{(1)} \dots \times_N U^{(N)}$

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233  
234  
235 low ranked joint subspaces across a wide range of task, architecture and modalities. In summary,  
236 we present a total of **eight** set of analysis and applications, including tasks like image classification,  
237 natural language understanding, text to image generation, model merging, etc for different model  
238 architectures and modalities.

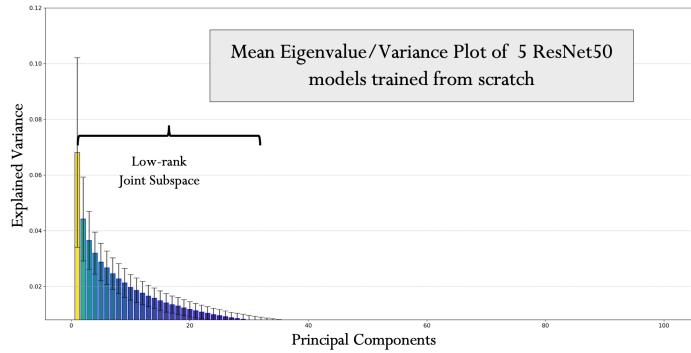
## 239 3.2.1 LOWER-RANK JOINT SUBSPACES IN CNNs, LORA AND FINETUNED MODELS

240 In smaller and conventional ar-  
241 chitectures such as CNNs, evi-  
242 dence for universal structure has  
243 been more limited but suggestive.  
244 Early work observed that the first  
245 convolutional layer often learns  
246 Gabor-like filters across diverse  
247 vision tasks (Krizhevsky et al.,  
248 2012). More recently, works  
249 report recurring eigenvectors in  
250 certain CNN layers trained on  
251 natural images (Guth et al., 2024;  
252 Guth & Ménard, 2024).

253 We extend these observations  
254 and examine whether a shared  
255 low-rank joint subspace emerges  
256 across tasks. Specifically, we  
257 train ResNet-50 models from ran-  
258 dom initialization for image clas-  
259 sification on five disjoint datasets  
260 (CIFAR-10, CIFAR-100, Im-  
261 ageNet, Oxford-IIIT Pets, and Eu-  
262 roSAT), ensuring no overlap in  
263 samples. While our theoretical  
264 analysis indicates that a small  
265 number of models may lead to  
266 an under-approximation of the  
267 joint universal subspace, training  
268 CNNs from scratch at scale con-  
269 strains the number of models we  
can include in this study.

(a) Comparison of model performance across datasets.

| Method        | ImageNet | EuroSat | CIFAR-10 | CIFAR-100 | Oxford Pets | Avg   |
|---------------|----------|---------|----------|-----------|-------------|-------|
| ResNet50      | 80.86    | 98.96   | 97.35    | 83.82     | 93.48       | 90.89 |
| Universal R50 | 77.89    | 98.83   | 95.89    | 81.49     | 83.81       | 87.58 |

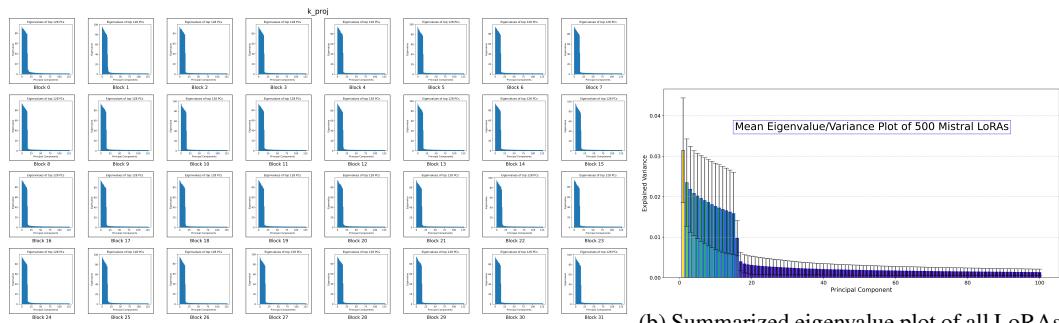


(b) Summarized (averaged for all layers) eigenvalue plot of all model weights corresponding to all 31 layers of 5 ResNet50 models. Mean refers to the fact that it has been averaged for all layers for conciseness. The vertical axis is Explained Variance (for *all* models) and X axis indicated Principal Components. We will follow this setup throughout the paper. We also refer to the low-ranked shared subspace as 'Universal' subspace and may refer to a specific model consisting of extracted basis as the 'Universal variant'.

Figure 2: **Proving existence of universal subspaces in CNNs.** Decomposing 5 ResNet50 models trained on different tasks shows the emergence of a low rank, universal subspace where the majority of the information is present in only 16 (or fewer) distinct subspace directions for all layers of the network.

270 Despite these limitations, Figure 2b reports the average explained variance across all layers of ResNet-  
 271 50 and reveals a distinct, shared low-rank structure spanning these disjoint tasks. Moreover, even  
 272 when the estimated universal subspace is relatively coarse, projecting to this subspace to obtain a  
 273 low-rank ResNet-50 (thereby reducing parameters) preserves competitive performance relative to full  
 274 fine-tuning, further supporting the presence and utility of a joint subspace (2a).  
 275

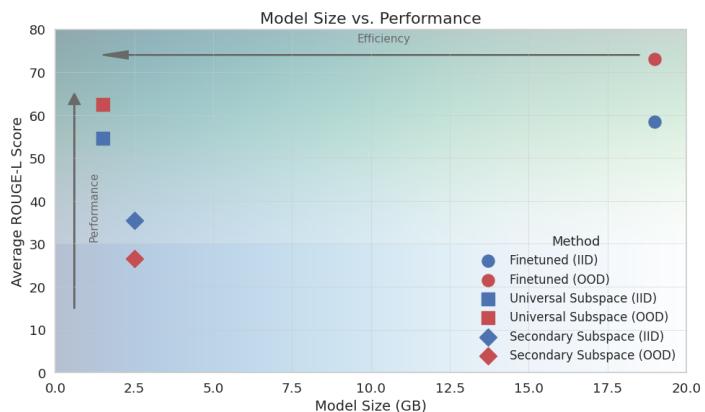
276 In order to conduct a more real-world experiment, we choose to run the subspace analysis for  
 277 LoRA Hu et al. (2021) models simply because they are available in abundance in public domain.  
 278 Given LoRA models distinctly capture task specific directions as they show weak alignment with  
 279 the original weights Hu et al. (2021), they form a good main model parameter alternative to run our  
 280 subspace analysis and verify whether this holds true. We spectrally decompose (Section 3.1) LoRA’s  
 281 submatrices individually, each concatenated across all the available finetuned LoRAs and choose top  
 282  $k$  spectral basis. This setup allows us to truly stress test the Universal Subspace.  
 283



284 (a) Eigenvalue/Variance plot for Orthogonal Spectral Components  
 285 corresponding to all 31 layers of all 500 Mis-  
 286 tral-7B models  
 287  
 288 (b) Summarized eigenvalue plot of all LoRAs  
 289 corresponding to all 31 layers of all 500 Mis-  
 290 tral 7B models  
 291  
 292

293  
 294 Figure 3: **Proving existence of universal subspaces in deep networks.** Decomposing 500 sets of  
 295 LoRAs trained on different tasks using the Mistral-7B model shows the emergence of a low rank,  
 296 universal subspace where the majority of the information is present in only 16 (or less) distinct  
 297 subspace directions for all layers of the network. Plots of other layers are present in the Section B.1.  
 298  
 299

300 We first study **500 LoRA**  
 301 **models** trained on distinct  
 302 Natural Instructions (Wang  
 303 et al., 2022) using Mistral-  
 304 7B-Instruct-v0.2 (Jiang  
 305 et al., 2023) as the  
 306 base (Brüel-Gabrielsson  
 307 et al., 2024). Each LoRA  
 308 has at least rank 16. Fig-  
 309 ure 3 shows that the top  
 310 spectral components cap-  
 311 ture most of the variance  
 312 in each layer, indicating a  
 313 low-rank structure shared  
 314 across tasks. Figure 3a  
 315 visualizes the eigenvalue  
 316 decay per layer, while Figure 3b  
 317 summarizes the pattern across all layers and models.



320 Figure 4: Lots of LoRAs Model Size vs Performance plot.  
 321  
 322

323 To test subspace expressiveness, we reconstruct LoRA weights for both seen (IID) and unseen (OOD)  
 324 tasks by projecting them into the universal subspace. As shown in Figure 4, the reconstructed models  
 325 retain high performance in both cases. In contrast, projection into the residual *Secondary Subspace*  
 326 leads to a sharp performance drop, underscoring the importance of the principal subspace. Our method  
 327 is also **19× more memory-efficient**, as it eliminates the need to store all 500 LoRAs.  
 328

329 We extend our analysis to **text-to-image generation** using Stable Diffusion-XL (Podell et al., 2023).  
 330 A universal subspace is extracted from publicly available LoRAs on HuggingFace (von Platen et al.,

222). When projecting individual LoRAs into this subspace, the resulting generations preserve 223 visual quality and style (Figure 5). CLIP-based evaluations (Table 1) show that the universal subspace 224 even outperforms individual LoRAs in some cases, possibly due to denoising effects previously 225 observed in (Sharma et al., 2023).

Table 1: CLIP scores (higher is better) of images generated using SDXL.

| Method              | Style 1 | Style 2 | Style 3 | Style 4 | Style 5 | Style 6 | Style 7 | Style 8 | Style 9 | Style 10 | Avg          |
|---------------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|----------|--------------|
| LoRA                | 21.95   | 15.59   | 22.18   | 18.84   | 16.65   | 17.99   | 24.66   | 17.47   | 22.07   | 19.93    | 19.73        |
| Universal SDXL LoRA | 21.96   | 16.07   | 22.07   | 18.79   | 16.68   | 17.99   | 24.66   | 17.56   | 22.46   | 20.09    | <b>19.83</b> |

Figure 5: Text-to-Image Generation Results for Individual models vs. our Universal Subspace model. We notice no visual reduction in style quality despite significant reduction in total model size.

In order to test the ability of condensing many models into a single universal subspace, we compare our method with SOTA model merging/combination methods in Table 2. We compare our universal subspace inspired combination approach against six state-of-the-art, gradient-free baselines: RegMean (Jin et al., 2023), Task Arithmetic (TA) (Ilharco et al., 2023), TIES (Yadav et al., 2023), DARE-TIES (Yu et al., 2024), KnOTS-TIES, and KnOTS-DARE-TIES (Stoica et al., 2025). RegMean aligns task-specific updates by solving a layer-wise linear regression problem, requiring transformation matrices for each model. TA merges models by linearly combining parameters, but relies on tuning scaling coefficients on a validation set for optimal performance. TIES extends TA with magnitude-based pruning and sign conflict resolution, introducing additional hyperparameters such as pruning thresholds, while DARE-TIES combines random Bernoulli pruning with TIES’ sign resolution, also requiring tuning of pruning probability. KnOTS-TIES and KnOTS-DARE-TIES further apply SVD-based subspace alignment before merging, but still inherit the need for coefficient or pruning hyperparameter selection. In contrast, our universal subspace method, analytically computes the merging coefficients based solely on the geometry of a shared, low-rank universal subspace identified across models, requiring no iterative tuning or validation data—although optional finetuning is possible if data is available. Furthermore, because our subspace is intrinsically low-rank, the merged model contains significantly fewer parameters than any individual models, offering both computational efficiency and theoretical alignment guarantees not present in the baselines. Empirically, our approach achieves higher average accuracy (see Table 2), while reducing parameter count, thus enabling scalable and robust model merging without heuristic pruning or validation overhead. We note that we did not optimize our merging process and better results nearing finetuned performance may be achieved.

In summary, these four experiments provide strong empirical support for our universal subspace hypothesis and demonstrate its practical advantages in terms of memory efficiency, model merging, model reusability, and scalable deployment across diverse tasks and modalities.

### 3.3 LOW RANK SHARED UNIVERSAL SUBSPACES IN CLASSICAL WEIGHTS

While aforementioned experiments on CNNs trained from scratch, and LoRAs provide strong evidence for the presence of the joint subspace, we further rigorously test on large scale finetuned models (500 pretrained ViT, 50 LLaMA3-8B models, 177 GPT-2 and Flan-T5).

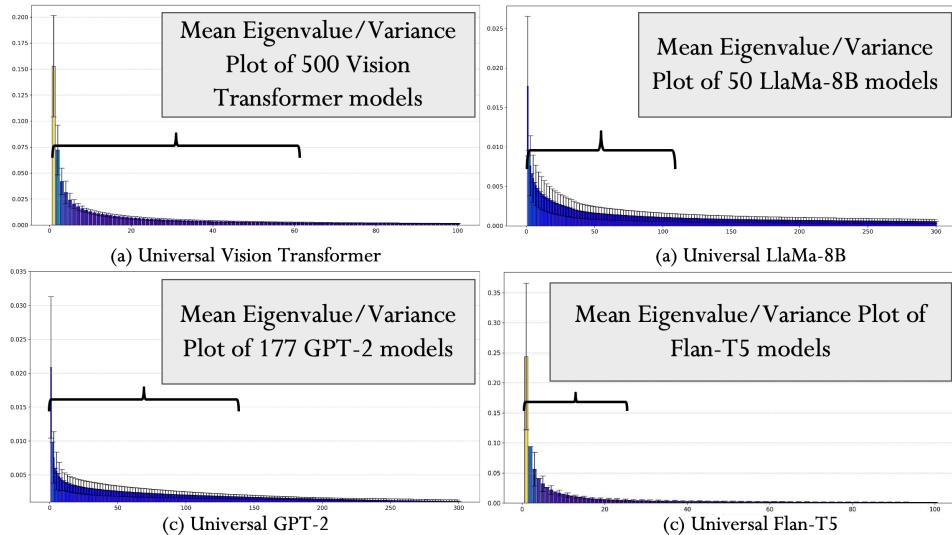
First, we collect  $\sim 500$  pretrained Vision Transformer (ViT) models from HuggingFace, spanning diverse domains - medical imaging, satellite data, and synthetic - and

Table 3: Image Classification Accuracy

| Method        | IID            | OOD            |
|---------------|----------------|----------------|
| Full Training | $94.4 \pm 1.7$ | $91.3 \pm 2.1$ |
| Universal ViT | $94.1 \pm 2.0$ | $87.8 \pm 1.5$ |

378  
 379  
 380  
 381  
 382 Table 2: Per-task results for eight ViT-B/32 models, each finetuned with LoRA on a different image classification  
 383 dataset. "Finetuned" indicates the accuracy of each model on its respective training dataset. For each merging  
 384 baseline, we report the normalized accuracy on every task, as well as the average across all tasks.  
 385  
 386

| 382 <b>Method</b>   | 383 <b>Datasets</b> |                |                    |                  |                  |                     |                   |                 | 384 <b>Avg</b> |
|---|---------------------|----------------|--------------------|------------------|------------------|---------------------|-------------------|-----------------|----------------|
|   | 385 <b>Cars</b>     | 386 <b>DTD</b> | 387 <b>EuroSAT</b> | 388 <b>GTSRB</b> | 389 <b>MNIST</b> | 390 <b>RESISC45</b> | 391 <b>SUN397</b> | 392 <b>SVHN</b> |                |
| <b>Per-Task Absolute Accuracies (%)</b>   |                     |                |                    |                  |                  |                     |                   |                 |                |
| Finetuned   | 74.0                | 58.3           | 99.0               | 92.7             | 99.3             | 88.4                | 64.5              | 96.2            | 84.1           |
| <b>Per-Task Accuracies of Combined Models Normalized Against Finetuned Models (%)</b> |                     |                |                    |                  |                  |                     |                   |                 |                |
| RegMean   | 80.2                | 71.3           | 37.9               | 47.3             | 43.1             | 70.5                | 99.3              | 43.0            | 60.9           |
| TA  | 82.0                | 73.6           | 48.8               | 42.1             | 53.1             | 71.5                | 97.5              | 41.2            | 63.7           |
| TIES  | 82.4                | 72.8           | 50.8               | 39.0             | 50.3             | 70.9                | 99.4              | 40.5            | 63.7           |
| DARE-TIES   | 81.4                | 74.5           | 50.8               | 39.2             | 55.0             | 70.7                | 96.7              | 40.4            | 63.7           |
| KnOTS-TIES  | 82.7                | 73.7           | 49.3               | 48.9             | 70.9             | 95.5                | 53.8              | 68.0            | 68.0           |
| KnOTS-DARE-TIES   | 81.8                | 75.9           | 50.7               | 40.3             | 53.2             | 70.2                | 97.9              | 41.0            | 63.9           |
| <b>Ours</b>   | <b>88.1</b>         | <b>82.3</b>    | <b>65.9</b>        | <b>61.3</b>      | <b>88.3</b>      | <b>98.1</b>         | <b>98.5</b>       | <b>85.1</b>     | <b>83.5</b>    |



411  
 412 **Figure 6: Universal Subspaces in Classical Weights.** Spectral decomposition of weight matrices from (a)  
 413 ~500 Vision Transformers (b) 50 LLaMa-8B models (c) 177 GPT-2 models (d) GLUE Flan-T5 models - each  
 414 trained independently across diverse tasks, datasets, and configurations - reveals a consistent low-rank structure:  
 415 most variance is captured by the top few spectral basis. This suggests that, despite significant variation in training  
 416 conditions, the learned weights consistently align along a shared low-dimensional subspace. For visualization  
 417 clarity, only a fraction of the basis are shown; extended plots are provided in the Section B.2.

418 trained with varying losses, optimizers, and initializations. These models are used as-is, without  
 419 curation or access to training data, to reflect real-world variability. See Section B.2 for details.  
 420 Following our method (Section 3.1), we spectrally decompose all layers (excluding first and last) and  
 421 observe, in Figure 6, that the majority of variance is captured by the top few spectral components,  
 422 revealing a highly compressible, shared subspace across layers. Only the top 100 components are  
 423 visualized for clarity.

424 To evaluate universal generalization, we project five held-out ViT models onto this 16-dim subspace  
 425 and measure classification accuracy. As shown in Table 3, performance remains robust, indicating  
 426 that a shared low-rank subspace spans a wide range of ViT model configurations and domains.

427 A major outcome of this experiment is that we can replace these 500 ViT models with a single  
 428 Universal Subspace model. Ignoring the task-variable first and last layer (weight matrices vary due to  
 429 different number of categories and input size and formats), we observe a requirement of **100× less**  
 430 **memory**, and these savings are prone to increase as the number of trained models increases. We note  
 431 that we are, to the best of our knowledge, the first work, to be able to *merge* 500 (and theoretically

more) Vision Transformer into a single universal subspace model. This result implies that hundreds of ViTs can be represented using a single subspace model - excluding task-specific layers - yielding up to **100× memory reduction**. To our knowledge, this is the first demonstration of merging over 500 ViTs into a single universal representation.

We further extend this analysis to 50 finetuned LLaMA3-8B models, 177 GPT-2 models, and Flan-T5 models (trained on GLUE Wang et al. (2019) datasets) again sourced from HuggingFace without filtering. As shown in Figure 6, a small number of directions capture dominant structure across models spanning diverse and distinct datasets and tasks. More details are provided in the Section B.2. This is, to our knowledge, the first instance of compressing such a large and diverse collection of foundation models into a unified subspace, highlighting its potential for large-scale model reuse and environmental efficiency.

### 3.3.1 FINDING UNIVERSAL SUBSPACES AND APPLYING THEM TO FUTURE TASKS

In this section, the low-rank shared subspaces estimated from a set of available tasks are leveraged to adapt to new, previously unseen tasks. While we do not make theoretical guarantees about reuse on unseen tasks, our experiments show that the approximate shared subspace is empirically reusable across a wide range of practical settings. Concretely, we reuse the shared principal directions and learn only their task-specific coefficients for the new task. Learning these low-rank coefficients is substantially cheaper than optimizing full-rank weights of size, reducing both computation and memory. The resulting trainable parameter counts are reported in Table 5. We find our universal subspace models can have significant impact on the carbon footprint issues of large AI models by making the training, inference and scaling of these models efficient and cheap. As shown in the previous section, we can effectively recycle and replace available pretrained models with a universal subspace model with every individual being represented by a sparse set of coefficients. In this section, we show a set of experiments where we utilize the universal subspaces to learn new tasks by freezing the components and simply learning the coefficients using gradient descent. We find that since we are only learning the coefficients, it drastically cuts down the number of parameters required to train the new models. Further, since these coefficients are simply linear scaling values, the optimization is smoother and faster.

Table 4: Performance on the GLUE Benchmark.

| Method            | Speedup | CoLA  | MRPC  | RTE   | QNLI  | SST-2 | STS-B | Avg   |
|-------------------|---------|-------|-------|-------|-------|-------|-------|-------|
| LoRA              | 1×      | 59.56 | 86.76 | 77.61 | 92.53 | 94.72 | 90.81 | 83.67 |
| Universal order-2 | 2×      | 61.82 | 87.25 | 77.62 | 92.71 | 94.15 | 90.48 | 84.01 |
| HOOI (order-2)    | 2×      | 61.96 | 87.55 | 77.50 | 92.83 | 94.45 | 90.40 | 84.12 |
| Universal order-3 | 1.8×    | 62.06 | 86.52 | 75.81 | 92.98 | 94.26 | 90.39 | 83.67 |

We present two experiments - Image Classification using ViT-base and Natural Language Understanding using GLUE benchmark Wang et al. (2019) with RoBERTa-base model. Both involve creating a universal subspace using publicly available LoRA adapters. Details are provided in the Section C. For the GLUE benchmark, we follow the same setup as (Kopitzko et al., 2023) considering the 6 tasks - CoLA, MRPC, SST-2, QNLI, RTE and STS-B while omitting the time-intensive MNLI and QQP tasks. We initialize our universal subspace using a leave-one-out-setup, where the subspace is calculated using components of all but one LoRA adapter for which the coefficients are learned. For image classification, we utilize publicly available ViT LoRAs to extract our universal subspaces taking care that the data any of these pretrained LoRAs have not seen the data we will be training our coefficients on. Table 5 and Table 4 show that our universal subspace enables significantly more

Table 5: Image Classification with Vision Transformer.

| # Training Params | CIFAR100 | Food101 | Flowers102 | CIFAR10 | Pets |
|-------------------|----------|---------|------------|---------|------|
| Full Training     | 86M      | 92.8    | 90.7       | 98.82   | 99.0 |
| Universal ViT     | 10K      | 90.1    | 89.1       | 90.1    | 96.7 |

efficient and effective learning since only compact coefficients are trained. The storage required to

486 save all these models is also drastically reduced. The ViT models require 150 GB and LLaMA models  
 487 require 1.6TB of memory in total. Our universal subspace reduces that memory requirement by more  
 488 than **100 $\times$** .  
 489

490 **4 DISCUSSION**  
 491

492 This work provides, to the best of our knowledge, the first large-scale, cross-domain analysis showing  
 493 that neural networks trained across diverse tasks, modalities, initializations, and hyperparameters  
 494 consistently exhibit an architecture-specific shared low-rank universal subspace at the layer level.  
 495 Concretely, by performing layer-wise spectral decompositions and retaining only the leading principal  
 496 directions, an accurate approximation of these universal subspaces can be extracted. Empirically, this  
 497 behavior emerges broadly: in fully finetuned models and LoRA-based adapters, in models trained  
 498 from scratch, in both generative and discriminative settings, and in multimodal configurations. More-  
 499 over, the approximated subspaces generalize to out-of-distribution tasks, where projecting models  
 500 and learning only a small set of coefficients suffices to recover strong performance. This enables  
 501 adapting to new tasks without retraining or storing full weights, and supports robust multi-task  
 502 learning, scalable fine-tuning, and principled model merging within a single unifying framework.  
 503

504 The practical implications are substantial. By learning only lightweight coefficients for shared layer-  
 505 wise principal directions, large models can be extended with dramatically reduced computational and  
 506 memory overhead. This lowers deployment costs while enabling more accessible AI development  
 507 and data-free model merging. These results suggest a path toward scalable model reuse grounded in a  
 508 simple geometric principle: most task variation lies in a shared, low-dimensional subspace.

509 **Why do these universal subspaces emerge?** Neural networks may exhibit spectral bias toward  
 510 low-frequency functions, potentially creating polynomial eigenvalue decay that concentrates learning  
 511 dynamics in a small number of dominant directions. Modern architectures also impose strong  
 512 inductive biases - convolutional structures might favor local patterns, attention mechanisms could  
 513 prioritize relational reasoning - that may constrain parameter variations to similar subspaces across  
 514 tasks. The ubiquity of gradient-based optimization, with its inherent preference for smooth solutions,  
 515 could further channel different learning trajectories toward shared geometric structures. If true, this  
 516 would suggest that the universal subspace captures fundamental computational patterns that transcend  
 517 specific tasks - potentially explaining why transfer learning works and why diverse problems often  
 518 benefit from similar architectural modifications. However, the precise mechanisms remain an open  
 519 question, making our empirical investigation all the more important to understand this surprising  
 520 regularity in neural network learning.

521 **5 LIMITATIONS AND FUTURE WORK**  
 522

523 Although we provide conclusive results towards the existence and utility of universal shared subspaces,  
 524 the current analysis has scope for future research, such as limited interpretability of the shared  
 525 subspace and the corresponding directions. While it is a critical area of research, it is extremely  
 526 cumbersome to demonstrate interpretability of the principal directions for each layer of the network.  
 527 To the best of our knowledge we are not aware of any other literature that performs such an in-  
 528 depth analysis of the weight space of large models across diverse tasks, data modalities and model  
 529 architectures. The current approach to approximating a universal subspace relies on pretrained task-  
 530 specific models (predictors) for tasks, which may not be readily available for new tasks. An interesting  
 531 direction for future research would be to explore model independent methods for learning a universal  
 532 shared subspace, potentially derived directly from data. Furthermore, the conditions proposed in  
 533 Ortiz-Jimenez et al. (2023) for enabling task arithmetic rely on localized eigenfunctions which are not  
 534 conducive to learning a shared universal subspace. As a result, performing task arithmetic within the  
 535 current framework of a shared universal subspace is non-trivial and warrants further investigation.

536 **REFERENCES**  
 537

538 Peter L. Bartlett and Shahar Mendelson. Rademacher and gaussian complexities: risk bounds and  
 539 structural results. *J. Mach. Learn. Res.*, 3(null):463–482, March 2003. ISSN 1532-4435.

540 Yuval Belfer, Amnon Geifman, Meirav Galun, and Ronen Basri. Spectral analysis of the neural  
 541 tangent kernel for deep residual networks. *Journal of Machine Learning Research*, 25(184):1–49,  
 542 2024. URL <http://jmlr.org/papers/v25/22-0597.html>.

543

544 Alberto Bietti, Grégoire Mialon, Dexiong Chen, and Julien Mairal. A kernel perspective for regular-  
 545 izing deep neural networks, 2019. URL <https://arxiv.org/abs/1810.00363>.

546 Lukas Bossard, Matthieu Guillaumin, and Luc Van Gool. Food-101 – mining discriminative compo-  
 547 nents with random forests. In *European Conference on Computer Vision*, 2014.

548

549 Rickard Brüel-Gabrielsson, Jiacheng Zhu, Onkar Bhardwaj, Leshem Choshen, Kristjan Greenewald,  
 550 Mikhail Yurochkin, and Justin Solomon. Compress then serve: Serving thousands of lora adapters  
 551 with little overhead, 2024. URL <https://arxiv.org/abs/2407.00066>.

552 Lin Chen and Sheng Xu. Deep neural tangent kernel and laplace kernel have the same {rkhs}. In  
 553 *International Conference on Learning Representations*, 2021. URL <https://openreview.net/forum?id=vK9WrZ0QYQ>.

554

555 Gong Cheng, Junwei Han, and Xiaoqiang Lu. Remote sensing image scene classification: Benchmark  
 556 and state of the art. *Proceedings of the IEEE*, 105(10):1865–1883, October 2017. ISSN 1558-2256.  
 557 doi: 10.1109/jproc.2017.2675998. URL <http://dx.doi.org/10.1109/JPROC.2017.2675998>.

558

559 Bilal Chughtai, Lawrence Chan, and Neel Nanda. A toy model of universality: Reverse engineer-  
 560 ing how networks learn group operations, 2023. URL <https://arxiv.org/abs/2302.03025>.

561

562

563 Mircea Cimpoi, Subhransu Maji, Iasonas Kokkinos, Sammy Mohamed, and Andrea Vedaldi. De-  
 564 scribing textures in the wild, 2013. URL <https://arxiv.org/abs/1311.3618>.

565

566 Jonathan Frankle and Michael Carbin. The lottery ticket hypothesis: Finding sparse, trainable neural  
 567 networks. In *International Conference on Learning Representations (ICLR)*, 2019.

568

569 Timur Garipov, Pavel Izmailov, Dmitrii Podoprikhin, Dmitry Vetrov, and Andrew Gordon Wilson.  
 570 Loss surfaces, mode connectivity, and fast ensembling of dnns. In *Advances in Neural Information  
 571 Processing Systems*, volume 31, pp. 8789–8798, 2018.

572

573 Matan Gavish and David L. Donoho. The optimal hard threshold for singular values is  $4/\sqrt{3}$ , 2014.  
 574 URL <https://arxiv.org/abs/1305.5870>.

575

576 Florentin Guth and Brice Ménard. On the universality of neural encodings in cnns, 2024. URL  
 577 <https://arxiv.org/abs/2409.19460>.

578

579 Florentin Guth, Brice Ménard, Gaspar Rochette, and Stéphane Mallat. A rainbow in deep network  
 580 black boxes. *Journal of Machine Learning Research*, 25(350):1–59, 2024. URL <http://jmlr.org/papers/v25/23-1573.html>.

581

582 Patrick Helber, Benjamin Bischke, Andreas Dengel, and Damian Borth. Eurosat: A novel dataset  
 583 and deep learning benchmark for land use and land cover classification, 2019. URL <https://arxiv.org/abs/1709.00029>.

584

585 Edward J Hu, Yelong Shen, Phillip Wallis, Zeyuan Allen-Zhu, Yuanzhi Li, Shean Wang, Lu Wang,  
 586 and Weizhu Chen. Lora: Low-rank adaptation of large language models. *arXiv preprint  
 587 arXiv:2106.09685*, 2021.

588

589 Gabriel Ilharco, Marco Tulio Ribeiro, Mitchell Wortsman, Ludwig Schmidt, Hannaneh Hajishirzi,  
 590 and Ali Farhadi. Editing models with task arithmetic. In *The Eleventh International Confer-  
 591 ence on Learning Representations*, 2023. URL <https://openreview.net/forum?id=6t0Kwf8-jrj>.

592

593 Arthur Jacot, Franck Gabriel, and Clément Hongler. Neural tangent kernel: Convergence and gener-  
 594 alization in neural networks. In *Advances in Neural Information Processing Systems*, volume 31,  
 595 2018.

594 Albert Q. Jiang, Alexandre Sablayrolles, Arthur Mensch, Chris Bamford, Devendra Singh Chaplot,  
 595 Diego de las Casas, Florian Bressand, Gianna Lengyel, Guillaume Lample, Lucile Saulnier,  
 596 Lélio Renard Lavaud, Marie-Anne Lachaux, Pierre Stock, Teven Le Scao, Thibaut Lavril, Thomas  
 597 Wang, Timothée Lacroix, and William El Sayed. Mistral 7b, 2023. URL <https://arxiv.org/abs/2310.06825>.

598

599 Xisen Jin, Xiang Ren, Daniel Preotiuc-Pietro, and Pengxiang Cheng. Dataless knowledge fusion  
 600 by merging weights of language models. In *The Eleventh International Conference on Learning  
 601 Representations*, 2023. URL <https://openreview.net/forum?id=FCnohuR6AnM>.

602

603 Prakhar Kaushik, Ankit Vaidya, Shravan Chaudhari, and Alan Yuille. Eigenlorax: Recycling  
 604 adapters to find principal subspaces for resource-efficient adaptation and inference, 2025. URL  
 605 <https://arxiv.org/abs/2502.04700>.

606

607 Vladimir Koltchinskii and Karim Lounici. Concentration inequalities and moment bounds for sample  
 608 covariance operators, 2014. URL <https://arxiv.org/abs/1405.2468>.

609

610 Dawid Jan Kopiczko, Tijmen Blankevoort, and Yuki M. Asano. VeRA: Vector-based Random Matrix  
 611 Adaptation. October 2023. URL <https://openreview.net/forum?id=NjNfLdxr3A>.

612

613 Jonathan Krause, Michael Stark, Jia Deng, and Li Fei-Fei. 3d object representations for fine-grained  
 614 categorization. In *4th International IEEE Workshop on 3D Representation and Recognition  
 (3dRR-13)*, Sydney, Australia, 2013.

615

616 Alex Krizhevsky, Vinod Nair, and Geoffrey Hinton. Cifar-100 (canadian institute for advanced  
 617 research). 2009. URL <http://www.cs.toronto.edu/~kriz/cifar.html>.

618

619 Alex Krizhevsky, Ilya Sutskever, and Geoffrey E Hinton. Imagenet classification with deep  
 620 convolutional neural networks. In F. Pereira, C.J. Burges, L. Bottou, and K.Q. Wein-  
 621 berger (eds.), *Advances in Neural Information Processing Systems*, volume 25. Curran Asso-  
 622 ciates, Inc., 2012. URL [https://proceedings.neurips.cc/paper\\_files/paper/2012/file/c399862d3b9d6b76c8436e924a68c45b-Paper.pdf](https://proceedings.neurips.cc/paper_files/paper/2012/file/c399862d3b9d6b76c8436e924a68c45b-Paper.pdf).

623

624 Yann LeCun, Corinna Cortes, and CJ Burges. Mnist handwritten digit database. *ATT Labs [Online]*.  
 Available: <http://yann.lecun.com/exdb/mnist>, 2, 2010.

625

626 Wei Ma and Jun Lu. An equivalence of fully connected layer and convolutional layer, 2017. URL  
<https://arxiv.org/abs/1712.01252>.

627

628 Charles Martin, Tongsu Peng, and Michael Mahoney. Predicting trends in the quality of state-of-the-  
 629 art neural networks without access to training or testing data. *Nature Communications*, 12:4122,  
 07 2021. doi: 10.1038/s41467-021-24025-8.

630

631 Stanislav Minsker. On some extensions of bernstein's inequality for self-adjoint operators, 2017.  
 632 URL <https://arxiv.org/abs/1112.5448>.

633

634 Yuval Netzer, Tao Wang, Adam Coates, Alessandro Bissacco, Bo Wu, and Andrew Y. Ng. Reading  
 635 digits in natural images with unsupervised feature learning. In *NIPS Workshop on Deep Learning  
 636 and Unsupervised Feature Learning 2011*, 2011. URL [http://ufldl.stanford.edu/housenumbers/nips2011\\_housenumbers.pdf](http://ufldl.stanford.edu/housenumbers/nips2011_housenumbers.pdf).

637

638 Maria-Elena Nilsback and Andrew Zisserman. Automated flower classification over a large number  
 639 of classes. In *Indian Conference on Computer Vision, Graphics and Image Processing*, Dec 2008.

640

641 Chris Olah, Nick Cammarata, Ludwig Schubert, Gabriel Goh, Michael Petrov, and Shan Carter.  
 642 Zoom in: An introduction to circuits. *Distill*, 2020. doi: 10.23915/distill.00024.001.  
<https://distill.pub/2020/circuits/zoom-in>.

643

644 Guillermo Ortiz-Jimenez, Alessandro Favero, and Pascal Frossard. Task arithmetic in the tangent  
 645 space: Improved editing of pre-trained models. In *Thirty-seventh Conference on Neural Information  
 646 Processing Systems*, 2023. URL <https://openreview.net/forum?id=0A9f2jZDGW>.

647

Omkar M. Parkhi, Andrea Vedaldi, Andrew Zisserman, and C. V. Jawahar. Cats and dogs. In *IEEE  
 Conference on Computer Vision and Pattern Recognition*, 2012.

648 Dustin Podell, Zion English, Kyle Lacey, Andreas Blattmann, Tim Dockhorn, Jonas Müller, Joe  
 649 Penna, and Robin Rombach. Sdxl: Improving latent diffusion models for high-resolution image  
 650 synthesis, 2023. URL <https://arxiv.org/abs/2307.01952>.

651

652 Konstantin Sch"urholt, Michael W. Mahoney, and Damian Borth. Towards scalable and versatile  
 653 weight space learning. In *Proceedings of the 41st International Conference on Machine Learning*  
 654 (*ICML*). PMLR, 2024.

655 Pratyusha Sharma, Jordan T. Ash, and Dipendra Misra. The Truth is in There: Improving Reasoning  
 656 in Language Models with Layer-Selective Rank Reduction, December 2023. URL <http://arxiv.org/abs/2312.13558> [cs].

657

658 Johannes Stallkamp, Marc Schlipsing, Jan Salmen, and Christian Igel. The German Traffic Sign  
 659 Recognition Benchmark: A multi-class classification competition. In *IEEE International Joint*  
 660 *Conference on Neural Networks*, pp. 1453–1460, 2011.

661

662 George Stoica, Pratik Ramesh, Boglarka Ecsedi, Leshem Choshen, and Judy Hoffman. Model  
 663 merging with SVD to tie the knots. In *The Thirteenth International Conference on Learning*  
 664 *Representations*, 2025. URL <https://openreview.net/forum?id=67X93aZHII>.

665

666 Patrick von Platen, Suraj Patil, Anton Lozhkov, Pedro Cuenca, Nathan Lambert, Kashif Rasul,  
 667 Mishig Davaadorj, Dhruv Nair, Sayak Paul, William Berman, Yiyi Xu, Steven Liu, and Thomas  
 668 Wolf. Diffusers: State-of-the-art diffusion models. <https://github.com/huggingface/diffusers>, 2022.

669

670 Alex Wang, Amanpreet Singh, Julian Michael, Felix Hill, Omer Levy, and Samuel R. Bowman. Glue:  
 671 A multi-task benchmark and analysis platform for natural language understanding, 2019. URL  
 672 <https://arxiv.org/abs/1804.07461>.

673

674 Yizhong Wang, Swaroop Mishra, Pegah Alipoormolabashi, Yeganeh Kordi, Amirreza Mirzaei,  
 675 Atharva Naik, Arjun Ashok, Arut Selvan Dhanasekaran, Anjana Arunkumar, David Stap, Eshaan  
 676 Pathak, Giannis Karamanolakis, Haizhi Lai, Ishan Purohit, Ishani Mondal, Jacob Anderson,  
 677 Kirby Kuznia, Krima Doshi, Kuntal Kumar Pal, Maitreya Patel, Mehrad Moradshahi, Mihir  
 678 Parmar, Mirali Purohit, Neeraj Varshney, Phani Rohitha Kaza, Pukit Verma, Ravsehaj Singh Puri,  
 679 Rushang Karia, Savan Doshi, Shailaja Keyur Sampat, Siddhartha Mishra, Sujan Reddy A, Sumanta  
 680 Patro, Tanay Dixit, and Xudong Shen. Super-NaturalInstructions: Generalization via declarative  
 681 instructions on 1600+ NLP tasks. In Yoav Goldberg, Zornitsa Kozareva, and Yue Zhang (eds.),  
 682 *Proceedings of the 2022 Conference on Empirical Methods in Natural Language Processing*, pp.  
 683 5085–5109, Abu Dhabi, United Arab Emirates, December 2022. Association for Computational  
 684 Linguistics. doi: 10.18653/v1/2022.emnlp-main.340. URL <https://aclanthology.org/2022.emnlp-main.340/>.

685

686 Colin Wei, Jason D. Lee, Qiang Liu, and Tengyu Ma. *Regularization matters: generalization and*  
 687 *optimization of neural nets v.s. their induced kernel*. Curran Associates Inc., Red Hook, NY, USA,  
 688 2019.

689

690 J. Xiao, J. Hays, K. A. Ehinger, A. Oliva, and A. Torralba. Sun database: Large-scale scene  
 691 recognition from abbey to zoo. In *2010 IEEE Computer Society Conference on Computer Vision*  
 692 *and Pattern Recognition*, pp. 3485–3492, June 2010. doi: 10.1109/CVPR.2010.5539970.

693

694 Prateek Yadav, Derek Tam, Leshem Choshen, Colin Raffel, and Mohit Bansal. TIES-merging:  
 695 Resolving interference when merging models. In *Thirty-seventh Conference on Neural Information*  
 696 *Processing Systems*, 2023. URL <https://openreview.net/forum?id=xtaX3WyCj1>.

697

698 Le Yu, Bowen Yu, Haiyang Yu, Fei Huang, and Yongbin Li. Language models are super mario:  
 699 Absorbing abilities from homologous models as a free lunch. In *ICML*, 2024. URL <https://openreview.net/forum?id=fq0NaiU8Ex>.

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701

Table 6: Notation reference.

| Notation                                   | Description  |
|--|--|
| $\mathcal{H}$                              | Separable Hilbert space with inner product $\langle \cdot, \cdot \rangle$ , norm $\  \cdot \ $ .       |
| $a \otimes b$                              | Rank-one operator $g \mapsto \langle b, g \rangle a$ , $\ a \otimes b\ _{\text{op}} = \ a\  \ b\ $ .   |
| $T$  | Number of tasks.   |
| $\mathcal{T}$                              | Distribution over tasks.   |
| $D_t$                                      | Data distribution for task $t$ .   |
| $S_t = \{(x_{t,i}, y_{t,i})\}_{i=1}^{n_t}$ | Dataset of size $n_t$ for task $t$ .   |
| $f_t^* \in \mathcal{H}$                    | Ground-truth predictor for task $t$ .  |
| $\hat{f}_t \in \mathcal{H}$                | Learned predictor for task $t$ .   |
| $B$  | Uniform bound: $\ f_t^*\  \leq B$ almost surely.   |
| $\mathcal{R}_{n_t, D_t}(\mathcal{H})$      | Per-task estimation error rate (e.g. $\tilde{O}(1/\sqrt{n_t})$ ).                                      |
| $\eta_t$                                   | Per-task error: $\eta_t := \mathcal{R}_{n_t, D_t}(\mathcal{H}) + \sqrt{\frac{\ln(2T/\delta)}{2n_t}}$ . |
| $\bar{\eta}$                               | Average error: $\frac{1}{T} \sum_{t=1}^T \eta_t$ .   |
| $\eta_t^2$                                 | Average squared error: $\frac{1}{T} \sum_{t=1}^T \eta_t^2$ .   |
| $\mathcal{S}$                              | Population operator: $\mathcal{S} = \mathbb{E}_{t \sim \mathcal{T}}[f_t^* \otimes f_t^*]$ .            |
| $\hat{\mathcal{S}}$                        | Empirical operator (true predictors): $\frac{1}{T} \sum_{t=1}^T f_t^* \otimes f_t^*$ .                 |
| $\tilde{\mathcal{S}}$                      | Empirical operator (learned predictors): $\frac{1}{T} \sum_{t=1}^T \hat{f}_t \otimes \hat{f}_t$ .      |
| $\lambda_1 \geq \lambda_2 \geq \dots$      | Eigenvalues of $\mathcal{S}$ .   |
| $\phi_i$                                   | Orthonormal eigenvectors of $\mathcal{S}$ .  |
| $P_k$                                      | Projector onto top- $k$ eigenspace of $\mathcal{S}$ .  |
| $\tilde{P}_k$                              | Projector onto top- $k$ eigenspace of $\tilde{\mathcal{S}}$ .  |
| $\gamma_k$                                 | Eigengap: $\gamma_k := \lambda_k - \lambda_{k+1} > 0$ .  |
| $\ A\ _{\text{op}}$                        | Operator (spectral) norm.  |
| $\ A\ _{HS}$                               | Hilbert–Schmidt norm.  |
| $r(V)$                                     | Intrinsic/Effective rank: $\text{tr}(V)/\ V\ _{\text{op}}$ .   |
| $X_t$                                      | Centered operator: $X_t := f_t^* \otimes f_t^* - \mathcal{S}$ .  |
| $V$  | Variance operator: $V := \sum_{t=1}^T \mathbb{E}[X_t^2]$ .   |
| $\delta, \delta_t, \delta_T$               | Failure probabilities (global, per-task, across-task).   |

## A APPENDIX

### A.1 RELATED WORK

Several lines of prior research support the core intuition behind our universal subspace hypothesis, though they do not provide a unified, scalable framework for identifying and leveraging such subspaces across architectures, tasks, and modalities. The Neural Tangent Kernel framework reinforces this idea, demonstrating that, in the infinite-width regime, training dynamics are governed by a kernel largely invariant to task specifics, implying the presence of common functional subspaces. (Jacot et al., 2018). This result implies that training is implicitly constrained to a shared function space, suggesting the existence of low-dimensional structures that generalize across tasks. Complementing this, works in mechanistic interpretability has uncovered modular and recurring patterns that consistently re-emerge in independently trained models (Olah et al., 2020; Chughtai et al., 2023), supporting the notion of structural universality in network representations.

Empirical studies further strengthen this perspective. The lottery ticket hypothesis (Frankle & Carbin, 2019) demonstrates that overparameterized networks contain sparse subnetworks capable of matching full-model performance, implying that task-relevant information resides in a small, structured subset of weights. Similarly, mode connectivity studies (Garipov et al., 2018) reveal that seemingly isolated optima in parameter space are often connected by low-loss paths, suggesting that task solutions lie on a shared manifold. In convolutional models, Krizhevsky et al. (Krizhevsky et al., 2012) famously observed that early layers consistently learn Gabor-like filters, indicating a universal inductive bias in early representations. More recent works (Guth et al., 2024; Guth & Ménard, 2024) extends this

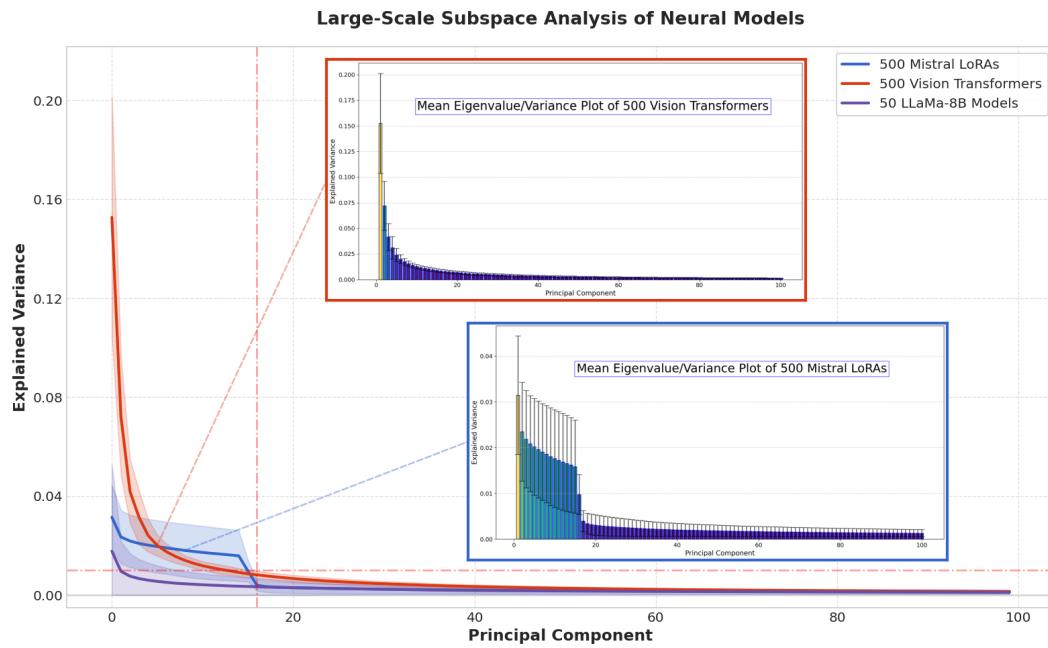


Figure 7: **Empirical Evidence for (Universal) Joint Weight Subspaces.** This figure illustrates the existence of joint low-dimensional subspaces across models trained on diverse tasks. We plot the average explained variance of the top few principal components of weight matrices from 500 Mistral-7B LoRAs, 500 Vision Transformers, and 50 LLaMA-8B models. Despite differences in modality, data, and training objective, all models exhibit rapid spectral decay - indicating that a small number of directions dominate across layers and settings. This consistent structure provides strong evidence for the presence of joint/universal subspaces, supporting our hypothesis that deep networks systematically reuse a common representational basis. Often, this shared subspace can be seen distinctly. The presence of the subspace has significant implications for deep learning. Not only can large number of models be compressed into a single, lighter Universal model with difference represented as lightweight coefficients, training on future tasks simply becomes tuning those coefficients. Since the basis are fixed, training becomes simpler and quicker. However, this convergence to similar subspace raises few important questions - is it possible to recover the "true" Universal Subspace without learning with huge amounts of data? Is this lack of diversity a bottleneck from current family of deep models?

observation to deeper layers, showing that certain eigenvectors of trained convolutional layers recur across networks trained on different datasets.

While these studies are suggestive of shared structures in neural representations or parameters, they remain limited in their focus, application and analysis. Our work fills this critical gap by presenting a principled and empirically validated method for discovering and utilizing universal parametric subspaces that span across architectures, tasks, and modalities. By conducting large-scale spectral analyses of over large number of diverse architectures, models and tasks, we demonstrate that a small number of principal directions consistently capture the majority of task-relevant variation. We then operationalize these findings by developing a practical framework for reusing these subspaces for parameter-efficient finetuning, task adaptation, and model merging, achieving competitive performance while dramatically reducing memory and compute requirements.

## A.2 THEORETICAL ANALYSIS

We apply a standard generalization bound over the squared error between the task function and its projection onto the shared subspace:

$$\ell(f_t, x) = \|f_t(x) - f_{t,k}(x)\|^2$$

To justify the application of PAC-style bounds, we verify that this loss is bounded. We assume that each task predictor  $f_t$  lies in a Reproducing Kernel Hilbert Space (RKHS) with norm bounded by  $B$ , i.e.,  $\|f_t\|_{\mathcal{H}} \leq B$ , and that the projection  $f_{t,k}$  onto the learned shared subspace  $\hat{\mathcal{H}}_k$  also satisfies  $\|f_{t,k}\|_{\mathcal{H}} \leq B$ .

Using the reproducing property and assuming a kernel bound  $\kappa^2 = \sup_{x \in \mathcal{X}} \|\phi(x)\|^2$ , we have for any  $x$ :

$$\|f_t(x)\| \leq \kappa B \quad \text{and} \quad \|f_{t,k}(x)\| \leq \kappa B$$

Thus, the pointwise squared loss is bounded as:

$$\|f_t(x) - f_{t,k}(x)\|^2 \leq (\|f_t(x)\| + \|f_{t,k}(x)\|)^2 \leq (2\kappa B)^2 = 4\kappa^2 B^2$$

Therefore, the loss function is bounded in  $[0, 4\kappa^2 B^2]$ , satisfying the conditions required for PAC-style generalization bounds to hold.

**Lemma A.1** (Matrix Bernstein for self-adjoint operators). *There exist absolute constants  $C > 0$  such that, for any  $\delta_T \in (0, 1)$ , we have with probability at least  $1 - \delta_T$ ,*

$$\|\hat{\mathcal{S}} - \mathcal{S}\|_{\text{op}} \leq C B^2 \left[ \sqrt{\frac{\ln(c/\delta_T)}{T}} + \frac{\ln(c/\delta_T)}{T} \right]$$

*Proof.* Operator Bernstein (intrinsic form).

Let  $X_1, \dots, X_T$  be independent, mean-zero, self-adjoint, bounded operators on a separable Hilbert space. Suppose

$$\|X_t\|_{\text{op}} \leq L \quad \text{a.s. for all } t.$$

Then from (Minsker, 2017; Koltchinskii & Lounici, 2014) there exist absolute constants  $C, c > 0$  such that for every  $\delta \in (0, 1)$ ,

$$\left\| \frac{1}{T} \sum_{t=1}^T X_t \right\|_{\text{op}} \leq C \left[ \sqrt{\frac{\left\| \sum_{t=1}^T \mathbb{E}[X_t^2] \right\|_{\text{op}}}{T^2} \ln \left( \frac{c \left( 1 + \frac{\text{tr}(\sum_{t=1}^T \mathbb{E}[X_t^2])}{\left\| \sum_{t=1}^T \mathbb{E}[X_t^2] \right\|_{\text{op}}} \right)}{\delta_T} \right)} + \frac{L}{T} \ln \left( \frac{c \left( 1 + \frac{\text{tr}(\sum_{t=1}^T \mathbb{E}[X_t^2])}{\left\| \sum_{t=1}^T \mathbb{E}[X_t^2] \right\|_{\text{op}}} \right)}{\delta_T} \right) \right]$$

with probability at least  $1 - \delta_T$ .

Application to  $X_t = f_t^* \otimes f_t^* - \mathcal{S}$  with  $\|f_t^*\| \leq B$  a.s.

We have

$$\|X_t\|_{\text{op}} \leq \|f_t^*\|^2 + \|\mathcal{S}\|_{\text{op}} \leq B^2 + \mathbb{E}\|f_t^*\|^2 \leq 2B^2.$$

so  $L \leq 2B^2$ . Moreover, for  $X_t = f_t^* \otimes f_t^* - \mathcal{S}$  we have

$$\mathbb{E}[X_t^2] \preceq 2B^2 \mathcal{S}.$$

Hence

$$\left\| \sum_{t=1}^T \mathbb{E}[X_t^2] \right\|_{\text{op}} \leq 2TB^2 \|\mathcal{S}\|_{\text{op}}, \quad \text{tr} \left( \sum_{t=1}^T \mathbb{E}[X_t^2] \right) \leq 2TB^2 \text{tr}(\mathcal{S}).$$

By assumption 2.3,

$$\frac{\text{tr}(\sum_{t=1}^T \mathbb{E}[X_t^2])}{\left\| \sum_{t=1}^T \mathbb{E}[X_t^2] \right\|_{\text{op}}} \leq \frac{\text{tr}(\mathcal{S})}{\|\mathcal{S}\|_{\text{op}}} \leq \kappa.$$

Therefore the intrinsic logarithmic factor in Bernstein reduces to

$$\ln \left( \frac{c(1 + \kappa)}{\delta_T} \right),$$

and since  $\kappa$  is a fixed constant,  $1 + \kappa$  can be absorbed into  $c$ .

864 Plugging into Bernstein gives  
 865

$$866 \|\hat{\mathcal{S}} - \mathcal{S}\|_{\text{op}} \leq C \left[ \sqrt{\frac{2B^2 \|\mathcal{S}\|_{\text{op}} \ln(c/\delta_T)}{T}} + \frac{2B^2 \ln(c/\delta_T)}{T} \right],$$

868 with probability at least  $1 - \delta_T$ .  
 869

□

871 **Lemma A.2** (Davis–Kahan, sin- $\Theta$ ). *Let  $\gamma_k > 0$ . Then*  
 872

$$873 \|\tilde{P}_k - P_k\|_{\text{op}} \leq \frac{2}{\gamma_k} \|\tilde{\mathcal{S}} - \mathcal{S}\|_{\text{op}}.$$

875 using definition of  $\gamma_k$  from definition 2.1.

876 **Theorem A.3** (Restating Two-level convergence to the shared subspace theorem). *Assume 2.3–2.4.  
 877 Let  $c_1, c_2$  be any absolute constants. For any  $\delta \in (0, 1)$ , choose  $\delta_t = \delta/(2T)$  and set  $\delta_T = \delta/2$ . With  
 878 probability at least  $1 - \delta$  (over tasks and all per-task samples),*

$$879 \|\tilde{\mathcal{S}} - \mathcal{S}\|_{\text{op}} \leq c_1 B^2 \sqrt{\frac{\ln(c_2/\delta)}{T}} + (2B\bar{\eta} + \bar{\eta}^2) \quad (3)$$

881 If moreover  $\gamma_k > 0$ , then

$$883 \|\tilde{P}_k - P_k\|_{\text{op}} \leq \frac{2}{\gamma_k} \left( c_1 B^2 \sqrt{\frac{\ln(c_2/\delta)}{T}} + (2B\bar{\eta} + \bar{\eta}^2) \right). \quad (4)$$

885 where  $\bar{\eta} = \frac{1}{T} \sum_{t=1}^T \eta_t$ ,  $\bar{\eta}^2 = \frac{1}{T} \sum_{t=1}^T \eta_t^2$  and  $\eta_t$  is defined same as in assumption 2.4

887 *Proof of Theorem 2.5. (i) Triangle split.*  $\|\tilde{\mathcal{S}} - \mathcal{S}\|_{\text{op}} \leq \|\tilde{\mathcal{S}} - \hat{\mathcal{S}}\|_{\text{op}} + \|\hat{\mathcal{S}} - \mathcal{S}\|_{\text{op}}$ .

889 **(ii) Within-task term.** We know that,

$$891 \|\hat{f}_t \otimes \hat{f}_t - f_t^* \otimes f_t^*\|_{\text{op}} \leq \|\hat{f}_t - f_t^*\| (\|\hat{f}_t\| + \|f_t^*\|) \\ 892 \leq \|\hat{f}_t - f_t^*\| (\|\hat{f}_t\| + \|f_t^*\|) \\ 893 \leq \eta_t (2B + \eta_t) \quad (\text{since } \|\hat{f}_t\| \leq \|f_t^*\| + \|\hat{f}_t - f_t^*\| \leq B + \eta_t) \\ 894 = 2B\eta_t + \eta_t^2.$$

897 Averaging and using the triangle inequality for operator norms,

$$899 \|\tilde{\mathcal{S}} - \hat{\mathcal{S}}\|_{\text{op}} \leq 2B\bar{\eta} + \bar{\eta}^2$$

902 This holds on the event  $\bigcap_{t=1}^T \{\|\hat{f}_t - f_t^*\| \leq \eta_t\}$ , whose probability is at least  $1 - \sum_t \delta_t = 1 - \delta/2$ .

904 **(iii) Across-task term.** Let  $X_t := f_t^* \otimes f_t^* - \mathbb{E}[f^* \otimes f^*]$ . Then  $X_t$  are independent, mean-zero,  
 905 self-adjoint, and  $\|X_t\|_{\text{op}} \leq \|f_t^*\|^2 + \|\mathcal{S}\|_{\text{op}} \leq 2B^2$ . Lemma A.1 (with  $R \asymp B^2$ ) yields

$$907 \|\hat{\mathcal{S}} - \mathcal{S}\|_{\text{op}} \leq c_1 B^2 \sqrt{\frac{\ln(c_2/\delta)}{T}} \\ 908 \\ 909 \|\tilde{\mathcal{S}} - \mathcal{S}\|_{\text{op}} \leq c_1 B^2 \sqrt{\frac{\ln(c_2/\delta)}{T}} + 2B \left( \sum_{t=1}^T \mathcal{R}_{n_t, D_t}(\mathcal{H}) + \sqrt{\frac{\ln(2T/\delta)}{2n_t}} \right) + \left( \sum_{t=1}^T \mathcal{R}_{n_t, D_t}^2(\mathcal{H}) + \frac{\ln(2T/\delta)}{2n_t} \right) \\ 910 \\ 911 \\ 912 \\ 913 \\ 914 \\ 915 \\ 916 \\ 917$$

$$\leq c_1 B^2 \sqrt{\frac{\ln(c_2/\delta)}{T}} + O \left( \sum_{t=1}^T \mathcal{R}_{n_t, D_t}^2(\mathcal{H}) + \frac{\ln(2T/\delta)}{2n_t} \right)$$

with probability at least  $1 - \delta_T = 1 - \delta/2$ .

**(iv) Union bound and Davis–Kahan.** Combining (ii)–(iii) with a union bound gives equation 1.  
 Lemma A.2 then implies equation 2. □

918 **Definition A.4** (Population projection risk). For a  $k$ -dimensional subspace  $\mathcal{H}_k^* \subset \mathcal{H}$ , define  
 919

$$920 \quad \mathcal{R}(\mathcal{H}_k^*) := \mathbb{E}_{t \sim \tau} \|f_t^* - P_{\mathcal{H}_k^*} f_t^*\|^2.$$

921 **Corollary A.5** (Excess projection risk of the learned subspace). *Under the event of Theorem 2.5,*  
 922

$$923 \quad \mathcal{R}(\tilde{\mathcal{H}}_k) \leq \sum_{i>k} \lambda_i + \frac{2 \operatorname{tr}(S)}{\gamma_k} \left( c_1 B^2 \sqrt{\frac{\ln(c_2/\delta)}{T}} + 2B\bar{\eta} + \bar{\eta}^2 \right).$$

926 *Proof.* Optimality of  $P_k$  gives  $\mathcal{R}(\mathcal{H}_k^*) = \sum_{i>k} \mu_i$ . Moreover,  
 927

$$928 \quad \mathcal{R}(\tilde{\mathcal{H}}_k) - \mathcal{R}(\mathcal{H}_k^*) = \mathbb{E} \left\langle f_t^*, (P_k - \tilde{P}_k) f_t^* \right\rangle \leq \left\| \tilde{P}_k - P_k \right\|_{\text{op}} \mathbb{E} \|f_t^*\|^2 = \operatorname{tr}(S) \left\| \tilde{P}_k - P_k \right\|_{\text{op}}.$$

931 Apply equation 2. □

932 **Remark A.6** (Where Rademacher complexity enters). Assumption 2.4 is instantiated by your learning  
 933 procedure. For strongly-convex ERM (e.g., kernel ridge), a standard Rademacher-based excess-risk  
 934 bound together with curvature yields an  $\eta_t = \eta_t(n_t, \delta_t)$  that vanishes with  $n_t$ . Plugging these  $\eta_t$  into  
 935  $\bar{\eta}$  and  $\bar{\eta}^2$  makes the rate explicit.  
 936

## 937 B UNIVERSAL SUBSPACE ANALYSIS

939 Similar methodology is followed for subspace analysis for both LoRA and classical weight models.  
 940 In fact, LoRA analysis' results can be theoretically extended to classical weights, as LoRA weights  
 941 can be construed to be simple translations from a mean weight matrix. However, in order to solidify  
 942 our universal subspace hypothesis, we conduct extensive experiments for both types of models. LoRA  
 943 is chosen because of the recent spurt in the availability of LoRA models trained on diverse kinds  
 944 of datasets and models. **We do this universal subspace analysis on all weight parameters in every**  
 945 **neural network layer except the first (or few initial) and last neural network layer. This is because**  
 946 **these layers may differ across models due to differences in input shapes and types, loss functions,**  
 947 **and the tasks being trained.** We also focus our analysis on linear/fully-connected and matrix weights,  
 948 as the analysis done on these are straightforward and the results observed can be trivially extended to  
 949 other types of neural parameters (Ma & Lu, 2017).  
 950

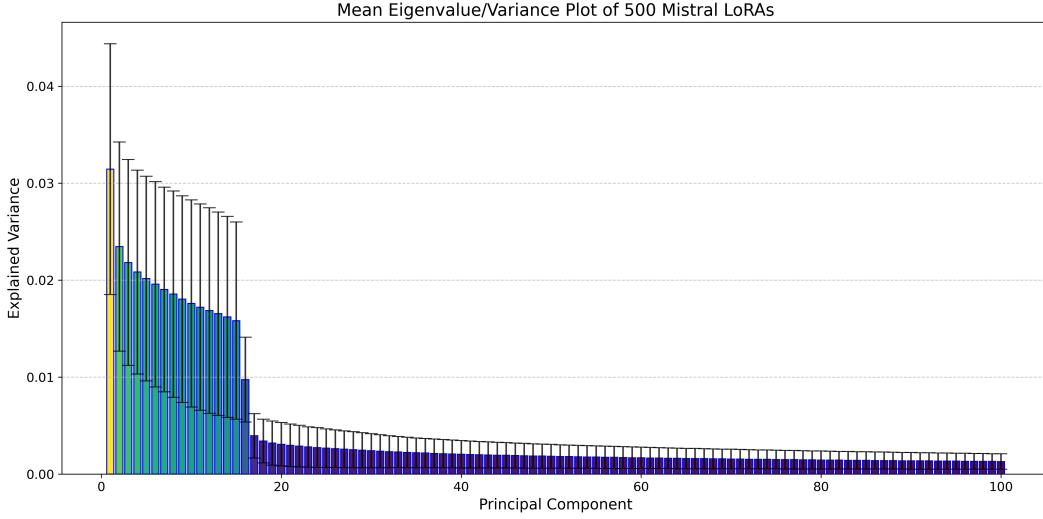
951 **Secondary Subspace** refers to the residual subspace that remains after removing the top  $k$  principal  
 952 directions associated with the low-rank universal subspace. This subspace is orthogonal to the universal  
 953 subspace and serves as a control for evaluating the uniqueness and effectiveness of the learned  
 954 shared subspace. To make computation tractable when the residual subspace is high-dimensional,  
 955 we focus on the top components beyond rank  $k$ , as computing a full SVD is often impractical. This  
 956 approximation is justified, since the lower components typically capture noise, which has been shown  
 957 to degrade performance (Sharma et al., 2023).  
 958

959 **How to choose top  $k$  components?** As shown in all eigenvalue (scree) plots, a trivial way to choose  
 960 is a simple visual inspection, since we can see a discontinuity in the spectral analysis. Another way is  
 961 to define a threshold on the explained variance, all components whose explained variance is close  
 962 to zero <.01 are considered secondary subspace, and can be discarded. A more structured way is to  
 963 define an optimal singular value threshold for the HOSVD, as found by previous works (Gavish &  
 964 Donoho, 2014).  
 965

### 966 B.1 LOWER RANK SHARED UNIVERSAL SUBSPACES WITHIN LOW RANK ADAPTATION (LoRA) 967 MODELS

968 Spectral Decomposition is employed to extract the top  $k$  principal directions for each of the LoRA  
 969 matrices  $B$  and  $A$ , which are concatenated across all available models. Subsequently, the top  $k$   
 970 principal directions are selected to define the low-rank subspace shared among the LoRA matrices.  
 971 This process is conducted separately for each layer of the model to derive a low-rank approximated  
 972 shared subspace for every individual layer. In practice, for every layer, the rank vectors of all available

972 LoRA matrices are extracted and concatenated into a single matrix. This matrix is then normalized by  
 973 subtracting the feature-wise mean from each vector, after which principal directions are extracted. The  
 974 mode-1(order-1) variant of our method is mathematically equivalent to Principal Component Analysis  
 975 (PCA), hence we can use `torch.pca_lowrank` or `sklearn.decomposition.PCA` to ex-  
 976 tract the principal directions. The data matrix corresponding to a specific layer for 500 LoRA models  
 977 is structured as  $500r \times d$ , where  $r$  denotes the rank of each LoRA and  $d$  specifies the dimension of  
 978 each rank vector. The same calculation can be applied to the  $\mathbf{B}$  matrix instead of individually to  $\mathbf{B}$   
 979 and  $\mathbf{A}$ , thereby increasing the computational cost of the Spectral Decomposition without affecting the  
 980 outcome.



999 Figure 8: Spectral analysis of the Mistral-7B-Instruct-v0.2 model: Aggregated eigenvalue (scree)  
 1000 plot across 500 LoRA models and all layers. The plot demonstrates that the majority of the variance  
 1001 is consistently captured by the top 16 principal directions, indicating the presence of a shared low-  
 1002 dimensional universal subspace.

1003  
 1004 **Universal Mistral-7B/Lots of LoRAs experiment details** In our first experimental analysis, we  
 1005 use 500 LoRA models trained on distinct Natural Instructions (Wang et al., 2022) using Mistral-  
 1006 7B-Instruct-v0.2 (Jiang et al., 2023) as the base (Brüel-Gabrielsson et al., 2024). Please refer to  
 1007 Brüel-Gabrielsson et al. (2024) for more details on how the LoRA models were trained.

1008 Table 7: Models from HuggingFace for the Universal Mistral LoRA. Models in blue indicate the  
 1009 OOD models and the ones in red are the IID models used for evaluation.

|  |  |
|--|--|
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task391  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task290  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task442  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1598 |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task039  |  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task076  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task627  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task664  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task819  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1631 |  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task190  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1391 |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1342 | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task620  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task769  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1448 |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task247  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task513  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task875  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task515  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1534 | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1551 |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task583  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1431 |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task270  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1487 |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task679  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task456  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task385  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1607 |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task278  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task022  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task210  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task137  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task574  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task629  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1378 | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1194 |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1529 | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task453  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task102  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task460  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1204 | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1384 |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1572 | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task699  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1722 | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task580  |





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| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task228  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task209  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task128  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task710  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1322 | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task163  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task178  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task089  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task700  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1581 |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task927  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task101  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task123  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1321 |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task550  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task129  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task393  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1214 |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task277  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1447 |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task324  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task455  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task725  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task365  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1316 | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1199 |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task717  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task245  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task874  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task925  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task380  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1712 |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1504 | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task619  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task590  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1186 |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task736  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task069  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task377  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task181  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task859  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task144  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task632  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task641  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task064  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task630  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1154 | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task390  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1188 | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task625  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task607  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task495  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1189 | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task398  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task108  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1347 |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1541 | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task202  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1723 | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1669 |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1089 | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1584 |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task081  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task329  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task691  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task588  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1593 | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task724  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task149  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1449 |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1313 | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1453 |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task905  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task704  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task585  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1209 |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task249  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1386 |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1400 | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task751  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1332 | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task674  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task379  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task243  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1318 | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task428  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task488  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task705  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task698  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1601 |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task861  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1510 |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task077  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task509  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task734  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task720  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1210 | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task284  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task584  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task105  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task330  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task923  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task319  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task400  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task246  | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task726  |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1568 | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1442 |
| Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task1640 | Lots-of-LoRAs/Mistral-7B-Instruct-v0.2-4b-r16-task280  |

Figure 9 presents the aggregated results across all layers, with error bars representing the standard deviation. For reference, the eigenvalue (scree) plot from Figure 3b is also reproduced in Figure 9. This plot depicts the proportion of variance explained by each principal component, computed across all weight matrices and layers from 500 independently trained Mistral models. The concentration of variance within the top  $k$  components reveals the presence of a consistent low-dimensional subspace, offering strong empirical support for the universal subspace hypothesis.

The individual plots provide spectral analysis results for the key, query, and value matrices from all 32 layers of all 500 Mistral models. For clarity, only the top 128 principal directions are visualized, representing a subset of the full component basis. This truncation mitigates the visual distortion caused by the long tail of near-zero eigenvalues beyond the universal subspace, which would otherwise dominate the graph without contributing meaningful information.

To test subspace expressiveness, we reconstruct LoRA weights for both 5 seen (IID) and unseen (OOD) tasks by projecting them into the universal subspace. As shown in Figure 4, the reconstructed models retain high performance in both cases. In contrast, projection into the residual *Secondary Subspace* leads to a sharp performance drop, underscoring the importance of the principal subspace. Our method is also 19 $\times$  more memory efficient, as it eliminates the need to store all 500 LoRAs.

Table 8: Models from HuggingFace used for the Universal Stable Diffusion-XL subspace extraction

|      |                                      |                                    |                                     |                                 |
|------|--------------------------------------|------------------------------------|-------------------------------------|---------------------------------|
| 1188 | aphone-mucha-style                   | directors-coen-brothers-style      | larry-carlson-style                 | rene-magritte-style             |
| 1189 | beeple-mike-winkelmann-style         | director-sergei-eisenstein-style   | lascaux                             | richard-corben-style            |
| 1190 | character-design                     | director-sofia-coppola-style       | laurel-burch-style                  | richard-dadd-style              |
| 1191 | director-christopher-nolan-style     | director-terrence-malick-style     | lawrence-alma-tadema-style          | richard-hescox-style            |
| 1192 | director-lars-von-trier-style        | director-tim-burton-style          | leonid-afrimov-style                | richard-scarry-style            |
| 1193 | director-ridley-scott-style          | director-wes-anderson-style        | leonora-carrington-style            | robert-adams-style              |
| 1194 | director-stanley-kubrick-style       | director-wong-kar-wai-style        | levitating-cube                     | robert-crumb-style              |
| 1195 | director-zhang-yimou-style           | director-yorgos-lanthimos-style    | liam-wong-style                     | robert-rauschenberg-style       |
| 1196 | olafur-eliasson-style                | dixit-card-generator               | lotte-reiniger-style                | rodney-matthews-style           |
| 1197 | origami                              | dressed-animals                    | louis-comfort-tiffany-style         | roger-ballen-style              |
| 1198 | simone-martini-style                 | dripping-art                       | louis-cornin-style                  | roger-deakins-style             |
| 1199 | studio-ghibli-style                  | edward-gorey-style                 | lucas-cranach-style                 | romare-bearden-style            |
| 1200 | ukiyo-e-art                          | elizabeth-gadd-style               | luc-schuiten-style                  | ryoji-ikeda-style               |
| 1201 | wu-quanzhong-style                   | erik-johansson-style               | lyonel-feininger-style              | sacha-goldberger-style          |
| 1202 | 1987-action-figure-playset-packaging | erik-madigan-heck-style            | made-of-iridescent-foil             | salomon-van-ruysdael-style      |
| 1203 | aardman-animations-style             | euan-uglow-style                   | makoto-shinkai-style                | sam-spratt-style                |
| 1204 | akos-major-style                     | felipe-pantone-style               | marc-silvestri-style                | sandy-skoglund-style            |
| 1205 | albumen-print                        | filip-hodas-style                  | marianna-rothen-style               | santiago-caruso-style           |
| 1206 | alec-soth-style                      | folk-art                           | maria-sibylla-merian-style          | shaun-tan-style                 |
| 1207 | alejandro-jodorowsky-style           | gabriel-pacheco-style              | mark-catesby-style                  | shepard-fairey-style            |
| 1208 | alessandro-gottardo-style            | gemma-correll-style                | mark-ryden-style                    | sidney-nolan-style              |
| 1209 | alex-andreev-style                   | george-condo-style                 | martin-whatson-style                | simon-stalenhag-style           |
| 1210 | alex-gross-style                     | gilbert-garcin-style               | mary-cassatt-style                  | skottie-young-style             |
| 1211 | alfred-augustus-glendening-style     | gregory-crewson-style              | maurice-de-vlaminck-style           | sofonisba-anguissola-style      |
| 1212 | alex-pardee-style                    | gustave-dore-style                 | maurice-prendergast-style           | sophie-gengembre-anderson-style |
| 1213 | alternate-realities                  | hasui-kawase-style                 | maxfield-parrish-style              | stained-glass-portrait          |
| 1214 | ando-fuchs-style                     | hiroshi-nagai-style                | maxime-mauffra-style                | stanley-donwood-style           |
| 1215 | andre-derain-style                   | infrared-photos                    | mike-mignola-style                  | stephan-martiniere-style        |
| 1216 | andrei-tarkovsky-style               | isometric-cutaway                  | mikhail-vrubel-style                | stephen-gammell-style           |
| 1217 | andrew-wyeth-style                   | ivan-bilibin-style                 | moebius-jean-giraud-style           | stop-motion-animation           |
| 1218 | angus-mckie-style                    | james-c-christensen-style          | movie-poster                        | surreal-collage                 |
| 1219 | anna-maria-garthwaite-style          | james-jean-style                   | moving-meditations                  | surreal-harmony                 |
| 1220 | atey-ghaiian-style                   | james-r-eads-style                 | nadav-kander-style                  | surreal-plate                   |
| 1221 | audrey-kawasaki-style                | james-turrell-style                | natalia-goncharova-style            | syd-mead-style                  |
| 1222 | avant-garde-fashion                  | jan-brueghel-style                 | n-c-wyeth-style                     | synthwave-t-shirt               |
| 1223 | banky-style                          | jan-svankmajer-style               | needlepoint                         | teamlab-style                   |
| 1224 | bas-relief                           | jan-van-eyck-style                 | neon-night                          | terry-gilliam-style             |
| 1225 | century-botanical-illustration       | jan-van-goyen-style                | nicolas-poussin-style               | thomas-cole-style               |
| 1226 | christopher-balaskas-style           | j-c-leyendecker-style              | noah-bradley-style                  | thomas-kinkade-style            |
| 1227 | christopher-ryan-mckenney-style      | jean-baptiste-camille-corot-style  | ohara-koson-style                   | thomas-moran-style              |
| 1228 | clay-animation                       | jean-baptiste-monge-style          | okuda-san-miguel-style              | thomas-schaller-style           |
| 1229 | color-palette                        | jean-baptiste-simeon-chardin-style | olly-moss-style                     | tim-walker-style                |
| 1230 | craig-mullins-style                  | jean-metzinger-style               | op-art                              | tintoretto-style                |
| 1231 | crocheted                            | jean-michel-basquiat-style         | parallel-dimensions                 | todd-hido-style                 |
| 1232 | daniel-arsham-style                  | jessie-willcox-smith-style         | pascal-campion-style                | tove-jansson-style              |
| 1233 | dark-fantasy                         | jim-mahfood-style                  | paul-gustav-fischer-style           | tracie-grimwood-style           |
| 1234 | dave-mckean-style                    | john-albert-bauer-style            | paul-laffoley-style                 | vasily-vereschagin-style        |
| 1235 | diorama                              | john-berkey-style                  | paul-signac-style                   | vertical-landscapes             |
| 1236 | director-agnes-varda-style           | john-blanche-style                 | peter-doi-style                     | victor-brauner-style            |
| 1237 | death-stranding                      | john-constable-style               | peter-paul-rubens-style             | victor-moscoso-style            |
| 1238 | director-akira-kurosawa-style        | john-everett-millais-style         | philippe-druillet-style             | video-installation              |
| 1239 | director-andrei-zvyagintsev-style    | john-harris-style                  | photographer-eleena-helfrecht-style | vintage-postage-stamps          |
| 1240 | director-bong-joon-ho-style          | john-james-audubon-style           | photographer-flora-borsi-style      | weegee-style                    |
| 1241 | director-darren-aronofsky-style      | john-kenn-mortensen-style          | photographer-maren-klemp-style      | wendy-froud-style               |
| 1242 | director-david-fincher-style         | john-martin-style                  | photographer-martin-kimbell-style   | will-eisner-style               |
| 1243 | director-david-lynch-style           | john-singer-sargent-style          | photographer-reuben-wu-style        | willem-haenraets-style          |
| 1244 | cute-animals                         | john-singleton-copley-style        | pierre-auguste-renoir-style         | willem-van-aelst-style          |
| 1245 | ben-aronson-style                    | john-william-waterhouse-style      | pierre-bonnard-style                | william-langson-lathrop-style   |
| 1246 | director-emir-kusturica-style        | joseph-wright-of-derby-style       | pieter-claesz-style                 | william-mctaggart-style         |
| 1247 | director-gaspar-noe-style            | josh-agle-style                    | punk-collage                        | william-merritt-chase-style     |
| 1248 | director-jean-pierre-junet-style     | josh-kirby-style                   | quentin-blake-style                 | winslow-homer-style             |
| 1249 | director-krzysztof-kieslowski-style  | jules-bastien-lepage-style         | raimonds-staprans-style             | worthington-whittredge-style    |
| 1250 | director-martin-scorsese-style       | kate-greenaway-style               | ralph-bakshi-style                  | yaacov-agam-style               |
| 1251 | director-nicolas-winding-refn-style  | kay-nielsen-style                  | ralph-steadman-style                | yoh-nagao-style                 |
| 1252 | director-park-chan-wook-style        | kilian-eng-style                   | randolph-caldecott-style            | yves-klein-style                |
| 1253 | director-pedro-almodovar-style       | kirigami                           | ray-caesar-style                    | zanele-muholi-style             |
| 1254 | director-quentin-tarantino-style     | konstantin-korovin-style           | remedios-varo-style                 |                                 |

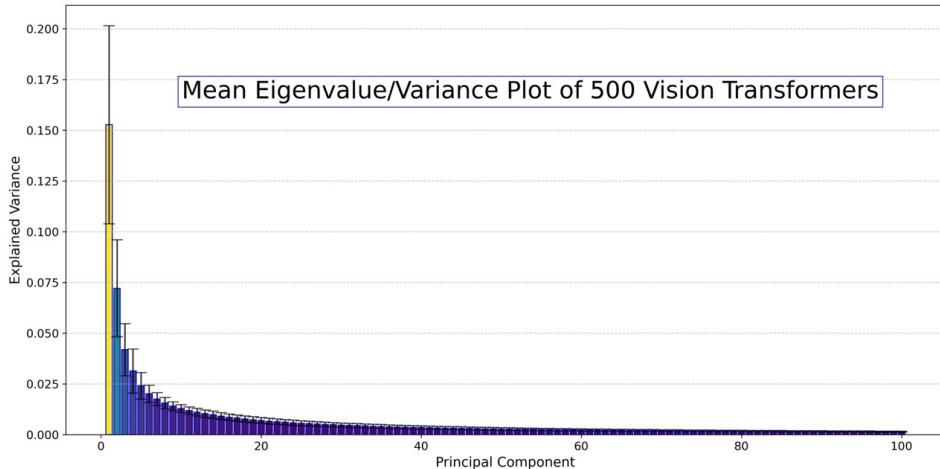


Figure 9: Layerwise Eigenvalue Plots of 500 Mistral-7B-Instruct-v0.2 models. Each layer has 3 sets of parameters -  $k_{proj}$ ,  $q_{proj}$ ,  $v_{proj}$

1296 **Universal SDXL experiment details** Our second experiment involves the complex and multimodal  
 1297 task of Text-to-Image generation using the Stable Diffusion-XL model Podell et al. (2023). We  
 1298 extract our low rank universal subspace from publicly available LoRA models on HuggingFace  
 1299 repository von Platen et al. (2022) - Table 8 lists all the SDXL models that we used to extract the  
 1300 Universal Subspace. As can be seen in Table 8, the models range wildly in styles on which they were  
 1301 finetuned. The fact that all these diverse models can be represented by a single low rank universal  
 1302 subspace model strongly verifies our hypothesis. We use top 16 components and 30 denoising steps.  
 1303 For each experiment model shown in Table 1 and Figure 5, that LoRA model is reconstructed using  
 1304 a universal subspace created using rest of the available LoRA adapters, essentially confirming the  
 1305 generalization capability of this subspace.

1306 We then use this single SDXL universal subspace to generate images with similar styles to evaluate  
 1307 whether this subspace is capable of doing so, by projecting randomly chosen LoRA models into this  
 1308 subspace. Figure 5 shows that our universal subspace matches the visual quality and style nuances of  
 1309 individual LoRAs, resulting in significant memory savings. Table 1 shows quantitative results for  
 1310 our Universal subspace in terms of CLIP scores, where interestingly we can see that our Universal  
 1311 Subspace outperforms the individual LoRA models. This improvement may be attributed to our  
 1312 Universal SDXL removing noise from the subspace - a phenomenon previously observed by Sharma  
 1313 et al. (2023). The styles used in Table 1, which are in Table 8 are (from Style 1 to Style 10) Ukiyo-e  
 1314 Style, Todd Hido Style, Olly Moss Style, Needlepoint Style, Studio Ghibli Style, Surreal Harmony  
 1315 Style, Dressed Animal Style, Lascaux Cave Art Style, Kirigami Style, Yaacov Agam Style.  
 1316

## B.2 LOW RANK SHARED UNIVERSAL SUBSPACES IN CLASSICAL WEIGHTS



1334 Figure 10: Spectral analysis of the Vision Transformer (ViT-base-patch16-224) model: Aggregated  
 1335 eigenvalue (scree) plot across 500 ViT models and all layers. The plot demonstrates that the majority  
 1336 of the variance is consistently captured by the top 16 principal directions, indicating the presence of a  
 1337 shared low-dimensional universal subspace.  
 1338

1339 In order to further solidify the evidence for our universal subspace hypothesis, we show that this  
 1340 universality does extend beyond adapter models to conventional weights. We do not focus on  
 1341 convolutional weight parameters as they can simply be equated with fully connected layers (Ma & Lu,  
 1342 2017), and have been shown, in limited scope, to match Gabor-like filters (Krizhevsky et al., 2012).  
 1343 Therefore, our analysis trivially extends to these kinds of parameters as well. However, there are a  
 1344 few practical differences between the low rank adapter and classical weight subspace analysis. The  
 1345 classical weight subspace analysis is more computationally expensive relative to the LoRA one due  
 1346 to high dimensionality of the parameters, but in effect, same. Additionally, the number of sufficiently  
 1347 well trained models is understandably fewer than LoRA models. Further, there is also higher variance  
 1348 in terms of model quality in the classical weights as it is harder to optimize these models as compared  
 1349 to LoRA which often are optimized from a good initialization point (the pretrained base model). An  
 outcome of this is that the universal subspace approximation that we obtain from the publicly

1350 available pretrained models are noisier than their LoRA counterparts. Inspite of this, our universal  
 1351 subspace hypothesis remains validated.  
 1352

1353 To further support our universal subspace hypothesis, we extend our analysis beyond adapter models  
 1354 to standard full-rank weights. We exclude convolutional parameters from explicit consideration, as  
 1355 they are functionally equivalent to fully connected layers under certain conditions (Ma & Lu, 2017),  
 1356 and their learned representations (e.g., Gabor-like filters) have been studied, in limited scope, in prior  
 1357 work (Krizhevsky et al., 2012). Consequently, our analysis generalizes naturally to convolutional  
 1358 weights as well.  
 1359

1360 There are, however, practical differences between the subspace analysis of full-rank model weights  
 1361 and that of low-rank adapters. First, analyzing conventional weight matrices is significantly more  
 1362 computationally intensive because of their higher dimensionality. Second, the availability of a large  
 1363 number of independently and sufficiently well-trained models is more limited compared to LoRA  
 1364 models. Third, the classical weight models exhibit greater variance in model quality, since they must  
 1365 be trained from scratch, often without the benefit of a well-optimized initialization, unlike LoRA  
 1366 which builds upon a strong pretrained base.  
 1367

1368 As a result, the subspaces estimated from classical weights tend to be noisier, and the universality  
 1369 signal is less pronounced. Despite these challenges, we still observe consistent structure in the leading  
 1370 components, lending further empirical support to the universal subspace hypothesis.  
 1371

1372 **Universal ViT-base-patch16-224 experiment details** We collect  $\sim 500$  pretrained ViT models  
 1373 from HuggingFace, shown in Table 9, spanning very diverse domains — many of which would  
 1374 be considered orthogonal to one another in terms of domain generalization. These models have  
 1375 been trained with varying losses, optimizers, and initializations. These models were used as-is,  
 1376 without curation or access to training data, to reflect real-world variability. Figure 10 shows the  
 1377 summarized scree plot for all relevant layers of ViT (sans first and last layers due to differences in  
 1378 shape and tasks) for all  $\sim 500$  ViT models showing that the majority of variance is captured by the  
 1379 top 16 principal directions, revealing a highly compressible, shared subspace across layers. Only  
 1380 the top 100 components are visualized for clarity, although the available subspace is significantly  
 1381 larger, underlying the sparsity of this universal subspace. We observe this for layerwise analysis in  
 1382 Figure 11 as well. For the experimental results presented in Table 3, we randomly choose 4-5 IID  
 1383 and 4-5 OOD models from Table 9 for which evaluation dataset is available, and reconstruct these  
 1384 model weights by projecting them into our 16 component universal subspace. For the OOD case, we  
 1385 ensure that the models being evaluated are not present in the subset used for creating the universal  
 1386 subspace approximation. As seen from the results, our extremely sparse subspace model performs  
 1387 competitively compared to the fully trained versions. It is likely that with more careful choice of  
 1388 principal directions per layer would allow for at par or even better performance.  
 1389

1390 Table 9: Finetuned Models from HuggingFace used for the Universal Vision Transformer subspace  
 1391 extraction (vit-base-patch16-224)  
 1392

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|--|---|
| 0.50-200Train-100Test-vit-base           | 2025-01-21-16-13-04-vit-base-patch16-224                  |
| 2025-02-05-14-22-36-vit-base-patch16-224 | 21BAI1229   |
| Accomodation_room_classification         | adam_ViT-B-p16-224-1e-4-batch_16_epoch_4_classes_24       |
| age_face_detection_base                  | AlvisionGuard-v2  |
| alea                                     | amns  |
| AnimeCharacterClassifierMark1            | autotrain-48ci8-roib9                                     |
| autotrain-80qr6-image0807-20             | autotrain-ap-pass-fail-v1                                 |
| autotrain-g2g80-iwcfm                    | autotrain-google-vit-13epoch                              |
| autotrain-ht4es-gbvtm                    | autotrain-image-classifier-cats-and-dogs                  |
| autotrain-phnku0-076h9                   | autotrain-80sds-erede                                     |
| autotrain-test-image-classification      | autotrain-vit-base-patch16-224-fog-or-smog-classification |
| beauty-ornot                             | beer-classifier   |
| bg-classif                               | bigger-chord-finetuned                                    |
| brain-tumor-44                           | ButterflyClasifModel                                      |
| camera-type                              | Caracam   |
| cards-vit-base-patch16-224-finetuned-v1  | carmodel  |
| cats123                                  | cats-dogs-2024  |
| cats-dogs-classification                 | CheXpert-ViT-U-MultiClass                                 |
| CheXpert-ViT-U-SelfTrained               | chord-final-model   |
| chord_ViT-finetuned                      | cifar10-lt  |
| city_multiclass_classification           | clasificador_masas  |
| corals_binary_classification             | custom  |
| detect_meme                              | dog-breeds-classification                                 |
| dog-cat-demo-20240815                    | dog-cats-model  |
| dummy_classification_model               | dvm-cars-vit-first-5k                                     |
| ecg-image-multilabel-classification      | emotion   |
| EmotionAgeModel                          | emotion_model   |
| emotion-recognition                      | emotion_recognition                                       |

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|---|---|
| emotion_recognition_results                                       | emotion-vit   |
| face_age_detection_base_v2  | face_age_detection_base_v3_weighted                                 |
| final-run   | finetune-cats   |
| fine-tuned  | finetuned-amazon  |
| fine-tuned-augmented  | finetuned-bin   |
| finetuned-cifar10   | finetuned-indian-food   |
| fine-tuned-model  | finetuned_model   |
| Fine-Tuned_Model  | Fine-Tuned_Model2   |
| Fine-Tuned_Model3   | Fine-Tuned_Model3_Transfer_learning                                 |
| finetune-vit-base-patch16-224                                     | finetune_vit_base_patch16_224_1epoch                                |
| Flowers   | food  |
| food-101-finetuned-model  | Freshness-Fruit_Veggies   |
| frost-vision-v2-google_vit-base-patch16-224                       | frost-vision-v2-google_vit-base-patch16-224-v2024-11-09             |
| frost-vision-v2-google_vit-base-patch16-224-v2024-11-11           | frost-vision-v2-google_vit-base-patch16-224-v2024-11-14             |
| fruit_classification  | fruits-360-16-7   |
| ft_stable_diffusion   | gender  |
| giecom-vit-model-classification-waste                             | google-vit-base-patch16-224-batch32-lr0.0005-standford-dogs         |
| google-vit-base-patch16-224-batch32-lr0.005-standford-dogs        | google-vit-base-patch16-224-batch32-lr5e-05-standford-dogs          |
| google-vit-base-patch16-224-batch64-lr0.005-standford-dogs        | google-vit-base-patch16-224-OrganicAndInorganicWaste-classification |
| google-vit-base-patch16-224-Waste-O-I-classification              | hf_vit_format_hap_pretrained_256_128                                |
| Human-Action-Recognition-VIT-Base-patch16-224                     | human-actions   |
| image-classification  | image_classification  |
| image_strawberry-peach_classifier                                 | isa-vit_model   |
| lixg_food_model001  | Maggi-Parle-G_Classifier  |
| mammals_multiclass_classification                                 | MemeDetector  |
| model   | Model   |
| model-vit-base-finetuned  | MRI_vit   |
| my_chest_xray_model   | myclass   |
| my_classification   | MyPetModel  |
| out   | outputs   |
| PagesClassificationModel  | physiotherapy-E2  |
| plant_disease_detection-beans                                     | pokemon_classification  |
| pokemon_model   | pokemon-vit   |
| recaptcha   | recycled_waste_classification                                       |
| results   | rmsprop_VitB-p16-224-2e-4-batch_16_epoch_4_classes_24               |
| rmsprop_VitB-p16-224-2e-4-batch_16_epoch_4_classes_24             | road-conditions   |
| rose_recognition  | rotated2  |
| Ruster  | S1_M1_R1_vit_42498800   |
| S1_M1_R1_vit_42509509   | S1_M1_R1_Vit_42616100   |
| S1_M1_R2_vit_42498972   | S1_M1_R2_ViT_42618476   |
| S1_M1_R3_vit_42499444   | S1_M1_R3_ViT_42618486   |
| S2_M1_R1_vit_42499480   | S2_M1_R1_ViT_42618522   |
| S2_M1_R2_vit_42499499   | S2_M1_R2_ViT_42618530   |
| S2_M1_R3_vit_42499514   | S2_M1_R3_ViT_42618549   |
| S5_M1_fold1_vit_42499955  | S5_M1_fold1_ViT_42618571  |
| S5_M1_fold2_vit_42499968  | S5_M1_fold2_ViT_42618583  |
| S5_M1_fold3_vit_42499983  | S5_M1_fold3_ViT_42618589  |
| S5_M1_fold4_vit_42499997  | S5_M1_fold4_ViT_42618593  |
| S5_M1_fold5_vit_42500027  | S5_M1_fold5_ViT_42621111  |
| Screenshots_detection_to_classification                           | sign-lan-model  |
| square_run_32_batch   | square_run_age_gender   |
| square_run_first_vote_full_pic_50                                 | square_run_first_vote_full_pic_50_age_gender                        |
| square_run_first_vote_full_pic_75                                 | square_run_first_vote_full_pic_75_age_gender                        |
| square_run_second_vote  | square_run_second_vote_full_pic_50                                  |
| square_run_second_vote_full_pic_50_age_gender                     | square_run_second_vote_full_pic_75                                  |
| square_run_second_vote_full_pic_75_age_gender                     | square_run_second_vote_full_pic_age_gender                          |
| square_run_second_vote_full_pic_stratified                        | square_run_square_run_first_vote_full_pic_25                        |
| square_run_square_run_first_vote_full_pic_25_age                  | square_run_square_run_first_vote_full_pic_25_age_gender             |
| square_run_square_run_first_vote_full_pic_25_age_gender_double_ch | square_run_square_run_second_vote_full_pic_25                       |
| square_run_square_run_second_vote_full_pic_25_age_gender          | square_run_with_16_batch_size                                       |
| square_run_with_actual_16_batch_size                              | stool-condition-classification                                      |
| swaddling-classifier  | swin-tiny-patch4-window7-224-finetuned-eurosat-kornia               |
| tarread   | telidermai  |
| test-cifar-10   | traffic-levels-image-classification                                 |
| Train-Augmentation-vit-base                                       | trainer_output  |
| Train-Test-Augmentation-V3D-vit-base                              | UL_base_classification  |
| UL_bedroom_classification   | UL_exterior_classification  |
| UL_interior_classification  | vehicle_multiclass_classification                                   |
| ViT_ASVspoof_DF   | vit-augmentation  |
| vit-b16-plant_village   | vit_base  |
| vit-base-1e-4-15ep  | vit-base-1e-4-20ep  |
| vit-base-1e-4-randaug   | vit-base-1stGen-Pokemon-Images                                      |
| vit-base-25ep   | vit-Base-30VN   |
| vit-base-3e-5-randaug   | vit-base-5e-4   |
| vit-base-add-2-decay  | vit-base-augment  |
| vit-base-batch-32   | vit-base-beans  |
| vit-base-brain-mri  | vit-base-cat_or_dog   |
| vit-base-change-arg   | vit-base-cocoa  |
| ViT-Base-Document-Classifier                                      | vit-base-fashion  |
| vit-base-finetuned-cephalometric                                  | vit-base-food101  |
| vit-base-fruits-360   | vit-base-hate-meme  |
| vit-base-nationality  | vit-base-org-plot   |
| vit-base-oxford-brain-tumor                                       | vit-base-oxford-brain-tumor_try_stuff                               |
| vit-base-oxford-brain-tumor_x-ray                                 | vit-base-oxford-iiit-pets   |
| vit-base-oxford-pets-krasuluk                                     | vit-base-patch16-224  |

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| vit-base-patch16-224-13_model  | vit-base-patch16-224-30-vit   |
| vit-base-patch16-224-9models   | vit-base-patch16-224-abhi1-finetuned                                    |
| vit-base-patch16-224_augmented-v2_fft                                      | vit-base-patch16-224_augmented-v2_t1                                    |
| vit-base-patch16-224-blur_vs_clean   | vit-base-patch16-224-brand  |
| vit-base-patch16-224-classifier  | vit-base-patch16-224-clothes-filter                                     |
| vit-base-patch16-224-cl-v1   | vit-base-patch16-224-crochets-clothes-classification                    |
| vit-base-patch16-224-Diastar   | vit-base-patch16-224-Diastarallclasses                                  |
| vit-base-patch16-224-dmae-va-U   | vit-base-patch16-224-dmae-va-U5-100-iN                                  |
| vit-base-patch16-224-dmae-va-U5-10-45-5e-05                                | vit-base-patch16-224-dmae-va-U5-20-45-5e-05                             |
| vit-base-patch16-224-dmae-va-U5-40-45-5e-05                                | vit-base-patch16-224-dmae-va-U5-42B                                     |
| vit-base-patch16-224-dmae-va-U5-42C  | vit-base-patch16-224-dmae-va-U5-42D                                     |
| vit-base-patch16-224-ethos   | vit-base-patch16-224-ethos-25   |
| vit-base-patch16-224-ethos-8   | vit-base-patch16-224-ethos-data   |
| vit-base-patch16-224-ethosreaddata   | vit-base-patch16-224-fatigue  |
| vit-base-patch16-224-finalterm   | vit-base-patch16-224-finetuned  |
| vit-base-patch16-224-finetuned-barkley                                     | vit-base-patch16-224-finetuned-brain-tumor-classification               |
| vit-base-patch16-224-finetuned-Brain-Tumor-Classification                  | vit-base-patch16-224-finetuned-cassava-leaf-disease                     |
| vit-base-patch16-224-finetuned-cedar                                       | vit-base-patch16-224-finetuned-cifar10                                  |
| vit-base-patch16-224-finetuned-combinedSpiders                             | vit-base-patch16-224-finetuned-context-classifier                       |
| vit-base-patch16-224-finetuned-covid_ct_set_full                           | vit-base-patch16-224-finetuned-covid_ct_set_resumed                     |
| vit-base-patch16-224-finetuned-crochets-clothes                            | vit-base-patch16-224-finetuned-dangerousSpiders                         |
| vit-base-patch16-224-finetuned-eurosat                                     | vit-base-patch16-224-finetuned-feature-maps-v3                          |
| vit-base-patch16-224-finetuned-feature-map-v2                              | vit-base-patch16-224-finetuned-fibre                                    |
| vit-base-patch16-224-finetuned-flower                                      | vit-base-patch16-224-finetuned-flower-classify                          |
| vit-base-patch16-224-finetuned-flowers                                     | vit-base-patch16-224-finetuned-food101                                  |
| vit-base-patch16-224-finetuned-food102                                     | vit-base-patch16-224-finetuned-foveated-features                        |
| vit-base-patch16-224-finetuned-foveated-features-v2                        | vit-base-patch16-224-finetuned-galaxy10-decals                          |
| vit-base-patch16-224-finetuned-hateful-meme-structured                     | vit-base-patch16-224-finetuned-hateful-meme-structured-balanced         |
| vit-base-patch16-224-finetuned-imagegpt                                    | vit-base-patch16-224-finetuned-ind-17-imbalanced-aadhaarMask            |
| vit-base-patch16-224-finetuned-ind-17-imbalanced-aadhaarMask-new-parameter | vit-base-patch16-224-finetuned-landscape-test                           |
| vit-base-patch16-224-finetuned-lora-oxford-pets                            | vit-base-patch16-224-finetuned-masked-hateful-meme-structured           |
| vit-base-patch16-224-finetuned-noh   | vit-base-patch16-224-finetuned-original-images                          |
| vit-base-patch16-224-finetuned-pneumonia-detection                         | vit-base-patch16-224-finetuned-polyterrasse                             |
| vit-base-patch16-224-finetuned-skin  | vit_base_patch16_224-finetuned-SkinDisease                              |
| vit-base-patch16-224-finetuned-teeth_dataset                               | vit-base-patch16-224-finetuned-trash-classifications-albumentations     |
| vit-base-patch16-224-finetuned-turquoise                                   | vit-base-patch16-224-finetuned-Visual-Emotional                         |
| vit-base-patch16-224-finetuned-vit   | vit-base-patch16-224-finetune_test                                      |
| vit-base-patch16-224-food101-16-7  | vit-base-patch16-224-food101-24-12                                      |
| vit-base-patch16-224-for_evaluation  | vit-base-patch16-224-fruits-360-16-7                                    |
| vit-base-patch16-224-high-vit  | vit-base-patch16-224-jvadamludi2  |
| vit-base-patch16-224-masaratti   | vit-base-patch16-224-mascotas   |
| vit-base-patch16-224-mascotas-DA   | vit-base-patch16-224-MSC-dmae   |
| vit-base-patch16-224-newly-trained   | vit-base-patch16-224-oxford-pets-classification                         |
| vit-base-patch16-224-perros-y-gatos  | vit-base-patch16-224-pure-VIT   |
| vit-base-patch16-224-R1-10   | vit-base-patch16-224-R1-40  |
| vit-base-patch16-224-Rado_5  | vit-base-patch16-224_rice-disease-02                                    |
| vit-base-patch16-224_rice-leaf-disease-augmented_fft                       | vit-base-patch16-224_rice-leaf-disease-augmented_tl                     |
| vit-base-patch16-224_rice-leaf-disease-augmented-v4_fft                    | vit-base-patch16-224_rice-leaf-disease-augmented-v4_tl                  |
| vit-base-patch16-224_rice-leaf-disease-augmented-v4_v5_fft                 | vit-base-patch16-224_rice-leaf-disease-augmented-v4_v5_pft              |
| vit-base-patch16-224-rotated-dungeons-v101                                 | vit-base-patch16-224-rotated-dungeons-v103                              |
| vit-base-patch16-224-RU2-10  | vit-base-patch16-224-RU2-40   |
| vit-base-patch16-224-RU3-10  | vit-base-patch16-224-RU3-40   |
| vit-base-patch16-224-RU4-10  | vit-base-patch16-224-RU4-40   |
| vit-base-patch16-224-RU5-10  | vit-base-patch16-224-RU5-10-8   |
| vit-base-patch16-224-RU5-40  | vit-base-patch16-224-RU9-24   |
| vit-base-patch16-224-RX1-24  | vit-base-patch16-224-RX2-12   |
| vit-base-patch16-224-RXL1-24   | vit-base-patch16-224-type   |
| vit-base-patch16-224-U6-10   | vit-base-patch16-224-U7-10  |
| vit-base-patch16-224-U8-10   | vit-base-patch16-224-U8-10b   |
| vit-base-patch16-224-U8-10c  | vit-base-patch16-224-U8-40  |
| vit-base-patch16-224-U8-40b  | vit-base-patch16-224-U8-40c   |
| vit-base-patch16-224-U8-40d  | vit-base-patch16-224-ve-b-U10-12  |
| vit-base-patch16-224-ve-b-U10-24   | vit-base-patch16-224-ve-b-U10-40  |
| vit-base-patch16-224-ve-U10-12   | vit-base-patch16-224-ve-U10-24  |
| vit-base-patch16-224-ve-U11-12   | vit-base-patch16-224-ve-U11-b-24  |
| vit-base-patch16-224-ve-U11-b-40   | vit-base-patch16-224-ve-U11-b-80  |
| vit-base-patch16-224-ve-U12-b-24   | vit-base-patch16-224-ve-U12-b-80  |
| vit-base-patch16-224-ve-U13-b-120  | vit-base-patch16-224-ve-U13-b-24  |
| vit-base-patch16-224-ve-U13-b-80   | vit-base-patch16-224-ve-U13b-80R  |
| vit-base-patch16-224-ve-U13b-80RX  | vit-base-patch16-224-ve-U13b-80RX1                                      |
| vit-base-patch16-224-ve-U13b-80RX3   | vit-base-patch16-224-ve-U13b-R  |
| vit-base-patch16-224-ve-U14-b-24   | vit-base-patch16-224-ve-U15-b-80  |
| vit-base-patch16-224-ve-U16-b-80   | vit-base-patch16-224-ve-Ub  |
| vit-base-patch16-224-vit   | vit-base-patch16-224-vit-base-patch16-224-vit-base-patch16-224-dogORnot |
| vit-base-pets  | vit-base-PICAI  |
| vit-base-seed-1e-4   | vit-base-seed-3e-4  |
| vit-base-travel-document-classification                                    | vit-base-v1-eval-epoch-maxgrad-decay-cosine                             |
| vit-beans-classifier   | vit-beta1-0.85  |
| vit-beta1-0.88   | vit-beta1-0.95  |
| vit-beta2-0.99   | vit-beta2-0.995   |
| vit-beta2-0.9995   | vit-bird  |

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|---|--|
| ViT_bloodmnist                                  | ViT_bloodmnist_std_0                                 |
| ViT_bloodmnist_std_15                           | ViT_bloodmnist_std_30                                |
| ViT_bloodmnist_std_45                           | ViT_bloodmnist_std_60                                |
| ViT_breastmnist                                 | ViT_breastmnist_std_0                                |
| ViT_breastmnist_std_15                          | ViT_breastmnist_std_30                               |
| ViT_breastmnist_std_45                          | ViT_breastmnist_std_60                               |
| VIT-cats-vs-dogs                                | vit-cifar10-fine-tuned                               |
| vit-class-weight                                | vit-cxr4   |
| vit-demo  | ViT_dog_food   |
| vit-dropout-0.2                                 | vit-dropout-0.3                                      |
| vit-dropout-0.4                                 | vit-dropout-0.5                                      |
| vit-ds-processed                                | vit-emotion-model                                    |
| vit-epsilon-1e-7                                | vit-epsilon-1e-9                                     |
| vit-epsilon-5e-9                                | vit-face-project-piyush                              |
| vit-fine-tune-classification-cats-vs-dogs       | vit-finetuned-1                                      |
| vit-food-classification-chrisis2                | vit-geometric-shapes-base                            |
| vit-google-model-30-classes                     | vit_google_vehicle_classification_model              |
| vit-historical-page                             | vit_Liveness_detection_v1.0                          |
| vit-lr-0.0001                                   | vit-lr-0.001   |
| vit-lr-0.01                                     | vit-lr-cosine-restarts                               |
| vit-lr-cosine-warm-restarts                     | vit-lr-cosine-warmup                                 |
| vit-lr-exponential                              | vit-lr-inverse-sqrt                                  |
| vit-lr-linear                                   | vit-lr-poly  |
| vit-lr-reduce-plateau                           | vit-lr-step  |
| vit-mae-base-finetuned-eurosat                  | vit-molecul  |
| vit-ori-dataset-exp                             | vit-plant-classification                             |
| vit-plantnet300k                                | vit-plants   |
| vit-real-fake-classification-v1                 | vit-real-fake-classification-v2                      |
| vit-real-fake-classification-v3                 | vit-real-fake-classification-v4                      |
| vit-skin-demo-v1                                | vit-skin-demo-v2                                     |
| vit-skin-demo-v3                                | vit-skin-demo-v4                                     |
| vit-skin-demo-v5                                | vit-spam   |
| vit-sports-cls                                  | vit-transfer-learning                                |
| vit_transformer_eye_disease                     | vit_tumor_classifier                                 |
| vit-vit   | vit-vit-base-patch16-224-finetuned-chest-xray        |
| vit-weight-decay-1e-2                           | vit-weight-decay-1e-3                                |
| vit-weight-decay-1e-4                           | vit-weight-decay-1e-5                                |
| wmc_v2_vit_base_wm811k_cls_contra_learning_0916 | wmc_v2_vit_base_wm811k_cls_contra_learning_0916_9cls |
| wmc-wmk811-v0-vit-special_map_det_0917          | WS800_ViT_42895082                                   |
| WS800_ViT_42895082                              | xraynewww  |
| yet-another-amber-mines                         | zdravJEM_CV_BERT                                     |

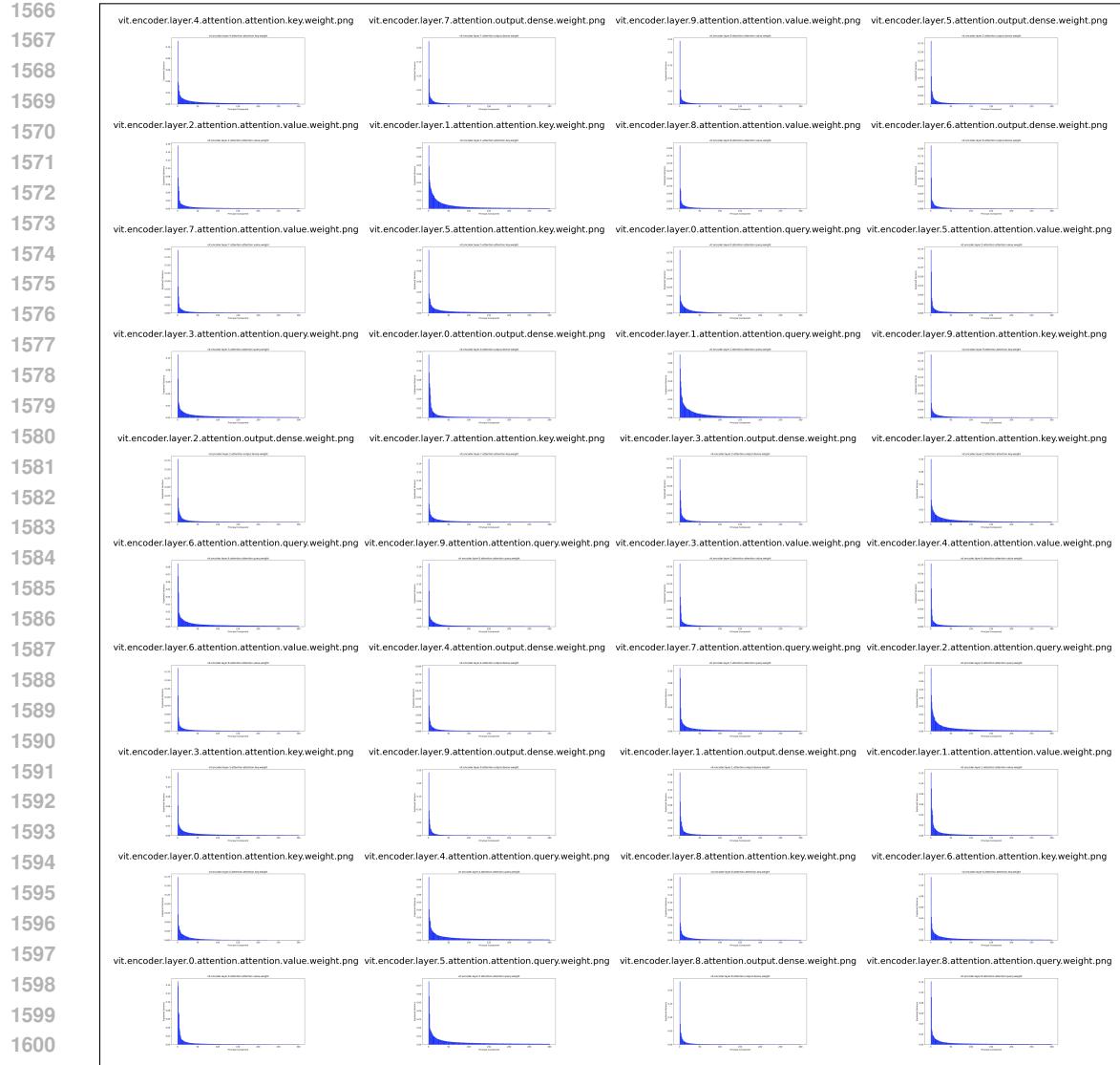


Figure 11: Layerwise Eigenvalue Plots of 500 ViT models.

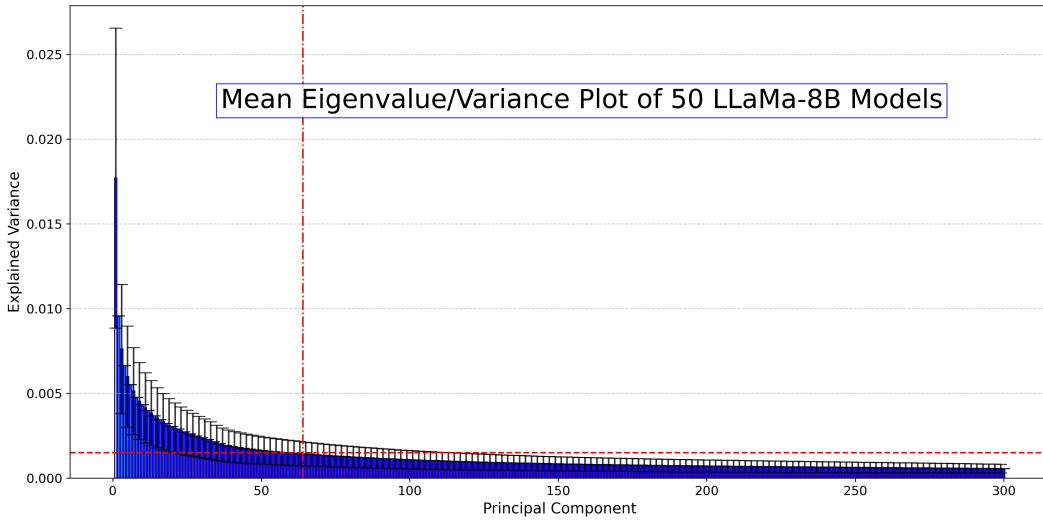
1605 **Universal LLaMA3-8B Experiment Details** To further stress-test our universal subspace hypothesis  
 1606 on classical weight matrices, we extract a shared subspace from approximately 50 finetuned  
 1607 LLaMA3 models, each with 8 billion parameters. These models were obtained from publicly available  
 1608 repositories on HuggingFace. Due to their scale, we do not apply any model selection or filtering,  
 1609 and instead include the entire available set.

1610 As shown in Figure 12, which presents the aggregated scree plot across all layers and all 50 models,  
 1611 the principal variance is concentrated in the top few components—consistent with the emergence of a  
 1612 low-rank universal subspace. For reference, the plot displays only the top 300 components, which  
 1613 represent a small fraction of the full rank, highlighting the inherently low-dimensional structure.

1614 The models included in this analysis span a diverse range of domains, including medical applications,  
 1615 multilingual dialogue systems, and general-purpose assistants, as listed in Table 10. To the best of  
 1616 our knowledge, this is the first work to demonstrate that such a large and heterogeneous collection of  
 1617 high-capacity language models can be jointly represented within a single low-rank subspace.

1618 The layerwise spectral analysis, shown in Figure 13, corroborates this finding: across all layers,  
 1619 the majority of eigenvalues fall below a threshold of  $< 0.001$ , indicating that most directions in

1620 parameter space contribute negligibly to variation across models. The plots are cropped to show only  
 1621 the leading components due to the large number of total dimensions. We recommend zooming in for  
 1622 clearer visualization.  
 1623



1641 Figure 12: Spectral analysis of 50 LLaMA-3-8B model: Aggregated eigenvalue (scree) plot across 50  
 1642 LLaMa-8B models and all layers. The plot demonstrates that the majority of the variance is consistently  
 1643 captured by few top principal directions, indicating the presence of a shared low-dimensional universal  
 1644 subspace.  
 1645

1646 Table 10: Models from HuggingFace used for the Universal LLaMa3-8B subspace extraction  
 1647

|                                      |   |   |                                    |
|--------------------------------------|---|---|------------------------------------|
| Meta-Llama-3-8B-Instruct-Jailbroken  | Llama-3-13B-Instruct                        | large_crafting_sft_success                          | suzume-llama-3-8B-multilingual     |
| summary-llama3-8B-T16-full           | Llama-3-13B-Instruct-v0.1                   | Llama-3-8B-ProLong-64k-Base                         | LLaMAntino-3-ANITA-8B-Inst-DPO-ITA |
| ai-medical-model-32bit               | filtered_crafting_train_data_shorter_length | Llama-3-portuguese-Tom-cat-8b-instruct              | Llama-3-MAAL-8B-Instruct-v0.1      |
| Human-Like-LLama3-8B-Instruct        | LLaMA-3-8B-Instruct-TR-DPO                  | CabraLlama3-8B                                      | chartgpt-llama3                    |
| KoLlama-3-8B-Instruct                | honeypot-llama3-8B                          | Llama-SEA-LION-v2-8B                                | TR                                 |
| Llama3-8B-Instruct-Turkish-Finetuned | Llama-3-15B-Instruct-zeroed                 | Llama-3-8B-Instruct-TAR-Bio-v2                      | Bio-Medical-Llama-3-8B             |
| filtered_construction_train_data     | shisa-v1-llama3-8B                          | REFUEL-Llama-3-Armo-iter_1                          | llama3-instrucTrans-enko-8b        |
| Llama-3-8B-Instruct-Ja               | llama3-paststhrough-chat                    | RoLlama3-8B-Instruct                                | Llora-SQL                          |
| Summary_L3_1000steps_1e7rate_SFT2    | CyberSentinel                               | Meta-Llama-3-8B-Instruct-function-calling-json-mode | MARS                               |
| Llama-3-8B-Instruct-Finance-RAG      | LLaMA3-Instruct-8B-FR-Spec                  | Llama-3-8B-Japanese-Instruct                        | Llama3-8B-Chinese-Chat             |
| llama-3-chinese-8b-instruct-v2       | Athena-RM-8B                                | Llama-3-OffsetBias-RM-8B                            | large_cooking_sft_success          |
| suzume-llama-3-8B-japanese           | llama-3-chinese-8b-instruct-v3              | Waktaverse-Llama-3-KO-8B-Instruct                   | llama-3-8b-gpt-4o-ru1.0            |
| Llama-3-ApIte-Instruct-4x8B-MoE      | Llama-3-8B-Instruct-DPO-v0.3                |   |                                    |

1655  
 1656 **Universal Flan-T5 Experiment Details** We collected Flan-T5 models fine-tuned on individual  
 1657 datasets from the GLUE (Wang et al., 2019) benchmark. We extract the joint subspace from these  
 1658 models and trends similar to those observed above are seen. This shows that across diverse datasets  
 1659 and tasks a low-rank subspace emerges.  
 1660

1661 Table 11: Finetuned Flan-T5 Models from HuggingFace used for the Universal Flan-T5 subspace  
 1662 extraction  
 1663

|                                 |                                 |
|---------------------------------|---------------------------------|
| tanganke/flan-t5-base_glue-cola | tanganke/flan-t5-base_glue-mnli |
| tanganke/flan-t5-base_glue-mrpc | tanganke/flan-t5-base_glue-qnli |
| tanganke/flan-t5-base_glue rte  | tanganke/flan-t5-base_glue-qqp  |
| tanganke/flan-t5-base_glue-sst2 | tanganke/flan-t5-base_glue-stsb |

### B.3 ABLATING NUMBER OF MODELS AND SUBSPACE EFFECTIVENESS

1669 Although this is implicitly addressed through our large-scale experiments (500 ViTs, 500 Mistral-7B  
 1670 and 300 Stable Diffusion LoRAs, 50 LLaMA3-8B, 177 GPT-2s, Flan-T5, and ResNet50 models)  
 1671 in all Figures and Tables, which demonstrate consistent behavior at different scales. Theorem 2.5  
 1672 provides insights on the saturation dynamics where we see that the rate of convergence of the shared  
 1673 subspace to the true subspace is in the order  $O(1/T)$ , where  $T$  is the number of tasks, indicating  
 increasingly effective coverage as  $T$  increases. In practice, the minimum number of models per

1674 Table 12: Lots of LoRAs (Mistral-7B) OOD evaluation per increasing number of models used to  
 1675 extract Universal Subspace  
 1676

| 1677 | Method          | Model Number | Rouge-L Score |
|------|-----------------|--------------|---------------|
| 1678 | Normal Model    | -            | 73.7          |
| 1679 | Universal model | 50           | 55.8          |
| 1680 | Universal model | 150          | 66.1          |
| 1681 | Universal model | 250          | 71.9          |
| 1682 | Universal model | 450          | 72.3          |

1683

1684

1685 architecture needed to achieve saturation point depends on the quality of the trained models, the  
 1686 diversity of data they have been trained on, and on the architecture itself. Ablating these would  
 1687 require access to all the data for all the models, and very careful training on every training for each  
 1688 data, and then running permutation with all possible combinations of models. All of this is out  
 1689 of reach for most researchers simply due to time, data and compute constraints. We, however, do  
 1690 provide an initial ablation here. For LoRA models shown in Table 7, we choose 9 random (OOD)  
 1691 tasks (39, 190, 280, 290, 391, 442, 1342, 1391, 1598) and extract the Universal Subspace from rest of  
 1692 the tasks, sampled randomly for increasing number of models. The coefficients for OOD tasks  
 1693 are analytically reconstructed to effectively evaluate the universal subspace created from varying  
 1694 number of models. Table 12 shows that the adequate principal components are quickly extracted, and  
 1695 increasing the number of models has diminishing returns.

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## C FINDING UNIVERSAL SUBSPACES AND APPLYING THEM TO FUTURE TASKS

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1699

1700 In this section, we present two tasks, GLUE (Wang et al., 2019) and Image Classification. For each  
 1701 experiment, the joint subspace is created using all other models in subset. For Image Classification,  
 1702 we use  $k = 4$  and train only 8 epochs using learning rate of 1e-4. Importantly, only the coefficients  
 1703 are trained for the experiment. It is important to note that our shared subspace model performs  
 1704 quite well despite using very few (4-5) models to extract the subspace. For GLUE, we use 16-32  
 1705 components for our subspace, with learning rate of 4e-4, batch size of 64, and 30-80 epochs for each  
 1706 task. In addition, it is likely that our model might perform similarly or better if trained longer or with  
 1707 optimized hyperparameters.

1708

1709

**Compute Resources** We conduct all our experiments using a single A5000 GPU, and a CPU with  
 1710 8 workers. For the universal subspace extraction, all calculation can be done on the CPU. However,  
 1711 GPU would increase the speed of calculation as the layerwise subspace extraction can be parallelized.

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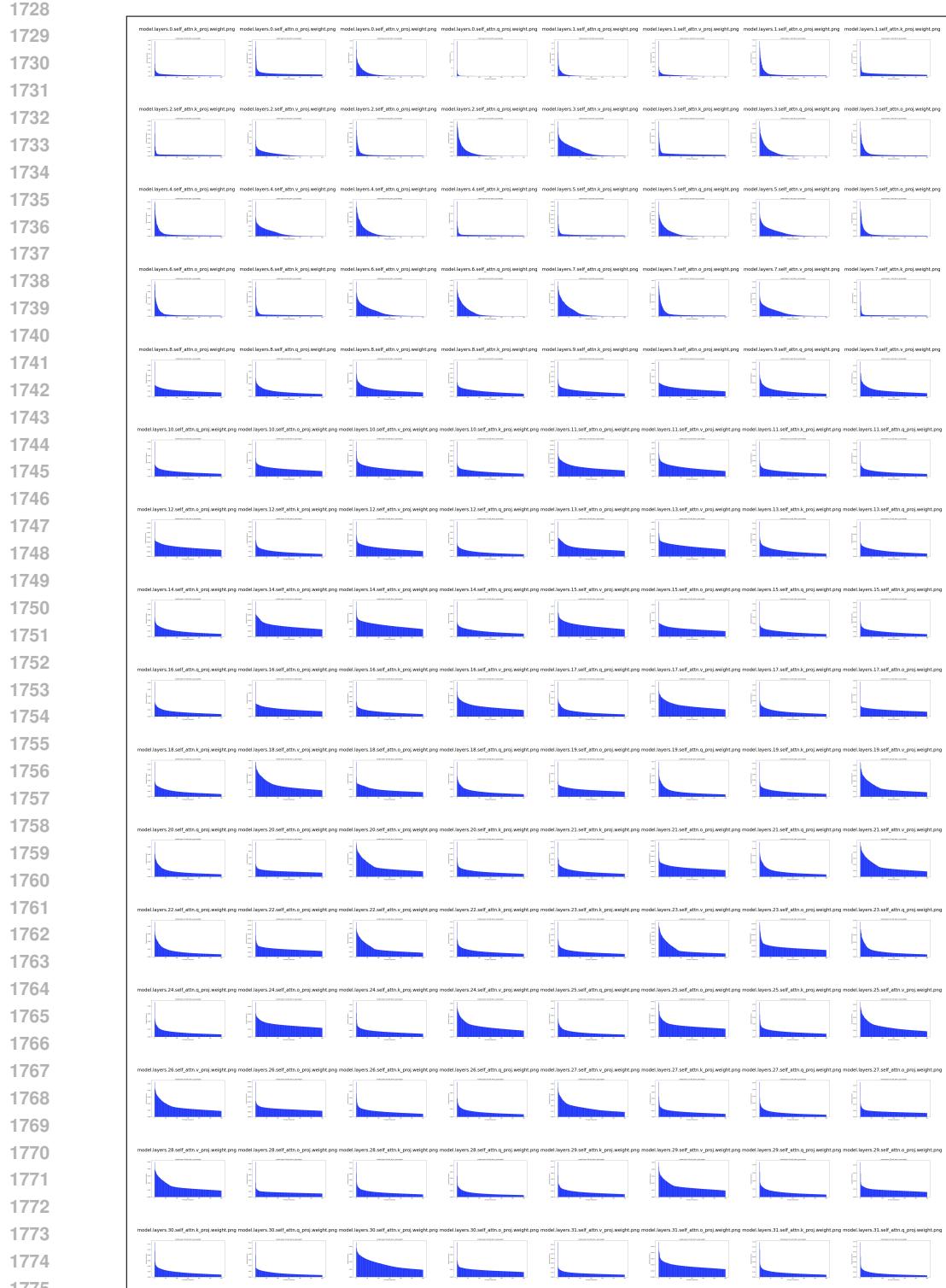


Figure 13: Layerwise Scree Plots for 50 LLaMA-3-8B Models. For enhanced clarity, each subplot presents a truncated view of the total possible principal directions. These plots consistently demonstrate that the dominant information, as represented by explained variance, resides within a small number of leading principal directions for all models. Components beyond this initial set are characterized by eigenvalues approaching zero, signifying their redundancy for the universal subspace.

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**D DISCUSSION AND BROADER IMPACT**

Our findings suggest that deep neural networks trained across diverse tasks and modalities systematically converge to shared, low-dimensional subspaces within their parameter space. The existence of such universal subspaces challenges conventional assumptions about the independence and diversity of model and task-specific finetuning trajectories. Instead, it highlights a powerful regularity in the way deep models encode task-specific knowledge - one that can be exploited for significantly improved training and deployment efficiency. By leveraging these subspaces, we demonstrate that models can be adapted to new tasks by learning only a small number of coefficients, rather than retraining or storing full sets of weights. This facilitates more robust multi-task learning, model merging, and scalable fine-tuning, with theoretical guarantees and empirical validation across multiple architectures.

The broader societal impact of this work is substantial. Our approach enables large-scale models to be reused and extended with dramatically reduced computational overhead, addressing both the financial and environmental costs associated with training and deploying deep learning systems. This contributes directly to the goals of sustainable and accessible AI. By lowering the hardware and energy requirements for adaptation and inference, we empower under-resourced researchers, institutions, and communities to build upon state-of-the-art models without needing extensive compute infrastructure. Furthermore, by supporting modular model design and data-free model merging, our work lays the foundation for more interpretable, maintainable, and equitable AI systems.