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Paper under double-blind review

ABSTRACT

Large Language Models (LLMs) achieve strong performance across diverse tasks but face prohibitive computational and memory costs. Pruning offers a promising path by inducing sparsity while preserving architectural flexibility. However, existing methods struggle to balance efficiency and robustness: local metric approaches prune layer by layer but often collapse under high sparsity, whereas global feedback methods enforce consistency at the cost of expensive weight updates or restrictive semi-structured formats. We present **UniPruning**, a unified post-training pruning framework that combines the speed of local saliency metrics with the stability of global coordination, enabled by a mirror descent based optimization, all **without updating model weights**. UniPruning leverages fast layer-wise scoring and a lightweight global controller to allocate a single sparsity budget, supporting both unstructured and semi-structured $N:M$ pruning within one framework. After a brief calibration, it can generate pruning masks for arbitrary sparsity levels in one shot, and adapts seamlessly to hardware-aware constraints. Extensive experiments on multiple pretrained LLM families and standard benchmarks show that UniPruning consistently delivers competitive or superior perplexity and zero-shot accuracy. Ablation studies further highlight the importance of mirror descent and local saliency anchoring. Overall, UniPruning provides an efficient, principled, and scalable solution for sparsifying large-scale LLMs.

1 INTRODUCTION

Large Language Models (LLMs) (Achiam et al., 2023; Touvron et al., 2023; Zhang et al., 2022) have redefined the frontier of natural language processing, achieving unprecedented capabilities across diverse tasks. Yet, their deployment at scale remains constrained by prohibitive computational and memory costs driven by their enormous parameter counts. To bridge this gap, model compression has emerged as a critical direction, with quantization (Lin et al., 2024), distillation (Gou et al., 2021), and pruning (Frantar & Alistarh, 2023) as key strategies. Among these, pruning stands out for its ability to induce sparsity while preserving architectural flexibility, thereby delivering substantial reductions in both memory footprint and computational demand.

Existing pruning paradigms for LLMs differ along two axes: **structural granularity and algorithmic coordination**. Structurally, pruning ranges from *unstructured pruning*, which removes individual weights for fine-grained control but suffers from limited hardware acceleration, to *structured pruning*, which eliminates entire channels or neurons to enable efficient execution on modern accelerators. *Semi-structured pruning* (Mishra et al., 2021), such as the widely adopted $N:M$ format, strikes a practical balance, enabling substantial sparsity with hardware-friendly patterns.

From an algorithmic perspective, pruning methods fall into two categories: local metric and global feedback. Local approaches, such as SparseGPT (Frantar & Alistarh, 2023) and Wanda (Sun et al., 2024), make layer-wise pruning decisions based on weight and activation statistics, offering simplicity but often failing under high sparsity due to ignored cross-layer dependencies. Global feedback methods address this by introducing model-wide coordination through regularization or mask learning, as seen in SparseLLM (Bai et al., 2024) and ProxSparse (Liu et al., 2025). While more consistent, these approaches can be computationally costly or restricted by specific sparsity formats.

In this paper, we introduce **UniPruning**, a unified pruning framework that combines the speed of local metrics with the consistency of global feedback, all without requiring weight updates. To integrate these two objectives, we adopt the mirror descent algorithm as a principled approach for joint optimization. UniPruning employs a fast, layer-wise scoring step to extract local evidence, coupled with a lightweight global controller that redistributes a single sparsity budget across layers using a mirror-descent projection (Beck & Teboulle, 2003; Nemirovsky & Yudin, 1983). Concretely, model weights evolve along a gradient flow while an auxiliary saliency variable Γ is updated under a sparsity-aware projection. This mechanism naturally supports both unstructured and semi-structured ($N:M$) pruning within one framework. After calibration, masks are generated by a single sorting operation on Γ and directly applied to the original pretrained weights, enabling one-shot extraction of multiple sparsity levels. To stabilize pruning decisions, UniPruning incorporates local saliency signals from a calibration set as robust local signals, while the global controller ensures balanced allocation across layers. This synergy yields pruning that is as efficient as local methods, yet globally consistent and structurally aware like feedback-based approaches.

To evaluate the effectiveness of our method, we conduct extensive experiments across a diverse set of large language models. We benchmark UniPruning under both unstructured and semi-structured sparsity regimes, comparing it against widely-used post-training pruning baselines. Our results show that UniPruning consistently achieves competitive or superior performance in terms of perplexity and zero-shot accuracy, especially under high sparsity levels. Notably, it maintains model stability where other methods degrade, and achieves strong average results across models and tasks—all while avoiding any weight updates during pruning. We also perform detailed ablation studies to validate the role of mirror descent and the choice of local saliency metrics. Our contributions are:

- **A unified view of local metric and global feedback pruning.** UniPruning offers a framework that keeps the simplicity of local metric, layer-wise evidence while coordinating a model-wide sparsity budget through a global regularizer and one-shot ranking. This unification maintains layer-level structure preservation and improves whole-model trade-offs under a common budget.
- **Mirror-descent pruning without weight update.** We extend mirror descent to LLM pruning by learning a saliency variable Γ jointly with weights and anchoring it to data-driven local saliency metrics. The same procedure supports both unstructured sparsity and semi-structured patterns. By avoiding weight updates and relying solely on learned saliency, the method remains lightweight, preserves accuracy, and is practical for scaling to large language models.
- **Extensive evaluation across models and sparsity.** We test UniPruning on multiple pretrained LLMs and standard benchmarks, comparing against previous state-of-the-art pruning baselines. The method consistently delivers strong accuracy at moderate-to-high sparsity while remaining efficient. Our results indicate that mirror-descent saliency, anchored to local metric, is a robust drop-in route for both unstructured and semi-structured ($N:M$) LLM sparsification.

2 RELATED WORK

LLM pruning. The tension between local efficiency and global coordination becomes even more pronounced when scaling pruning to large language models. On the one hand, lightweight local approaches such as Wanda (Sun et al., 2024), RIA (Zhang et al., 2024), stochRIA (Yi & Richtárik, 2025) show that activation-aware (relative activation-aware) scoring can prune both unstructured and semi-structured ($N:M$) patterns effectively, requiring only a small calibration set and minimal computation. On the other hand, methods like SparseGPT (Frantar & Alistarh, 2023) push pruning into the LLM regime by incorporating approximate second-order information: they prune many weights in one shot and apply a local least-squares correction to stabilize outputs. These advances confirm that post-training pruning can achieve strong efficiency–accuracy trade-offs at LLM scale, even without retraining. However, they remain fundamentally layer-local: each layer is pruned largely in isolation, with limited capacity to balance sparsity across the entire model. This gap highlights the central open question for LLM pruning: how to unify the speed and practicality of local methods with the robustness and balance of global coordination, especially under extreme sparsity or hardware-specific formats.

Mirror descent. Mirror descent (Nemirovsky & Yudin, 1983; Beck & Teboulle, 2003; Bubeck, 2015; Ding et al., 2025) is a general framework for constrained, geometry-aware optimization. It maps parameters into a dual space, takes gradient steps there, and projects back via a mirror map

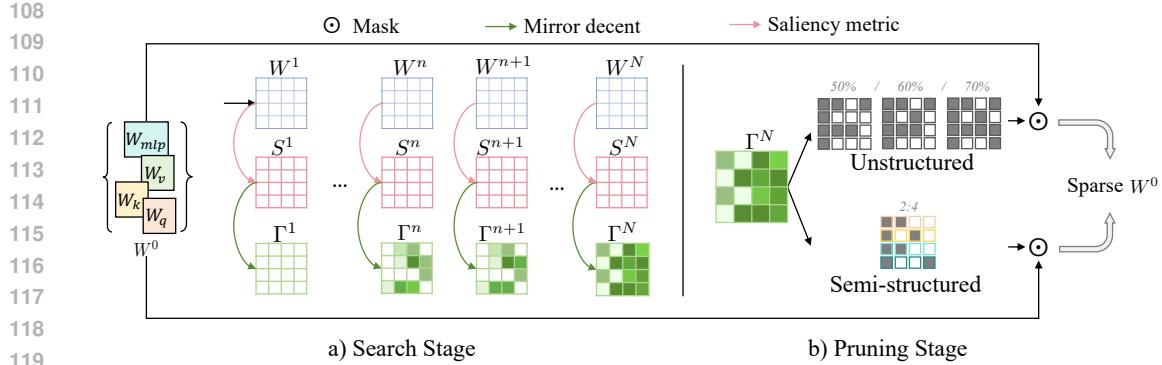


Figure 1: Overall framework of **Unified Pruning**. The framework targets pruning in two types of layers: **MLP layers** and **attention projection layers**. It operates in two stages (a) **Search Stage**: model weights W are iteratively updated while saliency variables Γ are jointly optimized with local metrics $S(W)$ via mirror descent, gradually accumulating pruning signals. (b) **Pruning Stage**: the final Γ^N is projected into unstructured or semi-structured sparsity masks, which are applied to the original pretrained weights W^0 to yield sparse models at arbitrary sparsity levels.

(Bregman projection). With a Euclidean map it reduces to projected gradient descent; with nonsmooth regularizers it yields proximal updates. This is particularly useful for pruning, where sparsity can be expressed as a regularizer (e.g., ℓ_1 , group, or block constraints) and enforced through a proximal step—stabilizing optimization while respecting structural constraints. Mirror descent also connects to continuous-time dynamics in differential-inclusion-based pruning, viewing proximal updates as discrete flows toward sparse sets. We build on this toolkit to unify local saliency with global sparsity budgets in an efficient, structure-aware pruning framework.

3 PROBLEM SETUP

Overview of Post-Training Pruning. Let W_0 be the pretrained parameters of an LLM and let $L(W)$ denote the task loss. Pruning seeks a sparse variable $\tilde{W} = M \odot W_0$ that preserves accuracy while reducing compute and memory, where $M \in \{0, 1\}^d$ is a mask with target sparsity $s \in [0, 1]$. We consider:

- Unstructured: elementwise masking with a global or per-layer budget.
- Semi-structured (N:M): in each contiguous M elements, at most N elements are kept.

A calibration set \mathcal{C} is typically used to guide the choice of M , ensuring that the pruned model behaves similarly to the dense model at the desired sparsity and structure. Such calibration sets are often drawn from common pretraining corpora such as C4 (Raffel et al., 2020), WikiText (Merity et al., 2016), or PTB (Marcinkiewicz, 1994).

Global feedback pruning. Global feedback pruning directs sparsity decisions using the model’s overall objective rather than isolated layer-wise signals. This avoids premature pruning, captures cross-layer dependencies, and produces sparser structures that better preserve performance. It uses a single, coordinated budget aligned with the model’s global objective, guiding pruning toward a near-optimal solution rather than suboptimal local choices. However, it has limitations: mask-based formulations are often restricted to specific patterns (e.g., fixed (N:M)); optimization can be complex (involving auxiliary variables, alternating solvers, or regularized mask learning); and each run typically targets a single sparsity level, requiring separate executions for different budgets.

Let \mathcal{D} measure the discrepancy between the pruned and dense models on \mathcal{C} , and let $\text{Cost}(M)$ be a global cost (e.g., nonzeros, FLOPs). A budgeted global objective is

$$\min_{M \in \mathcal{M}} \frac{1}{|\mathcal{C}|} \sum_{x \in \mathcal{C}} \mathcal{D}(f(x; W_0 \odot M), f(x; W_0)) \quad \text{s.t.} \quad \text{Cost}(M) \leq B, \quad (1)$$

162 **Algorithm 1** UniPruning: Mirror-Descent Pruning with Local Metric and Global Feedback

163

164 **Require:** Pretrained weights W_0 ; calibration set \mathcal{C} ; parameters $\rho > 0$, $\kappa > 0$; total steps N ; step sizes $\{\alpha_n\}_{n=0}^{N-1}$.

165

166 **Ensure:** Pruned weights $\widetilde{W}(B) = W_0 \odot \widehat{M}(B)$ for any global budget B .

167 1: **Local Saliency Statistics:** For each layer, run \mathcal{C} once to collect inputs X . Compute local metric $S(W, X)$.

168 2: **Initialize:** $W^0 \leftarrow W_0$, $\Gamma^0 \leftarrow 0$, $V^0 \leftarrow 0$.

169 3: **for** $n = 0$ **to** $N - 1$ **do**

170 4: $S(W^n, X) \leftarrow$ Local metric at current W^n ▷ Recompute local statistics every iteration

171 5: $g_{\text{task}} \leftarrow \nabla_W \mathcal{L}_{\text{task}}(W^n)$

172 6: $g_{\text{align}} \leftarrow \rho \cdot \nabla_W \frac{1}{2} \|\Gamma^n - S(W^n, X)\|_F^2$

173 7: $W^{n+1} \leftarrow W^n - \kappa \alpha_n \cdot (g_{\text{task}} + g_{\text{align}})$ ▷ Gradient step on W

174 8: **if** $N : M$ pruning **then**

175 9: $W^{n+1} \leftarrow \text{Prox}_{R_{2:4}}(W^{n+1})$

176 10: $R_{2:4}(w) = |w_1||w_2||w_3| + |w_2||w_3||w_4| + |w_3||w_4||w_1| + |w_4||w_1||w_2|$

177 11: $V^{n+1} \leftarrow V^n - \alpha_n \rho \cdot (\Gamma^n - S(W^n, X))$

178 12: $\Gamma^{n+1} \leftarrow \text{Prox}_{\Omega}(V^{n+1})$ ▷ Proximal update on Γ

179 13: $\Gamma^* \leftarrow \Gamma^N$ ▷ Final saliency scores

180

181 **Export (unstructured):** Sort $|\Gamma^*|$ once. For any global budget B , set threshold $\tau(B)$ to keep the top- B entries and define mask $\widehat{M}(B) = \mathbb{I}(|\Gamma^*| \geq \tau(B))$. Return $\widetilde{W}(B) = W_0 \odot \widehat{M}(B)$.

182 **Export ($N : M$):** In each block of size M , keep the top- N entries by $|\Gamma^*|$ and zero out the rest.

183 Return $\widetilde{W}_{NM} = W_0 \odot \widehat{M}_{NM}$.

186
187 or, in Lagrangian form,

$$\min_{M \in \mathcal{M}} \frac{1}{|\mathcal{C}|} \sum_{x \in \mathcal{C}} \mathcal{D}(f(x; W_0 \odot M), f(x; W_0)) + \lambda \text{Cost}(M). \quad (2)$$

191 Here, \mathcal{M} encodes the structure (unstructured or $N : M$). This captures the “one budget for the whole
192 model” view emphasized by global approaches.

193 **Local metric pruning.** Local metric pruning relies on simple heuristics, such as weight magnitude
194 or weight activation products, to prune parameters independently within each layer. This makes it
195 efficient and easy to apply without heavy global optimization or modifications to pretrained weights.
196 It naturally supports multiple sparsity patterns (unstructured and semi-structured) and can operate
197 in a single pass with only a small calibration set. However, this layer-wise independence comes
198 at a cost: it ignores cross-layer dependencies and trade-offs, which can lead to suboptimal sparse
199 structures and degraded model-level accuracy, particularly at high sparsity. Moreover, many criteria
200 remain heuristic and lack a unified optimization framework, limiting principled control over sparsity
201 allocation. So, its each layer ℓ selects a mask under its own budget B_ℓ , where g_ℓ represents the output
202 of layer ℓ :

$$\min_{M_\ell \in \mathcal{M}_\ell} \frac{1}{|\mathcal{C}|} \sum_{x \in \mathcal{C}} \mathcal{D}_\ell(g_\ell(x; W_{0,\ell} \odot M_\ell), g_\ell(x; W_{0,\ell})) \quad \text{s.t.} \quad \text{Cost}_\ell(M_\ell) \leq B_\ell. \quad (3)$$

203 **UniPruning.** Our method offers the best of both worlds by unifying global feedback and local metric
204 pruning within a mirror descent framework. It incorporates a lightweight, model-level controller that
205 dynamically allocates pruning budgets across layers and enforces target sparsity patterns through a
206 structured projection step. This design preserves the efficiency and flexibility of local pruning,
207 leverages the coordination of global feedback, and remains easily tunable across diverse sparsity
208 levels.

212 4 UNIPRUNING

213 We propose a mirror-descent pruning method that learns a saliency variable Γ together with a train-
214 able copy of the weights W (initialized from W_0). After training, we sort the final Γ once to derive

216 masks at any desired sparsity and apply those masks to W_0 . In this way, Γ acts as a data-driven
 217 pruning score. Because mask extraction is decoupled from training, the pretrained weights remain
 218 intact, which helps preserve performance without weight update.

219
 220 **4.1 ALGORITHM**
 221

222 **Local metric regularization.** We introduce a local metric regularizer that links the saliency score Γ
 223 to the current weights W by using a local importance metric $S(W, X)$, where X denotes the input
 224 statistics collected from a small calibration set \mathcal{C} . This map encodes the local significance of each
 225 weight based on its interaction with the input activations, assigning higher scores to connections
 226 associated with stronger or more frequently activated inputs.

227 This score increases the importance of weights connected to strongly activated inputs, while reducing
 228 the influence of weights tied to weak or rarely used inputs. The absolute value ensures the score
 229 reflects the strength of the connection regardless of its sign, focusing only on the magnitude of the
 230 weight and the scale of the input activation.

231 This design follows the local metric approach used in Wanda (Sun et al., 2024), and can also incorporate
 232 other local methods such as RIA (Zhang et al., 2024); additional experimental comparisons
 233 are provided in the appendix.

234 To align the learned saliency Γ with this local signal, we apply a simple alignment loss:
 235

$$236 \quad \|\Gamma - S(W, X)\|_F^2.$$

237 This encourages Γ to reflect meaningful, data-driven local importance metric without constraining
 238 the weights themselves, which remain free to be updated by the task loss.

239 **Objective.** We now describe the training objective used throughout the pruning stage. Let the task
 240 loss on \mathcal{C} be

$$242 \quad \mathcal{L}_{\text{task}}(W) = \frac{1}{|\mathcal{C}|} \sum_{x \in \mathcal{C}} \ell(f(x; W)).$$

244 We consider the composite energy

$$245 \quad \tilde{\mathcal{L}}_\rho(W, \Gamma) = \mathcal{L}_{\text{task}}(W) + \frac{\rho}{2} \|\Gamma - S(W)\|_F^2 + \Omega(\Gamma), \quad (4)$$

247 where Ω is a sparsity-inducing term. The second term injects local metric by aligning Γ with
 248 $S(W, X)$, ***ρ as a hyperparameter***; the third term imposes a global sparsity objective via Ω , guiding
 249 the gradual gradient updates to Γ and W with global feedback. Because Ω can be non-differentiable
 250 such as L_1 , we do not minimize equation 4 directly. Instead, we use a mirror-descent splitting that
 251 leads to the following dynamics.

252 **Dynamics and updates.** We now present the detailed algorithmic formulation of our proposed
 253 pruning method, UniPruning. During the sparsity training stage, we update the model weights W
 254 and the saliency scores Γ through a coupled dynamic process, ***with V acts as a conjugate variable (or***
 255 ***dual variable******.*** The weights W follow a smooth gradient descent, while Γ is updated via a proximal
 256 step guided by both local activation statistics and a global sparsity constraint.

$$257 \quad W^{k+1} = W^k - \kappa \alpha_k \left(\nabla_W \mathcal{L}_{\text{task}}(W^k) + \rho \nabla_W \frac{1}{2} \|\Gamma^k - S(W^k)\|_F^2 \right), \quad (5)$$

$$259 \quad V^{k+1} = V^k - \alpha_k \rho (\Gamma^k - S(W^k)), \quad (6)$$

$$260 \quad \Gamma^{k+1} = \text{Prox}_\Omega(V^{k+1}). \quad (7)$$

261 Here the proximal operator of Ω is

$$263 \quad \text{Prox}_\Omega(Z) = \arg \min_U \frac{1}{2} \|U - Z\|_F^2 + \Omega(U).$$

265 where ρ and κ are tunable hyperparameters, and $S(W)$ denotes a layer-wise saliency metric com-
 266 puted from activations. The procedure is initialized with $W^0 = W_0$, $\Gamma^0 = 0$, and $V^0 = 0$, and then
 267 proceeds iteratively as summarized in Algorithm 1.

268 The update on Γ in equation 7 yields a saliency map, not a trajectory we retain. During training,
 269 the alignment term pulls Γ_t toward the activation-aware scores $S(W_t)$, while the proximal map of Ω

enforces sparsity and structure. As optimization proceeds, entries that matter for the task loss grow in magnitude in Γ_t , and unimportant ones are pushed toward zero. After the dynamics stabilizes, we treat the final map Γ^* as a data-driven ranking of connections. Sorting and thresholding $|\Gamma^*|$ once produces masks at arbitrary sparsity levels, eliminating the need to retrain for each target.

After training converges, we discard all intermediate iterates and retain only the final Γ^* . This mapping is then used to derive pruning masks at arbitrary sparsity levels without further retraining. Specifically, we sort $|\Gamma^*|$ globally and select a threshold $\tau(B)$ that preserves the top- B entries. The resulting mask and pruned weights are given by:

$$\widehat{M}(B)_{ij} = \mathbb{I}(|\Gamma_{ij}^*| \geq \tau(B)), \quad \widetilde{W}(B) = W_0 \odot \widehat{M}(B).$$

Thus, a single training run suffices to generate pruning masks for any sparsity level, providing both flexibility and efficiency while avoiding repeated retraining.

4.2 DISCUSSION

Mirror descent stabilizes pruning by unifying local and global signals for robust high-sparsity performance. We give more insights here.

(1) *Why mirror descent is necessary.* Directly combining local metric and global feedback methods can introduce bias into the optimization process. To mitigate this, we decouple the model parameters from the sparsity objective, which stabilizes training and improves accuracy. This decoupling requires the introduction of an additional variable, Γ , to balance the trade-off between enforcing sparsity and preserving the convergence direction of the model. Ablation study of mirror descent is conducted in Section 5.3

(2) *Advantages of our method.* Our approach benefits from gradual, saliency-guided sparsification. By maintaining alignment with local metrics and leveraging the decoupled optimization framework, the model remains robust across architectures and pruning levels. As shown in our experiments 5, it maintains strong performance even at high sparsity ratios such as 60% and 70%, outperforming prior methods that often collapse under such conditions.

4.3 CONVERGENCE

Building on the mirror descent framework, we introduce a saliency variable Γ as part of a splitting strategy to decouple weight optimization from sparsity enforcement. While this approach is empirically effective, it may influence the convergence behavior of mirror descent—particularly because the regularizer Ω is not necessarily differentiable. To address this, we provide a rigorous convergence analysis of our algorithm.

We study the convergence of the composite objective defined in Eq. 4, under the following assumptions: (i) $\mathcal{L}_{\text{task}}$ has Lipschitz continuous gradients and is bounded below, (ii) $S(W)$ is smooth with Lipschitz Jacobian, and (iii) Ω is proper, convex, and lower semi-continuous.

Theorem 1 (Global convergence). *Under the above assumptions, if the step size α satisfies*

$$0 < \alpha < \frac{2}{\kappa(L_W + \rho L_S^2)},$$

then the sequence $\{(W^k, \Gamma^k)\}$ generated by updates Eq. 7 converges to a critical point of Eq. 4.

The proof of Theorem 1 is provided in Appendix A.5.1.

Remark. The convergence to a stationary point justifies extracting pruning masks directly from the limit Γ^* via global thresholding, enabling one-shot mask generation without retraining.

5 EXPERIMENTS

Experimental Setup. We evaluate on representative LLM families, including **LLaMA2** (Touvron et al., 2023), **Qwen2.5** (Team, 2025), and **Llama3** (Grattafiori et al., 2024) series, as well as distilled **DeepSeek** model (Guo et al., 2025). We consider several representative post-training pruning

324 Table 1: WikiText perplexity and zero-shot downstream benchmark results at 60% sparsity.
325

Model	Method	WikiText PPL	ARC-C	ARC-E	HellaSwag	OBQA	PIQA	SIQA	Avg
LLaMA2-13B	Dense	4.57	0.4846	0.7942	0.6003	0.3500	0.7900	0.4729	0.5820
	Magnitude	11.22	0.2713	0.5623	0.4465	0.2180	0.6872	0.3941	0.4299
	Wanda	11.90	0.3123	0.6460	0.4483	0.2740	0.7182	0.4243	0.4709
	RIA	7.57	0.3652	0.6970	0.5027	0.2840	0.7437	0.4524	0.5075
	UniPruning	7.82	0.3695	0.7003	0.5096	0.2840	0.7470	0.4529	0.5106
Qwen2.5-7B	Dense	6.39	0.4829	0.8047	0.6002	0.3360	0.7867	0.5481	0.5931
	Magnitude	3835.29	0.2295	0.3447	0.2594	0.1720	0.5305	0.3460	0.3137
	Wanda	14.06	0.3848	0.7163	0.4688	0.2680	0.7263	0.4780	0.5070
	RIA	12.09	0.3857	0.7344	0.4655	0.2600	0.7301	0.4621	0.5063
	UniPruning	11.87	0.3959	0.7306	0.4736	0.2620	0.7345	0.4703	0.5112
Qwen2.5-14B	Dense	4.93	0.5597	0.8241	0.6336	0.3480	0.8118	0.5537	0.6218
	Magnitude	117.74	0.3072	0.5455	0.4127	0.2900	0.6638	0.3828	0.4337
	Wanda	11.68	0.4266	0.7492	0.5070	0.3020	0.7595	0.4765	0.5368
	RIA	9.37	0.4360	0.7563	0.4991	0.2960	0.7601	0.4754	0.5378
	UniPruning	8.85	0.4531	0.7605	0.5070	0.3040	0.7601	0.4698	0.5424
Llama-3.2-1B	Dense	9.06	0.3157	0.6536	0.4774	0.2660	0.7459	0.4284	0.4812
	Magnitude	28096.59	0.1928	0.2643	0.2573	0.1380	0.5430	0.3373	0.2888
	Wanda	261.88	0.1852	0.2959	0.2701	0.1280	0.5555	0.3367	0.2952
	RIA	83.45	0.1928	0.3880	0.2884	0.1320	0.5941	0.3593	0.3258
	UniPruning	45.32	0.1886	0.4226	0.3104	0.1440	0.6153	0.3654	0.3411
Llama-3.2-3B	Dense	7.29	0.4224	0.7437	0.5532	0.3100	0.7666	0.4719	0.5446
	Magnitude	21913.05	0.2073	0.2639	0.2660	0.1380	0.5408	0.3296	0.2910
	Wanda	66.00	0.2193	0.4108	0.3132	0.1460	0.6121	0.3552	0.3428
	RIA	29.08	0.2483	0.5118	0.3555	0.1700	0.6659	0.3889	0.3901
	UniPruning	24.38	0.2654	0.5446	0.3714	0.1720	0.6774	0.4012	0.4053
DeepSeek-R1- Distill-Llama8B	Dense	11.86	0.4087	0.7037	0.5554	0.3140	0.7606	0.4473	0.5316
	Magnitude	8154.51	0.2005	0.3066	0.2736	0.1520	0.5560	0.3444	0.3055
	Wanda	50.66	0.2423	0.4676	0.3731	0.2000	0.6338	0.3864	0.3839
	RIA	28.16	0.2901	0.5358	0.4041	0.1960	0.6649	0.3987	0.4149
	UniPruning	24.50	0.3046	0.5804	0.4219	0.2140	0.6676	0.4186	0.4345

353 competitors which involve no weight update during pruning, keeping the same with our method:
354 (1) Magnitude (Zhu & Gupta, 2017), the most prevalent pruning approach; (2) Wanda (Sun et al.,
355 2024), which ranks weights using local metric scores and is applicable to both unstructured and
356 semi-structured settings; (3) RIA (Zhang et al., 2024), which combines relative importance with
357 activation norms to provide stable pruning decisions across different sparsity levels; and (4) ProxS-
358 parse (Liu et al., 2025), a proximal optimization framework that specifically targets semi-structured
359 $N:M$ pruning and achieves state-of-the-art results under 2:4 pattern.

360 For fairness, we adopt common calibration setup of 128 randomly sampled C4 datasets (Raffel
361 et al., 2020). Model quality is evaluated on both zero-shot reasoning tasks and language modeling:
362 zero-shot performance is measured with the EleutherAI LM-Eval-Harnesss (Gao et al., 2024) on
363 standard benchmark, while WikiText perplexity is reported as the language modeling metric (Merity
364 et al., 2016). For unstructured pruning we use stochRIA (Yi & Richtárik, 2025) as the local metric,
365 and for 2:4 semi-structured pruning we use Wanda (Sun et al., 2024). In both settings, we apply
366 an additional λL_1 regularization term Ω with $\lambda = 0.001$ (further discussed in Appendix A.4.3).
367 Ablations of the local metric are provided in Section 5.3. For all LLMs, we fix the context length
368 to 4096. All experiments are conducted on a single NVIDIA H200 GPU with 141GB of memory,
369 using a learning rate of 1e-4.

370 371 5.1 UNSTRUCTURED PRUNING

372 373 We evaluate unstructured pruning at 60% sparsity on six pretrained LLMs, comparing UniPruning
374 against Magnitude, Wanda, and RIA under the standard 128-sample C4 calibration. We report Wiki-
375 Text perplexity and zero-shot accuracy on ARC-C/E, HellaSwag, OBQA, PIQA, and SIQA (Gao
376 et al., 2024), and further include the average accuracy over all the evaluated benchmarks. Detailed
377 results shows in Table 1.

378 Table 2: WikiText perplexity across models and pruning methods under 2:4 semi-structured pruning.
379

Method	LLaMA2-13B	Qwen2.5-7B	Qwen2.5-14B	LLaMA-3.2-3B	DeepSeek-R1-Distill-Llama8B	DeepSeek-R1-Distill-Qwen-7B
Dense	4.57	6.39	4.93	7.29	11.86	21.73
Magnitude	8.32	inf	48.59	668.75	459.82	270.25
Wanda	8.37	14.77	11.69	32.86	29.77	49.83
RIA	7.85	13.81	10.87	33.38	30.10	43.11
ProxSparse	6.88	14.06	10.54	22.44	23.74	35.66
UniPruning	6.87	10.86	9.10	21.20	20.91	30.24

387
388 Across six models, UniPruning attains the best average accuracy on every model, with only three
389 per-task scores falling short of the top by small margins. For perplexity, UniPruning leads on 5
390 models, with the lone non-best case (LLaMA2-13B) trailing by just 0.25 (7.82 vs. 7.57).
391

392 In addition to average gains, Unified Pruning demonstrates stable improvements on commonsense-
393 style tasks compared with other pruning baselines at the same sparsity level. For example, on Llama-
394 3.2-3B, Unified Pruning improves ARC-C from 0.2193 (Wanda) and 0.2483 (RIA) to 0.2654, and
395 SIQA from 0.3552 (Wanda) and 0.3512 (RIA) to 0.4012. On DeepSeek-R1-Distill-Llama8, it raises
396 ARC-E from 0.4676 (Wanda) and 0.5358 (RIA) to 0.5804, and HellaSwag from 0.3731 (Wanda)
397 and 0.4041 (RIA) to 0.4219. These results indicate that under high sparsity, **a globally coordinated**
398 **budget allocation better preserves reasoning capacity.**

399 At 60% unstructured sparsity, UniPruning (i) sets **the best average zero-shot accuracy** on all re-
400 ported bases with leading PPL and no collapse; and (ii) sustains **task-wise robustness** on com-
401 monsense benchmarks. These results support coupling local metric with a global budget to achieve
402 balanced whole-model pruning.

403 5.2 SEMI-STRUCTURED $N:M$ PRUNING

405 For semi-structured pruning, we primarily evaluate the 2:4 sparsity pattern across models. To align
406 with this hardware-friendly pattern, we incorporate a 2:4 regularizer (Kübler et al., 2025) into
407 our algorithm. The detailed formulation and implementation of this adaptation are provided in
408 Algorithm 1.

409 As shown in Table 2, Unified Pruning achieves the best perplexity across all evaluated models, con-
410 sistently surpassing magnitude-based and importance-based baselines (e.g., Wanda, RIA). These
411 results highlight the benefit of combining global coordination with local saliency in the semi-
412 structured setting. Moreover, Unified Pruning also surpasses the current state-of-the-art ProxSparse,
413 demonstrating that global coordination further enhances performance even under semi-structured
414 constraints.

415 Beyond perplexity, we also evaluate zero-shot performance on downstream benchmarks under the
416 2:4 constraint, with results provided in Appendix A.2. Furthermore, to situate our approach against
417 stronger baselines, we conduct an additional comparison with SparseGPT (Frantar & Alistarh,
418 2023), which allows weight updates during pruning. The corresponding results are also reported
419 in Appendix A.2.

421 5.3 ABLATION STUDY

423 In this section, we conduct ablation studies to better understand the contribution of different compo-
424 nents in our framework. We focus on two key aspects: (i) the choice of local saliency metric, which
425 directly affects the pruning quality, and (ii) the role of mirror descent and the saliency variable,
426 which are introduced to stabilize optimization and balance sparsity with task performance. These
427 analyses provide deeper insights into the design choices underlying UniPruning and highlight their
428 necessity.

429 **Local Metric.** To evaluate the sensitivity of pruning performance to the choice of local saliency
430 metric, we conduct ablation studies comparing different saliency metric designs. Specifically, we
431 experiment with magnitude-based scoring, activation-aware scoring as used in Wanda (Sun et al.,
2024), and the combination of row/column norm-based relative importance with activation signals as

432 Table 3: WikiText perplexity of Qwen2.5-7B under different local metrics at varying sparsity.
433

434	Local Metric	50%	60%	70%
435	Magnitude	38.86	428.56	inf
436	Wanda	8.63	13.12	183.21
437	RIA	8.28	11.87	88.35
438	StochRIA	8.63	11.87	52.34

440 Table 4: WikiText perplexity of Qwen2.5-7B in different Ω and ρ at varying sparsity.
441

442	Sparsity	UniPruning	$\lambda = 0.01, \rho = 10^{-5}$	$\lambda = 0.01, \rho = 0$	$\lambda = 0, \rho = 10^{-5}$	$\lambda = 0, \rho = 0$
443	50%	8.63	11.39	20.45	29.86	35.59
444	60%	11.87	15.38	161.09	inf	inf

445 in RIA (Zhang et al., 2024). Furthermore, we incorporate the stochastic variant of RIA (stochRIA)
446 proposed by (Yi & Richtárik, 2025), which introduces randomness into the scoring process to mitigate
447 biases introduced by deterministic saliency measures and to enhance exploration during pruning.
448

449 Table 3 reports the results of different local saliency metrics on Qwen2.5-7B. Among the evaluated
450 methods, stochRIA demonstrates a balanced trade-off across sparsity levels. At 50% and 60% sparsity,
451 it performs comparably to RIA (8.63/11.87 vs. 8.28/11.87), and at 70% sparsity, it yields lower
452 perplexity (52.34) than other alternatives. These results suggest that incorporating stochasticity into
453 local importance estimation can improve robustness under high compression, making stochRIA a
454 viable option for use within our framework.
455

456 **The Necessity of Mirror Descent.** As discussed in Section 4.2, we argue that mirror descent and
457 the introduction of the saliency variable are necessary components of our framework. To verify
458 this claim, we **additionally conduct pruning experiments using only the local metric and global**
459 **feedback regularizers, without mirror descent or the saliency variable.** In other words, we
460 directly train with the following objective function:
461

$$462 \quad \bar{\mathcal{L}}_\rho(W) = \mathcal{L}_{\text{task}}(W) + \frac{\rho}{2} \|S(W)\|_F^2 + \Omega(W). \quad (8)$$

463 Since the L_1 regularizer is non-differentiable and cannot be directly integrated into the formulation
464 without mirror descent, we replace Ω with a λL_2 regularizer for this analysis. Table 4 reports
465 WikiText perplexity results on Qwen2.5-7B under different Ω and ρ configurations at sparsity levels
466 of 50% and 60%.
467

468 As shown in Table 4, similarly, using stochRIA as the local saliency metric but removing the mirror
469 descent update makes the optimization unstable and substantially increases perplexity, especially at
470 higher sparsity levels. In the absence of local metric regularizer coefficient ($\rho = 0$) or L_2 coefficient
471 ($\lambda = 0$), the model either diverges (infinite PPL) or converges to poor local minima. In contrast, our
472 proposed UniPruning integrates local metric with a mirror descent driven global budget controller
473 and achieves the lowest perplexity across all tested sparsity levels. These results demonstrate that
474 both local saliency metrics and global coordination are essential. Crucially, mirror descent serves as
475 the optimization bridge that enables their seamless integration in UniPruning, leading to stable and
476 effective sparse pruning.
477

478 6 CONCLUSION

481 We presented UniPruning, a mirror-descent framework that unifies local evidence with global coor-
482 dination to prune large language models. By introducing a saliency variable anchored to activation
483 statistics and enforcing a model-wise sparsity budget, our method naturally supports both unstruc-
484 tured and semi-structured $N : M$ patterns, avoids direct weight updates, and enables one-shot mask
485 extraction at multiple sparsity levels. Experiments show that Unified Pruning consistently delivers
486 strong performance compared to prior baselines, while ablations highlight the necessity of mirror

486
487 descent and local saliency metric. Overall, the framework offers a principled, efficient, and scalable
488 approach to LLM compression.
489

490 ETHICS STATEMENT

491
492 This work does not involve human subjects, sensitive personal data, or applications with foreseeable
493 societal risks. All experiments are conducted on publicly available datasets and widely used
494 pretrained models under their respective licenses. We have carefully considered issues related to
495 fairness, privacy, and potential misuse, and we believe our study poses minimal ethical concerns.
496 According to ICLR policy, this section is excluded from the page limit.
497

498 REPRODUCIBILITY STATEMENT

500 We have taken multiple steps to ensure reproducibility of our results. The paper clearly specifies
501 the model architectures, training objectives, sparsity settings, and evaluation protocols. Additional
502 implementation details, hyperparameters, and training scripts are provided in the supplementary materials.
503 We also include pseudocode and references to relevant sections of the appendix to facilitate
504 independent verification of our findings. According to ICLR policy, this section is excluded from
505 the page limit.
506

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594 **A APPENDIX**595 **A.1 USE OF LARGE LANGUAGE MODELS**

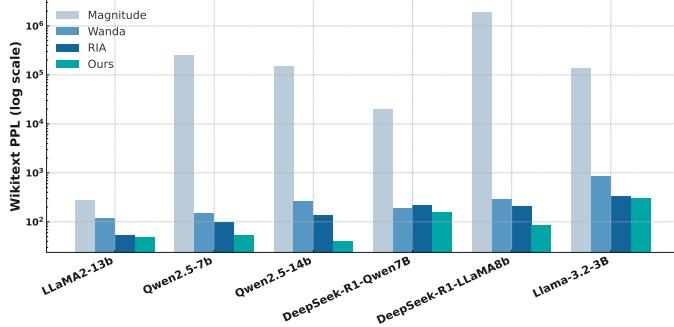
599 In preparing this paper, we leveraged GPT-4o to assist with polishing the writing, including grammar
600 correction, stylistic consistency, and clarity of expression. All substantive elements—problem def-
601 inition, methodological design, experimental execution, and analysis—were conceived and carried
602 out solely by the authors. Every section of the manuscript was carefully reviewed and revised by the
603 authors to guarantee fidelity to our original contributions. The authors take complete responsibility
604 for the accuracy and integrity of the final text.

605 **A.2 ADDITIONAL 2:4 PRUNING RESULTS**

606 As shown in Table 5, **Unified Pruning** consistently outperforms other pruning baselines across all
607 evaluated models on downstream tasks. While naive magnitude pruning leads to severe degradation
608 and Wanda or RIA offer only moderate improvements, Unified Pruning achieves the highest average
609 accuracy among sparse methods and remains close to the dense baseline. For instance, on Qwen2.5-
610 14B, Unified Pruning yields an average score of 0.5459, surpassing ProxSparse (0.5366). Similar
611 trends hold for LLaMA-2 and DeepSeek variants, highlighting the robustness of our method across
612 both medium- and large-scale LLMs.

613 Table 6 further evaluates language modeling perplexity under 2:4 semi-structured pruning. We ob-
614 serve that Unified Pruning outperforms SparseGPT on several benchmarks, such as Qwen2.5-7B
615 (10.86 vs. 11.42) and DeepSeek-R1-Distill-LLaMA-8B (12.45 vs. 13.07), showing clear improve-
616 ments. On other models the performance is slightly worse, e.g., LLaMA-3.2-3B (21.20 vs. 20.19),
617 but the gap remains small and does not lead to collapse. Overall, Unified Pruning achieves compet-
618 itive perplexity and can be comparable to, or even surpass, methods with weight update.

619 Taken together, these results demonstrate that coupling local saliency metrics with a unified global
620 budget not only improves task-wise robustness under sparsity but also mitigates the perplexity blow-
621 up commonly seen in post-training pruning methods. Unified Pruning thus offers a reliable path
622 toward structured sparsification of LLMs while maintaining downstream task performance.

623 **Figure 2: Wikitext perplexity comparison at 70% sparsity**643 **A.3 RESULTS ON ASCEND NPU**

644 The open-source Pangu model based on Ascend has demonstrated powerful capabilities in various
645 tasks. We conduct systematic empirical studies on the performance of the open-source Pangu model
646 across multiple domains using the Ascend platform. Results are shown in Table 7. Our method
647 outperforms others and interestingly finds the winning structure of the lottery ticket.

648
649 Table 5: Zero-shot downstream benchmark results under 2:4 pruning.
650

Model	Method	ARC-C	ARC-E	HellaSwag	OBQA	PIQA	SIQA	Avg
LLaMA2-13B	Dense	0.4846	0.7942	0.6003	0.3500	0.7900	0.4729	0.5820
	Magnitude	0.3174	0.6229	0.5011	0.2320	0.7171	0.4079	0.4664
	Wanda	0.3396	0.6856	0.4629	0.2460	0.7372	0.4243	0.4826
	RIA	0.3507	0.6987	0.4790	0.2600	0.7394	0.4294	0.4929
	ProxSparse	0.3695	0.6944	0.5300	0.2920	0.7427	0.4299	0.5098
	UniPruning	0.3712	0.7088	0.5255	0.2700	0.7535	0.4406	0.5116
Qwen2.5-7B	Dense	0.4829	0.8047	0.6002	0.3360	0.7867	0.5481	0.5931
	Magnitude	0.2432	0.3830	0.3006	0.2160	0.5560	0.3664	0.3442
	Wanda	0.3652	0.7142	0.4454	0.2600	0.7198	0.4703	0.4958
	RIA	0.3746	0.7176	0.4508	0.2680	0.7274	0.4678	0.5010
	ProxSparse	0.3985	0.7168	0.4803	0.2720	0.7296	0.4437	0.5068
	UniPruning	0.3959	0.7306	0.4736	0.2620	0.7345	0.4703	0.5112
Qwen2.5-14B	Dense	0.5597	0.8241	0.6336	0.3480	0.8118	0.5537	0.6218
	Magnitude	0.3584	0.6402	0.4176	0.2560	0.6790	0.4035	0.4591
	Wanda	0.3780	0.7231	0.4902	0.2840	0.7399	0.4386	0.5090
	RIA	0.3959	0.7386	0.4923	0.2720	0.7421	0.4545	0.5159
	ProxSparse	0.4428	0.7584	0.5208	0.2960	0.7470	0.4545	0.5366
	UniPruning	0.4531	0.7710	0.5268	0.2860	0.7617	0.4765	0.5459
Llama-3.2-3B	Dense	0.4224	0.7437	0.5532	0.3100	0.7666	0.4719	0.5446
	Magnitude	0.1954	0.3729	0.2837	0.1440	0.6055	0.3434	0.3242
	Wanda	0.2517	0.5093	0.3377	0.1640	0.6480	0.4675	0.3964
	RIA	0.2688	0.5311	0.3433	0.1760	0.6649	0.3823	0.3944
	ProxSparse	0.2611	0.5425	0.3877	0.1760	0.6768	0.3956	0.4066
	UniPruning	0.2602	0.5572	0.3797	0.1900	0.6746	0.3884	0.4084
DeepSeek-R1- Distill-Llama8B	Dense	0.4087	0.7037	0.5554	0.3140	0.7606	0.4473	0.5316
	Magnitude	0.2312	0.4242	0.3302	0.1340	0.6050	0.3495	0.3457
	Wanda	0.2756	0.5459	0.3852	0.1940	0.6534	0.4012	0.4092
	RIA	0.2816	0.5400	0.3846	0.1800	0.6659	0.3941	0.4077
	ProxSparse	0.2901	0.5366	0.4066	0.1760	0.6556	0.4002	0.4109
	UniPruning	0.2790	0.5614	0.4130	0.1880	0.6687	0.4069	0.4195
DeepSeek-R1- Distill-Qwen-7B	Dense	0.4232	0.6911	0.4637	0.26	0.7046	0.4248	0.4946
	Magnitude	0.2526	0.4848	0.3061	0.1640	0.5996	0.3562	0.3606
	Wanda	0.3046	0.5737	0.3660	0.1640	0.6491	0.3808	0.4064
	RIA	0.3166	0.5875	0.3686	0.1620	0.6583	0.3756	0.4114
	ProxSparse	0.3447	0.6330	0.3952	0.2120	0.6708	0.4023	0.4430
	UniPruning	0.3396	0.6545	0.4056	0.1940	0.6823	0.4023	0.4464

670
671 Table 6: Wikitext perplexity results across models and pruning methods under 2:4 pruning.
672

Method	Weight Update	LLaMA2-13B	Qwen2.5-7B	Qwen2.5-14B	LLaMA-3.2-3B	DeepSeek-R1- Distill-Llama8B	DeepSeek-R1- Distill-Qwen-7B
Dense	-	4.57	6.39	4.93	7.29	11.86	21.73
SparseGPT	✓	8.30	8.42	9.57	20.19	25.45	35.69
UniPruning	✗	6.87	10.86	9.10	21.20	20.91	30.24

687
688 A.4 ADDITIONAL ANALYSIS689
690 A.4.1 THE ROBUST TO HIGHER SPARSITY.691
692 We further evaluate pruning performance at a more aggressive **70% sparsity**, with results shown
693 in Fig. 2. The gap between methods becomes more evident in this regime. Magnitude and Wanda
694 both collapse under such high compression, leading to perplexities that grow by several orders of
695 magnitude. RIA is more stable but still suffers from noticeable degradation. In contrast, UniPrun-
696 ing remains well-behaved across all tested architectures, consistently yielding perplexities within a
697 reasonable range.698
699 A.4.2 INFERENCE EFFICIENCY700
701 We evaluated the throughput gain of applying 2:4 semi-structured sparsity to Qwen2.5-7B on an
NVIDIA H200 GPU (batch size 8, sequence length 128). The sparse kernels accelerate the main
compute-intensive modules: the self-attention projections (Q/K/V/O) achieve a **1.30×** speedup,

702
703 Table 7: Results On Ascend NPU.
704

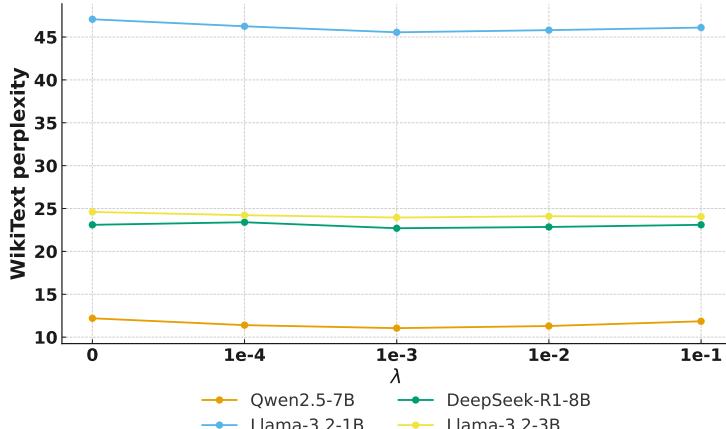
Model	Type	Method	ppl \downarrow	ARC-C \uparrow	ARC-E \uparrow	HellaSwag \uparrow	OBQA \uparrow	PIQA \uparrow	SIQA \uparrow	Avg \uparrow
openPangu-Embedded-7B-V1.1	—	dense	31.36	0.3302	0.5673	0.3946	0.4497	0.1980	0.6844	0.4374
	2:4	ria	208.04	0.2526	0.5034	0.3792	0.3645	0.1600	0.6344	0.3824
	2:4	wanda	237.32	0.2628	0.5109	0.3746	0.3593	0.1600	0.6328	0.3834
	2:4	UniPruning	106.21	0.2927	0.6002	0.3930	0.3778	0.1840	0.6518	0.4166
	50%	ria	59.75	0.3072	0.5488	0.3936	0.4211	0.1960	0.6632	0.4217
	50%	wanda	70.26	0.2944	0.5492	0.3930	0.4227	0.2060	0.6670	0.4221
	50%	UniPruning	49.73	0.3677	0.7054	0.4043	0.6700	0.2040	0.6959	0.5079

705 while the MLP blocks (up, down, and gating projections) reach **1.34** \times . When integrating all
711 components—including non-sparse operations such as softmax, normalization, and key–value I/O,
712 the overall end-to-end inference achieves a **1.27** \times throughput improvement. These results fall within
713 the typical **1.2–1.4** \times range reported for 2:4 sparsity on modern accelerators.

714
715 Table 8: Inference time analysis for Qwen2.5-7B.

Module name	Speedup ratio
self.attn Q/K/V/O	1.30 \times
MLP up/down/gate	1.34 \times
End-to-end inference	1.27 \times

721 A.4.3 HYPERPARAMETER ANALYSIS

722 Figure 3: WikiText perplexity of different models at 60% sparsity across λ values.

725 We further investigate the impact of key hyperparameters on the performance of our method. In par-
726 ticular, we examine the effects of the regularization weight λ . Results shown in Fig. 3. These experi-
727 ments provide insights into the sensitivity of UniPruning to hyperparameter choices and demonstrate
728 the robustness of our framework under different configurations.

729 A.4.4 LIMITATIONS OF OUR METHOD

730 While UniPruning achieves strong performance across various model families and sparsity regimes,
731 several limitations remain that warrant further investigation:

732

- 733 • **Additional hyperparameters.** Our framework introduces additional hyperparameters, in-
734 cluding the regularization coefficient λ and the choice of local saliency metric. As demon-
735 strated in our hyperparameter analysis, model performance can vary across different con-
736 figurations.
- 737 • **Limited architectural generalization.** Our experiments primarily focus on LLaMA and
738 Qwen model families, with limited exploration of other transformer architectures. It re-
739 mains an open question how well the proposed method generalizes to models with substan-
740 tially different design paradigms.

756 These limitations highlight promising directions for improving the robustness and applicability of
 757 Unified Pruning in broader model and deployment contexts.
 758

759 **A.5 PROOF OF THEOREM 1**
 760

761 First of all, we reformulate Eq. 4 into an equivalent form. Without loss of generality, consider
 762 $\Omega = L_1$ in the sequel. Denote $R(P) := \Omega(\Gamma)$, then our UniPruning Algorithm is equivalent to the
 763 following iterations,

764 $W_{k+1} = W_k - \kappa\alpha\nabla_W \bar{\mathcal{L}}(W_k, \Gamma_k), \quad (9a)$
 765

766 $\Gamma_{k+1} = \text{Prox}_{\kappa\Omega}(\Gamma_k + \kappa(g_k - \alpha\nabla_\Gamma \bar{\mathcal{L}}(W_k, \Gamma_k))), \quad (9b)$
 767

768 $g_{k+1} = g_k - \kappa^{-1}(\Gamma_{k+1} - \Gamma_k + \kappa\alpha \cdot \nabla_\Gamma \bar{\mathcal{L}}(W_k, \Gamma_k)). \quad (9c)$

769 where $p_k = [0, g_k]^T \in \partial R(P_k)$ and $g_k \in \partial\Omega(\Gamma_k)$. Thus

770 The global convergence of (M_k, Γ_k, g_k) can be established based on the Kurdyka-Łojasiewicz
 771 framework.

772 **A.5.1 SUFFICIENT DESCENT PROPERTY ALONG LYAPUNOV FUNCTION**

773 Let $P_k := (M_k, \Gamma_k)$, and $Q_k := (P_k, g_{k-1})$, $k \in \mathbb{N}$. In the following, we present the sufficient
 774 descent property of Q_k along the Lyapunov function F .

775 **Lemma.** Suppose that \mathcal{L} is continuously differentiable and $\nabla\mathcal{L}$ is Lipschitz continuous with a
 776 constant $Lip > 0, C = \max|W_0|$ is the max value of the pretrained model parameters W_0 . Let
 777 $\{Q_k\}$ be a sequence generated by SLBI with a finite initialization. If $0 < \alpha < \frac{2}{\kappa(Lip*C + \nu^{-1})}$, then

778 $F(Q_{k+1}) \leq F(Q_k) - \rho\|Q_{k+1} - Q_k\|_2^2,$

779 where $\rho := \frac{1}{\kappa} - \frac{\alpha(Lip*C + \nu^{-1})}{2}$.

780 *Proof.* By the optimality condition of equation 7 and also the inclusion $p_k = [0, g_k]^T \in \partial R(P_k)$,
 781 there holds

782 $\kappa(\alpha\nabla\bar{\mathcal{L}}(P_k) + p_{k+1} - p_k) + P_{k+1} - P_k = 0,$

783 which implies

784 $\nabla\hat{\mathcal{L}}(M) = \sum \nabla\mathcal{L}(\hat{W}) * W_0 \quad (10)$

785 Noting that $\bar{\mathcal{L}}(P) = \hat{\mathcal{L}}(M) + \frac{1}{2\nu}\|M - \Gamma\|_2^2 = \mathcal{L}(W_0 \odot M) + \frac{1}{2\nu}\|M - \Gamma\|_2^2$. Together with,

786 $\alpha\bar{\mathcal{L}}(P_{k+1}) + D(\Gamma_{k+1}, \Gamma_k) + \rho\|P_{k+1} - P_k\|_2^2 \leq \alpha\bar{\mathcal{L}}(P_k). \quad (11)$

787 Adding some terms in both sides of the above inequality and after some reformulations implies

788 $F(Q_{k+1}) \leq F(Q_k) - \rho\|P_{k+1} - P_k\|_2^2 - B_\Omega^{g_{k+1}}(\Gamma_k, \Gamma_{k-1}) - B_\Omega^{g_{k-1}}(\Gamma_k, \Gamma_{k-1}) \quad (12)$

789 $\leq F(Q_k) - \rho\|P_{k+1} - P_k\|_2^2, \quad (13)$

800 where the final equality holds for $D(\Gamma_{k+1}, \Gamma_k) - B_\Omega^{g_k}(\Gamma_{k+1}, \Gamma_k) = B_\Omega^{g_{k+1}}(\Gamma_k, \Gamma_{k-1})$.

801 Note that the final inequality holds for $B_\Omega^{g_{k+1}}(\Gamma_k, \Gamma_{k-1}) \geq 0$ and $B_\Omega^{g_{k-1}}(\Gamma_k, \Gamma_{k-1}) \geq 0$. Thus, we
 802 finish the proof of this lemma. \square

803 Based on Lemma A.5.1, we directly obtain the following lemma.

804 **Lemma 2.** Suppose that assumptions of Lemma A.5.1 hold. Then

805 (i) both $\alpha\{\bar{\mathcal{L}}(P_k)\}$ and $\{F(Q_k)\}$ converge to the same finite value, and
 806 $\lim_{k \rightarrow \infty} B_\Omega^{g_k}(\Gamma_{k+1}, \Gamma_k) = 0$.

810 (ii) the sequence $\{(M_k, \Gamma_k, g_k)\}$ is bounded,
 811
 812 (iii) $\lim_{k \rightarrow \infty} \|P_{k+1} - P_k\|_2^2 = 0$ and $\lim_{k \rightarrow \infty} D(\Gamma_{k+1}, \Gamma_k) = 0$,
 813
 814 (iv) $\frac{1}{K} \sum_{k=0}^K \|P_{k+1} - P_k\|_2^2 \rightarrow 0$ at a rate of $\mathcal{O}(1/K)$.

815 *Proof.* By (11), $\bar{\mathcal{L}}(P_k)$ is monotonically decreasing due to $D(\Gamma_{k+1}, \Gamma_k) \geq 0$. Similarly, by (13),
 816 $F(Q^k)$ is also monotonically decreasing. By the lower boundedness assumption of $\mathcal{L}(W)$, both
 817 $\bar{\mathcal{L}}(P)$ and $F(Q)$ are lower bounded by their definitions respectively. Therefore, both $\{\bar{\mathcal{L}}(P_k)\}$ and
 818 $\{F(Q_k)\}$ converge, and it is obvious that $\lim_{k \rightarrow \infty} F(Q_k) \geq \lim_{k \rightarrow \infty} \alpha \bar{\mathcal{L}}(P_k)$. By (12),
 819

$$820 B_{\Omega}^{g^{k-1}}(\Gamma_k, \Gamma_{k-1}) \leq F(Q_k) - F(Q_{k+1}), \quad k = 1, \dots$$

821 By the definition of $F(Q_k) = \alpha \bar{\mathcal{L}}(P_k) + B_{\Omega}^{g^{k-1}}(\Gamma_k, \Gamma_{k-1})$ and the above equality, it yields
 822

$$823 \lim_{k \rightarrow \infty} F(Q_k) = \lim_{k \rightarrow \infty} \alpha \bar{\mathcal{L}}(P_k).$$

824 Since $L(M)$ has bounded level sets, then M_k is bounded. By the definition of $\bar{\mathcal{L}}(M, \Gamma)$ and the
 825 finiteness of $\bar{\mathcal{L}}(M_k, \Gamma_k)$, Γ_k is also bounded due to M_k is bounded. The boundedness of g_k is due
 826 to $g_k \in \partial \Omega(\Gamma_k)$, condition (d), and the boundedness of Γ_k .

827 By (13), summing up (13) over $k = 0, 1, \dots, K$ yields

$$828 \sum_{k=0}^K (\rho \|P_{k+1} - P_k\|^2 + D(\Gamma_{k+1}, \Gamma_k)) < \alpha \bar{\mathcal{L}}(P_0) < \infty. \quad (14)$$

830 Letting $K \rightarrow \infty$ and noting that both $\|P_{k+1} - P_k\|^2$ and $D(\Gamma_{k+1}, \Gamma_k)$ are nonnegative, thus
 831

$$832 \lim_{k \rightarrow \infty} \|P_{k+1} - P_k\|^2 = 0, \quad \lim_{k \rightarrow \infty} D(\Gamma_{k+1}, \Gamma_k) = 0.$$

833 Again by (14),

$$834 \frac{1}{K} \sum_{k=0}^K (\rho \|P_{k+1} - P_k\|^2 + D(\Gamma_{k+1}, \Gamma_k)) < K^{-1} \alpha \bar{\mathcal{L}}(P_0),$$

835 which implies $\frac{1}{K} \sum_{k=0}^K \|P_{k+1} - P_k\|^2 \rightarrow 0$ at a rate of $\mathcal{O}(1/K)$. \square

836 A.5.2 RELATIVE ERROR PROPERTY

837 In this subsubsection, we provide the bound of subgradient by the discrepancy of two successive
 838 iterates.

$$839 H_{k+1} := \begin{pmatrix} \alpha \nabla_M \bar{\mathcal{L}}(M_{k+1}, \Gamma_{k+1}) \\ \alpha \nabla_{\Gamma} \bar{\mathcal{L}}(M_{k+1}, \Gamma_{k+1}) + g_{k+1} - g_k \\ \Gamma_k - \Gamma_{k+1} \end{pmatrix} \in \partial F(Q_{k+1}), \quad k \in \mathbb{N}. \quad (15)$$

840 **Lemma.** Under assumptions of Lemma 2, then

$$841 \|H_{k+1}\| \leq \rho_1 \|Q_{k+1} - Q_k\|, \quad \text{for } H_{k+1} \in \partial F(Q_{k+1}), \quad k \in \mathbb{N},$$

842 where $\rho_1 := 2\kappa^{-1} + 1 + \alpha(Lip * C + 2\nu^{-1})$. Moreover, $\frac{1}{K} \sum_{k=1}^K \|H_k\|^2 \rightarrow 0$ at a rate of $\mathcal{O}(1/K)$.

843 *Proof.* Note that

$$844 \nabla_M \bar{\mathcal{L}}(M_{k+1}, \Gamma_{k+1}) = (\nabla_M \bar{\mathcal{L}}(M_{k+1}, \Gamma_{k+1}) - \nabla_M \bar{\mathcal{L}}(M_{k+1}, \Gamma_k)) \\ 845 + (\nabla_M \bar{\mathcal{L}}(M_{k+1}, \Gamma_k) - \nabla_M \bar{\mathcal{L}}(M_k, \Gamma_k)) + \nabla_M \bar{\mathcal{L}}(M_k, \Gamma_k). \quad (16)$$

846 where the last inequality holds for the Lipschitz continuity of $\nabla \mathcal{L}$ with a constant $Lip > 0$, and
 847 $C = \max |W_0|$. By equation 9a,

$$848 \|\nabla_M \bar{\mathcal{L}}(M_k, \Gamma_k)\| = (\kappa \alpha)^{-1} \|M_{k+1} - M_k\|.$$

864 Substituting the above (in)equalities into (16) yields
 865
 866

$$\|\nabla_M \bar{\mathcal{L}}(M_{k+1}, \Gamma_{k+1})\| \leq [(\kappa\alpha)^{-1} + \text{Lip} * C + \nu^{-1}] \cdot \|M_{k+1} - M_k\| + \nu^{-1} \|\Gamma_{k+1} - \Gamma_k\|$$

867 Thus,

$$\|\alpha \nabla_M \bar{\mathcal{L}}(M_{k+1}, \Gamma_{k+1})\| \leq [\kappa^{-1} + \alpha(\text{Lip} * C + \nu^{-1})] \cdot \|M_{k+1} - M_k\| + \alpha \nu^{-1} \|\Gamma_{k+1} - \Gamma_k\|. \quad (17)$$

871 Noting that $\nabla_\Gamma \bar{\mathcal{L}}(M_k, \Gamma_k) = \nu^{-1}(\Gamma_k - M_k)$, and after some simplifications yields
 872
 873

$$\begin{aligned} \|\alpha \nabla_\Gamma \bar{\mathcal{L}}(M_{k+1}, \Gamma_{k+1}) + g_{k+1} - g_k\| &= \|(\kappa^{-1} - \alpha \nu^{-1}) \cdot (\Gamma_k - \Gamma_{k+1}) + \alpha \nu^{-1} (M_k - M_{k+1})\| \\ &\leq \alpha \nu^{-1} \|M_k - M_{k+1}\| + (\kappa^{-1} - \alpha \nu^{-1}) \|\Gamma_k - \Gamma_{k+1}\|, \end{aligned} \quad (18)$$

874 where the last inequality holds for the triangle inequality and $\kappa^{-1} > \alpha \nu^{-1}$ by the assumption.
 875

876 By (17), (18), and the definition of H_{k+1} (15), there holds
 877

$$\begin{aligned} \|H_{k+1}\| &\leq [\kappa^{-1} + \alpha(\text{Lip} * C + 2\nu^{-1})] \cdot \|M_{k+1} - M_k\| + (\kappa^{-1} + 1) \|\Gamma_{k+1} - \Gamma_k\| \\ &\leq [2\kappa^{-1} + 1 + \alpha(\text{Lip} * C + 2\nu^{-1})] \cdot \|Q_{k+1} - Q_k\|. \end{aligned}$$

882 By Lemma 2(iv), $\frac{1}{K} \sum_{k=1}^K \|H_k\|^2 \rightarrow 0$ at a rate of $\mathcal{O}(1/K)$.
 883

884 This finishes the proof of this lemma. \square

885 A.5.3 KURDYKA-ŁOJASIEWICZ PROPERTY

887 To introduce the definition of the Kurdyka-Łojasiewicz (KL) property, we need some notions and
 888 notations from variational analysis.

889 The notion of subdifferential plays a central role in the following definitions. For each $\mathbf{x} \in$
 890 $\text{dom}(h) := \{\mathbf{x} \in \mathbb{R}^p : h(\mathbf{x}) < +\infty\}$, the *Fréchet subdifferential* of h at \mathbf{x} , written $\widehat{\partial}h(\mathbf{x})$, is
 891 the set of vectors $\mathbf{v} \in \mathbb{R}^p$ which satisfy
 892

$$\liminf_{\mathbf{y} \neq \mathbf{x}, \mathbf{y} \rightarrow \mathbf{x}} \frac{h(\mathbf{y}) - h(\mathbf{x}) - \langle \mathbf{v}, \mathbf{y} - \mathbf{x} \rangle}{\|\mathbf{y} - \mathbf{x}\|} \geq 0.$$

893 When $\mathbf{x} \notin \text{dom}(h)$, we set $\widehat{\partial}h(\mathbf{x}) = \emptyset$. The *limiting-subdifferential* (or simply *subdifferential*) of
 894 h , written $\partial h(\mathbf{x})$ at $\mathbf{x} \in \text{dom}(h)$, is defined by
 895

$$\partial h(\mathbf{x}) := \{\mathbf{v} \in \mathbb{R}^p : \exists \mathbf{x}^k \rightarrow \mathbf{x}, h(\mathbf{x}^k) \rightarrow h(\mathbf{x}), \mathbf{v}^k \in \widehat{\partial}h(\mathbf{x}^k) \rightarrow \mathbf{v}\}. \quad (19)$$

896 A necessary (but not sufficient) condition for $\mathbf{x} \in \mathbb{R}^p$ to be a minimizer of h is $\mathbf{0} \in \partial h(\mathbf{x})$. A point
 897 that satisfies this inclusion is called *limiting-critical* or simply *critical*. The distance between a point
 898 \mathbf{x} to a subset \mathcal{S} of \mathbb{R}^p , written $\text{dist}(\mathbf{x}, \mathcal{S})$, is defined by $\text{dist}(\mathbf{x}, \mathcal{S}) = \inf\{\|\mathbf{x} - \mathbf{s}\| : \mathbf{s} \in \mathcal{S}\}$, where
 899 $\|\cdot\|$ represents the Euclidean norm.

900 The *graph* is defined by

$$\begin{aligned} \text{Graph}(h) &:= \{(\mathbf{x}, y) \in \mathbb{R}^p \times \mathbb{R} : y = h(\mathbf{x})\}, \\ \text{(resp. } \text{Graph}(h) &:= \{(\mathbf{x}, \mathbf{y}) \in \mathbb{R}^p \times \mathbb{R}^q : \mathbf{y} \in h(\mathbf{x})\}), \end{aligned}$$

901 and its domain by $\text{dom}(h) := \{\mathbf{x} \in \mathbb{R}^p : h(\mathbf{x}) < +\infty\}$ (resp. $\text{dom}(h) := \{\mathbf{x} \in \mathbb{R}^p : h(\mathbf{x}) \neq \emptyset\}$).
 902 When h is a proper function, i.e., when $\text{dom}(h) \neq \emptyset$, the set of its global minimizers (possibly
 903 empty) is denoted by

$$\arg \min h := \{\mathbf{x} \in \mathbb{R}^p : h(\mathbf{x}) = \inf h\}.$$

904 **Definition 3.** [Kurdyka-Łojasiewicz property] A function h is said to have the Kurdyka-Łojasiewicz
 905 (KL) property at $\bar{u} \in \text{dom}(\partial h) := \{v \in \mathbb{R}^n | \partial h(v) \neq \emptyset\}$, if there exists a constant $\eta \in (0, \infty)$,
 906 a neighborhood \mathcal{N} of \bar{u} and a function $\phi : [0, \eta] \rightarrow \mathbb{R}_+$, which is a concave function that is
 907 continuous at 0 and satisfies $\phi(0) = 0$, $\phi \in \mathcal{C}^1((0, \eta))$, i.e., ϕ is continuous differentiable on $(0, \eta)$,
 908 and $\phi'(s) > 0$ for all $s \in (0, \eta)$, such that for all $u \in \mathcal{N} \cap \{u \in \mathbb{R}^n | h(\bar{u}) < h(u) < h(\bar{u}) + \eta\}$, the
 909 following inequality holds

$$\phi'(h(u) - h(\bar{u})) \cdot \text{dist}(0, \partial h(u)) \geq 1. \quad (20)$$

910 If h satisfies the KL property at each point of $\text{dom}(\partial h)$, h is called a KL function.
 911

918 **Definition 4.** [Semialgebraic]
 919

920 (a) A function $h : \mathbb{R}^p \rightarrow \mathbb{R} \cup \{+\infty\}$ (resp. a point-to-set mapping $h : \mathbb{R}^p \rightrightarrows \mathbb{R}^q$) is called
 921 semialgebraic if its graph $\text{Graph}(h)$ is a semialgebraic set.

922 (b) A set $\mathcal{D} \subset \mathbb{R}^p$ is called semialgebraic if it can be represented as

$$924 \quad \mathcal{D} = \bigcup_{i=1}^s \bigcap_{j=1}^t \{\mathbf{x} \in \mathbb{R}^p : P_{ij}(\mathbf{x}) = 0, Q_{ij}(\mathbf{x}) > 0\},$$

925 where P_{ij}, Q_{ij} are real polynomial functions for $1 \leq i \leq s, 1 \leq j \leq t$.
 926

927 The class of semialgebraic sets are stable under the operation of finite union, finite intersection,
 928 Cartesian product or complementation. Some typical examples include polynomial functions, the
 929 indicator function of a semialgebraic set, and the Euclidean norm.

930 **Definition 5.** [Real analytic] A function h with domain an open set $U \subset \mathbb{R}$ and range the set
 931 of either all real or complex numbers, is said to be **real analytic** at u if the function h may be
 932 represented by a convergent power series on some interval of positive radius centered at u : $h(x) =$
 933 $\sum_{j=0}^{\infty} \alpha_j (x - u)^j$, for some $\{\alpha_j\} \subset \mathbb{R}$. The function is said to be **real analytic** on $V \subset U$ if it is
 934 real analytic at each $u \in V$. The real analytic function f over \mathbb{R}^p for some positive integer $p > 1$
 935 can be defined similarly.

936 Typical real analytic functions include polynomials, exponential functions, and the logarithm,
 937 trigonometric and power functions on any open set of their domains. One can verify whether a
 938 multivariable real function $h(\mathbf{x})$ on \mathbb{R}^p is analytic by checking the analyticity of $g(t) := h(\mathbf{x} + t\mathbf{y})$
 939 for any $\mathbf{x}, \mathbf{y} \in \mathbb{R}^p$.

940 Let $(\bar{W}, \bar{\Gamma}, \bar{g})$ be a critical point of F , then the following holds

$$941 \quad \begin{aligned} \partial_M F(\bar{M}, \bar{\Gamma}, \bar{g}) &= \alpha(\nabla \mathcal{L}(\bar{M}) + \nu^{-1}(\bar{M} - \bar{\Gamma})) = 0, \\ 942 \quad \partial_{\Gamma} F(\bar{M}, \bar{\Gamma}, \bar{g}) &= \alpha \nu^{-1}(\bar{\Gamma} - \bar{M}) + \partial \Omega(\bar{\Gamma}) - \bar{g} \ni 0, \\ 943 \quad \partial_g F(\bar{M}, \bar{\Gamma}, \bar{g}) &= \bar{\Gamma} - \partial \Omega^*(\bar{g}) \ni 0. \end{aligned} \quad (21)$$

944 By the final inclusion and the convexity of Ω , it implies $\bar{g} \in \partial \Omega(\bar{\Gamma})$. Plugging this inclusion into the
 945 second inclusion yields $\alpha \nu^{-1}(\bar{\Gamma} - \bar{M}) = 0$. Together with the first equality implies
 946

$$947 \quad \nabla \bar{\mathcal{L}}(\bar{M}, \bar{\Gamma}) = 0, \quad \nabla \mathcal{L}(\bar{M}) = 0.$$

948 This finishes the proof of this theorem.
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