Every Token Counts: Generalizing 16M Ultra-Long Context in Large Language Models

Anonymous ACL submission

Abstract

This work explores efficient ultra-long context modeling. We posit that an effective solution requires three fundamental properties: **sparsity**, **random-access flexibility**, and **length generalization**. To achieve this, we leverage Hierarchical Sparse Attention (HSA), a novel attention mechanism that satisfies all three properties. We integrate HSA into the Transformer architecture to develop HSA-UltraLong, an 8B-parameter Mixture-of-Experts (MoE) model trained on over 8 trillion tokens. We rigorously evaluate the model across tasks with both in-domain and out-of-domain context lengths to validate its capabilities. Our model demonstrates comparable performance to full-attention baselines on in-domain sequence lengths. Crucially, it achieves over 90% accuracy on most in-context retrieval tasks with contexts up to 512 times the pre-training context length. This work reports our findings and remaining issues throughout the experiments, offering insights for future research in ultra-long context modeling.

1 Introduction

Despite the impressive capabilities of Large Language Models (LLMs) (Brown et al., 2020; Achiam et al., 2023; Touvron et al., 2023), their world knowledge is confined to static parameters, making it inflexible to update and impossible to learn dynamically from daily user interactions. This limitation motivates a fundamental question: how can we build machines that truly remember? Effective memory is critical for future AI agents, enabling each user to have a personalized agent that accumulates unique experiences over time. Human memory spans the entire context from birth to the present, suggesting that the problem of machine memory is closely related to ultralong context modeling. Imagine if Transformers could efficiently handle infinite-length contexts—encompassing all pre-trained tokens—so that most world knowledge

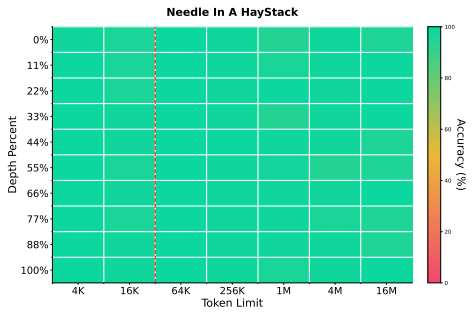


Figure 1: Despite being pre-trained with an 8K context window and mid-trained up to 32K, HSA-UltraLong achieves near-perfect accuracy on S-NIAH even at a 16M-token context length. The red dashed line at 32K marks the boundary between in-domain (left) and out-of-domain (right).

can be retrieved from context rather than compressed into model parameters. Furthermore, skills and the latest information could be acquired via in-context learning rather than through costly model retraining. Such advances would dramatically improve the online learning of knowledge and skills.

However, the Transformer (Vaswani et al., 2017) architecture, the backbone of modern LLMs, faces a fundamental efficiency challenge when processing ultra-long sequences, due to both poor length generalization and the quadratic computational complexity of full attention. Supporting longer contexts requires training models with extended context windows, yet simply scaling context length is computationally prohibitive. If we consider the extreme case of extending ultra-long context modeling to infinite context modeling, the following three points become necessary:

- **Sparsity**: Drawing inspiration from human long-term memory, which operates via selective activation and retrieval of relevant fragments (Cowan, 2008), full-attention for infinitely long contexts is clearly infeasible. Therefore, sparsity is a necessary prerequisite.
- **Random-Access Flexibility**: The utility of spar-

sity is predicated on the accurate retrieval of relevant past information. This necessitates designing an *intrinsic retrieval mechanism* within the model and optimizing it end-to-end under the guidance of an auto-regressive loss.

- **Length Generalization**: Pretraining with an infinite context is impossible. To achieve the goal, the path must involve generalizing retrieval ability from short to long contexts.

While several approaches show promising paths to achieve the goal, each presents notable shortcomings. Recurrent architectures, such as Mamba (Gu and Dao, 2023; Dao and Gu, 2024) and Linear Attentions (Katharopoulos et al., 2020; Yang et al., 2025b), compress past variable-length information into a fixed-dimensional state vector. This introduces an information bottleneck and sacrifices random access to distant tokens. Similarly, sliding-window attention (Beltagy et al., 2020) suffers from the same fundamental constraint on distant context accessibility. Sparse attention approaches like NSA (Yuan et al., 2025) and MoBA (Lu et al., 2025) improve training and inference efficiency over long sequences, but our empirical studies show they suffer from inaccurate chunk selection, which leads to both in-domain and out-of-domain performance degradation on in-context retrieval tasks.

A recent line of work that combines model-inherent retrieval (Mohtashami and Jaggi, 2023; Hu et al., 2025b) with chunk-wise sparse attention, such as Hierarchical Sparse Attention (HSA) (Hu et al., 2025a), has shown promising results in long-context modeling. Empirical studies (Leng et al., 2025) report that an HSA-based model pre-trained with a 4K context length can extrapolate to more than 10M context length while keeping high accuracy on the RULER (Hsieh et al., 2024) and BabiLong (Kuratov et al., 2024) benchmark, which simultaneously satisfies **sparsity**, **random-access flexibility** and **length generalization**. The method partitions text into fixed-length chunks with landmark representations; each token retrieves top-k relevant past chunks via these landmarks. The core innovation of HSA is to conduct attention with each chunk *separately*, and then *fuse the results weighted by the retrieval scores*. The overall process closely resembles the Mixture-of-Experts (MoE) (Shazeer et al., 2017), as illustrated in Figure 2. This design allows the retrieval scores to be integrated into the forward pass, enabling them to receive gradient updates during backpropagation. As a result, the model learns to assign higher retrieval scores to chunks that are more helpful for next token prediction. However, current work in this area is limited in scale and lacks results on data and parameter scaling.

We introduce HSA-UltraLong, an architecture combining sliding-window attention with HSA, and validate its effectiveness by training a 0.5B dense model and an 8BA1B MoE model from scratch on 8 trillion tokens. Through long-context extension and annealing, we verify that these models significantly enhance out-of-domain long-context capabilities while preserving in-domain performance without degradation. Our key findings include:

- Effective length generalization requires the combination of chunk-wise attention, retrieval score–based fusion, and NoPE (No Positional Encoding); all three are essential.
- Sliding-window attention and HSA interact in nontrivial ways. HSA's long-range generalization arises from learning to retrieve over short contexts and transferring that ability to long contexts. However, an overly large sliding window can weaken HSA's learning of short-range dependencies, degrading generalization.
- The effective context length in the training corpus strongly influences length extrapolation.

This work makes the first empirical demonstration of training-free length generalization—scaling from a 32K pretraining context to 16M tokens—in a model with billions of parameters trained on trillions of tokens. It details the training recipe and associated challenges, offering key insights for modeling ultra-long contexts.

2 Preliminary

2.1 Limitations of Chunk Selection in NSA

NSA is a highly inspiring contribution to sparse attention. However, our empirical study in Table 1 shows that its chunk selection mechanism does not always pick the most relevant chunk. On the RULER benchmark (Hsieh et al., 2024), NSA fails to achieve perfect accuracy even on in-domain context lengths such as Multi-Query NIAH. We trace this to a key limitation: the inaccurate chunk selection action. Further, our analysis of NSA's extrapolation ability indicates that performance on in-context retrieval degrades rapidly as context length increases. Regarding positional encoding, we also

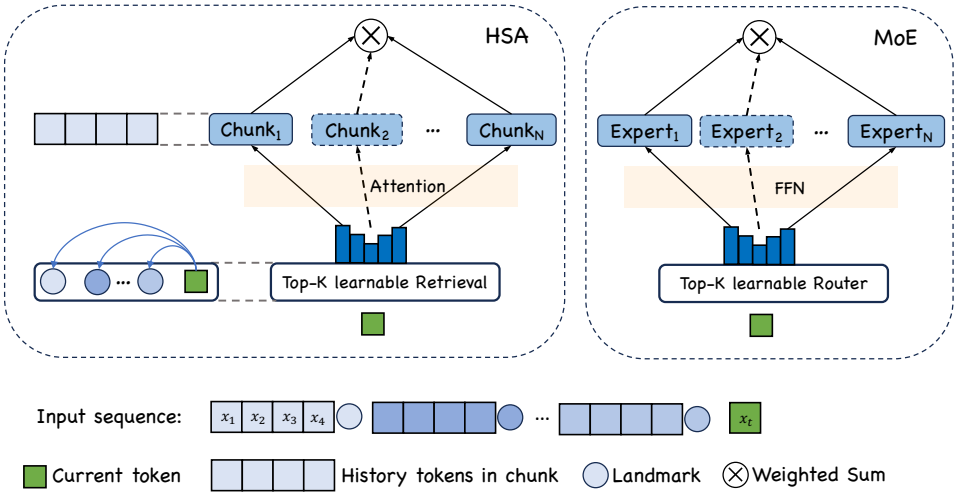


Figure 2: Hierarchical Sparse Attention (HSA) operates in a manner analogous to Mixture of Experts (MoE). First, the current token x_t computes dot products with the landmark representations of past chunks as retrieval scores, from which the top-k chunks are selected—similar to how MoE uses a router to select top-k experts. Subsequently, x_t performs attention with each of the k retrieved chunks **separately**, mirroring the process in MoE where x_t independently conducts Feedforward with k experts. Finally, the attention outputs from each chunk are weighted by the softmax-normalized retrieval scores and summed, which is functionally equivalent to MoE's fusion of outputs from the selected FFNs.

Table 1: NSA ablation with 4K as the in-domain length. The higher scores are shown in **bold**.

Models	#params	Single-NIAH (ACC ↑)					MQ-NIAH(ACC ↑)				
		4K	8K	16K	32K	64K	4K	8K	16K	32K	64K
NSA(w/ RoPE)	370M	97.0	90.0	83.0	73.0	60.0	72.0	50.0	24.0	15.0	4.0
NSA(w/o RoPE)	370M	**99.0**	**96.0**	**88.0**	**84.0**	**73.0**	**83.0**	**66.0**	**51.0**	**40.0**	**12.0**

find that No Positional Encoding (NoPE) supports extrapolation better than RoPE (Su et al., 2024).

2.2 Attention with Intrinsic Chunk Retrieval

As we mentioned, the challenge of sparse attention lies in accurately retrieving the previous chunks. Hierarchical Sparse Attention (HSA) addresses the challenge by jointly learning chunk selection and attention in an end-to-end manner. Compared to NSA, HSA mainly makes two contributions:

- **Retrieval-oriented sparse attention.** Specifically, each token conducts attention with each past chunk **separately** and then fuses the attention results via retrieval scores.
- **RoPE for short, NoPE for long.** To mitigate the negative impact of RoPE on extrapolation, the sliding-window attention's KV cache employs RoPE, while the HSA uses NoPE.

For an input sequence $\mathbf{S} = \{x_0, x_1, ..., x_n\}$, where n is the length of the sequence, we denote the hidden states of tokens as $\mathbf{H} \in \mathbb{R}^{n \times d}$, where d is the hidden dimension. The whole sequence is split into chunks according to a fixed length S, which is set to 64 by default to better align with hardware, thus we have $\frac{n}{S}$ chunks in total. We use indices with $[\cdot]$ to indicate that it is indexed by chunk rather than by token, e.g., $\mathbf{H}_{[i]} := \mathbf{H}_{iS:(i+1)S} \in \mathbb{R}^{S \times d}$. For each chunk, it has its own KV cache as $\mathbf{K}_{[i]}, \mathbf{V}_{[i]} \in \mathbb{R}^{S \times h \times d_h}$, with h as the number of heads satisfying $h \times d_h = d$, and its landmark representation as $\mathbf{L}_i \in \mathbb{R}^d$, which serves to summarize the content of the chunk. For each token, it uses $\mathbf{Q}_t^{slc} \in \mathbb{R}^d$ to retrieve chunks and $\mathbf{Q}_t^{attn} \in \mathbb{R}^{h \times d_h}$ to conduct attention with tokens inside chunks, both of which are derived from $\mathbf{H}_i$ via linear transformations.

$$
\begin{aligned}
\mathbf{Q}^{slc} &= \mathbf{W}^{slc}\mathbf{H}, \quad \mathbf{Q}^{attn} = \mathbf{W}^{attn}\mathbf{H} \\
s_{t,i} &= \begin{cases} \mathbf{Q}_t^{slc\top}\mathbf{L}_i/\sqrt{d}, & i \le \lfloor \frac{t}{S} \rfloor, \\ -\infty, & i > \lfloor \frac{t}{S} \rfloor, \end{cases} \\
\mathcal{I}_t &= \{\, i \mid \mathrm{rank}(s_{t,i}) < K \,\},
\end{aligned}
\tag{1}
$$

where $\mathrm{rank}(\cdot)$ denotes the ranking position in descending order, and $\mathcal{I}_t$ is the indices of K chunks

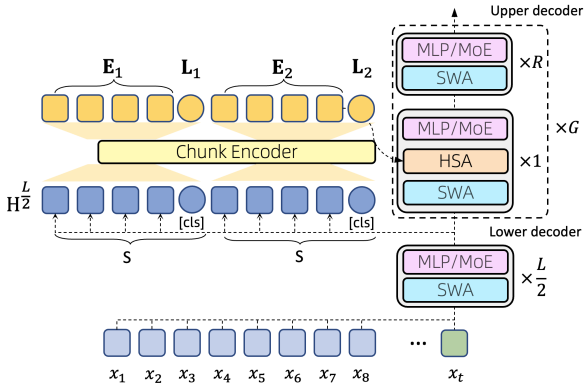


Figure 3: HSA-UltraLong model architecture.

with the highest relevance scores for x_t.

$$\begin{aligned}\bar{\mathbf{O}}_{t,i} &= \mathrm{Attention}\big(\mathbf{Q}_t^{attn}, \mathbf{K}_{[i]}, \mathbf{V}_{[i]}\big) \\ &= \underbrace{\mathrm{Softmax}\left(\frac{\mathrm{norm}(\mathbf{Q}_t^{attn})\ \mathrm{norm}(\mathbf{K}_{[i]}^{\top})}{\sqrt{d_h}}\right)\mathbf{V}_{[i]}}_{\textit{intra-chunk attention}},\end{aligned}$$

$$w_{t,i} = \frac{\exp(s_{t,i})}{\sum_{k\in\mathcal{I}_t}\exp(s_{t,k})}, \quad \mathbf{O_t} = \underbrace{\sum_{k\in\mathcal{I}_t} w_{t,k}\bar{\mathbf{O}}^{l}_{t,k}}_{\textit{inter-chunk fusion}}. \quad (2)$$

norm is the Query-Key Normalization (Dehghani et al., 2023; Wortsman et al., 2023), which we find to be very important for the stability of HSA in practical trillion-token scale training.

3 Methodology

In terms of model design, we use SWA for local information access and HSA for global information access, fusing both local and global information through a stacking approach. A key challenge of long sequence inference is that the KV cache grows with the sequence length. Previous works (Wu and Tu, 2024; Rubin and Berant, 2024; Sun et al., 2024) have demonstrated that sharing the KV cache can significantly compress its size while maintaining comparable results. Inspired by these works, we share the intermediate layer KV cache among all HSA modules to serve as context memory.

3.1 Model Architecture

Overall, as shown in Figure 3, our architecture contains L layers, partitioned into an upper decoder and a lower decoder.

- The lower decoder is composed of $\frac{L}{2}$ standard Transformer layers, each utilizing Sliding Windowed Attention (SWA).
- The upper decoder is divided into G groups. Each group consists of one Transformer layer featuring both SWA and Head-based Sparse Attention (HSA), followed by several layers employing SWA only.

We denote $\mathbf{H}^l$ as the output hidden states of the l-th layer. To build compressed long-context memories, the intermediate layer output $\mathbf{H}^{\frac{L}{2}}$ is fed to a bidirectional encoder chunk by chunk. Specifically, each $\mathbf{H}^{\frac{L}{2}}_{[i]} \in \mathbb{R}^{S\times d}$ is followed by a [CLS] (acting as the landmark token) before being processed by the bi-directional encoder. This process yields the chunk embedding $\mathbf{E}_{[i]} \in \mathbb{R}^{S\times d}$ and the landmark embedding $\mathbf{L}_i \in \mathbb{R}^d$ used in Equatin 1. Finally, the keys and values used in Equation 2 are obtained by applying a linear transformation to $\mathbf{E}_{[i]}$. Regarding MoE, we follow the design of Ling-2.0 (Ling-Team et al., 2025), where the first layer of the model adopts a dense MLP structure and all subsequent layers use MoE. Each MoE block has one shared expert, following the design in DeepSeek V3 (DeepSeek-AI et al., 2024). We use training-free balance strategy (Wang et al., 2024) as the expert balancing strategy.

3.2 Training

Previous work (Leng et al., 2025) demonstrates that with a 512-token sliding window, HSA-based models pre-trained on a 4K context length can generalize to 32M on the RULER task with high accuracy. However, such a small sliding window often sacrifices downstream performance. To address this, we increase the sliding window size to 4K tokens. However, when training models from scratch with a 4K sliding window, we observe that ***they fail to generalize beyond the 4K context length.*** We hypothesize that HSA's length generalization ability arises from its inherent retrieval mechanism, which learns to generalize from short to long contexts. An oversized sliding window covers all short-range information, rendering short-range retrieval unnecessary for HSA. This inactivates HSA, preventing it from learning the crucial short-context retrieval mechanism required for generalization to longer contexts. To overcome this limitation, we introduce a warmup stage before the pre-training phase. The whole pre-training procedure is as follows:

- **Warm-up.** We use a short sliding window attention (SWA) of 512 tokens with HSA, whose $topk$ is set large enough to cover all tokens, satisfying $topk \times chunk\_size = seq\_len$. Synthetic RULER tasks are randomly inserted into 1% of training samples. The warm-up phase is considered complete once the model achieves high

needle-in-a-haystack retrieval accuracy on contexts well beyond the 512-token window. At this step, the context length is set to 16K.

- **Pre-training.** Following the warm-up, training will resume from the checkpoint with the following parameter adjustments: the SWA window size is expanded to 4K, and the HSA's $topk$ is decreased such that $topk \times chunk\_size = 4K$. The overall context length remains 16K.
- **Long-context mid-training.** Switch to corpora with longer effective contexts and increase HSA $topk$ to cover all tokens. The context length is expanded to 32K.
- **Annealing.** Perform annealing on high-quality data while keeping a 32K context length.
- **Supervised fine-tuning.** Perform supervised fine-tuning (SFT) with an 8K context length.

4 Experiments

4.1 Small-scale Preliminary Experiments

In this section, we detail the architecture of HSA-UltraLong and validate its ability to balance in-domain performance with length extrapolation capabilities through small-scale experiments. We compared our model against Base Language Model (BaseLM), which uses full attention across all layers. HSA-UltraLong employs SWA with a window size of 4K across all layers, while strategically replacing two layers with HSA layers. The HSA layers maintain a SWA window size of 512 and, following our findings in Section 2.1, we removed positional encoding information from these layers. Each HSA layer includes an additional encoder sublayer, resulting in less than 5% parameter increase compared to BaseLM. Within the HSA layers, we set the chunk size to 64 and top-k to 64, establishing a fixed historical context window of 4,096 tokens.

Our initial experiments revealed limited length extrapolation capabilities when training the model directly with such configuration. We hypothesized that this limitation stemmed from the predominance of pretraining data requiring only short-range modeling capabilities within the 4K window of SWA layers, leaving the HSA modules insufficiently trained. To address this issue, we explored two warm-up strategies:

- **Self-copy warm-up:** We keep the model architecture unchanged and initialize training with a self-copy objective. Given an input sequence $\mathbf{S} = \{x_1, \ldots, x_n\}$, we construct a target sequence $\mathbf{S}' = \{x_1, \ldots, x_n, x_1, \ldots, x_n\}$ by concatenating $\mathbf{S}$ with itself. This objective encourages the model to attend to and retrieve long-range prefix information, enabling it to reconstruct the second half of the sequence.
- **Full HSA + Short SWA warm-up:** Setting top-k in HSA layers to 256 and sliding window size to 512 during the initial training phase.

All experiments were conducted on a 0.5B parameter dense model trained on 100B tokens with a pre-training context length of 16k. We incorporated 1% ruler-specific synthetic data into the pre-training data to facilitate evaluation using ruler benchmarks. Performance was evaluated based on the perplexity of the last 4k tokens on the PG19 dataset and accuracy on the Multi-key NIAH (MK-NIAH) task within the ruler benchmark.

The results in Table 2 demonstrated that the self-copy warm-up strategy yielded the best length extrapolation performance, albeit with some negative impact on in-domain performance. The full HSA + short SWA warm-up approach achieved a better balance, maintaining in-domain performance while delivering reasonable length extrapolation capabilities.

4.2 Large-scale experiments

4.2.1 Training & Evaluation Details

We detail the pre-training data ratios, hyperparameters, and evaluation datasets in the Appendix A.

4.2.2 General Tasks Evaluation

For HSA-UltraLong, we developed two variants: a 0.5B dense model and an 8B MoE model with 1B activated parameters. We compared the MoE variant against a standard Transformer-based model (TRM-MoE) with similar parameter count—trained on identical data with matching hyperparameters. The architectures are largely consistent, with only one structural difference: HSA-UltraLong modifies the MoE configuration from 32-expert/2-activated to 64-expert/4-activated with halved expert dimensions. Additionally, HSA-UltraLong uses 16K pretraining context compared to TRM-MoE's 4K. For evaluation, we used the TRM-MoE's 8T-token checkpoint (pre-annealing).

We benchmarked the dense variant against Qwen 2.5-0.5B (Yang et al., 2024) and Qwen3-0.6B (Yang et al., 2025a). These comparison mod-

Table 2: Preliminary experiments on HSA-UltraLong-Base with a training context length of 16k tokens. The highest and second-best scores are shown in **bold** and underlined, respectively.

Models	#params	Warmup	PG19 (PPL ↓)			MQ-NIAH(ACC ↑)			
			4K	8K	16K	4K	8K	64K	1M
BaseLM	519.6M	-	18.61	17.53	16.77	89.0	23.0	5.0	0.0
SWA+HSA	537.7M	self-copy	18.87	17.44	16.50	**100.0**	**96.0**	**93.0**	**93.0**
SWA+HSA	537.7M	short-swa,full-hsa	**18.30**	**17.13**	**15.96**	99.0	95.0	90.0	66.0

Table 3: Comparison among HSA-UltraLong-Base (HSA-UL-Base) and other baselines. All models were evaluated under a unified framework for fair comparison.

	Qwen2.5 Annealing	Qwen3 Annealing	HSA-UL Annealing	TRM-MoE Base	HSA-UL Base	HSA-UL Annealing
Architecture	Dense	Dense	Dense	MoE	MoE	MoE
# Total Params	0.5B	0.6B	0.5B	8B	8B	8B
# Activated Params	0.5B	0.6B	0.5B	1B	1B	1B
# Training Tokens	18T	36T	4T	8T	8T	8T
General Tasks						
BBH	32.27	41.28	18.15	50.34	51.70	60.11
ARC-C	55.25	66.10	46.10	72.20	67.80	71.53
AGIEval	30.01	33.58	29.29	38.64	36.52	44.08
HellaSwag	48.05	48.88	44.48	67.69	67.39	67.43
PIQA	70.46	71.33	70.29	77.48	78.84	80.69
MMLU	49.73	54.40	41.76	58.74	57.83	60.71
CMMLU	52.10	51.97	42.08	57.68	57.49	64.41
C-Eval	54.17	54.57	44.30	56.87	58.36	65.98
Math Tasks						
GSM8K	41.32	60.88	37.45	66.41	67.02	72.93
MATH	18.14	31.44	20.66	37.96	41.98	48.00
CMATH	52.09	66.67	60.75	74.59	74.13	82.88
Coding Tasks						
HumanEval+	24.39	26.83	29.27	48.17	50.61	61.59
MBPP+	32.80	38.36	20.63	50.26	55.82	62.17
CRUX-O	14.38	31.62	22.56	35.12	36.31	40.75
AVG	41.08	48.42	37.70	56.58	57.27	63.09

els have similar parameter counts but were trained on substantially larger datasets—4.5 times and 9 times our training data volume, respectively.

Our primary evaluation focused on assessing model performance within the pretraining context length across standard benchmarks. Results in Table 3 show that the HSA-UltraLong-MoE achieved parity with TRM-MoE in average performance scores, while the dense variant demonstrated only a 3.3-point deficit compared to Qwen 2.5-0.5B, despite having significantly less training data.

Additionally, we evaluated the MoE and Dense models after supervised fine-tuning. The results in Table 4 indicate that while a few general tasks showed no significant performance improvement after supervised fine-tuning, most tasks—particularly math and coding tasks—demonstrated substantial enhancements compared to the base models. Notably, our HSA-UltraLong-MoE achieved scores averaging 1.3 points higher than Qwen3-1.7B (Non-thinking), despite requiring fewer training flops. Similarly, our dense variant performed competitively, scoring only approximately 4 points below Qwen3-0.6B, despite being trained on a dataset merely one-ninth the size.

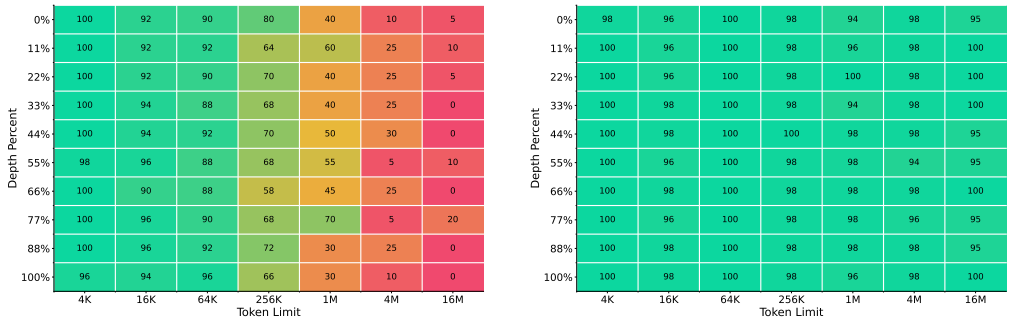
These findings demonstrate that HSA-UltraLong models maintain their capabilities within standard contexts while extending their effective context length to 16M tokens, further highlighting the superiority of our architectural approach.

4.2.3 Long-context Evaluation

During training, we randomly convert samples into RULER tasks with a 1% probability by inserting a "needle" in a long context and appending the Needle-in-a-Haystack (NIAH) prompt and answer at the end of the sample so the model can follow the NIAH instructions. This modification serves as a probe task to evaluate the model's extrapolation ability while having minimal impact on training.

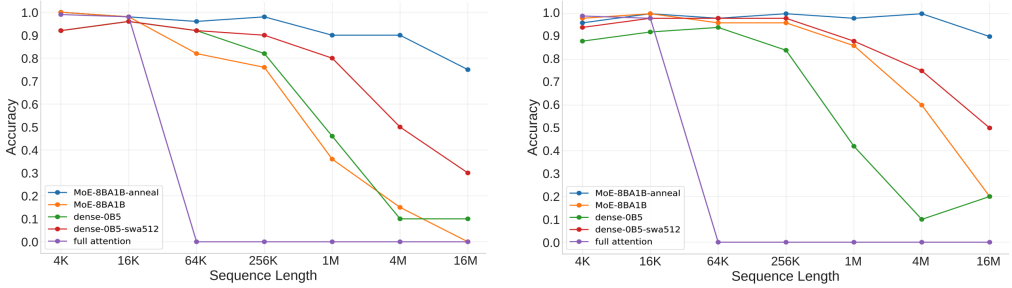
Table 4: Comparison among HSA-UltraLong-Inst (HSA-UL-Inst) and Qwen3 (Non-thinking) after supervised fine-tuning. All models were evaluated under a unified framework for fair comparison.

	Qwen3-Inst	HSA-UL-Inst	Qwen3-Inst	HSA-UL-Inst
Architecture	Dense	Dense	Dense	MoE
# Total Params	0.6B	0.5B	1.7B	8B
# Activated Params	0.6B	0.5B	1.7B	1B
# Training Tokens	36T	4T	36T	8T
General Tasks				
BBH	42.56	26.25	59.48	57.25
MMLU	45.87	42.24	63.05	61.34
CMMLU	41.64	43.33	60.84	64.06
C-Eval	43.81	45.41	62.70	62.86
Math Tasks				
GSM8K	55.65	55.42	79.00	82.94
MATH	45.26	40.76	64.32	61.56
MATH500	53.00	41.00	73.20	71.00
OlympiadBench	16.89	8.74	36.30	27.85
Coding Tasks				
HumanEval	40.24	39.63	65.24	71.95
MBPP	29.20	34.40	51.00	57.00
HumanEval+	35.37	37.20	61.59	70.73
MBPP+	34.39	39.95	59.52	65.87
CRUX-O	28.00	23.25	50.00	50.75
Alignment Tasks				
IFEval Strict Prompt	55.08	33.09	64.33	63.22
AVG	40.50	36.48	60.76	62.03



(a) Before long-context mid-training.

(b) After long-context mid-training.



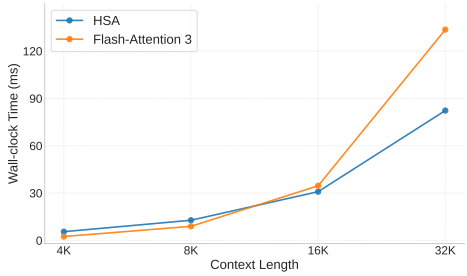
(c) Multi-Query NIAH Task.

(d) Variable Tracking Task.

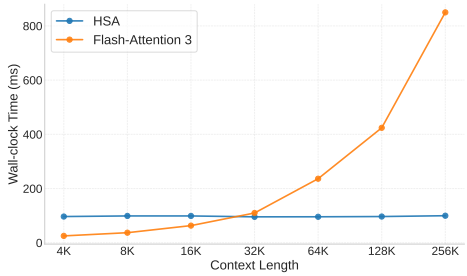
Figure 4: Evaluation of length generalization using the Needle-in-a-Haystack test. (a) and (b) present the results of the HSA-UltraLong-MoE before and after the long-context continued training phase on the Single-NIAH task at various depths. In (c) and (d), we evaluate the performance of different models on the Multi-Query NIAH Task (2 queries, 6 key-value pairs) and the Variable Tracking Task.

The results for RULER tasks are reported in Figure 4, we identify three key findings:

- **Effective context length of training data is critical for HSA extrapolation.** As shown in Figure4(a), models pretrained on standard corpora exhibit a progressive decline in retrieval accuracy with longer contexts. This occurs despite a 16K pretraining context window, because the effective context length of the data is often much shorter. In contrast, training on data with longer effective



(a) Training efficiency.



(b) Inference efficiency.

Figure 5: Comparison of training/inference efficiency between HSA kernel and Flash-attention 3.

contexts (>32K), as in Figure4(b), yields substantially improved extrapolation. This principle underpins the trends in Figures 4(c) and (d).

- **A seesaw effect exists between HSA and Sliding Window Attention.** Figures 4(c) and (d) indicate that a smaller SWA window (512) during continued pretraining leads to better HSA extrapolation than a larger window (4K). Given that training from scratch with a 4K window fails to develop extrapolative HSA, we conclude that larger SWA windows impair HSA's long-range generalization. We posit that HSA learns a form of retrieval-based extrapolation. Large SWA windows handle most short-range dependencies inherently, reducing the incentive for HSA to learn them and thus weakening its ability to generalize to longer sequences.

- **HSA capability scales with parameter size in reasoning-retrieval tasks.** While MoE-8B-A1B and Dense-0.5B exhibit comparable performance on the pure retrieval task (MQ-NIAH; Figure 4(c)), MoE-8B-A1B consistently outperforms Dense-0.5B in the variable-tracking task (Figure 4(d)), demonstrating that larger models better support joint reasoning and retrieval.

4.3 Training/Inference Efficiency

To further evaluate the efficiency of the sparse attention module, we benchmark the HSA operator against the FlashAttention-3 (Shah et al., 2024) operator on H800 for both training and inference, with HSA implemented using TileLang (Wang et al., 2025). As shown in Figure 5, at shorter sequence lengths, FlashAttention-3 still leads in both training and inference, and HSA only gains an advantage with longer contexts. We attribute this to two factors: (1) the sparsity in HSA causes the kernel to incur more memory accesses compared to FlashAttention-3; and (2) FlashAttention-3 is implemented in CUDA, enabling it to better leverage the features of the Hopper architecture.

5 Related Works

Many works have explored sparse attention for efficient long-context modeling, which broadly fall into two lines: token-level sparsity (Lou et al., 2024; Gonçalves et al., 2025; DeepSeek-AI et al., 2025) and chunk-level sparsity. A central challenge in chunk-level sparse attention is how to accurately select relevant past chunks. NSA (Yuan et al., 2025) compresses the key–value pairs within each chunk into a single key–value pair and uses the resulting compressed attention to guide chunk selection. MoBA (Lu et al., 2025) heuristically selects chunks by summing unnormalized attention logits within each chunk. Seer Attention (Gao et al., 2025) learns sparse patterns via distillation from full attention. Our work is mainly based on HSA (Hu et al., 2025a), a chunk-level sparse attention mechanism. The original HSA framework is coupled with Mamba layers; however, in our scaling experiments, we observe that extrapolation degrades as model size increases. To stabilize extrapolation under scaling, we replace Mamba with SWA. Compared with NSA, MoBA, and Seer Attention, the key distinction of HSA is end-to-end learned chunk retrieval, without distillation or heuristic approximations, which enables efficient training while achieving higher in-domain accuracy and preserving extrapolation capability.

6 Conclusion

In this work, HSA-UltraLong presents a highly promising paradigm for long-context processing. The core insight of HSA is to *perform attention chunk by chunk and fuse the results via retrieval scores*, rather than selecting chunks and then concatenating them for attention. The experimental results provide a meaningful step toward effectively handling infinite-long context, advancing progress on long-term memory in machines.

Limitations

Although HSA has shown promising extrapolation capabilities, several challenges remain:

- The HSA/SWA seesaw problem. After training on short SFT data, extrapolation can degrade. The main reason is that an excessively long sliding-window attention reduces the need for HSA to learn short-range dependencies, which in turn hampers its ability to extrapolate to long-range dependencies.
- The head ratio constraint. HSA currently requires a 16:1 ratio of query heads to key–value heads, creating a severe information bottleneck. Future work should pursue kernel-level optimizations to alleviate this constraint.
- When sequences are short, training and inference show no clear advantage over FlashAttention-3; further kernel-level optimizations are needed to improve efficiency.

Furthermore, there is a slight discrepancy in the comparison between HSA-UL and TRM-MoE in Table 3. While the total MoE parameters are identical, the baseline employs a top-2 routing strategy among 32 experts (each with 1024 dimensions), whereas HSA-UL employs a top-4 strategy among 64 experts (each with 512 dimensions). However, this difference does not significantly impact the overall conclusion: HSA-based models maintain efficient extrapolation capabilities even as parameter counts and data volumes scale up.

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A Pre-training setups

Training Data In the first phase of general pre-training, we utilized a large-scale deduplicated, multi-domain dataset totaling 10T tokens, with differential sampling ratios across various sub-datasets from different domains. This distribution comprised predominantly Web content (50%), followed by Code (14.4%), Math (12.0%), Code-nlp (5.6%), Reason (5%), Multilingual (4.0%), Books (2.0%), Wikipedia (1.5%), and Others (5.5%). During this phase, the MoE model processed 8T tokens while the dense model was trained on 4T tokens. The second phase uses a dataset of 32K-length long-text sequences totaling 175B tokens and the third phase consisted of 400B tokens with a high proportion of reasoning data. During the Supervised Fine-tuning phase, we utilized the same dataset as described in (Wu et al., 2025).

Hyperparameters All models are trained using AdamW optimizer with a weight decay of 0.01, $\beta_1 = 0.9$, $\beta_2 = 0.95$, and gradient clipping norm of 1.0. We use FSDP2 for distributed training. For the MoE model, we employ a learning rate of 3.87e-4, sequence length of 16,384, and batch size of 16.8M tokens. The dense model is trained with a learning rate of 4.96e-4 and batch size of 5.2M tokens. The learning rate schedule begins with a linear warmup phase followed by a constant learning rate maintained until training completion. For the Supervised Fine-tuning stage, we adopt a cosine decay learning rate schedule. The dense model uses a learning rate of 5.5e-5 and was trained for up to 5 epochs, while the MoE model uses a learning rate of 3.87e-4 and was trained for up to 3 epochs. In both cases, we select the checkpoint from the epoch that yields the best performance.

Evaluation Benchmarks To conduct a comprehensive evaluation of the model, we selected a diverse range of assessment tasks, encompassing four major categories: general tasks, mathematical tasks, coding tasks, and alignment tasks:

- **General Tasks:** MMLU (Hendrycks et al., 2020), CMMLU (Li et al., 2023), C-Eval (Huang et al., 2023), ARC (Bhakthavatsalam et al., 2021), AGIEval (Zhong et al., 2024), PIQA (Bisk et al., 2020), HellaSwag (Zellers et al., 2019) and BBH (Suzgun et al., 2022).
- **Math Tasks:** GSM8K (Cobbe et al., 2021), MATH (Hendrycks et al., 2021), CMATH (Wei et al., 2023), MATH-500 (Lightman et al., 2023) and Olympiad-Bench (He et al., 2024).
- **Coding Tasks:** HumanEval (Chen et al., 2021), HumanEval+ (Liu et al., 2023), MBPP (Tao et al., 2024), MBPP+ (Liu et al., 2023), and CRUX-O (Gu et al., 2024).
- **Alignment Tasks:** We report the average prompt-level strict accuracy of IFEval (Zhou et al., 2023)