Reinforcement Learning Improves Traversal of Hierarchical Knowledge in LLMs

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Abstract

Reinforcement learning (RL) is often credited with improving language model reasoning and generalization at the expense of degrading memorized knowledge. We challenge this narrative by observing that **RL-enhanced models consistently outperform their base and supervised fine-tuned (SFT) counterparts on pure knowledge recall tasks**, particularly those requiring traversal of hierarchical, structured knowledge (e.g., medical codes). We hypothesize these gains stem not from newly acquired data, but from improved procedural skills in navigating and searching existing knowledge hierarchies within the model parameters. To support this hypothesis, we show that structured prompting—which explicitly guides SFTed models through hierarchical traversal—recovers most of the performance gap (reducing 24pp to 7pp on MedConceptsQA for DeepSeek-V3/R1). We further find that while prompting improves final-answer accuracy, RL-enhanced models retain superior ability to recall correct procedural paths on deep-retrieval tasks. Finally our layer-wise internal activation analysis reveals that while factual representations (e.g., activations for the statement "code 57.95 refers to urinary infection") maintain high cosine similarity between SFT and RL models, query representations (e.g., "what is code 57.95") diverge noticeably, indicating that **RL primarily transforms how models traverse knowledge rather than the knowledge representation itself**.

1 Introduction

Large Language Models (LLMs) acquire vast parametric knowledge during pre-training, encoding facts, concepts, and their relationships across billions of parameters. Post-training techniques—including supervised fine-tuning (SFT), Reinforcement Learning from Human Feedback (RLHF), and specialized reasoning-focused RL—are then applied to transform these base models into instruction-following agents capable of complex reasoning (Yu et al., 2025; Wang et al., 2025c; Bai et al., 2022). While these methods improve performance on reasoning benchmarks and user preference metrics, a growing body of evidence reveals a concerning trade-off known as the "alignment tax" (Lin et al., 2024; Askell et al., 2021; Sorensen et al., 2025): models sacrifice factual memorization capabilities to optimize for other objectives, leading to reduced performance on knowledge-intensive benchmarks (Yuan et al., 2024; Gekhman et al., 2024). However, existing work has primarily focused on direct factual recall tasks over unstructured knowledge, leaving a critical gap: *do these degradation patterns hold for all forms of parametric knowledge retrieval tasks?*

To address this question, we investigate tasks where retrieval demands navigating hierarchical structures encoded within the model's parameters. Consider medical code lookup (Figure 3): to identify that ICD-9-CM code 57.95 refers to "Replacement of indwelling urinary catheter," a model can attempt direct recall—often failing due to the vast code space—or systematically traverse the taxonomy (Chapter 11 → codes 57.0-57.99 → specific procedure). Surprisingly, reasoning-enhanced models outperform their base counterparts by 24 percentage points on MedConceptsQA, directly **challenging the conventional wisdom that RL sacrifices memorization for reasoning** (Ghosh et al., 2024; Chu et al., 2025). We hypothesize these models succeed through systematic hierarchical navigation rather than direct recall, proposing that **reinforcement learning enhances navigation of ex-**

isting parametric knowledge rather than adding new factual content.

To disentangle knowledge acquisition from navigation, we design three complementary experiments. First, inspired by work showing prompt optimization can match RL gains (Agrawal et al., 2025; Khattab et al., 2023; Ziems et al., 2025), we develop structured prompting that explicitly guides base models through hierarchical traversal. If knowledge exists in base models, prompting should surface it. **Structured prompting reduces the 24pp gap between DeepSeek-V3 and DeepSeek-R1 to 7pp, suggesting information is present but inaccessible without proper navigation** (Figure 3, right-hand side). Second, to validate that improved traversal drives these gains, we introduce a complexity-stratified patent classification dataset and Path Matching Score metric measuring traversal accuracy. We show that as recall depth increases (from fewer than 3 hops to more than 5), **reasoning models demonstrate superior path recall accuracy**, with the performance gap widening from 5pp to 9pp, demonstrating that reasoning models excel at complex hierarchical navigation (Table 5).

Third, to provide internal validation, we conduct layer-wise representational analysis inspired by work examining how post-training modifies internal model structure (Mukherjee et al., 2025; Skean et al., 2025; Huan et al., 2025; Agarwal et al., 2024). We extract layer-wise representations for matched query-answer pairs, comparing interrogative queries (e.g., "What is the medical code 57.95?") versus declarative statements (e.g., "Code 57.95 refers to urinary catheter replacement"). We find a striking pattern (Figure 2): declarative statements maintain high cosine similarity (0.85-0.92) between base and RL models throughout most layers, while interrogative queries diverge substantially (similarity dropping to 0.65-0.73 in middle layers). This asymmetry reveals that **RL and instruction tuning primarily transforms how models process *questions* while leaving factual knowledge representations intact**, consistent with our hypothesis that RL enhances navigation mechanisms rather than knowledge content.

We further conduct ablation studies comparing distilled R1 models to R1 and base models (Kim et al., 2025; Chen et al., 2025a), finding that distilled models capture only surface-level improvements without acquiring robust navigation capabilities—achieving intermediate performance on complex retrieval tasks. Structured prompting provides minimal gains for distilled models, and layer-wise analysis reveals greater representational changes than instruction-tuned variants, yet without improved deep-retrieval navigation.

Our findings carry important implications: RL-enhanced models succeed not through expanded knowledge but through improved cognitive scaffolding—the ability to systematically traverse structures already encoded during pretraining, which is inline with recent work showing that RL surfaces intelligence (Huan et al., 2025; Wu et al., 2025). While our experiments focus on two datasets (MedConceptsQA and IPC) and specific model families (Qwen2.5, DeepSeek, Mistral), the patterns suggest more efficient training paradigms separating knowledge acquisition (pretraining) from organization (post-training). We encourage future work to investigate these phenomena across broader domains and develop RL mechanisms that explicitly optimize for hierarchical navigation.

Research Questions

1. **RQ1: Does explicit prompting close the performance gap?** If instruction-tuned models contain the required knowledge, can structured prompts that explicitly instruct hierarchical traversal match the performance of RL-enhanced models?
2. **RQ2: Do reasoning models navigate deeper hierarchies better?** On tasks requiring multi-step hierarchical traversal, do reasoning models demonstrate superior path accuracy beyond what prompting achieves?
3. **RQ3: How do internal representations differ?** Do reasoning models transform how they encode queries, factual knowledge, or both?

2 Experimental Methodology

Our investigation into how reinforcement learning enhances hierarchical knowledge traversal is guided by three research questions:

We address these questions through three complementary experiments. Section 2.1

demonstrates that structured prompting can induce hierarchical reasoning in instruction-tuned models, reducing the performance gap by up to 68%. Section 2.2 introduces retrieval tasks of varying complexity with a path matching metric, revealing that reasoning models excel particularly on deep-retrieval tasks requiring extensive hierarchical navigation. Section 2.3 presents layer-wise activation analysis showing that while factual representations remain largely unchanged, query processing diverges substantially between SFT and RL models, supporting our hypothesis that RL primarily enhances navigation mechanisms rather than knowledge content.

2.1 Hierarchical Navigation Through Structured Prompting

We investigate tasks requiring pure information recall without multi-step computation or logical deduction, to determine whether the performance gap between base and reasoning models can be mitigated through prompting strategies alone. Remarkably, structured prompting reduces the performance gap for 671B base models such as DeepSeek-V3 from 23.7 pp to 7.5 pp (a 68% gap reduction), demonstrating the effectiveness of our method.

Datasets

- **MedConceptsQA:** A multiple-choice question answering dataset focused on biomedical and clinical concepts. The questions are designed to test factual recall of medical terminology, concept definitions, and their relationships, without reasoning over patient cases or performing calculations.
- **International Patent Classification (IPC):** A dataset consists of queries mapped to patent classification codes. The task requires identifying the correct category for a given technical description, relying on recalling standardized knowledge of patent domains rather than multi-step reasoning.

Prompting

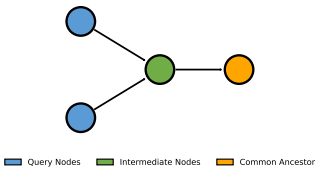
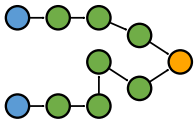
- **Direct QA Prompting**: This baseline requires the model to provide only a single-letter answer to each multiple-choice question without any explanation.
- **Standard CoT Prompting**: This template requests both a final answer and a supporting explanation, aiming to capture the model's intrinsic reasoning without imposing any procedural constraints.
- **Structured Prompting**: We introduce hierarchical instructions that enforce systematic reasoning. This strategy involves a two-stage process: (1) recall the hierarchical structural breakdown of the relevant medical code or concept, and (2) systematically evaluate each option with justification before elimination. This approach tests our hypothesis that enforcing structured knowledge recall and stepwise elimination can reduce performance gaps (see Appendix B.1 for complete prompt templates).

Models We evaluate a diverse set of large language models, focusing on comparisons between base, instruction-tuned, reasoning, and distilled models. The first group includes instruction-tuned models, such as the Qwen2.5 family (7B, 14B, 32B, and 72B parameters) (Team et al., 2024) and Mistral-Small-3.1-24B-Instruct (Karamcheti et al., 2021), each paired with their respective base models. The second group consists of reasoning models, including QwQ-32B (reasoning-enhanced Qwen2.5-32B), DeepSeek-R1 (from DeepSeek-V3), Magistral (from Mistral-Small-3.1-24B), and the reasoning model of Qwen3-235B-A22B (Liu et al., 2024; Guo et al., 2025a; Yang et al., 2025a). The third group includes models distilled from DeepSeek-R1: Qwen2.5-Math-7B, Qwen2.5-32B, and Llama3.3-70B, each compared against their pre-distillation ones (Grattafiori et al., 2024). We sample from all models using a temperature of 0.8 and top-p of 0.7 across three independent runs. Performance is reported as both mean accuracy ($\pm$ standard deviation) and majority-voted accuracy, where majority voting selects the most frequent answer among the three runs for each question.

2.2 Hierarchical Navigation Across Retrieval Complexity

While we previously conclude that reasoning models use hierarchical navigation that can be externalized through structured prompting, a fundamental question

Table 1: Stratification of the "Nearest Common Ancestor" task by retrieval complexity, defined by the number of unique ancestor nodes (traversals) recalled to find the common node.

Task Complexity	Traversals	Figure Example	Example
Memory-Light	< 3		**Question:** Nearest common ancestor of H04B 1/7075 and H04B 1/7083 is: A) H04B 1/707 B) H04B 1/7073 C) H04B 1/69 D) H04B **Hierarchical Paths:** • H04B 1/7075 → H04B 1/7073 • H04B 1/7083 → H04B 1/7073 **Answer:** B
Memory-Heavy	5+		**Question:** Nearest common ancestor of A01B 3/421 and A01B 15/06 is: A) A01B 3/00 B) A01B 15/00 C) A01B D) A01 **Hierarchical Paths:** • A01B 3/421 → A01B 3/42 → A01B 3/40 → A01B 3/36 → A01B 3/00 → A01B • A01B 15/06 → A01B 15/04 → A01B 15/02 → A01B 15/00 → A01B **Answer:** C

remains: *do reasoning models merely execute these strategies more consistently, or are there tasks that they execute fundamentally better?* To address this, we need to analyze not just whether models retrieve correct answers but how they traverse knowledge hierarchies to reach those answers. Therefore, we extend the original IPC dataset to stratify it by retrieval complexity and introduce a new metric to measure path traversal quality. Subsequent results reveal that reasoning models show superior hierarchical traversal-an ability that emerges on complex tasks requiring deeper knowledge navigation.

IPC Multi-Level Retrieval Dataset As shown in Table 1, this expanded dataset tests basic structural knowledge, including identifying common ancestors of a given pair of nodes. The questions are categorized by retrieval complexity, defined as the total number of ancestor nodes that must be recalled along both hierarchical paths (excluding the initial query nodes) to reach the nearest common ancestor. This stratification allows us to isolate the effect of retrieval depth on model performance.

- **Memory-Light (ML)** tasks require recalling < 3 ancestor nodes total across both paths to reach the common ancestor.
- **Memory-Heavy (MH)** tasks demand recalling ≥ 5 ancestor nodes across both paths.

Path Matching Score To evaluate the quality of predicted hierarchical paths for IPC codes, we propose the path matching score, which combines two metrics:

- **F1-Score:** Measures precision and recall of hierarchical ancestor identification, defined as $F_1 = \frac{2 \times P \times R}{P+R}$, where P and R denote precision and recall over the set of hierarchical ancestors (Buckland and Gey, 1994).
- **Common Subsequence Score (CSS):** Evaluates structural integrity of sequential paths via the ratio of the Longest Common Subsequence (LCS) (Paterson and Dančík, 1994) between the predicted and true paths to the length of the true path: $\text{CSS} = \frac{|\text{LCS}(\text{predicted,ground truth})|}{|\text{ground truth ancestors}|}$.

The path matching score combines both components via harmonic mean: $\text{Path Matching} = \frac{2 \times F_1 \times \text{CSS}}{F_1 + \text{CSS}}$. This metric captures both structural accuracy and hierarchical coherence in patent classification navigation.

Models To analyze the impact of retrieval complexity, we conduct a case study using the DeepSeek-V3 and R1 pair on our expanded IPC dataset. While a broader evaluation would be ideal, we select the DeepSeek pair due to their instruction-following capabilities suitable for a reliable analysis.

2.3 Hierarchical Navigation in Internal Representations

To investigate whether base and specialized models[1] possess equivalent knowledge

[1]Here "base models" refer to the foundation model from which "specialized models" (instruction-tuned/reasoning/distilled) variants are derived. We adopt this terminology throughout the section to clearly distinguish the two categories.

Table 2: Performance comparison of Instruct vs. Reasoning models on MedConceptsQA and IPC datasets. The first column indicates the dataset. Models are evaluated across three prompt templates (QA, CoT, Structured). Metrics shown are majority voting accuracy (Maj. Vote Acc.) and mean accuracy (Mean Acc.). Mean accuracy is reported as Mean Acc. $_{\text{(Std.)}}$, with the standard deviation in subscripted parentheses. For each model pair, a Δ row shows the gap from the reasoning model for both Maj. Acc. (red) and Mean Acc. (green). Bold values indicate the best performance within each model pair. Δ values are highlighted, with darker shades indicating larger gaps.

Dataset	Model	Model Type	Maj. Vote Acc.			Mean Acc.$_{\text{(Std.)}}$		
			QA	CoT	Structured	QA	CoT	Structured
MedConceptsQA	Qwen2.5-32B	Instruct	0.379	0.475	0.469	$0.371_{(.012)}$	$0.449_{(.010)}$	$0.454_{(.007)}$
		Reasoning	**0.482**	**0.513**	**0.505**	$\mathbf{0.470_{(.012)}}$	$\mathbf{0.487_{(.009)}}$	$\mathbf{0.481_{(.005)}}$
		Δ	**+0.103**	**+0.038**	**+0.036**	**+0.099**	**+0.038**	**+0.027**
	Qwen3-235B-A22B	Instruct	0.542	0.548	**0.631**	$0.503_{(.004)}$	$0.528_{(.005)}$	$\mathbf{0.589_{(.007)}}$
		Reasoning	**0.641**	**0.656**	0.580	$\mathbf{0.599_{(.003)}}$	$\mathbf{0.617_{(.003)}}$	$0.554_{(.008)}$
		Δ	**+0.099**	**+0.108**	**-0.051**	**+0.096**	**+0.089**	**-0.035**
	DeepSeek-V3	Instruct	0.541	0.632	0.717	$0.551_{(.014)}$	$0.636_{(.049)}$	$0.701_{(.026)}$
		Reasoning	**0.778**	**0.790**	**0.792**	$\mathbf{0.830_{(.006)}}$	$\mathbf{0.774_{(.013)}}$	$\mathbf{0.775_{(.026)}}$
		Δ	**+0.237**	**+0.158**	**+0.075**	**+0.279**	**+0.138**	**+0.074**
IPC Codes	Qwen2.5-32B	Instruct	0.759	0.754	0.774	$0.759_{(.007)}$	$0.754_{(.000)}$	$0.774_{(.007)}$
		Reasoning	**0.777**	**0.875**	**0.790**	$\mathbf{0.713_{(.015)}}$	$\mathbf{0.754_{(.070)}}$	$\mathbf{0.769_{(.033)}}$
		Δ	**+0.018**	**+0.121**	**+0.016**	**-0.046**	**+0.000**	**-0.005**
	Qwen3-235B-A22B	Instruct	0.800	**0.846**	0.846	$0.800_{(.013)}$	$\mathbf{0.846_{(.013)}}$	$0.846_{(.013)}$
		Reasoning	**0.908**	0.877	**0.893**	$\mathbf{0.846_{(.013)}}$	$0.836_{(.026)}$	$\mathbf{0.851_{(.015)}}$
		Δ	**+0.108**	**+0.031**	**+0.047**	**+0.046**	**-0.010**	**+0.005**
	DeepSeek-V3	Instruct	0.831	**0.923**	0.877	$0.846_{(.000)}$	$\mathbf{0.882_{(.007)}}$	$0.872_{(.007)}$
		Reasoning	**0.923**	0.892	**0.923**	$\mathbf{0.913_{(.019)}}$	$0.867_{(.026)}$	$\mathbf{0.903_{(.007)}}$
		Δ	**+0.092**	**-0.031**	**+0.046**	**+0.067**	**-0.015**	**+0.031**

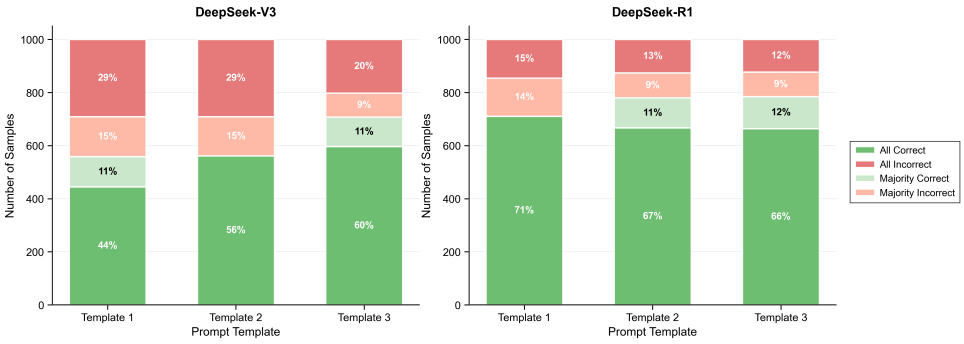


Figure 1: Comparative performance analysis of DeepSeek-V3 and DeepSeek-R1 across prompt strategies: direct question-answering (Template 1), chain-of-thought (Template 2), and structured prompting (Template 3) on MedConceptsQA dataset. Four categories are defined based on the number of correct votes across three independent runs: "All Incorrect" (0/3 correct), "Majority Incorrect" (1/3 correct), "Majority Correct" (2/3 correct), and "All Correct" (3/3 correct).

for hierarchical reasoning, we analyze their internal activations on MedConceptsQA using contrastive question-answer pairs. We conduct two complementary analyses: an *inter-model* comparison to show how enhancement modifies representations relative to the base model, and an *intra-model* comparison to trace how individual models transform questions into answers across layers. Our findings show that enhancement refines query processing while preserving factual knowledge.

Probe Construction. We construct probes from the MedConceptsQA dataset, which spans five medical vocabularies: ATC, ICD9CM, ICD10CM, ICD9PROC, and ICD10PROC. To ensure balanced representation, we randomly sample 100 question-answer pairs from each vocabulary. Each probe consists of a factual question and its corresponding ground-truth answer, formatted as declarative statements. For example, a probe for medical code `0QD20Z` from `ICD10PROC` takes the following form:

> *Question:* What is the description of the medical code `0QD20Z` in `ICD10PROC`?
>
> *Answer:* The description of the medical code `0QD20Z` in `ICD10PROC` is extraction of right pelvic bone, open approach.

We process questions and answers inde-

Table 3: Performance comparison of Base vs. Instruct models on MedConceptsQA and IPC datasets. The first column indicates the dataset. Models are evaluated across three prompt templates (QA, CoT, Structured). Metrics shown are majority voting accuracy (Maj. Vote Acc.) and mean accuracy (Mean Acc.). Mean accuracy is reported as Mean Acc.$_{(\text{Std.})}$, with the standard deviation in subscripted parentheses. For each model pair, an Δ row shows the gap from the instruct model for both Maj. Acc. (red) and Mean Acc. (green). Bold values indicate the best performance within each model pair. Δ values are highlighted, with darker shades indicating larger gaps. **This gap shrinks as we optimize the prompt, showing that the knowledge exists in the instruct model, it just needs to surface.**

Dataset	Model	Model Type	Maj. Vote Acc.			Mean Acc.$_{(\text{Std.})}$		
			QA	CoT	Structured	QA	CoT	Structured
MedConceptsQA	Qwen2.5-7B	Base	0.148	0.277	0.286	0.159$_{(.007)}$	0.239$_{(.036)}$	0.270$_{(.012)}$
		Instruct	**0.295**	**0.329**	**0.313**	**0.289$_{(.006)}$**	**0.316$_{(.008)}$**	**0.307$_{(.015)}$**
		Δ	**+0.147**	**+0.052**	**+0.027**	**+0.130**	**+0.077**	**+0.037**
	Qwen2.5-14B	Base	0.335	0.332	0.386	0.316$_{(.015)}$	0.293$_{(.025)}$	0.372$_{(.007)}$
		Instruct	**0.395**	**0.420**	**0.420**	**0.385$_{(.006)}$**	**0.415$_{(.007)}$**	**0.409$_{(.012)}$**
		Δ	**+0.060**	**+0.088**	**+0.034**	**+0.069**	**+0.122**	**+0.037**
	Qwen2.5-32B	Base	0.221	0.332	0.404	0.219$_{(.012)}$	0.260$_{(.071)}$	0.372$_{(.007)}$
		Instruct	**0.379**	**0.475**	**0.469**	**0.371$_{(.012)}$**	**0.449$_{(.010)}$**	**0.454$_{(.007)}$**
		Δ	**+0.158**	**+0.143**	**+0.065**	**+0.152**	**+0.189**	**+0.082**
	Qwen2.5-72B	Base	0.443	0.351	0.468	0.389$_{(.005)}$	0.305$_{(.028)}$	0.418$_{(.008)}$
		Instruct	**0.546**	**0.520**	**0.546**	**0.519$_{(.007)}$**	**0.512$_{(.005)}$**	**0.537$_{(.008)}$**
		Δ	**+0.103**	**+0.169**	**+0.078**	**+0.130**	**+0.207**	**+0.119**
IPC Codes	Qwen2.5-7B	Base	0.463	0.436	**0.588**	0.349$_{(.040)}$	0.364$_{(.038)}$	**0.585$_{(.038)}$**
		Instruct	**0.615**	**0.554**	0.574	**0.615$_{(.025)}$**	**0.554$_{(.013)}$**	0.574$_{(.015)}$
		Δ	**+0.152**	**+0.118**	**-0.014**	**+0.266**	**+0.190**	**-0.011**
	Qwen2.5-14B	Base	0.526	0.608	0.609	0.421$_{(.038)}$	0.492$_{(.033)}$	0.600$_{(.013)}$
		Instruct	**0.708**	**0.691**	**0.718**	**0.708$_{(.025)}$**	**0.687$_{(.029)}$**	**0.718$_{(.007)}$**
		Δ	**+0.182**	**+0.083**	**+0.109**	**+0.287**	**+0.195**	**+0.118**
	Qwen2.5-32B	Base	0.644	0.641	**0.777**	0.482$_{(.059)}$	0.503$_{(.038)}$	0.769$_{(.013)}$
		Instruct	**0.759**	**0.754**	0.774	**0.759$_{(.007)}$**	**0.754$_{(.000)}$**	**0.774$_{(.007)}$**
		Δ	**+0.115**	**+0.113**	**-0.003**	**+0.277**	**+0.251**	**+0.005**

Table 4: Performance of distilled models compared to the **DeepSeek-R1 (reasoning model)**. Each cell for a distilled model shows its absolute score, followed in parentheses by the Δ gap (reasoning - distilled). Δ values for Maj. Vote Acc. are shaded red, and Δ values for Mean Acc. are shaded green. Darker shades indicate a larger performance gap. All Δ values are positive, showing the gap to the stronger R1 model.

Dataset	Model	Maj. Vote Acc. (Δ vs. R1)			Mean Acc.$_{(\text{Std.})}$ (Δ vs. R1)		
		QA	CoT	Structured	QA	CoT	Structured
MedConceptsQA	**DeepSeek-R1 (Reasoning)**	**0.778**	**0.790**	**0.792**	**0.830$_{(.006)}$**	**0.774$_{(.013)}$**	**0.775$_{(.026)}$**
	Qwen2.5-Math-7B (Dist.)	0.296 (**+0.482**)	0.256 (**+0.534**)	0.282 (**+0.510**)	0.292$_{(.010)}$ (**+0.538**)	0.250$_{(.017)}$ (**+0.524**)	0.289$_{(.017)}$ (**+0.486**)
	Qwen2.5-32B (Dist.)	0.375 (**+0.403**)	0.380 (**+0.410**)	0.447 (**+0.345**)	0.351$_{(.009)}$ (**+0.479**)	0.369$_{(.005)}$ (**+0.405**)	0.420$_{(.002)}$ (**+0.355**)
	Llama3.3-70B (Dist.)	0.537 (**+0.241**)	0.633 (**+0.157**)	0.610 (**+0.182**)	0.495$_{(.002)}$ (**+0.335**)	0.609$_{(.011)}$ (**+0.165**)	0.596$_{(.012)}$ (**+0.179**)
IPC Codes	**DeepSeek-R1 (Reasoning)**	**0.923**	**0.892**	**0.923**	**0.913$_{(.019)}$**	**0.867$_{(.026)}$**	**0.903$_{(.007)}$**
	Qwen2.5-32B (Dist.)	0.778 (**+0.145**)	0.730 (**+0.162**)	0.788 (**+0.135**)	0.754$_{(.038)}$ (**+0.159**)	0.667$_{(.019)}$ (**+0.200**)	0.780$_{(.019)}$ (**+0.123**)
	Llama3.3-70B (Dist.)	0.785 (**+0.138**)	0.831 (**+0.061**)	0.815 (**+0.108**)	0.785$_{(.015)}$ (**+0.128**)	0.785$_{(.041)}$ (**+0.082**)	0.790$_{(.018)}$ (**+0.113**)

pendently through each model to extract their respective layer-wise representations, enabling both inter-model and intra-model comparative analyses.

Representation Extraction. For a model with L layers and hidden dimension d, we extract the hidden state at the final token position for each layer $\ell \in \{1, \ldots, L\}$ as the layer's representation vector $\mathbf{h}_\ell \in \mathbb{R}^d$. This representation attends to all preceding tokens, thereby capturing the full input context at that layer.

Representation Analysis. We quantify representational differences across and within models using *inter-model* and *intra-model* analyses:

- **Inter-Model Analysis.** By comparing the question–question (Q–Q) and answer–answer (A–A) representations between the base and specialized models, we assess how they differ at understanding query and retrieving factual knowledge.
- **Intra-Model Comparison.** This analysis investigates the internal transformation of information within a single model. By comparing a model's question and answer representations layer by layer, we trace how internal activations evolve from encoding a problem to producing a solution.

Comparison Metric For each layer $\ell \in \{1, \ldots, L\}$, we use cosine similarity, a mea-

Table 5: Comparison of structured prompting performance by task complexity for DeepSeek-R1 and DeepSeek-V3 models. Memory-Light tasks (1-2 hierarchical recalls); Memory-Heavy tasks (5+ hierarchical recalls). Bold values indicate the best performance for each metric within each complexity category. **As we move to retrieval heavier tasks with structure, the gap between path matching score of R1 and V3 increases.**

Task Complexity	Model	Accuracy (%)	Path Matching Score
Memory-Light	DeepSeek-R1	**44.8**	**0.681**
	DeepSeek-V3	37.9	0.627
Memory-Heavy	DeepSeek-R1	**67.7**	**0.597**
	DeepSeek-V3	**67.7**	0.503

sure of directional alignment, to define representation similarity:

$$d_{\cos}^{(a,b)}(\ell) = 1 - \frac{1}{N}\sum_{i=1}^{N} \frac{\mathbf{h}_\ell^{(a)}(i)^\top \mathbf{h}_\ell^{(b)}(i)}{\|\mathbf{h}_\ell^{(a)}(i)\|_2 \, \|\mathbf{h}_\ell^{(b)}(i)\|_2}, \tag{1}$$

Here, $\mathbf{h}_\ell^{(s)}(i)$ denotes the layer-ℓ hidden representation for probe i from a source s. The set of sources $\mathcal{S} = \{\mathrm{Q}^{\text{base}}, \mathrm{A}^{\text{base}}, \mathrm{Q}^{\text{specialized}}, \mathrm{A}^{\text{specialized}}\}$ includes representations for both the question (Q) and answer (A) components from the base and specialized models. Pair (a, b) represents either inter-model (e.g., Q^{base} vs $\mathrm{Q}^{\text{specialized}}$, A^{base} vs $\mathrm{A}^{\text{specialized}}$) or intra-model comparisons (e.g., Q^{base} vs A^{base}, $\mathrm{Q}^{\text{specialized}}$ vs $\mathrm{A}^{\text{specialized}}$). Results are reported per vocabulary using $N = 100$ probes.

Models We compare Qwen2.5-32B (base) against three specialized variants: Qwen2.5-32B-Instruct (instruction-tuned), DeepSeek-R1-Distill-Qwen-32B (distilled), and QwQ-32B (reasoning). We select this 32B parameter family because it spans multiple enhancement methods while remaining computationally tractable for single-GPU inference. A supplementary analysis comparing variants of the Mistral-Small-24B family (base, instruct, and reasoning) is included in Appendix C.

3 Experimental Results

3.1 Hierarchical Navigation Through Structured Prompting

Hierarchical navigation and stepwise elimination strategies systematically narrow the accuracy gap between base models and their reasoning-enhanced, or instruction-tuned versions across both MedConceptsQA and IPC code datasets. For example, on MedConceptsQA, structured prompting allows the Qwen3-235B Instruct model (Table 2) to outperform its reasoning counterpart, reversing a +0.108 majority vote accuracy gap (CoT) to a -0.051 advantage. Similarly, on the IPC dataset (Table 3), this prompting reduces the gap between the Qwen2.5-32B base and instruct models from +0.115 (QA) to -0.003. However, this effect is less pronounced for distilled models (Table 4), where the performance gap relative to the reasoning model remains substantial, even with structured prompts (e.g., Llama3.3-70B on MedConceptsQA, +0.182 gap).

To understand the mechanisms underlying structured prompting's effectiveness, we examine response consistency patterns. Figure 1 presents results for DeepSeek-V3 and R1 across three independent runs under majority voting on MedConceptsQA. When transitioning from the baseline to the structured prompt, DeepSeek-V3 shows significant sample migration: questions initially categorized as "All Incorrect" and "Majority Incorrect" shift toward "Majority Correct" and "All Correct". In contrast, R1 exhibits static distribution across these categories, suggesting it already operates near its ceiling. This redistribution in V3 indicates that explicit structural guidance improves the consistency of the model's internal reasoning and that its underlying knowledge is sufficient. Therefore, the primary role of specialized post-training is not to introduce entirely novel knowledge, but rather to enhance the procedural consistency and strategic reasoning of existing knowledge structures.

3.2 Hierarchical Navigation Across Retrieval Complexity

Stratifying performance by retrieval complexity highlights a distinction between the base and reasoning models. Despite similar overall accuracy, R1 consistently achieves a higher path matching score, particularly on complex tasks such as common ancestor identification, suggesting it can correctly navigate the hierarchy step-by-step (Table 5). This is a deeper form of understanding that goes beyond simple memorization. Ultimately, R1 understands the process of navigating a knowledge hierarchy better

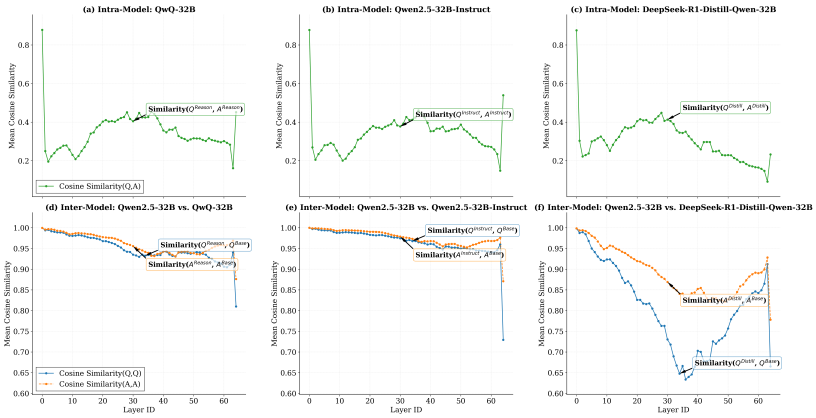


Figure 2: Layerwise Representation Similarity for ICD9PROC Vocabulary from MedConceptsQA. Plots compare last-token hidden state representations across layers (x-axis) using cosine similarity. Top Row (Intra-Model): Question vs. Answer representation similarity within QwQ-32B, Qwen2.5-32B-Instruct, and DeepSeek-R1-Distill-Qwen-32B. Bottom Row (Inter-Model): Similarity between the base model (Qwen2.5-32B) and each respective advanced model, comparing Question representations (Q_{Reason} vs. Q_{Base}) and Answer representations (A_{Reason} vs. A_{Base}) separately. **The representations of questions diverge more, specially in the last layer, compared to the answers. This hints at the knowledge being encoded similarly in base and reasoning models, but navigated differently.**

than the base model (V3), even when their final-answer accuracy is similar.

3.3 Hierarchical Navigation in Internal Representations

Intra-Model Representational Similarity. Within each model, representations for questions and answers are initially highly similar, but this similarity decreases in later layers, suggesting that the representations accumulate increasingly distinct features.

Inter-Model Representational Similarity. Instruction-tuned and reasoning models show strong directional alignment with the base model for both question and answer representations, whereas the distilled model shows much greater divergence (Figure 2(d-f)). Notably, question representations diverge more than answer representations across all specialized models, suggesting that performance gains arise primarily from refining question understanding rather than reorganizing factual knowledge.

4 Conclusion

This work challenges the view that reinforcement learning enhances reasoning at the expense of memorization. We demonstrate that RL-enhanced models outperform base counterparts by 24pp on hierarchical knowledge tasks, not through acquiring new knowledge, but by improving navigation of existing structures. Structured prompting reduces this gap to 7pp on simple tasks, yet reasoning models maintain superior path traversal on complex deep-retrieval tasks (5pp to 9pp gap widening). Layer-wise analysis reveals that RL transforms query processing (similarity drops to 0.65-0.73) while preserving factual representations (0.85-0.92), confirming that improvements stem from enhanced navigation mechanisms rather than knowledge content changes.

Several open questions warrant investigation. First, do similar navigation mechanisms underlie RL improvements on other structured reasoning tasks such as mathematical proof generation, code debugging, or multi-hop question answering? Second, can we develop RL objectives that explicitly optimize for hierarchical navigation rather than relying on implicit emergence? Third, how do these findings extend to knowledge domains with different structural properties—flat versus deeply nested hierarchies, dense versus sparse connectivity? Finally, can we design hybrid approaches that combine the efficiency of structured prompting with the robustness of RL-trained navigation for practical deployment? Addressing these questions will deepen our understanding of how language models organize and access parametric knowledge, ultimately enabling more capable and efficient reasoning systems.

Limitations

Limitations A key limitation of this work is that the experiments are restricted to a small set of structured knowledge domains, which may limit the generalizability of the findings to other tasks or less hierarchical settings.

There is a risk that structured prompting and path-based metrics overestimate true understanding, as models may learn superficial traversal patterns rather than robust, transferable navigation strategies.

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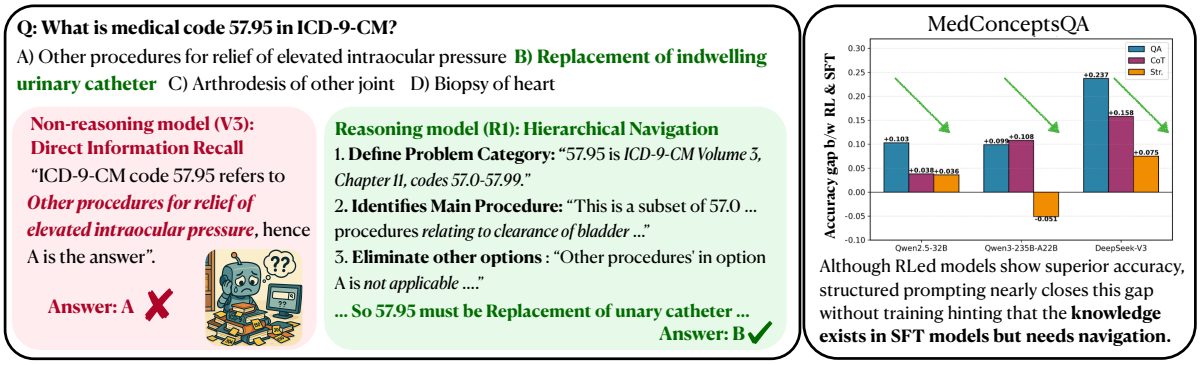


Figure 3: (Left) Overview of our main observation: When querying structured medical codes, non-reasoning models (DeepSeek-V3) rely on direct memorization attempts, often selecting incorrect answers (here choosing A). In contrast, reasoning-enhanced RL models (DeepSeek-R1) employ systematic hierarchical navigation—first categorizing the problem domain, then identifying relevant procedures, and finally interpreting ambiguous terminology—to successfully retrieve the correct answer (B). (Right) Reasoning models consistently outperform their instruction-tuned counterparts when prompted with conventional QA templates. This gap decreases when we optimize the prompt and is minimized with our hand-crafted structured prompt, hinting that the necessary knowledge exists in the instruct models.

A Related Work

A.1 The Alignment Tax and Factual Degradation

The trade-off between alignment and factual accuracy has been extensively explored. Lin et al. (2024) introduced the concept of "alignment tax", showing systematic degradation on factual benchmarks as RLHF reward strength increases. Achiam et al. (2023) similarly reported that RLHF "does not improve exam performance (without active effort, it actually degrades it)" and can reduce calibration. Mechanistic analyses in Ghosh et al. (2024) reveal that instruction tuning primarily adjusts style rather than new knowledge, with responses generated from pre-trained knowledge consistently outperforming those from models learning new knowledge through instruction tuning. Both Li et al. (2025) and Kirk et al. (2023) show that base models' parametric knowledge originates from pre-training while aligned models learn how to express it—training directly from base models mitigates knowledge forgetting and alignment tax incurred by SFT-based distillation. Recent work by Phan et al. (2025) reveals that optimizing for narrow verifiable rewards in reasoning-focused RL leads to regression in general capabilities, with models exhibiting increased hallucinations despite improved reasoning.

While these studies document factual degradation from alignment, our work reveals a contrasting phenomenon: RL-enhanced models *outperform* their base counterparts on structured knowledge recall tasks. This apparent contradiction suggests that alignment tax may not uniformly affect all forms of parametric knowledge retrieval—particularly when retrieval demands systematic navigation through hierarchical structures rather than direct factual recall.

A.2 Reasoning Enhancement Through RL

RL is commonly viewed as a means of amplifying reasoning ability. Process supervision and reward-driven methods (Lightman et al., 2023; Ye et al., 2025) demonstrate clear improvements on reasoning tasks, with process-supervised models solving substantially more problems than outcome-supervised variants. However, recent work hints at a more nuanced picture. Zelikman et al. (2024) introduce Quiet-STaR, showing that training models to generate internal rationales improves downstream reasoning by teaching systematic exploration of solution spaces—essentially navigation skills that achieve zero-shot improvements from 5.9% to 10.9% on GSM8K. Shinn et al. (2023) demonstrate that reinforcement learning primarily helps models learn from feedback to refine their search through problem spaces, rather than acquiring new problem-solving rules. Most strikingly, Guo et al.

(2025b) show that DeepSeek-R1 develops self-reflection, verification, and dynamic strategy adaptation through RL alone, without human-labeled reasoning trajectories, increasing pass@1 scores on AIME 2024 from 15.6% to 71.0%. Recent work on search-augmented reasoning (Jin et al., 2025; Chen et al., 2025b) demonstrates models learning to autonomously generate search queries and self-correct, with behaviors like pausing when detecting knowledge gaps emerging naturally.

These findings align with our hypothesis that RL enhances navigation of existing knowledge structures. However, while prior work focuses on mathematical and algorithmic reasoning, we examine whether these navigation improvements extend to retrieval from structured factual hierarchies, providing complementary evidence that RL's benefits stem from improved access patterns rather than new knowledge acquisition.

A.3 Hierarchical Reasoning and Structured Navigation

Hierarchical reasoning frameworks further support our knowledge navigation hypothesis. Wang et al. (2025a) present the Hierarchical Reasoning Model (HRM), a brain-inspired recurrent architecture that achieves near-perfect performance on complex tasks with only 27 million parameters trained on 1000 samples, without pre-training or chain-of-thought data. HRM's architecture features interdependent modules for high-level abstract planning and low-level detailed computation, achieving 40.3% on ARC-AGI—precisely the type of structured traversal we hypothesize enables medical code lookup. Yang et al. (2025b) show that hierarchical reinforcement learning on template sequences rather than long chain-of-thought data achieves 91.2% on MATH, outperforming models trained on detailed reasoning traces. Wang et al. (2025b) reveal RL training induces emergent separation between high-level strategic planning and low-level procedural execution, with two-phase learning of procedural consolidation followed by strategic exploration.

In the medical domain, structured approaches demonstrate substantial gains. Liao et al. (2025) report that EHR-R1 achieves over 30 percentage points improvement on MIMIC-Bench (F1 of 0.6744 vs 0.3155 for GPT-4o) through graph-driven structured medical reasoning that converts raw EHR records into thinking graphs encoding temporal relations and causal hypotheses. Work on ICD code classification (Liu et al., 2022; Sen et al., 2021) shows that leveraging hierarchical structure through label-wise attention and multi-class reformulation improves classification, particularly at higher hierarchy levels.

While these works demonstrate that hierarchical architectures and structured representations improve reasoning, they typically attribute gains to enhanced reasoning capabilities. Our work provides an alternative interpretation: these improvements may stem from better *navigation* of knowledge hierarchies already encoded during pretraining, rather than acquiring new reasoning abilities. We test this by showing that structured prompting—which explicitly guides traversal without modifying model parameters—recovers most performance gaps between base and RL models.

A.4 Prompting as an Alternative to RL

The possibility of achieving RL-like benefits through prompting has gained increasing attention. (Agrawal et al., 2024) demonstrate that Genetic-Evolution Prompt Alignment (GEPA) can outperform Group Relative Policy Optimization by up to 20% while using 35× fewer computational resources. They argue that "the interpretable nature of language provides a richer learning medium than sparse scalar rewards." (Wei et al., 2022) show that chain-of-thought prompting can match fine-tuned performance on reasoning tasks, while (Zhou et al., 2023) demonstrate that optimized prompts can exceed supervised fine-tuning. The "Invisible Leash" phenomenon (Wu et al., 2025) reveals that much of RLHF's apparent benefit comes from teaching models to follow implicit formatting patterns—effects reproducible through prompting.

A.5 Knowledge Storage versus Knowledge Access

The distinction between knowledge acquisition and knowledge retrieval is crucial to our thesis. (Ovadia et al., 2024) show that models fine-tuned on new knowledge often

"hallucinate" by incorrectly combining existing knowledge rather than storing new information. (Ruis et al., 2024) provide key insights with their finding that models rely on procedural knowledge extracted from documents involving similar reasoning processes rather than memorizing new facts. This aligns with our hypothesis that RL enhances navigation strategies rather than expanding knowledge. (Berglund et al., 2024) further support this through their "Reversal Curse" findings—models trained on "A is B" cannot infer "B is A," suggesting that training affects access patterns rather than creating bidirectional knowledge representations.

A.6 Retrieval Complexity in Knowledge-Intensive Tasks

Recent work has begun to to examine the relationship between retrieval complexity and model performance in knowledge-intensive tasks. (Gabburo et al., 2024) show that retrieval complexity extend beyond simple multi-hop reasoning—including temporal (15%), comparative (10%), and aggregate (16%) questions—suggesting that different types of knowledge organization require distinct retrieval strategies. (Min et al., 2023) demonstrate that in long-form generation, factual accuracy in biographies drops as entity rarity increases, suggesting that retrieval difficulty directly impacts knowledge accessibility.

B Technical Appendices and Supplementary Material

B.1 Zero-Shot Prompt Templates

We present three prompt templates used in MedConceptsQA and IPC, which are designed to elicit specific responses from language models. These templates request:

- Direct answers, both with and without explanations.
- Structural recall of codes and a stepwise elimination of incorrect options.

Prompt Template 1: MCQ with Final Answer Only

Answer only A,B,C,D according to the answer to this multiple choice question.
[... Insert Question Text Here ...]
Answer (only the letter of your choice (A, B, C, or D)):

Prompt Template 2: MCQ with Explanation

You are a medical research assistant. Read the following multiple-choice question carefully. Your task is to:

1. Answer each question with one of A/B/C/D, which corresponds to the four options.
2. For my convenience, please give me a list of ANSWERs for the given instances in the format 'Answer: ...', with additional explanation for each answer in the format 'Explanation: ...'.

Respond in the following format:
`Answer: <A/B/C/D>`
`Explanation: <your explanation here>`

[... Insert Question Text Here ...]

Answer:
Explanation:

Prompt Template 3: MCQ with Stepwise Reasoning

You are a medical classification expert. For each option, first **recall the general category and structure breakdown of the medical code**, then explain **why it might be wrong**. Finally pick the correct one.

[... Insert Question Text Here ...]
Steps to follow:

1. Recall the general category and structural break down of the code.
2. Evaluate each option (A–D) briefly.
3. Choose the best option and justify.

Answer format:

```
Step 1: ...
Step 2A: ...
Step 2B: ...
Step 2C: ...
Step 2D: ...
Final Answer:
[A/B/C/D] because ...
```

C Layer-wise Representation Analysis

C.1 Question-Answer Pairwise Probing

This section provides supplementary results for the layer-wise representation divergence analysis presented in Figure 2, extending the comparison across additional MedConceptsQA vocabularies for two model families.

C.1.1 Qwen2.5 Series

Figure 5 presents the analysis for the Qwen2.5-32B base model compared against its instruction-tuned (-Instruct), distilled (DeepSeek-R1-Distill-), and reasoning-enhanced (QwQ-32B) variants across the ATC, ICD10PROC, ICD9CM, and ICD10CM vocabularies.

C.1.2 Mistral-Small-24B Series

Figure 4 shows the corresponding analysis for the Mistral-Small model family, comparing the base (-Base-2503), instruction-tuned (-Instruct-2503), and reasoning-enhanced (Magistral-Small-2507) variants across all five MedConceptsQA vocabularies (ATC, ICD9PROC, ICD9CM, ICD10CM, ICD10PROC).

C.2 CoT Prompt Stepwise Probing

To analyze model representations under chain-of-thought (CoT) prompting, we construct a series of hierarchical prompts. For example, for the question "What is the description of the medical code 743.63 in ICD9CM?", the CoT series builds incrementally:

- "hmm let me think. 001-999.99 refers to diseases and injuries"
- "hmm let me think. 001-999.99 refers to diseases and injuries, and 740-759.99 refers to congenital anomalies"
- ...
- "hmm let me think. ...and 743.63 refers to other specified congenital anomalies of eyelid"

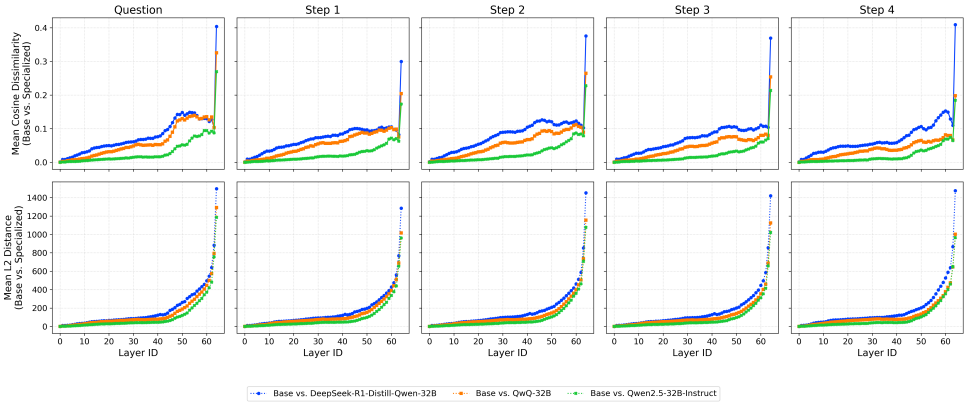
For each prompt in this series, we extract the activations from each layer of the model and group them by their corresponding vocabularies.

Additionally, we use **L2 distance** captures both directional and magnitude differences:

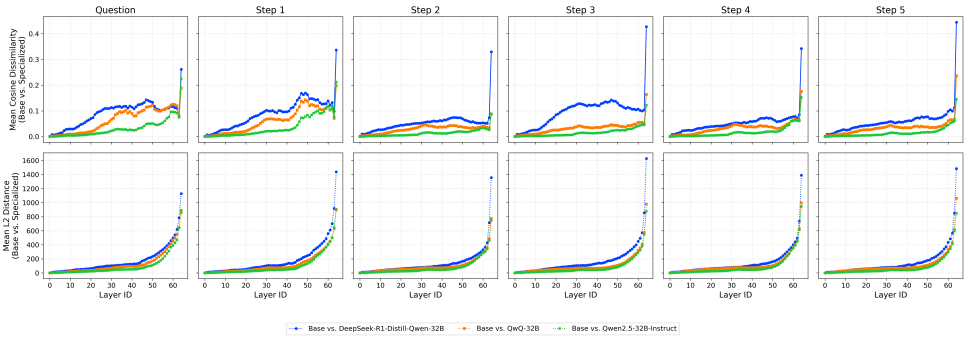
$$d_{\mathrm{L2}}^{(a,b)}(\ell) = \frac{1}{N}\sum_{i=1}^{N} \left\|\mathbf{h}_{\ell}^{(a)}(i) - \mathbf{h}_{\ell}^{(b)}(i)\right\|_2. \quad (2)$$

The number of CoT steps varies across vocabularies. To standardize this, we predefine all CoT sequences to be 5 steps long, with the exception of ICD10PROC, which uses 6 steps due to its more deeply embedded code structure (e.g., 0Q894Z). After grouping the activations by vocabulary for each layer, we compute the layerwise cosine similarity and L2 norm between the base and specialized models, following the methodology in Section 2.3.

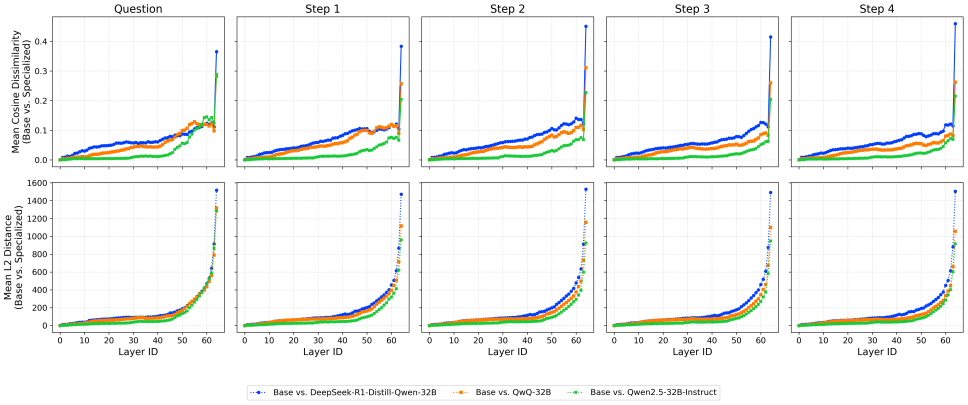
Layer-wise Similarity per COT Step: ATC Vocabulary



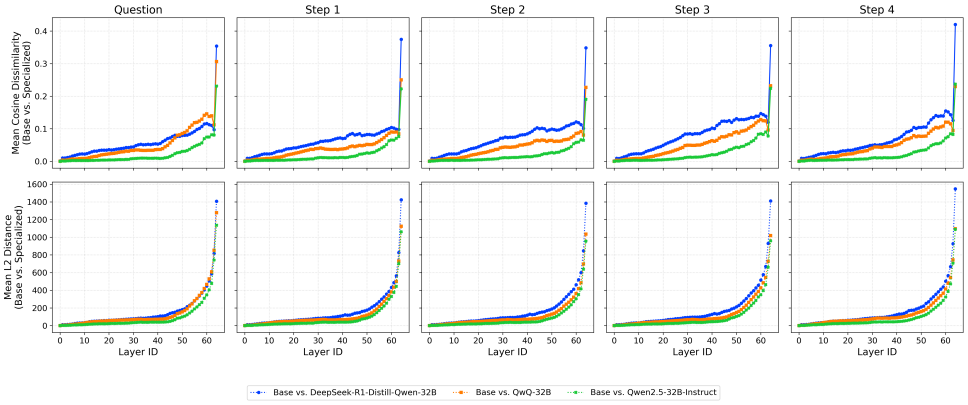
Layer-wise Similarity per COT Step: ICD10PROC Vocabulary



Layer-wise Similarity per COT Step: ICD9CM Vocabulary



Layer-wise Similarity per COT Step: ICD10CM Vocabulary



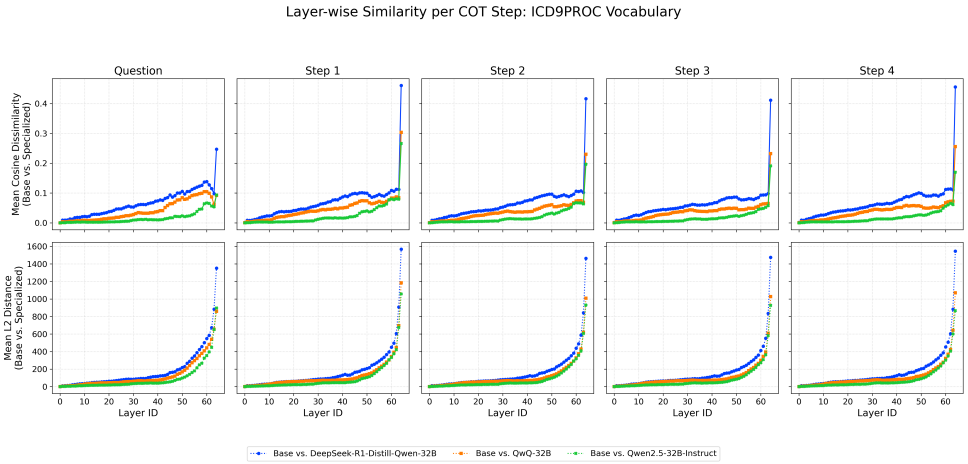
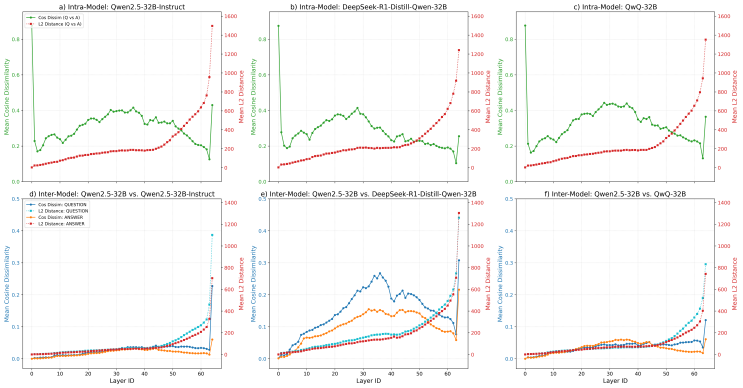


Figure 4: **Layer-wise Representation Divergence Across CoT Steps for All MedConceptsQA Vocabularies.** This figure shows the divergence analysis results for the ATC, ICD9PROC, ICD10PROC, ICD9CM, and ICD10CM vocabularies. The top and bottom rows correspond to mean cosine similarity and L2 distance, respectively. Each column represents a distinct step in the Chain-of-Thought (CoT) process, from Step 0 (the original question) to the final step (the original question plus the complete hierarchical traversal to the correct answer).

ATC



ICD10PROC

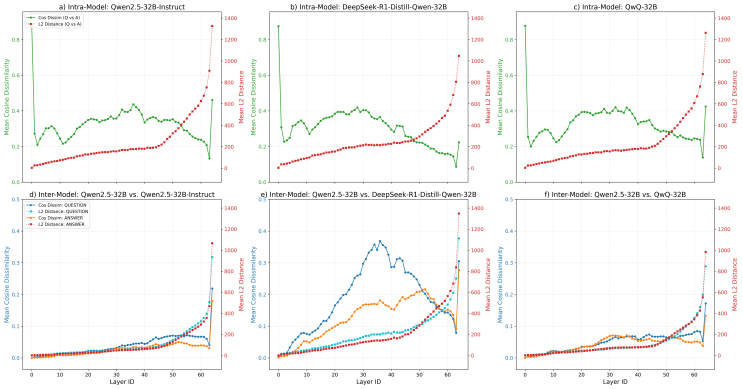
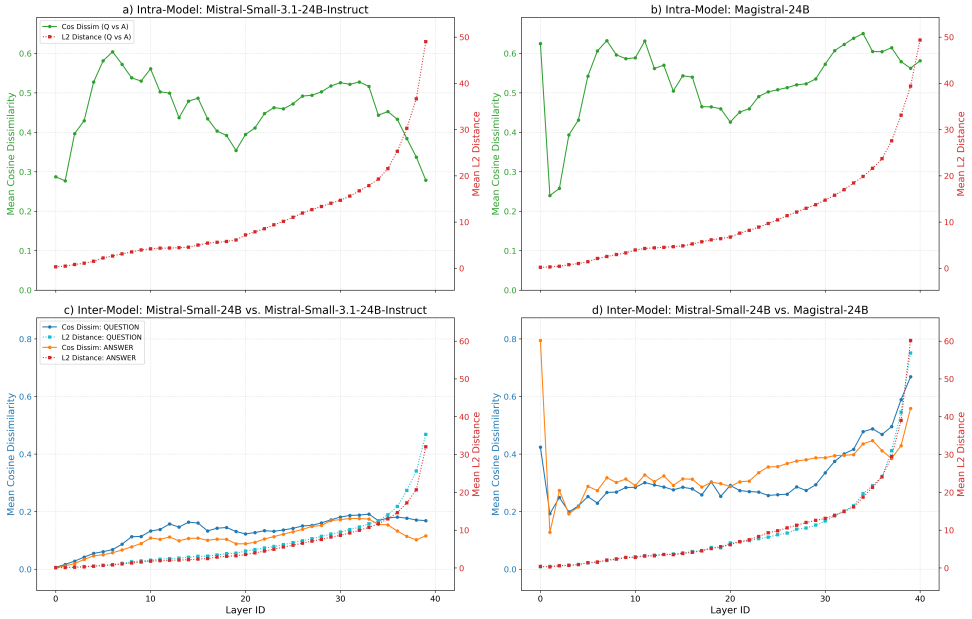
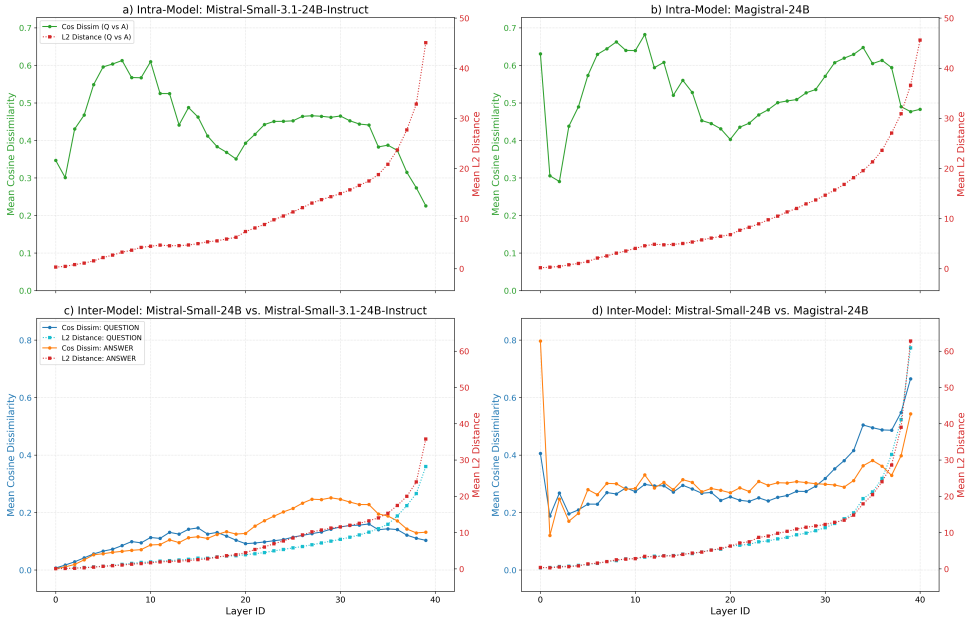


Figure 5: **Layer-wise Representation Divergence Across Remaining MedConceptsQA Vocabularies.** Same visualization format as Figure 2, showing results for ATC, ICD10PROC, ICD9CM, and ICD10CM vocabularies. Top and bottom rows correspond to intra- and inter-model divergence, respectively.

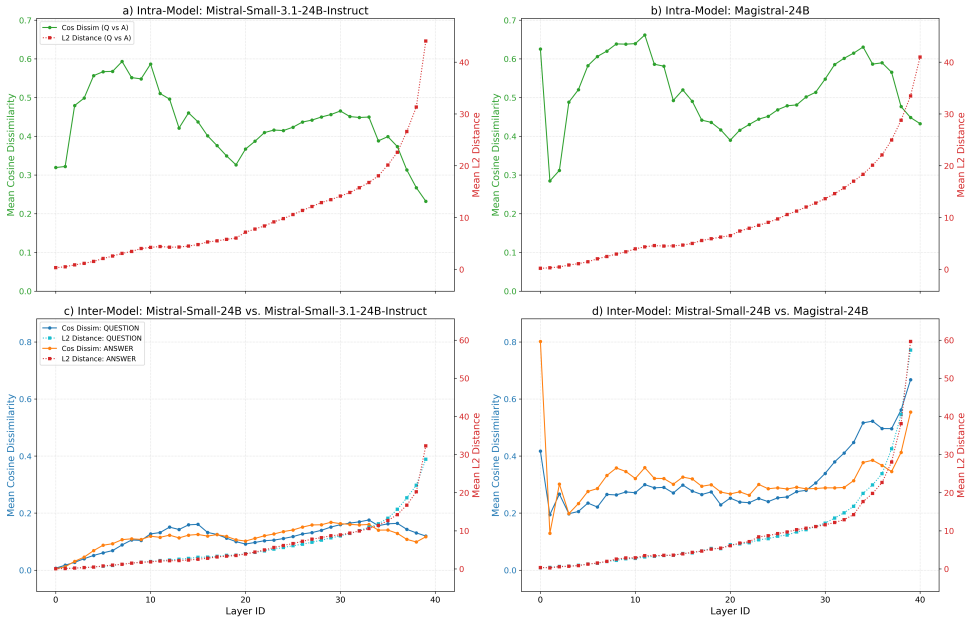
ATC



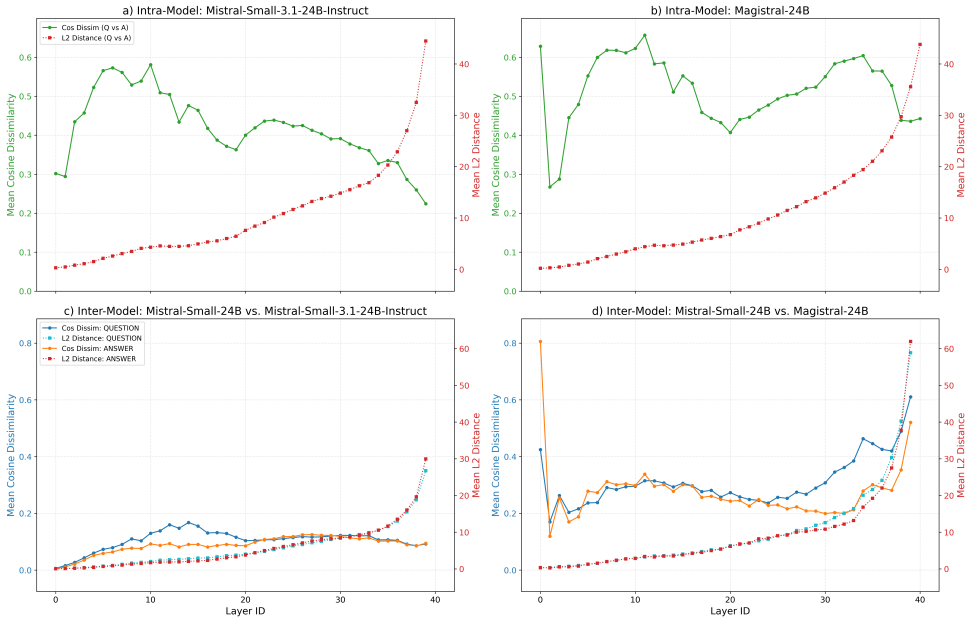
ICD9PROC



ICD10PROC



ICD9CM



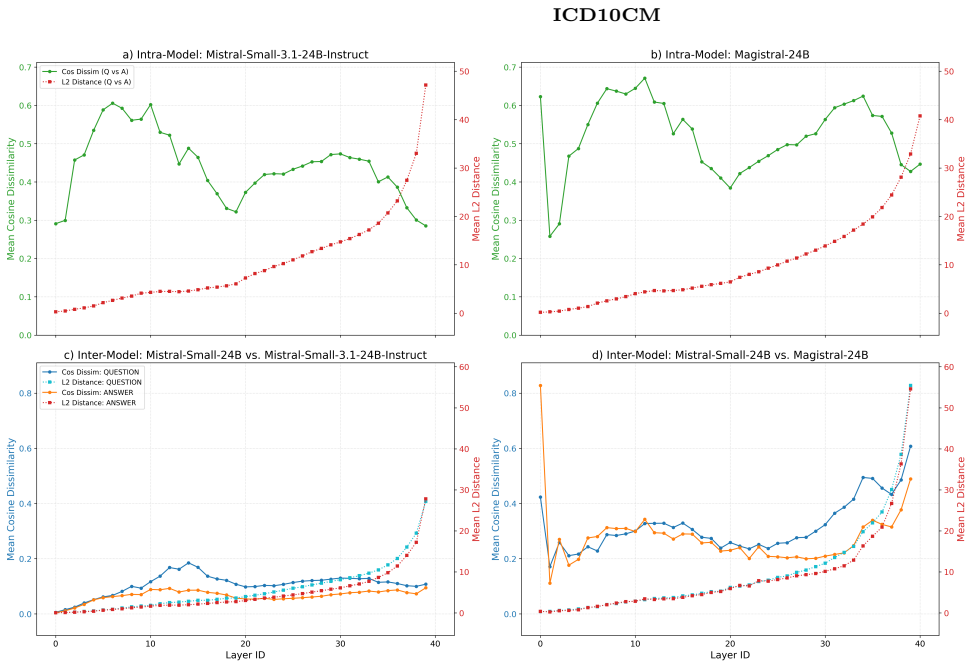


Figure 6: **Layer-wise Representation Divergence Across Remaining MedConceptsQA Vocabularies.** Same visualization format as Figure 2, showing results for ATC, ICD10PROC, ICD9CM, and ICD10CM vocabularies. Top and bottom rows correspond to intra- and inter-model divergence, respectively.