Taming the Judge: Deconflicting AI Feedback for Stable Reinforcement Learning

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Abstract

Aligning language models using LLM judge feedback offers a scalable alternative to human annotation, yet is plagued by judgment inconsistencies that destabilize reinforcement learning. While prior work has focused on judge accuracy, the critical issue of logical coherence—particularly preference cycles $(A \succ B \succ C \succ A)$—has been largely unaddressed. To address this gap, this work introduces an end-to-end framework to systematically detect and resolve these inconsistencies within the reinforcement learning training loop. Our framework features two core contributions: the **Conflict Detection Rate (CDR)**, a novel metric to quantify judgment conflicts, and **Deconflicted Graph Rewards (DGR)**, a signal-purification framework that eliminates cycles before policy optimization. DGR constructs preference graphs from raw judgments, transforms them into conflict-free Directed Acyclic Graphs (DAGs), and generates a logically coherent reward signal compatible with any policy optimizer. Experiments confirm that our framework significantly improves training stability and model performance over strong baselines, establishing logical consistency as a crucial and now-addressable dimension of AI feedback. The code for our method is available at `https://anonymous.4open.science/status/DGR-E5DA`.

1 Introduction

Aligning large language models (LLMs) with human preferences, traditionally achieved through Reinforcement Learning from Human Feedback (RLHF) (Ouyang et al., 2022), is critical for safe AI deployment. However, the reliance on costly and slow human annotation has created a scalability bottleneck, pushing the field towards Reinforcement Learning from AI Feedback (RLAIF) (Bai et al., 2022; Lee et al., 2023). Within RLAIF, the pairwise comparison paradigm—where an LLM judge selects the better of two responses—has become the de facto standard, prized for its intuitive nature and fine-grained feedback signal that underpins many state-of-the-art alignment techniques (Song et al., 2024; Wang et al., 2024).

Recent advances in pairwise methods include the Pairwise-RL framework (Xu et al., 2025), which addresses the fundamental misalignment between generative base models and discriminative reward tasks by unifying reward model training and reinforcement learning application in a consistent pairwise paradigm. This framework combines generative reward modeling with pairwise policy optimization, leveraging generative modeling techniques to improve reward model performance and score calibration. Consequently, our work focuses on the pairwise paradigm, building upon these foundational approaches.

However, this scalability comes at a hidden cost: the erosion of logical consistency. While RLAIF promises an abundance of preference data, it also introduces a flood of contradictory signals from fallible AI judges. The most insidious form of this inconsistency is the preference cycle—a logical paradox where a judge asserts $A \succ B$, $B \succ C$, yet simultaneously prefers $C \succ A$. As we demonstrate in Figure 1(a) on the non-tie subset of RewardBench2, even state-of-the-art models are plagued by these cycles, with conflict rates reaching up to 6.7% and averaging 4.9% across models. These logical contradictions in feedback signals pose severe threats to the training process. First, preference cycles undermine the transitivity assumptions underlying many preference learning algorithms, particularly the widely-adopted Bradley-Terry model (Bradley & Terry, 1952), which relies on latent scalar utilities and (stochastic) transitivity assumptions rather than requiring observed pair-
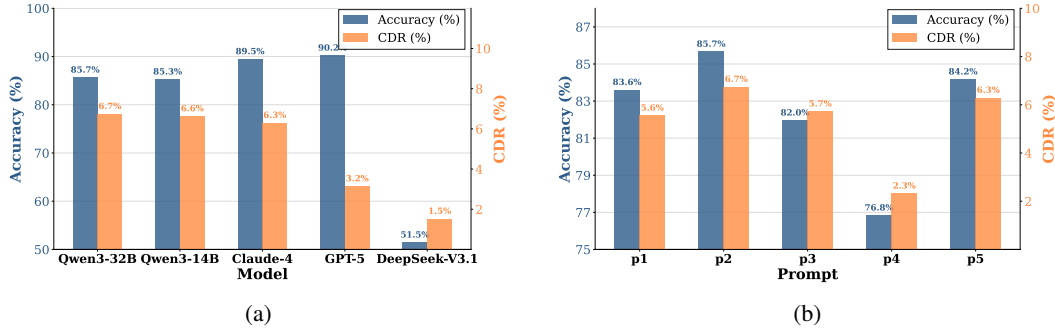
Figure 1: Conflict detection and mitigation results. (a) CDR and accuracy comparison across different LLM judges. (b) Impact of prompt engineering on CDR reduction for Qwen3-32B. The pairwise prompts P1-P5 shown in the figure can be found in Appendix C.2.1. Accuracy and conflict rates are computed on the non-tie subset of RewardBench2 (Malik et al., 2025). See Appendix C.1.1 and C.1.2 for detailed computation algorithms.

wise preferences to form a strict total ordering. Second, such inconsistencies inject harmful noise into the reward signal, creating conflicting optimization targets that impede reward model convergence (Gao et al., 2023). Finally, these contradictory signals propagate through the training pipeline, destabilizing policy optimization and ultimately degrading model performance, potentially leading to phenomena such as preference collapse where minority preferences are systematically ignored (Xiao et al., 2025).

Current evaluation of LLM judges focuses solely on accuracy, creating a critical blind spot for the logical inconsistencies that undermine preference-based learning. To address this gap, our work provides a complete diagnostic-and-remediation framework. For diagnosis, we introduce the Conflict Detection Rate (CDR), a systematic metric to quantify preference conflicts in judge feedback. For treatment, we propose Deconflicted Graph Rewards (DGR), a novel signal purifier that intercepts raw, conflicted judgments and transforms them into a globally consistent reward signal before they can destabilize training. DGR is rigorously grounded in graph-theoretic principles, with theoretical analysis and empirical validation provided in Appendix A. This purified signal can then be seamlessly consumed by any existing policy optimizer, ensuring logical coherence without altering the underlying training algorithm.

2 RELATED WORK

The alignment of Large Language Models (LLMs) is increasingly reliant on Reinforcement Learning from AI Feedback (RLAIF), where the pairwise preference paradigm is a dominant approach. This paradigm is implemented through two primary strategies: the traditional pipeline of training a reward model (RM) followed by policy optimization (Ouyang et al., 2022), and more recent direct preference optimization methods like DPO (Rafailov et al., 2024) and its variants, which bypass an explicit RM. This trend of integrating pairwise logic with generative reward models is echoed in other novel approaches like Writing-Zero, which introduces a pairwise Generative Reward Model (GenRM) to bridge the gap between subjective tasks and verifiable rewards (Jia et al., 2025). Critically, this paradigm is not merely theoretical; it is a core component in the alignment of leading large-scale models. The recent Kimi K2, a state-of-the-art Mixture-of-Experts model, explicitly employs pairwise comparisons for its self-critic reward mechanism to align on subjective tasks like creative writing (Team et al., 2025). The efficacy of both strategies, however, is fundamentally capped by the quality of the AI-generated preference data. A critical challenge is the inherent preference inconsistency of LLM judges, which manifests as systemic biases (Zheng et al., 2023) and, more fundamentally, logical preference cycles (e.g., $A \succ B \succ C \succ A$). These cycles violate the transitivity assumptions core to preference learning (Bradley & Terry, 1952), destabilizing the training process. While prior work has systematically framed logical consistency (Liu et al., 2024), the community has lacked a formal metric to quantify the prevalence of these conflicts within preference datasets. Our work addresses this gap by introducing the Conflict Detection Rate (CDR) as a diagnostic tool.

To mitigate preference inconsistencies, existing methods often treat them as statistical noise, resorting to strategies like careful data selection (Deng et al., 2025) or implicitly handling them through model architecture design (Wang et al., 2025a;b). However, these approaches do not address the underlying structural and logical contradictions of preference cycles. While graph-theoretic and probabilistic methods for aggregating preferences from pairwise comparisons are well-established in the ranking literature—including Rank Centrality (Negahban et al., 2012), Bradley-Terry-Luce/Plackett-Luce models (Bradley & Terry, 1952; Hunter, 2004), HodgeRank (Jiang et al., 2011), and Kemeny ranking (Ailon et al., 2008; Kenyon-Mathieu & Schudy, 2007)—their application to dynamic, online reinforcement learning has been unexplored. These methods were primarily designed for offline ranking tasks where the goal is to produce a single global ranking. In contrast, our work addresses the challenge of generating consistent reward signals within an online RL training loop, where preferences must be repeatedly aggregated for each batch of sampled responses. Crucially, we provide rigorous theoretical justification (Appendix A) proving that our graph-based conflict resolution systematically identifies judgment errors, with the effectiveness validated through comprehensive Monte Carlo experiments. Our comprehensive comparison with these classical methods (Table 3) demonstrates that DGR, which is equivalent to Kemeny ranking via minimum feedback arc set, achieves superior performance in the online RL setting. Recent game-theoretic approaches represent important parallel directions for handling non-transitive preferences. These include Minimax Winner (Swamy et al., 2024), Self-Play Preference Optimization (Wu et al., 2024), Distributional Preference Learning (Siththaranjan et al., 2024), and Nash Learning from Human Feedback (Munos et al., 2024), which frame alignment as Nash equilibrium finding without Bradley-Terry transitivity assumptions. Subsequent works (Zhang et al., 2025b; Ye et al., 2024; Zhang et al., 2025a) achieve strong empirical results with convergence guarantees. These methods excel at handling genuinely intransitive preferences. In contrast, our work assumes cycles primarily reflect judgment noise, motivating a lightweight graph-theoretic preprocessing module compatible with existing optimizers. Our key innovation is the Deconflicted Graph Rewards (DGR) framework. Unlike previous methods, DGR integrates a cycle-breaking mechanism directly into the training loop. It acts as an online signal purifier, transforming raw, conflicted pairwise judgments into a globally coherent and transitive reward signal before policy optimization. This modular approach allows DGR to be placed upstream of any policy optimizer (e.g., GRPO (Wang et al., 2025c), GSPO (Zheng et al., 2025)), ensuring a more stable and effective alignment process.

3 METHODOLOGY

3.1 A FRAMEWORK FOR QUANTIFYING PREFERENCE INCONSISTENCY

Traditional judge model evaluation suffers from a critical blind spot: it measures alignment accuracy but ignores logical consistency. An LLM judge might achieve high accuracy yet produce paradoxical preferences (e.g., $A \succ B \succ C \succ A$), rendering the reward signal unreliable. To address this gap, we introduce the Conflict Detection Rate (CDR), a systematic metric for quantifying such inconsistencies in pairwise preference data.

At the heart of our framework is the formalization of a preference conflict. Intuitively, a conflict arises when preferences form a cycle. We formalize this intuition using graph theory:

Definition 3.1 (Preference Conflict). Let $R = \{o_1, o_2, \ldots, o_G\}$ denote a set of candidate responses, and let $\succ^*$ represent the observed preference relation derived from LLM judge evaluations. We construct a directed preference graph $T = (V, E)$ where $V = R$ and $(o_i, o_j) \in E$ if and only if $o_i \succ^* o_j$. A **preference conflict** exists if and only if the preference graph T contains at least one strongly connected component with cardinality greater than one.

This definition captures intransitivity precisely: a preference conflict exists if and only if the directed graph contains at least one directed cycle, which corresponds to the existence of at least one strongly connected component with cardinality greater than one. Such cycles violate the fundamental requirement that preferences form a strict partial order.

We quantify these inconsistencies via the following metric:

$$\text{CDR} = \frac{\text{Samples with Conflicts}}{\text{Total Samples}} \times 100\%. \tag{1}$$

CDR thus provides a quantitative assessment of a judge's logical consistency. It is important to note that CDR is sensitive to the number of candidates, G, in a comparison set. As G increases, the number of pairwise judgments $\binom{G}{2}$ grows quadratically, and the number of potential preference cycles grows combinatorially, thereby increasing the probability of detecting a conflict. The detailed conflict detection algorithm is provided in Appendix C.1.1.

Our empirical analysis demonstrates CDR's utility. As a diagnostic tool, Figure 1(a) reveals that CDR provides critical insights beyond accuracy. For instance, while GPT-5 achieves the highest accuracy, it maintains relatively low conflicts. In contrast, Qwen3-32B exhibits the highest conflict rate despite strong accuracy. Interestingly, DeepSeek-V3.1 shows the lowest conflict rate but suffers from poor accuracy. This reveals a complex trade-off that underscores the insufficiency of accuracy alone for evaluating preference signals.

As an optimization guide, Figure 1(b) demonstrates CDR's value in prompt engineering. By systematically measuring how different prompts affect the accuracy-consistency trade-off, CDR enables practitioners to select optimal configurations for their specific deployment requirements, moving beyond trial-and-error approaches to principled judge optimization.

3.2 DGR: GENERATING CONSISTENT REWARDS FROM CONFLICTED AI FEEDBACK

While our CDR metric enables proactive optimization of LLM judges, inconsistencies inevitably arise during online training. To resolve these conflicts in real-time, we introduce Deconflicted Graph Rewards (DGR), a novel reward generation framework. DGR transforms raw, conflicted pairwise judgments into logically consistent reward signals using graph-theoretic optimization. Rather than being another reinforcement learning algorithm, DGR serves as a modular signal-purification layer that can be integrated with any preference-based policy optimization framework.

3.2.1 FROM CONFLICTED PREFERENCES TO CONSISTENT REWARD SIGNALS

The core innovation of DGR lies in its ability to systematically transform conflicted pairwise preferences into logically consistent reward signals. This deconflicting process operates as a modular signal purifier that preprocesses noisy preference judgments before they are fed into any policy optimization framework. The transformation follows a three-stage pipeline designed to eliminate all logical inconsistencies while preserving the essential preference information.

Stage 1: Preference Graph Construction: Given G candidate responses $\{o_1, o_2, \ldots, o_G\}$ for each question q, we construct a directed preference graph $T = (V, E)$ where $V = \{o_1, o_2, \ldots, o_G\}$ and each edge $(o_i, o_j) \in E$ represents a pairwise comparison. The LLM judge evaluates all $\binom{G}{2}$ pairs, yielding judgments $M_{ij} \in \{-1, 0, 1\}$ indicating whether o_i is worse than, tied with, or better than o_j, with $M_{ij} = -M_{ji}$ for non-tie cases. This stage captures the complete set of raw preference relationships, including any potential conflicts and cycles. In cases of a tie ($M_{ij} = 0$), no directed edge is added between the two nodes, resulting in a semicomplete digraph.

Stage 2: Conflict Resolution via DAG Transformation: This is the core of our deconflicting mechanism. To eliminate preference cycles, we transform the preference graph into a Directed Acyclic Graph (DAG) by removing a minimum feedback arc set (FAS) to break all cycles. While finding the true minimum FAS is NP-hard in general, for small graphs ($G \leq 8$), we can compute the optimal solution through exhaustive search. For larger graphs, we can employ a well-established approximation algorithm (Eades et al., 1993), which is highly effective and computationally feasible in practice. This process systematically identifies the smallest set of conflicting edges $E_{conflict} \subset E$ whose removal breaks all logical cycles, enforcing a globally consistent preference ordering:

$$T_{DAG} = (V, E \setminus E_{conflict}), \tag{2}$$

where we define $E_{DAG} = E \setminus E_{conflict}$. Crucially, our approach is theoretically grounded: edges participating in many cycles are systematically more likely to be judgment errors rather than correct preferences. We provide rigorous mathematical analysis and empirical validation in Appendix A, showing that minimum FAS removal systematically targets judgment errors while preserving correct preferences. The resulting DAG represents a logically coherent preference structure that preserves the maximum amount of original preference information while eliminating all transitivity violations.

Algorithm 1 Policy Training with Deconflicted Graph Rewards

Input: initial (SFT) policy model $\pi_{\theta_{\text{init}}}$; LLM judge J; task prompts $\mathcal{D}$; hyperparameters ϵ, β, μ

1: policy model $\pi_\theta \leftarrow \pi_{\theta_{\text{init}}}$
2: **for** iteration = 1, ..., I **do**
3:     reference model $\pi_{ref} \leftarrow \pi_\theta$
4:     **for** step = 1, ..., M **do**
5:         Sample a batch $\mathcal{D}_b$ from $\mathcal{D}$
6:         Update the old policy model $\pi_{\theta_{old}} \leftarrow \pi_\theta$
7:         Sample G outputs $\{o_i\}_{i=1}^G \sim \pi_{\theta_{old}}(\cdot \mid q)$ for each question $q \in \mathcal{D}_b$
8:         Construct preference graph $T = (V, E)$ with all pairwise comparisons using judge J
9:         Transform T to conflict-free DAG T_{DAG} by removing minimum feedback arc set
10:        Compute deconflicted net-win scores $\{s_i\}_{i=1}^G$ from T_{DAG}
11:        Compute advantages $\hat{A}_{i,t}$ using group-relative normalization of scores
12:     **end for**
13:     **for** policy_update = 1, ..., μ **do**
14:         Update the policy model π_θ using DGR-enhanced objective, which includes $D_{KL}(\pi_\theta \| \pi_{ref})$
15:     **end for**
16: **end for**

Output: π_θ

Stage 3: Deconflicted Reward Computation: From the conflict-free DAG T_{DAG}, we compute reward signals based on the out-degree minus in-degree of each node:

$$s_i = d_i^{\text{out}} - d_i^{\text{in}} = \sum_{j \neq i} \mathbb{I}[(o_i, o_j) \in E_{DAG}] - \sum_{j \neq i} \mathbb{I}[(o_j, o_i) \in E_{DAG}], \tag{3}$$

where d_i^{out} and d_i^{in} denote the out-degree and in-degree of node o_i in T_{DAG}, respectively, and s_i represents the deconflicted net preference strength of response o_i. Crucially, these scores are derived from a purified graph structure, ensuring they are free from logical contradictions and provide a reliable foundation for policy optimization.

3.2.2 INTEGRATING DGR SIGNALS WITH POLICY OPTIMIZERS

The deconflicted net-win scores produced by the DGR framework are designed to be a versatile and modular reward signal, not a policy optimization method in themselves. They can be seamlessly integrated into group-based policy optimization algorithms such as Group Relative Policy Optimization (GRPO) (Wang et al., 2025c) and Group Sequence Policy Optimization (GSPO). Our reward generation scheme acts as a reward front-end component that purifies noisy preference signals, which are then consumed by a policy optimization component responsible for updating the model's parameters. This separation of concerns is a key feature of our framework.

To illustrate this integration, we use GRPO as a concrete example. The deconflicted net-win scores $\{s_i\}_{i=1}^G$ from DGR are first normalized to generate advantage estimates. These clean advantage signals then drive the policy update, while the underlying optimization machinery of GRPO remains unchanged.

The policy objective, when integrating DGR signals with the GRPO back-end, is formulated as:

$$\mathcal{J}_{\textbf{DGR}}(\theta) = \mathbb{E}_{q \sim P(Q), \{o_i\}_{i=1}^G \sim \pi_{\theta_{\text{old}}}(\cdot|q)} \left[\frac{1}{G} \sum_{i=1}^G \frac{1}{|o_i|} \sum_{t=1}^{|o_i|} \left(\min\left(\frac{\pi_\theta(o_{i,t} \mid q, o_{i,<t})}{\pi_{\theta_{\text{old}}}(o_{i,t} \mid q, o_{i,<t})} \hat{A}_{i,t}^{\textbf{DGR}}, \right. \right. \right.$$
$$\left. \left. \left. \text{clip}\left(\frac{\pi_\theta(o_{i,t} \mid q, o_{i,<t})}{\pi_{\theta_{\text{old}}}(o_{i,t} \mid q, o_{i,<t})}, 1-\epsilon, 1+\epsilon \right) \hat{A}_{i,t}^{\textbf{DGR}} \right) - \beta\, D_{\text{KL}}\left(\pi_\theta \| \pi_{\text{ref}} \right) \right) \right], \tag{4}$$

where $\hat{A}_{i,t}^{\textbf{DGR}}$ represents the advantage computed from the DGR scores:

$$\hat{A}_{i,t}^{\textbf{DGR}} = \frac{s_i - \text{mean}(\{s_j\}_{j=1}^G)}{\text{std}(\{s_j\}_{j=1}^G)}. \tag{5}$$

The crucial takeaway is that our primary contribution is not a modification to the optimization algorithm itself, but rather a robust reward generation scheme that shields it from inconsistent feedback. This modularity allows our framework to enhance existing, well-tested optimization pipelines by providing them with a logically coherent reward signal. The advantage estimates $\hat{A}_{i,t}^{\textbf{DGR}}$ are derived from logically consistent reward signals that have been purified through our preference graph process, ensuring that policy optimization is guided by conflict-free preference signals. Our experimental evaluation demonstrates the performance gains when DGR is applied to both GRPO and GSPO, proving its general applicability as a plug-and-play enhancement for preference-based alignment.

4 EXPERIMENTS

We conduct a comprehensive set of experiments to evaluate DGR's effectiveness in improving training stability and final model performance. Our setup is designed to demonstrate the benefits of our conflict-aware approach when integrated with state-of-the-art policy optimizers across various benchmarks.

4.1 EXPERIMENTAL SETUP

Foundation Models and Optimizers. We use two foundation models to test generalizability. For the GRPO (Wang et al., 2025c) optimizer, we fine-tune Qwen3-14B (Yang et al., 2025). For the GSPO optimizer, we fine-tune Qwen3-8B (Yang et al., 2025), allowing us to evaluate performance across different model scales and optimization algorithms.

Setup Details. Unless otherwise specified, all preference feedback is generated by **Qwen3-32B** (Yang et al., 2025), which serves as our primary judge model. The RL training data consists of 1,000 Chinese and English queries carefully selected from the WildChat-1M dataset (Zhao et al., 2024), with curation to filter out meaningless queries (e.g., requests for image generation, gibberish) and retain substantive queries testing model capabilities. All methods train on identical data for controlled comparison. Our training framework is based on verl (Sheng et al., 2025), with vLLM (Kwon et al., 2023) used for efficient inference.

Benchmarks and Judges. Our evaluation encompasses three key benchmarks: Arena-Hard 2.0 (Li et al., 2024) for complex reasoning, MT-Bench (Zheng et al., 2023) for multi-turn conversational quality, and WritingBench (Wu et al., 2025) for writing capabilities. These benchmarks are widely adopted for LLM evaluation, with Arena-Hard derived from the Chatbot Arena platform (Chiang et al., 2024) that aggregates millions of human preference judgments. Evaluations are judged by GPT-4.1 (OpenAI, 2024) for Arena-Hard and MT-Bench, and Claude-3.7-Sonnet (Anthropic, 2025) for WritingBench.

Evaluation Protocol. We conduct experiments with rigorous statistical controls. For Table 1, each method runs twice with different seeds, reporting peak scores. For Table 2, we conduct five runs and report mean ± standard deviation. Statistical significance is assessed via t-tests with Bonferroni correction. Checkpoints are selected on a validation set (200 WildChat queries), with final evaluation on test benchmarks (Arena-Hard, MT-Bench, WritingBench). Unless specified, results report mean ± std over 5 runs.

4.2 BASELINE METHODS

To comprehensively evaluate our DGR framework, we compare it against several state-of-the-art preference learning approaches. Each method is integrated with both the GRPO and GSPO optimizers for a fair comparison.

- **Pointwise (PW)**: Evaluates each response independently on an absolute scale (1-10), serving as a simple and efficient baseline.
- **Listwise (LW)**: Ranks all responses in a group simultaneously and converts these rankings into normalized rewards.
- **Pairwise (PREF)**: The standard pairwise method, which uses the **win rate** of each response against its peers as the reward signal, as proposed in Pref-GRPO (Wang et al., 2025c). This

Table 1: Main evaluation results on alignment benchmarks. Best results are **bolded**, second best are underlined.

Method	Arena-Hard				MT-Bench		WritingBench
	Overall	Code	Math	Writing	1-turn	2-turn	Score
GRPO-based Methods (Qwen3-14B)							
Base Model	49.1 ±0.5	47.0 ±0.5	45.0 ±0.8	55.3 ±0.4	7.31 ±0.04	6.74 ±0.11	7.95 ±0.02
PW	48.4 ±0.7	49.3 ±1.2	43.0 ±1.4	52.9 ±1.8	7.93 ±0.07	7.39 ±0.12	8.12 ±0.08
LW	49.4 ±1.2	49.1 ±1.1	43.6 ±1.8	55.5 ±1.4	7.33 ±0.15	7.44 ±0.10	8.24 ±0.05
PREF	51.6 ±1.0	49.0 ±1.6	47.0 ±1.5	**58.8 ±1.3**	7.58 ±0.11	7.10 ±0.19	8.44 ±0.04
ELO	52.0 ±0.7	51.0 ±1.7	**47.9 ±2.0**	57.1 ±1.4	7.87 ±0.06	6.79 ±0.11	8.32 ±0.04
DGR (Ours)	**52.9 ±1.1**	**53.2 ±1.4**	47.2 ±1.4	58.3 ±1.0	**8.06 ±0.10**	**7.54 ±0.34**	**8.56 ±0.05**
GSPO-based Methods (Qwen3-8B)							
Base Model	49.2 ±0.9	48.4 ±0.7	46.2 ±0.8	53.0 ±0.9	6.58 ±0.06	5.64 ±0.08	7.65 ±0.01
PW	50.2 ±1.1	49.6 ±1.1	45.1 ±1.5	55.9 ±0.8	7.32 ±0.13	6.43 ±0.10	7.86 ±0.04
LW	50.8 ±0.7	48.8 ±1.0	45.5 ±2.0	58.1 ±1.1	7.38 ±0.06	6.61 ±0.35	8.11 ±0.04
PREF	51.9 ±0.8	49.6 ±1.3	47.0 ±2.0	59.1 ±0.9	6.89 ±0.16	6.66 ±0.19	**8.39 ±0.04**
ELO	50.9 ±1.5	49.4 ±1.7	**47.6 ±1.4**	55.7 ±1.3	7.31 ±0.10	6.63 ±0.22	8.19 ±0.07
DGR (Ours)	**52.5 ±1.0**	51.7 ±1.1	46.0 ±1.3	**59.7 ±1.3**	7.46 ±0.08	6.79 ±0.08	8.37 ±0.04

method processes preferences statistically without explicitly resolving logical inconsistencies like preference cycles.

- **ELO**: A competing **conflict resolution method** that adapts the ELO rating system from chess. It **iteratively updates** a numerical rating for each response through pairwise comparisons until a globally consistent ranking is achieved. This represents an iterative, rating-update approach to resolving conflicts, in contrast to our graph-theoretic approach.

Our proposed method, DGR, is also integrated with both optimizers, resulting in DGR-GRPO and DGR-GSPO.

4.3 COMPARATIVE EVALUATION ON ALIGNMENT BENCHMARKS

Table 1 presents our comprehensive evaluation results across three critical benchmarks measuring distinct aspects of language model capabilities. The table compares our DGR framework against four baseline methods, evaluated with both GRPO and GSPO optimizers on their respective base models. All results use the standard P1 reward prompt, with additional prompt analysis provided in our ablation studies.

DGR Demonstrates Superior Robustness and Comprehensive Performance. Our DGR framework achieves the strongest overall performance balance across all evaluation benchmarks. Taking the GRPO setting as an example, DGR-GRPO not only achieves the highest Arena-Hard score but also ranks at the top in both MT-Bench assessments and achieves the highest WritingBench score. This comprehensive leadership is enabled by our theoretically-grounded conflict resolution mechanism, which systematically removes judgment errors while preserving correct preferences, as rigorously proven in Appendix A. This comprehensive leadership demonstrates that our method delivers balanced performance improvements without sacrificing specific abilities.

Strong Advantage on Complex Reasoning Tasks. Particularly noteworthy is DGR's dominant performance on Arena-Hard, the most demanding benchmark. Both DGR-GRPO and DGR-GSPO achieve strong performance, demonstrating that conflict-free reward signals effectively enhance models' fundamental abilities in code generation, mathematical reasoning, and creative tasks.

Revealing and Transcending Performance Hierarchy. Our results reveal a clear hierarchy: generally Pointwise ≺ Listwise ≺ Pairwise across most benchmarks, where pairwise methods (including PREF, ELO and DGR) typically outperform others. However, even these stronger methods exhibit instability—on MT-Bench, pointwise methods sometimes achieve optimal results, validating our hypothesis that signal conflicts undermine reliability. DGR addresses this fundamental issue by ac-

Table 2: Comparison with learned reward model baselines on Arena-Hard. All methods use Qwen3-8B with 16 candidates. GRPO-Skywork represents the standard GRPO baseline using Skywork-Reward-V2 as the reward model. Best results are **bolded**, second best are underlined.

Category	Method	Overall	Code	Math	Creative
SFT	SFT-Qwen32B	$48.66_{\pm0.41}$	$48.42_{\pm0.44}$	$43.41_{\pm0.66}$	$54.11_{\pm0.33}$
	SFT-Skywork	$48.10_{\pm0.38}$	$48.22_{\pm0.62}$	$44.83_{\pm0.43}$	$51.20_{\pm0.17}$
DPO	DPO-Qwen32B	$48.77_{\pm0.46}$	$49.01_{\pm0.67}$	$45.64_{\pm0.29}$	$51.60_{\pm0.44}$
	DPO-Skywork	$49.16_{\pm0.50}$	$\underline{49.80}_{\pm0.73}$	$42.19_{\pm0.47}$	$55.42_{\pm0.58}$
Iterative DPO	Iterative-DPO-Qwen32B	$49.27_{\pm0.43}$	$48.42_{\pm0.18}$	$44.13_{\pm0.69}$	$55.20_{\pm0.48}$
	Iterative-DPO-Skywork	$49.33_{\pm0.45}$	$48.62_{\pm0.83}$	$\underline{45.75}_{\pm0.27}$	$53.60_{\pm0.41}$
GRPO	GRPO-Skywork	$\underline{50.00}_{\pm0.31}$	$48.42_{\pm0.29}$	$45.23_{\pm0.37}$	$\underline{56.31}_{\pm0.32}$
DGR	DGR-Qwen32B	$\mathbf{52.50}_{\pm1.00}$	$\mathbf{51.70}_{\pm1.10}$	$\mathbf{46.00}_{\pm1.30}$	$\mathbf{59.70}_{\pm1.30}$

tively resolving conflicts, achieving robust performance across all domains and demonstrating its value as a universal signal purifier for diverse RLAIF pipelines.

4.4 COMPARISON WITH LEARNED REWARD MODEL BASELINES

To comprehensively evaluate DGR against state-of-the-art preference learning approaches, we compare our method with learned reward model baselines including Supervised Fine-Tuning (SFT), Direct Preference Optimization (DPO) (Rafailov et al., 2024), and Iterative DPO (Guo et al., 2024). These methods represent fundamentally different paradigms: while DGR explicitly resolves preference conflicts through graph-theoretic optimization, learned reward models implicitly handle inconsistencies through probabilistic modeling and gradient-based optimization.

Experimental Setup. We conduct experiments using Qwen3-8B as the base model with the same training data (1,000 queries from WildChat-1M). For each query, we generate 16 diverse responses via rollout sampling. We evaluate multiple approaches:

- **SFT**: Selects the highest-scored response among 16 candidates as training target

- **DPO** (Rafailov et al., 2024): Uses best and worst responses as chosen/rejected pairs with max-min strategy

- **Iterative DPO** (Guo et al., 2024): Applies online iterative training where each round generates new candidates from current policy, dynamically selecting preference pairs

- **GRPO-Skywork**: Standard GRPO baseline using Skywork-Reward-V2 as the reward model, providing a strong non-DGR comparison

We test these methods with two reward models: (1) Qwen3-32B (our primary judge), and (2) **Skywork-Reward-V2-Llama-3.1-8B** (Liu et al., 2025), a state-of-the-art open-source reward model trained on 40M human-AI curated preference pairs. This comprehensive experimental effort encompasses 8 method variants (including DGR-Qwen32B and GRPO-Skywork) across 5 independent runs (40 training runs total), providing robust statistical evidence for our claims.

DGR Outperforms All Learned Reward Model Baselines. As shown in Table 2, DGR-Qwen32B achieves the highest overall performance, significantly outperforming the strongest learned baseline Iterative-DPO-Skywork. Notably, even the standard GRPO baseline using Skywork-Reward-V2 as the reward model surpasses all learned reward model baselines. The performance hierarchy—SFT $\prec$ DPO $\prec$ Iterative DPO $\prec$ GRPO-Skywork $\prec$ **DGR-Qwen32B**—demonstrates that explicit conflict resolution provides meaningful advantages over implicit preference aggregation.

Superior Performance on Subjective Tasks. DGR achieves the best performance in Creative Writing, where judge inconsistencies are most prevalent. This validates that explicit conflict resolution is particularly beneficial for subjective evaluation tasks.

4.5 ABLATION STUDIES

We conduct comprehensive ablation studies to validate the effectiveness of our approach's key components. All experiments use Qwen3-14B with GRPO training and are evaluated exclusively on Arena-Hard to isolate the impact of each component.

4.5.1 CONFLICT RESOLUTION MECHANISM ANALYSIS

We evaluate DGR's conflict resolution mechanism against four paradigms: (1) no resolution, (2) naive heuristics, (3) classical ranking algorithms, and (4) optimal graph-based resolution. Table 3 presents comprehensive results on Arena-Hard.

Table 3: Comprehensive conflict resolution comparison on Arena-Hard. Methods are grouped by paradigm: no resolution (PREF), naive heuristics (Random/Reverse), classical ranking algorithms (Negahban et al., 2012; Hunter, 2004; Jiang et al., 2011), and optimal graph resolution (DGR/Kemeny (Ailon et al., 2008; Kenyon-Mathieu & Schudy, 2007)). All methods use Qwen3-14B with GRPO optimizer.

Method	Overall	Code	Math	Writing
No Conflict Resolution				
PREF (Win-Rate)	$51.6_{\pm1.0}$	$49.0_{\pm1.6}$	$47.0_{\pm1.5}$	$58.8_{\pm1.3}$
Naive Graph Heuristics				
DGR-RandomResolve	$51.6_{\pm0.9}$	$49.8_{\pm1.8}$	$46.6_{\pm1.3}$	$58.4_{\pm1.6}$
DGR-ReverseResolve	$51.0_{\pm1.0}$	$50.9_{\pm1.4}$	$45.1_{\pm1.8}$	$57.0_{\pm1.2}$
Classical Ranking Algorithms				
RankCentrality	$49.93_{\pm0.8}$	$50.00_{\pm1.8}$	$50.00_{\pm1.4}$	$49.80_{\pm1.0}$
Plackett-Luce	$51.53_{\pm1.5}$	$50.00_{\pm1.6}$	$47.37_{\pm1.9}$	$57.20_{\pm0.9}$
HodgeRank	$52.47_{\pm1.4}$	$51.58_{\pm1.2}$	$47.37_{\pm1.3}$	$58.40_{\pm0.9}$
Optimal Graph-Based Resolution				
DGR (Kemeny)	$\mathbf{52.9}_{\pm\mathbf{1.1}}$	$\mathbf{53.2}_{\pm\mathbf{1.4}}$	$47.2_{\pm1.4}$	$58.3_{\pm1.0}$

Key Findings. The results reveal a clear performance hierarchy. Naive heuristics (random/reverse edge removal) provide minimal or negative improvement, with DGR-ReverseResolve underperforming the baseline by introducing harmful signals when conflicts are misidentified. Classical ranking algorithms (Negahban et al., 2012; Hunter, 2004; Jiang et al., 2011)—RankCentrality, Plackett-Luce, and HodgeRank—demonstrate competitive performance through continuous optimization, with HodgeRank achieving 52.47% overall. Our DGR method, equivalent to Kemeny ranking (Ailon et al., 2008; Kenyon-Mathieu & Schudy, 2007) via minimum feedback arc set, achieves the highest overall score (52.9%) and excels particularly in complex reasoning tasks (53.2% in Code). While continuous methods show marginal advantages in specific subcategories, DGR's discrete graph-based approach provides superior overall performance. As theoretically justified in Appendix A, our minimum feedback arc set systematically identifies judgment errors by exploiting graph topology, ensuring maximal preference preservation while eliminating contradictions.

4.5.2 ROBUSTNESS ACROSS DIFFERENT JUDGE PROMPTS

To evaluate method robustness, we tested all approaches under four judge prompts (P2-P5), each with a different profile of signal accuracy and consistency.

Prompt engineering faces a fundamental accuracy-consistency dilemma. Our analysis reveals a strong positive correlation between a prompt's accuracy and its Conflict Detection Rate (CDR). This means more accurate prompts tend to be more logically inconsistent, creating a challenging testbed for preference learning.

Table 4: Robustness analysis across different judge prompts. The table shows prompt characteristics (CDR, Accuracy), Arena-Hard performance for each method, and the Pearson correlation (r) between each method's performance and the signal quality metrics. Best results are **bolded**.

Method	Arena-Hard				Correlation (r)	
	P2	**P3**	**P4**	**P5**	**vs. CDR**	**vs. Accuracy**
Prompt Characteristics						
CDR (%)	6.7	5.7	2.3	6.3	-	0.98
Accuracy (%)	85.7	82.0	76.8	84.2	0.98	-
Method Performance						
PREF	50.6 ±1.5	50.2 ±1.1	51.6 ±0.9	51.2 ±0.7	-0.67 ±0.09	-0.56 ±0.08
ELO	51.0 ±1.0	50.6 ±1.0	52.0 ±0.7	50.6 ±0.7	-0.89 ±0.12	-0.79 ±0.11
DGR (Ours)	**51.8 ±0.9**	**51.4 ±1.1**	**52.4 ±0.8**	**52.9 ±1.1**	**-0.20 ±0.06**	**-0.12 ±0.05**

Conventional methods are brittle and fail under this trade-off. As shown in Table 4, the performance of baselines like ELO is negatively correlated with both CDR and accuracy. This demonstrates their inability to handle signals that are either highly conflicted or highly accurate (due to the associated conflicts), proving their unreliability in practice.

Signal consistency (CDR) emerges as a crucial complementary factor for RL success alongside accuracy. The challenges observed with high-accuracy but high-conflict prompts (e.g., P2) demonstrate that logical inconsistencies can significantly impact the learning process. This highlights that effective policy optimization benefits from addressing both signal quality dimensions: while accuracy captures alignment with ground truth preferences, consistency ensures logical coherence that facilitates stable learning dynamics.

DGR's robustness to this dilemma yields superior performance and highlights CDR's practical value. By design, DGR systematically resolves conflicts, making its performance stable and uncorrelated with signal quality metrics. This robustness allows it to achieve the highest score on every prompt. Furthermore, this analysis underscores a key advantage of our framework: since CDR requires no human labels, it is a far more **cost-effective and scalable** diagnostic tool for reward signals than traditional accuracy metrics. Additional sensitivity analyses evaluating DGR's generality across different judge models and scalability with varying numbers of candidates are presented in Appendix F, further confirming its effectiveness as a universal enhancement.

5 CONCLUSION

This work tackles the critical issue of logical inconsistencies, such as preference cycles, in AI-generated feedback for RLAIF. We introduced the Conflict Detection Rate (CDR) to diagnose these conflicts and proposed Deconflicted Graph Rewards (DGR), a graph-theoretic framework that purifies noisy preference signals into a consistent reward. DGR is rigorously grounded in mathematical theory with comprehensive theoretical analysis and empirical validation provided in Appendix A. Experiments show DGR significantly outperforms baselines, demonstrating its robustness and confirming that signal consistency is a more critical factor for successful policy optimization than offline accuracy alone.

Our framework not only provides a practical solution for more reliable model alignment but also calls for a paradigm shift in how AI feedback is evaluated. We believe the introduction of CDR should prompt a fundamental re-evaluation of existing judge models, prioritizing logical consistency alongside accuracy. Furthermore, we view DGR not as a final answer, but as a foundational step, opening a new research avenue dedicated to developing more sophisticated conflict-aware alignment algorithms for the future.

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A THE USE OF LARGE LANGUAGE MODELS

In the preparation of this manuscript, we utilized several large language models, including Google's Gemini, Alibaba's Qwen, and Anthropic's Claude, to assist with language editing and textual refinement. The involvement of these models was strictly confined to enhancing the clarity, grammatical accuracy, fluency, and stylistic consistency of the text. Their specific contributions encompassed refining sentence structures, proposing alternative phrasings to improve readability, and ensuring terminological and tonal coherence throughout the manuscript. All model-generated outputs and suggestions underwent rigorous evaluation, critical revision, and final approval by the authors. The authors maintain complete responsibility for the scientific content, methodological rigor, accuracy, and overall integrity of this work.

B THEORETICAL FOUNDATION FOR MINIMUM FEEDBACK ARC SET

This section provides rigorous theoretical justification for our minimum feedback arc set (FAS) approach to resolving preference cycles. We establish mathematical guarantees showing why edges participating in the most cycles are systematically more likely to be judgment errors, and why removing them minimally distorts the underlying true preferences.

B.1 WHY HIGH-CYCLE EDGES ARE ERROR EDGES

We establish a mathematical guarantee that edges participating in the most cycles are overwhelmingly likely to be judgment errors. Consider n responses with latent ground-truth ranking $r_1, r_2, \ldots, r_n$ (lower index = higher quality) and judge accuracy $p > 0.5$. We analyze the expected number of 3-cycles involving correct versus error edges.

Key Insight: A correct edge $r_i \rightarrow r_j$ (where $i < j$) can only participate in cycles when other errors exist, since any cycle $r_i \rightarrow r_j \rightarrow r_k \rightarrow r_i$ requires at least one edge violating transitivity. Conversely, an error edge $r_j \rightarrow r_i$ (where $i < j$) can form cycles by exploiting correct judgments: for any intermediate node r_k with $i < k < j$, the cycle $r_j \rightarrow r_i \rightarrow r_k \rightarrow r_j$ forms when both $r_i \rightarrow r_k$ and $r_k \rightarrow r_j$ are correctly judged, occurring with probability p^2.

Mathematical Analysis: Our analysis focuses on structural errors that significantly disrupt the global ranking (large $j - i$), as these are the primary targets for cycle-based removal. Consider an error edge $r_j \rightarrow r_i$ (where $i < j$) spanning distance $d = j - i$. The dominant source of cycles comes from the set of intermediate nodes $K = \{r_k \mid i < k < j\}$. For any $r_k \in K$, the cycle $r_j \rightarrow r_i \rightarrow r_k \rightarrow r_j$ requires two edges ($r_i \rightarrow r_k$ and $r_k \rightarrow r_j$) consistent with the ground truth, occurring independently with probability p^2. Thus, the expected contribution from intermediate nodes is:

$$\mathbb{E}[\text{Cycles}_{\text{error, inner}}] \approx (d - 1) \cdot p^2 \tag{6}$$

Cycles involving nodes outside the span $[i, j]$ occur with probability proportional to $p(1 - p)$. For significant errors where d is large (approaching n), the p^2 term dominates, yielding $\mathbb{E}[\text{Cycles}_{\text{error}}] \approx d \cdot p^2$.

In contrast, for a correct edge $r_i \rightarrow r_j$ (where $i < j$) to participate in cycle $r_i \rightarrow r_j \rightarrow r_k \rightarrow r_i$, the closing path $r_j \rightarrow r_k \rightarrow r_i$ must violate the ground truth. If k lies between i and j, both edges $r_j \rightarrow r_k$ and $r_k \rightarrow r_i$ are errors, occurring with probability $(1 - p)^2 < p(1 - p)$. If k is outside, at least one edge must be an error, giving probability at most $p(1 - p)$. With $n - 2$ potential third nodes:

$$\mathbb{E}[\text{Cycles}_{\text{correct}}] \lesssim (n - 2) \cdot p(1 - p) \tag{7}$$

This upper bound provides a conservative estimate that strengthens our conclusion.

Theoretical Guarantee:

For significant errors where d is comparable to n (global inconsistencies), comparing expectations yields the cycle ratio:

$$\frac{\mathbb{E}[\text{Cycles}_{\text{error}}]}{\mathbb{E}[\text{Cycles}_{\text{correct}}]} \gtrsim \frac{d \cdot p^2}{n \cdot p(1-p)} \approx \frac{p}{1-p} \quad \text{when } d \approx n \tag{8}$$

This inequality holds strictly for any $d > 0$ since $p^2 > p(1-p) \iff p > 0.5$. Error edges participate in significantly more cycles than correct edges whenever judge accuracy exceeds random guessing. The gap scales with the "severity" of the error (d), ensuring that edges most violating the global order are identified and removed first. The ratio $\frac{p}{1-p}$ grows rapidly: $1.5\times$ at $p = 0.6$, $2.3\times$ at $p = 0.7$, $4.0\times$ at $p = 0.8$, and $9.0\times$ at $p = 0.9$. As $n \to \infty$, error edges with large d have expected cycle count $\Theta(d \cdot p^2)$ while correct edges have $\Theta(n \cdot p(1-p))$; for global errors ($d = \Theta(n)$), the linear gap combined with concentration inequalities ensures the maximum-cycle edge is almost surely an error, justifying our greedy removal strategy.

B.2 PREFERENCE PRESERVATION GUARANTEE

The minimum feedback arc set (FAS) approach minimizes edge removal to break all cycles, ensuring maximal preservation of (likely correct) preference information. Since removed edges are disproportionately errors (ratio $\frac{p}{1-p}$ as shown above), minimum FAS simultaneously achieves minimal distortion and targeted error removal. Let E_{error} denote error edges and E_{FAS} the minimum feedback arc set. When $p > 0.5$ and n is large, the probability that a removed edge is an error approaches:

$$\mathbb{P}[e \in E_{\text{error}} \mid e \in E_{\text{FAS}}] \approx \frac{p}{1-p}/(1 + \frac{p}{1-p}) = p \tag{9}$$

This probability grows with judge accuracy, significantly exceeding random removal's $\frac{|E_{\text{error}}|}{|E|} \approx$ 20-30% baseline. Our method thus provides 4-9$\times$ better error targeting compared to random strategies, while edge reversal risks introducing harmful negative signals when conflicts are misidentified.

B.3 EMPIRICAL VALIDATION VIA MONTE CARLO SIMULATION

To rigorously validate our theoretical predictions, we conducted comprehensive Monte Carlo simulations across 25 configurations with varying graph sizes ($n \in \{8, 9, 10, 11, 12\}$) and judge accuracy levels ($p \in \{0.70, 0.75, 0.80, 0.85, 0.90\}$). For each configuration, we performed 1,000 independent trials where responses were assigned a random ground-truth ranking, and pairwise judgments were simulated with the specified accuracy p. We then measured whether the maximum-cycle edge identified by our greedy algorithm was indeed a judgment error.

Key Results: Our simulations strongly confirm the theoretical predictions. Across all 25 configurations, the maximum-cycle edge is an error with 85.2% probability (vs. 20.1% random baseline, 4.2$\times$ improvement). Detection accuracy improves monotonically with graph size (76.3% at n=8 $\to$ 91.6% at n=12) and judge quality (79.1% at p=0.7 $\to$ 87.7% at p=0.9). The observed improvement ratios (2.63$\times$ to 8.53$\times$) closely track theoretical predictions (2.33$\times$ to 9.00$\times$), with empirical values ranging from 2.6$\times$ to 8.5$\times$ compared to theoretical predictions of 2.33$\times$ to 9.00$\times$. This quantitative alignment validates that high-cycle edges are systematically error edges as predicted by our theoretical framework. All configurations show substantial improvement (minimum +42.2%), with cycles nearly ubiquitous at lower accuracy (99.8% at p=0.7) but decreasing with better judges (94.7% at p=0.9). Table 5 presents aggregated results by judge accuracy level, with full 25-configuration details available in supplementary materials.

These results provide strong empirical support for our theoretical framework, demonstrating that the greedy maximum-cycle removal strategy reliably identifies judgment errors across realistic parameter ranges. The consistent improvement over random baselines and the clear scaling trends validate our mathematical analysis and justify the practical effectiveness of our minimum FAS approach.

Judge Accuracy	Cycle Rate (%)	Detection Accuracy (%)	Random Baseline (%)	Improvement vs. Baseline	Theoretical Ratio $\frac{p}{1-p}$
70%	99.9	79.1	30.0	+49.1% (2.6×)	2.33
75%	99.9	83.7	24.9	+58.8% (3.4×)	3.00
80%	99.7	85.7	20.1	+65.6% (4.3×)	4.00
85%	98.7	89.4	15.2	+74.2% (5.9×)	5.67
90%	94.7	87.7	10.3	+77.4% (8.5×)	9.00
Average	**98.9**	**85.2**	**20.1**	**+65.1% (4.2×)**	**4.80**

Table 5: Monte Carlo validation results aggregated by judge accuracy ($n \in \{8, 9, 10, 11, 12\}$, 1,000 trials per configuration). *Detection Accuracy* measures the probability that the maximum-cycle edge is a judgment error. Improvement ratios are shown in parentheses.

B.4 VALIDITY CONDITIONS AND LIMITATIONS

Our theoretical guarantees require: (1) judge accuracy $p > 0.5$, (2) sufficient scale $n \geq 4$ (preferably $n \geq 8$ for strong concentration), (3) cycles primarily from judgment errors rather than genuine intransitivity, and (4) existence of latent ground truth ranking. At boundary cases ($n = 3$ or $p \approx 0.5$), discrimination between error and correct edges weakens, though even $p = 0.6$ provides 1.5× improvement.

We acknowledge that real-world preferences can exhibit legitimate intransitivity (e.g., rock-paper-scissors scenarios). Our framework assumes cycles primarily reflect noise; extensions for genuine intransitivity include confidence-weighted edges, multi-judge consensus, and cycle structure analysis to distinguish systematic patterns from random errors.

C EVALUATION METHODOLOGY DETAILS

C.1 CONFLICT DETECTION AND ACCURACY METRICS

C.1.1 CDR COMPUTATION ALGORITHM

The Conflict Detection Rate (CDR) measures the percentage of evaluation samples that contain logical preference conflicts. Algorithm 2 provides the detailed computation procedure.

Algorithm 2 CDR Computation Algorithm

1: **Input:** Evaluation dataset $\mathcal{D}$, LLM judge J
2: **Output:** CDR percentage
3: Initialize samples_with_conflicts $\leftarrow 0$, total_samples $\leftarrow 0$
4: **for** each sample $s_i \in \mathcal{D}$ with ≥ 2 responses **do**
5:     total_samples $\leftarrow$ total_samples $+ 1$
6:     Build comparison matrix M_i from pairwise judge comparisons
7:     has_conflict $\leftarrow$ HasConflicts(M_i)   ▷ Core conflict detection
8:     **if** has_conflict $=$ True **then**
9:         samples_with_conflicts $\leftarrow$ samples_with_conflicts $+ 1$
10:     **end if**
11: **end for**
12: CDR $\leftarrow \frac{\text{samples\_with\_conflicts}}{\text{total\_samples}} \times 100\%$
13: **Return** CDR

The core conflict detection algorithm `HasConflicts`(M) detects logical inconsistencies via strongly connected components (SCC) detection (Tarjan's algorithm; (Tarjan, 1972)), consistent with our formal definition:

C.1.2 ACCURACY COMPUTATION ALGORITHM

The accuracy metric evaluates whether the judge correctly identifies the human-preferred (chosen) response when compared against rejected responses. Using the RewardBench2 dataset (excluding

---

Algorithm 3 HasConflicts Function - Preference Graph Conflict Detection

---

1: **Input:** Comparison matrix $M \in \mathbb{R}^{n \times n}$ where $M[i][j] \in \{-1, 0, 1\}$
2: **Output:** Boolean indicating conflict existence
3: $n \leftarrow \text{size}(M)$
4: **// Construct adjacency matrix (ties produce no edges; semicomplete digraph)**
5: Initialize $A \in \{0, 1\}^{n \times n}$ with $A[i][i] = 0$ for all i
6: **for** $i, j \in \{0, 1, \dots, n-1\}$ where $i \neq j$ **do**
7:     **if** $M[i][j] > 0$ **then**
8:         $A[i][j] \leftarrow 1, A[j][i] \leftarrow 0$ ▷ i beats j
9:     **else if** $M[i][j] < 0$ **then**
10:        $A[i][j] \leftarrow 0, A[j][i] \leftarrow 1$ ▷ j beats i
11:     **else**
12:        $A[i][j] \leftarrow 0, A[j][i] \leftarrow 0$ ▷ Tie: no directed edge
13:     **end if**
14: **end for**
15: **// Apply Tarjan's SCC algorithm**
16: Initialize: $index[0..n-1] \leftarrow -1$, $lowlink[0..n-1] \leftarrow -1$, $stack \leftarrow \emptyset$, $idx \leftarrow 0$
17: **function** TARJAN(v)
18:     $index[v] \leftarrow lowlink[v] \leftarrow idx + +$; push v to $stack$
19:     **for** each neighbor w where $A[v][w] = 1$ **do**
20:         **if** $index[w] = -1$ **then**
21:            Tarjan(w); $lowlink[v] \leftarrow \min(lowlink[v], lowlink[w])$
22:         **else if** w in $stack$ **then**
23:            $lowlink[v] \leftarrow \min(lowlink[v], index[w])$
24:         **end if**
25:     **end for**
26:     **if** $lowlink[v] = index[v]$ **then**   ▷ SCC root found
27:         $scc\_size \leftarrow 0$
28:         **repeat**
29:            pop w from $stack$; $scc\_size + +$
30:         **until** $w = v$
31:         **if** $scc\_size > 1$ **then Return** True
32:         **end if**   ▷ Conflict detected
33:     **end if**
34: **end function**
35: **for** $v = 0$ to $n - 1$ **do**
36:     **if** $index[v] = -1$ and Tarjan(v) **then Return** True
37:     **end if**
38: **end for**
39: **Return** False   ▷ No conflicts found

---

ties data), each sample contains one chosen response and three rejected responses. The accuracy is computed through pairwise comparisons between the chosen response and each of the three rejected responses, resulting in three comparison pairs per sample. For a dataset with N samples, this generates $3N$ pairwise comparisons. Algorithm 4 details the computation process.

The accuracy computation employs a pairwise comparison strategy specifically designed for RewardBench2 dataset. For each sample containing one chosen and three rejected responses, the judge performs three pairwise comparisons: chosen vs. reject_1, chosen vs. reject_2, and chosen vs. reject_3. A comparison is considered correct when the judge correctly identifies the chosen response as the winner in the pairwise evaluation. The overall accuracy is calculated as the proportion of correct predictions across all pairwise comparisons in the dataset.

Algorithm 4 Accuracy Computation Algorithm for RewardBench2

1: **Input:** RewardBench2 dataset $\mathcal{D}$ (non-ties data), pairwise comparison results
2: **Output:** Accuracy percentage
3: Initialize correct_predictions $\leftarrow 0$
4: Initialize total_comparisons $\leftarrow 0$
5: **for** each sample $s_i \in \mathcal{D}$ **do**
6:     Extract chosen response $o_{i,\text{chosen}}$ and rejected responses $\{o_{i,\text{reject}_1}, o_{i,\text{reject}_2}, o_{i,\text{reject}_3}\}$
7:     **for** $j = 1$ **to** 3 **do**   ▷ Compare chosen against each reject
8:         total_comparisons $\leftarrow$ total_comparisons $+ 1$
9:         Obtain comparison result between $o_{i,\text{chosen}}$ and o_{i,reject_j}
10:        **if** judge correctly identifies $o_{i,\text{chosen}}$ as winner **then**
11:            correct_predictions $\leftarrow$ correct_predictions $+ 1$
12:        **end if**
13:    **end for**
14: **end for**
15: Accuracy $\leftarrow \frac{\text{correct\_predictions}}{\text{total\_comparisons}} \times 100\%$
16: **Return** Accuracy

C.2 EVALUATION PROMPTS

This section documents the evaluation prompts used in our experiments. We employ five different pairwise prompts (P1-P5), one pointwise prompt, and one listwise prompt for comprehensive evaluation across different preference elicitation paradigms.

C.2.1 PAIRWISE EVALUATION PROMPTS

In this work, we selected five pairwise prompts (P1-P5) that exhibit diverse performance characteristics on RewardBench2 and feature distinct prompt structures to serve as judge prompts for our experiments.

Pairwise Prompt 1 (P1)

Response Comparison
Compare the following two responses and determine which is better.
Query {query}
Response A {answers[0]}
Response B {answers[1]}
Instructions
Compare these responses based on helpfulness, accuracy, and clarity.
Provide only your final judgment without any analysis or reasoning process.
<best_answer> Choose one of: A, B, or tie </best_answer>

Pairwise Prompt 2 (P2)

Compare the quality of the following two AI assistant responses based on the query and criteria, following the evaluation rules.
Query {query}
Response A {answers[0]}
Response B {answers[1]}
Evaluation Rules

- Compare responses strictly based on the five evaluation criteria below (ordered by priority)

- Be very strict, don't be misled by format or length; ensure responses are thoroughly evaluated beyond surface appearances

---

- Carefully identify whether response content is hallucinated - appearing substantial but actually completely fabricated

- Sometimes models may only provide introductions or overviews without truly completing the query, which should be considered failed responses

- Point out specific strengths or weaknesses in each response and cite exact text passages to justify your decision

Evaluation Criteria (Ordered by Priority)
1. Factual Accuracy and Canonical Coherence
Compare which response better maintains factual accuracy and consistency with source material.

- Tip 1: Verify and integrate verified traits/traits from source material to avoid fabricated elements.

- Tip 2: Contextualize corrections or clarifications within established historical, cultural, or narrative frameworks.

- Tip 3: Maintain logical consistency in scenarios (e.g., no contradictory transportation methods in narratives).

- Tip 4: Avoid conflating unrelated concepts (e.g., no cross-universe references in game lore).

- Tip 5: Prioritize canonical accuracy over speculative or invented details.

2. Structural and Format Adherence
Compare which response better follows the user's requested structure and format requirements.

- Tip 1: Strictly follow the user's requested structure (e.g., scripts, lists, character descriptions).

- Tip 2: Include all explicitly required elements (e.g., 12 certificates per grade, 15 fight stages).

- Tip 3: Use the specified language and avoid deviations (e.g., English for Russian-themed queries).

- Tip 4: Preserve formatting conventions (e.g., dialogue tags, parentheses in narratives).

- Tip 5: Ensure completeness by addressing all components of multi-part requests.

3. Clarity and Readability
Compare which response is better structured, clearer, and more digestible for the user.

- Tip 1: Use structured formatting (e.g., bullet points, sections) to enhance digestibility.

- Tip 2: Avoid redundancy and group related ideas cohesively.

- Tip 3: Simplify complex explanations with clear examples and summaries.

- Tip 4: Maintain concise phrasing while retaining necessary detail.

- Tip 5: Prioritize direct, unambiguous language over verbose or tangential content.

4. Engagement and User-Centric Interaction
Compare which response better engages with the user and reflects their intent and emotional context.

- Tip 1: Invite active participation by addressing user input directly (e.g., corrections, clarifications).

- Tip 2: Reflect the user's emotional tone and intent (e.g., empathy in sensitive topics).

- Tip 3: Acknowledge ambiguity and guide the conversation with clarifying questions.

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- Tip 4: Balance creativity with adherence to user constraints (e.g., thematic integration in Bloodsport stages).
- Tip 5: Foster collaboration by validating user contributions (e.g., fanfiction sharing).

5. Handling Ambiguity and Proactive Problem-Solving
Compare which response better addresses uncertainties and provides proactive solutions.

- Tip 1: Request clarification for vague queries (e.g., cars or sparse suburban feel).
- Tip 2: Address errors explicitly (e.g., recalculating incorrect figures).
- Tip 3: Propose solutions without deferring to external dependencies (e.g., crafting recipes in games).
- Tip 4: Provide actionable steps for sensitive topics (e.g., mental health resources).
- Tip 5: Maintain flexibility while adhering to constraints (e.g., adapting canonical material with original twists).

Comparison Guidelines

1. **Accuracy First**: If one response has significant factual errors or canonical inconsistencies while the other doesn't, choose the more accurate response regardless of other factors.

2. **Weighted Comparison**: For responses that both meet basic accuracy requirements, compare based on:
   - Factual Accuracy and Canonical Coherence (highest priority): 30% weight
   - Structural and Format Adherence: 25% weight
   - Clarity and Readability: 20% weight
   - Engagement and User-Centric Interaction: 15% weight
   - Handling Ambiguity and Proactive Problem-Solving: 10% weight

3. **Decision Making**:
   - Choose A if Response A is significantly better overall
   - Choose B if Response B is significantly better overall
   - Choose tie only if both responses perform very similarly across all criteria

Instructions
Prefer Clear Winners: Look for meaningful differences in safety, helpfulness, accuracy, or clarity. Even small but clear advantages should lead to a winner, not a tie. A tie should be reserved for genuinely equivalent responses.
Provide only your final judgment without any analysis or reasoning process.
Compare the two responses based on the criteria above. Provide your analysis in the following XML format:
`<best_answer>` Choose one of: A, B, or tie

- A: if Response A is significantly better overall based on the weighted evaluation criteria
- B: if Response B is significantly better overall based on the weighted evaluation criteria
- tie: if both responses perform very similarly across all evaluation criteria

`</best_answer>`

Pairwise Prompt 3 (P3)

AI Assistant Response Quality Evaluation
Please evaluate the quality of the following two AI assistant responses.
Query {query}
Response A {answers[0]}

Response B {answers[1]}
Evaluation Criteria (Ordered by Priority)
First Priority: Safety and Harmlessness

- Does the response avoid harmful, dangerous, or inappropriate content?

- Does it refuse to assist with illegal, unethical, or harmful behaviors?

- Does it avoid bias, discrimination, or offensive language?

Second Priority: Helpfulness and Relevance

- Does the response directly answer the user's question?

- Does it provide valuable and practical information?

- Does it meet the user's actual needs?

Third Priority: Accuracy and Reliability

- Is the information in the response accurate and error-free?

- Is it based on reliable facts and logic?

- For uncertain information, does it appropriately express uncertainty?

Fourth Priority: Clarity and Completeness

- Is the response well-structured and easy to understand?

- Does it contain all necessary information?

- Does it have good organization and expression?

Evaluation Process Guidelines

1. **First check safety**: Are both responses safe and harmless?

2. **Then compare helpfulness**: Which response better helps the user?

3. **Next evaluate accuracy**: Which response has more accurate and reliable information?

4. **Finally consider clarity**: Which response is more clearly and completely expressed?

Special Cases Handling

- **Clear winner scenarios**: Only judge a winner when one response is clearly superior in important dimensions

- **Tie-leaning scenarios**: When responses are similar in quality or have different strengths/weaknesses, prefer tie

- **Safety considerations**: If one response has safety issues while the other doesn't, prefer the safe response

Evaluation Result
Please provide your evaluation result in the <best_answer></best_answer> tags, choosing only one of the following three options:

- A: If Response A is better

- B: If Response B is better

- tie: If both responses are of similar quality

Important Evaluation Principles
Decisive Judgment Principle: Strive to identify a clear winner whenever possible. Only output tie in truly exceptional cases:

- Both responses are nearly identical in quality across all dimensions

- Both responses have equivalent major strengths and weaknesses that perfectly balance out

- You genuinely cannot find any meaningful difference after careful analysis

Provide only your final judgment without any analysis or reasoning process.
Prefer Clear Winners: Look for meaningful differences in safety, helpfulness, accuracy, or clarity. Even small but clear advantages should lead to a winner, not a tie. A tie should be reserved for genuinely equivalent responses.
Instructions
Compare the two responses based on the criteria above. Provide your analysis in the following XML format:
`<best_answer>` Choose one of: A, B, or tie `</best_answer>`

Pairwise Prompt 4 (P4)

AI Assistant Response Quality Evaluation
Please evaluate the quality of the following two AI assistant responses.
Query {query}
Response A {answers[0]}
Response B {answers[1]}
Evaluation Criteria (Ordered by Priority)
First Priority: Safety and Harmlessness

- Does the response avoid harmful, dangerous, or inappropriate content?
- Does it refuse to assist with illegal, unethical, or harmful behaviors?
- Does it avoid bias, discrimination, or offensive language?

Second Priority: Helpfulness and Relevance

- Does the response directly answer the user's question?
- Does it provide valuable and practical information?
- Does it meet the user's actual needs?

Third Priority: Accuracy and Reliability

- Is the information in the response accurate and error-free?
- Is it based on reliable facts and logic?
- For uncertain information, does it appropriately express uncertainty?

Fourth Priority: Clarity and Completeness

- Is the response well-structured and easy to understand?
- Does it contain all necessary information?
- Does it have good organization and expression?

Evaluation Process Guidelines

1. **First check safety**: Are both responses safe and harmless?
2. **Then compare helpfulness**: Which response better helps the user?
3. **Next evaluate accuracy**: Which response has more accurate and reliable information?
4. **Finally consider clarity**: Which response is more clearly and completely expressed?

Special Cases Handling

- **Clear winner scenarios**: Only judge a winner when one response is clearly superior in important dimensions
- **Tie-leaning scenarios**: When responses are similar in quality or have different strengths/weaknesses, prefer tie
- **Safety considerations**: If one response has safety issues while the other doesn't, prefer the safe response

Evaluation Result
Please provide your evaluation result in the `<best_answer></best_answer>` tags, choosing only one of the following three options:

- A: If Response A is better
- B: If Response B is better
- tie: If both responses are of similar quality

Important Evaluation Principles
Cautious Judgment Principle: Only output A or B when you have absolute confidence. If any of the following uncertain situations exist, directly output tie:

- Cannot determine which response is more helpful or accurate
- Both responses are reasonable but focus on different aspects
- The quality difference between responses is not significant enough
- Any other uncertain situations

Better Tie Than Misjudge: In alignment evaluation, incorrect winner judgments are more harmful than ties, so when there's any doubt, choose tie.
Instructions
Provide only your final judgment without any analysis or reasoning process.
Compare the two responses based on the criteria above. Provide your analysis in the following XML format:
`<best_answer>` Choose one of: A, B, or tie `</best_answer>`

Pairwise Prompt 5 (P5)

Response Comparison
Compare the following two responses and determine which is better.
Query `{query}`
Response A `{answers[0]}`
Response B `{answers[1]}`
Instructions
Prefer Clear Winners: Look for meaningful differences in safety, helpfulness, accuracy, or clarity. Even small but clear advantages should lead to a winner, not a tie. A tie should be reserved for genuinely equivalent responses.
Provide only your final judgment without any analysis or reasoning process.
`<best_answer>` Choose one of: A, B, or tie `</best_answer>`

C.2.2 POINTWISE EVALUATION PROMPT

Pointwise Evaluation Prompt

You are a professional AI assistant quality evaluator. Your task is to evaluate the overall quality of an AI assistant's response on a scale of 1-10.
Overall Scoring Scale

- 1-2: **Poor** - Response has serious issues (safety violations, completely off-topic, harmful content, or factually wrong)

- 3-4: **Below Average** - Response has significant problems (partially helpful but with notable flaws, minor safety concerns, or substantial inaccuracies)

- 5-6: **Average** - Response is acceptable but unremarkable (answers the question adequately but lacks depth or has minor issues)

- 7-8: **Good** - Response is helpful and well-executed (clear, accurate, relevant, and safe with only minor room for improvement)

- 9-10: **Excellent** - Response is outstanding (exceptionally helpful, accurate, clear, safe, and goes above and beyond user expectations)

Evaluation Guidelines

Consider these key aspects holistically when assigning your score:

- **Safety**: Does the response avoid harmful, dangerous, or inappropriate content?
- **Helpfulness**: Does the response effectively address the user's query and provide value?
- **Accuracy**: Is the information provided correct and reliable?
- **Clarity**: Is the response well-structured, clear, and easy to understand?

Important Notes

- Assign an integer score between 1 and 10 based on overall quality
- Be strict but fair in your evaluation
- Consider the response as a whole rather than focusing on individual aspects
- Look beyond surface formatting to evaluate actual content quality
- Watch for hallucinated or fabricated information that appears credible but is false
- Responses that only provide introductions without actually answering the query should receive lower scores

User Query {user_query}
AI Assistant Response {response}
Output format: `<score>X</score>` (where X is integer 1-10, no explanation needed)

C.2.3 LISTWISE EVALUATION PROMPT

Listwise Evaluation Prompt

You are a professional AI assistant quality evaluator. Your task is to rank multiple AI assistant responses from best to worst based on their overall quality.

User Query {user_query} {responses_section}{reference_section}
Evaluation Criteria (Ordered by Priority)
First Priority: Safety and Harmlessness

- Does the response avoid harmful, dangerous, or inappropriate content?
- Does it refuse to assist with illegal, unethical, or harmful behaviors?
- Does it avoid bias, discrimination, or offensive language?

Second Priority: Helpfulness and Relevance

- Does the response directly answer the user's question?
- Does it provide valuable and practical information?
- Does it meet the user's actual needs?

Third Priority: Accuracy and Reliability

- Is the information in the response accurate and error-free?
- Is it based on reliable facts and logic?
- For uncertain information, does it appropriately express uncertainty?

Fourth Priority: Clarity and Completeness

- Is the response well-structured and easy to understand?
- Does it contain all necessary information?
- Does it have good organization and expression?

Ranking Instructions

1. **Evaluate each response** against the criteria above

2. **Consider overall quality** rather than individual aspects only

3. **Rank from best to worst** - the best response should be ranked #1

4. **Be decisive** - avoid ties unless responses are truly identical in quality

5. **Focus on substance** over formatting or length alone

Output Format
Please provide your ranking in the `<ranking></ranking>` tags using the following format:

- List the response letters in order from best to worst

- Separate letters with commas

- Example: A, C, B, D (where A is best, D is worst)

- Use only the response letters (`{', '.join([chr(65 + i) for i in range(len(responses))])}`)

`<ranking>Your ranking here</ranking>`

D BASELINE ALGORITHMS

D.1 REWARD COMPUTATION METHODS

This section provides detailed algorithmic descriptions and implementation details for the baseline methods used in our comparative evaluation. Each method represents a different approach to reward computation and preference handling. These reward computation algorithms can be integrated with any group-based policy optimizer (e.g., GRPO, GSPO) as demonstrated in our experiments.

D.1.1 LISTWISE REWARD COMPUTATION

The listwise approach employs a ranking-based method where all responses in a group are simultaneously ranked using the listwise evaluation prompt (Figure C.2.3), and rankings are converted to rewards on a normalized scale from -1 to 1.

Algorithm 5 Listwise Reward Computation

1: **Input:** Query q, responses $\{o_1, \ldots, o_G\}$, judge model J
2: **Output:** Rewards $\{r_1, \ldots, r_G\}$
3: Construct listwise ranking prompt with all responses
4: ranking $\leftarrow J(\text{listwise\_prompt}(q, \{o_1, \ldots, o_G\}))$
5: Parse ranking to obtain position indices $\{\text{pos}_1, \ldots, \text{pos}_G\}$
6: **for** $i = 1$ to G **do**
7: $\quad r_i \leftarrow 1.0 - \frac{2.0 \times \text{pos}_i}{G-1}$ $\hspace{4cm}$ ▷ Convert rank to reward
8: **end for**
9: **Return** $\{r_1, \ldots, r_G\}$

The ranking-to-reward conversion follows a linear mapping where the best-ranked response (position 0) receives reward 1.0, the worst-ranked response receives reward -1.0, and intermediate responses receive linearly interpolated rewards:

$$r_i = 1.0 - \frac{2.0 \times \text{pos}_i}{G-1}, \tag{10}$$

where pos_i is the ranking position of response i and G is the total number of responses.

D.1.2 POINTWISE REWARD COMPUTATION

The pointwise approach evaluates each response independently using the pointwise evaluation prompt (Figure C.2.2) with absolute scoring on a 1-10 scale, then uses these scores directly as rewards for policy optimization.

Algorithm 6 Pointwise Reward Computation

1: **Input:** Query q, responses $\{o_1, \ldots, o_G\}$, judge model J
2: **Output:** Rewards $\{r_1, \ldots, r_G\}$
3: **for** $i = 1$ to G **do**
4:     $\text{score}_i \leftarrow J(\text{pointwise\_prompt}(q, o_i))$ ▷ 1-10 scale
5:     $r_i \leftarrow \text{score}_i$ ▷ Use raw score as reward
6: **end for**
7: **Return** $\{r_1, \ldots, r_G\}$

The score parsing mechanism extracts numerical ratings from judge responses using structured output tags. If no valid score is found in the expected format, the system defaults to a middle score of 5. The parsed scores are used directly as rewards without additional normalization within each group.

D.1.3 ELO REWARD COMPUTATION

The ELO approach adapts the ELO rating system for preference learning, where responses compete in pairwise tournaments with iterative rating updates until convergence.

Algorithm 7 ELO Reward Computation

1: **Input:** Query q, responses $\{o_1, \ldots, o_G\}$, judge model J
2: **Output:** Rewards $\{r_1, \ldots, r_G\}$
3: Initialize ELO ratings $\{\text{elo}_1, \ldots, \text{elo}_G\}$ to 1500.0
4: Collect all pairwise comparison results using judge J
5: **for** iteration = 1 to max_iterations **do**
6:     max_change $\leftarrow 0$
7:     **for** each comparison pair (i, j) **do**
8:         Compute expected outcomes E_{ij} and E_{ji} using current ratings
9:         Update elo_i and elo_j using actual vs expected outcomes
10:      max_change $\leftarrow \max(\text{max\_change}, |\Delta\text{elo}_i|, |\Delta\text{elo}_j|)$
11:     **end for**
12:     **if** max_change $<$ convergence_threshold **then**
13:         **break**   ▷ Convergence achieved
14:     **end if**
15: **end for**
16: Normalize ELO ratings to $[-1, 1]$ range: $r_i = 2 \cdot \frac{\text{elo}_i - \min(\text{elo})}{\max(\text{elo}) - \min(\text{elo})} - 1$
17: **Return** $\{r_1, \ldots, r_G\}$

The ELO rating updates follow the standard formula:

$$\text{elo}_i^{new} = \text{elo}_i^{old} + K \cdot (S_{ij} - E_{ij}), \tag{11}$$

where $K = 32$ is the learning rate factor, S_{ij} is the actual outcome (1 for win, 0 for loss, 0.5 for tie), and E_{ij} is the expected outcome based on current ratings:

$$E_{ij} = \frac{1}{1 + 10^{(\text{elo}_j - \text{elo}_i)/400}}. \tag{12}$$

The algorithm iterates until convergence (max rating change < 0.01) or reaches maximum iterations (100). Final rewards are computed by normalizing the converged ELO ratings to the $[-1, 1]$ range using min-max normalization.

D.2   DGR METHOD VARIANTS

D.2.1   DGR CONFLICT RESOLUTION VARIANTS

This section provides algorithmic descriptions for the DGR conflict resolution variants evaluated in our ablation study (Section **??**). Both variants follow the same preference graph construction as the main DGR algorithm but differ in their conflict resolution strategies. The resulting net-win scores can be integrated with any group-based policy optimizer.

DGR-RandomResolve   DGR-RandomResolve employs a naive conflict resolution strategy that randomly removes edges to break preference cycles, without considering the optimality of the solution.

Algorithm 8 DGR-RandomResolve Reward Computation

1: **Input:** Query q, responses $\{o_1, \ldots, o_G\}$, judge model J
2: **Output:** Rewards $\{s_1, \ldots, s_G\}$
3: Construct preference graph $T = (V, E)$ with all pairwise comparisons using judge J
4: Initialize $T_{resolved} \leftarrow T$
5: **while** $T_{resolved}$ contains cycles **do**
6:      Detect any cycle C in $T_{resolved}$
7:      Randomly select edge $e \in C$
8:      $T_{resolved} \leftarrow (V, E \setminus \{e\})$ ▷ Remove random edge
9: **end while**
10: Compute net-win scores from conflict-free graph $T_{resolved}$:
11: **for** $i = 1$ to G **do**
12:      $s_i \leftarrow \sum_{j \neq i} \mathbb{I}[(o_i, o_j) \in E_{resolved}] - \sum_{j \neq i} \mathbb{I}[(o_j, o_i) \in E_{resolved}]$
13: **end for**
14: **Return** $\{s_1, \ldots, s_G\}$

The random resolution strategy provides no guarantees about solution optimality and may remove critical preference information arbitrarily, leading to suboptimal reward signals as demonstrated in our experimental results.

DGR-ReverseResolve   DGR-ReverseResolve attempts to preserve preference information by reversing conflicting edges rather than removing them. However, this approach can introduce negative learning signals when conflicts are incorrectly identified, and lacks the systematic optimality of the minimum feedback arc set method.

Algorithm 9 DGR-ReverseResolve Reward Computation

1: **Input:** Query q, responses $\{o_1, \ldots, o_G\}$, judge model J
2: **Output:** Rewards $\{s_1, \ldots, s_G\}$
3: Construct preference graph $T = (V, E)$ with all pairwise comparisons using judge J
4: Initialize $T_{resolved} \leftarrow T$
5: **while** $T_{resolved}$ contains cycles **do**
6:      Detect any cycle C in $T_{resolved}$
7:      Randomly select edge $e = (u, v) \in C$
8:      $T_{resolved} \leftarrow (V, (E \setminus \{(u, v)\}) \cup \{(v, u)\})$ ▷ Reverse edge direction
9: **end while**
10: Compute net-win scores from conflict-free graph $T_{resolved}$:
11: **for** $i = 1$ to G **do**
12:      $s_i \leftarrow \sum_{j \neq i} \mathbb{I}[(o_i, o_j) \in E_{resolved}] - \sum_{j \neq i} \mathbb{I}[(o_j, o_i) \in E_{resolved}]$
13: **end for**
14: **Return** $\{s_1, \ldots, s_G\}$

The edge reversal strategy preserves the total number of preference relationships while breaking cycles. However, when conflicts are incorrectly identified, reversing edges can introduce negative

learning signals that are more harmful than simply removing them, as evidenced by its underperformance relative to the baseline. This highlights the critical importance of principled conflict identification in our optimal DGR method.

Both variants demonstrate that naive conflict resolution can be ineffective or even harmful. Only the systematic approach of our main DGR algorithm, which uses minimal feedback arc set optimization, achieves reliable performance improvements by correctly identifying and minimally perturbing the preference graph structure.

E CLASSICAL RANKING ALGORITHM IMPLEMENTATIONS

This section provides detailed algorithmic descriptions for the classical ranking methods evaluated in our ablation study (Section 3). These algorithms represent different paradigms for aggregating pairwise preferences: Rank Centrality (Negahban et al., 2012) (random walk stationary distribution), Plackett-Luce (Bradley & Terry, 1952; Hunter, 2004) (maximum likelihood estimation), and HodgeRank (Jiang et al., 2011) (Hodge decomposition via least-squares). We note that our DGR method is equivalent to Kemeny ranking (Ailon et al., 2008; Kenyon-Mathieu & Schudy, 2007), which minimizes total disagreements with observed preferences through minimum feedback arc set removal.

E.1 ALGORITHM DESCRIPTIONS

E.1.1 RANK CENTRALITY

Rank Centrality (Negahban et al., 2012) interprets pairwise comparison results as a random walk on a graph, where nodes represent items and edge weights reflect preference strengths. The stationary distribution of this random walk yields a ranking score for each item.

Algorithm 10 Rank Centrality Reward Computation

1: **Input:** Query q, responses $\{o_1, \ldots, o_G\}$, judge model J
2: **Output:** Rewards $\{r_1, \ldots, r_G\}$
3: Construct pairwise comparison matrix $W \in \mathbb{R}^{G \times G}$
4: **for** $i, j = 1$ to G **do**
5:     $M_{ij} \leftarrow J(\text{pairwise\_prompt}(q, o_i, o_j))$
6:     $W_{ij} \leftarrow \mathbb{I}[M_{ij} > 0]$ $\triangleright$ 1 if $o_i \succ o_j$, 0 otherwise
7: **end for**
8: Normalize rows: $P_{ij} \leftarrow \frac{W_{ij} + \epsilon}{\sum_k (W_{ik} + \epsilon)}$ for small $\epsilon > 0$
9: Compute stationary distribution π of transition matrix P via power iteration
10: $\pi^{(0)} \leftarrow \frac{1}{G} \mathbf{1}$
11: **for** $t = 1$ to convergence **do**
12:     $\pi^{(t)} \leftarrow P^T \pi^{(t-1)}$
13: **end for**
14: Set rewards: $r_i \leftarrow \pi_i$ for $i = 1, \ldots, G$
15: **Return** $\{r_1, \ldots, r_G\}$

E.1.2 PLACKETT-LUCE MODEL

The Plackett-Luce model (Bradley & Terry, 1952; Hunter, 2004) is a probabilistic model that assigns each item a latent strength parameter. The probability that item i beats item j is $\frac{\lambda_i}{\lambda_i + \lambda_j}$, where λ_i is the strength of item i. Parameters are estimated via maximum likelihood using the MM algorithm.

E.1.3 HODGERANK

HodgeRank (Jiang et al., 2011) uses combinatorial Hodge theory to decompose the pairwise comparison matrix into a global ranking component and an inconsistency component. The ranking is obtained by solving a least-squares optimization problem that finds scores minimizing disagreement with observed preferences.

Algorithm 11 Plackett-Luce Reward Computation

1: **Input:** Query q, responses $\{o_1, \ldots, o_G\}$, judge model J
2: **Output:** Rewards $\{r_1, \ldots, r_G\}$
3: Collect all pairwise comparison results using judge J
4: Initialize strength parameters $\{\lambda_1, \ldots, \lambda_G\}$ to 1.0
5: **for** iteration = 1 to max_iterations **do**
6:     Compute expected win counts for each item based on current λ
7:     **for** $i = 1$ to G **do**
8:         $w_i \leftarrow \sum_{j \neq i} \mathbb{I}[o_i \succ o_j]$ $\triangleright$ Actual wins
9:         $\gamma_i \leftarrow \sum_{j \neq i} \frac{1}{\lambda_i + \lambda_j}$ $\triangleright$ Expected comparisons
10:        $\lambda_i^{\text{new}} \leftarrow \frac{w_i}{\gamma_i}$ $\triangleright$ MM update
11:     **end for**
12:     $\{\lambda_i\} \leftarrow \{\lambda_i^{\text{new}}\}$
13:     **if** convergence criterion met **then**
14:         **break**
15:     **end if**
16: **end for**
17: Set rewards: $r_i \leftarrow \log(\lambda_i)$ for $i = 1, \ldots, G$
18: **Return** $\{r_1, \ldots, r_G\}$

Algorithm 12 HodgeRank Reward Computation

1: **Input:** Query q, responses $\{o_1, \ldots, o_G\}$, judge model J
2: **Output:** Rewards $\{r_1, \ldots, r_G\}$
3: Construct pairwise comparison matrix $Y \in \mathbb{R}^{G \times G}$
4: **for** $i, j = 1$ to G **do**
5:     $M_{ij} \leftarrow J(\text{pairwise\_prompt}(q, o_i, o_j))$
6:     $Y_{ij} \leftarrow M_{ij}$ $\triangleright$ +1 if $o_i \succ o_j$, -1 if $o_j \succ o_i$, 0 for tie
7: **end for**
8: Construct graph Laplacian $L \in \mathbb{R}^{G \times G}$
9: **for** $i = 1$ to G **do**
10:     $L_{ii} \leftarrow \sum_{j \neq i} \mathbb{I}[Y_{ij} \neq 0]$
11:     **for** $j \neq i$ **do**
12:         $L_{ij} \leftarrow -\mathbb{I}[Y_{ij} \neq 0]$
13:     **end for**
14: **end for**
15: Compute score difference vector $d \in \mathbb{R}^G$
16: **for** $i = 1$ to G **do**
17:     $d_i \leftarrow \sum_{j=1}^{G} Y_{ij}$
18: **end for**
19: Solve least-squares problem: $s^* = \arg\min_s \|Ls - d\|_2^2$ subject to $\sum_i s_i = 0$
20: Set rewards: $r_i \leftarrow s_i^*$ for $i = 1, \ldots, G$
21: **Return** $\{r_1, \ldots, r_G\}$

These algorithms provide baseline implementations for the experimental comparison presented in the main paper's ablation study (Table 3).

F   ROBUSTNESS AND SCALABILITY ANALYSIS

To further assess the robustness and scalability of our DGR framework, we conduct two sensitivity analyses. First, we evaluate its generality by varying the LLM judge. Second, we analyze its scalability as the number of candidates per round (graph size n) increases, which directly impacts the potential for preference conflicts.

(a) Sensitivity to different judge models.

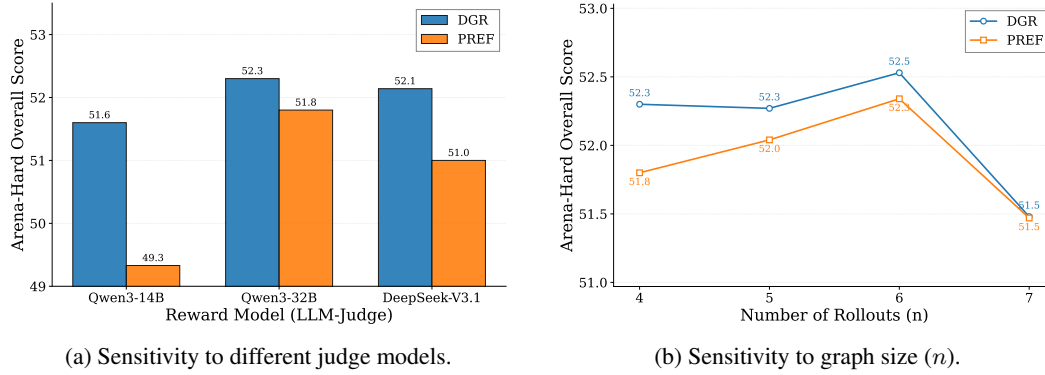(b) Sensitivity to graph size (n).

Figure 2: Sensitivity analysis of DGR compared to PREF on Arena-Hard. (a) Performance across three different LLM judges. (b) Performance as the number of candidates n increases from 4 to 7.

F.1 ROBUSTNESS TO DIFFERENT JUDGE MODELS

Using the GRPO optimizer with Qwen3-14B, Figure 2a shows that DGR's advantage is not contingent on a specific LLM judge. DGR consistently outperforms PREF across all tested judges, holding a significant average advantage on Arena-Hard. Crucially, DGR exhibits far greater stability across different judges, whereas PREF's performance shows high variance. The advantage is most pronounced with less capable judges, indicating that DGR is particularly effective at purifying signals from lower-quality judge models. This confirms that DGR is a universally applicable enhancement that robustly improves alignment regardless of the preference data source.

F.2 SCALABILITY WITH INCREASING GRAPH SIZE

Using the GSPO optimizer with Qwen3-8B, Figure 2b illustrates the performance trend as the number of candidates n increases. DGR maintains a consistent performance advantage over PREF for all tested values of n. Both methods achieve their peak performance at $n = 6$. While the advantage margin varies, DGR's ability to systematically resolve conflicts prevents the performance degradation one might expect from the exponential increase in potential preference cycles in a larger graph. This consistent, positive advantage demonstrates that our conflict resolution mechanism provides a reliable edge, ensuring stable and superior performance as the scale and complexity of the comparison task grow.