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System-Identification for Regular Water Waves

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Abstract

In this paper, we study and estimate the regular water-wave identification methods by use of the Nonlinear auto-regressive model (NARM) and Hammerstein-Wiener model. We analyze and optimize the parameters in multi-regressions. Under the nonlinear group regression model, we selected three common models, such as wavelet transform, decision tree model, and support vector machine model with Gaussian process. Finally, the Hammerstein-Wiener shows a great performance on identification processes. Specifically, we achieve a maximum accuracy of 88% on our validation set. We used the AIC index and NMSE to measure the superiority of the model.

Keywords: *System identification; Nonlinear auto-regressive model; Wave prediction; Hammerstein-Wiener model;*

1. INTRODUCTION

Wave forecasting involves predicting or reporting future sea conditions based on the wind and natural geographic conditions of a particular ocean area. A better understanding of wave models is crucial for accurate wave prediction. Many researchers have applied economic methods to study the nonlinear interactions of waves and explain their evolution from a frequency-dependent perspective (Hasselmann et al., 1985; Rogers et al., 2002). With the emphasis on sustainable development, there have been many studies focusing on the environmental and climatic impact of waves, particularly in port construction and biological effects (Silva et al., 2018). Identifying more important parameterized models or extracting more important dimensional features is to make better prediction algorithms.

Despite the availability of semi-empirical, theoretical, and numerical methods for predicting ocean waves, these methods are not always practical in the field of ocean engineering. However, with the advancement of information science, more and more approaches based on machine learning and deep learning have been proposed for accurate wave prediction.

Among the related studies, Deo et al. (2001) developed a simple 3-layered feed forward type of network to obtain the output of significant wave heights and average wave periods from the input of generating wind speeds. More recently, James et al., (2018) developed a machine learning framework by using multi-layer perception and support vector machine to estimate ocean wave conditions. Duan et al., (2020) applied a artificial neural network-based wave prediction (ANN-WP) model to predict irregular wave elevations. Law et al., (2020) presented a framework by the use of data-driven model to predict the evolution of wave fields in the time domain from a given wave field record. Huang et al. (2022) and Jing et al. (2022) proposed a regional wind wave prediction surrogate model based on a convolutional neural network (CNN), which took historical wind and wave data as input to realize the prediction of current waves. Meanwhile, the real-time nonlinear wave prediction with quantified was also achieved by the Bayesian machine learning (Zhang et al., 2022), which was crucial for marine engineering such as wave energy converters (WECs). Liu et al., (2022) developed a deep learning wave prediction (Deep-WP) based on the long short-term memory (LSTM) and the sequence to sequence (S2S), which were common in the natural language processing (NLP).

System identification, which involves modeling non-parametric models using data-driven methods, is another approach to understanding the laws of wave changing with water depth. Huang (1989) adopted a trace theorem for time periodical operator. dos Santos and Perdicoúlis (2021) successfully applied the methods of polynomial functions, orthogonal sinusoid and Gaussian regression to the state space equation.

Based on artificial intelligence, a new system identification method is proposed in this paper, which combines linear and nonlinear theory and the physical form of simple string motion inherent in water waves. The time series characteristic expression is added. We evaluate and optimize the performance of the two strong nonlinear models and propose our own evaluation matrices on datasets.

2. Model Selection and Comparison[1]

2.1 Nonlinear auto-regressive model (NARM) models selection

In terms of mathematical relationship, we can consider the wave elevation as an observable measurement y. According to the prior physical knowledge, the wave evolution change is a dynamic process about $\boldsymbol{u}$ such as time, position and water depth, which is very similar to the NARM model. We model it by generating discrete-time t as following in Eq. [1] :

$$y(t) = f(\boldsymbol{u}(t-1), y(t-1), \boldsymbol{u}(t-2)\ldots) \quad [1]$$

where f is a non-linear operation function in this equation.

The nonlinear model is known as identified nonlinear models. These models represent nonlinear systems with coefficients that are identified by using measured input $\boldsymbol{u}$ and output y data. In this study, the model is multi-input and single output system. For the NARM estimator, Figure 1 illustrates the structure and principle of the model. The regresses have chronology differential relationship with different input and output.

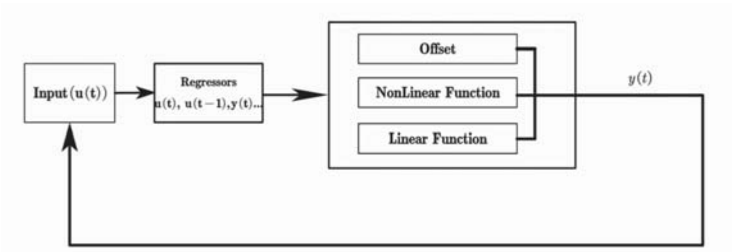

Figure 1. The NARM model structure for the wave identification. The Input has multi variables. In this case, the number of input variables is three and the number of output is one.

Output is relevant to the static offset, nonlinear functions and linear functions. The mathematical expression is as follows in Eq. [2].

$$F(\boldsymbol{x}) = d + L(\boldsymbol{x} - r) + g(Q(\boldsymbol{x} - r)) \quad [2]$$

where $\mathbf{x}$ is a vector about regresses and r is the mean of regresses $\mathbf{x}$. d is a scalar offset that is added to this equation. Q is a projection matrix which the calculations could be well influenced. The linear function is L^T and the nonlinear function is g. Ideally, the parameters of the model should minimize the mean square error (MSE), given systematic error and random error (Variance).

However, using MSE as a train loss function is nothing to do with the prior physical knowledge and the dynamic time term of the system and increase the high uncertainty burden. For the present wave prediction problem, adding proper regularization is a better way to improve the fitting effect, as shown below in Eq. [3].

$$V_N(\boldsymbol{\theta}) = \frac{1}{N}\epsilon^2(t, \boldsymbol{\theta}) + \frac{1}{N}\lambda|\boldsymbol{\theta}|^2 \quad [3]$$

where t is the time variable, N is the number of data samples, $\boldsymbol{\theta}$ is a parameter and $\epsilon^2(t, \boldsymbol{\theta})$ is the predicted error computed as the difference between the observed output and the predicted output. λ represents the confidence in the prior knowledge of the unknown parameters. This implies that the larger the value, the higher the confidence. In the following experiments, we pick the λ as 1 by empirical experience.

[1] All the results of this paper, including data and codes are linked on GitHub. https://github.com/liangaomng/paper.git.

2.2 Characteristics and Model metrics in the following experiments

For the train and valid set, we choose the data-set which is produced by closed from linear water wave equations of finite depth in Eq. [4] and Eq. [5].

$$\beta = A \cdot \cos(k \cdot x - \omega \cdot t + \theta) \quad [4]$$

$$\omega^2 = k \cdot g \cdot \tanh(k \cdot d) \quad [5]$$

In Eq. [4], β, *A*, *k*, ω, x and θ are the wave elevation, amplitude, wave number, frequency, position and initial phase. In Eq. [5], *g* and *d* are the gravity acceleration and water depth respectively. The purpose of this work is to find a suitable model, preferably with the output term being water depth and the output term being data with a fixed position and time on the wave elevation.

For the wave elevation, we select three inputs as local water depth d, time t and the position x. The Output y is the wave elevation. In this section, we choose regresses as four items in three inputs: $\boldsymbol{u}(t)$,$\boldsymbol{u}(t\text{-}1)$, $\boldsymbol{u}(t\text{-}2)$ and $y(t\text{-}1)$, respectively. The non-linear function is about fourth-order Fourier decomposition terms in this case. For example, for input $\boldsymbol{u}(t)$, we set the cosine function to as a regressor item. such as sin(1*$\boldsymbol{u}(t)$), sin(2*$\boldsymbol{u}(t)$), sin(3*$\boldsymbol{u}(t)$), sin(4*$\boldsymbol{u}(t)$). In order to evaluate the final result, the normalized root mean square error (NMSE) is adopted in Eq. [6]. When the value of NMSE is close to 1, it indicates that the model is correct. Additionally, Akaike information criterion (AIC) in Eq. [7] is a standard for the size of the parameter space in models, and the criteria is based on the concept of entropy. The smaller the AIC, the better the model. Usually, the model with the smallest AIC is selected.

$$NMSE = 100(1 - \frac{||y - \hat{y}||}{||y - y_{mean}||}) \quad [6]$$

$$AIC = 2K - 2\ln(L) \quad [7]$$

where K is equal to the number of parameters and L is a likelihood function.

In this section, we selected the fourth-order regresses and different non-linear functions in wavelet networks, tree partitions and SVM combined Gaussian process named Model A, Model B, Model C.

For the wavelet Nonlinear function model (Zhang, 1997), the basic mathematics principle is as follows in Eq. [8]:

$$y(\text{t}) = y_0 + X(t)^T(\boldsymbol{P}L) + W(X(t) + S(X(t)) \quad [8]$$

where $X(t)$ is an m by 1 vector of regresses. y_0 is an offset. $\boldsymbol{P}$ is m by p projection matrix, m is the number of regresses and *p* is linear weights. L is *p* by 1 vector of weights. $W(X)$ and $S(X)$ constitute wavelet network function. $S(X)$ is a sum of dilated and translated functions. $W(X)$ is a sum of a sum of dilated and translated wavelets.

For the model B, which is a tree-partition method, a non-linear function is shown as following in Eq. [9].

$$F(x) = xL + [1, x]C_k + d \quad [9]$$

where x belongs to the partition P_k, The mapping $F(x)$ is defined by a dyadic partition P of the space, such that on each partition element P_k, P_k is a linear mapping. L is a 1-by-m vector, C_k is a 1-by-m+1 vector and k is a value of the total nodes. In “tree” model, the AIC is None.

For the model C, which is the support vector Machine combined with Gaussian process, the principle is as following in Eq. [10].

$$y(t) = S\big(X(t)\big) = \sum_{i=1}^{N} \alpha_n\, G(x_n, X) + b \quad [10]$$

where X is an m-by-1 vector of inputs, and N is the number of support vectors in the trained model. x_n is the *n*th support vector in the model. α is the weight associated with each support vector. G is the Gram matrix that results from the operation of the specified kernel function on X and x_n and b is the offset of the trained model. Basically, we choose Gaussian kernel for computing the Gram matrix.

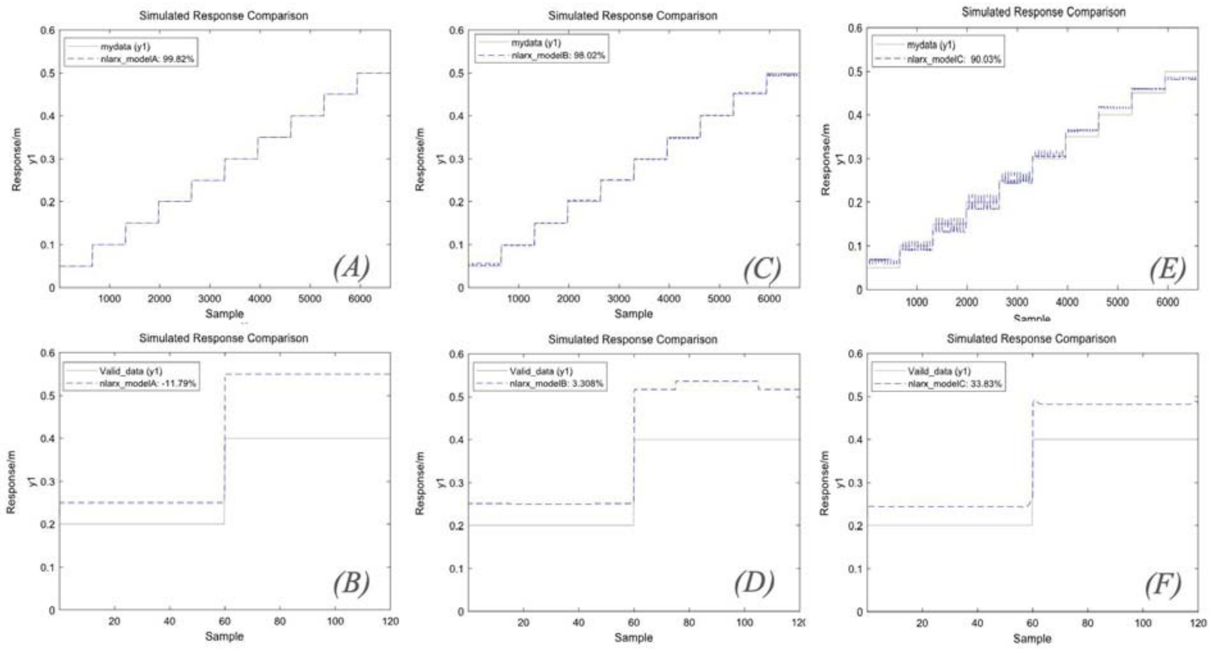

Figure2. The fourth-order NARM model fitting result. (A), (C) and (E) show that the fitness in the train data set of model A, B and C. (B), (D) and (F) show that the fitness in the valid data

Table1 shows that although the model A has the best fitting characterize in NMSE, the effect in the test set is not as good as model C. However, in terms of model size, the AIC index of model C is higher, which means that more variable space is required.

Table 1. NARM Performance in Training and valid set.

MODEL	MSE	NMSE IN TEST	AIC	NMSE IN VALID
A	6.80e-8	98.82%	-4.5e+6	-11.79%
B	7.24e-6	98.02%	None	3.308%
C	2.09e-4	90.10%	-1.86e+6	33.83%

3. Hammerstein-Wiener Model Selection and Experiments of Regular Waves

3.1 Hammerstein-Wiener Model Selection

The Hammerstein-Wiener model is a model to describe a dynamic model by using three parts which includes linear transform and non-linear transform (Brouri et al., 2022; Kumar et al., 2022; Xie et al., 2021). The basic structure is as follows in Figure 4.

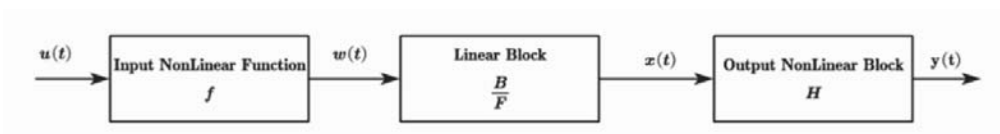

Figure 3. The structure of the Hammerstein-Wiener model

It is worth noting that the middle linear layer is a discrete and dynamic combination of $w(t)$. Relatively speaking, the middle linear layer represents a dynamic layer. f acts on the input port of the linear block, this function is called the input non-linearity. Similarly, because H acts on the output port of the linear block, this function is called the output non-linearity.

The f is a nonlinear function that transforms input data $\boldsymbol{u}(t)$ as $\boldsymbol{w}(t) = f(u(t))$. The linear block is a linear transfer function that transforms as $\boldsymbol{x}(t) = (B/F)\boldsymbol{w}(t)$. B/F represents discreteness, determining the order of dynamics of the system. H is a nonlinear function that maps the output of the linear block $\boldsymbol{x}(t)$ to the system output $\boldsymbol{y}(t)$ as $\boldsymbol{y}(t) = H\,(x(t))$.

3.2 Result of HW model in identification

In this experiment, we utilized linear transformation. When selecting the intervals of the linear transformation, we evenly divide them into 10 and 100 intervals as model D and E, and selected the second-order linear dynamic layer in the middle. The final results are shown in the Figure 4.

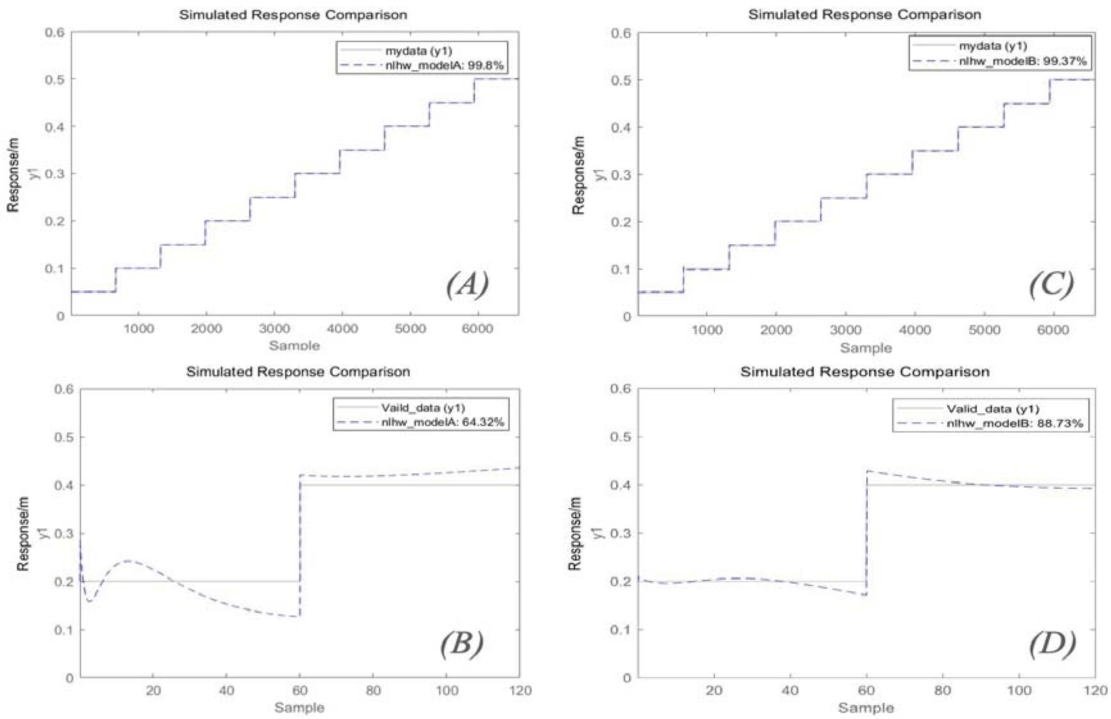

Figure4. (A) and (B) is the HW model fitting result, (A) and (C) show that the fitness in the train dataset. (C)and (D) shows that fitness in the valid dataset

As shown in Figure 5, the more the number of segmented linear intervals, the easier it is to achieve identification results, and the verification of the latter is 88.73%.

In Table 2 below, in order to compare the effect of linear interval segmentation between these two quantity sets, we did not choose a large linear interval in this comparison because the calculation time of the response will increase exponentially according to the AIC.

Table 2. HW Performance in Training and valid set.

MODEL	MSE	NMSE IN TEST	AIC	NMSE IN VALID
D	8.315e-8	98.8%	-4.45e+6	64.32%
E	3.165e-6	98.7%	-3.24e+6	88.73%

4. CONCLUSIONS

In this paper, we compare the system identification of the regular water wave elevation of two non-linear models with different dynamic function terms in Model D and Model E. As more linear segments are segmented, the fitting effect of the model will also improve. Results show that the HW model has very superior properties. The optimization of the former model has not been carried out, because the identification time increases with the increasement of power items, but has reached 88% of the effect in model E, which shows the application of system identification in water waves. The four-order model effects of the NARM are much poorer than those of the HW model and are not suitable for simulating waves. Higher model orders require significantly higher computational costs, and we did not seek the optimal order, which is also the next step that needs to be done. The goal of this paper will generate a new model which is fit for strong non-linear wave forms, with integrating dynamic theories.

5. ACKNOWLEDGEMENTS

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