

Skip-Layer Attention: Bridging Abstract and Detailed Dependencies in Transformers

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Abstract

The Transformer architecture has significantly advanced deep learning, particularly in natural language processing, by effectively managing long-range dependencies. However, as the demand for understanding complex relationships grows, refining the Transformer’s architecture becomes critical. This paper introduces Skip-Layer Attention (SLA) to enhance Transformer models by enabling direct attention between non-adjacent layers. This method improves the model’s ability to capture dependencies between high-level abstract features and low-level details. By facilitating direct attention between these diverse feature levels, our approach overcomes the limitations of current Transformers, which often rely on suboptimal intra-layer attention. Our implementation extends the Transformer’s functionality by enabling queries in a given layer to interact with keys and values from both the current layer and one preceding layer, thus enhancing the diversity of multi-head attention without additional computational burden. Extensive experiments demonstrate that our enhanced Transformer model achieves superior performance in language modeling tasks, highlighting the effectiveness of our skip-layer attention mechanism.

1 Introduction

The Transformer architecture has made notable strides in the field of large language models (LLMs) (Devlin et al., 2019; Radford and Narasimhan, 2018; Radford et al., 2019; Brown et al., 2020; Ouyang et al., 2022; OpenAI, 2023). These models have impressively tackled a variety of tasks, including natural language understanding (Hendrycks et al., 2021), general question answering (Rein et al., 2023), coding (Chen et al., 2021), mathematics (Cobbe et al., 2021), and scientific knowledge (Chen et al., 2023). However, as data grows more complex and relationships more intricate (Rein

et al., 2023), there’s a need for ongoing improvements in the architecture to keep up with these challenges.

The primary strength of the Transformer lies in its self-attention mechanism, which allows each element in the input sequence to compare directly with every other element, thereby capturing dependencies regardless of their distance (Vaswani et al., 2017). Nevertheless, this design faces limitations when handling more complex relationships. The original intra-layer attention in Transformers is often inadequate for capturing the deeper interactions (i.e., high-level abstract features and low-level details) demanded by more complex tasks (Tenenbaum, 2018; Yang et al., 2016).

To address these limitations, researchers have explored various methods employed in earlier models such as ResNet (He et al., 2016) and Highway Networks (Srivastava et al., 2015). Our goal is to refine inter-layer interactions within Transformers. Drawing inspiration from DenseNet (Huang et al., 2017) in convolutional neural networks (CNNs), which employs dense cross-layer connections to facilitate feature propagation, we propose a novel Skip-Layer Attention (SLA) approach to enhance the Transformer model. Our implementation augments the Transformer’s capabilities by permitting queries in a given layer to interact not only with keys and values from the current layer but also from the preceding layer. This method enriches the diversity of multi-head attention, while maintaining the same computational efficiency. Unlike DenseNets, which focus on identical tokens across layers, our strategy connects both identical and distinct tokens, thereby enhancing the model’s capacity to capture and incorporate both abstract and detailed dependencies. Our contributions are as follows:

- We propose a novel mechanism that enables direct attention between non-adjacent layers, enhancing the ability to capture dependencies

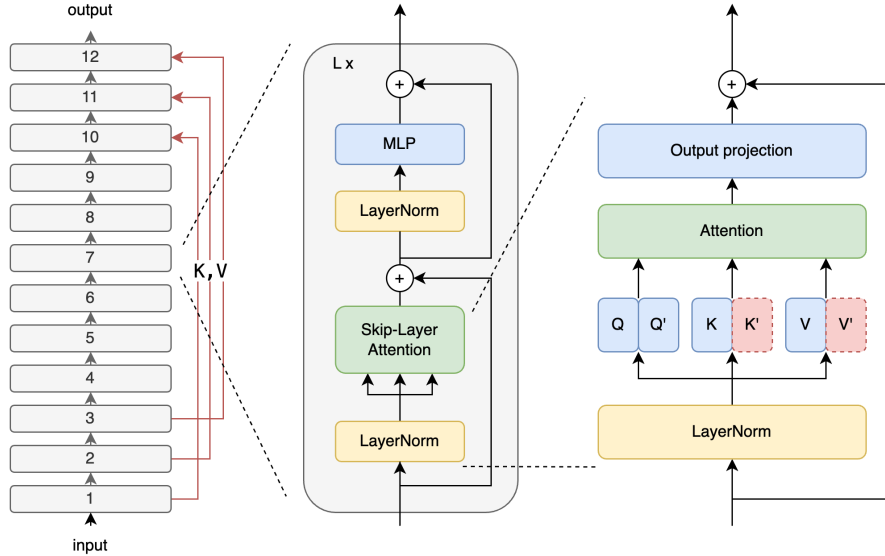


Figure 1: Model architecture of the Transformer with skip-layer attention. The left figure illustrates a Transformer model with 12 layers, each equipped with an additional skip-layer attention connection (e.g., layer 1 to layer 10, layer 2 to layer 11, layer 3 to layer 12). The center figure provides a zoomed-in view of each layer, highlighting the skip-layer attention and MLP sublayers. The right figure details the skip-layer attention mechanism, with red indicating keys and values from the preceding layer.

082 between high-level abstract features and low- 108
 083 level details. 109

- 084 • Our method extends the Transformer’s func- 110
 085 tionality without significantly increasing com- 111
 086 putational complexity, making it practical for 112
 087 large-scale applications. 113
- 088 • Through extensive experiments against Trans- 114
 089 former baselines, we demonstrate the effec- 115
 090 tiveness of our enhanced architecture in lan- 116
 091 guage modeling tasks. 117

092 2 Related Work 120

093 The concept of enhancing network connectivity 122
 094 originates from earlier architectures such as ResNet 123
 095 (He et al., 2016), which introduces residual con- 124
 096 nections. These residual connections enable the train- 125
 097 ing of much deeper networks by facilitating the 126
 098 flow of gradients during backpropagation. High- 127
 099 way Networks (Srivastava et al., 2015) introduce 128
 100 gated connections to regulate information flow 129
 101 across layers, making the end-to-end training of 130
 102 deep networks more feasible. 131

103 DenseNet (Huang et al., 2017) advances this idea 132
 104 by creating an intricate connectivity pattern where 133
 105 each layer connects to every other layer in a feed- 134
 106 forward manner. This dense connectivity promotes 135
 107 feature reuse and significantly reduces the number

of parameters, directly inspiring the skip-layer con- 108
 nectivity pattern explored in our work. However, 109
 DenseNet primarily targets CNNs and is mainly 110
 applied to computer vision tasks, with connections 111
 occurring between the same tokens in subsequent 112
 layers. Our approach extends this concept to Trans- 113
 formers by incorporating direct connections among 114
 both identical and distinct tokens. 115

More recently, Brandon et al. (2024) propose 116
 sharing key/value heads across adjacent layers to 117
 reduce the size of the KV cache. This strategy 118
 draws inspiration from the success of Multi-Query 119
 Attention (Shazeer, 2019) and Grouped-Query At- 120
 tention (Ainslie et al., 2023). Our skip-layer at- 121
 tention, however, enables direct attention between 122
 non-adjacent layers, thus bridging the dependen- 123
 cies between high-level abstract features and low- 124
 level details. This approach encapsulates the depth 125
 of interactions required by more demanding tasks. 126

127 3 Method 127

128 Our novel Transformer model enhances the stan- 128
 129 dard Transformer architecture by incorporating 129
 skip-layer attention, aimed at improving the in- 130
 formation flow between non-adjacent layers. As 131
 shown in Figure 1, our approach replaces the con- 132
 ventional multi-head attention sublayer with skip- 133
 layer attention sublayer. This upgrade establishes 134

direct connections between layers that are not immediately adjacent, thereby promoting a more efficient and comprehensive exchange of information across the entire network.

Our model retains the Transformer’s original multi-head attention mechanism, utilizing queries, keys, and values, but extends this framework by enabling queries in a given layer to interact with keys and values from both the current layer and one preceding layer. This extension is fundamental to the implementation of skip-layer attention.

We denote the queries after linear projection for all heads as the tensor $Q \in \mathbb{R}^{h \times T \times d}$, where h is the number of heads, T is the sequence length, and d is the hidden size for each head. Similarly, the keys and values are represented as tensors $K \in \mathbb{R}^{h \times T \times d}$ and $V \in \mathbb{R}^{h \times T \times d}$, respectively. The number of skip layers is denoted as n_l , and the number of skip heads as n_h .

The attention mechanism for each head is described by the following equations:

$$H_i^l = \begin{cases} \text{Att}(Q_i^l, K_i^l, V_i^l) & \text{if } i \in \{1, \dots, h - n_l\} \\ \text{Att}(Q_i^l, K_i^{l-n_l}, V_i^{l-n_l}) & \text{if } i \in \{h - n_l + 1, \dots, h\} \end{cases}$$

where i represents the index of the head and l represents the index of the layer. This formulation allows our model to effectively bridge abstract and detailed dependencies and improve information flow throughout the network. The core implementation in PyTorch can be found in Appendix A.1.

4 Experimental Setup

4.1 Dataset

We use the OpenWebText corpus¹, an open-source recreation of the WebText dataset. It comprises approximately 8 million documents sourced from Reddit-linked web content. The corpus is divided into a training set with about 9 billion tokens and a validation set with around 4 million tokens.

4.2 Training Setup

Training batches contain 524,288 tokens, stabilizing the process across model scales. The training spans a maximum of 18,000 steps, processing roughly 9.4 billion tokens, equivalent to one full epoch on the training set. We experiment with three sizes of the GPT-2 architecture: GPT2 (124M), GPT2-Medium (350M) and GPT2-Large (774M).

¹<https://skylion007.github.io/OpenWebTextCorpus/>

Table 1: Optimal number of skip layers.

#SkipLayer	#SkipHead	Loss	Abs. Impr.
0 (Baseline)	0	3.3826	-
1	6	3.4030	-0.0204
3	6	3.2958	0.0868
6	6	3.2853	0.0973
9	6	3.2750	0.1076
11	6	3.3734	0.0092

Table 2: Optimal number of skip heads.

#SkipLayer	#SkipHead	Loss	Abs. Impr.
0 (Baseline)	0	3.3826	-
9	3	3.3543	0.0283
9	6	3.2750	0.1076
9	9	3.2497	0.1329
9	12	3.2643	0.1183

Flash Attention (Dao, 2023) is incorporated to accelerate attention operations. Training is conducted on NVIDIA V100 GPUs with 32GB of memory. We experiment with sequence lengths of 4096, 8192, and 16,384 tokens to explore long-range dependencies. Code implementation and optimization are managed using the nanoGPT framework².

All models start with a learning rate of 1.5e-4, determined to offer a good balance between rapid convergence and stability. Model performance is evaluated based on the loss on the validation dataset.

5 Result

5.1 Number of skip layers

We initially explore the impact of varying the number of skip layers using a GPT-2 model (124M parameters) as our default backbone. This model has a hidden size of 768, 12 heads, 12 layers, and supports a sequence length of 16,384. The default number of skip heads is set to 6. As shown in Table 1, our findings indicate that the optimal performance enhancement via the skip-layer attention method is achieved with 9 skip layers, resulting in a substantial absolute improvement of 0.1076 over the baseline. Configurations with 3 and 6 skip layers also demonstrate notable progress. However, employing just a single skip layer yields no benefit; this might be attributed to the similarity between the key and value heads among adjacent layers, as discussed in Brandon et al. (2024). Similarly, setting the number of skip layers to 11 does

²<https://github.com/karpathy/nanoGPT/>

Table 3: Model size and sequence length variations.

Model	Length	Baseline	Skip-Layer Attention	Abs. Impr.	Training Speedup
GPT2(124M)	4096	3.1858	3.1762	0.0096	-1.69%
	8192	3.2077	3.2020	0.0057	-0.28%
	16384	3.3826	3.2497	0.1329	0.83%
GPT2-Medium(350M)	4096	2.9538	2.9399	0.0139	-0.76%
	8192	3.0335	2.9506	0.0829	-2.34%
GPT2-Large(774M)	4096	2.8271	2.8156	0.0115	-1.24%

not produce noticeable advancements, possibly due to the presence of only one skip-layer attention in this setup. Based on these results, we recommend that 3/4 of the total number of layers is the most effective number of skip layers.

5.2 Number of skip heads

Subsequently, we investigate the impact of varying the number of skip heads, also using a GPT-2 model (124M) and a sequence length of 16,384 as our default backbone. The default number of skip layers is set to 9, based on the optimal configuration identified in the previous section. As presented in Table 2, our results indicate that the optimal performance enhancement is observed with 9 skip heads, yielding a significant absolute improvement of 0.1329 over the baseline. Configurations with 6 and 12 skip heads also demonstrate commendable improvements. Notably, the configuration with 12 skip heads suggests that the keys and values from the last 3 layers all use the keys and values from the first 3 layers. This implies that direct attention modeling between high-level abstract features and low-level detail features is more important than attention modeling purely between high-level abstract features. Conversely, employing only 3 skip heads shows no substantial benefit, indicating that more skip-attention heads are necessary than the original attention heads. Based on these findings, we recommend setting the number of skip heads to 3/4 of the total number of heads.

5.3 Model Size and Sequence Length Variations

In this section, we explore the effects of varying model sizes and sequence lengths on performance. We maintain the default configuration of using 3/4 of the total number of layers as skip layers and 3/4 of the total number of heads as skip heads. Due to the 32GB memory limit of the V100 GPU,

we restrict our tests to model sizes and sequence lengths that do not trigger out-of-memory (OOM) errors when using Distributed Data Parallel (DDP).

As shown in Table 3, we observe an absolute improvement of 0.1329 over the baseline when using GPT-2 (124M) with a sequence length of 16,384. However, no significant improvements are noted for sequence lengths of 4,096 and 8,192. This suggests that longer sequences benefit more from our skip-layer attention method, likely because they encompass more abstract and detailed dependencies. Furthermore, for a sequence length of 8,192, GPT-2 Medium (350M) achieves an absolute improvement of 0.0829 over the baseline, while GPT-2 (124M) shows no noticeable improvement. This indicates that larger models gain a greater advantage from our skip-layer attention method.

The training time with our skip-layer attention method is slightly longer than the baseline, experiencing a maximum decrease in training speed of 2.34%. This is likely due to the additional storage requirements for keys and values in the lower layers.

6 Conclusion

In this paper, we propose a Skip-Layer Attention (SLA) mechanism to enhance the Transformer architecture’s ability to capture complex dependencies within input data. By enabling direct attention between non-adjacent layers, our approach improves the model’s capacity to integrate high-level abstract features with low-level details without significantly increasing computational complexity. Extensive experiments demonstrate that our enhanced Transformer model outperforms standard Transformer baselines in language modeling tasks, validating the effectiveness of our method. Our findings pave the way for further innovations in optimizing neural network architectures.

7 Limitation

While our research has demonstrated the effectiveness of skip-layer attention in Transformer models, several avenues for future work remain to be explored:

Scaling to Larger Models: Future research could extend skip-layer attention to larger Transformer architectures, such as GPT-3 (Brown et al., 2020) and beyond, to assess its effectiveness across different scales and complexity levels. Evaluating the performance and efficiency on these larger models will offer valuable insights into its scalability.

Real-World Applications: Evaluating the skip-layer attention mechanism in various real-world applications, such as natural language understanding (Hendrycks et al., 2021), general question answering (Rein et al., 2023), coding (Chen et al., 2021), and mathematics (Cobbe et al., 2021), will be critical to fully understand its practical benefits and limitations.

Beyond Text: Extending the applicability of the skip-layer attention mechanism to other domains, such as computer vision and speech processing, will help determine its versatility and potential for cross-modal impact.

Ablation Studies: Conducting comprehensive ablation studies to understand the contributions of different components within the skip-layer attention mechanism could provide deeper insights. For instance, exploring the impact of connections to multiple preceding layers rather than just one could reveal additional enhancements and inform design choices.

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A Appendix

A.1 Code

In this paper, the model code is based on nanoGPT, which is publicly available on GitHub³. The implementation of the skip-layer attention method is detailed in the code listing provided below.

```
import torch.nn.functional as F

class CausalSelfSkipLayerAttention(nn.Module):

    def __init__(self, config):
        super().__init__()
        assert config.n_embd % config.n_head == 0
        # key, query, value projections for all heads, but in a batch
        self.c_attn = nn.Linear(config.n_embd, 3 * config.n_embd, bias=config.bias)
        # output projection
        self.c_proj = nn.Linear(config.n_embd, config.n_embd, bias=config.bias)
        # regularization
        self.attn_dropout = nn.Dropout(config.dropout)
        self.resid_dropout = nn.Dropout(config.dropout)
        self.n_head = config.n_head
        self.n_embd = config.n_embd
        self.dropout = config.dropout
        self.num_skip_layer = config.num_skip_layer
        self.split_num = config.n_head - config.num_skip_layer

    def forward(self, x, prev_k_list=[], prev_v_list=[]):
        B, T, C = x.size() # batch size, sequence length, n_embd

        q, k, v = self.c_attn(x).split(self.n_embd, dim=2)
        k = k.view(B, T, self.n_head, C // self.n_head).transpose(1, 2) # (B, nh, T, hs)
        q = q.view(B, T, self.n_head, C // self.n_head).transpose(1, 2) # (B, nh, T, hs)
        v = v.view(B, T, self.n_head, C // self.n_head).transpose(1, 2) # (B, nh, T, hs)

        # causal self-attention; Self-attend: (B, nh, T, hs) x (B, nh, hs, T) -> (B, nh, T, T)
        # efficient attention using Flash Attention CUDA kernels
        if len(prev_k_list) >= self.num_skip_layer and len(prev_v_list) >= self.num_skip_layer:
            k_combine = torch.cat([k[:, :, self.split_num:, :], prev_k_list[-self.num_skip_layer:]], dim=1)
            v_combine = torch.cat([v[:, :, self.split_num:, :], prev_v_list[-self.num_skip_layer:]], dim=1)
            y = F.scaled_dot_product_attention(q, k_combine, v_combine, attn_mask=None,
                                              dropout_p=self.dropout if self.training else 0,
                                              is_causal=True)
        else:
            y = F.scaled_dot_product_attention(q, k, v, attn_mask=None,
                                              dropout_p=self.dropout if self.training else 0,
                                              is_causal=True)

        y = y.transpose(1, 2).contiguous().view(B, T, C)
        # output projection
        y = self.resid_dropout(self.c_proj(y))
        return y, k[:, :, self.split_num:, :], v[:, :, self.split_num:, :]
```

³<https://github.com/karpathy/nanoGPT/>