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A note on the method of equal shares

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ABSTRACT

The Method of Equal Shares (ES) is a popular approval-based multi-winner voting rule, which was originally proposed by Peters and Skowron. It satisfies several well-known representation axioms, like extended justified representation (EJR) and priceability, and it can be computed in polynomial time. Further, it has already been employed in several real participatory budgeting elections. In this note, we prove that ES is an instance of the EJR-Exact family of voting rules that also satisfy EJR and were proposed by Aziz et al. 2018 before the work of Peters and Skowron.

1. Introduction

Approval-based multi-winner voting has received a significant amount of attention by the computational social choice community (see [19] for an overview of multi-winner voting challenges and [22] for a survey on approval-based multi-winner voting). A major concern here is to achieve (proportional) representation: in many scenarios, it is considered highly desirable that the selected committee faithfully represents the voters. The topic of achieving representation with approval-based multi-winner voting has been studied in many research papers, both theoretically [1,29,2,24,4,26,5,25,30,11,13,10] and empirically [16,8,13,18,23,6].

A seminal work in the study of representation in approval-based multi-winner voting was that of Aziz et al. [1]. They proposed two axioms to capture the notion of representation, that they called *justified representation* (JR), and *extended justified representation* (EJR). Informally, when there are n voters that participate in an election to select a committee of k candidates, JR requires that for any group of voters of size at least $\frac{n}{k}$ such that all the voters in the group approve a candidate in common, at least one of the voters in the group must approve one of the candidates in the elected committee. EJR requires that for any group of voters of size at least $\ell\frac{n}{k}$, with $1 \leq \ell \leq k$, such that all the voters in the group approve ℓ candidates in common, at least one of the voters in the group must approve at least ℓ candidates in the elected committee. EJR is a strengthening of JR: any committee that satisfies EJR also satisfies JR. A voting rule that always outputs committees that satisfy JR (respectively, EJR) is said to satisfy JR (respectively, EJR).

Aziz et al. proved that committees that satisfy JR or EJR always exist. In particular, they identified a voting rule satisfying each axiom. In the case of JR, they identified a rule called GreedyAV that satisfies JR and can be computed in polynomial time. Informally, GreedyAV is an iterative algorithm that, at each iteration, selects the most approved candidate among those voters who are still unrepresented in the committee. Unfortunately, this rule may output committees that do not satisfy EJR.

In the case of EJR, Aziz et al. proved that Proportional Approval Voting (PAV) [34], proposed by the Danish mathematician Thorvald N. Thiele, in the nineteenth century, satisfies EJR. Unfortunately, PAV is known to be NP-hard to compute [3,31]. In summary, by the time when [1] was first published, it was not known whether it was possible to compute committees that satisfy EJR in polynomial time.

This issue was closed by Aziz et al. [2] who identified two different approaches to computing committees that satisfy EJR in polynomial time. The first one is a local search algorithm for PAV, fixing a minimum increase in the PAV score at each iteration to guarantee that the number of iterations of the local search algorithm grows polynomial in the worst case. The second approach is a family of voting rules called EJR-Exact (the operation of the EJR-Exact family of voting rules is reviewed in detail later). All the voting rules in the EJR-Exact family satisfy EJR, and run in polynomial time as long as the algorithms that identify each rule within the family are polynomial.

Later, Peters and Skowron [26] proposed an appealing voting rule, originally called Rule X and later renamed as the Method of Equal Shares (ES), that, among other interesting properties,[1] satisfies EJR and can be computed in polynomial time. ES has received a significant amount of attention by the computational social choice community (see for instance [25,33,21,9,17,7,12,14,15,6]). Further, ES has also been used

[1] The Method of Equal Shares satisfies two other axioms proposed also in [26], called *laminar proportionality* and *priceability*.

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in real elections. In particular, a variant of ES for participatory budgeting has been used in participatory budgeting elections in Wieliczka (Poland), Aarau (Switzerland) and Świecie (Poland).[2]

In this note, we prove that the Method of Equal Shares is a particular instance of the EJR-Exact family of voting rules proposed by Aziz et al. [2].

The rest of this note is organised as follows. In the next section, we introduce some notation and review the operation of ES. The following two sections are devoted to presenting the EJR-Exact family of voting rules and to prove that ES is an instance of the EJR-Exact family. We will finish this note with a short discussion.

2. Preliminaries

An approval-based multi-winner election can be represented with a tuple $(N, C, \mathcal{A}, k)$, where $N = \{1, \ldots, n\}$ is the set of *voters*, C is the set of *candidates*, $\mathcal{A} = (A_1, \ldots, A_n)$ is the *ballot profile*, and k, such that $1 \leq k \leq |C|$, is the *target committee size*. Approval voting means that voters do not order the candidates according to their preferences but select the candidates they approve. Thus, the ballot A_i of a voter i is a subset of the set of candidates C. For each candidate c in C, let N_c be the set of voters that approve c.

An approval-based multi-winner voting rule R takes as input an election $(N, C, \mathcal{A}, k)$ and outputs a nonempty set of winners (the selected committee) $W \subseteq C$ of size k.

2.1. The method of equal shares (ES)

ES [26] is an iterative algorithm that starts with an empty set of winners W and adds a new candidate to W at each iteration until a stop condition is met.

Initially, each voter possesses one vote, and voters can use fractions of their votes to back candidates. To add a candidate to the set of winners, the voters that approve such a candidate must possess at least $\frac{n}{k}$ votes altogether, and this amount will be subtracted from the votes possessed by the voters who approve the candidate that is added to the set of winners at each iteration.

Denote by $\rho_j(i)$ the fraction of a vote possessed by voter i after j candidates have been added to the set of winners. Thus, $\rho_0(i) = 1$ for each voter i.

Peters and Skowron explain in [26] that ES is designed so that the candidate selected at each iteration and the procedure chosen to remove fractions of votes are done to share the cost of selecting a candidate "as equally as possible among the voters who approve the candidate".

To that extent, at iteration $j+1$, the candidate that is added to the set of winners is the candidate c that minimizes the value $\gamma_{j+1}(c)$, defined as:

$$\gamma_{j+1}(c) = \min_x \left\{ x : \sum_{i: c \in A_i} \min\{x, \rho_j(i)\} = \frac{n}{k} \right\}$$

The value of $\rho_{j+1}(i)$ is equal to $\rho_j(i) - \min\{\gamma_{j+1}(c), \rho_j(i)\}$ for each voter i that approves candidate c and is equal to $\rho_j(i)$ otherwise. If, after a certain iteration, no candidate exists such that the sum of the fractions of the votes left by the voters that approve such candidate is at least $\frac{n}{k}$, the algorithm stops and outputs the candidates that have been added to the set of winners so far.

It may happen that at the end of this algorithm, the number of candidates in the set of winners is strictly smaller than k. Peters and Skowron proved that regardless of how the set of winners obtained with the above algorithm is completed, the final set of winners will satisfy EJR. Peters and Skowron suggested as a possible way to complete ES to run another rule known as seq-Phragmén [10,20] to complete the set of winners after the basic ES algorithm has finished. Other approaches for completing committees have been studied in the literature [14]. In any case, the procedure used to complete the committee output by ES does not affect the result that we present in this note.

3. The EJR-exact family of voting rules

The operation of a significant number of voting rules existing in the literature consists of an iterative procedure in which, at each iteration, a candidate is added to the set of winners, and (fractions of) a subset of the votes are removed from the election. The candidate added to the committee is supposed to represent the voters whose votes or fractions of their votes are removed from the election. In what follows, we refer to this scheme as "add-candidates-and-remove-votes". The definition of the Method of Equal Shares that we have given in the previous section follows the add-candidates-and-remove-votes scheme.

Examples of multi-winner voting rules that were proposed before the introduction of the EJR-Exact family, and that can be defined following the add-candidates-and-remove-votes scheme[3] are: the Single Transferable Vote (STV) (for ranked ballots), Greedy Monroe, originally proposed by Skowron et al. [32] for ranked ballots and later adapted for approval ballots by Sánchez-Fernández et al. [29], and HareAV, GreedyAV, and the rule of Eneström-Phragmén for approval ballots. HareAV was proposed by Aziz et al. [1]. GreedyAV is a particular instance of the **w**-SeqPAV family of voting rules [1], and its name is also due to Aziz et al. [1]. Finally, the rule of Eneström-Phragmén was proposed in the nineteenth century by the Swedish mathematicians Lars Edvard Phragmén and/or Gustaf Eneström [10,20]. None of the approval-based multi-winner voting rules mentioned in this paragraph satisfies EJR.[4] For the apportionment problem, the operation of quota methods [27] (like the largest remainders method) can also be defined following the add-candidates-and-remove-votes scheme.

An interesting feature of voting rules whose operation follows the add-candidates-and-remove-votes scheme is that they are polynomial time computable as long as the computations required at each iteration can be done in polynomial time. In fact, all the rules mentioned in the previous paragraph are polynomial time computable, and some of them are greedy approximation algorithms for voting rules that require solving NP-hard problems as part of their computation. Thus, the goal of EJR-Exact, proposed by Aziz et al. [2], was to identify a set of sufficient conditions such that any voting rule whose operation follows the add-candidates-and-remove-votes scheme and satisfies such conditions would satisfy EJR, in the hope that some (or many) of these rules could be polynomial time computable.

Under the operation of EJR-Exact rules, initially, each voter possesses one vote (we use here the same notation as for ES before), and voters can use fractions of their votes to back candidates. At each iteration, a candidate is added to the set of winners and fractions of the votes possessed by the voters are removed from the election. For each $j \in \{0, \ldots, k\}$ and each voter $i \in N$, we denote by $\rho_j(i)$ the fraction of vote that voter i retains after the jth iteration (again, we use here the same notation that we have used for ES). $\rho_0(i)$ is equal to 1 for each voter i, and $1 \geq \rho_j(i) \geq 0$ for each voter i and each iteration j.

It is convenient to review the definition of the EJR+ axiom at this point due to Brill and Peters [13]. The EJR+ axiom was defined after the EJR-Exact family, but its definition is related to some concepts that we need to introduce.[5]

2 https://equalshares.net/ (visited on 13/6/2024).

3 Some of the voting rules mentioned here admit alternative definitions of their operation that do not require the use of the add-candidates-and-remove-votes scheme.

4 Aziz et al. [1] conjectured that HareAV satisfied EJR if ties were broken to favour committees that satisfy EJR. However, Sánchez-Fernández et al. [28] proved that this conjecture is wrong.

5 The proof that every voting rule in the EJR-Exact family satisfies EJR given in [2] allows also to say that every voting rule in the EJR-Exact family satisfies EJR+.

Definition 1. Consider an approval-based multi-winner election $(N, C, \mathcal{A}, k)$. A subset W of C such that $|W| \leq k$ satisfies EJR+ if there is no candidate $c \in C \setminus W$, group of voters $N' \subseteq N$, and $\ell \in \mathbb{N}$, with $|N'| \geq \ell \frac{n}{k}$, such that

$$c \in \bigcap_{i \in N'} A_i \text{ and } |A_i \cap W| < \ell \text{ for all } i \in N'.$$

Given an approval-based multi-winner election and a set of candidates W, we can compute for each candidate $c \in C \setminus W$ the maximum value of ℓ such that there exists a group of voters N' of size at least $\ell \frac{n}{k}$ such that c and N' witness a violation of EJR+. We call this value the EJR+ demand[6] of candidate c. Further, if for a given candidate c no group of voters N' exists such that c and N' would witness a violation of EJR+, we say that the EJR+ demand of candidate c is 0. Clearly, if the EJR+ demand of each candidate in $C \setminus W$ is 0, then W satisfies EJR+ (and EJR). Intuitively, the EJR+ demand of a candidate is a measure of the size of the group of unsatisfied voters that approve this candidate. These ideas are formalised in the following definition.

Definition 2. The EJR+ demand $\mathrm{d}(c, W)$ of a candidate $c \in C \setminus W$ with respect to a set of candidates $W \subseteq C$ such that $|W| \leq k$, is the maximum non-negative integer such that a group of voters N' exists satisfying (i) c belongs to A_i for each voter i in N'; (ii) $|N'|\frac{k}{n} \geq \mathrm{d}(c, W)$; and (iii) $|A_i \cap W| < \mathrm{d}(c, W)$ for each voter i in N'. If conditions (i), (ii), and (iii) are not satisfied for any N' and any positive value of $\mathrm{d}(c, W)$, then $\mathrm{d}(c, W) = 0$.

Definition 3. The entitlement $en(i, W)$ of a voter i with respect to a set of candidates $W \subseteq C$ is

$$en(i, W) = \max_{c \in A_i \setminus W} \mathrm{d}(c, W)$$

The entitlement of a voter is the largest EJR+ violation of which the voter is part.

By the definitions of EJR+ demand and entitlement it is easy to see that both notions decrease monotonically when we add candidates to W, that is, $\mathrm{d}(c, W \cup \{c'\}) \leq \mathrm{d}(c, W)$ and $en(i, W \cup \{c'\}) \leq en(i, W)$ for any candidates $c, c' \in C \setminus W$ and any voter i.

If, at a particular iteration, we add a candidate c to the set of winners and the entitlement of a specific voter i in N_c after adding this candidate is greater than the number of candidates in the set of winners that this voter approves, then we may need to add later to the set of winners other additional candidates that this voter approves. To that end, we must save a fraction of this voter's vote to back other candidates that should be added later to the winning committee.

For each $j \in \{0, \ldots, k-1\}$, each candidate c in $C \setminus W$, and each voter i in N_c such that $en(i, W \cup \{c\}) > |A_i \cap (W \cup \{c\})|$, we define the *buffer* $g_i^{j+1}(c)$ of voter i if candidate c is added to the set of winners at iteration $j+1$ as

$$g_i^{j+1}(c) = 1 - \frac{|A_i \cap (W \cup \{c\})|}{en(i, W \cup \{c\})}.$$

If $en(i, W \cup \{c\}) \leq |A_i \cap (W \cup \{c\})|$, then $g_i^{j+1}(c) = 0$.

Intuitively, to avoid that voter i witness an EJR+ violation we may need to add some candidates to W. The total number of candidates in W that voter i must approve is bounded by $en(i, W \cup \{c\})$, but it can be smaller. Further, suppose voter i must approve $x \leq en(i, W \cup \{c\})$ candidates in W to avoid witnessing an EJR+ violation. In that case, each of these x candidates must be approved by at least $x\frac{n}{k}$ unsatisfied voters, according to Definition 1. The definition of $g_i^{j+1}(c)$ allows to guarantee that voter i could pay at least $\frac{\frac{n}{k}}{x\frac{n}{k}} = \frac{1}{x}$ for each of the additional candidates, because

$$\begin{aligned}
g_i^{j+1}(c) - (x - |A_i \cap (W \cup \{c\})|)\frac{1}{x} &= \\
1 - \frac{|A_i \cap (W \cup \{c\})|}{en(i, W \cup \{c\})} - (x - |A_i \cap (W \cup \{c\})|)\frac{1}{x} &= \\
-\frac{|A_i \cap (W \cup \{c\})|}{en(i, W \cup \{c\})} + \frac{|A_i \cap (W \cup \{c\})|}{x} &= \\
|A_i \cap (W \cup \{c\})|(\frac{1}{x} - \frac{1}{en(i, W \cup \{c\})}) &\geq 0
\end{aligned}$$

Definition 4. After iteration $j \in \{0, \ldots, k-1\}$, we say that a candidate is

- weak if $\sum_{i \in N_c} \rho_j(i) < \frac{n}{k}$,
- risky if $\frac{n}{k} \leq \sum_{i \in N_c} \rho_j(i) < \frac{n}{k} + \sum_{i \in N_c} g_i^{j+1}(c)$, and
- safe if $\sum_{i \in N_c} \rho_j(i) \geq \frac{n}{k} + \sum_{i \in N_c} g_i^{j+1}(c)$.

A distinguishing characteristic of voting rules that belong to the EJR-Exact family is that they never add risky candidates to the set of winners because their addition may cause EJR violations. However, other rules that follow the add-candidates-and-remove-votes scheme and do not belong to the EJR-Exact family often add risky candidates to the set of winners.

We say that an iteration is *safe* if the following conditions hold for such iteration (c is the candidate added to the set of winners in such iteration):

1. $\sum_{i \in N} \rho_j(i) - \sum_{i \in N} \rho_{j+1}(i) = \frac{n}{k}$,
2. $\rho_{j+1}(i) \geq g_i^{j+1}(c)$, for each voter i in N_c, and
3. $\rho_{j+1}(i) = \rho_j(i)$, for each voter i that does not approve c.

Observe that the conditions above can only be fulfilled if the selected candidate is safe.

Definition 5. A rule belongs to the EJR-Exact family if it repeatedly performs safe iterations until all remaining candidates are weak.

Aziz et al. [2] proved that, if, at a particular iteration, there are no safe candidates left in $C \setminus W$, then all the remaining candidates in $C \setminus W$ are weak, and any committee that contains W satisfies EJR.

4. Main result

In the proof that ES is an instance of the EJR-Exact family, we will use the following proposition[7] which was proved by Aziz et al. [2].

Proposition 1. *Consider any voting rule that is an iterative algorithm where, at each iteration, a candidate is added to the set of winners and fractions of the votes are removed from the election. Suppose the first j iterations of such a voting rule for a given election are safe. Then, for each voter $i \in N$ with $en(i, W) > |A_i \cap W|$ we have*

$$\rho_j(i) \geq 1 - \frac{|A_i \cap W|}{en(i, W)}$$

Theorem 1. *The rule of Equal Shares is an instance of the EJR-Exact family of voting rules.*

Proof. We prove that ES belongs to the EJR-Exact family by induction. For $j = 0, \ldots, k-1$, we prove that under the assumption that the first j

[6] Aziz et al. [2] refer to this notion as the plausibility of a candidate.

[7] Proposition 1 is stated as Lemma 2 in [2].

iterations are safe ($j = 0$ corresponds to the point before the first iteration of ES starts, and therefore, for $j = 0$ no assumption is made), then iteration $j + 1$ is also safe or no safe candidates remain after the first j iterations.

First, we observe that, by the definition of ES, conditions 1 and 3 for safe iterations are always satisfied, so to prove that an iteration of ES is safe, we only need to prove that condition 2 holds.

Suppose first that the EJR+ demand of all the remaining candidates in $C \setminus W$ is 0. This implies that $en(i, W \cup \{c\}) = 0$ for each voter i, and each candidate c in $C \setminus W$, and therefore that also $g_i^{j+1}(c) = 0$ for each voter i, and each candidate c in $C \setminus W$. Therefore, if there is some candidate c in $C \setminus W$ such that $\sum_{i \in N_c} \rho_j(i) \geq \frac{n}{k}$, then iteration $j + 1$ is safe because condition 2 does not apply, and otherwise all the remaining candidates are weak.

Suppose now that after the first j iterations, some candidate exists in $C \setminus W$ with EJR+ demand of at least 1. Consider a candidate c in $C \setminus W$ with maximum EJR+ demand value $\ell \geq 1$. By the definition of EJR+ demand, this implies that there exists a set of voters $N' \subseteq N_c$ such that $|N'| \geq \ell \cdot \frac{n}{k}$, and $|A_i \cap W| < \ell$ for each $i \in N'$. By Proposition 1,

$$\rho_j(i) \geq 1 - \frac{|A_i \cap W|}{\mathrm{en}(i, W)} = 1 - \frac{|A_i \cap W|}{\ell} \geq \frac{1}{\ell}, \text{for each voter } i \text{ in } N'.$$

Thus,

$$\sum_{i \in N_c} \rho_j(i) \geq \sum_{i \in N'} \rho_j(i) \geq \ell \cdot \frac{n}{k}\frac{1}{\ell} = \frac{n}{k}, \text{and } \gamma_{j+1}(c) \leq \frac{1}{\ell}.$$

Let c' be the candidate added by ES at iteration $j + 1$. Since ES selects the candidate with minimum $\gamma_{j+1}(c')$, this implies that $\gamma_{j+1}(c') \leq \gamma_{j+1}(c) \leq \frac{1}{\ell}$.

This implies that iteration $j + 1$ is safe because

$$\begin{aligned}
\rho_{j+1}(i) &= \rho_j(i) - \gamma_{j+1}(c') \geq 1 - \frac{|A_i \cap W|}{en(i, W)} - \frac{1}{\ell} \\
&\geq 1 - \frac{|A_i \cap W|}{en(i, W \cup \{c'\})} - \frac{1}{en(i, W \cup \{c'\})} = 1 - \frac{|A_i \cap (W \cup \{c'\})|}{en(i, W \cup \{c'\})} \\
&= g_i^{j+1}(c')
\end{aligned}$$

for each voter i in $N_{c'}$ such that $en(i, W \cup \{c'\}) > |A_i \cap (W \cup \{c'\})|$. We again used Proposition 1 in the first inequality. The second inequality holds by the monotonicity of entitlement and because $en(i, W \cup \{c'\}) \leq \ell$. □

5. Discussion

In this note, we have proved that the Method of Equal Shares is an instance of the EJR-Exact family of voting rules. This result illustrates that the conditions imposed by EJR-Exact are quite general. We do not dare to conjecture that such conditions are an axiomatic characterisation for voting rules that satisfy EJR and follow the add-candidates-and-remove-votes scheme, but we certainly do not know any rule whose operation follows the add-candidates-and-remove-votes scheme and satisfies EJR, but it is not an instance of the EJR-Exact family.

Other voting rules proposed in the literature that are also instances of the EJR-Exact family include the rule Simple EJR, proposed by Sánchez-Fernández et al. [28], that selects at each iteration the candidate with a higher EJR+ demand. This rule was later and independently rediscovered and renamed as the Greedy Justified Candidate Rule by Brill and Peters [13].

Declaration of generative AI and AI-assisted technologies in the writing process

While preparing this work, the author used Grammarly® to improve the writing. After using this tool, the author(s) reviewed and edited the content as needed and take(s) full responsibility for the publication's content.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

No data was used for the research described in the article.

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