

Fibration symmetry-breaking supports functional transitions in a brain network engaged in language

Keywords: Brain, Connectome, Functional Networks, Fibrations, Cluster Synchronization

Extended Abstract

The network of internal connections crucially shapes collective phenomena in complex dynamical systems [1]. In particular, synchronisation can be exhibited as a global state [2] in which all units follow the same trajectory or via clustered states where the system splits into subsets of units synchronised. In the latter phenomenon, known as cluster synchronisation (CS) [3], a key role in determining the composition of the clusters is played by the symmetries inherent to the network structure of connections. We found [4] that the cluster synchronisation observed in the human brain at the mesoscopic scales of regions of interest (ROIs) measured by functional magnetic resonance imaging (fMRI) is deeply intertwined with the symmetries of the brain network. These symmetries explain how the structural connections among the system's ROIs (connectome or structural network) determine the emergent dynamical synchronisation expressed in the functional network in the resting state (RS) and during a cognitive language task. The symmetries we find in the human brain are not those of physical systems. Physical (and geometrical) symmetries are automorphisms and form symmetry groups. Instead, the symmetries in the brain network are symmetry fibrations [5]. Fibrations are less restricted symmetries than automorphisms because they are local symmetries that preserve only the colour isomorphic inputs of nodes. Consequently, they preserve the dynamical evolution leading to cluster synchronisation in the network. We implemented a symmetry-driven algorithm based on a mixed integer linear programming to infer the structural network that sustains the cluster synchronisation of the functional language network obtained experimentally in different processes, the outcome of which is the human language ability. In analogy to the theory of phase transitions between states in physical systems, we describe recruiting communication resources across different brain states as a process of network symmetry breaking.

References

- [1] Baruch Barzel and Albert-László Barabási. “Universality in network dynamics”. In: *Nature physics* 9.10 (2013), pp. 673–681.
- [2] Alex Arenas et al. “Synchronization in complex networks”. In: *Physics reports* 469.3 (2008), pp. 93–153.
- [3] Francesco Sorrentino et al. “Complete characterization of the stability of cluster synchronization in complex dynamical networks”. In: *Science advances* 2.4 (2016), e1501737.
- [4] Tommaso Gili et al. “Fibration symmetry-breaking supports functional transitions in a brain network engaged in language”. In: *ArXiv* (2024), arXiv–2409.
- [5] Paolo Boldi and Sebastiano Vigna. “Fibrations of graphs”. In: *Discrete Mathematics* 243.1-3 (2002), pp. 21–66.

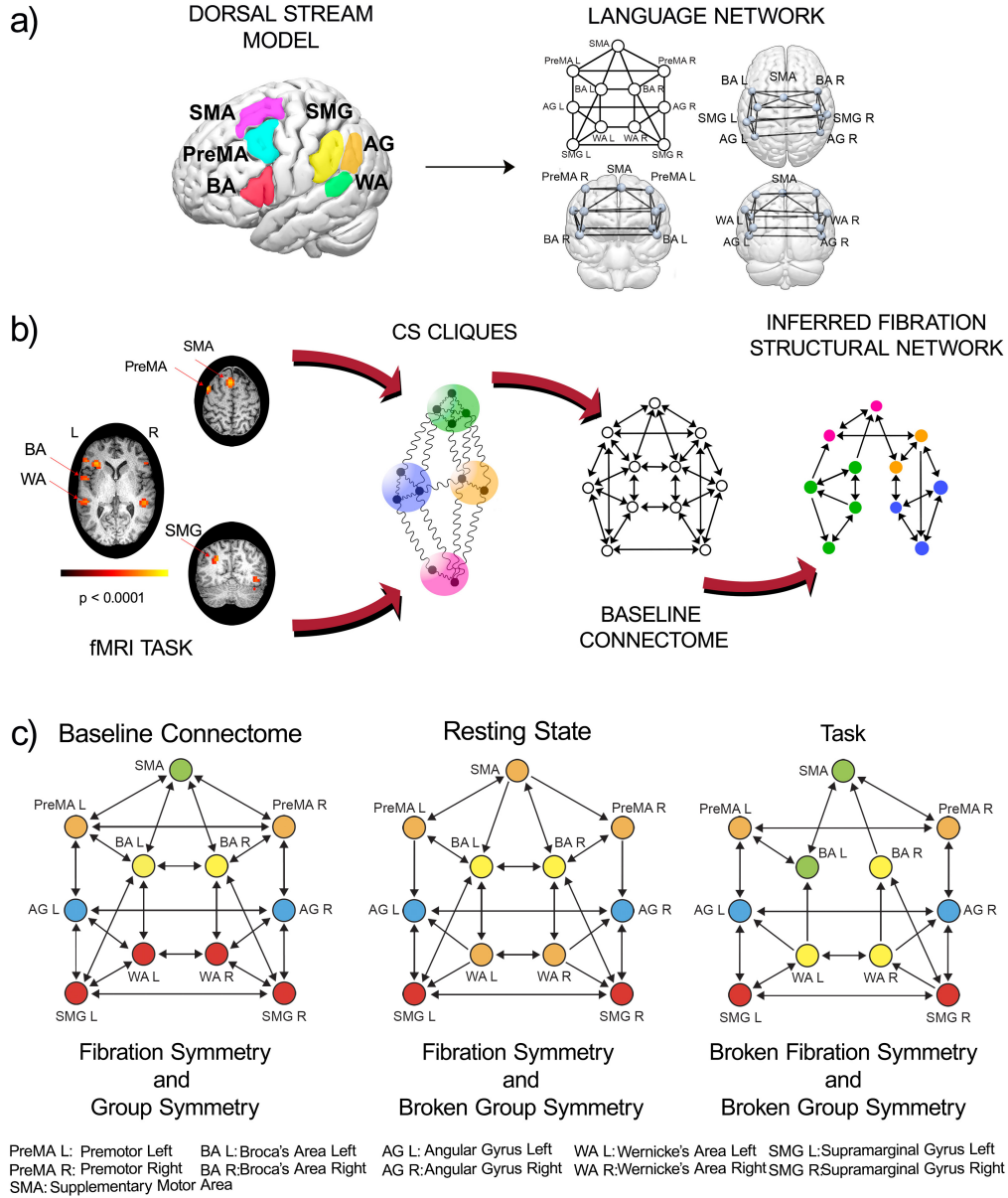


Figure 1: a) **Regions of Interest of the study.** Left: ROIs of the primary language network are given by the dorsal stream of the dual-stream model localized in the brain. Right: dorsal stream baseline connectome showing the fiber tracks between the ROIs. (b) **Pipeline for inference of the structural network from CS data.** Left: fMRI images for RS or a task over many subjects are taken as input to calculate the group-average CS cliques among ROIs. The CS cliques are identified with the colors assigned to nodes in the baseline connectome. A mixed integer programming algorithm is employed to optimally infer the structural network (right) that sustains the coloring cluster pattern obtained from the dynamics. (c) **Minimal balanced coloring in the baseline connectome.** This network has the highest symmetry: a global automorphism group, which is the same as the local fibration symmetry with five orbits equal to fibers (five balanced colors). **Inferred Resting State structural network using the CS from fMRI data.** The network has only local fibration symmetry with four fibers but no global symmetry, which is broken with respect to the connectome under the RS dynamics. **Inferred language task network.** The lateralization of function under the language task breaks the fibration symmetry of the resting state connectome showing less symmetry (more fibers). The group symmetry remains broken.