# Towards Self-Organizing Swarms Composed of Swarmalators with Distributed Couplings

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## I. Introduction

Synchronizing systems are the essence of the coupled oscillators field and have been studied extensively throughout the past half century in the context of mathematics[1], physics, biology [2], and robotics [3]. The coupled oscillators field has shown through extensive studies that a system of distributed agents can achieve synchrony even when there is sparse coupling between constituents. The coupled oscillators field has also demonstrated that oscillations are universal throughout natural and artificial systems. In the context of collectives, coupled oscillations can be exploited to enable large numbers of agents to produce interesting self-organized behaviors [4]; popular examples in nature include flashing fireflies [5], firing neurons [6], and clapping audiences [7]. Although many researchers have investigated the emergent temporal self-organization that occurs in static arrays of coupled oscillators, little is known about the dynamics of mobile oscillators and their emergent self-organization. Studying this further could enable us to create versatile robot swarms capable of switching between diverse formations; this would be especially important in the area of microrobotics, where there is almost no onboard sensing and computation because of the inherent size limitation, and therefore little information can be shared with other agents.

The swarming oscillators, or swarmalators, field formally began in 2017 by O'Keeffe et al. [8]; a swarming model was introduced in which mobile agents' motion is a function of their phase interactions, and their phase interactions are also a function of their motion relative to each other. Since then, researchers have investigated the swarmalators further when there is local coupling [9], thermal noise [10], confined motion on a ring [11], and external force perturbations [12]; a robotic demonstration of the key behaviors from the original swarmalator model has also demonstrated that these behaviors translate from the simulated predictions to a physical system [13]. More recently, Ceron et al. introduced a new and more general form of the swarmalator model and demonstrated a multitude of new swarming behaviors that mimicked the natural and artificial swarming systems' behaviors [14]. Through this work it became evident that the swarmalators could be used to replicate and characterize the swarming behaviors of many different types of systems including spermatozoa [15], social amoebae [16], and magnetic microrobot (micron scale) swarms [17]. The swarmalator model is powerful and could enable us to not only better study and characterize the behaviors of diverse swarming systems but also design more advanced microrobots that exploit various types of physical pairwise interactions to enable diverse collective behaviors. Indeed, Ref. [18] demonstrates that the swarmalator model can be used to design and characterize the self-organizing behavior of magnetic microrobot collectives heterogeneous by size. In these microrobot swarms, there is no room for onboard computation; instead, all collective behaviors result from the local physical pairwise interactions between constituents; the swarmalator model abstracts away the physical interactions and reproduces the microrobots' self-organizing behavior. As we look towards the future of the swarmalators field and its extended potential applications in the microrobotics field, we must study in more detail the emergent self-organization when there is non-identical coupling throughout the collective. Throughout most swarmalator studies, the pairwise coupling remains constant across the collective which means collective behaviors may be tuned through this global parameter. In the context of microrobotics, however, it would be more useful to study systems in which there is non-identical coupling across the collective. Heterogeneous microrobot collective systems are being developed so that non-homogeneous ensembles exploit asymmetric pairwise interactions to enable various behaviors including self-organization, collective morphology reconfiguration, and object manipulation which would not be possible with a homogeneous microrobot collective. In these microrobot collective systems the coupling is agent-specific; distributed coupling would enable us to imitate this type of agent-specific interaction through a general framework like the swarmalator model. Distributed coupling would also usher new self-organization behaviors that enable the collective to exhibit several types of swarming behaviors simultaneously.

We introduce the reader to the swarmalator framework and present exciting preliminary results related to distributed coupling; however, the full development of this study is a full research program that will consist of various phases across many papers and studies. One phase will enable us to refine distributed coupling swarmalator models so that we can achieve specific swarming behaviors. The next phase uses our distributed coupling models to design robot collective systems that realize the swarming behavior predicted through the swarmalator model. In the first phase, this work will

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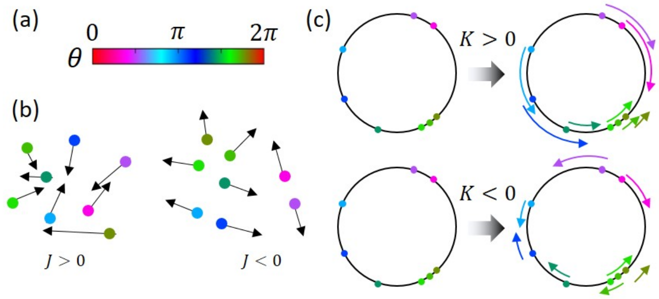


Fig. 1: Introduction to swarmalators. (a) Agents' phase values are between 0 and $2\pi$ and are mapped to a color bar. (b) Agents spatially attract toward other agents with a similar phase. (c) Agents inherently oscillate their phase at their natural frequecny $\omega_i$ and couple to their neighbor's phase through the term $K\sin(\theta_j - \theta_i)$. When $K > 0$, agents' phase tend to move towards synchrony (top); when $K < 0$, agents' phases tend to move towards asynchrony (bottom).

demonstrate a major thrust for the swarmalators field and will have the potential to impact diverse fields ranging from swarm robotics, to mobile networking systems, to characterization of natural swarming phenomena.

## II. The Original Model

We first introduce the original swarmalator model since this is the basis off which most of the swarmalator field has investigated the dynamics of the swarmalator framework, and it is the base off which we introduce distributed coupling.

$$\dot{\mathbf{x}}_i = \frac{1}{N}\sum_{j\neq i}^{N}\left[\frac{\mathbf{x}_j - \mathbf{x}_i}{|\mathbf{x}_j - \mathbf{x}_i|}\Big(A + J\cos\big(\theta_j - \theta_i\big)\Big) - B\frac{\mathbf{x}_j - \mathbf{x}_i}{|\mathbf{x}_j - \mathbf{x}_i|^2}\right] \tag{1}$$

$$\dot{\theta}_i = \omega_i + \frac{K}{N}\sum_{j\neq i}^{N}\frac{\sin\big(\theta_j - \theta_i\big)}{|\mathbf{x}_j - \mathbf{x}_i|} \tag{2}$$

Here, $N$ agents follow an equation of motion (Eq. 1) that is a function of an agent $i$'s position ($\mathbf{x}_i$) and phase ($\theta_i$), and its pairwise interaction with other agents' ($j$) position ($\mathbf{x}_j$) and phase ($\theta_j$). All agents have a phase between 0 and $2\pi$ and are initialized with random phases and positions within a box of side length 2. As shown in Fig. 1a, agents' phases are represented by a color according to the colorbar. Global spatial attraction is enabled through the unit vector portion of the model and scaled by the coefficient $A$; spatial repulsion is enabled by a power law model and is scaled by the coefficient $B$; both coefficients hold a value of 1. The spatial-phase coupling parameter $J$ holds a value between -1 and 1; it determines how much agents will move towards other agents with similar phases. As shown in Fig. 1b, when $J > 0$, agents with similar phases will aggregate, and when $J < 0$, like-phased agents will repel each other. $J$ has held a constant value for all agents throughout all previous swarmalator studies; however, here we introduce a distributed spatial-phase coupling and define it more clearly in the following section.

Swarmalators also demonstrate temporal self-organization

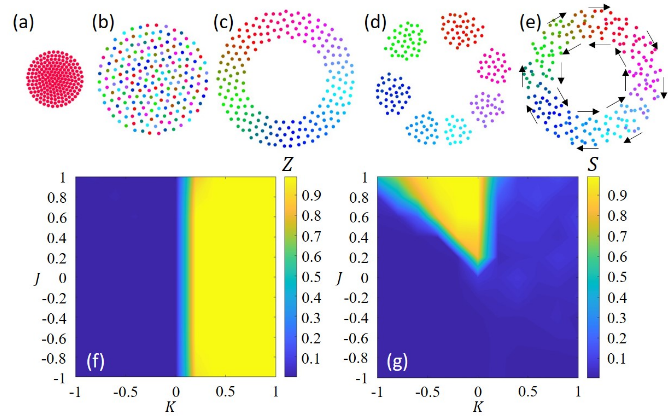


Fig. 2: (a-e) Collective behaviors of the original swarmalator model. (a) Synchronized cluster ($K = 1$, $J = 1$). (b) Asynchronous cluster ($K = 0$, $J = -1$). (c) Static phase wave ($K = 0$, $J = 1$). (d) Splintered phase wave ($K = -0.05$, $J = 1$). (e) Active phase wave ($K = -0.4$, $J = 1$). (f-j) Collective behaviors of the new swarmalator model. (f) Heat map of the S order in K - J parameter space. (g) Heat map of phase coherence (Z).

through their phase coupling interactions as dictated by the distance-dependent Kuramoto model in Eq. 2. Here, $\omega_i$ is the natural frquency of each agent $i$; it is the inherent rate at which an agent cycles its internal phase. The coupling parameter $K$ dictates how much agents will adjust their oscillating phase to each other. As shown in Fig. 1c, agents' phases can be mapped to a unit circle; when $K > 0$, agents move towards phase synchrony, and when $K < 0$, agents move towards phase asynchrony. $K$ is also a global coupling parameter throughout most swarmalator studies; however, here we demonstrate how the collective can split into several subgroups with distinct behaviors when $K$ becomes a distributed coupling parameter.

We briefly summarize in Figs. 2a-e the collective behaviors when $K$ and $J$ are global coupling parameters; for clarity in this introductory paper, $\omega_i = 0$ for all agents. High $K$ and $J$ values enable the collective to aggregate into a phase-synchronized circular cluster, and low $K$ and $J$ enable the circular cluster to increase in radius and move towards phase asynchrony. When $K = 0$ and $J = 1$, agents do not affect each others' cycles but they move towards agents with a low phase difference (i.e. a low value for $\cos(\theta_j - \theta_i)$); since there is a uniform distribution of phases between 0 and $2\pi$, the collective creates an annulus formation, termed a phase wave. If $J = 1$ but $K$ is slightly less than 0 ($K = -0.05$), agents anti-phase couple slightly such that they form tight clusters in petal-like formations, this collective state is termed the splintered phase wave. When $K$ is lower ($K = -0.4$), the collective enters an active phase wave state in which all agents move around the collective centroid; this happens because agents anti-phase couple yet are spatially attracted to other agents with similar phases. These are the five general collective behaviors that result from the original swarmalator model.

We can characterize the transition between these various behaviors through two order parameters that measure phase coherence ($Z$) and circumferential spatial-phase correlation

($S$), shown in Eqs. 3 and 4; both order parameters have a value between 0 and 1.

$$Z = |\frac{1}{N}\sum_{j=1}^{N} e^{i(\theta_j)}| \tag{3}$$

$$S = \max|\left(\frac{1}{N}\sum_{j=1}^{N} e^{i(\phi_j \pm \theta_j)}\right)| \tag{4}$$

$Z$ measures how close the agents' phases are to each other and $S$ measures how well the collective is organized by phase around its centroid. $Z$ and $S$ are mapped across the K - J parameter space in Figs. 2f and g. The heat maps help us map out the general behaviors that occur when the collective experiences these various global coupling parameters. We can also see distinct collective behavior transitions within these heat maps as the order parameters transition from low to high values. We present these order parameters to showcase how we can quantitatively and qualitatively distinguish between different collective behaviors exhibiting spatial and temporal self-organization.

## III. Distributed Coupling

Since this is the beginning of a set of studies that will explore how various forms of distributed coupling affect the swarmalators' behaviors, we will simply summarize some of the forms of distributed coupling that could potentially exhibit the most interesting results.

We can define $K$ and $J$ as $K_{ij}$ and $J_{ij}$, respectively. This means each pair of agents has a distinct coupling coefficient that determines how much they will phase couple to each other and how much they will spatially attract to each other as a function of their phase interactions. Looking toward the future studies of this research program, we can consider various possibilities for studying the self-organization behaviors when there is distributed coupling. We can model the original swarmalator as distributed coupling by having all agents be coupled to all other agents (Fig. 3a). We may consider systems in which all agents couple to a single agent or a very small number of agents (Fig. 3b). This will yield very interesting results since it will enable us to study how the collective behaves when there is a 'leader' agent influencing all other agents; we can study how the collective's self-organized behaviors change as the number of 'leader' agents increases. This is portrayed in Fig. 3c; the whole collective is connected to a subset of the group. Fig. 3d shows an unconnected graph architecture, where the half of the swarmalators would interact with only half of the swarmalators, and they would remain in two separate groups. This may be useful for robot swarms that need to separate into some number of groups and exhibit different formations within each group; however, each of the formations would be the same as in the original swarmalator model since each group is essentially its own segregated collective. A slightly more interesting twist to these self-organized behaviors might come from a graph architecture similar to what is shown in Fig. 3e. Here, there are also two main groups, but a single connection or a few connections can enable the two groups to affect each others' behaviors.

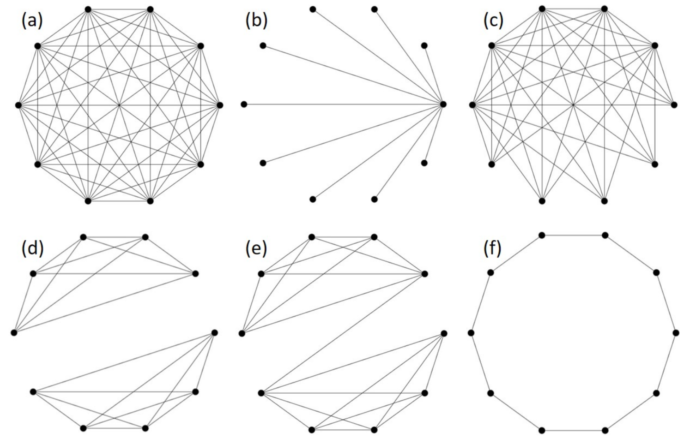


Fig. 3: Some of the potentially most interesting forms of distributed coupling for the swarmalator framework. (a) All agents coupled with all other agents; this is the baseline and what is already shown through the original swarmalator model. (b) All agents coupled to a single agent. (c) A subset of the collective coupled to another subset of the collective. (d) A disconnected graph in which the collective is segregated into separate coupling groups. (e) The same scenario as what is shown in (d), but with a single or small number of coupling connections that make it a connected graph. (f) Each agent couples to a subset of the collective.

Fig. 3f shows a graph architecture in which each agent is affected by two other agents. In a scaled up graph architecture we could study the self-organization behavior of the collective as the number of agents that each agent couples to increases. This type of distributed coupling architecture would enable us to study how many other agents each agent needs to be connected to for specific formations to exist in the collective. Within this research program, there are a multitude of graph coupling architectures that we may consider in future studies including random distributed coupling, time-varying distributed coupling, and directed distributed coupling. The time-varying distributed coupling could be used to evolve from one architecture to another; two immediately interesting directions of study would be to (1) apriori program the evolution of the coupling architecture and (2) evolve from one architecture to another as a function of the collective system's state. The directed coupling would enable agent $i$ to exert a coupling interaction on agent $j$ that is not necessarily the same as the coupling from agent $j$ to agent $i$; this would enable asymmetric pairwise spatial-phase interactions. A directed graph architecture would enable the collective to exhibit entirely new behaviors than what is exhibited in the undirected graph architectures. Each of these additional distributed coupling architectures could enable us to explore fundamental and application-level concepts of self-organizing swarming systems that could enable robot swarms to have greater control over their shape formations without necessarily having to communicate information to the entire group.

## IV. Preliminary Results

Our preliminary results overview the diverse self-organized behaviors that happen when the distributed coupling architecture mimics some of the architectures outlined in Fig. 3. The first two rows of Fig. 4 demonstrate that the collective can exhibit two states when the collective has a distributed coupling architecture in which each couples to about half of the collective. In these distributed coupling architectures it is very simple to partition the group and quantify phase coherence and spatial-phase correlation parameters like $Z$ and $S$. Detailed studies of some of these behaviors will enable us to learn how we can control the position of specific parts of the group with respect to other parts of the group through distributed coupling architecture. More specifically, we will be able to determine how coupling architectures similar to the ones depicted in Figs. 3c and d will enable a synchronized cluster to form in the center of the collective while being encapsulated by an asynchronous cluster, as shown in Fig. 4a. We will also learn if the distributed coupling architecture can be flipped so that the asynchronous cluster forms in the center and the synchronized cluster encapsulates it. In these cases it is also immediately evident that the behaviors from the original swarmalator model transfer over even when agents do not couple to all other agents in the collective.

Rows 3 and 4 of Fig. 4 demonstrate the behavior when each agent connects to a specific number of agents; this is similar to the architecture depicted in Fig. 3f. Through these formations, we observe similarities with self-organized behaviors realized through the original swarmalator model; however, several behaviors are not present at once as was the case in rows 1 and 2. As we increase the number coupling connections for each agent, the collective transitions from a disordered state to a more ordered state. It is interesting that in collectives of 200 swarmalators, when agents just have 50 coupling connections, the group is already able to create a formation close to one of the self-organized behaviors present in the original swarmalator model. Our future studies will investigate if this behavior is stable and what the lower limit of coupling connections per agent may be when the collective is of a given size. It is not clear yet if this is a snap transition from a disordered cluster to one of the self-organized behaviors or if we can quantify the amount of order as the number of coupling connections is increased.

## V. Discussion

We introduce the notion of distributed coupling in the swarmalator system. Our preliminary results show interesting formations that demonstrate the collective can reach several steady states when it begins with its agents having random positions and phases. Throughout these preliminary results, the distributed couplings are kept constant; however, controllable distributed coupling between agents would enable us to control various parameters in a self-organizing system, such as the distance between nearby agents, their phase-locking behavior, and the motion of the entire group. Nonetheless, the diverse behaviors shown in Fig. 4 demonstrate the wide range

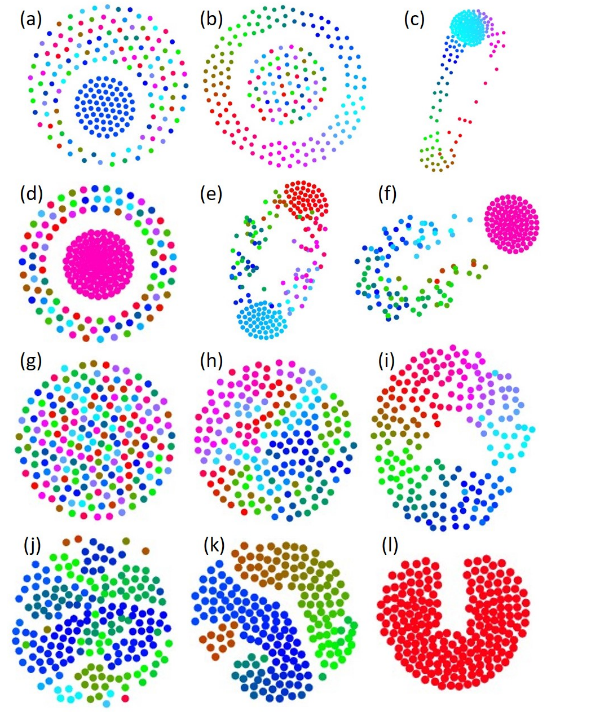


Fig. 4: Diverse behavior enabled by various distributed coupling architectures. (a-c) Self-organization when the distributed coupling is similar to the graph architecture depicted in Fig. 3c. (d-f) Self-organization when the collective is split into two groups like the graph architecture depicted in Fig 3d. (g-l) Self-organization when the distributed coupling is similar to the graph architecture depicted in Fig. 3f. (g-i) $K_{ij} = 0$, $J_{ij} = 1$; number of agents each agent is coupling to is 1 (g), 20 (h), 50 (i). (j-l) $K_{ij} = 1$, $J_{ij} = 1$; number of agents each agent is coupling to is 10 (j), 20 (k), and 50 (l).

of possibilities in studying self-organizing swarmalators in the context of networked systems. We are expanding on our preliminary results by: (1) characterizing the systems' global and local spatial and temporal self-organization through $Z$, $S$, and other order parameters for specific distributed coupling architectures; (2) studying whether any of the self-organized states can be proved to be stable; (3) developing a robotic implementation to reproduce some of the swarmalator behaviors with $> 10$ robots. We are using the Sphero Bots to study how the distributed coupling architecture translates to a physical system. Within this phase of the research program we are also demonstrating how the individual dynamics of the robotic agents affects the final self-organization.

We hope the introduction of this line of studies on distributed coupling in swarmalators will excite many swarm robotocists to investigate how distribute coupling can be used in the context of this swarming framework and other frameworks. This line of research has the potential to enable large developments in the field of microrobot swarms and any other kind of self-organizing robot collective system in which each agent is capable of transmitting very little information to other agents.

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